Nonlinear Effects of School Quality on House Prices

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Nonlinear Hedonics and the Search for School Quality*

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Abstract

We reexamine the relationship between school quality and house prices and find it to be nonlinear. Unlike most studies in the literature, we find that the price premium parents must pay to buy a house associated with a better school increases as school quality increases. This is true even after controlling for neighborhood characteristics, such as the racial composition of neighborhoods, which is also capitalized into house prices. In contrast with previous studies that use the boundary discontinuity approach, we find that the price premium from school quality remains substantially large, particulary for neighborhoods associated with high-quality schools. [JEL: C21, I20, R21]

Keywords: education capitalization, house prices, hedonics, boundary discontinuity.

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1 Introduction

Since the pioneering work of Tiebout (1956), economists have recognized that the quality of public services, especially schools, influences house prices. The relationship between house prices and public school quality has been widely studied in the literature, dating back to Oates' (1969) seminal paper. In the analysis of school quality, researchers have often applied the hedonic pricing model developed by Rosen (1974).[1] In that model, the implicit price of a house is a function of its comparable characteristics, as well as measures of school quality and a set of neighborhood characteristics. Examples of this approach include Haurin and Brasington (1996), Bogart and Cromwell (1997), Hayes and Taylor (1996), Weimer and Wolkoff (2001), and Cheshire and Sheppard (2002). The estimated coefficients from the regression represent the *capitalization* of these components into house values.[2]

A prevalent concern of capitalization studies is the possibility of omitted variable bias, induced by failing to account for the correlation between school quality and unobserved neighborhood characteristics, as better schools tend to be located in better neighborhoods. Black (1999) circumvented this problem by restricting the sample to examine only houses near the boundaries between school attendance zones and controlling for neighborhood characteristics with boundary fixed effects in several large Boston suburbs.[3] She rationalized that, whereas school quality measures such as standardized test scores make a discrete jump at the boundary, neighborhood characteristics change more smoothly. Some authors are also concerned about the potential endogeneity of school quality when it is measured by indicators of student performance.[4]

The hedonic approach, including Black's (1999) variation, often specifies a linear relationship between the measure of school quality and the natural logarithm of house prices.[5] The linear specification, however, presupposes that the marginal valuation of low-quality schools is equal to the valuation of high-quality schools and results in a constant premium on school quality.

In this paper, we argue that the relationship between school quality and house prices in the boundary discontinuity framework is better characterized as a nonlinear relationship. We test for nonlinear effects from

[1] Sheppard (1999) provides a survey of the literature of hedonic models of housing markets.

[2] Ross and Yinger (1999) and Gibbons and Machin (2008) provide surveys of the capitalization literature.

[3] A rudimentary precursor of this idea was analyzed by Gill (1983), who studied a sample of houses in Columbus, Ohio, restricting observations to neighborhoods with similar characteristics. More to the point, Cushing (1984) analyzed house price differentials between adjacent blocks at the border of two jurisdictions in the Detroit, Michigan, metropolitan area.

[4] Gibbons and Machin (2003), for example, argue that better school performance in neighborhoods with high house prices may reflect that wealthy parents buy bigger houses with more amenities and therefore devote more resources to their children.

[5] Nonlinear effects are, however, routinely allowed among some house characteristics, such as the number of bathrooms and the age of the building.

school quality measured with standardized test scores for elementary schools in the St. Louis, Missouri, metropolitan area.[6]

To the best of our knowledge, nonlinear hedonics from school quality have been explored only by Cheshire and Sheppard (2004) in a study of primary and secondary schools in the United Kingdom. The authors estimate a full-sample, standard hedonic regression modified to include Box-Cox transformations of house prices, house characteristics, and measures of school quality. Their evidence suggests that the price-quality relationship is highly nonlinear.

We argue that the nonlinearity with respect to school quality illustrates two aspects of the market for public education that are reflected in the housing market.[7] First, in an environment in which potential buyers are heterogeneous in the intensity of their preferences for school quality and neighborhood characteristics, buyers with a stronger preference for education quality may concentrate their buying search for a house in the highest-quality attendance zones.[8] As school quality increases, competition from other buyers would create an increasingly tight housing market, because the housing supply in these areas is often very inelastic, as most metropolitan areas have a fixed housing stock in the short run.[9] Second, alternative schooling arrangements (e.g., private schools, home school, magnet schools) can provide home buyers with high-quality education even if they choose to live in lower-quality public school attendance zones, allowing for a reduced price premium in these neighborhoods. The existence of these options underlies our belief that a constant premium across the range of school quality is not realistic.

The above arguments rely on the heterogeneity of preferences for school quality and neighborhood characteristics among the population of prospective home buyers, a feature that has been widely documented in the literature. Bayer, Ferreira, and McMillan (2007), for example, argue that there is a considerable degree of heterogeneity in homeowners' preferences for schools and racial composition of neighborhoods.

[6]A previous study by Ridker and Henning (1967) found no evidence of education capitalization in St. Louis house prices. Although their main concern was to determine the negative effect of air pollution on housing prices, they included a dummy variable that indicated residents' attitudes about the quality of the schools (above average, average, and below average). Ridker and Henning acknowledge that their study may suffer from small-sample bias that could explain this seemingly contradictory finding. Kain and Quigley (1970) is an early study analyzing the components of a hedonic price index for housing in the St. Louis metropolitan area, but it does not consider measures of school quality.

[7]This argument can be motivated with a search model of the housing market in the spirit of Wheaton (1990) and Williams (1995).

[8]A school's attendance zone delimits the geographic area around the school the residents' children would attend. A *school district* is an administrative unit in the public-school system often comprising several schools.

[9]This argument is similar to that proposed by Hilber and Mayer (2009). They argue that scarcity of land confounds identification of the education premium. Brasington (2002) and Hilber and Mayer (2009) have also noted that the extent of capitalization in a hedonic framework may vary depending on whether houses are located near the interior or the edge of an urban area. They find that capitalization is weaker toward the edge, where housing supply elasticities and developer activity are greater.

