Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium

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Abstract

In this paper, we present a dynamic optimizing model that allows explicitly for imperfect substitutability between different financial assets. This is specified in a manner which captures Tobin's (1969) view that an expansion of one asset's supply affects both the yield on that asset and the spread or "risk premium" between returns on that asset and alternative assets. Our estimates of this model on U.S. data confirm that some of the observed deviations of long-term rates from the expectations theory of the term structure can be traced to movements in the relative stocks of financial assets. The richer aggregate demand and asset specifications imply that there exists an additional channel of monetary policy. Our results suggest that central bank operations exercise a modest influence on the relative prices of alternative financial securities, and so exert an extra effect on long-term yields and aggregate demand separate from their effect on the expected path of short-term rates.

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1   Introduction

A central message of James Tobin's "General Equilibrium Approach to Monetary Theory" was that "[t]here is no reason to think that the impact [of monetary policy] will be captured in any single [variable]..., whether it is a monetary stock or a market interest rate" (Tobin 1969, p. 29). This message was a departure from both the simplest quantity-theory setup—where nominal aggregate demand moves in step with the nominal stock of money—and the traditional IS-LM framework, where the aggregate demand for output depends on a single, representative interest rate.

In many respects, the "New Keynesian" or dynamic stochastic general equilibrium models (DSGE models) used today (see Walsh 2003, and Woodford 2003, for extended treatments) represent advances on the models that Tobin criticized. For example, in contrast to the simple quantity theory, the LM relationship in the New Keynesian model implies an interest-elastic and stochastic velocity function. And as Rotemberg and Woodford (1999) and Svensson (2000) emphasize, the presence of forward-looking behavior in the optimizing IS equation means that aggregate demand can be interpreted as depending on a type of long-term real interest rate. In that sense, modern models do admit a distinction between different asset yields. In addition, in

contrast to Tobin's work, these functions are worked out from explicit sto-
chastic optimization problems of agents, while the aggregate supply portion
of the model—typically based on Calvo (1983) staggered price contracts—
improves on the absence of an aggregate supply specification in Tobin (1969).

At a deeper level, however, New Keynesian systems are vulnerable to
Tobin's criticism of earlier-generation models. While a (real) "long-term
rate" appears in the model, it does so only as a stand-in for the expectation of
the path of the current (real) short rate. Deviations of the long-term interest
rate from the expectations theory of the term structure are not recognized.
In effect, the arbitrage relations in the model restore the two-asset structure
of traditional IS-LM, leaving a framework like that Tobin criticized, where
"all nonmonetary assets and debts are... taken to be perfect substitutes at a
common interest rate plus or minus exogenous interest differentials" (Tobin
1982, p. 179).

In this paper, we develop the New Keynesian model to allow explicitly
for imperfect substitutability between different financial assets. As Kashyap
(1999, p. 190) noted, "Tobin has long pushed the view that different secu-
rities should be treated differently," and we represent this view by allowing
for imperfect substitutability between short-term and long-term financial se-

curities. Furthermore, we specify the imperfect substitutability in a manner which allows for Tobin's view that an expansion of one asset's supply affects both the yield on that asset and the spread or "risk premium" between returns on that asset and alternative assets.

Policy debates in recent years have given these issues a prominence that was absent when Tobin's paper was first published. Macroeconomic modeling after 1969 tended not to follow up the implications of imperfect substitution between assets, with notable exceptions including Brunner and Meltzer (1973) and B. Friedman (1976, 1978). This probably reflected the convenience of the perfect-substitute baseline, especially for dynamic general equilibrium analysis; and also the fact that many of the key debates of the past three decades—e.g. the natural rate hypothesis, staggered contracts, and inflation bias—focused on the aggregate supply specification (i.e. price/output interaction, rather than output/interest-rate interaction). For these debates, how private reactions split a policy-induced injection of nominal spending into prices and output was of first-order importance; how monetary policy creates the additional spending is second-order.

Recent discussions of the monetary transmission mechanism have restored the specification of aggregate demand to a first-order issue. The possibility

that short- and long-term securities are imperfect substitutes has become an issue in monetary policy discussions in the U.S. In a speech on November 21, 2002, Federal Open Market Committee member Ben S. Bernanke considered the channels for monetary expansion available to the Federal Reserve beyond lowering the federal funds rate. He observed: "One relatively straightforward extension of current procedures would be to try to stimulate spending by lowering rates further out along the Treasury term structure—that is, rates on government bonds of longer maturities." While noting that one route through which the Fed might achieve this goal is by the term-structure expectations channel, Bernanke suggested that historical experience suggested a second, less conventional channel was available. This was one where money creation exerts additional effects on the long rate, for a given path of the short rate, so that central bank purchases of long-term securities (his proposed operation) reduce long rates relative to the expected path of short rates . In emphasizing this channel, Bernanke was endorsing a central message of Tobin (1969): the influence of central bank actions on aggregate demand cannot be summarized by a single yield, the short-term interest rate, but are reflected in a variety of asset yields.[1] That position, in turn, rests on a model where

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[1] This position is, of course, closely related to the monetarist transmission mechanism advanced in (e.g.) Brunner and Meltzer (1973) and Friedman and Schwartz (1982). B.

different securities are imperfect substitutes for one another.[2]

More formally, in this paper we modify our previous model (Andrés, López-Salido and Nelson 2003, henceforth ALSN) in three main respects. First, we add a long-term bond market. Second, we will have two kinds of agents, consisting of a fraction of unrestricted households who can trade in both short-term and long-term securities markets, with the remaining fraction only able to trade in the market for long-term bonds. Finally, we allow for deviations from the expectations theory. In particular, we depart from the expectations theory in two respects. First, we include an exogenous term-premium shock. Secondly, and more important, we allow for the presence of a portfolio-balance term which creates a role for money (or a money/long-term debt ratio) in the equation linking short and long rates. Together, these modifications to the standard model make long rates matter directly, not only via the relation of these to the expectations of short rates, in both the IS and the LM functions. This produces an additional channel of monetary

Friedman (1976) provides an early discussion of the links between Tobin's and the monetarists' views on monetary transmission.

[2]The sense in which our model validates Bernanke's experiment is discussed further in Section 4. The Federal Reserve and the U.S. Treasury did seek to influence long-term government bond rates for a given path of short rates during the "Operation Twist" program of the Kennedy Administration. This operation, which attempted to alter the relative supplies of government debt, was influenced by Tobin's early work on imperfect substitution (e.g. Tobin, 1961). It differed, however, from the operation described by Bernanke because no expansion of the monetary base was involved in Operation Twist.

policy, as base money expansion now relieves portfolio constraints and lowers long rates in the short run relative to the average of expected future short rates.

Our paper proceeds as follows. Section 2 lays out our modification of the standard New Keynesian model to allow for imperfect substitutability between assets. This changes the LM and term-structure relationships in the model, but still leaves a single interest rate in the IS function. Section 3 introduces a further modification in the form of heterogeneity between agents, and so puts multiple rates in the aggregate IS relation. Section 4 provides empirical estimates of the model and studies its quantitative properties. Section 5 concludes.

2 A Model with Imperfect Asset Substitutability

We assume a continuum of infinitely-lived households, indexed by $i \in [0, 1]$, a government, and a continuum of producing firms indexed by $j \in [0, 1]$. We abstract from capital accumulation. The model will display sufficient symmetry for our analysis to focus on the behavior of both a representative consumer and a goods-producing firm. The model is a generalization of that

we used in ALSN (2003). The generalizations allow for a distinct long-term securities market, as well as sufficient frictions to incorporate long rates in the LM function (and, in Section 3, the IS function). We consider first a *perfect-substitute* baseline version of the model. Then we introduce financial frictions that make short- and long-term bonds *imperfect substitutes*, and we discuss the implications that alternative preference specifications have on both the money demand relationship and the term structure of interest rates. We will show that they imply that now nominal long rates play an explicit role in the LM function.

Except for the presence of alternative degree of sustitutability among assets, our framework consists of a standard dynamic stochastic general equilibrium model with staggered price setting à la Calvo (1983). We now describe the objectives and constraints of these different agents, paying special attention to the specification of household's preferences.

2.1   Households

Households have access to both short- and long-term securities markets, i.e. they can trade in markets for both one-period and L-period securities. We will start with our baseline economy, where we assume that assets are perfect substitutes, then we model imperfect sustitutability among assets, bringing

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out the implications for money demand and the term structure of interest rates.

