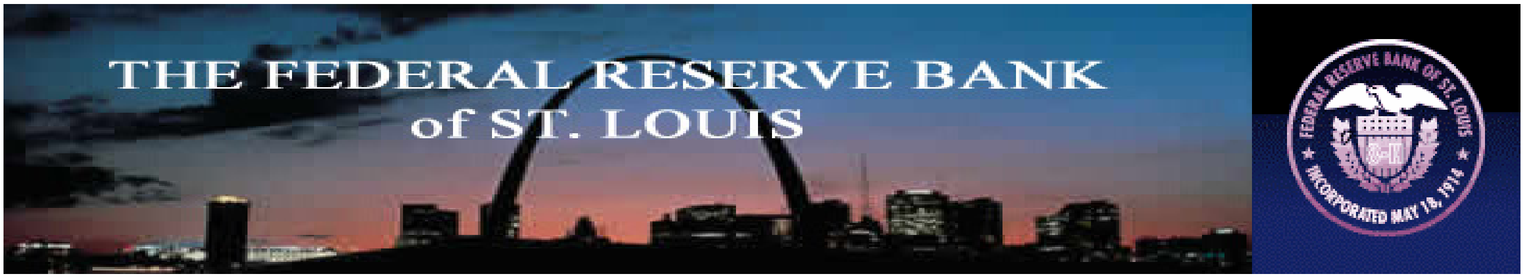

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James Bullard
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Stefano Eusepi

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

Did the Great Inflation Occur Despite Policymaker Commitment to a Taylor Rule?

James Bullard[1]

and

Stefano Eusepi[2]

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We study the hypothesis that misperceptions of trend productivity growth during the onset of the productivity slowdown in the U.S. caused much of the great inflation of the 1970s. We use the general equilibrium, sticky price framework of Woodford (2003), augmented with learning using the techniques of Evans and Honkapohja (2001). We allow for endogenous investment as well as explicit, exogenous growth in productivity and the labor input. We assume the monetary policymaker is committed to using a Taylor-type policy rule. We study how this economy reacts to an unexpected change in the trend productivity growth rate under learning. We find that a substantial portion of the observed increase in inflation during the 1970s can be attributed to this source. *JEL* Classification Numbers E4, E5.

Key Words: Monetary policy rules; productivity slowdown; learning.

[1]Research Department, Federal Reserve Bank of St. Louis, 411 Locust Street, St. Louis, MO 63102. E-mail: bullard@stls.frb.org.

[2]Domestic Research Function, Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045. Email: stefano.eusepi@ny.frb.org.

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1. INTRODUCTION

1.1. The great inflation and the productivity slowdown

Broadly speaking, U.S. inflation was low in the early 1960s, then high in the 1970s and early 1980s, and then lower again during the last twenty years. Figure 1 shows one measure of the dramatic rise and fall, which is sometimes called the *great inflation*. We investigate the hypothesis that much of the great inflation was due to a misperception on the part of economic actors—both the private sector and the Federal Reserve—concerning the trend pace of productivity growth. This hypothesis is associated most closely (and most recently) with Orphanides (2000, 2001, 2002, 2003). The broad idea is that it was initially very difficult for the economy's participants to detect that the productivity slowdown had occurred—that is, agents had to learn about it. The misperception caused the central bank to overestimate the size of the output gap, leading through a Taylor-type policy rule to lower-than-intended interest rates and, subsequently, higher-than-intended inflation.

Our vehicle for analysis is a version of the general equilibrium, sticky price model of Woodford (2003). We allow for endogenous investment along with explicit, exogenous growth, both of which we view as essential for discussion of this issue. We include learning to capture the idea that it took some time for the economy's participants to evaluate the changing nature of the nation's balanced growth path dictated by the productivity slowdown. Our learning methodology is that of Evans and Honkapohja (2001).

For the purposes of this paper, we use a Perron (1989)-style characterization of the productivity data, in which trend-stationarity is buffeted by rare breaks in trend, occurring perhaps once or twice in the postwar data.[4] Thus the process driving productivity growth is actually nonstationary, but the permanent shock occurs only rarely, not each period. The agents in our model—both the central bank and the private sector—understand that such structural change may occur, and employ learning algorithms to ensure that they will be able to adjust following such shocks. In this sense the agents in our model are protecting themselves against the possibility of *structural change*—permanent changes in key aspects of the economy, like the pace of productivity growth—by re-estimating their perceived laws of motion for the economy each period. When there is no structural change for a period of time, our systems will simply converge to a small neighborhood of the rational expectations equilibrium, balanced growth path. But when structural change occurs, the agents will be able to learn the new balanced growth path. Thus, learning will act as the glue that holds the piecewise balanced growth paths of our model together. Our paper concerns a quantitative assessment of the reaction of key macroeconomic variables to permanent productivity growth shocks in

[4] For recent empirical papers concerning trend breaks in productivity, see Bae, Lumsdaine, and Stock (1998) and the survey by Hansen (2001). Our reading of the empirical literature is that it is difficult to reconcile a completely trend-stationary view with the macroeconomic data.

this environment.

Of the many recent hypotheses for the 1970s inflation experience, the "misperceived change-in-trend" view has some of the more jarring policy implications. It suggests the possibility that even a determined and knowledgeable central bank—today's policymakers—could end up with a lot of inflation. The monetary authority is determined and knowledgeable in the sense that they are committed to using a Taylor-type policy rule that would be optimal or near-optimal in some stationary contexts where structural change never occurs.[5] Since productivity growth is notoriously hard to track, because of noise and measurement problems in the data, and since, at the same time, productivity is critically important for economic growth, it seems quite reasonable that some learning about rare changes in its mean growth rate must occur. Thus it is not out of the question, under this hypothesis, that the inflation experience of the 1970s could be repeated, given the right type of shock. Understanding this shock and what might be done should it occur again would then be a key concern for policymakers.

A backdrop to this issue is the more recent improvement in U.S. productivity growth during the 1990s, the so-called "new economy." To the extent that private sector actors and policymakers had to again learn about the changing nature of the balanced growth path, this event might have been expected to lead to lower inflation through a Taylor-type policy rule. We provide an assessment of this hypothesis as well.

1.2. Model summary

We use a modification of Woodford's (2003) general equilibrium, sticky price model with endogenous investment. We include explicit, exogenous growth in the model, driven by growth in the labor force as well as productivity. We maintain the assumption of firm-specific labor inputs, but we allow for a homogeneous capital input, traded in a perfectly competitive and economy-wide capital market. The main reason for the homogeneous capital assumption is to keep the model relatively simple and comparable with models currently used for policy analysis. The model economy has a well-defined rational expectations equilibrium characterized as a balanced growth path. We assume the economy begins on such a path. Our experiment is to unexpectedly alter the rate of growth of productivity along this path, and allow the economy's actors to adapt to the new rational expectations equilibrium. So long as the system is expectationally stable (which we verify), the disturbance to productivity growth will only temporarily cause the system to depart from the rational expectations equilibrium, as the agents will eventually learn the new equilibrium.

We include learning in the model using the methodology of Evans and Honkapohja (2001). Expectations are formed by agents using well-specified vector autore-

[5] The argument that monetary policy during the 1970s was essentially the same as the policy recommended as optimal or near-optimal in the recent literature has been made by Orphanides (2002).

gressions updated each period as new information becomes available. The regressions are well-specified in the sense that they are consistent with the rational expectations equilibrium law of motion for the economy.

1.3. Main findings

We find that a one-time, unexpected change in productivity growth of the magnitude observed in the early 1970s generates a lot of inflation in our model, arguably a large portion of the persistent inflationary acceleration during this period. Thus our assessment is that the "misperceived-change-in-trend" view has considerable merit, even in the context of a micro-founded, forward-looking, general equilibrium model with endogenous investment.

We also find, however, that the inflation generated by the productivity slowdown in this model is far too persistent, as it does not fall rapidly enough by the early 1980s to provide a satisfactory account of the data. The onset of the new economy in the 1990s does lead to disinflation, but the model still misses the sharp disinflation of the early 1980s. This suggests to us that a change in the policy rule occurred during the first portion of the Volker era. When we add a modest, unexpected reduction in the target rate of inflation in 1980,[6] which might be viewed as a good approximation to policy developments at that time, then the model can capture the medium run movements in inflation from 1970 to the present depicted in Figure 1. We also show that other features of the simulated economy, including the pattern of inflation expectations and the pace of output growth, are close to observations from the postwar U.S. data.

We find that an important component of our quantitative results is that we allow both the central bank and the private sector to learn about the change in the balanced growth path. If the private sector has rational expectations while the central bank does not, then they understand more than the central bank both about the shock that has hit the economy and about the nature of the central bank's reaction to that shock. That is, they understand that the central bank is setting the nominal interest rate at "too low" of a level and accordingly they take actions that mitigate the inflation that would otherwise occur. We discuss this and other variations of the model in the results section of the paper.

1.4. Recent related literature

A number of authors have recently put forward formal models offering an explanation for the postwar U.S. inflation experience. Examples include Clarida, Gali, and Gertler (2000), Christiano and Gust (1999), Ireland (1999), and Albanesi, Chari, and Christiano (2002). The studies of Sargent (1999), Cho, Williams, and Sargent (2002), Bullard and Cho (2003), Primaceri (2004), and Williams (2003) emphasize escape dynamics and possession of a misspecified model on the part of policymakers. Our learning methodology is similar, but in our systems the learning

[6] In all other respects the policy rule remains unchanged.

dynamics simply converge to the economy's unique balanced growth path following a disturbance (that is, we have the mean dynamics of Sargent (1999) and Evans and Honkapohja (2001), not the escape dynamics).

Orphanides (2000, 2001, 2002, 2003) has written extensively on the 1970s U.S. inflation experience from the perspective of policymakers at the time. His work suggests that the perceived output gap was quite large during this period, and that this influenced policy appreciably. Lansing (2002) studies the interaction of monetary policy and trend growth changes in a simplified version of Fuhrer and Moore (1995), obtaining a modest increase in inflation in response to a permanent technology shock. Tambalotti (2003) studies the great inflation in a general equilibrium model were the central bank responds to an incorrect measure of the output gap while the private sector is learning. For research concerning optimal monetary policy, see Tambalotti (2002), who studies policymaker reactions to persistent, but not permanent, shocks to technology, in a model without capital. Tambalotti (2002) does not consider the policy 'mistake' discussed in Orphanides' papers. Collard and Dellas (2004) study the great inflation from a perspective similar to the one presented in this paper, but with alternative representations of the nature of productivity change, the learning rules in place, and the policy rule of the monetary authorities. They compare the misperceived-change-in-trend theory with rival theories, and find that it is difficult to distinguish between the alternatives. Nelson and Nikolov (2002) study the productivity slowdown-inflation nexus in the U.K.