An alternative hypothesis that can generate nonlinearities is that school quality can be considered a luxury good, and therefore, at higher-quality schools (and therefore richer neighborhoods), people would be willing to pay more for the same marginal increase in school quality.[10]

Studies in the literature often differ in terms of the level of geographic detail at which school quality is measured: at the school-*district* level or at the *school* level. Many papers that do not use the boundary discontinuity approach measure education quality at the school-district level, as opposed to considering schools individually. These studies also face the challenge of devising appropriate definitions of *neighborhoods* to match the geographic level at which school quality is measured. In our paper, we measure school quality at the individual school level. We also use Census blocks (as opposed to the larger block groups or tracts) to measure neighborhood demographics.

The boundary discontinuity approach has received some criticism in recent works, motivated primarily by the concern about removal of any remaining omitted spatial fixed effects (Cheshire and Sheppard, 2004), or the possibility of discontinuous changes in neighborhood characteristics, which also depends on the definition of *neighborhood* that is adopted (Kane, Riegg, and Staiger, 2003; and Bayer, Ferreira, and McMillan, 2007). However, barring the availability of repeat sales data or information on boundary redistricting or policy changes to supply the exogenous variation required for identification, in the case of stable boundary definitions and cross-sectional data, the boundary discontinuity approach remains a useful methodology.

In this paper, we use Black's (1999) method of considering only houses located near attendance zone boundaries and regressing house prices on physical characteristics of houses and a full set of pairwise boundary dummies to control for unobserved neighborhood characteristics. Additionally, in response to the criticisms of the boundary discontinuity approach, we augment the estimation by controlling for a set of demographic characteristics defined at the Census-block level.

We find, as did Black (1999), that controlling for unobserved neighborhood characteristics with boundary fixed effects reduces the premium estimates from test scores relative to the hedonic regression with the full sample of observations. We also find, however, that the linear specification for test scores underestimates the premium at high levels of school quality and overestimates the premium at low levels of school quality. In contrast to Black (1999) and many subsequent studies in the literature, we find that the effects of school quality on housing prices remain substantially large even after controlling for neighborhood demographics and boundary fixed effects. We also find that the racial composition of neighborhoods has a statistically

[10]We thank an anonymous referee for this suggestion.

significant effect on house prices.

This paper is organized as follows. Section 2 describes the econometric methodology used to estimate the model. Section 3 describes our data. Section 4 presents the empirical results, and Section 5 concludes.

2 Econometric Methodology

Our intention is to estimate the value that higher school performance adds to house prices. Specifically, we are interested in estimating the dollar value difference in home prices for a quantified increase in school quality. We discuss three alternative specifications that include two different identification techniques to disentangle neighborhood quality from school quality.

2.1 Pure hedonic pricing model

As a benchmark, we introduce a hedonic pricing equation in which the sale price is described as a function of the characteristics of the house and its location-specific attributes, including the quality of the school associated with it. The basic hedonic function can be described as follows:

$$\ln(p_{iaj}) = \kappa + \mathbf{X_i}'\beta + \mathbf{Z_j}'\delta + \mu_a \psi^H + \epsilon_{iaj}, \tag{1}$$

where p_{iaj} is the price of house i in attendance zone a in neighborhood j. The vector $\mathbf{X_i}$ represents the comparable aspects of house i (i.e., the number of bedrooms, bathrooms, and so on) and vector $\mathbf{Z_j}$ represents local characteristics. The value μ_a is the quality of the school in attendance zone a. In this paper, we measure school quality with an index constructed from test scores, defined at the school level and expressed in standard deviations from the mean.[11] The quantity of interest ψ^H is the education capitalization premium and represents the percentage increment in house prices from increasing school test scores by 1 standard deviation.

[11]Various studies in the hedonic analysis tradition have used input-based measures of education quality, such as per-pupil spending. Hanushek (1986, 1997) found that school inputs have no apparent impact on student achievement and are therefore inappropriate as measures of school quality. The research on education production functions also makes the case that *value-added* measures of achievement–the marginal improvement in a particular cohort's performance over a period of time–would be more appropriate as measures of quality in capitalization studies. However, constructing value-added measures requires tracking groups of students over time and implies more sophistication in the decision making process of potential buyers, as value-added measures are not readily available to the public. Brasington (1999), Downes and Zabel (2002), and Brasington and Haurin (2006) found little support for using value-added school quality measures in the capitalization model; they argue that home buyers favor, in contrast, more traditional measures of school quality in their housing valuations.

Thus, the house price reflects all relevant attributes; that is, physical and location-specific characteristics of the home are capitalized into the house value even if they are not directly consumable by the current tenants (because of their effects on the resale value of the house).[12] One potential problem with this specification is that the comparable house characteristics, $\mathbf{X_i}$, do not fully capture the quality of the house (updates, condition, landscaping, layout, and so on), the quality of the surrounding neighborhood, and various other factors. The hedonic pricing function attempts to capture these factors with the inclusion of the $\mathbf{Z_j}$ vector. The success with which the model captures these unobserved factors often depends on how coarsely the geographic area encompassed by $\mathbf{Z_j}$ is defined (i.e., for how small a vicinity around the house $\mathbf{Z_j}$ provides variation).

2.2 Linear boundary fixed effects model

The methodology of adding the location characteristics vector, $\mathbf{Z_j}$, may reduce but not entirely account for all of the variation that can be introduced on a neighborhood level. Suppose that the neighborhood characteristics gradient is large in absolute value. This implies that houses a few blocks away from each other can vary a great deal in atmosphere and, therefore, in price. This variation can by related to distance to amenities, mass transit, and thoroughfares (i.e., highway access), proximity to commercial and industrial zoning, single-family housing density, and so on. The vector $\mathbf{Z_j}$ may be unable to account for all the unobserved neighborhood variation that confounds the estimate of the capitalization premium because of the potential correlation with school quality. As Black (1999) points out, much of this variation (though admittedly not all) can be corrected for by analyzing houses that are geographically close.

