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Working Paper Number	2004-007B
Citable Link	https://doi.org/10.20955/wp.2004.007
Suggested Citation	Dittmar, R.D., Gavin, W.T.; Inflation-Targeting, Price-Path Targeting and Indeterminacy, Federal Reserve Bank of St. Louis Working Paper 2004-007. URL https://doi.org/10.20955/wp.2004.007

Published In	Economics Letters
Publisher Link	https://doi.org/10.1016/j.econlet.2005.03.003

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Abstract

In this paper, we examine the areas of indeterminacy in a flexible price RBC model with shopping time role for money and a central bank that uses an interest rate rule to target inflation and/or the price level. We present analytical results showing that, although inflation targeting often results in real indeterminacy, a price level target generally delivers a unique equilibrium for a relevant range of policy parameters.

Keywords: Inflation targeting, price-path targeting, indeterminacy

JEL Classification: C62, E52

Original Date: September 26, 2003, Revised December 21, 2004

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The views expressed here are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis or of the Federal Reserve System. We thank Jim Bullard and Chuck Carlstrom for helpful comments on an earlier draft.

1. **Introduction**

Economists working in theoretical models have long understood that finding solutions to models with interest rate rules and forward-looking agents can be problematic because a relevant range of parameter values often leads to multiple solutions (or, equivalently, areas of indeterminacy). Sargent and Wallace (1975) showed that interest rates rules could lead to price level indeterminacy in models with forward-looking agents. McCallum (1981, 1986) showed that modifying the rule to include the lagged money supply was one of many possible ways to eliminate indeterminacy in the price level. He has argued that many cases of indeterminacy are the result of inappropriate specification choices.

However, the issue about indeterminacy in monetary models is still interesting because it turns out that the conditions for real determinacy in dynamic general equilibrium models depend critically on model specifications of policy rules that are thought to correspond to features in modern economies.

Benhabib and Farmer (1999) summarize literature in which researchers have used models with indeterminacy to explain sticky prices and real effects of monetary policy. For the policy economist, the issue is whether inappropriately designed policy institutions increase the probability of asset pricing bubbles and other self-fulfilling prophecies that can be generated in models with multiple equilibria. If so, then policymakers want to design institutions that eliminate this potential source of instability.[1]

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[1] For a discussion of indeterminacy in the context of policy design see McCallum (2003), Woodford (2003a,b) and Benhabib, Schmitt-Grohe and Uribe (2003).

Giannoni (2000) investigates the optimality of alternative interest rate rules in a New Keynesian economy. Using a sticky-price model, he shows that while indeterminacy is present for some range of parameters in the inflation targeting regimes, they disappear when the central bank targets a price path. In this paper we show that a similar result also holds in a flexible-price model.

2. The Economic Model

The model we use here is a slighted modified version of the monetary business cycle model developed in Dittmar, Gavin and Kydland (2004). In each period, infinitely-lived consumers decide how to allocate time between work, leisure, and transaction-related activities such as trips to the bank, shopping, and so on. Larger money balances carried in from the previous period make the shopping activity less time consuming, leaving more time for work and leisure. New money enters the economy as a government transfer but does not reduce shopping time until the next period.[2] The government sets a target for the nominal interest rate on bonds and transfers whatever amount of money is necessary to achieve it.

Many identical households inhabit the model economy. Each household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t), \tag{1}$$

where $0 < \beta < 1$ is a discount factor, c_t is consumption expenditure, and ℓ_t is leisure time. The current-period utility function is linear in leisure but otherwise general.

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[2] Note that this is timing convention used in Kydland (1989).

The household's stock of capital, k, is governed by the law of motion,

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{2}$$

where $0 < \delta < 1$, δ is the depreciation rate, and i_t is investment.

Household time spent on transactions-related activities in period t is given by

$$\omega_0 - \Omega\left(\frac{m_t}{P_t}\right)^{\omega}, \tag{3}$$

m_t is the nominal stock of money, and P_t is the price of physical goods relative to that of money. The parameter, Ω, represents the level of shopping time technology. By restricting Ω and ω to have the same sign and $\omega < 1$, the amount of time saved increases as a function of real money holdings, but at a decreasing rate. Note that this shopping time function differs from Dittmar, Gavin and Kydland (2003) because it does not include consumption expenditures. With leisure linear in utility, this assumption allows us to reduce our dynamic system to only two equations.

Leisure in period t is

$$\ell_t = T - n_t - \omega_0 + \Omega\left(\frac{m_t}{P_t}\right)^{\omega}, \tag{4}$$

where T is the total time available and n_t is time spent in market production.

Aggregate output, Y_t, is produced using labor and capital inputs:

$$Y_t = C_t + I_t = z\ N_t^{\theta} K_t^{1-\theta}, \tag{5}$$

where I_t is the total of investment expenditures and z is the level of technology. We assume a Cobb-Douglas production function with labor share $\theta : 0 < \theta < 1$.