2. THE ENVIRONMENT

2.1. Exogenous growth

We study a model economy due to Woodford (2003) but with exogenous growth added. The economy is populated by a continuum of households indexed by h. The size of the aggregate labor force is described by N_t which grows according to

$$N_t = \eta N_{t-1},$$

with $\eta \geq 1$ and $N_0 = 1$, so that η is the gross rate of growth in the labor force for the economy. We assume this growth is equally distributed across households h, so that the size of each household also grows at gross rate η.

We also assume explicit technological progress. We analyze the most standard case by assuming that this progress, defined in terms of efficiency units X_t, affects only labor productivity. Productivity is assumed to grow according to

$$X_t = \gamma X_{t-1},$$

with $\gamma \geq 1$ and $X_0 = 1$. We assume labor productivity improvements apply equally to all households.

2.2. Household decisions

Each household h makes expected utility-maximizing decisions regarding consumption, labor supply, and asset holding, with expected utility given by

$$E_t \sum_{i=0}^{\infty} \beta^i \eta^i \left[\ln C_{t+i}^h - a \int_0^1 \frac{\left[L_{t+i}^h(f)\right]^{1+\nu}}{1+\nu} df + \Psi\left(\frac{M_{t+i}^h}{P_{t+i}}\right)\right]$$

where the parameters $a > 0$ and $\nu \geq 0$, and the discount factor $\beta \in (0,1)$. We will denote $\tilde{\beta} \equiv \beta\eta$, and call it the 'effective' discount factor. The variable C_{t+i}^h is an index of household h consumption at date $t+i$. In the economy a continuum of differentiated goods are supplied, indexed by $f \in (0,1)$. Each household consumes some units of each good produced. Each individual good $C_t(f)$ is produced by using capital and a specialized labor input—labor of type f produces the differentiated good indexed by f. We use the standard consumption aggregator

$$C_t^h = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is a parameter that controls the price elasticity of demand. Following Woodford (2003), every household h simultaneously supplies all types of firm-specific labor $L_t^h(f)$. The function $\Psi(\cdot)$ denotes the utility derived from holding real balances M_t^h/P_t, where M_t^h is nominal money balances and P_t is the price index associated with C_t^h at time t, defined by

$$P_t = \left[\int_0^1 P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{1}$$

where $P_t(f)$ is the price associated with good $C_t(f)$. We will denote gross inflation as $\Pi_t = P_t/P_{t-1}$.

The operator E_t denotes agents' rational expectations about the future evolution of the economic variables. For most of the analysis households, firms and the central bank are assumed to have *identical* expectations about the future evolution of the model's variables. For this reason we do not index the expectation operator. Later in the paper, we will replace the agents' rational expectations with recursive learning algorithms following the standard analysis of Evans and Honkapohja (2001), but maintaining the identical expectations assumption, as is the standard in the learning literature.[7] At the end of the paper, we will allow the central bank to learn in a manner somewhat different from the private sector, and at that point we will label the expectations of the actors separately.

Each household can transform investment into productive capital and rents capital to firms in a perfectly competitive capital market. A household's budget

[7] The homogeneous learning assumption has some appeal because, in industrialized countries, there is a forecasting community that essentially makes econometric forecasts for all actors in the economy.

constraint can be written in nominal terms as

$$P_t C_t^h + M_t^h + B_t^h = M_{t-1}^h + (1 + i_{t-1}) B_{t-1}^h + \Gamma_t^h + P_t R_t^K K_t^h - P_t I_t^h + \int_0^1 N_t L_t^h(f) W_t(f) \, df,$$

where Γ_t^h defines the nominal profits from holding a share of each firm in the economy, and $W_t(f)$ is the nominal wage paid by the firm that uses labor of type f. We let B_t^h denote household holdings of financial assets other than money.[8] The short-term nominal interest rate, i_t, is assumed to be controlled directly by the monetary authority. Finally, R_t^K is the real rental rate of capital K_t^h and I_t^h denotes investment.

Each agent faces convex adjustment costs defined by

$$I_t^h = I\left(\frac{K_{t+1}^h}{K_t^h}\right) K_t^h \tag{2}$$

where $I(\cdot)$ is the adjustment cost function and I_t^h is total investment expenditure of the household. The investment good I_t^h is assumed to be an aggregate of all goods in the economy, with the same constant elasticity of substitution as the consumption aggregate. The function $I(\cdot)$ has the following properties

$$\begin{aligned} I(\gamma\eta) &= \gamma\eta - 1 + \delta, \\ I'(\gamma\eta) &= 1, \\ I''(\gamma\eta) &= \epsilon_\psi > 0. \end{aligned}$$

where ϵ_ψ denotes the cost of adjusting the capital stock.[9]

Summing up, in each period t households choose $\{C_t(f), L_t^h(f), B_t^h, M_t^h, I_t^h\}$, taking prices and profits $\{P_t(f), W_t(f), i_t, R_t^K, \Gamma_t^h\}$ as given. We emphasize that under the maintained assumptions the consumption decision of every household h is identical because each household receives the same flow of income and shares the same initial money and other asset holdings.

2.3. Firm behavior

Each firm produces a differentiated good and has some market power. As mentioned above, we assume that each good is produced using capital and labor. We assume that capital is homogenous and traded in a perfectly competitive, economy-wide market. Labor is firm-specific: each firm, in order to produce its differentiated good, uses a different type of labor. The households supply labor hours $L_t(f) = \int L_t^h(f) \, dh$ to the firm producing good f, given the wage offered by the firm. We assume that labor markets are competitive.[10]

[8] Here we assume complete markets. For details, see Woodford (2003, Chapter 2).
[9] See Woodford (2003) for a discussion.
[10] See Woodford (2003) for a discussion.

Firm f produces the good $Y_t(f)$ using the technology

$$Y_t(f) = K_t(f)^{\alpha} \left[X_t N_t L_t(f)\right]^{1-\alpha}, \tag{3}$$

where $K_t(f)$ is the capital stock used for production at time t by firm f and $L_t(f)$ is the amount of hours each worker supplies for production. The optimal allocation of household spending across differentiated goods implies the demand curve[11] for each firm f,

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\theta} Y_t, \tag{4}$$

where $\theta/(\theta-1)$ denotes the firm's markup and where Y_t denotes aggregate demand for consumption and investment goods.

We assume Calvo-type sticky prices. A fraction $1-\xi$ of firms is allowed to optimally change prices each period, while the remaining fraction ξ are not allowed to optimally set prices but instead make a simple inflation adjustment.

Firm f decides its optimal price $P_t(f)$ by maximizing expected profits

$$E_t \sum_{j=0}^{\infty} \tilde{\beta}\xi^j Q_{t,t+j} \left[\frac{P_t(f)}{P_t}\bar{\Pi}^j - \frac{\theta}{(\theta-1)} S_{t+j}(f)\right] \left(\frac{P_t(f)}{P_{t+j}}\right)^{-\theta} Y_{t+j} \tag{5}$$

where $S_t(f)$ denotes the real marginal cost of firm f, obtained from minimizing costs subject to (3). The term $Q_{t,t+j}$ is the stochastic discount factor defined in the appendix. The parameter $\bar{\Pi}$ is the gross target inflation rate of the central bank. This objective is consistent with the hypothesis that firms not choosing the optimal price in period t adjust their prices according to

$$P_t(f) = \bar{\Pi} P_{t-1}(f),$$

which corresponds to automatic adjustment by the amount of the inflation target.[12]

Summing up, each firm chooses $P_t(f)$, $L(f)$, and $K(f)$ to maximize expected future profits according to (5), taking as given $\{P_T,\ Q_{t,T},\ Y_T,\ R_T^K,\ W_T(f)\}$ for every $T > t$.

2.4. The central bank

We assume that monetary policy is conducted according to a time-invariant, Taylor-type policy rule. The central bank sets the short-term nominal interest rate in response to deviations of inflation and output growth from the inflation target and the long-run growth trend, respectively. The rule is given by

$$i_t = \rho i_{t-1} + (1-\rho)\left[i + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (g_{y,t} - g_y)\right] \tag{6}$$

[11] This is a standard result implied by the consumption aggregator.

[12] This updating rule is chosen to keep the model simple. It could be objected that in periods of high inflation firms should change prices every period according to past inflation, as suggested by Christiano, *et al.*, (2001). We choose instead to keep the inertia in the inflation rate generated by the model under rational expectations as low as possible, in order to be able to better assess the role of learning in generating persistence.

where i_t is the net nominal interest rate. We let $\bar{\pi}$ denote the central bank's net inflation target (we define $\pi \equiv \ln \Pi$). The parameters ϕ_π and ϕ_y describe the strength of the central bank's reaction to inflation and output growth deviations in the Taylor-type rule. The parameter ρ describes the degree of sluggishness or inertia in the policy rule, which we will set to be consistent with estimates from the literature. We let $g_{y,t} = y_t - y_{t-1} + \ln \gamma\eta$ denote the observed output growth rate, and g_y is the growth trend. Finally, i is the level of the nominal interest rate consistent with output growing along the balanced growth path and inflation at target.