2.1.1 Baseline Case: Perfect Asset Substitution

Let C_t and N_t represent consumption and hours worked by households in period t.[3] Preferences are defined by the discount factor $\beta \in (0,1)$ and a period utility function. These households seek to maximize

$$\max_{C_t, N_t, M_t, B_t, B_{L,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t \left[U\left(\frac{C_t}{C_{t-1}^h}\right) + V\left(\frac{M_t}{e_t P_t}\right) - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right] - G(.) \right\} \tag{1}$$

where, in what follows, we specialize the period utility to take the form

$$U(.) = \frac{1}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma} , \quad V(.) = \frac{1}{1-\delta}\left(\frac{M_t}{e_t P_t}\right)^{1-\delta}$$

$$G(.) = \frac{d}{2}\left\{ \exp\left[c\left\{ \frac{M_t/P_t}{M_{t-1}/P_{t-1}} - 1 \right\} \right] + \exp\left[-c\left\{ \frac{M_t/P_t}{M_{t-1}/P_{t-1}} - 1 \right\} \right] - 2 \right\} \tag{2}$$

where M_t/P_t represents real balances of the household; a_t is a preference shock, and e_t is a shock to the household's demand for real balances. The parameter $\beta \in (0,1)$ is a discount factor, $\sigma > 0$ governs relative risk aversion, $\varphi \geq 0$ represents the inverse of the Frisch labor supply elasticity, and finally $\delta > 0$, $d > 0$, and $c > 0$. In line with the empirical evidence provided by

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[3]Because there is a continuum of consumption goods available for purchase (see Section 2.2), C_t corresponds to a Dixit-Stiglitz aggregate of consumption.

Ireland (2002), Andrés, López-Salido, and Vallés (2001), and our own work, ALSN (2003), we impose *separability* among consumption, real balances, and hours, and allow for habit formation in consumption. In addition, we incorporate the presence of portfolio adjustment cost through the function $G(.)$. This functional form for portfolio adjustment costs, used by Nelson (2002) and ALSN (2003), is that of Christiano and Gust (1999), modified to refer to real balances and applied to a model without "limited participation" features.[4] Below we will analyze its implications for money demand.

The budget constraint each period is:

$$\frac{M_{t-1} + B_{t-1} + B_{L,t-L} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{\frac{B_t}{r_t} + \frac{B_{L,t}}{(r_{L,t})^L} + M_t}{P_t} \quad (3)$$

Households enter period t with money holdings M_{t-1} and maturing one-period bond holdings B_{t-1}, and maturing L-period bonds, $B_{L,t-L}$, that they purchased in period $t - L$. At the beginning of the period, they receive

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[4]Whether adjustment costs should be expressed in real or nominal terms was a concern of the classic study of Goldfeld (1973). If the transaction motive is the sole reason for holding money, and the costs correspond to literal payments for converting nonmoney assets into cash, then a nominal specification is appropriate. On the other hand, if money provides a service as a safe asset distinct from its transaction role (a "temporary abode of purchasing power" in the words of Friedman and Schwartz (1982), or Modigliani's (1944) "reserve against contingencies"), this service can motivate a cost function specified in real terms, and this is the specification we favor. Throughout this paper, just as we specify the basic services from money using a money-in-the-utility function specification, we specify adjustment costs involving money directly in the utility function. Specifying the costs as appearing in households' intertemporal budget constraints would deliver similar results, at the expense of more cumbersome algebra.

lump-sum nominal transfers T_t, labor income $W_t N_t$, where W_t denotes the nominal wage, and a nominal dividend D_t from the firms. They use some of these funds to purchase new one-period and L-period bonds at nominal cost $\frac{B_t}{r_t}$ and $\frac{B_{L,t}}{\left(r_{L,t}\right)^L}$, where r_t and $r_{L,t}$ denote their gross nominal interest rate between t and $t+1$, respectively. The household carries M_t units of money into period $t+1$.

Long-Term Bonds Long-term securities are modeled, following Sargent (1987, pp. 102–105), as zero-coupon bonds purchased by households at the nominal price (gross nominal interest rate) of $(r_{L,t})^L$, $L > 1$, $r_{L,t} > 1$, and each redeemed for one dollar in period $t + L$. There is no secondary market for these bonds, and so they must be held by their purchaser to maturity. Both short- and long-term securities are solely for loans to the government.[5]

Relative to long-term securities traded in practice, our specification features two obvious simplifications. First, we have no coupon payments received by agents during the period they hold the bond. Second, there is no secondary market for long-term bonds, and so no possibility of obtaining a capital gain or loss by trading in existing securities. Both assumptions are

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[5]As usual, the government bond market specified in the model is really a stand-in for the markets for loans to both government and large corporations that exist in practice.

for simplicity, but have some further justification. The absence of a coupon payment is in line with much of the treatment of long-term bonds in macroeconomic models (e.g. Svensson 2000). The absence of a secondary market can be justified by the fact that a large fraction of the nonbank private sector holds long-term securities with the intention of keeping them to maturity.[6] In addition, several empirical money demand studies have found a role for the nominal long-term interest rate as an opportunity-cost variable. This is consistent with the horizon of money demand decisions being long enough that the reported yield on long-term bonds is the relevant opportunity cost. It is inconsistent with money demand decisions being driven by movements in the secondary prices of long-term assets, since then money demand should be related to the holding-period yield rather than the reported yield on long assets (Mishkin 1983). Finally, we note that if we did allow for our long-term bonds both to be traded and yielding coupon payments, then for an L-period bond there would be an additional $2L$ terms in the intertemporal budget con-

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[6] Kuttner and Lown (1999) report that data are not available that indicate precisely how holding of government debt by maturity is split across agents. However, they note that commercial banks' demand is concentrated on short-term Treasury securities. Pension funds, on the other hand are "significant buyers of long-maturity securities," as Bruskin, Sanders, and Sykes (2000, p. 15) note. Moreover, this demand comes precisely because long-term government bonds are high-quality assets that can be held for a long maturity: when in 2001 the prospect was raised of all U.S. government debt eventually being paid off, this was seen as a dilemma for pension funds and insurance companies, as switching to short-maturity assets would create a mismatch between the maturity of their assets and liabilities (IMF 2001, p. 95).

straint and in the optimality condition for long-term bond holding, clearly an intractable specification for large L.

Optimality Conditions The first-order conditions for the optimizing consumer's problem can be written as:

$$\Lambda_t = a_t U_{t,C_t} + \beta E_t \{ a_{t+1} U_{t+1,C_t} \} \tag{4}$$

$$a_t (N_t)^\varphi = \Lambda_t \left(\frac{W_t}{P_t} \right) \tag{5}$$

$$\left(\frac{\Lambda_t}{P_t} \right) = \beta^L (r_{L,t})^L E_t \left(\frac{\Lambda_{t+L}}{P_{t+L}} \right) \tag{6}$$

$$\left(\frac{\Lambda_t}{P_t} \right) = \beta r_t E_t \left(\frac{\Lambda_{t+1}}{P_{t+1}} \right) \tag{7}$$

$$a_t V_{t,M_t} - \{ G_{t,M_t} + \beta E_t \{ G_{t,M_t} \} \} = \left(\frac{1}{P_t} \right) \Lambda_t - \beta E_t \left(\frac{1}{P_{t+1}} \right) \Lambda_{t+1} \tag{8}$$

where $U_{t,C_t} = \frac{\partial U_t}{\partial C_t}$, $U_{t+1,C_t} = \frac{\partial U_{t+1}}{\partial C_t}$, $V_{t,M_t} = \frac{\partial V_t}{\partial M_t}$, $G_{t,M_t} = \frac{\partial G_t}{\partial M_t}$ and $G_{t+1,M_t} = \frac{\partial G_{t+1}}{\partial M_t}$.

Equation (4) is the standard expression for the marginal utility of wealth (i.e., the Lagrange multiplier for the budget constraint), which, in the presence of habit formation, will depend upon both the marginal utility of consumption today and the expected marginal utility of consumption tomorrow. This relationship is affected by the presence of preference shocks at time t and time $t + 1$. Expression (5) is the labor supply schedule, relating real

wages to the marginal rate of substitution between consumption and hours. Expressions (6) and (7) correspond to the Euler equations for bond holdings at different maturities, and so link the marginal utility of wealth across periods. As we will show below, implicitly in those expressions is a term-structure relationship linking the interest rates on short-term and long-term bonds.