The boundary discontinuities refinement considers only those houses that are geographically close to school attendance zone boundaries and replaces the vector of local characteristics with a full set of *pair-wise* boundary dummies.[13] Each house in this reduced sample is associated with the nearest, and hence unique, attendance zone boundary. This yields

$$\ln(p_{iab}) = \kappa + \mathbf{X_i}'\beta + \mathbf{K_b}'\phi + \mu_a\psi^L + \epsilon_{iab}, \tag{2}$$

where $\mathbf{K_b}$ is the vector of boundary dummies and the subscript b indexes the set of boundaries. The resulting

[12]For example, if the current tenants have no school-aged children.

[13]Clapp, Nanda, and Ross (2008), in contrast, measure school quality at the school-district level and use Census-tract fixed effects to control for omitted neighborhood characteristics. Brasington and Haurin (2006) also measure school quality at the school-district level but use spatial statistics rather than fixed effects to control for neighborhood characteristics.

education premium calculated with the linear boundary fixed effects model is ψ^L. Equation (2), then, is equivalent to calculating differences in house prices on opposite sides of attendance boundaries while controlling for house characteristics and relating the premium to test score information.

The boundary dummies allow us to account for unobserved neighborhood characteristics of houses on either side of an attendance boundary because two homes next to each other generally would have the same atmosphere. For this approach to be successful, particular care must be taken to exclude from the sample attendance zones whose boundaries coincide with administrative boundaries, rivers, parks, highways, or other landmarks that clearly divide neighborhoods, as neighborhood characteristics in these cases would be expected to vary discontinuously at the boundary.[14]

2.3 Nonlinear boundary fixed effects models

As an alternative to the linear model outlined in the previous section, we consider the possibility that the capitalization premium is not constant over the range of school qualities. This is accomplished by testing whether the education capitalization term enters nonlinearly. Consider the following pricing equation:

$$\ln(p_{iab}) = \kappa + \mathbf{X_i}'\beta + \mathbf{K_b}'\phi + f(\mu_a) + \epsilon_{iab}, \tag{3}$$

where $f(\mu_a)$ represents a potentially nonlinear function of school quality. For simplicity, suppose the function $f(\mu_a)$ is composed of a linear polynomial term and higher-order polynomial terms in school quality. That is,

$$f(\mu_a) = \psi_1\mu_a + \psi_2\mu_a^2 + \psi_3\mu_a^3 \tag{4}$$

where ψ_m, $m = 1, 2, 3$, are scalar parameters. We then rewrite equation (3) as

$$\ln(p_{iab}) = \kappa + \mathbf{X_i}'\beta + \mathbf{K_b}'\phi + \psi_1\mu_a + \psi_2\mu_a^2 + \psi_3\mu_a^3 + \epsilon_{iab}. \tag{5}$$

Specification (5) has a number of advantages over the linear form (2). First, the rate at which the

[14] In addition to boundary discontinuities (Leech and Campos, 2003; Kane, Staiger, and Samms, 2003; Kane, Staiger, and Riegg, 2005; Gibbons and Machin, 2003, 2006; Fack and Grenet, 2007; Davidoff and Leigh, 2007) recent studies have used various methods of addressing the omitted variables and endogeneity issues, including time variation (Bogart and Cromwell, 2000; Downes and Zabel, 2002; Figlio and Lucas, 2004; and Reback, 2005, among others), natural experiments (Bogart and Cromwell, 2000; Kane, Staiger, and Riegg, 2005), spatial statistics (Gibbons and Machin, 2003; Brasington and Haurin, 2006), or instrumental variables (Rosenthal, 2003; Bayer, Ferreira, and McMillan, 2007).

nominal premium varies across the range of school quality is not fixed. This allows us to differentiate the incremental effects on house prices of low- versus high-quality school attendance zones. Second, with a constant premium the linear model *penalizes* houses in low-quality school attendance zones by valuing them below what would be predicted by their comparable attributes.[15] Moreover, the penalty increases as the school quality worsens. This is unappealing because, as mentioned in the introduction, potential buyers who value education quality often can find substitute arrangements outside the public school system. Our prediction is that houses in lower-quality attendance zones command a smaller premium; in other words, the price function should be flatter for areas with lower test scores and steeper for those with higher test scores. This possibility is explicitly excluded in the linear model.

2.4 A note on the estimation

We estimated regression equations (1), (2), and (5) with ordinary least squares. In all cases, we computed robust standard errors clustered at the school level. For completeness, the Results section also presents the estimation of the nonlinear models using the full sample. We included boundary dummies in the regression equation and estimated the coefficients for these variables directly.

In an attempt to reduce any remaining bias from omitted characteristics, some recent studies, such as Bayer, Ferreira, and McMillan (2007), have supplemented their analysis by including demographic controls in the regressions. We therefore present results of the boundary fixed effects regressions in which the vector $\mathbf{Z_j}$ of neighborhood characteristics has been reinserted in the estimation. In particular, we control for the racial composition of neighborhoods. Studies that specifically consider the racial composition of neighborhoods include Bogart and Cromwell (2000); Downes and Zabel (2002); Cheshire and Sheppard (2004); Kane, Staiger, and Riegg (2005); Reback (2005); Clapp, Nanda, and Ross (2007); and Bayer, Ferreira, and McMillan (2007).

3 Data

In this analysis, we restrict our attention to single-family residences and elementary school attendance zones. Each observation corresponds to a house and is described by variables reflecting its physical characteristics,

[15] We adopt the convention that an increase in school quality induces a *premium* on house prices, whereas a decrease in school quality imposes a *penalty* on house prices.

the quality of the local elementary school that children in the household would attend, and the characteristics of the neighborhood in which the house is located—namely, demographic indicators measured at the Census-block level and property tax rates measured at the school-district level.