The choice of a rule that responds to the output growth gap instead of to the output gap is dictated by the particular model used in the paper. It can be shown that the steady state level of output per efficiency unit is negatively related to the growth rate of productivity (see the calculation in the appendix, especially equation (47)). Hence, if the monetary authority was assumed to respond to deviations of output per efficient worker from its estimated steady state value, a decrease in the productivity growth rate would actually lead the central bank to *increase* the interest rate, not to decrease it. The effect would go in the wrong direction from the perspective of evaluating the "misperceived-change-in-trend" hypothesis. This problem is avoided if the bank is assumed to respond to deviations of the output growth rate from the long-run trend growth rate.[13]

2.5. Market clearing, general equilibrium, and linearization

We can now define a stationary rational expectations equilibrium for this model. The analysis of learning and structural change will then occur relative to this baseline equilibrium. The equilibrium requires that in each period households maximize utility, firms maximize profits, markets clear, and expectations are consistent with the distributions generated by the decisions of the firms and households and the stochastic structure of the economy. The market clearing assumption means

$$\int \int L^h (f)\, dh df = \int L(f)\, df$$

for the labor markets,

$$\int K_t (f)\, df = \int K_t^h dh$$

for the capital market, and

$$Y_t = \int \left(C_t^h + I_t^h \right) dh$$

for the goods market. By linearizing the first order conditions for the households and the firms around the balanced growth path, aggregating the individual decision

[13] Orphanides (2001) argues that a Taylor rule that reacts to output growth may be more stabilizing than a rule that responds to the output gap, in the sense that the implied policy mistake is smaller. This conclusion does not necessary hold in the present model because the central bank will be directly estimating the trend growth rate instead of the level of potential output.

rules, using market clearing conditions, and imposing rational expectations, we can express the system in four equations. These equations involve the following variables: (1) y_t, the logarithm of output expressed in efficiency units,[14] (2) π_t, the inflation rate, (3) k_t, capital expressed in efficiency units, and (4) i_t, the nominal interest rate, as determined by (6). We provide the complete derivation of the equilibrium as a set of linearized equations in the technical appendix. The linearized system is given by

$$\begin{aligned} y_t &= a_{y,0} + a_{y,1} E_t \left(i_t - \pi_{t+1} \right) + a_{y,2} E_t y_{t+1} \\ &\quad - a_{y,3} E_t \left(k_{t+2} - k_{t+1} \right) + a_{y,4} \left(k_{t+1} - k_t \right), \end{aligned} \tag{7}$$

$$\pi_t = a_{\pi,0} + a_{\pi,1} y_t + a_{\pi,2} k_{t+1} + a_{\pi,3} k_t + a_{\pi,4} E_t \pi_{t+1}, \tag{8}$$

$$k_{t+1} = a_{k,0} + a_{k,1} E_t y_{t+1} + a_{k,2} E_t k_{t+2} + a_{k,3} y_t + a_{k,4} k_t, \tag{9}$$

$$i_t = a_{i,0} + a_{i,1} \pi_t + a_{i,2} y_t + a_{i,3} y_{t-1} + a_{i,4} i_{t-1}. \tag{10}$$

where the coefficients $a_{\ell,j}$, $\ell = y, \pi, k, i$; $j = 1, 2, 3, 4$; are composites of the underlying parameters of the model as described in the technical appendix. The first two equations are versions of the IS and Phillips curve equations often discussed in the New Keynesian literature, the third equation is a law of motion for the capital stock, and the fourth equation is the assumed policy rule. We stress that we have written these equations with steady state values of the variables embodied in constant terms for each equation (that is, $a_{\ell,0}$, $\ell = y, \pi, k, i$).[15]

Equation (7) differs from the more commonly studied version of the model, which has no capital accumulation, in that investment appears. An expected increase in investment has a negative effect on consumption and therefore output. Equation (8) resembles standard versions obtained in models without capital, with the addition that capital affects the average marginal cost. One important aspect of this equation is that higher investment will tend to put upward pressure on inflation. On one hand, higher investment means additional capital and thus lower future marginal costs, which has a negative effect on inflation. On the other hand, higher investment means higher aggregate demand, which implies upward pressure on inflation. It is possible to show that the latter effect dominates.

The system (7)-(10) has a unique minimum state variable solution under rational expectations, which can be expressed in matrix form as

$$V_t = \Omega_0^\star + \Omega_1^\star V_{t-1} + \epsilon_t, \tag{11}$$

where $V_t = [y_t,\, \pi_t,\, k_{t+1},\, i_t]'$, $\Omega_0^\star$ and $\Omega_1^\star$ are conformable matrices whose elements are composites of underlying parameters, and ϵ_t is a vector of *i.i.d.* shocks. We have not provided a description of the nature of the fundamental shocks in this economy contained in the vector ϵ_t. This is because we wish to set these shocks

[14] That is $y_t = \ln (Y_t / X_t N_t)$.

[15] This will be important under learning, as the productivity slowdown will change the values of these constant terms, and we wish to force the agents to learn the new values through recursive updating.

to zero in order to isolate the nature of the transition of the economy to a new rational expectations equilibrium following a productivity slowdown. We describe our methodology for calculating the transition path in the next section.

3. STRUCTURAL CHANGE AND LEARNING

3.1. Recursive learning

In this section we introduce structural change and learning using the methodology of Evans and Honkapohja (2001). We discuss this in general terms in this subsection, and then provide a more technical discussion beginning in the next subsection. From equations (7) and (10), the evolution of the state of the system depends in part on agents's expectations. We assume that the agents do not know the true structure of the economic model, and so they do not initially have rational expectations. Instead, they behave as econometricians. We endow the agents with a parametric model of the economy that they use for prediction. This parametric model is also known as a *perceived law of motion*. The agents observe the main macroeconomic variables each period and then re-estimate the coefficients in their parametric model. The agents' model is appropriately specified in that it is consistent with the rational expectations equilibrium (11). It includes the variables in V_t and has the same linear form as (11). Convergence to rational expectations equilibrium occurs if the agents are eventually able to estimate the correct, rational expectations, coefficients of (11). If the learning process converges, the agents have learned how to make rational forecasts, and they behave as if they have rational expectations. The work of Evans and Honkapohja (2001) describes these ideas in a variety of macroeconomic contexts and provides theorems for the convergence of such systems to rational expectations equilibrium. Convergence is not a simple matter since the expectations of the agents help determine actual economic behavior, and thus the elements of V_t, and these data then feed back into the recursive updating of the agents. The systems we study below will be stable in this learning process. We will think in terms of an economy which is initially at a rational expectations equilibrium, is perturbed slightly creating a new rational expectations equilibrium and destroying the old one, and then converges to the new rational expectations equilibrium.

We wish to use this learning concept to study the effects of a one-time unexpected change in one of the key parameters underlying the model. It could be any parameter, but we are particularly interested in a one-time change in the growth rate of productivity. The derivation in the appendix makes it clear that a change in this parameter is going to change the rational expectations coefficients embodied in the matrices $\Omega_0^{\star}$ and $\Omega_1^{\star}$, let's say to new values $\Omega_0^{\star,\prime}$ and $\Omega_1^{\star,\prime}$. However, we will equip the agents with a method of recursive estimation that will allow the agents to learn the values of $\Omega_0^{\star,\prime}$ and $\Omega_1^{\star,\prime}$ using the new information V_t that arrives each period. Hence, the agents will be able to learn the new rational expectations equilibrium without having any special information about what has caused the shift from $\Omega_0^\star$ and $\Omega_1^\star$ to $\Omega_0^{\star,\prime}$ and $\Omega_1^{\star,\prime}$.

During the transition to the new rational expectations equilibrium, prices and quantities are determined by equations (7) through (10) along with the evolution of the agents' expectations. This larger, joint system depends on how we specify the recursive updating process of the agents, and is known as the *actual law of motion*. When actual and perceived laws of motion coincide, rational expectations equilibrium is achieved. Market clearing conditions are satisfied during the transition but the economic system does not converge to a new stationary rational expectations equilibrium until the agents' parametric model converges to the correct model.

The policy rule (6) is written in terms of g_y and i. When there is no structural change (that is, there are no changes in the rate of productivity growth) these terms are constant. But when structural change occurs, we will require all agents to form forecasts of the new long-run growth rate and the new nominal interest rate consistent with the new balanced growth path, because both of these values will change when the productivity growth rate changes. Thus we will place expectations on g_y and i in the learning analysis described below. The hypothesis we wish to investigate then will work as follows. As the productivity slowdown hits the economy, the monetary authority observes a decrease in the current growth rate of output while its current estimates of the growth trend g_y and the nominal interest rate consistent with the balanced growth path, i, initially remain unchanged. Hence, the central bank will initially interpret the productivity slowdown as a negative "output growth gap" and reduce the current setting of the nominal interest rate i_t. Our conjecture is that this will lead to abnormally high inflation. Our objective is to obtain a quantitative assessment of this hypothesis.

We also note that i appears in equation (7), the consumption Euler equation of the households. In order to preserve the symmetry between the monetary authority and the private sector, we will need to place an expectations operator on this term to complete the analysis under learning and structural change. We now turn to describing the system under learning and structural change more specifically.

3.2. Two-sided learning

3.2.1. The perceived law of motion and recursive updating

Under two-sided learning we do not distinguish between the monetary authority and the private sector concerning the nature of the perceived law of motion or the recursive updating in use. Following Evans and Honkapohja (2001), we endow all market participants with a perceived law of motion for the economy. In particular, we assume the agents believe the economy evolves according to

$$V_t = \Omega_0 + \Omega_1 V_{t-1} + e_t, \tag{12}$$

where e_t is an unobservable *i.i.d.* shock. The agents must estimate the elements of the matrices Ω_0 and Ω_1 recursively. This perceived law of motion encompasses the rational expectations equilibrium of (11). We stress that the presence of the constant implies that the agents do not know the steady state value of V, so that when the steady state shifts, the agents will have to learn about it. Using the model (12), the agents form expectations according to[16]

$$E_{t-1}V_t = \Omega_{0,t-1} + \Omega_{1,t-1}V_{t-1},$$

and

$$E_{t-1}V_{t+1} = \Omega_{0,t-1} + \Omega_{1,t-1}\Omega_{0,t-1} + \Omega_{1,t-1}^2 V_{t-1}. \tag{13}$$

The agents update the estimates of the model's parameters using new observations available each period on V_t. We assume the agents use stochastic gradient learning, so that the parameters are recursively estimated according to

$$\theta_t = \theta_{t-1} + \zeta Z_{t-1}\left(V_t - \theta_{t-1}' Z_{t-1}\right)' \tag{14}$$

where $\theta_t' = (\Omega_{0,t}, \Omega_{1,t})$, $Z_t = [1, y_t, \pi_t, k_{t+1}, i_t]$, and ζ is the constant gain. As in nearly all learning schemes employed in macroeconomics, today's parameter estimates are revised upwards or downwards, depending on a function of the difference between the observed and the predicted variables.

We chose the stochastic gradient specification for two main reasons. It is a gradient descent algorithm which is less complex than recursive least squares, and thus might be viewed as a more plausible description of learning in the macroeconomy. Secondly, we found in the simulations that under recursive least squares, the system quite often leaves the basin of attraction of the rational expectations equilibrium and diverges, even if it is, technically, locally stable. In other words, the basin of attraction is quantitatively small under recursive least squares. In order to achieve stability of the learning process under recursive least squares, we needed to use an extremely small value for the gain parameter, which in turn slowed down the learning process to an empirically implausible rate. Use of stochastic gradient learning avoids this problem.