Finally, combining equations (7) and (8), we can obtain an expression for money demand, i.e. a relation linking nominal interest rates to the marginal rate of substitution between money and wealth:

$$a_t V_{t,M_t} - \{G_{t,M_t} + \beta E_t\{G_{t+1,M_t}\}\} = \frac{\Lambda_t}{P_t}\left(\frac{r_t - 1}{r_t}\right) \tag{9}$$

Notice that the presence of the portfolio adjustment costs, $G(.)$, shifts the standard money-demand decision from being static to one where expectations of real income and nominal interest rates matter for today's portfolio decision (see the term in braces in the previous expression).

2.1.2 Imperfect Substitutability

We now modify our baseline model to allow for imperfect substitutability between assets. In the modified version, households face two frictions when participating in these markets. *First*, there are time-varying, stochastic trans-

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action costs in the long bond market, so households pay $(1 + \zeta_t)$ instead of 1 for each dollar of long-term bond purchases in t, where ζ_t is a zero-mean disturbance. *Second*, households perceive entering the long bond market as "riskier," entailing a loss of liquidity, relative to the same investment in short-term securities. As they purchase long-term securities, they hold additional currency to compensate themselves for the loss of liquidity. In effect, these agents have self-imposed "reserve requirements" on their long-term investments. Formally, we specify the second friction as an additional cost function in the households' decision problem regarding their purchases in the long-term bond market. The cost function is specified in terms of relative asset holdings:

$$-\frac{v}{2}\left[\frac{M_t}{B_{L,t}}\kappa - 1\right]^2 \tag{10}$$

where $v > 0$, and κ is the inverse of the steady-state money to debt ratio (which implies that this new function has a zero steady-state value). Given this specification, the equilibrium conditions (8) and (6) now become

$$a_t V_{t,M_t} - \{G_{t,M_t} + \beta E_t\{G_{t+1,M_t}\}\} - \frac{v\kappa}{B_{L,t}}\left[\frac{M_t}{B_{L,t}}\kappa - 1\right]$$
$$= \frac{\Lambda_t}{P_t}(\frac{r_t - 1}{r_t}) \tag{11}$$

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$$-v\kappa\frac{M_t}{(B_{L,t})^2}\left[\frac{M_t}{B_{L,t}}\kappa - 1\right] + \frac{(1+\zeta_t)}{(r_{L,t})^L}\left(\frac{\Lambda_t}{P_t}\right)$$
$$= \beta^L E_t\left(\frac{\Lambda_{t+L}}{P_{t+L}}\right). \tag{12}$$

As can be seen from expression (12), these two frictions aim to capture two deviations from the pure term-structure theory of interest rates. The deviations were both referred to in Tobin's work. The ζ_t series corresponds to Tobin's "exogenous interest differentials." He believed that these shocks were, nevertheless, only part of the wedges that created fluctuations in the relative prices of different assets. In particular, he regarded spreads between interest rates as functions of the relative quantities of assets. This is captured here by the presence of our second friction, the household reserve requirement, with "liquid assets" corresponding to the monetary base, and illiquid assets to long-term securities. Hence, in expression (12), the term structure of interest rates is shifted by the ratio of money to long-term bond holdings. To induce the public to hold an increase in the relative supply of the more illiquid assets, the spread between illiquid and liquid assets is bid up. It would probably be closer to Tobin's position to define liquid assets more broadly, for example to include short-term securities.[7] But base money

[7]Such an definition would be in line with the approach of Canzoneri and Diba (2003). In their treatment, all bonds are short-term, and additional holdings of bonds make agents feel *more* liquid for any given holding of base money. Note that, because of the different

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certainly belongs in the total, and this narrow definition of liquidity is sufficient to capture an essential feature of Tobin's framework, namely the extra channel of monetary policy recently invoked by Bernanke (2002).[8] Finally, notice that this household reserve requirement also makes the relative supply of long-term bonds matter for money demand decisions; in particular, an increase in the relative supply of more illiquid assets increases the demand for money.

2.2 Firm Behavior and Price Setting

The production function for firm j is

$$Y_t(j) = z_t N_t(j)^{1-\alpha} \tag{13}$$

where $Y_t(j)$ is output, $N_t(j)$ represents the number of work-hours hired from the household (i.e. $N_t = \int_0^1 N_t(j) \ dj$), z_t is a common technology shock and $(1-\alpha)$ parameterizes the technology. We define aggregate output as $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \ dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$.

The representative firm sells its output in a monopolistically competitive

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characteristics of short and long-term securities, there is no incompatibility of their approach with our specification, in which additional long-term asset holdings make agents feel less liquid for any given holding of base money.

[8]In Eggertsson and Woodford's (2003) treatment of imperfect substitutability, state-contingent claims markets provide insurance that effectively restores the perfect-substitute baseline. The friction we introduce can be regarded as arising from agents attempting to compensate for the absence or imperfection of these markets in practice.

market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm resets its price with probability $1 - \theta$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1 - \theta$ of producers resets their prices, while a fraction θ simply adjusts prices at the pace of steady-state inflation, π (i.e., non-adjusting firms simply follow the rule: $P_t(j) = P_{t-1}(j)\pi$). Hence, θ^k will be the probability that the price set at time t will still hold at time $t+k$. Notice that, if there were no constraints on the adjustment of prices, the typical firm would set a price according to the rule $P_t(j) = (\frac{\varepsilon}{\varepsilon-1})MC_t(j)$, where $MC_t(j) = \frac{W_t}{\frac{\partial Y_t(j)}{\partial N_t(j)}}$ is the nominal marginal cost and $\frac{\varepsilon}{\varepsilon-1}$ is the steady-state price markup.

A firm resetting its price in period t will seek to maximize

$$\max_{P_t^*} \; E_t \sum_{k=0}^{\infty} \theta^k \; E_t \left\{ \Lambda_{t,t+k} \; Y_{t+k}(j) \; (P_t^* - P_{t+k} \; MC_{t+k}) \right\}$$

subject to the sequence of demand constraints $Y_{t+k}(j) = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$, with P_t^* denoting the price chosen by firms resetting prices at time t. The first-order condition for the above problem is:

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ \Lambda_{t,t+k} \; Y_{t+k}(j) \; \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} \; P_{t+k} \; MC_{t+k} \right) \right\} = 0 \qquad (14)$$

Finally, the equation describing the dynamics for the aggregate price level is

given by:

$$P_t = \left[\theta \ P_{t-1}^{1-\varepsilon} + (1 - \theta) \ (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{15}$$

2.3 Government Budget Constraint

Transfer payments minus seignorage revenues are financed issuing long-term and short-term bonds. Assuming that there is no government spending in the model, the public sector budget constraint is given by

$$\frac{\left(M_t + \frac{B_t}{r_t} + \frac{B_{L,t}}{\left(r_{L,t} \right)^L} \right) - (M_{t-1} + B_{t-1} + B_{L,t-L})}{P_t} = \frac{T_t}{P_t} \tag{16}$$

For simplicity we shall assume that long-term bonds follow a simple AR(1) process,

$$\frac{B_{L,t}}{P_t} = \left(\frac{B_{L,t-1}}{P_{t-1}} \right)^{\varkappa_{b_L}} \exp(\epsilon_{B_L,t}) \tag{17}$$

where $\varkappa_{b_L} \in [0,1)$, and $\epsilon_{B_L,t}$ is an i.i.d. exogenous perturbation. Thus, short-term debt is used as a residual means of public financing. To guarantee dynamic stability and a unique equilibrium in the model, in which prices are determined by monetary policy, we also assume that transfers are set according to the following fiscal rule:

$$\frac{T_t}{P_t} = -\varkappa \frac{B_{t-1}}{P_{t-1}} + \epsilon_t \tag{18}$$

where $\varkappa \in (0,1)$. Finally, the market-clearing condition implies $Y_t = C_t$.

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2.4   Monetary Policy Rule

To close the model, we assume (as in Ireland 2002) that the central bank sets the nominal interest rate following an augmented Taylor-type interest-rate rule. In particular, the nominal rate responds not only to the interest rate in the previous period and to deviations of output and inflation from their steady-state values, but also to nominal money growth:

$$\ln(r_t/r) \;=\; \rho_r \ln(r_{t-1}/r) + (1 - \rho_r)\rho_\pi \ln(\pi_t/\pi) + (1 - \rho_r)\rho_y \ln(y_t/y) \quad (19)$$

$$+ (1 - \rho_r)\rho_\mu \ln(\mu_t/\mu) + \varepsilon_{r_t}$$

where the innovation ε_{r_t} is normally distributed with standard deviation σ_r, and

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t \qquad (20)$$

is the rate of money growth.[9] An interest-rate rule that depends on money growth (or changes in real balances) might be rationalized, as in Rudebusch and Svensson (2002), as a result of an optimal policy exercise when money-growth variability appears in the central bank's loss function. Alternatively, the response to money might be rationalized by money's usefulness in forecasting inflation.