3.1 Real estate prices and housing characteristics

We obtained house price and house characteristics data from First American Real Estate, Inc. The observations selected correspond to a cross section of single-family residences sold during the 1998-2001 period in the St. Louis, Missouri, metropolitan area. After eliminating from the original dataset observations with missing or outlier house prices (outside a bound of 3.5 standard deviations from the mean unadjusted house price), our sample includes 38,656 single-family residences.

We deflated house prices to 1998 dollars with the Office of Federal Housing Enterprise Oversight repeat-sales price index for the entire St. Louis metropolitan area.[16] In the full sample the resulting adjusted house price has a mean of $148,082 and a standard deviation of $161,397. House characteristics include the total number of rooms, number of bedrooms, number of bathrooms, lot size, internal square footage, age of the structure, and number of stories in the house.

3.2 Attendance zones

For the boundary discontinuity analysis, we obtained the definitions of 121 attendance zones for elementary schools in 15 school districts in St. Louis County. Most of these were obtained by contacting the school districts directly. Boundaries were variously provided as listings of streets, maps, and in some isolated cases as geo-coded files. We, in turn, geo-coded all the attendance zones and determined the boundary for every pair of adjacent schools, as in Black (1999). We also geo-coded each house in our sample using the street address. We then selected houses within a 0.1-mile buffer of the boundaries and assigned them to the nearest (and therefore unique) pair-wise boundary.[17] We also eliminated from the boundary sample observations in St. Louis County that were associated with the boundaries of St. Louis City schools because we have no house price observations for the city. The final boundary sample consisted of 10,190 single-family residences.

[16] House prices were deflated using the average price index corresponding to the quarter of sale. The results were qualitatively unaffected if the National Association of Realtors price index was used instead.

[17] Black considers a number of different boundary width ranges and finds no significant differences. Our sample does not permit wider boundaries as these would encompass some attendance zones almost entirely.

3.3 Neighborhood characteristics

Houses were also matched to Census blocks as the geographic unit at which we measured neighborhood demographics. We used the publicly available population tables at the block level from the Census 2000 Summary File 1, which includes counts by age, sex, and race, to construct the following measures: percent of females, percent of school-aged children (aged between 5 and 14 years), and percent of nonwhite population (defined as the total population count minus the count of white people).

Additionally, we include as neighborhood controls the property tax rates defined at the school-district level for the years 1998 through 2001. In this case, each house was matched to the tax rate prevailing during the year of sale in its associated school district.[18] Table 1 presents summary statistics for house prices and characteristics with neighborhood characteristics for both the full and boundary samples.

3.4 Test scores

As the measure of school quality we use a school-level index generated by the Missouri Department of Elementary and Secondary Education. This index is computed from test score data from the Missouri Assessment Program (MAP). The MAP test includes a *Mathematics* section, a *Communication Arts* section (which includes a *Reading* portion), a *Science* section, and a *Social Studies* section.

Neither individual student scores nor school-level averages of these scores are publicly available. Instead, for each content area, the publicly available data provide the overall school-level MAP index. This index is obtained with a state-defined formula as the weighted sum of the percentages of students in each of five performance categories (Advanced, Proficient, Nearing Proficient, Progressing, and Step 1). The formula is MAP index = (percent in Step1)*1 + (percent in Progressing)*1.5 + (percent in Nearing Proficient)*2 + (percent in Proficient)*2.5 + (percent in Advanced)*3. The weights are exogenously determined by the Missouri Department of Elementary and Secondary Education.[19]

For our study we chose the Math MAP index for elementary schools only (fourth grade) as our measure of school quality.[20] This measure was then averaged over the period 1998-2001 to remove any year-to-year noise in the component variables (as in Bayer, Ferreira, and McMillan, 2007). Because our housing data are

[18] The analysis was not affected qualitatively if an average over the period was used instead.

[19] This formula was updated in 2007 when the number of performance categories was reduced to 4.

[20] We consider the math score a measure of school quality superior to the reading or science measures. First, the math scores are arguably the more objective measure. Second, the distribution of the school Math MAP index among the schools was contained almost entirely within two standard deviations of the mean. In contrast, the reading and science indexes contained a large number of outliers, particularly in the lower tail. We did not consider the social sciences scores.

essentially cross sectional, this procedure provides one consistent score for each school in the sample.

Table 2 presents summary statistics for MAP indexes along with property tax rates among the schools and school districts included in the sample.

[Table 1 about here.]

[Table 2 about here.]

4 Empirical Results

4.1 Standard hedonic regression

Table 3 presents the regression results using the full sample, which includes neighborhood demographic controls but excludes, of course, the boundary fixed effects. In addition to the traditional linear model, we have included the quadratic and cubic specifications in test scores for completeness.

We find that housing characteristics enter the pricing equation with the expected sign. Increases in living area, lot size, and the total number of rooms increase the price of a house on average. Similarly, the number of bathrooms and the number of stories have a positive and statistically significant effect. The number of bedrooms, the number of bathrooms squared, the age of the building, and its square do not seem to have a statistically significant effect in the full sample.

Among the neighborhood demographics only the percent of the nonwhite population (measured at the block level) is capitalized into house prices with a negative and statistically significant effect. The estimated coefficients indicate that an increase of 1 percentage point in the proportion of the nonwhite population decreases house prices by about 22 (in the linear model) to 27 (in the quadratic model) basis points. The property tax rate does not have a statistically significant effect.

As expected, the regressions illustrate a strong relation between school quality and house prices. The coefficient of 0.21734 in the traditional linear model (1) reveals that an increase in school test scores of half a standard deviation results in a house premium of about 11 percent (0.21734/2 = 10.867%) or about $16,000 at the mean price. A half standard deviation increase is equivalent to an increase of 4.6 percent in the Math MAP index.