The constant gain aspect of the learning algorithm (14) has been widely explored in the recent learning literature.[17] An algorithm like recursive least squares puts equal weight on all observations in the data set by setting the gain to $1/t$. A constant gain algorithm puts more weight on recent observations, and discounts the past. A constant gain can be interpreted as allowing decision-makers to allow for the possibility of structural change, because when structural change occurs, they are able to react more quickly via the extra weight given to more recent data. This is exactly the situation in the model we have.[18]

[16] When studying the learning process we follow Evans and Honkapohja (2001) and assume that the economic agents take expectations using $t-1$ information. This is the "dating of expectations" issue that often arises in learning environments. Evans and Honkapohja (2001) have viewed $(t-1)$-dating as more realistic in a learning environment. The assumption of t-dating under learning can be employed at the cost of some complications, but then date t variables are being used to form expectations and are also being determined by the system at date t. The simultaneity is common in models solved under RE, but is less satisfactory in a learning environment.

[17] For extensive discussion and intuition, see Sargent (1999).

[18] In the case of constant gain algorithms, the learning proces does not completely converge to

3.2.2. Learning i and g_y

In the simulations below we consider two ways of calculating the trend in output growth. Under the *simple method*, we assume that the agents estimate the trend of output growth by minimizing the squared deviations of output growth from the constant trend. This leads to the following recursive estimate of g_y,

$$\begin{aligned} E_t g_y &= E_{t-1} g_y + \zeta \left(g_{y,t} - E_{t-1} g_y \right) \\ &= E_{t-1} g_y + \zeta \left(y_t - y_{t-1} + \ln \gamma\eta - E_{t-1} g_y \right) \end{aligned} \tag{15}$$

where $E_t g_y$ is the estimate of the growth rate at time t.

Under this method the agents do not efficiently use all the information available to estimate the growth trend. They might use other relevant variables to improve the estimates, as under the model consistent method outlined below. But in order to do that the agents need to know more details about the model of the economy. The simple estimation procedure does not require such precise knowledge about the economy. In this sense, the simple method keeps the assumption of bounded rationality applicable to the monetary authority.

A more ambitious approach, which we call the *model consistent method*, uses more sources of information to estimate the output growth trend. At the rational expectations equilibrium, it is possible to express the growth rate of output as a function of past output, capital and the interest rate. We can then assume that the perceived low of motion of the growth rate of output is

$$E_t g_y = \omega_0 + \omega_1 y_{t-1} + \omega_2 k_{t-1} + \omega_4 i_{t-1}, \tag{16}$$

where ω_i, $i = 1, 2, 3, 4$ are coefficients to be estimated. By estimating this equation the central bank and the private sector can form an estimate of the trend growth rate by considering its steady state

$$g_y = \omega_0 + \omega_1 y + \omega_2 k + \omega_4 i.$$

Naturally, the agents do not know the steady state values of the system and need estimates for them. These estimates can be found by using the estimated θ_t.

We also assume that the monetary authority and the agents employ (17) together with the simple or the model consistent estimate of the output growth rate to estimate the long run nominal interest rate. Both the central bank and the private sector need an estimate of the long run nominal interest rate in order to have a benchmark rate i for the application of the Taylor-type policy rule. In order to simplify the analysis, we assume that the agents use the steady state relation between the growth rate of output and the nominal interest rate in order to estimate

the rational expectations equilibrium, because the gain sequence does not decline to zero, so that the agents keep re-estimating the model's parameters. The parameter estimates converge instead to an invariant distribution around the rational expectations equilibrium values. For details, see Evans and Honkapohja (2001). We found this effect to be small in this model, and so we do not discuss it further here.

the long run nominal rate. In steady state

$$g_y - \ln \eta\beta + \bar{\pi} = i.$$

Hence, both the central bank and the private sector can use the following estimate of the long run nominal interest rate

$$E_{t-1} i = E_{t-1} g_y - \ln \eta\beta + \bar{\pi}. \tag{17}$$

Here we are implicitly assuming that the agents have already learned the value $-\ln \eta\beta + \bar{\pi}$. This is a constant which does not change when the rate of productivity growth changes. We start our systems on a balanced growth path, and so this seems like a reasonable assumption. In some simulations we will change the inflation target $\bar{\pi}$ unexpectedly. We are assuming that once this change is announced all agents immediately adjust their nominal interest rate target i downward. Thus the inflation target change is viewed as fully credible when announced. If the agents were learning about the inflation target, the system would not have any nominal anchor (that is, the steady state inflation rate would not be uniquely determined). This is because both the private sector and the central bank are learning about the real rate and in the model it is the Taylor-type policy rule that fixes the long run equilibrium value of the nominal interest rate. The assumption of perfect knowledge of the inflation target is thus necessary, even though this implies that any change in the inflation target is assumed to be fully credible.

3.2.3. The model under learning

The linearized model can be expressed in matrix notation. The model can be re-written to a four dimensional system of equations expressed as

$$V_t = B_1 + B_2 \begin{bmatrix} E_{t-1} g_y \\ E_{t-1} i \end{bmatrix} + B_3 E_{t-1} V_{t+1} + B_4 V_{t-1} + \epsilon_t, \tag{18}$$

where again $V_t = [y_t,\ \pi_t,\ k_{t+1},\ i_t]'$ and where $B_i,\ i = 1, 2, 3, 4$ are conformable matrices. The agents in the model need to forecast the one period ahead evolution of V_t and the long-run values of inflation and the growth rate of output. Inserting the expectations (13), (17) and either (15) or (16) in (18) we obtain the *actual law of motion* (ALM) of the economic system, which describes the evolution of the macrovariables over time in the system under learning.[19] We do not provide a formal proof for the stability of this system under learning. However, we did verify stability with simulations.

3.3. One-sided learning

In the quantitative analysis we consider the case in which only the central bank is learning while the private sector has rational expectations.[20] The implications

[19] We provide more detail on the actual law of motion under alternative assumptions in the appendix.

[20] This assumption was used by Lansing (2001).

of this assumption in the present model are strong. Rational expectations on the part of the private sector means that each firm can observe not only the change in its own productivity but also the change in productivity in all other firms. This conflicts with the hypothesis of decentralized markets. Also, each household in making consumption and investment decisions is assumed to perfectly monitor the change in productivity. Hence the average household knows more than the monetary authority about economy-wide firm productivity. Despite these implications, we want to compare the dynamics of inflation, the output growth gap, and inflation expectations in this case to see whether it is more in line with the data.

4. QUANTITATIVE DYNAMICS IN THE BASELINE ECONOMY

4.1. The baseline economy

We have outlined several possible versions of the model. In in order to organize the discussion, we will begin by presenting results for a baseline case. We think of the baseline economy as having two-sided learning—both the central bank and the private sector learn using the same perceived law of motion and the same recursive updating methodology. We assume the simple learning method (as opposed to the model consistent method) for the growth trend and the long-run level of the nominal interest rate. Later in the paper, we consider variations on this baseline and show how results are affected.

4.2. Calibration

We do not wish to challenge the calibration of this model due to Woodford (2003) for the purposes of this paper. Therefore, we simply follow Woodford (2003) closely for the calibration of the following parameters of the model to quarterly data. The parameter choices are designed to allow the model to match empirical impulse-response patterns following a monetary policy shock under rational expectations. We assume the disutility from labor to be nearly linear, assigning $\nu = 0.11$. We set the discount factor $\beta = 0.9987$. Concerning the production side, we set the capital share in the production function $\alpha = 0.25$, the depreciation rate of capital $\delta = 0.012$, and the adjustment cost coefficient $\epsilon_{\psi} = 3$. Also, we set $\theta = 7.88$, which implies a mark-up of about 15 percent, and $a = 1.34$. In contrast to Woodford (2003), we set the probability of not changing the price $\xi = 0.78$, which is in line with the macroeconomic evidence but higher than Woodford's choice. This is a consequence of our assumption of homogeneous capital, and it lowers the degree of persistence of inflation.[21] These parameters imply that $\omega = 0.47$ and $\bar{\omega} = 0.08$.

[21] If we added firm-specific capital in the model, we could set $\xi = 0.66$, which is the estimated parameter in Woodford (2003), and still get the same persistence in the inflation rate. The choice in the text is still below the estimate of Gali and Gertler (1999). They set $\xi = 0.83$, under a homogeneous labor assumption. Sbordone (2002) estimates $\xi = 0.66$ but assumes fixed capital at the firm level. A model with firm-specific capital and $\xi = 0.66$ would give an inflation equation equivalent to our formulation.

This parameterization implies that firms re-optimize their price every 4.5 quarters on average. We stress that firms *do* change their prices every quarter in the model, even if not optimally at each date. Taylor (1999) characterized the literature on price change as suggesting that prices change about once per year on average. Our prices change more often than this. Bils and Klenow (2002) examine an extensive data set on price change from 1995 to 1997. They report that half of consumption prices change more often than once every 4.3 months. This is closer to the type of assumption made here. We remark that this model cannot effectively address the heterogeneity in the frequency of price change that is a prominent feature of the data Bils and Klenow study. Also, the Bils and Klenow study addresses frequency of price change instead of the frequency of price re-optimization, which might be viewed as the more relevant concept. Eichenbaum and Fisher (2003) suggest that the standard Calvo model of pricing can provide a good empirical fit to macroeconomic data, but only if firms are viewed as re-optimizing their price just once every $2\frac{1}{2}$ years. Their extended version of the Calvo pricing model with immobile capital and non-constant elasticity of demand implies firms re-optimize more often, about once per year. This latter estimate is closer to the assumption made here, and the model is closer to the one we study. We do not have immobile capital but firms do face capital stock adjustment costs.

We assume that the monetary authority uses the same Taylor-type policy rule for the whole sample. For our baseline simulations we choose a value of $\rho = 0.2$, in line with the estimate of Erceg and Levin (2001) for a similar rule. We set ϕ_π to 1.5, as in the standard Taylor rule and the coefficient on output growth, $\phi_y = 0.5$, consistent with the choice of Woodford (2003) for a similar rule. This is also consistent with the assumption that the Federal Reserve emphasized output stabilization in the seventies (see, for instance, the evidence in Orphanides (2001)). We set the central bank's inflation target to 4 percent, the approximate level of inflation before the onset of the productivity slowdown.

