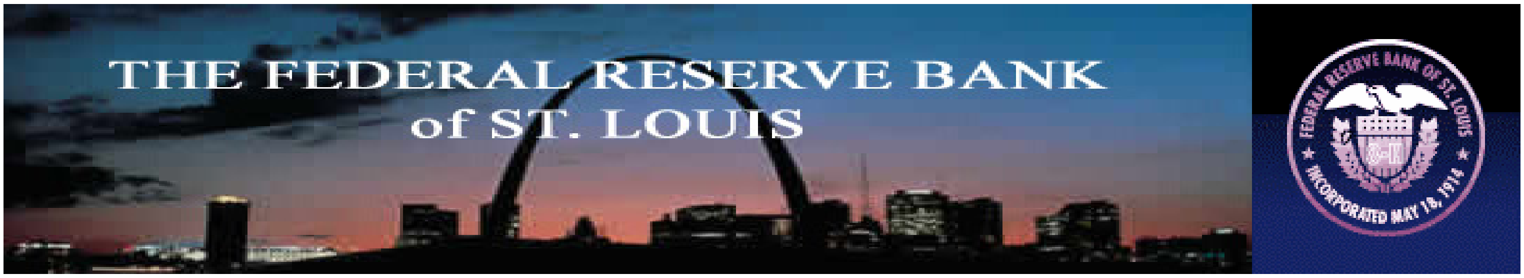

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Predictions of Short-Term Rates and the Expectations Hypothesis of the Term Structure of Interest Rates

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Abstract

Despite its important role in macroeconomics and finance, the expectations hypothesis (EH) of the term structure of interest rates has received little empirical support. While the EH's poor performance has been attributed to a variety of sources, none appear to account for the EH's poor performance. Recent evidence (Diebold and Li, 2003; Duffee, 2002; and Carriero, et al., 2003) suggests the possibility that the EH's poor performance may be due to market participants' relative inability to forecast the short-term rate. This possibility is investigated by comparing *h*-month ahead forecasts for the 1-month Treasury yield implied by the EH with the forecasts from both a random-walk model and a three factor model of the term structure.

JEL Classification: E40, E52

Key Words: expectations theory, random walk, time-varying risk premium

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> "The forecasting of short term interest rates by long term interest is, in general, so bad that the student may well begin to wonder whether, in fact, there really is any attempt to forecast."—Macaulay (1938, p. 33)

1. Introduction

The expectations hypothesis (EH) of the term structure of interest rates—the proposition that the long-term rate is determined by the market's expectation of the short-term rate over the holding period of the long-term asset plus a constant risk premium—has been tested extensively using a wide variety of interest rates, over a variety of time periods and monetary policy regimes. As Diebold, et al., (2003, p. 14) note, "severe failures of the expectations hypothesis have been documented at least since Macaulay (1938)." The EH is frequently tested using single-equation approach.[1] Within the context of this approach, the EH's failure may be attributed to biases in the test statistic due to (i) a time-varying risk premium, (ii) irrational expectations (e.g., the overreaction hypothesis), and (iii) statistical biases (e.g., peso problems, measurement error, etc.) Generally, these explanations are unable to fully account for the empirical shortcoming of the EH (e.g., Simon, 1990; Campbell and Shiller, 1991; Hardouvelis, 1994, Dotsey and Otrok, 1995; Balduzzi, et al., 1997; Roberds and Whiteman, 1999; Bekaert, et al., 1997, 2001). Indeed, Bekaert and Hodrick (2001) note that "the literature has had surprisingly little success generating risk premiums that explain the empirical evidence."[2]

Fundamentally, the EH depends on the market's ability to predict the future short-term rate. Consequently, recent evidence indicating that it is very difficult to improve on a random walk forecasts of short-term rates (e.g., Duffee, 2002; Diebold and Li, 2003;

[1] An alternative VAR approach, pioneered by Campbell and Shiller (1987) has been used infrequently. See Bekaert and Hodrick (2001) and Dittmar and Thornton (2003) for applications of the VAR test.

and Carriero, et al., 2003) suggests the low predictive power of the long-term/short-term rate spread may stem from the inability of market participants to predict the future short-term rate significantly beyond its current level. Rather than testing the EH directly, this paper investigates the possibility that the lack of support for the EH stems from the market's inability to adequately predict the future short-term rate. This is done by calculating the theoretical t-period expectation of the 1-period rate for periods $t+h$, $h=1,2,\ldots H$, under the assumption that the EH holds. The forecast errors associated with the "theoretical expectations" are compared with those from a simple random walk model and with forecasts from Diebold and Li's three factor model of the term structure. Because these estimates depend solely on observed values of the long-term and short-term rates they must reflect the market's expectation for the future short-term rate. The approach has the advantage that no assumption is required regarding the expectation generating process, as is the case with most tests of the EH.

To anticipate the results, the estimated theoretical forecasts of the 1-month rate one to three months ahead do not differ significantly from the forecasts obtained from a random walk model of the 1-month rate or from forecasts of the 1-month rate from Diebold and Li's three factor model of the term structure. These results are shown to be robust to whether the risk premiums are taken to be constant over the sample period or permitted to vary considerably over time. Further analysis shows that the current rate spread has power for predicting the *expected change* in the short-term rate, as measured by Diebold and Li's three factor term structure model. Unfortunately, these expectations contain relatively little incremental information about the future short-term rate relative

[2] Bekaert and Hodrick (2001, p. 1358).

to the current short-term rate. Consequently, the behavior of the long-term rate has little power for predicting the actual change in the short-term rate.

The outline of the paper is as follows. Section 2 presents the EH and the predictability of the spread between the long-term and short-term rate. Section 3 presents the methodology for estimating the *t*-period expectation for the 1-month rate *h* periods ahead. The theoretical expectations are compared with the forecasts with those from the random walk model. Section 4 presents Diebold and Li's (2003) three factor term structure model. Forecasts from this model are compared with the theoretical forecasts. The conclusions and implications are presented in Section 5.

2. The EH and the Predictability of the Short-term Rate

The EH asserts that

(1) $$r_t^n = (1/k)\sum_{i=0}^{k-1} E_t r_{t+mi}^m + \pi^{n,m},$$

where r^n denotes the long-term (*n*-period) rate and r^m denotes the short-term (*m*-period) rate, $k = n/m$ is an integer, and $\pi^{n,m}$ denotes a term-specific but constant risk premium. The most widely used test of the EH is obtained by subtracting r_t^m from both sides of (1) and rearranging terms to yield

(2) $$(1/k)\sum_{i=0}^{k-1} E_t r_{t+i}^m - r_t^m = -\pi^{n,m} + (r_t^n - r_t^m)\,.$$

The test of the EH is obtained by assuming that market participants' expectations are rational in the sense that

(3) $$E_t r_{t+i}^m = r_{t+i}^m + \nu_t^i\,,$$

where ν_t^i is distributed i.i.d. (0, σ_i^2). Substituting (3) into (2) and parameterizing the resulting expression yields

(4) $$(1/k)\sum_{i=0}^{k-1} r_{t+i}^m - r_t^m = \alpha + \beta(r_t^n - r_t^m) + \varpi_t \ ,$$

where $\omega_t = -(1/k)\sum_{i=0}^{k-1} \nu_{t+i}$. The EH is tested by estimating (4) and testing the hypothesis $\beta = 1$, its value if the EH holds.