The nominal budget constraint for the typical individual is

$$P_t c_t + P_t k_t + b_{t+1} + m_{t+1} = P_t z n_t^{\theta} k_t^{1-\theta} + P_t(1-\delta)k_t + R_t b_t + m_t + v_t, \tag{6}$$

where b_{t+1} are nominal bonds carried into period t, and v_t is a nominal lump-sum transfer from the government. R_t is the gross nominal interest rate earned on bonds which are assumed to be in zero net supply in equilibrium. The government transfers money balances directly to households according to its policy rule. It produces money and conducts policy at zero cost and does not have any expenditures or revenues.

A law of motion analogous to that for individual capital describes the aggregate quantity of capital. The distinction between individual and aggregate variables is represented here by lowercase and uppercase letters, respectively. Competitive factor markets will imply that in equilibrium each factor receives its marginal product.

The central bank manipulates the monetary transfer to implement an interest rate rule of the type:

$$\ln R_{t+1} = \kappa + \nu_1 \ln P_{t+1} + \nu_2(\ln P_{t+1} - \ln P_t), \tag{7}$$

where $\ln R_{t+1}$ is the period t+1 nominal interest rate target chosen by the central bank. The interest rate is set as a function of the price level and the inflation rate.

The Dynamic System. The agent's choice of any four variables–say leisure, labor, capital and bonds–will determine the others via his budget and time constraints. Here, we assume that utility is linear in leisure:

$$\frac{\partial u}{\partial l}(\bullet_t) \equiv \varepsilon \tag{8}$$

where ε is an arbitrary constant. We define the labor-capital ratio as $x_t = n_t / k_t$ and compute the first order conditions for labor, capital, and bonds in terms of this ratio. The first order condition for labor is given as:

$$\frac{\partial u}{\partial c}(\bullet_t) \equiv \frac{\varepsilon}{z\theta x_t^{\theta-1}}, \tag{9}$$

which equates the marginal utility of consumption to the marginal utility of labor relative to the marginal product of labor. The first order condition for capital is given as:

$$\beta^t \frac{\partial u}{\partial c}(\bullet_t) + \beta^{t+1} E_t \left\{ \left[z(1-\theta) x_{t+1}^{\theta} + 1 - \delta \right] \frac{\partial u}{\partial c}(\bullet_{t+1}) \right\} = 0, \tag{10}$$

which equates the intertemporal rate of substitution of consumption in utility to the intertemporal rate of transformation of output in production. The first order condition for bonds is given as:

$$\frac{1}{P_t} \frac{\partial u}{\partial c}(\bullet_t) = E_t \left[R_{t+1} \beta \frac{1}{P_{t+1}} \frac{\partial u}{\partial c}(\bullet_{t+1}) \right], \tag{11}$$

which relates the nominal interest rate to the inflation rate and the intertemporal rate of substitution in consumption.

Substituting the first order condition for labor (9) into the first order conditions for capital (10) and bonds (11), we get:

$$x_t^{1-\theta} = \beta E_t \left\{ x_{t+1}^{1-\theta} \left[z(1-\theta) x_{t+1}^{\theta} + 1 - \delta \right] \right\}, \text{ and} \tag{12}$$

$$\frac{1}{P_t} x_t^{1-\theta} = \beta E_t \left(\frac{R_{t+1}}{P_{t+1}} x_{t+1}^{1-\theta} \right). \tag{13}$$

In the steady state, $1/\beta = z(1-\theta)x_{t+1}^{\theta} + 1 - \delta$ and $R = 1/\beta$. After defining $\tilde{x}_t = \ln x_t$ and $\tilde{P}_t = \ln P_t$, we log linearize (12) and (13) to get two first order difference equations in the log of the price level and the log of the labor-capital ratio:

$$\Delta\tilde{x}_t = \left(\frac{1-\beta\theta(1-\delta)}{1-\theta} \right) E_t \Delta\tilde{x}_{t+1}, \text{ and} \tag{14}$$

$$-\Delta\tilde{P}_t + (1-\theta)\Delta\tilde{x}_t = E_t\left[\Delta\tilde{R}_{t+1} - \Delta\tilde{P}_{t+1} + (1-\theta)\Delta\tilde{x}_{t+1}\right], \tag{15}$$

We log-linearize the interest-rate rule (7) to get:

$$E_t\Delta\tilde{R}_{t+1} = (\nu_1 + \nu_2)E_t\Delta\tilde{P}_{t+1} - \nu_2\Delta\tilde{P}_t. \tag{16}$$

Substituting (16) into (15), we get:

$$\Delta\tilde{P}_t + (1-\theta)\Delta\tilde{x}_t = E_t\left[(\nu_1 + \nu_2 - 1)\Delta\tilde{P}_{t+1} - \nu_2\Delta\tilde{P}_t + (1-\theta)\Delta\tilde{x}_{t+1}\right], \tag{17}$$