The quadratic and cubic models in columns 2 and 3, respectively, also indicate a large and positive linear coefficient of school quality on house prices. The coefficient for the square of the Math score is, however, not statistically significant in columns 2 and 3. Interestingly, the cubic coefficient in column 3 is statistically significant, but it enters with a negative sign, which indicates that the house price premium does not monotonically increase over the range of school quality. In any case, these models suggest that nonlinearities are relevant. This is confirmed by a battery of Wald specification tests (Table 4). These tests reject the null hypothesis of a model with a constant education premium. We find that the restriction of not including a quadratic or cubic term ($\psi_2 = \psi_3 = 0$) is rejected at the 1 percent level, while not including a cubic term ($\psi_3 = 0$) is rejected at the 5 percent level. However, the restriction of no quadratic term ($\psi_2 = 0$) is not rejected. Thus, we find evidence that the preferred specification for the education premium in the full sample is the cubic model.

[Table 3 about here.]

[Table 4 about here.]

4.2 Boundary discontinuity models

Table 5 presents the results for the restricted boundary sample (omitting the estimated coefficient for the boundary fixed effects). As in the full sample, house characteristics are statistically significant and with the expected sign. In contrast to the full sample results, the age of the building and its square, along with the square of the number of bathrooms, are statistically significant. Compared with the full sample results, the estimated coefficients for house characteristics are smaller in magnitude but very stable across specifications.

In the linear model in column 1, school quality is a statistically significant contributor to house prices and enters with the expected positive sign. Compared with the results from the full sample regression, the estimated coefficient declines in magnitude by a factor of about four. The estimate of the education premium implies that a half standard deviation increase in the average school score leads to an increase of about 3.2 percent in house prices, or about $4,766 evaluated at the full sample mean price. This value is only slightly higher than that estimated by Black (1999).[21]

[Table 5 about here.]

The two specifications of the nonlinear boundary fixed effects models in columns 2 and 3 indicate that the quadratic coefficient of school quality is statistically significant, but the cubic coefficient is not. The

[21] Black reports a 2.1 percent increase (or $3,948 at her sample mean) in house prices for a 5 percent increase in test scores.

positive sign of the quadratic coefficient indicates that the capitalization effect of school quality is increasing over the range of test scores.

Specifications 1, 2, and 3 do not include additional controls for neighborhood quality other than the boundary fixed effects. As mentioned previously, some authors have raised concerns about whether the boundary discontinuity approach fails to control for omitted neighborhood characteristics and suggest that explicit additional controls be included in the estimation. We therefore include the same demographic controls as in the full sample regression—namely, the percent of female population, the percent of nonwhite population, and the percent of school-aged children, all measured at the block level. We also include the school district property tax rate.

Columns 4, 5, and 6 in Table 5 show that these additional variables are directly capitalized into house prices. The percent of nonwhite population is statistically significant and enters with a negative sign as in the full sample results. The magnitude of the effect is similar to the full sample results and indicates a decline of about 22 basis points in house prices for a 1-percentage-point increase in the proportion of the nonwhite population. We interpret the significance of this variable, as in other papers, as evidence of preferences about the racial composition of neighborhoods.

In contrast to the full sample results, the percent of school-aged children is statistically significant and indicates an increase in house prices of about 15 basis points for a 1-percentage-point increase in the proportion of children aged between 5 and 14 years. The property tax rate is also statistically significant and enters with a negative sign.

The inclusion of explicit neighborhood controls does not affect the magnitude of the coefficients of the housing characteristics, but it decreases the magnitude of the linear test score coefficient by almost half. The quadratic coefficient declines only slightly. The linear coefficient on school quality remains, nevertheless, statistically significant and the results suggest that the magnitude of the effect of school quality on house prices remains substantially large.

Wald specification tests (Table 6) confirm that with or without the inclusion of additional neighborhood controls, the preferred specification is the quadratic model. These tests also reject, as in the full sample regressions, the null hypothesis of a model with a constant education premium. We find that the restriction of not including a quadratic or cubic term ($\psi_2 = \psi_3 = 0$) is rejected at the 5 percent level. However, the restriction of no cubic term ($\psi_3 = 0$) is not rejected.

[Table 6 about here.]

4.3 Implicit housing premia

Figure 1 illustrates the preferred specification for the house pricing function with the more conservative model with boundary fixed effects resulting from the inclusion of additional neighborhood controls. The plot includes one-standard-error bands.[22]

We argued in the introduction that competition in the housing market generates increasing tightness in areas associated with higher school quality, but that competition is not as prevalent in areas associated with lower school quality. The pricing function in Figure 1 confirms our argument.

The premium from school quality on housing prices is better illustrated in Figure 2. This figure is constructed from the pricing function of specification 5 in Table 5 and represents the percentage increase in house prices in response to a half a standard deviation increase in Math test scores plotted along the range of school scores contained within two standard deviations of the mean.

The plotted function reveals a monotonically increasing premium across the spectrum of school quality. The plot indicates that, even when using the most conservative estimates, the premium for houses in areas associated with high-quality schools remains substantially large. The plot also reveals a much smaller premium for houses in areas associated with low-quality schools, where house prices seem to be driven almost entirely by housing and neighborhood characteristics other than public school quality.

[Figure 1 about here.]

[Figure 2 about here.]

Table 7 summarizes the implied school quality premia for all models and provides the dollar equivalent of the implied percentage increase in house prices relative to the mean house prices in the full and boundary samples that results from a half standard deviation increase in test scores.

[Table 7 about here.]

The linear model with the full sample regression results in a constant premium of 10.87 percent or about $16,000 at the mean house price. The cubic model in the full sample, which the specification tests

[22] The asymptotic variance of the price function was computed using the delta method as: $\mathrm{AsyVar}(f(\mu;\beta)) = \frac{\partial f(\mu;\beta)}{\partial \beta'}\mathrm{AsyVar}(\beta)(\frac{\partial f(\mu;\beta)}{\partial \beta'})'$.

suggest is the preferred one, illustrates a nonmonotonic premium that ranges from 11.53 percent for houses in areas where school quality is one standard deviation below the mean to 15.78 percent in areas where school quality coincides with the average, and finally to 9.23 percent in areas where school quality is one standard deviation above the mean.