We calibrate the change in productivity using the estimated trend under learning calculated by Bullard and Duffy (2002). They find a productivity break in the third quarter of 1973. Productivity growth falls (in annual rates) from 2.47 percent to 1.21 percent. They also estimate the growth rate of the labor force. For the period that we consider they find a growth rate of 1.88 percent (the quarterly gross rate is $\eta = 1.00467$). This leads to a change in the real output trend growth rate from 4.36 percent to 3.10 percent, at an annual rate, following the productivity slowdown. We also include an increase in productivity growth (the "new economy") beginning in the third quarter of 1993. Bullard and Duffy (2002) estimate an increase in productivity growth to an annual rate of 1.86 percent at that date. This implies a trend output growth rate of 3.75 percent. The corresponding quarterly values for γ_i, $i = 0, 1, 2$ (corresponding to the gross rate of productivity growth before the productivity slowdown, after the productivity slowdown, and after the onset of the new economy) are given by $(\gamma_0, \gamma_1, \gamma_2) = (1.00612, 1.00301, 1.00462)$.

We calibrate the gain ζ (assumed to be the same for both the central bank and the private sector) to 0.03. This yields a plausible speed of learning, implying that the central bank almost fully detects the productivity break by 1980. This is consistent with the discussion in Orphanides (2003).

Our goal is to study the transitional behavior of key macroeconomic variables under the recursive learning assumption following a change in the productivity growth rate. The learning assumption requires that a stochastic structure (four shocks, one for each equation) be specified for the economy in order to allow recursive estimation to proceed.[22] One method would be to include fundamental shocks to the economy, say to technology, monetary policy, and other aspects, simulate the model with these shocks along with the changes in trend many times, and average the result. We could then trace out the average effect of a change in productivity growth on the key variables in the economy. However, we would have to take a stand on the nature of the fundamental shocks in order to proceed in this fashion, and we think it might be more difficult to interpret the results. Therefore, we do not pursue this approach.[23] Instead, we trace out the average effect more directly by adding a shock with low variance to each equation (7) through (10).[24] These shocks are necessary to allow the recursive estimation we have assumed, but have such small variance that they do not materially affect the transition paths we report below. Other aspects of the simulation, when changed, *do* materially affect the transition paths, and we focus on those aspects in the next section.

4.3. Main results for the baseline economy

4.3.1. Inflation

We first discuss the effects of an unexpected slowdown in productivity of the magnitude observed during the 1970s on output growth and inflation. We begin with the inflation process. One of the primary findings is that the model can generate a sizable increase in inflation following this type of shock. As shown in Figure 2, the model predicts an increase in inflation from four percent in 1970 (the steady state) to more than seven percent in 1976. Thus the benchmark economy generates an increase in inflation peaking more than 300 basis points above steady state in response to the productivity slowdown. For comparison purposes, we have also plotted the personal consumption expenditure inflation data from Figure 1 in this figure. We conclude that the basic economic mechanism—a misperceived change in trend interacting with a Taylor-type policy rule—causes a significant amount of unintendedly high inflation in the baseline case.

The increase in inflation following the productivity slowdown, while significant, is also far too persistent. According to Figure 2, inflation does not fall sharply until a second productivity growth shock occurs, which is the beginning of the "new economy" in 1993. The model economy therefore misses the sharp disinflation of

[22] Otherwise, the model variables are perfectly colinear and recursive estimation breaks down.

[23] See Bullard and Duffy (2002) for an application like this.

[24] We use a standard deviation of 1×10^{-5}.

the early 1980s.[25] We conclude that, according to this model, the productivity slowdown could have sparked much of the observed increase in inflation during the 1970s, but that without other structural changes, the inflation rate would have stayed relatively high for many more years. The intuition for this result seems clear. The policy rule is designed to return inflation to target slowly following normal deviations caused by business cycle shocks. The rule assumes that policymakers have a good guess about the nature of the balanced growth path for the economy. It is not designed to cope with large inflation deviations caused by misperceptions of the nature of the balanced growth path. This is both what allows the run-up in inflation following the productivity slowdown, and also what allows that increase to be so persistent. Better policy rules might be designed, but they would have to take into account the possibility that the productivity growth rate may be subject to significant but rare changes in mean.[26]

If we take the view that the model is a reasonable approximation of the economy, the overly persistent response of inflation might be understood as evidence that the Taylor-type policy rule changed in some way after the 1970s.[27] We consider just one possible change, namely that the central bank credibly changes the inflation target to two percent in 1980. As Figure 3 demonstrates, this change helps capture aspects of the Volker disinflation. Inflation falls appreciably in the early 1980s and later disinflation is associated with the productivity acceleration beginning in the 1990s. This change in the policy rule is arbitrary and we do not wish to place too much emphasis on it. We do think it is suggestive of the idea that productivity growth changes can be coupled with policy changes to help match the postwar inflation data.

Inflation is only one variable in the model. We now turn to checking other aspects of the baseline case to see if they are at odds with the postwar data or not.

4.3.2. Output growth

After the productivity slowdown, the growth rate of real output decreases. Figure 4 shows the real output growth trend implied by the baseline model with a change in the inflation target against the actual growth rates in the U.S. data. In general, the growth trend from the model tends to track the longer run behavior of output growth in the data quite well. Perhaps not surprisingly, actual output growth rates are quite volatile compared to the trends coming from the model. The intuition behind Figure 4 is that the exogenous growth assumption combined with learning produces average growth rates for the model economy which fit the data

[25] Despite the appearances of the figure, it would eventually converge back to the steady state rate of four percent—but that process takes many years.

[26] For a discussion of optimal policy in a much simpler but related context of this type, see Bullard and Schaling (2002).

[27] Possible changes could be a downward shift in the inflation target, as in Huh and Lansing (1999) and Erceg and Levin (2001), or a change in the parameters of the Taylor rule. Orphanides (2001) suggests that the coefficient on the output gap has decreased after the 1980. Also, Clarida, Gali and Gertler (2000) find that the inflation coefficient is higher in the post-Volker sample.

better than an assumption of balanced growth at a constant rate, since trend breaks are allowed and the learning adjustment is relatively smooth.

4.3.3. Inflation expectations

The hypothesis we examine in this paper is one where expectations play a key role. An important question is whether the expectations suggested by the model bear any resemblence to the data on macroeconomic expectations during the post-war era, to the extent that we have such data. A key finding is that the model captures the behavior of actual and expected inflation surprisingly well, as shown in Figures 5 and 6. Figure 5 displays inflation and inflation expectations data for the U.S.—we stress that both lines in this figure are actual data. The expected inflation series is expected GDP deflator inflation four quarters ahead as measured by the median projection from the Survey of Professional Forecasters. The inflation series is GDP deflator inflation (from NIPA). This inflation data is a different measure than what is shown in Figure 1, in order to keep the measured expectations matched with the actual inflation rate professional forecasters were surveyed about. As is well known, inflation expectations were below actual inflation until the early 1980s, and then stay consistently above during much of the remainder of the sample. We take this as the stylized fact from the U.S. data on inflation expectations.

Figure 6 shows inflation and inflation expectations from the model. These inflation expectations are in line with the data in the sense that inflation expectations are too low in the 1970s, and then too high in the remainder of the sample. Our theory suggests that this feature of the data is exactly what we should expect to observe if economic actors have to learn about structural changes in productivity. The economic intuition for this result is clear: If the agents believe that productivity growth is higher than it actually is, then they will associate less inflation with Federal Reserve policy than is warranted, resulting in expected inflation which is too low relative to actual outcomes.

4.3.4. Policy mistakes

Orphanides (2000, 2001, 2002, 2003) suggests that the misperception of the policy authorities concerning the nature of the balanced growth path was substantial during the 1970s. We can assess the magnitude of the policy implied mistake using this model. Figure 7 shows the misperception of the output growth gap, on which the policy mistakes of the central bank are based. Because the central bank and the private sector actors are learning, it takes time for the monetary authority to detect the decreased rate of output growth following the productivity slowdown. Observing slow output growth, the authorities initially conjecture a larger output growth gap than is warranted. This causes them to set a lower interest rate and induce a higher inflation rate than what would otherwise occur. This effect is reinforced by the higher inflation expectations of the private sector.

4.4. One-sided learning

Our results for the baseline economy are based on the assumption that both the private sector and the central bank are learning. In this subsection we consider the case where only the central bank learns, and the private sector makes rational forecasts, knowing that the central bank is learning. We wish to assess how important it is for our baseline results that we assume the private sector must learn about structural changes as well. The actual law of motion for this case is outlined in the appendix. It turns out that this version of the model does not provide a qualitative match to the data. First, under the assumption that the private sector fully adjusts to the decrease in the real rate implied by the productivity slowdown, the model predicts a *drop* in inflation rather than an increase in inflation! This because the private sector agents are aware that the monetary authority is using a constant in the Taylor-type policy rule which is too high. In fact, the real interest rate drops after the productivity slowdown, requiring a lower constant in the Taylor rule.

We also simulate the model under two additional assumptions: (1) that both the policy authorities and the private sector observe the initial change in the real interest rate, and (2) that the private sector has to estimate the real rate of interest but that they make the same rational forecasts in all other respects. The former assumption is common in the literature.[28] Under these two additional assumptions, the model with only central bank learning predicts a smaller inflation, peaking about 100 basis points higher than steady state following the productivity slowdown. This simulation has two difficulties. One is that, given that the private sector is assumed to know the mistake of the central bank about the output growth gap, inflation expectations are predicted to be higher than actual inflation, which conflicts with the evidence presented in Figure 5. A second problem is that the model does not predict the downturn in the growth rate of output that is present in the data.

This leads us to the conclusion that the hypothesis of both the policymaker and the private sector learning is the most appropriate one to capture the behavior of the U.S. economy after the productivity slowdown.

4.5. Model consistent growth estimates by the central bank

In our baseline economy, the central bank uses the simple method to estimate a trend growth rate for the economy. In this section we describe how our findings change if the central bank uses the model-consistent estimator of the growth rate. First of all, and perhaps not surprisingly, such an estimator seems to be more efficient in estimating changes in the trend rate of output growth. In general, it takes less time to learn about most of the change in trend under the model consistent method. In addition, the inflation rate is lower through the whole sample. Nevertheless, as Figure 8 suggests, the long run behavior of the inflation process seems to be quite consistent with the data. This shows that even a more efficient

[28] See, for example, Orphanides (2001).

use of information than we have assumed in our baseline economy would not have avoided most of the inflation observed in the U.S. data.

5. DISCUSSION

We have analyzed the effects of permanent changes in productivity trends on inflation, when the economy's actors must learn about the changes in trend and the central bank is committed to using a Taylor-type policy rule. We find that a productivity slowdown of the magnitude observed in the 1970s causes a significant and persistent rise in inflation in the model economy, peaking at more than 300 basis points. An increase in the rate of productivity growth—the "new economy" of the 1990s—then causes a reduction in inflation. These effects alone are not sufficient to provide a qualitative match to the U.S. inflation experience, because of the sharp decline in inflation observed in the early 1980s. However, by adding an unexpected reduction in the inflation target of the central bank, we were able to provide a qualitative match for the data. We conclude that the misperceived-change-in-trend hypothesis has considerable merit in explaining the medium-run dynamics of inflation in the U.S. since 1970.