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[9]The symbol $\widehat{}$ represents percentage deviations of a variable from its steady-state value. See below.

2.5 Log-linear Approximation

We now proceed to log-linearize the previous equations around the steady state. The first equation is the one for the aggregate Lagrange multiplier which is obtained by log-linearizing equation (4),

$$\widehat{\Lambda}_t = \phi_1 \widehat{y}_{t-1} + \beta \phi_1 E_t \widehat{y}_{t+1} - \phi_2 \widehat{y}_t + \frac{1 - \beta h \rho_a}{1 - \beta h} \widehat{a}_t \tag{21}$$

where $\phi_1 = \frac{(\sigma - 1)h}{1 - \beta h}$, $\phi_2 = \frac{\sigma + (\sigma - 1)\beta h^2 - \beta h}{1 - \beta h}$.

Log-linearizing equations (12) and (7) we obtain

$$\widehat{\Lambda}_t = L \widehat{rr}_{L,t} + E_t \widehat{\Lambda}_{t+L} - \zeta_t + \tau(\widehat{m}_t - \widehat{b}_{L,t}) \tag{22}$$

$$\widehat{\Lambda}_t = \{\widehat{r}_t - E_t \widehat{\pi}_{t+1}\} + E_t \widehat{\Lambda}_{t+1} = \widehat{rr}_t + E_t \widehat{\Lambda}_{t+1} \tag{23}$$

where

$$\widehat{rr}_t = \widehat{r}_t - E_t \widehat{\pi}_{t+1} \tag{24}$$

$$\widehat{rr}_{L,t} = \widehat{r}_{L,t} - \frac{1}{L} \sum_{j=0}^{L-1} \widehat{\pi}_{t+j+1} \tag{25}$$

are the short-term real interest rate, and the long-term real interest rate, respectively; and where $\widehat{m}_t$ and $\widehat{b}_{L,t}$ are the log-deviation of households real balances and long-term debt holdings, respectively. The parameter τ is defined as $\tau = \frac{v(r_L)^L}{\Lambda b_L}$, with b_L as the steady state level of household

real long-term bond holdings. Notice that, using the steady-state condition $\left(\frac{M}{P}\right)^{-\delta} = m^{-\delta} = \Lambda\left(\frac{r-1}{r}\right)$, we can write $\tau = \frac{v(r_L)^L(r-1)}{r b_L m^{-\delta}}$.

From (23) and (22), we obtain the following expression for the term structure:

$$\widehat{rr}_{L,t} = \frac{1}{L}\sum_{j=0}^{L-1}\{\widehat{rr}_{t+j}\} + \frac{1}{L}\{\zeta_t - \tau(\widehat{m}_t - \widehat{b}_{L,t})\} \tag{26}$$

which implies that there is a deviation from the expectations theory of the term structure. This deviation depends upon an exogenous risk premium term and an endogenous term related to the ratio of money to long-term bonds. Note that in the absence of the exogenous term, ζ_t, and with no costs, $v = 0$, we obtain the standard *term-structure* equation:

$$\widehat{rr}_{L,t} = \frac{1}{L}\sum_{j=0}^{L-1}\widehat{rr}_{t+j} \tag{27}$$

It can also be shown that a log-linear approximation to expression (11) yields

$$
\begin{aligned}
\widehat{m}_t &= \mu_1\widehat{m}_{t-1} + \mu_2 E_t\widehat{m}_{t+1} + \mu_3(\widehat{\Lambda}_t - \widehat{a}_t) + \mu_4\widehat{r}_t + \mu_5\widehat{e}_t \\
&\quad - \left(\frac{vm^\delta}{b_L\delta\left(1 + \delta_o(1+\beta)\right)}\right)(\widehat{m}_t - \widehat{b}_{L,t})
\end{aligned} \tag{28}
$$

where $\delta_o = \frac{dc^2}{\delta m^{(1-\delta)}}$, $\mu_1 = \frac{\delta_o}{1+\delta_o(1+\beta)}$, $\mu_2 = \beta\,\mu_1$, $\mu_3 = \frac{-1}{\delta(1+\delta_o(1+\beta))}$, $\mu_4 = \frac{-1}{\delta(r-1)(1+\delta_o(1+\beta))}$, $\mu_5 = \left(\frac{1-\delta}{\delta}\right)\frac{\delta_o}{1+\delta_o(1+\beta)}$. This is a generalization of the standard expression for money demand. The generalization has two aspects. First, the

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presence of imperfect substitutability implies that real balances depend upon the relative quantity of money and long-term bonds. Second, the presence of portfolio adjustment costs, $G(.)$, implies that real balances also depend on past and expected future real balances (i.e. by setting these costs equal to zero, $d = 0$, the previous expression will collapse into an static money demand equation).

Finally, notice that using (26), we can substitute out the relative asset quantity, and so money demand can be written as a function of the present discounted value of short-term rates as well as long-term rates:

$$\widehat{m}_t = \mu_1 \widehat{m}_{t-1} + \mu_2 E_t \widehat{m}_{t+1} + \mu_3 (\widehat{\Lambda}_t - \widehat{a}_t) + \mu_4 \widehat{r}_t + \mu_5 \widehat{e}_t$$

$$+ \mu_6 \{ L \widehat{rr}_{L,t} - \sum_{j=0}^{L-1} \{ \widehat{rr}_{t+j} \} - \zeta_t \} \tag{29}$$

where $\mu_6 = \left(\frac{r}{r-1} \right) \frac{1}{(r_L)^L \delta(1+\delta_o(1+\beta))}$. This specification is therefore a generalization of the standard money demand relationships examined in the literature, where money demand now incorporates forward-looking elements, and, due to the existence of imperfect substitution between short and long securities, an explicit influence of both short- and long-term interest rates. Note that only the exogenous component of the transaction costs, ζ_t, will appear as part of the money demand function; the remaining terms can be expressed in the form of interest rates.

Completing the model

It can be shown that around a zero steady-state inflation rate, the log-linearized supply-side equations are given by

$$\widehat{\pi}_t = \beta E_t\{\widehat{\pi}_{t+1}\} + \widetilde{\lambda}\widehat{mc}_t \tag{30}$$

$$\widehat{mc}_t = \left(\chi + \phi_2\right)\widehat{y}_t - \phi_1\widehat{y}_{t-1} - \beta\phi_1 E_t\widehat{y}_{t+1} - \frac{\beta h(1-\rho_a)}{(1-\beta h)}\widehat{a}_t - (1+\chi)\,\widehat{z}_t \tag{31}$$

where $\chi = \frac{\varphi+\alpha}{1-\alpha}$, $\widetilde{\lambda} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\xi$, and $\xi = \frac{(1-\alpha)}{1+\alpha(\varepsilon-1)}$ (see, for instance, Galí, Gertler, and López-Salido 2001, and Andrés, López-Salido, and Vallés 2001). Because of the presence of habits formation, marginal costs are not a linear function of output, but instead also depend on past and future output, as well as preference and technology shocks.

We will assume that the shocks follow univariate first-order autorregressive processes, i.e.,

$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \varepsilon_{a_t} \tag{32}$$

$$\widehat{e}_t = \rho_e\widehat{e}_{t-1} + \varepsilon_{e_t} \tag{33}$$

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \varepsilon_{z_t} \tag{34}$$

$$\zeta_t = \rho_\zeta\zeta_{t-1} + \varepsilon_{\zeta_t} \tag{35}$$

3   A Model with Heterogeneous Agents

The preceding framework has introduced sufficient frictions to introduce an endogenous wedge into the relative price of alternative financial assets. Long-term interest rates cannot therefore be treated interchangeably with the expectation of short rates. But this modification is *not* sufficient to move the DSGE framework away from a single-interest-rate model of aggregate demand determination. The reason is that, notwithstanding the existence of long-term securities, there is no compelling reason why all households cannot "bypass" the long-term market altogether, and simply enforce their consumption plans by trading in sequences of short-term investments. In that case, only the expectation of short rates would appear in the IS equation, and deviations from the expectations theory of the term structure would have no implications for aggregate demand behavior.