While estimates of β are frequently positive and statistically significant, the null hypothesis $\beta = 1$ is frequently rejected at very low significance levels. For example, Table 1 reports the estimates of β obtained by estimating (4) using the 1-month rate as the short-term rate and the 2-, 3-, and 4-month rates as alternative long-term rates. Table 1 also reports tests of the hypotheses that $\beta = 0$ and $\beta = 1$. The data are end-of-period monthly observations compiled by (McCulloch and Kwon, 1993) and cover the period 1952.01 through 1991.02. Estimates of β are positive and significantly different from zero in all three cases; nevertheless, the null hypothesis $\beta = 1$ is easily rejected.

Moreover, estimates of $\bar{R}^2$ suggest that the spread between the longer-term and the 1-month rate explain a relatively small proportion of future changes in the 1-month rate. The spread between the 2-month and 1-month rate accounts for about 10 percent of the observed change in the 1-month rate next month. The proportion of change in the 1-month rate accounted for by the rate spread declines as the horizon increases from 1 to 3 months.

Recently, Kool and Thornton (2004) and Thornton (2005ab) have shown that estimates of (4) can generate misleading results. Moreover, based on their predictive experiments, Carriero et al. (2003) suggest that the common practice of using the actual short-term rate as a proxy for the tth-period expectation of the short-term rate (i.e., the assumption given by Equation 3) is inappropriate. The EH per se places no restrictions on how the market participants' expectations of the short-term rate are formed. Consequently it is useful to investigate the EH using a procedure that does not require an expectational assumption but, by construction, reflects market participants' expectations for the short-term rate.

3. Estimating the Theoretical Expected Future Short-Term Rate

However market participants' expectations are formed, for the EH to be useful to policymakers and market analysts the predictions of the future short-term interest rates made by market participants must be significantly better than those made by a simple random walk model. Hence, it is interesting to compare the *theoretical forecast errors*, obtained under the assumption that the EH holds, with the forecast errors from a random walk model of the short-term rate. If market participants are able to predict the future short-term rate, the theoretical forecast errors should be significantly smaller than the random walk errors.

In order to make such a comparison, however, one must identify the theoretical expected short-term rate under the EH. To see how the expected rate can be estimated it is convenient to consider the case where $n = 2$ and $m = 1$, so that (1) is rewritten as

(5) $$2r_t^2 - r_t^1 = E_t r_{t+1}^1 + 2\pi^{2,1} .$$

Since both r_t^2 and r_t^1 are observed $E_t r_{t+1}^1$ can be estimated up to a constant risk premium under the assumption that the EH holds.

In general,

(6) $$jr_t^j - (j-1)r_t^{j-1} = E_t r_{t+j}^1 + j\pi^{j,1} - (j-1)\pi^{(j-1),1} ,$$

for all $j \geq 2$, where $\pi^{1,1} = 0$.

3.1 *Identifying the Constant Risk Premiums*

Because (5) identifies $\tilde{E}_t r_{t+1}^1$ only up to a constant risk premium, an additional identifying assumption is required. Note that the mean forecast error for $E_t r_{t+1}^1$ is given by

(7) $$(1/T)\sum_{t=1}^{T}[r_{t+1}^1 - (2r_t^2 - r_t^1)] = (1/T)\sum_{t=1}^{T}[r_{t+1}^1 - E_t r_{t+1}^1] - 2\pi^{2,1}$$

If it is assumed that expectations are unbiased on average over a long period of time, i.e.,

(8) $$(1/T)\sum_{t=1}^{T}[r_{t+1}^1 - E_t r_{t+1}^1] = 0,$$

the constant risk premium can be estimated as

(9) $$\hat{\pi}^{2,1} = -(1/2T)\sum_{t=1}^{T}[r_{t+1}^1 - (2r_t^2 - r_t^1)].$$

Estimates of the other risk premiums can be obtained sequentially in the same way. Generally,

(10) $$\hat{\pi}^{j,1} = -(1/jT)\sum_{t=1}^{T}[r_{t+j-1}^1 - (jr_t^j - (j-1)r_t^{j-1})] + ((j-1)/j)\hat{\pi}^{j-1,1}.$$

Given estimates of the risk premiums, Er_{t+j}^1 can be estimated as

(11) $$\tilde{E}_t r_{t+j}^1 = jr_t^j - (j-1)r_t^{j-1} - j\hat{\pi}^{j,1} + (j-1)\hat{\pi}^{j-1,1}.$$

3.2 *Comparison of Theoretical and Random Walk Forecasts*

The theoretical *m*-period ahead expected rate can be calculated only for rates whose maturities increase consecutively with the *m*-period rate. For McCulloch and Kwan's data, this occurs only for 2-, 3-, and 4-month yields. Hence, the theoretical expected 1-month rate is calculated for 1-, 2-, and 3-month horizons.

Estimates of the risk premiums are $\hat{\pi}^{2,1} = 0.1916$, $\hat{\pi}^{3,1} = 0.3153$, and $\hat{\pi}^{4,1} = 0.4081$. As expected, the estimated risk premiums increase at a decreasing rate as the term to maturity lengthens. While these estimates are a little larger than anticipated, they are not unreasonable.

Table 2 presents summary statistics for the monthly theoretical forecast errors, $r^1_{t+i} - \tilde{E}_t r^1_{t+i}$, for all three horizons along with the corresponding forecast errors from the random walk model, RW_{t+i}. The former errors represent the forecast errors that market participants made as implied by the observed long-term rate, while the latter are the forecast errors made under the assumption that the 1-month rate is unpredictable beyond its current level. The theoretical forecasts were generated under the assumption that the mean forecast error is zero over the sample period. The random walk mean forecast errors were likewise small and insignificantly different from zero; hence, the mean forecast errors are not reported. The median forecast errors suggest a tendency by market participants to underpredict the 1-month rate over the sample period. That the same is true of the random walk forecasts is indicative of the fact that interest rates generally rose over the sample period. In any event, this tendency tends to increase with the forecast horizon. The absolute level and standard deviations of the forecast errors also tend to increase with the forecast horizon.

Figure 1 presents the theoretical forecast errors (black lines) and the corresponding forecast errors under the random walk hypothesis (gray lines) for the 1, 2 and 3 month horizons for the period January, 1952 - November, 1990. The figures suggest a high degree of correspondence between the theoretical and random walk forecasts over the period, which is reflected by the correlation between the theoretical and random walk errors shown in the figures.