Equations (14) and (17) form a two-equation system with two roots:

$$A\begin{bmatrix} E_t\Delta\tilde{x}_{t+1} \\ E_t\Delta\tilde{P}_{t+1} \end{bmatrix} = B\begin{bmatrix} \Delta\tilde{x}_t \\ \Delta\tilde{P}_t \end{bmatrix},$$

Where

$$A = \begin{bmatrix} 1-\beta\theta(1-\delta) & 0 \\ 1-\theta & \nu_1+\nu_2-1 \end{bmatrix} \text{and } B = \begin{bmatrix} 1-\theta & 0 \\ 1-\theta & \nu_2-1 \end{bmatrix}.$$

For a unique solution to this system we need one eigenvalue inside the unit circle and the other outside. The eigenvalues of $A^{-1}B$ are given by $(1-\theta)/[1-\beta(1-\delta)\theta]$ and $(1-\nu_2)/(1-\nu_1-\nu_2)$. The first is between zero and one by our model assumptions and the second is greater than unity for the combination of values shown in the shaded regions in figure 1. If there is no weight on inflation, then the model is determined for $0 < \nu_1 < 2$. The general conditions for determinacy are:

$$\nu_1 > 0 \to \left(-\infty < \nu_2 < 1 - \frac{\nu_1}{2}\right), \text{ and}$$

$$\nu_1 < 0 \to \left(1 - \frac{\nu_1}{2} < \nu_2 < \infty\right).$$

Of special interest is the case of a pure inflation target, i.e. $\nu_1 = 0$. In this case, we can show that *there is indeterminacy everywhere*. To see this we first note that if $\nu_1 = 0$, then we can rewrite equation (17) as follows:

$$\left(1-\theta\right)\Delta\tilde{x}_t = \left(\nu_2 - 1\right)E_t(\Delta\tilde{P}_{t+1} - \Delta\tilde{P}_t) + \left(1-\theta\right)E_t\Delta\tilde{x}_{t+1}. \tag{18}$$

If we let $\pi_{t+1} = P_{t+1} / P_t$ be the inflation rate at time *t+1* and $\tilde{\pi}_{t+1} = \ln \pi_{t+1}$, then by definition we have $E_t\Delta\tilde{\pi}_{t+1} = E_t\Delta\tilde{P}_{t+1} - \Delta\tilde{P}_t$. Therefore equation (18) above can be written as a difference equation in $\Delta\tilde{x}_t$, $E_t\Delta\tilde{\pi}_{t+1}$, and $E_t\Delta\tilde{x}_{t+1}$, namely:

$$\left(1-\theta\right)\Delta\tilde{x}_t = \left(\nu_2 - 1\right)E_t\Delta\tilde{\pi}_{t+1} + \left(1-\theta\right)E_t\Delta\tilde{x}_{t+1}. \tag{19}$$

Combining equations (19) and (14) as above again gives a two variable system of difference equations that can be written as:

$$A\begin{bmatrix} E_t\Delta\tilde{x}_{t+1} \\ E_t\Delta\tilde{\pi}_{t+1} \end{bmatrix} = B\begin{bmatrix} \Delta\tilde{x}_t \\ \Delta\tilde{\pi}_t \end{bmatrix},$$

where

$$A = \begin{bmatrix} 1-\beta\theta\left(1-\delta\right) & 0 \\ 1-\theta & \nu_1 - 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1-\theta & 0 \\ 1-\theta & 0 \end{bmatrix}.$$

For a unique solution, we again would need one eigenvalue of $A^{-1}B$ to be inside the unit circle and one eigenvalue outside the unit circle. Here, however, one eigenvalue of $A^{-1}B$ is given by $\left(1-\theta\right)/\left[1-\beta(1-\delta)\theta\right]$ which is positive and strictly less than 1 by our model assumptions while the other eigenvalue is exactly 0. The model is therefore indeterminate with a pure inflation targeting rule for any value of ν_1.[3]

[3] Note that this result appears to be at odds with Woodford (2003, proposition 2.6), but it is not. He has a cashless economy with determinacy conditions that are the same as a model in which money balances at the

3. Concluding Remarks

We find that the indeterminacy that can arise with inflation targeting disappears when the inflation target is replaced with a path for the price level. In the real world central banks do not set paths for the price level, but they do tend to target inflation averaged over multiple periods. Countries often adopted inflation targeting in order to eliminate high and variable inflation. As inflation fell, these countries seem to have settled on a single target that is repeated year after year. For example, the Bank of Canada has had the same inflation targeting range, one to three percent, since 1995 and the Bank of England has had a single inflation target, 2.5 percent, since 1998. Our model suggests that doing so may be a good idea if the implementation of such a regime approximates a target path for the price level.

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end of the period enter utility. In our model, only money balances brought into the period enter the shopping time function (and current period utility). See Carlstrom and Fuerst (2001a) for a detailed analysis of how this timing assumption affects the determinacy conditions. Carlstom and Fuerst (2001b) present results for contemporaneous policy rules.

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Figure 1: Determinacy Conditions for Policy Parameters