We therefore introduce an additional modification to the standard model, in the form of two kinds of households. The households are similar except that only a fraction λ of them can trade in the both short- and long-term bond markets, i.e. they can purchase both one-period and L-period securities. We use the term *unrestricted households* to refer to that subset of households. The remaining fraction $1 - \lambda$ of households can only trade in

the *L*-period securities market. We refer to these agents as the *restricted households*. Furthermore, in this modified version of the model, only the *unrestricted households* face the two frictions when participating in long-term bond markets, i.e. only the unrestricted agents regard long-term bonds as imperfect substitutes for money.

Before introducing this modification formally, we discuss the practical implications of the modification and defend its realism. The practical effect of this modification is that the real long-term interest rate unambiguously "matters" in the aggregate IS equation, in a distinct manner from the expectation of short rates. Together with the assumption of imperfect substitutability, this means that central-bank open-market purchases exert two distinct influences on aggregate demand: effects operating via the reaction of current and expected future values of the real short-term interest rate; and effects on the risk premium connecting the real long-term interest rate to expected short rates.

Regarding the realism of this modification, we note that while households in practice do not literally dichotomize into restricted and unrestricted agents, our specification does capture important aspects of the holding of debt by the private sector. The unrestricted agents in our model can be

thought of as standing in for that portion of the private sector that carries out most of its saving decisions through commercial bank deposits; the restricted households, as those agents who subscribe directly to long-term government bonds or who save heavily through such agencies as pension funds. Commercial banks tend to be averse to holding long-term securities, due to their perceived lack of liquidity; nonbank holders of long-term government debt, on the other hand, do not see the same risks in holding long-term assets, mainly because they plan to "cash in" these assets at maturity and do not plan to dispose of them prior to the maturity date. In light of such factors, Congdon (1982, p. 42) notes of the UK situation, "Treasury bills are taken up predominantly be the banking system, and gilts [long-term government bonds] by the nonbank public..." Similarly, for the U.S., Kuttner and Lown (1999, pp. 170–171) report that as of 1997 the share of Federal government debt held by commercial banks and pension funds was roughly equal, and argue that "a major factor behind banks' demand" is the "exceptional liquidity of [short-maturity] Treasury securities..." Our setup reflects key aspects of this situation: a fraction of agents who can deal in both short- and long-term instruments but regard the latter as risky; and another fraction who prefer long-term bonds as their savings vehicle and for whom these risk

considerations are not present.

Formally, the unrestricted households' problem is the one we have solved so far. The restricted households' optimality conditions, on the other hand, are different since the Euler equation on short-term bonds does not apply to them.

3.1   Implications for Aggregate Demand

We first obtain an expression for the aggregate Lagrange multiplier. We proceed as follows. The Lagrange multiplier for the *unrestricted households* satisfies equations (22) and (23). Without loss of generality, we can write these equations in the following way:

$$\widehat{\Lambda}_t^u = L\widehat{rr}_{L,t} + E_t\widehat{\Lambda}_{t+L}^u - \zeta_t + \Phi_t^u \tag{36}$$

$$\widehat{\Lambda}_t^u = \widehat{rr}_t + E_t\widehat{\Lambda}_{t+1}^u \tag{37}$$

where $\Phi_t^u = \tau(\widehat{m}_t^u - \widehat{b}_{L,t}^u)$ is a term that captures the degree of imperfect substitution between money and long-term bonds. In addition, the equilibrium condition for the Lagrange multiplier of the *restricted households* is

$$\widehat{\Lambda}_t^r = L\widehat{rr}_{L,t} + E_t\widehat{\Lambda}_{t+L}^r \tag{38}$$

Define the aggregate Lagrange multiplier as $\widehat{\Lambda}_t = \lambda\widehat{\Lambda}_t^u + (1-\lambda)\widehat{\Lambda}_t^r$. From

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expressions (36) and (37), we have that term-structure behavior is given by

$$\widehat{rr}_{L,t} = \frac{1}{L}\sum_{j=0}^{L-1}\widehat{rr}_{t+j} + \frac{1}{L}\{\zeta_t - \Phi_t^u\} \tag{39}$$

Hence, from the definition of the aggregate Lagrange multiplier and expressions (36) and (38) we obtain an aggregate multiplier given by

$$\widehat{\Lambda}_t = \lambda\{\Phi_t^u - \zeta_t\} + L\widehat{rr}_{L,t} + E_t\widehat{\Lambda}_{t+L} \tag{40}$$

Notice that from expression (39) we have that

$$\{\Phi_t^u - \zeta_t\} = \sum_{j=0}^{L-1}\widehat{rr}_{t+j} - L\widehat{rr}_{L,t}$$

which leads to the following expression for $\widehat{\Lambda}_t$:

$$\widehat{\Lambda}_t = \lambda\left[\sum_{j=0}^{L-1}\widehat{rr}_{t+j}\right] + (1-\lambda)L\widehat{rr}_{L,t} + E_t\widehat{\Lambda}_{t+L} \tag{41}$$

From the previous expression, it is straightforward to obtain the implications for the dynamic IS equation of the model. Combining equations (21) and (41) yields to an expression for the IS equation, which is a function of both short-term and long-term real rates:

$$\phi_1\widehat{y}_{t-1} + \beta\phi_1 E_t\widehat{y}_{t+1} - \phi_2\widehat{y}_t + \frac{1-\beta h\rho_a}{1-\beta h}\widehat{a}_t$$

$$= \lambda\left[\sum_{j=0}^{L-1}\widehat{rr}_{t+j}\right] + (1-\lambda)\ L\widehat{rr}_{L,t} +$$

$$\phi_1\widehat{y}_{t+L-1} + \beta\phi_1 E_t\widehat{y}_{t+L+1} - \phi_2\widehat{y}_{t+L} + \left(\frac{1-\beta h\rho_a}{1-\beta h}\right)\rho_a^L\ \widehat{a}_t$$

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This expression can be written in a compact way as follows:

$$\phi_2(1 - F^L)\widehat{y}_t = \phi_1(1 - F^L)\widehat{y}_{t-1} + \beta\phi_1(1 - F^L)E_t\widehat{y}_{t+1}$$

$$-\lambda\left[\sum_{j=0}^{j=L-1}\widehat{rr}_{t+j}\right] - (1 - \lambda)L\widehat{rr}_{L,t} + \frac{1 - \beta h\rho_a}{1 - \beta h}(1 - \rho_a^L)\widehat{a}_t \qquad (42)$$

where F is the forward operator. As in a standard IS equation, aggregate demand is written as a function of real rates and an IS shock; and as in the standard Euler equation, spending decisions are forward-looking. With homogeneous agents and perfect substitution between assets, it would be possible to collapse this expression into a second-order expectational difference equation involving output and the short real rate. With imperfect asset substitution and heterogeneity, the conditions for these simplifications are not satisfied; and a more general relationship, linking output to two distinct interest rates and L-period-ahead expected output, prevails.

We now derive an expression for aggregate money demand (an LM function). We first note that the unrestricted agents' money demand equation may be written as:

$$\widehat{m}_t^u = \mu_1\widehat{m}_{t-1}^u + \mu_2 E_t\widehat{m}_{t+1}^u + \frac{\mu_3}{1 - \beta}\widehat{\Lambda}_t^u - \mu_3\widehat{a}_t - \frac{\beta\mu_3}{1 - \beta}E_t\widehat{\Lambda}_{t+1}^u$$

$$+\frac{\beta\mu_3}{1 - \beta}\widehat{\pi}_{t+1} + \mu_5\widehat{e}_t + \mu_6\{L\widehat{rr}_{L,t} - \sum_{j=0}^{L-1}\{\widehat{rr}_{t+j}\} - \zeta_t\} \qquad (43)$$

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while restricted agents' money demand is the simpler expression:

$$\widehat{m}_t^r = \mu_1 \widehat{m}_{t-1}^r + \mu_2 E_t \widehat{m}_{t+1}^r + \frac{\mu_3}{1-\beta} \widehat{\Lambda}_t^r - \mu_3 \widehat{a}_t - \frac{\beta \mu_3}{1-\beta} E_t \widehat{\Lambda}_{t+1}^r$$

$$+ \frac{\beta \mu_3}{1-\beta} E_t \widehat{\pi}_{t+1} + \mu_5 \widehat{e}_t \tag{44}$$

The aggregate money demand equation is therefore:

$$\widehat{m}_t = \mu_1 \widehat{m}_{t-1} + \mu_2 E_t \widehat{m}_{t+1} + \frac{\mu_3}{1-\beta} \widehat{\Lambda}_t - \mu_3 \widehat{a}_t - \frac{\beta \mu_3}{1-\beta} E_t \widehat{\Lambda}_{t+1} + \frac{\beta \mu_3}{1-\beta} E_t \widehat{\pi}_{t+1}$$

$$+ \mu_5 \widehat{e}_t + \lambda \mu_6 \{ L \widehat{rr}_{L,t} - \sum_{j=0}^{L-1} \{\widehat{rr}_{t+j}\} - \zeta_t \}. \tag{45}$$

With no adjustment costs, perfect asset substitution, and agent homogeneity, this condition would collapse into a static money demand relationship linking real balances to current output and the current nominal interest rate. Relative to this baseline, there are two extensions: (i) portfolio adjustment costs mean that lagged and expected future real balances appear; and (ii) imperfect asset substitution puts long rates in money demand.