We think the policy conclusions from this exercise may be quite important. Our analysis suggests that, should a shock of the magnitude of the productivity slowdown occur again in the future, it could generate a considerable inflation disturbance, even if policymakers do their best to remain committed to a Taylor-type policy rule and to estimate the changing growth trends in the economy. Thus, when evaluating Taylor-type policy rules, an additional criterion might be, *How well does the rule insulate the economy in the event of structural change?*

We have analyzed this economy under a hypothesis of rare, permanent shocks to productivity growth. We think this is a good characterization of the data based on our reading of the econometric literature concerning structural change. Agents protect themselves against the possibility of such a rare shock by employing a version of constant gain learning. Because the system is stable under learning, the agents can then adapt to the new rational expectations equilibrium following a structural break. We view this approach as one model-consistent method of addressing issues like this. There is another method, which we think is also interesting, but ultimately less satisfactory. That method retains the rational expectations assumption, and models the permanent shocks that appear to be in the data as a regime-switching process. Agents understand that there are two (or more) regimes, and rationally infer which regime they are in and how likely they are to transit to an alternative regime when making decisions. In this approach, the rational expectations assumption is completely consistent with the model, and the dynamics of the economy are completely characterized by a rational expectations equilibrium. Again, we think this is an interesting approach. The drawback is that the agents in the model must have specific alternative regimes in mind, along with the associated transition probabilities, when making decisions. Thus, as usual, the informational demands of the rational expectations assumption are stringent. In reality, there are many possible alternative regimes, most of which have rarely, if ever, occurred. Under the learning approach we have used here, the agents are in some sense prepared to adapt to *any* type of structural change that might occur in the economy, so long as it is not so disruptive as to destabilize the system. Therefore, we think the approach we have outlined here provides the most reasonable modelling strategy.

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APPENDIX A: DERIVATION OF THE LINEARIZED MODEL

A.1. Notation

Given the many changes of variables we use below a few comments about the notation are needed. Because of the exogenous growth assumption, a given variable Y_t may possibly be non-stationary in our model. Aggregate output and aggregate capital, for instance, will grow over time. Where necessary, we denote such a variable in stationary form as $\hat{y}_t \equiv \frac{Y_t}{X_t N_t}$. We will take logarithmic deviations from a steady state for some purposes, and so we define $\tilde{y}_t \equiv \ln\left(\frac{\hat{y}_t}{\bar{y}}\right)$, where $\bar{y}$ is the steady state value of the stationary variable $\hat{y}$. And finally, for our learning model we will want to refer to the logarithm of the steady state component of this deviation separately, and so we denote $y_t = \ln \hat{y}_t$ and $y = \ln \bar{y}$.

A.2. Household behavior

A.2.1. First order conditions

From the asset accumulation decision, we obtain the Euler equation

$$1 + i_t = \tilde{\beta}^{-1} \Lambda_t \left(E_t \Lambda_{t+1}\right)^{-1} \tag{19}$$

where

$$\Lambda_t = C_t^{-1}/P_t \tag{20}$$

is the Lagrange multiplier associated with the optimization problem and $C_t = \int C_t^h dh = C_t^h$. By defining $\hat{\lambda}_t = \Lambda_t P_t \left(X_t N_t\right)$ we can express (19) as the stationary equation

$$1 + i_t = \gamma \beta^{-1} \hat{\lambda}_t \left(E_t \Pi_{t+1} \hat{\lambda}_{t+1}\right)^{-1} \tag{21}$$

where $\Pi_t = P_t/P_{t-1}$. The labor supply decision for each type of labor f is determined by

$$\frac{a\left[L_t\left(f\right)\right]^{\nu}}{N_t}\left(\frac{W_t\left(f\right)}{P_t}\right)^{-1} = \Lambda_t P_t \tag{22}$$

where $\frac{W_t(f)}{P_t}$ is the real wage. We assume that the labor market is competitive, so that the households take the wage as given. Equation (22) can also be expressed in stationary form, which is

$$\frac{a\left[L_t\left(f\right)\right]^{\nu}}{\hat{w}_t\left(f\right)} = \hat{\lambda}_t. \tag{23}$$

where $\hat{w}_t\left(f\right)$ represents the real wage per efficiency unit $W_t\left(f\right)/P_t N_t X_t$. We have dropped the subscript h because the labor supply decision is identical across households, given that they have identical preferences and make identical consumption decisions (in particular, $L_t(f) = \int L_t^h\left(f\right) dh = L_t^h\left(f\right)$). Moreover, bond and money holdings will be the same for every agent. The first order condition for capital, expressed in per-efficiency units $\hat{k}_{t+1}$, is

$$I'\left(\frac{\gamma\eta\hat{k}^h_{t+1}}{\hat{k}^h_t}\right) = E_t Q_{t,t+1}\Pi_{t+1}\left[R^K_{t+1} + \frac{\gamma\eta\hat{k}^h_{t+2}}{\hat{k}^h_{t+1}} I'\left(\frac{\gamma\eta\hat{k}^h_{t+2}}{\hat{k}^h_{t+1}}\right) - I\left(\frac{\gamma\eta\hat{k}^h_{t+2}}{\hat{k}^h_{t+1}}\right)\right] \tag{24}$$

where

$$Q_{t,t+1} = \frac{U'\left(\hat{c}_{t+1}\right) P_t}{U'\left(\hat{c}_t\right) P_{t+1}} \frac{\tilde{\beta}}{\gamma}$$

is the stochastic discount factor as defined in Woodford (2003).

A.2.2. Linearization

We wish to linearize the model about the nonstochastic balanced growth path in order to be able to apply the learning methodology of Evans and Honkapohja (2001). We begin by considering household behavior.[29] Concerning the labor supply we obtain

$$\tilde{w}_t\left(f\right) = \nu \tilde{L}_t\left(f\right) - \tilde{\lambda}_t \tag{25}$$

where we recall that $\tilde{x}_t$ denotes a logarithmic deviation of a stationary variable $\hat{x}_t$ from the deterministic balanced growth path value $\bar{x}$. The Euler equation becomes

$$\tilde{\lambda}_t = E_t\left(\tilde{\imath}_t - \tilde{\pi}_{t+1}\right) + E_t\tilde{\lambda}_{t+1}. \tag{26}$$

Linearizing (24) and averaging across households, we find the following approximation for capital dynamics

$$\tilde{\lambda}_t + \epsilon_\psi\left(\tilde{k}_{t+1} - \tilde{k}_t\right) = E_t\tilde{\lambda}_{t+1} + \left[1 - \frac{\tilde{\beta}}{\gamma}\left(1-\delta\right)\right] E_t\tilde{R}^K_{t+1} + \frac{\tilde{\beta}}{\gamma}\epsilon_\psi E_t\left(\tilde{k}_{t+2} - \tilde{k}_{t+1}\right). \tag{27}$$

Finally, linearizing (2) we obtain

$$\tilde{I}_t = \frac{\bar{k}}{\bar{y}}\left[\gamma\eta\tilde{k}_{t+1}\left(f\right) - \left(1-\delta\right)\tilde{k}_t\left(f\right)\right], \tag{28}$$

A.3. Firm behavior

A.3.1. First order conditions

Cost minimization for firm f implies the following first order condition with respect to labor and capital services

$$S_t\left(f\right) = \frac{W_t\left(f\right)}{P_t}\frac{N_t}{MPL_t\left(f\right)}. \tag{29}$$

where $S_t\left(f\right)$ is real marginal cost and the marginal product of labor is given by

$$MPL_t\left(f\right) = \left(1-\alpha\right) K_t\left(f\right)^\alpha \left(X_t N_t\right)^{1-\alpha} L_t\left(f\right)^{-\alpha}. \tag{30}$$

[29] We provide the steady state values of variables in terms of parameters later in this appendix.

The real rental rate of capital is

$$R_t^K = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t\left(f\right)}{P_t}\frac{N_t L_t\left(f\right)}{K_t\left(f\right)}. \tag{31}$$

Given the assumption of an economy-wide capital market each firm faces the *same* rental price of capital, and thus it is not indexed by f. At the same time, each firm faces a different wage in the labor market for the type of labor needed.

A.3.2. Linearization

Linearization of (31) gives

$$\tilde{R}_t^K = \tilde{w}_t\left(f\right) + \tilde{L}_t\left(f\right) - \tilde{k}_t\left(f\right), \tag{32}$$

while linearizing the production function and the marginal cost gives

$$\tilde{y}_t\left(f\right) = \alpha\tilde{k}_t\left(f\right) + \left(1-\alpha\right)\tilde{L}_t\left(f\right), \tag{33}$$

and

$$\tilde{S}_t\left(f\right) = \tilde{w}_t\left(f\right) - \alpha\tilde{k}_t\left(f\right) + \alpha\tilde{L}_t\left(f\right) \tag{34}$$

respectively. The linearized price setting equation is

$$E_t\sum_{j=0}^{\infty}\left(\tilde{\beta}\xi\right)^j\left[\tilde{P}_t^\star - \tilde{S}_{t+j}\left(f\right) + \left(\sum_{i=1}^{j}\bar{\pi} - \sum_{i=1}^{j}\pi_{t+i}\right)\right] = 0 \tag{35}$$

where $P_t^\star$ is the optimal relative price.