Figure 1 suggests the possibility that there is no significant difference between the theoretical forecast errors and the random walk model forecast errors over the period. This proposition is investigated formally using a Diebold and Mariano (1995) test

$$(12) \qquad DM = \frac{\bar{d}}{\sqrt{V(\bar{d})}}$$

where $\bar{d}$ is an average over T observations of a differential loss function, d_t, and $V(\bar{d})$ is the variance of $\bar{d}$. The *DM* statistic has an asymptotic standard normal distribution under the null hypothesis that $\bar{d} = 0$. Following standard practice, the variance of $\bar{d}$ is estimated using a heteroskedastic-autocorrelation consistent estimator

$$(13) \qquad \hat{V}(\bar{d}) = T^{-1}[\hat{\varphi}_0 + 2T^{-1}\sum_{j=1}^{K}(T-k)\hat{\varphi}_j],$$

where $\varphi_j = (T-j)^{-1}\sum_{t=j+1}^{T}(d_t - \bar{d})(d_{t-j} - \bar{d})$. Based on the findings of Harvey, et al. (1997) the modified Diebold-Mariano test,

$$(14) \qquad MDM = \left[\frac{T+1-2K+T^{-1}K(k-1)}{T}\right]DM$$

is used. The *MDM* statistic corrects for size distortions associated with the *DM* statistic.[3]

Two differential loss functions are considered—the absolute forecast error and the squared forecast error. Table 3 presents the *MDM* statistics for both differential loss

[3] Harvey, et al, (1997, 1998) also recommend using the critical values from the Student’s t distribution rather than those from the normal distribution. The sample sizes used here are large enough, however, that the distinction is trivial.

functions for the three forecast horizons.[4] While the theoretical forecast errors were larger than the random walk forecast errors in all but one instance, the differences are not statistically significant for either loss function.

3.3 *Time-Varying Risk Premiums*

A frequent explanation for the failure of the EH is that the risk premium is not constant, as the EH requires, but time varying. With the exception of Dai and Singleton (2002) and Tzavalis and Wickens (1997), who use approaches that are flexible enough to account for nearly all of the time variation in the observed risk premiums, time-varying-risk-premium explanations for the lack of empirical success of the EH have been relatively unsuccessful (e.g., Hardouvelis, 1994; Dotsey and Otrok, 1995; Rudebusch, 1995; Bekaert, et al., 1997; and Roberds and Whiteman, 1999).[5]

In any event, the analysis in the preceding section is based on the assumption that the risk premium is constant over the sample period. Hence, it is important to investigate how the forecast errors are affected by this assumption. To this end, the theoretical forecast errors were estimated using a rolling window of *P* observations. It is obvious from (10) that the estimated risk premiums are likely to vary considerably when estimated over short samples. A number of window sizes were considered. While the degree of time variation in the estimated risk premiums varied considerably with the choice of P, the estimated forecast errors were relatively insensitive to the window size. Consequently, the results are presented for a very short window, i.e., $P = 10$. Rolling

[4] In all cases the truncation lag, K, is 20.

[5] Dai and Singleton (2002) show this using a dynamic term structure model that allows for considerable flexibility in the specification of the market price of risk. Tzavalis and Wickens, on the other hand, assume that the term premiums associated with different maturities are determined by a common factor. Tzavalis and Wickens show that when their estimate of the risk premium is included in (4), the estimate of β is

estimates of $\pi^{2,1}$, $\pi^{3,1}$ and $\pi^{4,1}$ are presented in Figure 2. The estimates suggest that, when estimated over a very short window, the estimated risk premiums at the three horizons vary considerably over time and are highly correlated. The risk premiums appear to be stationary, however. This is confirmed by augmented Dickey-Fuller tests, where the null hypothesis of a unit root is easily rejected at the 1-percent significance level for each horizon.[6]

Figure 3 compares the forecast errors under the constant and time-varying risk premium assumptions. Despite the variability of the estimated risk premiums, there is relatively little difference in the forecast errors obtained from the rolling window of 10 observations and those obtained under the assumption that the risk premium was constant. Formal *MDM* tests of the equality of the forecast errors (for the two differential loss functions) under these alternative assumptions confirm this impression. These statistics are reported in Table 4. With one exception, the differential loss functions were smaller on average for the constant risk premium. In no instance, however, was the difference statistically significant for either differential loss function.

The fact that the forecast errors change little when the risk premiums are allowed to vary over time suggests that the effect of variation in the risk premium on the forecast errors is modest relative to the new information reflected in the observed value of the short-term rate. That is, the forecast errors are dominated by news, which the market is unable to forecast.

4. Comparison with Econometric Forecasts

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insignificantly different from unity. Dai and Singleton show an analogous result for an alternative single-equation test of the EH.

[6] The Dickey-Fuller test statistics are -4.236, -4.695, and -5.649 for $\hat{\pi}^{2,1}$, $\hat{\pi}^{3,1}$, and $\hat{\pi}^{4,1}$, respectively. The lag order for these statistics is 4; however, the qualitative results were insensitive to the lag order used.

These results support the idea that the empirical shortcomings of the EH are likely due to market participants' inability to improve on the random walk forecast of the 1-month rate, at least over horizons up to three months.[7] This result is not too surprising in light of recent evidence suggesting that it is difficult to improve on random walk forecasts of interest rates. For example, no-arbitrage factor models (e.g., Dai and Singleton, 2000; Chen and Scott, 1993) appear to fit the cross-sectional yields in a static framework; however, Duffee (2002) shows that such "completely affine" term structure models forecast poorly by a root-mean-square-error criterion relative to a random walk model in out-of-sample forecasts. Duffee (2002) shows that a class of "essentially affine" models can improve on random walk forecasts by this criterion, where the improvement generally increases with the length of the forecast horizon. Duffee (2002) does not test whether the differences in forecasts are statistically significant.

Diebold and Li (2003) also show that some improvement in out-of-sample forecasts of the short-term rate can be obtained by using a three factor model of the Nelson and Siegel (1987) exponential components framework for estimating the yield curve. Similar to Duffee (2002), the improvement over random walk forecasts is better for longer forecast horizons. Recently, however, Carriero, et al. (2003) report some improvement over the random walk model at short horizons using the Diebold-Li approach. Like Duffee (2002), Carriero, et al. (2003) provide no formal statistical analysis of the improvement relative to the random walk forecasts.[8]

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[7] Given the high degree of persistence observed in interest rates in general, it is reasonable to conjecture that it may be difficult to beat a random walk forecasting model at relatively short horizons. At longer horizons; however, information embedded in the structure may be more useful. Consistent with this conjecture, Diebold and Li (2003) note that their model outperforms the random walk model only at relatively long horizons.

[8] Carriero, et al. (2003) find essentially no improvement in the forecasts when the model is augmented with economic variables, specifically, the CPI-inflation and unemployment rates.