Finally, we note that once agent heterogeneity is allowed for, the portfolio term that creates deviations from the expectations theory of the term structure is the unrestricted agents' real money/real long term-debt ratio, rather than the aggregate ratio. In order to have a model suitable for estimation with aggregate data, our estimated system of equations uses aggregate real money rather than unrestricted agents' money in this condition. We have

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verified by simulation that, if imperfect substitution is present, one should expect to find a significant positive estimated value of τ even when proxying unrestricted agents' money holdings in this condition by aggregate real money.

3.2   An Alternative Specification of Heterogeneity

In this section we show the implications of an alternative specification of the restricted agents' decision problem.   In this modified version, we could treat the two types of agents more symmetrically by making money and long-term bonds imperfect substitutes for both.   What are the implications for aggregate demand of a version of the model in which both the *unrestricted households and the restricted households* face the two frictions when participating in the long-term bonds market?

Formally, this will imply that the endogenous term $\Phi_t = \tau(\widehat{m}_t - \widehat{b}_{L,t})$, is common to all households; equations (36) and (38) therefore become identical, and so the aggregate expression for the Lagrange multiplier is as follows:

$$\widehat{\Lambda}_t = L\widehat{rr}_{L,t} + E_t\widehat{\Lambda}_{t+L} - \zeta_t + \Phi_t$$

In addition, using expression (39) we can substitute out the stochastic term $-\zeta_t + \Phi_t$ from the previous expression. Hence, the aggregate Lagrange mul-

tiplier evolves according to:

$$\widehat{\Lambda}_t = \left[\sum_{j=0}^{L-1} \widehat{rr}_{t+j} \right] + E_t \widehat{\Lambda}_{t+L}$$

Therefore, aggregate demand can be written either as depending on the long-term rate and the term $-\zeta_t + \Phi_t$, or as depending on the short-rate sum $\left[\sum_{j=0}^{L-1} \widehat{rr}_{t+j} \right]$. The latter representation shows that *if* attitudes to risk are perfectly symmetric across households, a single-interest-rate IS equation is restored. Our result that the relative price of financial assets is a function of money and debt stocks remains, but there are nevertheless no implications for the transmission mechanism of monetary policy. This result demonstrates the three key modifications of the DSGE framework we have introduced that deliver a multiple-channels model of monetary policy: (i) agent heterogeneity in the manner we have specified; (ii) imperfect substitution on the part of the unrestricted agents; and (iii) a lower degree of imperfect substitutability for the restricted agents, i.e. those agents who have access to long-term markets only, do not regard these assets with as much risk as do those who trade in both markets.[10] All *three* conditions are required; if either (i) or

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[10]The formal requirement for condition (iii) is that the parameter v is lower for the restricted agents. In line with this requirement, in our specification of imperfect substitutability other than in this section, we have imposed $v = 0$ for the restricted agents. A value of v that was positive for the both agent types but lower for the restricted agents, would also deliver multiple channels of monetary policy. The reason that this is not our

(iii) is violated, the single-interest-rate mechanism case $\lambda = 1$ obtains; if (ii) is violated, both short and long rates matter for aggregate demand, and the wedge between short and long rates is time-varying, but monetary policy cannot affect this wedge.

3.3   Discussion

Note that our model embodies three special cases:

Baseline two-asset model $(\lambda = 1, \Phi_t^u = 0)$.. This is the perfect-substitute version of the model. All households are unrestricted. Monetary policy operates on long rates only via the expectations channel, and long rates only matter for aggregate demand via their relation to short rates.[11]

Exogenous interest differentials model $(\lambda < 1, \Phi_t^u = 0)$ This is the imperfect-substitute model without the second friction. Deviations from the expectations theory of short rates matter for real long rates and so for aggregate demand, but the deviations are not related to other macroeconomic

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baseline specification of the restricted agents' problem is that money and debt stocks then appear directly in the IS equation, which seems contrary to Tobin's (and others') view of the effect of imperfect substitutability. Tobin (1974, p. 89) maintained that it did not imply "strong, direct effects of the *quantity* of money... which bypass credit and securities markets..." Rather, as in our preferred specification, imperfect substitutability changes the asset-pricing relationships and not the IS equation. In addition, econometric evidence is unfavorable for money terms appearing directly in the optimizing IS equation (Ireland 2002; ALSN 2003).

[11] As noted in Section 3.2, the case $(\lambda = 1, \Phi_t^u > 0)$ also implies a single interest-rate channel, and occurs if both types of agents face identical costs in switching between money and long-term bonds.

aggregates. Monetary policy continues to operate only via its effect on the expected path of short rates.

Multiple-channels model ($\lambda < 1$, $\Phi_t^u \neq 0$). Base money expansion now matters for the deviations of long rates from the expected path of short rates. Monetary policy operates by both the expectations channel (the path of current and expected future short rates) and this additional channel. As in Tobin's framework, interest-rate spreads (specifically, the deviations from the pure expectations theory of the term structure) are an endogenous function of the relative quantities of assets supplied.

4 Maximum Likelihood Estimates

The maximum likelihood estimation follows Hansen and Sargent (1997) and recent applications can be found in Kim (2000) and Ireland (2001, 2002, 2003). The procedure involves expressing the stationary solution of the model in state-space form and estimating the model's parameters using a recursive Kalman filter algorithm (see Ireland 2002 for details).[12]

We use U.S. quarterly data for 1980:1–1999:2. The series used in the estimation are: real GDP; the quarterly average of Anderson and Rasche's (2000)

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[12]A detailed description of the solution and estimation methods is available upon request.

domestic monetary base series; quarterly average population; the quarterly average of the seasonally adjusted CPI; and the quarterly average of the nominal Federal funds rate. Finally, we choose $L = 12$ to represent the maturity of long-term interest rates, and accordingly use the 3-Year Treasury Constant Maturity Rate three-year nominal long rate, obtained as the quarterly average of the 3-Year Treasury Constant Maturity Rate. The sources for these data are the appendix to Ireland (2002) and the Federal Reserve Bank of St. Louis's FRED database.[13] Figure 1 displays the data we use in estimation.[14]

The log-linearized optimizing model that we estimate refers to deviations of variables from their steady-state values (or steady-state growth paths in the case of output and real money), rather than the levels of variables. Following Ireland (2002), we have detrended output and real balances separately prior to estimation. Inflation and nominal interest rates also exhibit a (downward) trend over our sample; nevertheless, we continue to use the (demeaned) levels of these variables in estimation, on the grounds that the

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[13] http://research.stlouisfed.org/fred2/data/GS3.txt

[14] We do not include public debt in the estimated model; therefore, the dynamics of the risk premium (ζ_t) should be interpreted as incorporating both the exogenous term-premium disturbance and the dynamics of long-term bonds. This simplification reduces the number of exogenous sources of fluctuations in the model, but it does not affect the identification of the structural parameters.

trends may be reduced or eliminated when these variables are cast as linear combinations (e.g. as a real interest rate). The model consists of equations (19), (20), (24), (25), (30), (31), (32)-(35), (39), (41) and (45), and we are interested in estimating the parameters σ, h, β, δ, δ_0, λ, τ, χ, $\widetilde{\lambda}$, ρ_π, ρ_y, ρ_μ, ρ_r, ρ_e, ρ_a, ρ_z, ρ_ζ, σ_a, σ_e, σ_z, σ_r, σ_ζ.