Combining (31), (32), (33) and (34), we obtain the following expression for the individual firm's real marginal cost

$$\tilde{S}_t\left(f\right) = \omega(\tilde{y}_t\left(f\right) - \tilde{k}_t\left(f\right)) + \nu\tilde{k}_t\left(f\right) - \tilde{\lambda}_t, \tag{36}$$

where $\omega = \omega_p + \omega_w$, $\omega_p = \frac{\alpha}{1-\alpha}$, and $\omega_w = \frac{\nu}{1-\alpha}$. The average real marginal cost can thus be expressed as

$$\tilde{S}_t = \omega(\tilde{y}_t - \tilde{k}_t) + \nu\tilde{k}_t - \tilde{\lambda}_t. \tag{37}$$

Also, the equation for the real rental price of capital becomes

$$\tilde{R}_t^K = \omega_w\tilde{y}_t\left(f\right) + \tilde{c}_t + \frac{\omega_w}{\nu}\tilde{y}_t\left(f\right) - \nu\left(\frac{\omega_w}{\nu} - 1\right)\tilde{k}_t\left(f\right) - \omega_p\tilde{k}_t\left(f\right) - \tilde{k}_t\left(f\right).$$

Simplifying this expression we get

$$\begin{aligned} \tilde{R}_t^K &= \omega_w\left(\frac{\nu+1}{\nu}\right)\tilde{y}_t\left(f\right) - \left[\nu\left(\frac{\omega_w}{\nu} - 1\right) + \omega_p + 1\right]\tilde{k}_t\left(f\right) - \tilde{\lambda}_t \\ &= \left(\omega+1\right)\left(\tilde{y}_t\left(f\right) - \tilde{k}_t\left(f\right)\right) - \nu\tilde{k}_t\left(f\right) - \tilde{\lambda}_t \\ &= \rho_y\tilde{y}_t\left(f\right) - \rho_k\tilde{k}_t\left(f\right) - \tilde{\lambda}_t \end{aligned} \tag{38}$$

where $\rho_y = (\omega + 1)$ and $\rho_k = \rho_y - \nu$. Substituting (38) in (36) to eliminate the capital stock variable we can express the marginal cost in terms of firm f output and the average marginal cost, which is equation

$$\tilde{S}_t = \bar{\omega}\left(\tilde{y}_t - \tilde{y}_t\left(f\right)\right) + \tilde{S}_t\left(f\right) \tag{39}$$

where

$$\bar{\omega} = \frac{\rho_y - \rho_k}{\rho_k}.$$

Linearizing (1) and inserting the optimal price we obtain

$$\tilde{P}_t^{\star} = \frac{\xi}{1-\xi}\tilde{\pi}_t. \tag{40}$$

Using the linearized demand for output of firm f and substituting (40) and (39) in the price setting equation and quasi-differentiating we get the inflation equation

$$\tilde{\pi}_t = \psi\tilde{S}_t + \tilde{\beta}E_t\tilde{\pi}_{t+1} \tag{41}$$

where

$$\psi = \left(\frac{1-\xi}{\xi}\right)\left(\frac{1-\xi\tilde{\beta}}{1+\theta\bar{\omega}}\right).$$

Using (28) in the economy's resource constraint and substituting for consumption in (20) after linearizing we obtain

$$\tilde{\lambda}_t = -\frac{\bar{y}}{\bar{c}}\left[\tilde{y}_t - \frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+1} - (1-\delta)\,\tilde{k}_t\right)\right]. \tag{42}$$

Substituting (42) in (37) we obtain

$$\tilde{S}_t = \left(\omega + \sigma^{-1}\right)\tilde{y}_t - \sigma^{-1}\frac{\bar{k}}{\bar{y}}\gamma\eta\tilde{k}_{t+1} + \left[\sigma^{-1}\frac{\bar{k}}{\bar{y}}(1-\delta) - (\omega - \nu)\right]\tilde{k}_t. \tag{43}$$

where, following Woodford (2003), we define σ as the intertemporal elasticity of substitution in consumption (which is equal to one here because consumption preferences are logarithmic) times $\bar{c}/\bar{y}$.

A.4. The reduced form model

We can now express the model in compact form and compare it to standard models without investment dynamics. However, the deviations form of the model is still not sufficient for our purposes, because we want to analyze learning. When our agents learn, they do not know the steady state values of each economic variable. In order to force them to learn these values when the productivity growth rate changes, we put the logarithms of steady state values into a constant term in a "log-levels" form for each equation. In the model with learning, the constant terms will be estimated recursively by the agents each period.[30]

[30] If we did not do this, we would in effect be telling the agents the value of the new steady state in the event of a structural change. This does not seem very reasonable, and would in any event be contrary to the hypothesis we are trying to investigate.

A.4.1. The output equation

Putting together the equations (26) and (42) implies

$$\tilde{y}_t = -\sigma E_t \left(\tilde{\imath}_t - \tilde{\pi}_{t+1}\right) + E_t \tilde{y}_{t+1} - \\ E_t \frac{\bar{k}}{\bar{y}} \left(\gamma\eta\tilde{k}_{t+2} - (1-\delta)\,\tilde{k}_{t+1}\right) + \frac{\bar{k}}{\bar{y}} \left(\gamma\eta\tilde{k}_{t+1} - (1-\delta)\,\tilde{k}_t\right),$$

which can be also written in terms of investment

$$\tilde{y}_t = -\sigma^{-1} E_t \left(\tilde{\imath}_t - \tilde{\pi}_{t+1}\right) + E_t \tilde{y}_{t+1} - E_t \tilde{I}_{t+1} + \tilde{I}_t$$

as deviations from steady state. This equation can be expressed in log-levels form as

$$y_t = \sigma\left(i - \pi\right) - \sigma E_t \left(i_t - \pi_{t+1}\right) + E_t y_{t+1} - \\ E_t \frac{\bar{k}}{\bar{y}} \left(\gamma\eta k_{t+2} - (1-\delta)\,k_{t+1}\right) + \frac{\bar{k}}{\bar{y}} \left(\gamma\eta k_{t+1} - (1-\delta)\,k_t\right).$$

This equation differs from the standard version of the forward-looking IS curve in that investment appears. An expected increase in investment has a negative effect on consumption and therefore output. Notice that, in order to preserve the symmetry between the central bank and the private sector's information set, we will need to assume that the households need an estimate of the long run nominal interest rate under learning in order to take consumption decisions (that is, the i in the first term on the right hand side will require an expectations operator). Writing this equation is in simpler notation yields

$$y_t = a_{y,0} + a_{y,1} i + a_{y,2} E_t \left(i_t - \pi_{t+1}\right) + \\ a_{y,3} E_t y_{t+1} - a_{y,4} E_t \left(k_{t+2} - k_{t+1}\right) + a_{y,5} \left(k_{t+1} - k_t\right),$$

where $a_{y,0} = -\sigma\pi$, $a_{y,1} = a_{y,2} = \sigma$, $a_{y,3} = 1$, $a_{y,4} = \frac{\bar{k}}{\bar{y}}\gamma\eta$, and $a_{y5} = \frac{\bar{k}}{\bar{y}}\left(1-\delta\right)$.

A.4.2. The inflation equation

Substituting equation (43) for the average marginal cost in (41) we obtain

$$\tilde{\pi}_t = \psi\left[\left(\omega + \sigma^{-1}\right)\tilde{y}_t - \sigma^{-1}\tilde{I}_t - (\omega - \nu)\tilde{k}_t\right] + \tilde{\beta} E_t \tilde{\pi}_{t+1},$$

where the marginal cost depends also on investment. Expressed in terms of capital only, this equation becomes

$$\tilde{\pi}_t = \psi\left(\omega + \sigma^{-1}\right)\tilde{y}_t - \psi\sigma^{-1}\frac{\bar{k}}{\bar{y}}\gamma\eta\tilde{k}_{t+1} \\ + \psi\left[\sigma^{-1}\frac{\bar{k}}{\bar{y}}\left(1-\delta\right) - (\omega - \nu)\right]\tilde{k}_t + \tilde{\beta} E_t \tilde{\pi}_{t+1}.$$

Following the same process as for the output equation we can define the equation in log-levels as

$$\pi_t = a_{\pi,0} + a_{\pi,1}y_t + a_{\pi,2}k_{t+1} + a_{\pi,3}k_t + a_{\pi,4}E_t\pi_{t+1},$$

where $a_{\pi,0} = \left(1-\tilde{\beta}\right)\bar{\pi} - a_{11}y - (a_{12}+a_{13})\,k$, $a_{\pi,1} = \psi\left(\omega+\sigma^{-1}\right)$, $a_{\pi,2} = -\psi\sigma^{-1}\gamma\eta\frac{\bar{k}}{\bar{y}}$, $a_{\pi,3} = \psi\left[\sigma^{-1}\frac{\bar{k}}{\bar{y}}(1-\delta) - (\omega-\nu)\right]$, and $a_{\pi,4} = \tilde{\beta}$.

A.4.3. The capital equation

Combining the average demand of capital from (38) with (27) and using the market clearing condition $\int \tilde{k}_t^h = \tilde{k}_t = \int \tilde{k}_t(f)$ we obtain

$$\begin{aligned}\tilde{\lambda}_t + \epsilon_\psi\left(\tilde{k}_{t+1}-\tilde{k}_t\right) &= \frac{\tilde{\beta}}{\gamma}(1-\delta)\,E_t\tilde{\lambda}_{t+1} + \\ &\left[1-\frac{\tilde{\beta}}{\gamma}(1-\delta)\right]E_t\left(\rho_y\tilde{y}_{t+1}-\rho_k\tilde{k}_{t+1}\right) + \frac{\tilde{\beta}}{\gamma}\epsilon_\psi E_t\left(\tilde{k}_{t+2}-\tilde{k}_{t+1}\right). \quad (44)\end{aligned}$$

Substituting (42) in (44) we obtain

$$\begin{aligned}\epsilon_\psi\left(\tilde{k}_{t+1}-\tilde{k}_t\right) = \\ &\frac{\tilde{\beta}}{\gamma}(1-\delta)\,E_t\left(-\sigma^{-1}\left[\tilde{y}_{t+1}-\frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+2}-(1-\delta)\,\tilde{k}_{t+1}\right)\right]\right) \\ &\quad + \sigma^{-1}\left[\tilde{y}_t-\frac{\bar{k}}{\bar{y}}\left(\gamma\eta\tilde{k}_{t+1}-(1-\delta)\,\tilde{k}_t\right)\right] \\ &\quad + \left[1-\frac{\tilde{\beta}}{\gamma}(1-\delta)\right]E_t\left(\rho_y\tilde{y}_{t+1}-\rho_k\tilde{k}_{t+1}\right) \\ &\quad + \frac{\tilde{\beta}}{\gamma}\epsilon_\psi E_t\left(\tilde{k}_{t+2}-\tilde{k}_{t+1}\right),\end{aligned}$$

which, in log-levels, can be expressed as

$$k_{t+1} = a_{k,0} + a_{k,1}E_ty_{t+1} + a_{k,2}E_tk_{t+2} + a_{k,3}y_t + a_{k,4}k_t$$

where

$$\begin{aligned}a_{k,0} &= (1-a_{22}-a_{24})\,k-(a_{21}+a_{23})\,y, \\ a_{k,1} &= \bar{a}^{-1}\left[-\frac{\tilde{\beta}\sigma(1-\delta)}{\gamma}\left(\sigma^{-1}+\rho_y\right)+\rho_y\right], \\ a_{k,2} &= \bar{a}^{-1}\left(\frac{\tilde{\beta}}{\gamma}\frac{(1-\delta)\,\sigma^{-1}k\gamma\eta}{y}+\frac{\tilde{\beta}}{\gamma}\epsilon_\psi\right), \\ a_{k,3} &= \bar{a}^{-1}\sigma^{-1}, \\ a_{k,4} &= \bar{a}^{-1}\left[\epsilon_\psi+\sigma^{-1}\frac{\bar{k}}{\bar{y}}(1-\delta)\right],\end{aligned}$$

and

$$\bar{a} = \epsilon_\psi + \sigma^{-1}\frac{\bar{k}}{\bar{y}}\gamma\eta - \left[-\frac{\tilde{\beta}}{\gamma}(\gamma\eta - \delta)\left(\sigma^{-1}\frac{\bar{k}}{\bar{y}}(1-\delta) + \rho_k\right) + \frac{\tilde{\beta}}{\gamma}\epsilon_\psi + \rho_k\right].$$