The boundary sample models with and without additional neighborhood controls indicate that the premium is severely overestimated in the traditional hedonic regressions, even accounting for nonlinearities. Nevertheless, even in the most conservative estimates, the premium remains substantially large especially for areas associated with very high-quality schools. The table also shows that in the quadratic specification (the middle column of the the third panel in Table 7), the premium is very small (about 0.13 percent or less than $200) in areas where test scores are one standard deviation below the mean, and monotonically increases in areas with higher test scores (about 2.34 percent or $3,468 in areas with average test scores, and 4.55 percent or $6,739 in areas with test scores at one standard deviation above the mean).

5 Conclusion

Traditional empirical models of the capitalization of education quality on house prices have established that the quality of primary school education is positively correlated with house prices.

Recent capitalization studies have dealt in various ways with concerns about omitted variable bias induced by failing to account for the correlation between school quality and unobserved neighborhood characteristics, Most of these variations on the traditional hedonic approach (including the boundary discontinuity regression) have made, however, similar assumptions on the shape of the house pricing premium over school quality: The rate at which the premium increases is restricted to be constant because in all these models the contribution from school quality on house prices is constrained to be linear.

In this paper, we propose an alternative formulation that allows for nonlinear effects of school quality. We show that this formulation is preferred by the data over a baseline linear boundary fixed effects model and that the rate at which the house price premium rises increases over the range of school quality. In other words, we find that the standard linear specification for test scores overestimates the premium at low levels of school quality and underestimates the premium at high levels of school quality.

In the St. Louis metropolitan area, houses associated with a school ranked at one standard deviation below the mean are essentially priced on physical characteristics only. In contrast, houses associated with higher-quality schools command a much higher price premium.

Interestingly, and in contrast to many studies in the literature, the price premium remains substantially large, especially for houses associated with above-average schools. This is true even in our most conservative estimates, which complement the boundary discontinuity approach by explicitly controlling for neighborhood demographics. These estimates also reveal that the racial composition of neighborhoods is capitalized directly into house prices.

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Figure 1: Implied Price Function

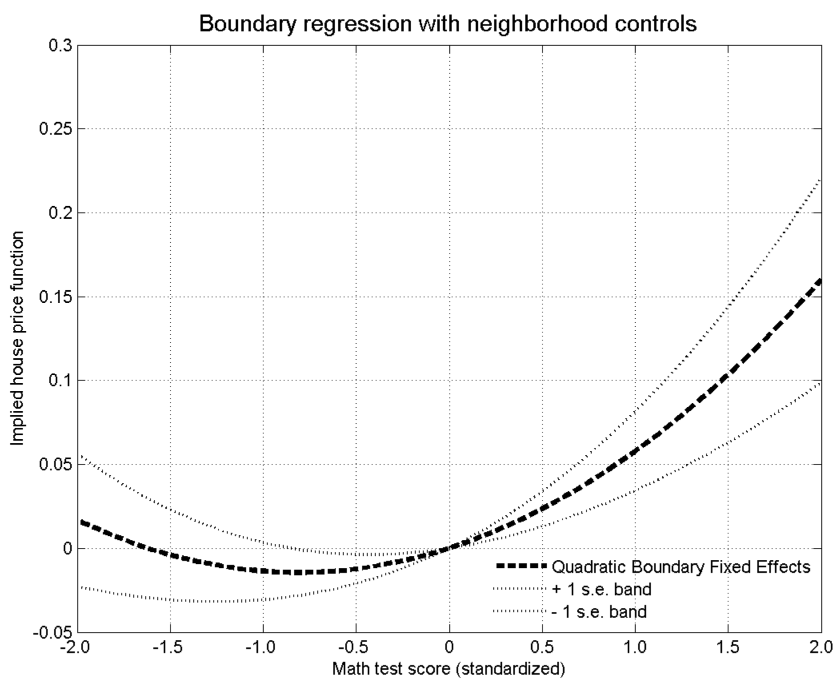

Figure 2: Implied Premium Function (to an increase in 0.5 s.d. in test scores)