Given the demonstrated ability of the Diebold-Li approach to improve on the random walk model in out-of-sample forecasts and its relative ease of estimation, out-of-sample forecasts from the Diebold-Li model are compared with the theoretical forecasts.[9]

Diebold and Li (2003) use the following modified version of the Nelson and Siegel (1987, 1988) forward rate curve to approximate the yield curve

$$(15)\qquad r_t^j = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t j}}{\lambda_t j}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t j}}{\lambda_t j} - e^{-\lambda_t j}\right).$$

The parameter λ_t governs the exponential decay rate. Small values produce slow decay and a better fit at longer maturities, while large values tend to provide a better fit at short maturities. λ_t also governs where the loading on β_{3t} achieves it maximum. Because the loading on β_{1t} is 1 and, hence, does not decay, Diebold and Li interpret it to be the long-term factor corresponding to the *level* of the term structure. Because the factor loading on β_{2t} decays monotonically from 1 to zero, it is viewed as the short-term factor, corresponding to the *slope* of the yield curve. In contrast, the factor loading on β_{3t} rises from zero and then decays back to zero. Hence, Diebold and Li suggest that this factor corresponds to the *curvature* of the yield curve.[10]

Rather than estimating (15) by nonlinear least squares, Diebold and Li fix the value of λ_t and estimate (15) for each period using ordinary least squares. They argue that this not only greatly simplifies the estimation, but likely results in more trustworthy estimates of the level, slope and curvature factors. Diebold and Li set $\lambda = 0.0609$, precisely the value where the loading on the curvature factor reaches it maximum on the assumption that the curvature of the yield curve reaches its maximum at 30 months.

[9] See Diebold and Li (2003) for comparative evaluation of 6 competing models.
[10] See Diebold and Li (2003) for a more detailed analysis of and interpretation of these three factors.

Diebold and Li make out-of-sample forecasts of rates at all maturities along the yield curve by estimating (15) over the first N observations and then forecasting the yields going forward according to,

$$(16) \qquad \hat{r}^{j}_{t+h} = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\lambda j}}{\lambda j}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\lambda j}}{\lambda j} - e^{-\lambda j}\right),$$

where forecasts of the three factors are obtained by estimating

$$(17) \qquad \beta_{i,t} = c + \gamma \beta_{i,t-1}, \quad i = 1,2,3$$

over the first N observations. The process is updated recursively to generate T-N out-of-sample forecasts of each rate over horizons, $h = 1, 2, \ldots, H$.

This procedure was used to forecast the 1-month rate for $H = 3$. To estimate the three factor model, in addition to the 1- to 4-month yields, the McCulloch and Kwon yields on maturities of 6, 9, 12, 24, 36, 48, 60, and 120 months are used. The procedure is initialized using the period January, 1952 - December, 1969, and updated recursively through November, 1990.[11] These factors are presented in Figure 4. While not shown here, just as in Diebold and Li (2003), these factors correspond very closely to estimates of the level, slope, and curvature factors obtained from the first three principal components obtained from these rates. Unlike principal components estimates, these factors are correlated; however, the correlations are low.[12]

The theoretical and Diebold-Li forecast errors for the three forecast horizons are presented in Figure 5, along with the correlation between these forecast errors at each horizon. Theoretical forecasts appear to correspond closely to the Diebold-Li forecasts. Moreover, while the differences are not large, the theoretical forecast errors are more

[11] This is the longest sample for all three forecast horizons.

highly correlated with the Diebold-Li forecast errors than with the random walk forecast errors over this period—the latter correlations being 0.812, 0.872, and 0.893 for the 1-, 2- and 3-month horizons, respectively.

In any event, Figure 5 suggests that statistically significant differences between the theoretical and Diebold-Li forecasts over the period are unlikely. This conjecture is confirmed by the *MDM* tests, reported in Table 5. The mean differences in the forecast errors are very small for both the absolute and squared error differential loss functions. Furthermore, in no instance is the difference significantly different from zero at the 5 percent significance level.

4.1 *Encompassing Tests*

A more stringent forecasting requirement is that the forecasts from the competing models contain no useful forecasting information that is not contained in the preferred model. Harvey, et al., (1998) have shown that the *MDM* test can used to test for forecast encompassing. Specifically, if the loss function sequence is defined as

(18) $$d_t = (e_t^P - e_t^A)e_t^P ,$$

where e_t^P and e_t^A are the errors from the preferred and alternative models, respectively, one can test whether the preferred model encompasses the alternative model by testing the hypothesis $\bar{d} = 0$. Failure to reject the null hypothesis implies that the alternative model does not contain information useful for forecasting that is not contained in the preferred model. To be conclusive, however, the null hypothesis must be rejected when the model being analyzed is the preferred model and not rejected when it is the alternative model.

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[12] Specifically, $Corr(\hat{\beta}_1\hat{\beta}_2) = -0.138$, $Corr(\hat{\beta}_1\hat{\beta}_3) = 0.244$, and $Corr(\hat{\beta}_2\hat{\beta}_3) = 0.174$

The results of the encompassing test between the theoretical forecasts and the forecasts from the Diebold-Li three factor model are presented in Table 6. At the 1-month horizon the results suggest that there is useful information in both the theoretical and Diebold-Li forecasts that is not contained in the other. However, at the 2- and 3-month horizons there appears to be no statistically significant information in one forecast that is not contained in the other. Hence, neither model encompasses the other.

4.2 *The Rate Spread as a Predictor of the Expected Short-term Rate*

Clearly long-term rates are forward looking. The evidence above, however, suggests that the markets have a relatively difficult time forecasting the 1-month rate—at least for horizons of three months or shorter. Hence, the results reported in Table 1, that show that the longer-term rate is a relatively poor predictor of the future change in the 1-month rate does not address the question: How well do long-term rates reflect the *expected change* in the short-term rate?

This question is investigated by estimating the equation

$$(19) \qquad (1/k)\sum_{i=0}^{k-1}(E_t r_{t+i}^m)^{DL} - r_t^m = \alpha + \beta(r_t^n - r_t^m) + \varepsilon_t\,,$$

where $(E_t r_{t+i}^m)^{DL}$, for $m=1$, is the out-of-sample forecast of the 1-month rate from the Diebold-Li model for $i \geq 1$. Note that $(E_t r_t^m)^{DL} = r_t^m$. Note that (19) is identical to (4) except that the actual 1-month rate in period $t+i$ is replaced by the prediction from the Diebold-Li model.

Estimates of (19) for n = 2, 3, and 4 are presented in Table 7. The estimates of β are positive and statistically significant; however, the null hypothesis that $\beta = 1$ is easily rejected. Nevertheless, the spread between the longer-term rates and the 1-month rate accounts for a large percent of the predicted change in the short-term rate, suggesting that—just as the EH implies—the spread between the long-term and short-term rate provides useful information about the markets' expectation for the change in the short-term rate. The problem for the EH is that these predictions improve only slightly over the predictions from a random walk model. Consequently, the spread between the long-term and short-term rate provides relatively little information about the actual change in the short-term rate. This is evidenced in the lower half of Table 7, which presents the estimates of (4) for the identical sample period. While the spread between the long-term and short-term rate is a relatively good predictor of the expected change in the short-term rate from the Diebold_Li model, it is a relatively poor predictor of the actual change in the short-term rate.[13]

5. Conclusions and Implications

Recent empirical work suggests the low predictive power of the long-term/short-term rate spread may stem from the inability of market participants to predict the future short-term rate significantly beyond its current level. This paper investigated this possibility by estimating the theoretical expected 1-month rate 1, 2 and 3 months ahead using a relatively mild identifying restriction. The theoretical forecast errors are then compared with forecast errors from a random walk model and the three factor term structure model of Diebold and Li (2003).