In Table 1, we present the parameter estimates for the model incorporating all theoretical restrictions described above. Initial attempts to estimate all the parameters led to implausible combinations of the preference parameters σ and h and to values of ρ_r that were high compared to other studies. We therefore have set these parameters' values equal to 2, 0.9 and 0.75 respectively (in line with the evidence provided by ALSN, 2003); constraining σ also follows Ireland (2002). The main result in Table 1 is that we find supporting evidence for our heterogeneous-agent imperfect-substitution framework: in particular the value of $\tau = 0.54$ is clearly significant, and the fraction of unrestricted agents (λ) is estimated to be around 0.29. This fraction is in keeping with the notion that a subset of the private sector deals predominantly with both long-term and short-term investments; according to our estimate, these agents form about 29 percent of households.

We find substantial deviations from the expectations theory of the term

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structure. These take the form both of exogenous, persistent deviations with an AR(1) parameter of 0.8, and of endogenous variations related to asset stocks. The latter, implied by our positive estimate of τ, confirms the presence of the imperfect substitutability that Tobin (1969) emphasized, and the existence of the channel of monetary transmission invoked in recent U.S. policy debates. This channel is of a reasonable magnitude—the partial impact of a 1 percent increase in real monetary base on the annualized nominal long-term interest rate, given expected short rates, is $4\tau/L = 0.18$, i.e. about 18 basis points—and serves to supplement the traditional expectations channel. This is broadly consistent with Evans and Marshall (1998, pp. 73–74), who find that there is some support for the position that monetary policy shocks affect long rates both by the expectations channel and by effects on term premia.

From the estimated parameters ϕ_2, μ_3, and μ_4 we can draw an estimate of the long-run income elasticity of money demand close to 1 that is consistent with many previous U.S. studies.[15] Our estimates also confirm that money demand has a non-negligible forward-looking element, as the significant value

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[15] The assumption of separability between consumption and real balances implies that the income and interest elasticity of money demand, are very tightly related (see e.g. Ireland 2002). Thus, the implied interest rate elasticity is around 2.

of δ_0 obtained indicates. This effect can be rationalized in terms of the importance of adjustment costs for holding money balances, in line with the previous results in ALSN (2003).

Another interesting result is the one related to the slope of the aggregate supply equation. We find a value of $\widetilde{\lambda}$ close to 0.015, which is in line with the values reported by Galí, Gertler, and López-Salido (2001). In particular, under a standard value of 2/3 for the elasticity of output with respect to hours, $1 - \alpha$, and assuming a 10 percent steady state markup (see Basu and Fernald 1997), our estimate of the slope of the Phillips curve implies a value for the degree of price stickiness $\theta = 0.74$, that is, firms change prices every four quarters, a value generally used to calibrate New Keynesian Phillips curve (see, for instance, Woodford 2003). Finally, from the estimated value of the parameter χ we can infer an elasticity of labor supply around 1.6, again close to the benchmark value used in the business cycle literature (e.g. Hall 1997).

The estimated interest-rate rule also displays many similarities with rules already estimated in the literature. There is a modest, but significant, interest-rate response to output ($\rho_y = 0.35$). The response of the nominal rate to the inflation rate is well above 1.0 (i.e., $\rho_\pi \approx 2$); and. finally, we also

find that money growth is significant in the interest-rate rule ($\rho_\mu \approx 1.3$).[16]

Overall, our estimates support the multiple-transmission model of monetary policy over both the perfect-substitute and exogenous-interest-differential alternatives. It does so in a model that shares many quantitative features with calibrated as well as with estimated general equilibrium monetary models of the U.S. economy. We now illustrate some dynamic implications of the estimated model.

4.1   Comparative Dynamics

To illustrate the richer transmission of monetary policy in our imperfect-substitution framework, we characterize the adjustment of the economy to exogenous monetary policy shocks. We consider first a shock to the interest rate. Figure 2 displays the responses of the main variables to a monetary policy contraction under the estimated rule. In each panel we plot the dynamic adjustment to the variables in the estimated imperfect-substitution model (circled line) and in the standard two-asset model (continuous line). Notice that the estimated model has sufficient price stickiness to produce a liquidity effect, i.e., both nominal and real short-term interest rates move in

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[16]This term may be approximating either genuine money targeting by the central bank, or a way of targeting future inflation, by responding to information beyond that contained in current π_t.

opposite directions to money growth (and real balances). In addition, the change in the relative supply of assets generates a substantial increase in both nominal and real long- term interest rates that generates a more pronounced output and a slightly higher inflation reduction. In particular, the maximum output decline is around 0.5 percentage points at around the third quarter after the shock, and inflation initially falls by slightly less than 0.2 percent, then progressively is restored to its steady-state value. It is interesting to note that inflation response is quite persistent due to the combination of a highly protracted output decline (because of habit formation), and a low labor supply elasticity.

In Figure 3, we show the impulse responses when the monetary policy rule is changed to an exogenous univariate money growth process. In particular, we focus on the adjustment of the economy to an exogenous, permanent 1 percent increase in the nominal money stock. This experiment corresponds closely to the "injection of reserves" experiment in Rudebusch and Svensson (1999, p. 238). Notice that this experiment corresponds to an injection of money in exchange for lump-sum transfers, while the proposed operation described in Bernanke (2002) is of a switch of money for a portion of the private sector's holdings of long-term government bonds. Nevertheless, the

experiment depicted here shows that our model captures Bernanke's proposed transmission channel in a manner that the standard model does not. Our model generalizes the specification of private sector behavior in a way that makes the money/long bond stock ratio matter for long rates. Any operation that adds to this ratio in the short run exploits this extra channel. With prices sticky, such operations include adding to the nominal money stock for a given debt stock (our experiment), a simultaneous addition to the nominal money stock and a reduction in the debt stock (Bernanke's operation), or reducing the nominal long-term debt stock for given money. In the standard model, none of these operations will be effective in providing stimulus except via their effect on the path of the short-term interest rate; in our model, all three provide stimulus via the imperfect-substitution channel.

5 Concluding Remarks

In this paper we have generalized the standard sticky-price dynamic general equilibrium model to incorporate a richer aggregate demand specification. Following Tobin (1969), we have introduced imperfect substitution between different types of securities. Together with other model features, this has the effect of putting long-term interest rates explicitly into the aggregate IS and

LM functions. Our estimates of this model on U.S. data confirm that some of the observed deviations of long-term rates from the expectations theory of the term structure can be traced to movements in the relative stocks of financial assets, just as claimed by Tobin (1969, 1982). The richer aggregate demand and asset specifications imply that there exists an additional channel of monetary policy. In the standard perfect-substitute baseline, monetary policy can operate on long-term interest rates only via affecting the expected path of short rates. But our estimates suggest that central bank operations also affect the relative price of alternative financial securities, and so exert an extra effect on long-term yields and aggregate demand. We have therefore provided an optimizing general equilibrium framework that supports the existence of "unconventional" or "quantitative" channels of monetary policy, of the type raised in policy discussions by Bernanke (2002).

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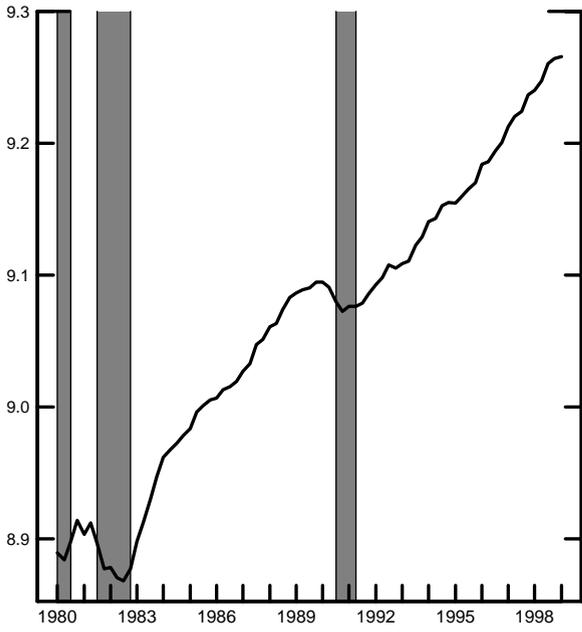
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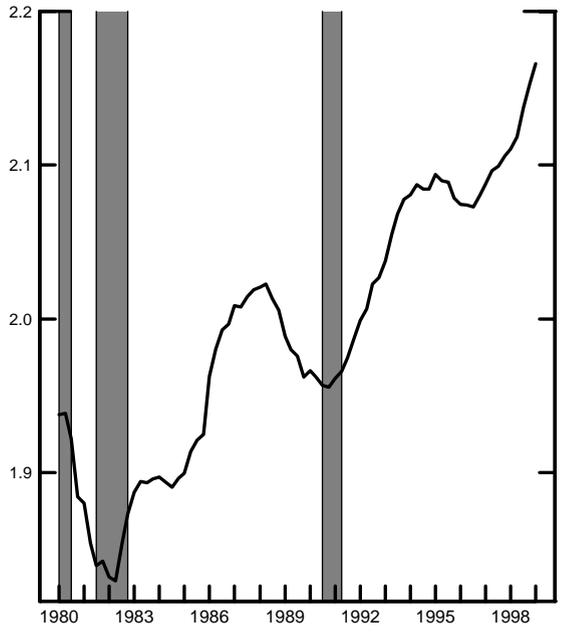
Table 1. Maximum Likelihood Estimates