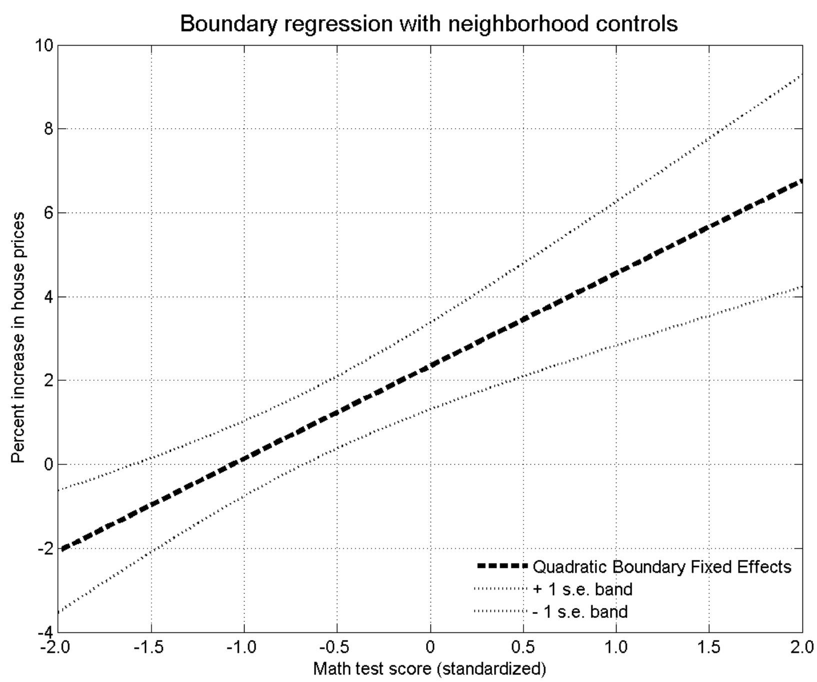

Table 1: Summary Statistics: House and Neighborhood Characteristics

	Full sample (N=38656)		Boundary sample (N=10190)	
House variables	**Mean**	**SD**	**Mean**	**SD**
Sale price (1998 US$)	148,081.67	161,397.24	142,033.42	176,191.20
Log of sale price	11.62	0.73	11.56	0.75
Number of bedrooms	2.96	0.84	2.9	0.84
Number of bathrooms	2.01	0.95	1.95	0.93
Number of bathrooms (squared)	4.97	5.05	4.66	5.04
Age of building	38.91	20.63	40.72	21.27
Age of building (squared)	1,939.38	1,922.87	2,110.15	2,028.41
Lot area (1000s of sq. ft.)	14.75	38.35	13.61	39.20
Living area (1000s of sq. ft.)	1.16	0.44	1.13	0.42
Number of stories	1.24	0.42	1.23	0.41
Total number of rooms	6.38	1.60	6.26	1.57
	Full sample (N=6360 blocks)		Boundary sample (N=2560 blocks)	
Census variables	**Mean**	**SD**	**Mean**	**SD**
Percent female population	51.17	11.22	51.34	11.33
Percent nonwhite population	20.43	29.29	22.42	30.67
Percent population aged 5 to 14	9.34	9.58	9.98	9.38

Table 2: Summary Statistics: Test Scores and Property Tax

Test scores (N=121 schools)				
Variable	**Mean**	**SD**	**Min.**	**Max.**
Math MAP score	211.45	19.44	168.14	250.18
Science MAP score	211.88	22.56	100.00	242.61
Reading MAP score	200.73	20.15	100.00	228.94
Property tax (N=15 school districts)				
Variable	**Mean**	**SD**	**Min.**	**Max.**
Property tax rate ($1/$1,000 of assessed house value)	4.23	0.91	2.60	5.74

Table 3: Education Regressions: Full Sample.

	(1) log_adj_price	(2) log_adj_price	(3) log_adj_price
test_math	0.21734***	0.22192***	0.31693***
	(7.79)	(7.13)	(7.70)
test_math_sq		0.03002	0.01555
		(1.48)	(0.76)
test_math_cb			-0.03606**
			(-2.60)
house_bedrooms	0.01062	0.01502	0.01575
	(1.09)	(1.52)	(1.62)
house_bathrooms	0.14086***	0.14413***	0.13458***
	(4.75)	(4.93)	(4.44)
house_baths_sq	-0.00612	-0.00740	-0.00501
	(-1.14)	(-1.37)	(-0.89)
house_age	0.00065	0.00057	0.00123
	(0.37)	(0.31)	(0.67)
house_age_sq	0.00002	0.00002	0.00002
	(1.35)	(1.31)	(1.03)
house_lotarea	0.00123***	0.00120***	0.00119***
	(4.21)	(4.27)	(4.17)
house_livingarea	0.45365***	0.44475***	0.43526***
	(20.02)	(17.35)	(19.05)
house_stories	0.39693***	0.38775***	0.37835***
	(11.29)	(10.58)	(10.87)
house_rooms	0.07484***	0.07421***	0.07245***
	(10.10)	(10.21)	(10.11)
cenbl_fem	-0.00061	-0.00050	-0.00053
	(-0.88)	(-0.73)	(-0.79)
cenbl_nonwhite	-0.00221***	-0.00277***	-0.00257***
	(-3.62)	(-5.06)	(-4.57)
cenbl_p5_14	-0.00017	-0.00033	-0.00021
	(-0.19)	(-0.38)	(-0.24)
tax	-0.04636	-0.04457	-0.03562
	(-1.65)	(-1.51)	(-1.28)
_cons	10.00143***	9.99065***	9.96337***
	(59.89)	(57.55)	(58.13)
N	38,656	38,656	38,656
R^2	0.697	0.699	0.702
Adjusted R^2	0.697	0.698	0.702

t-Statistics in parentheses.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 4: Specification Tests: Full Sample

Premium Model $f(Y) = \psi_1 Y + \psi_2 Y^2 + \psi_3 Y^3$			
	Linear	**Quadratic**	**Cubic**
With Neighborhood Controls			
Null hypothesis	$\psi_1 = 0$	$\psi_1 = \psi_2 = 0$	$\psi_1 = \psi_2 = \psi_3 = 0$
Wald *F*-Statistic	60.757***	27.686***	30.665***
Null hypothesis		$\psi_2 = 0$	$\psi_2 = \psi_3 = 0$
Wald *F*-Statistic		2.192	7.446***
Null hypothesis			$\psi_3 = 0$
Wald *F*-Statistic			6.754**
* Significant at 10%; ** significant at 5%; *** significant at 1%			