The evidence suggests that the theoretical forecasts implied by the EH do not differ appreciably from the random walk or term structure forecasts. Moreover, it is shown that, just as the EH implies, long-term rates reflect significant information about

[13] In the case where $n = 4$, β is rather imprecisely estimated and the hypothesis $\beta = 1$ is not rejected.

the markets' expectation for the short-term rate. That is, to the extent that the market is able to forecast the future short-term rate, long-term rates reflect that information. The difficultly arises from the fact that the observed short-term rates are dominated by new information that appears to be difficult to forecast. For this reason, the spread between the long-term and short-term rate is a relatively poor predictor of the future short-term rate.

Hence, while the EH is fundamentally correct—longer-term rates incorporate the markets' expectation for the future short-term rate—its usefulness for financial market analysts and policymakers is doubtful. Of course, policymakers targeting short-term interest rates might increase the predictability of the rate spread by making short-term rates more predictable. Indeed, some recent evidence (e.g., Lange, et al., 2003; Poole, et al., 2002; and Watson, 2002) indicates that the predictability of the federal funds rate has increased since the Fed began announcing its funds rate target in 1994.

The fact that the markets' expectations for the 1-month rate *h*-periods ahead differ significantly from the observed future 1-month rate is not a violation of rational expectations. Indeed, it may be that the markets' expectations incorporate all of the relevant information for forecasting the future 1-month rate except the news that affects the rate that cannot be forecast. This possibility is supported by the fact that the theoretical forecast errors are relatively unaffected by allowing for greater time variation in the risk premiums and by the fact that the spread between the long-term and short-term rate is a relatively good predictor of the model-based expected change in the short-term rate.

The findings presented here also support Carriero, et al.'s (2003) conclusion that researchers should be wary about using the ex-post short-term rate to proxy for the markets' ex-ante expectation is common practice, e.g., using (4) to test the EH.

Finally, these results only apply to relatively short-term rates and forecast horizons. Given the extreme persistence in interest rates and the evidence that the forecasts tend to improve relative to a random walk as the forecast horizon lengthens (e.g., Diebold and Li, 2003; Duffee, 2002; and Carriero et al. 2003), it may be that the theoretical forecasts will be significantly better than random walk forecasts at longer horizons. Such a finding would be consistent with the evidence (e.g., Campbell and Shiller, 1991, and Dittmar and Thornton, 2003) that the EH fares better at the longer end of the maturity spectrum.

References:

Balduzzi, P., G. Bertola and S. Foresi. (1997) "A Model of Target Changes and the Term Structure of Interest Rates," *Journal of Monetary Economics 39*(2), 223-49.

Bekaert, G. and R. J. Hodrick. (2001) "Expectations Hypotheses Tests," *Journal of Finance 56*(4), 1357-94.

Bekaert, G., R. J. Hodrick, and D. A. Marshall. (1997) "Peso Problem Explanations for Term Structure Anomalies, *Journal of Monetary Economics 48*(2), 241-70.

Campbell, John Y. and Robert J. Shiller. (1987) "Cointegration and Tests of Present Value Models," *Journal of Political Economy 95*(5), 1062-88.

Campbell, John Y. and Robert J. Shiller. (1991) "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies 58*(3), 495-514.

Carriero, A., C.A. Favero, and I. Kaminska. (2003) "Financial Factors, Macroeconomic Information and the Expectations Theory of the Term Structure of Interest Rates," unpublished manuscript.

Chen, R.R., and L. Scott. (1993) "Maximum Likelihood Estimation for a Multi-Factor Model of the Term Structure of Interest Rates." *Journal of Fixed Income 4*, 14-31.

Dai, Q., and K. J. Singleton. (2002) "Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure," *Journal of Financial Economics 63*(3), 415-441.

Dai, Q., and K. J. Singleton. (2000) "Specification Analysis of Affine Term Structure Models," *Journal of Finance 55*(5), 1943-78.

Diebold, F.X., and Li, C. (2003), "Forecasting the Term Structure of Government Bond Yields," NBER working paper, no. 10048.

Diebold, F.X. and R.S. Mariano. (1995) "Comparing Predictive Accuracy," *Journal of Business and Economic Statistics 13*(3), 253-63.

Diebold, F.X., G.D. Rudebusch, and S.B. Aruoba. (2003) "The Macroeconomy and the Yield Curve: A Nonstructural Analysis," CSF working paper #2003/31.

Dittmar, R. D. and D. L. Thornton. (2003) "New Evidence on the Expectations Hypothesis of Bond Yields," Federal Reserve Bank of St. Louis Working Paper 2003-021A.

Dotsey, M. and C. Otrok. (1995) "The Rational Expectations Hypothesis of the Term Structure, Monetary Policy, and Time-Varying Term Premia," Federal Reserve Bank of Richmond *Economic Quarterly 81*(1), 65-81.

Duffee, G. (2002), "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance 57*(1), 405-443.

Hardouvelis, G. A. (1994) "The Term Structure Spread and Future Changes in Long and Short Rates in the G7 Countries: Is There a Puzzle?" *Journal of Monetary Economics 33*(2), 255-83.

Harvey, D., S. Leybourne, and P. Newbold. (1997) "Testing the Equality of Prediction Mean Squared Errors," *International Journal of Forecasting 13*(2), 281-91.

Harvey, D., S. Leybourne, and P. Newbold. (1998) "Tests for Forecast Encompassing," *Journal of Business and Economics Statistics, 16*(2), 254-59.

Kool, C.J.M., Thornton, D.L., 2004. "A Note on the Expectations Hypothesis at the Founding of the Fed," *Journal of Banking and Finance 28*(12), 3055-3068.

Lange, J., B. Sack, and W. Whitesell. (2003) "Anticipations of Monetary Policy in Financial Markets," *Journal of Money, Credit and Banking 35*(6), Part 1, 889-910.

Macaulay, Fredrick. (1938) *Some Theoretical Problems Suggested by Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856*, National Bureau of Economic Research, New York.

McCulloch, J.H., and H-C. Kwon. (1993) "U.S. Term Structure Data, 1947-1991," Ohio State University Working Paper #93-6.

Nelson, C.R., and A.F. Siegel. (1988) "Long-Term Behavior Yield Curves," *Journal of Financial and Quantitative Analysis*, *23*(1), 105-10.

Nelson, C.R., and A.F. Siegel. (1987) "Parsimonious Modeling of Yield Curves," *Journal of Business*, *60*(4), 473-89.