Parameter	Estimated Value
β	0.991
δ	4.36 (0.12)
σ	2
h	0.9
δ_0	1.82 (0.29)
λ	0.29 (0.14)
τ	0.54 (0.08)
χ	1.36 (0.43)
$\widetilde{\lambda}$	0.014 (0.006)
ρ_r	0.75
ρ_y	0.36 (0.09)
ρ_π	1.97 (0.39)
ρ_μ	1.38 (0.27)
ρ_a	0.89 (0.007)
ρ_e	0.99 (0.01)
ρ_z	0.97 (0.03)
ρ_ζ	0.80 (0.19)
σ_a	0.039 (0.021)
σ_e	0.054 (0.007)
σ_z	0.011 (0.003)
σ_r	0.009 (0.007)
σ_ζ	0.004 (0.007)
Log-Likelihood	1740.7

Figure 1

Log output

Log real balances

Inflation

Short and long rates

Short rate
Long rate

Figure 2
Impulse Responses to a Monetary Policy Shock
Estimated Interest Rates Rule

Note: Each panel shows percentage deviations of the variable from its steady state value. Circle line estimated model, and continuous line perfect asset substitution model.
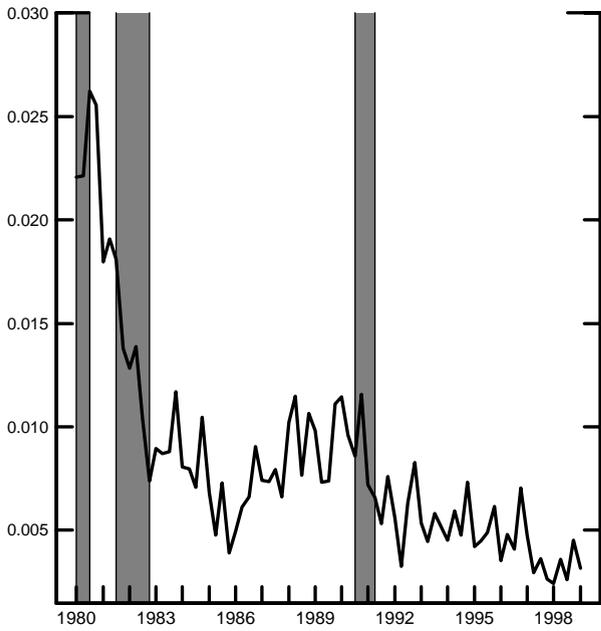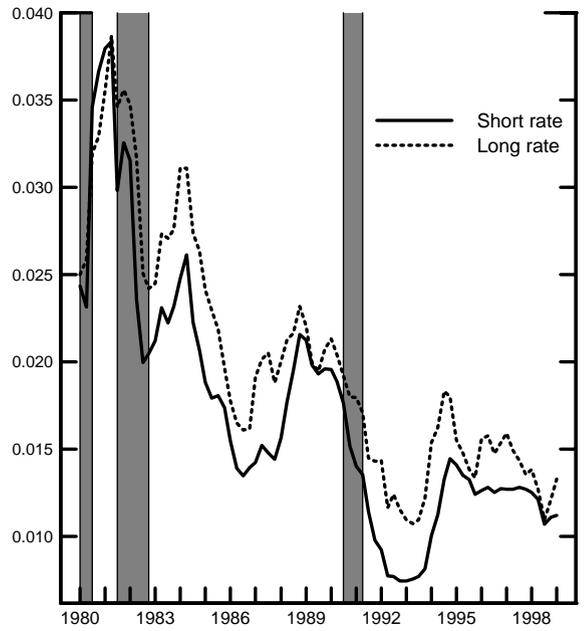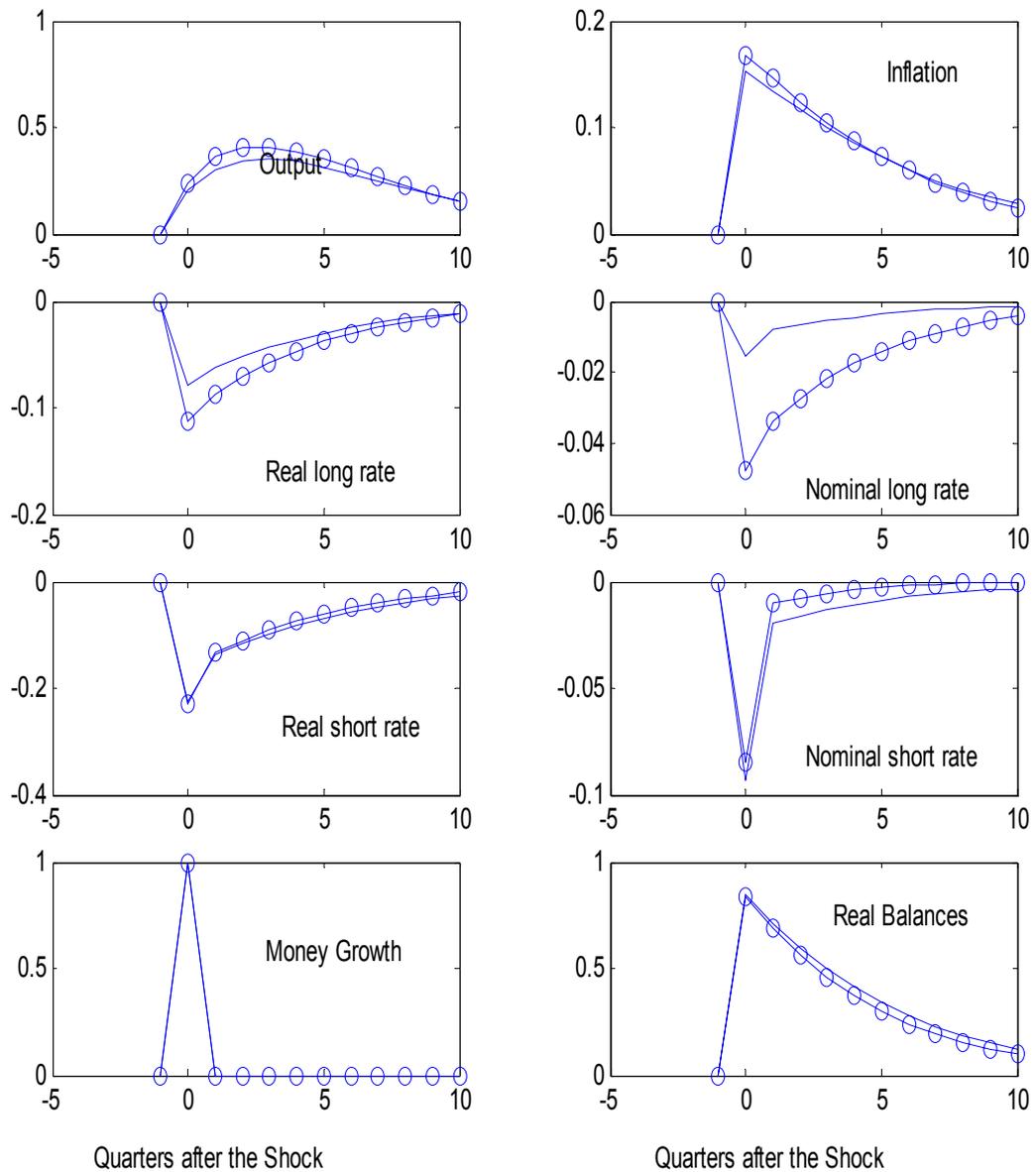
Figure 3
Impulse Responses to a Money Growth Shock

Note: Each panel shows percentage deviations of the variable from its steady state value. Circle line estimated model parameters and a money growth rule, and continuous line perfect asset substitution model.