Table 5: Education Regressions: Boundary Sample

	(1)	(2)	(3)	(4)	(5)	(6)
	log_adj_price	log_adj_price	log_adj_price	log_adj_price	log_adj_price	log_adj_price
test_math	0.06437**	0.06274***	0.04659	0.03227*	0.03579*	0.03172
	(2.58)	(2.90)	(1.64)	(1.78)	(1.93)	(1.20)
test_math_sq		0.02656**	0.02909**		0.02209**	0.02284**
		(2.47)	(2.47)		(2.48)	(2.40)
test_math_cb			0.00514			0.00137
			(0.73)			(0.21)
house_bedrooms	0.03726***	0.03730***	0.03749***	0.03816***	0.03805***	0.03809***
	(3.88)	(3.89)	(3.90)	(4.02)	(4.01)	(4.00)
house_bathrooms	0.10834***	0.10785***	0.10792***	0.10349***	0.10318***	0.10320***
	(5.78)	(5.80)	(5.82)	(5.81)	(5.82)	(5.83)
house_baths_sq	-0.00529*	-0.00533*	-0.00535*	-0.00488	-0.00491	-0.00491
	(-1.68)	(-1.70)	(-1.71)	(-1.58)	(-1.60)	(-1.60)
house_age	-0.00408***	-0.00411***	-0.00412***	-0.00453***	-0.00454***	-0.00454***
	(-2.73)	(-2.75)	(-2.76)	(-3.11)	(-3.13)	(-3.14)
house_age_sq	0.00004***	0.00004***	0.00004***	0.00004***	0.00004***	0.00004***
	(2.89)	(2.91)	(2.92)	(3.15)	(3.16)	(3.17)
house_lotarea	0.00089**	0.00089**	0.00089**	0.00088**	0.00088**	0.00088**
	(2.41)	(2.41)	(2.41)	(2.39)	(2.40)	(2.39)
house_livingarea	0.35315***	0.35228***	0.35236***	0.34332***	0.34297***	0.34301***
	(15.43)	(15.29)	(15.29)	(15.52)	(15.49)	(15.49)
house_stories	0.27574***	0.27559***	0.27558***	0.26621***	0.26625***	0.26626***
	(9.30)	(9.30)	(9.31)	(9.55)	(9.57)	(9.57)
house_rooms	0.05974***	0.05952***	0.05945***	0.05902***	0.05893***	0.05891***
	(7.38)	(7.33)	(7.31)	(7.43)	(7.40)	(7.39)
cenbl_fem				-0.00044	-0.00039	-0.00039
				(-0.66)	(-0.59)	(-0.59)
cenbl_nonwhite				-0.00219***	-0.00223***	-0.00222***
				(-3.50)	(-3.56)	(-3.55)
cenbl_p5_14				0.00154**	0.00153**	0.00154**
				(2.25)	(2.24)	(2.25)
tax				-0.06787***	-0.05526***	-0.05465***
				(-3.21)	(-2.88)	(-2.73)
_cons	11.13260***	11.12998***	11.13935***	8.86314***	8.72871***	8.72454***
	(32.85)	(32.96)	(32.97)	(59.31)	(62.61)	(60.54)
N	10190	10190	10190	10182	10182	10182
R^2	0.769	0.770	0.770	0.772	0.772	0.772
Adjusted R^2	0.763	0.763	0.763	0.766	0.766	0.766
Boundary fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

t-Statistics in parentheses.

* Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 6: Specification Tests: Boundary Sample

Premium Model $f(Y) = \psi_1 Y + \psi_2 Y^2 + \psi_3 Y^3$			
	Linear	**Quadratic**	**Cubic**
Without Neighborhood Controls			
Null hypothesis	$\psi_1 = 0$	$\psi_1 = \psi_2 = 0$	$\psi_1 = \psi_2 = \psi_3 = 0$
Wald *F*-Statistic	6.632**	4.658**	3.130**
Null hypothesis		$\psi_2 = 0$	$\psi_2 = \psi_3 = 0$
Wald *F*-Statistic		6.115**	3.114**
Null hypothesis			$\psi_3 = 0$
Wald *F*-Statistic			0.527
With Neighborhood Controls			
Null hypothesis	$\psi_1 = 0$	$\psi_1 = \psi_2 = 0$	$\psi_1 = \psi_2 = \psi_3 = 0$
Wald *F*-Statistic	3.178*	3.581**	2.381*
Null hypothesis		$\psi_2 = 0$	$\psi_2 = \psi_3 = 0$
Wald *F*-Statistic		6.166**	3.102**
Null hypothesis			$\psi_3 = 0$
Wald *F*-Statistic			0.043
* Significant at 10%; ** significant at 5%; *** significant at 1%			

Table 7: Implied House Premia from Math Test Scores

Change in Math score (SD)			0.5	Equivalent percentage of mean					4.6%
Regression model	Full sample with neighborhood controls			Boundary sample with no neighborhood controls			Boundary sample with neighborhood controls		
Linear coefficient	0.21734	0.22192	0.31693	0.06437	0.06274	0.04659	0.03227	0.03579	0.03172
Quadratic coefficient	–	0.03002	0.01555	–	0.02656	0.02909	–	0.02209	0.02284
Cubic coefficient	–	–	-0.03606	–	–	0.00514	–	–	0.00137
Case 1 (Mean score - 1 SD)									
Percent increase in house price	10.87	8.84	11.53	3.22	1.15	0.60	1.61	0.13	-0.01
Dollar value at mean (full sample)	16,092	13,097	17,066	4,766	1,696	885	2,389	197	-11
Dollar value at mean (boundary sample)	15,435	12,562	16,369	4,571	1,626	849	2,292	189	-10
Case 2 (Mean score)									
Percent increase in house price	10.87	11.85	15.78	3.22	3.80	3.12	1.61	2.34	2.17
Dollar value at mean (full sample)	16,092	17,542	23,374	4,766	5,629	4,622	2,389	3,468	3,219
Dollar value at mean (boundary sample)	15,435	16,826	22,419	4,571	5,399	4,433	2,292	3,326	3,088
Case 2 (Mean score + 1 SD)									
Percent increase in house price	10.87	14.85	9.23	3.22	6.46	7.19	1.61	4.55	4.77
Dollar value at mean (full sample)	16,092	21,988	13,662	4,766	9,562	10,642	2,389	6,739	7,058
Dollar value at mean (boundary sample)	15,435	21,090	13,104	4,571	9,171	10,207	2,292	6,464	6,770

The premium is computed from the logarithm specification $\Delta p/p = \Delta ln(p) = \Delta f(\mu)$, so the percent change in house prices is given by $\Delta f(\mu) = f(\mu_1) - f(\mu_0)$ and the premium at the mean price is $\Delta f(\mu) \times \bar{p}$.