Poole, W., R.H. Rasche, and D.L. Thornton. (2002) "Market Anticipations of Monetary Policy Actions," Federal Reserve Bank of St. Louis *Review 84*(4), 65-93.

Roberds, W., and C. H. Whiteman. (1999) "Endogenous Term Premia and Anomalies in the Term Structure of Interest Rates: Explaining the Predictability Smile," *Journal of Monetary Economics 44*(3), 555-80.

Rudebusch, G.D. (1995) "Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure," *Journal of Monetary Economics 35*(2), 245-74.

Simon, D.P. (1990) "Expectations and the Treasury Bill-Federal Funds Rate Spread over Recent Monetary Policy Regimes," *Journal of Finance 45*(2), 567-77.

Thornton, D.L., 2005a. "Tests of the Expectations Hypothesis: Resolving the Campbell-Shiller Paradox," forthcoming, *Journal of Money, Credit, and Banking*.

Thornton, D.L., 2005b. "Tests of the Expectations Hypothesis: Resolving the Anomalies when the Short-Term Rate Is the Federal Funds Rate," forthcoming, *Journal of Banking and Finance*.

Tzavalis, E. and M. R. Wickens. (1997) "Explaining the Failures of the Term Spread Models of the Rational Expectations Hypothesis of the Term Structure," *Journal of Money, Credit and Banking 29*(3), 364-80.

Watson, M.W. (2002) "Commentary" Federal Reserve Bank of St. Louis *Review 84*(4), 95-97.

Table 1: Estimates of Equation 4: 1952.01 1991.02				
Horizon	β	$\bar{R}^2$	$\beta = 0$	$\beta = 1$
1 month	0.5016 (0.105)	0.107	4.777*	4.747*
2 months	0.4975 (0.143)	0.083	3.262*	3.514*
3 months	0.4708 (0.180)	0.073	2.163*	2.940*

HAC consistent standard errors in parentheses.
*Indicates statistical significance at the 5 percent level.

Table 2: Descriptive Statistics for Forecast Errors for the Estimated Theoretical 1-Month Rate						
	$r^1_{t+1} - \tilde{E}_t r^1_{t+1}$	RW_{t+1}	$r^1_{t+2} - \tilde{E}_t r^1_{t+2}$	RW_{t+2}	$r^1_{t+3} - \tilde{E}_t r^1_{t+3}$	RW_{t+3}
Median	0.109	0.042	0.141	0.082	0.160	0.111
Max.	1.788	2.737	4.186	3.645	3.910	5.194
Min.	-5.047	-4.682	-6.423	-7.224	-7.698	-8.406
S.D.	0.644	0.646	0.950	0.923	1.162	1.018
	Serial correlation					
ρ_1	0.258	0.023	0.512	0.502	0.714	0.634
ρ_2	0.076	-0.019	0.140	-0.066	0.351	0.220
ρ_3	0.063	-0.118	0.008	-0.163	0.059	-0.178
ρ_4	-0.009	-0.078	0.014	-0.142	-0.001	-0.194

Table 3: *MDM* Statistics for Theoretical and Random Walk Forecasts		
Horizon	AE	SE
1-Month	1.152	-0.016
2-Month	0.784	0.468
3-Month	1.174	0.450

A positive sign indicates that the mean of the differential loss function is larger for the theoretical forecast.

Table 4: *MDM* tests for Theoretical Forecasts: Time-Varying and Constant Risk Premium		
Horizon	AE	SE
1-Month	0.872	-0.504
2-Month	-0.488	-1.192
3-Month	-1.096	-1.427

A positive sign indicates that the mean of the differential loss function is larger for the constant-risk-premium theoretical forecast.

Table 5: *MDM* Statistics for Theoretical and Diebold-Li Forecasts		
Horizon	AE	SE
1-Month	0.103	-0.640
2-Month	-0.480	0.162
3-Month	-0.001	0.341

A positive sign indicates that the mean of the differential loss function is larger for the theoretical forecast.

Table 6: *MDM* Encompassing Tests for Theoretical and Diebold-Li Forecasts		
	Null Hypothesis	
Horizon	Diebold-Li	Theoretical
1-Month	2.956*	2.816*
2-Month	1.547	1.319
3-Month	1.147	1.147

*Indicates statistical significance at the 5 percent level.

Table 7: Estimates of Equation 19: 1970.01 1990.11				
Estimates of Equation 19				
Horizon	β	$\bar{R}^2$	$\beta = 0$	$\beta = 1$
1 month	0.4332 (0.016)	0.855	27.938*	35.425*
2 months	0.4239 (0.016)	0.806	26.656*	36.006*
3 months	0.4158 (0.023)	0.677	17.981*	25.400*
Estimates of Equation 4				
1 month	0.5637 (0.131)	0.113	4.319*	3.331*
2 months	0.5542 (0.182)	0.095	3.051*	2.449*
3 months	0.5841 (0.233)	0.090	2.512*	1.785

HAC consistent standard errors in parentheses.
*Indicates statistical significance at the 5 percent level.

Figure 1: Theoretical (black) and RW (gray) Forecast Errors

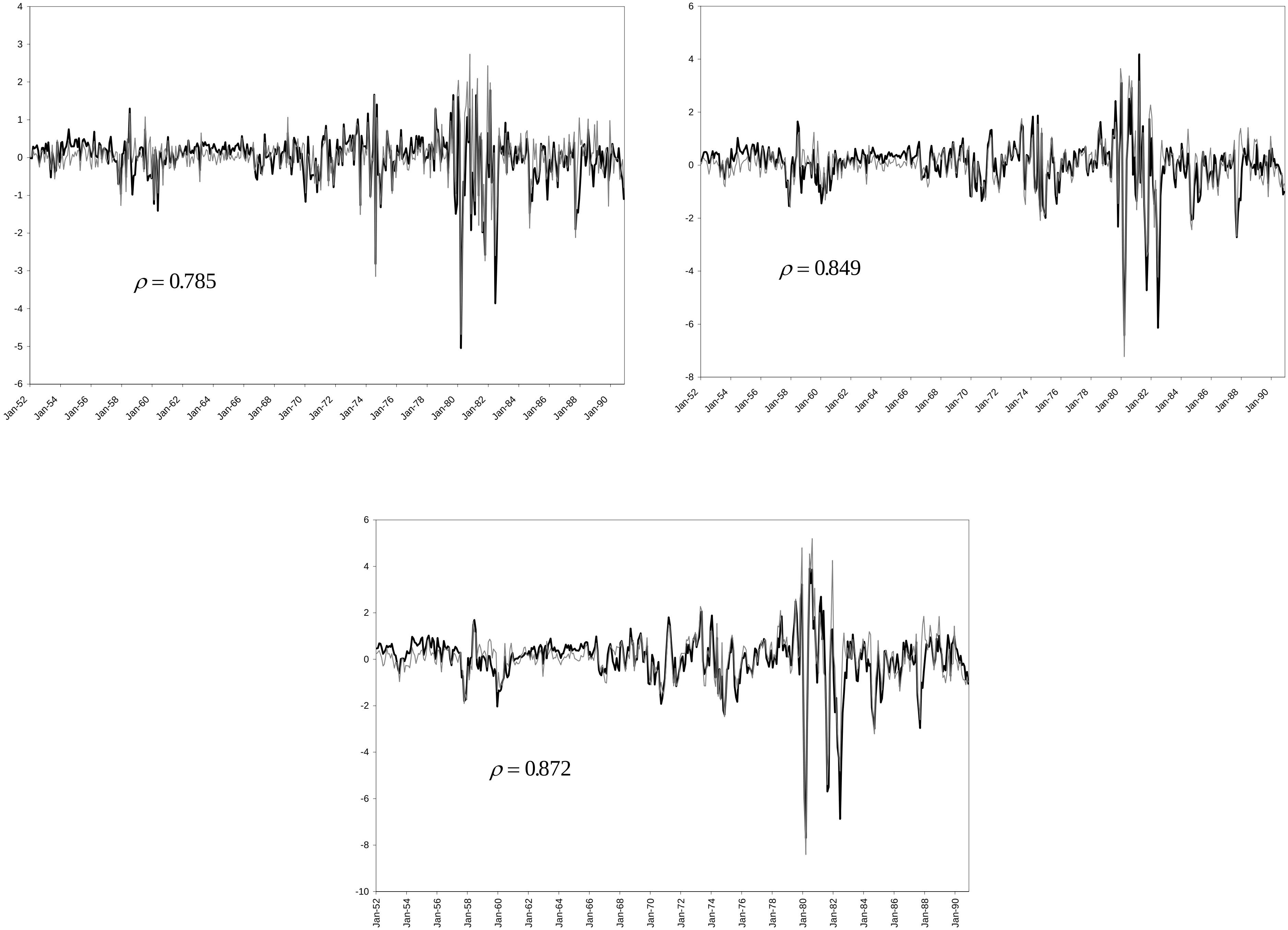

Figure 2: Rolling Window Estimates of the Risk Premiums

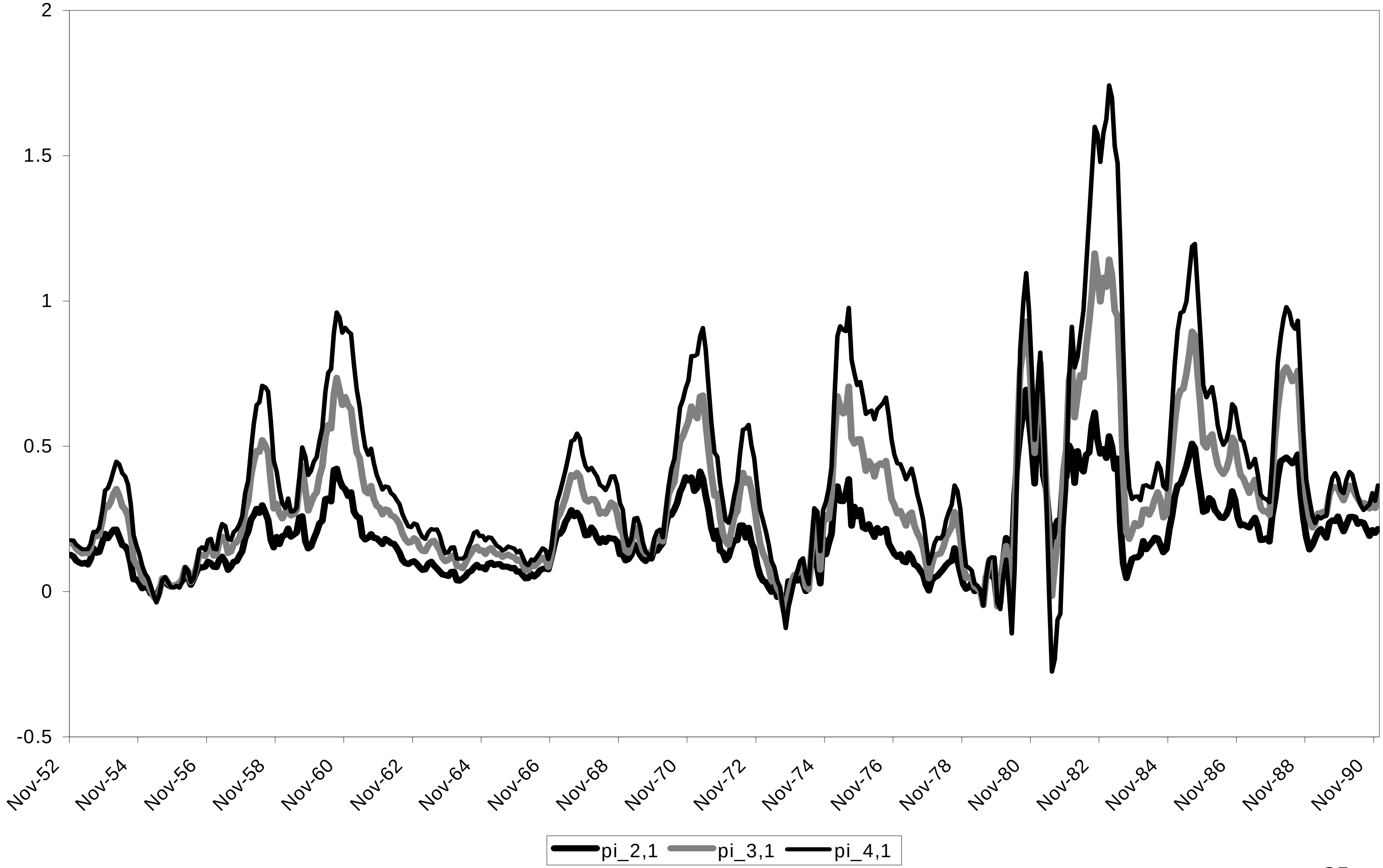

Figure 3: Theoretical Forecast Errors Assuming a Constant and a Time-Varying Risk

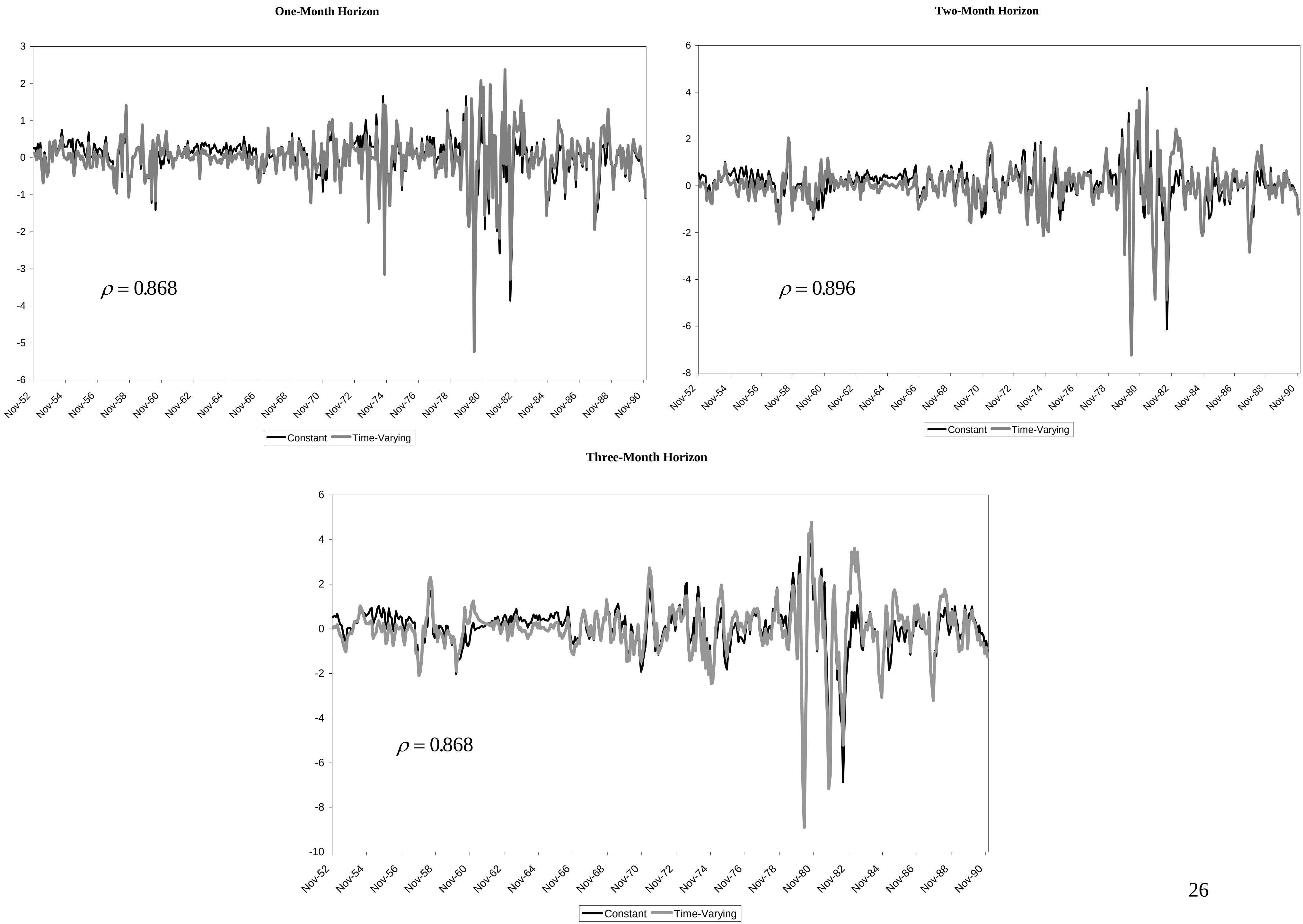

Figure 4: Estimates of the Level, Slope and Curvature Factors
(January, 1952 - February, 1991)

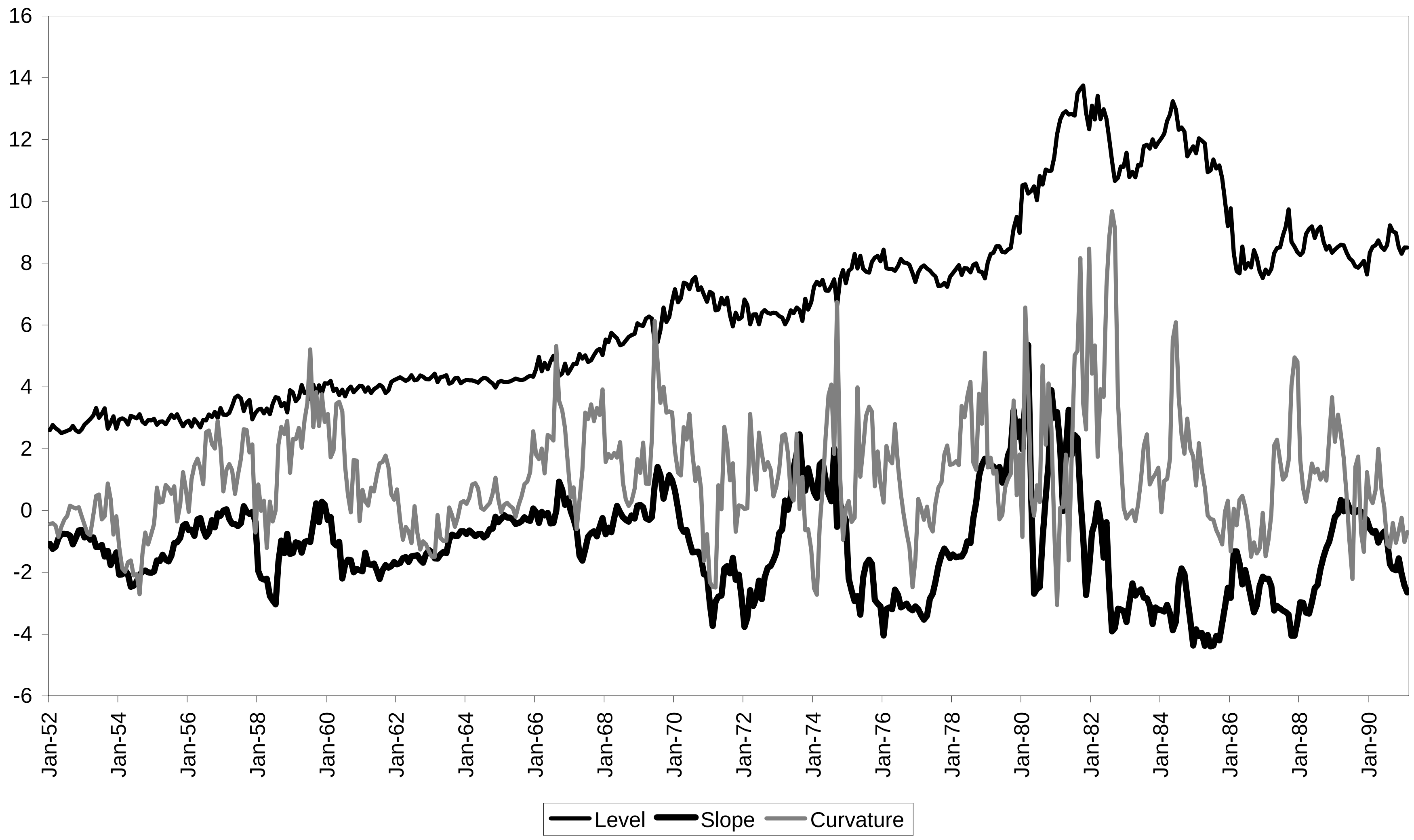

Figure 5: Theoretical and Diebold-Li Forecast Errors