A.5. The steady state

The real variables other than labor are expressed in stationary terms. We begin with

$$\bar{R}^K = \gamma\tilde{\beta}^{-1} - 1 + \delta,$$

where, again, a bar indicates a steady state value. From the investment equation

$$\frac{\bar{I}}{\bar{k}} = \gamma\eta - 1 + \delta.$$

From the Euler equation we have

$$\gamma\tilde{\beta}^{-1} = \frac{1+i}{\bar{\Pi}}.$$

From the evolution of the price index we know that

$$P_t = \left[(1-\xi)\left(P_t^\star\right)^{1-\theta} + \xi\left(\bar{\pi}P_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Dividing by P_t and and rearranging, the steady state value of the steady state relative price is equal to one, while the real marginal cost is equal to

$$\bar{s} = \frac{\theta - 1}{\theta}.$$

Form the firm's first order condition, we obtain the capital-labor ratio

$$\frac{\bar{k}}{\bar{L}} = \left(\frac{\alpha\bar{s}}{\gamma\tilde{\beta}^{-1} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}$$

Also, the output-labor ratio can be found from the production function

$$\frac{\bar{y}}{\bar{L}} = \left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}},$$

which implies an inverse relation between productivity and the output-labor ratio. Then $\frac{\bar{y}}{\bar{L}}\frac{\bar{L}}{\bar{k}}$ gives

$$\frac{\bar{y}}{\bar{k}} = \frac{\gamma\tilde{\beta}^{-1} - 1 + \delta}{\alpha\bar{s}}.$$

Also, from the household first order condition we obtain

$$\bar{L} = \left[\frac{\bar{w}}{a\bar{c}}\right]^{\frac{1}{\nu}}, \tag{45}$$

where

$$\bar{w} = (1-\alpha)\bar{s}\left(\frac{\alpha\bar{s}}{\gamma\tilde{\beta}^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}},$$

and, from the resource constraint

$$
\begin{aligned}
\frac{\bar{c}}{\bar{L}} &= \frac{\bar{y}}{\bar{L}} - \frac{\bar{k}}{\bar{L}}\frac{\bar{I}}{\bar{k}} \\
&= \left(\left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1} - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} (\gamma\eta - 1 + \delta) \right).
\end{aligned}
$$

Inserting this expression into (45) to substitute for $\bar{c}$ and rearranging we get:

$$
\begin{aligned}
\bar{L} &= \left[\frac{(1-\alpha)\bar{s}\left(\frac{\alpha\bar{s}}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{a\left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)^{\frac{\alpha}{1-\alpha}}\left[1-\alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)\right]} \right]^{\frac{1}{\nu+1}} && (46) \\
&= \left[\frac{(1-\alpha)\bar{s}}{a\left[1-\alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)\right]} \right]^{\frac{1}{\nu+1}}. && (47)
\end{aligned}
$$

From this last equation, it is easy to check that, provided $\beta^{-1} < \eta$ (which is verified in our parameterization), a *decrease* in productivity leads to a *increase* in total labor. The steady state output per effective worker is:

$$
\begin{aligned}
\bar{y} &= \left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1} - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left[\frac{(1-\alpha)\bar{s}}{a\left[1-\alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)\right]} \right]^{\frac{1}{\nu+1}} \\
&= \left[\frac{[(1-\alpha)\bar{s}]^{\nu+1}\left(\frac{\alpha s}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)^{\frac{\alpha(\nu+1)}{1-\alpha}}}{a\left[1-\alpha\bar{s}\left(\frac{\gamma\eta-1+\delta}{\gamma\tilde{\beta}^{-1}-1+\delta}\right)\right]} \right]^{\frac{1}{\nu+1}}.
\end{aligned}
$$

Because both $\bar{L}$ and $\bar{y}/\bar{L}$ are negatively related to productivity, $\bar{y}$ is also negatively related to productivity. Hence, a productivity slowdown will increase $\bar{y}$. Finally, we define $\sigma = \bar{c}/\bar{y}$ which is obtained from $\frac{\bar{c}}{\bar{L}}\left(\frac{\bar{y}}{\bar{L}}\right)^{-1}$.

A.6. The actual law of motion under learning

A.6.1. Under the simple method for estimating g_y

Inserting the expectations (13), (17) and (15) or (16) in (18) we obtain

$$
V_t = \breve{B}_1 + \breve{B}_2 E_{t-1} g_y + B_3 \left[\Omega_{0,t-1} + \Omega_{1,t-1}\Omega_{0,t-1} + \Omega_{1,t-1}^2 V_{t-1}\right] + B_4 V_{t-1} + B_5 \epsilon_t \quad (48)
$$

where $\breve{B}_1$ and $\breve{B}_{12}$ are suitable tranformations of B_1 and B_2. This can be expressed as

$$
V_t = T\left(E_{t-1} g_y, \Omega_{0,t-1}, \Omega_{1,t-1}\right) \left[\begin{array}{c} 1 \\ V_{t-1} \end{array} \right].
$$

Evans and Honkapohja (2001) discuss the conditions under which the dynamics of beliefs when the system is close to the rational expectations equilibrium are governed by the ordinary differential equation

$$\begin{pmatrix} \frac{dEg_y}{d\tau} \\ \dot{\Omega}_0 \\ \dot{\Omega}_1 \end{pmatrix} = \begin{pmatrix} M\left[T\left(Eg_y, \Omega_0, \Omega_1\right) - \left(Eg_y, \Omega_0, \Omega_1\right)\right] \\ Eg_y - g_y \end{pmatrix} \tag{49}$$

defined in notional time τ where M denotes the asymptotic second moments of the vector $[1, V_{t-1}]'$. The fixed point of (49) is the rational expectations equilibrium $(g_y, \Omega_0^\star, \Omega_1^\star)$. Evans and Honkapohja (2001) state the conditions under which the learning process is locally convergent to the rational expectations equilibrium if and only if the eigenvalues of $T' - I$ have real parts less than one.

A.6.2. Under the model consistent method for estimating g_y

The actual law of motion of the economy under the model consistent method mentioned in the text can be described in compact notation

$$\tilde{V}_t = C_1 + C_2 E_{t-1}\tilde{V} + C_3 E_{t-1}\tilde{V}_{t+1} + C_4\tilde{V}_{t-1} + C_5\epsilon_t$$

where $\tilde{V}_t = [y_t, \pi_t, k_t, i_t, g_{yt}]'$ and

$$E_{t-1}V = \left(I - \tilde{\Omega}_{1,t-1}\right)\tilde{\Omega}_{0,t-1}$$

where $\tilde{\Omega}$ includes the estimation of (16). Inserting the expectations we get

$$\begin{aligned} \tilde{V}_t = C_1 + C_2\left(I - \tilde{\Omega}_{1,t-1}\right)^{-1}\tilde{\Omega}_{0,t-1} + C_3\tilde{\Omega}_{0,t-1} + C_3\tilde{\Omega}_{1,t-1}\tilde{V}_{t-1} + \\ C_4\tilde{\Omega}_{0,t-1} + C_4\tilde{\Omega}_{1,t-1}\tilde{\Omega}_{0,t-1} + C_4\tilde{\Omega}^2_{1,t-1}\tilde{V}_{t-1} + C_5\tilde{V}_{t-1} + C_5\epsilon_t \end{aligned}$$

The remainder of the analysis follows as above.

A.6.3. When the private sector has rational expectations

Under the hypothesis of rational expectations of the private sector, the model can be put in structural form as

$$V_t = D_1 + D_2 E^{cb}_{t-1} g_y + D_3 E^{ps}_{t-1} V_{t+1} + D_4 V_{t-1} + D_5\epsilon_t \tag{50}$$

where the central bank and the private sector form different expectations. In order to find the rational forecast of the private sector we need a guess for the law of motion of the economy

$$V_t = \Omega_0^{ps} + \Omega_1^{ps} V_{t-1} + e_t$$

where e_t is a perceived *i.i.d.* disturbance. Forecasts by the private sector are then given by

$$\begin{aligned} E^{ps}_{t-1}V_{t+1} &= \Omega_0^{ps} + \Omega_1^{ps}\left(\Omega_0^{ps} + \Omega_1^{ps}V_{t-1}\right) && (51) \\ &= \left(\Omega_0^{ps} + \Omega_1^{ps}\Omega_0^{ps} + \left(\Omega_1^{ps}\right)^2 V_{t-1}\right). && (52) \end{aligned}$$

Inserting (51) in (50) we find the actual law of motion

$$V_t = D_1 + D_2 E^{cb}_{t-1} g_y + \\ D_3 \left(\Omega^{ps}_0 + \Omega^{ps}_1 \Omega^{ps}_0 + \left(\Omega^{ps}_1\right)^2 V_{t-1} \right) + D_4 V_{t-1} + D_5 \epsilon_t.$$

Using the method of undetermined coefficients we obtain

$$\Omega^{ps,\star}_1 = \left(D_3 \Omega^{ps,\star}_1 \right)^2 + D_4,$$

and

$$\Omega^{ps}_0 = D_1 + D_2 E^{cb}_{t-1} g_y + D_3 \left(\Omega^{ps}_0 + \Omega^{ps}_1 \Omega^{ps}_0 \right), \tag{53}$$

which gives

$$\Omega^{ps,\star}_0 = \left(I - D_3 - D_3 \Omega^{ps,\star}_1 \right)^{-1} \left(D_1 + D_2 E^{cb}_{t-1} g_y \right), \tag{54}$$

where the $(\star)$ indicates the rational expectations solution coefficients.

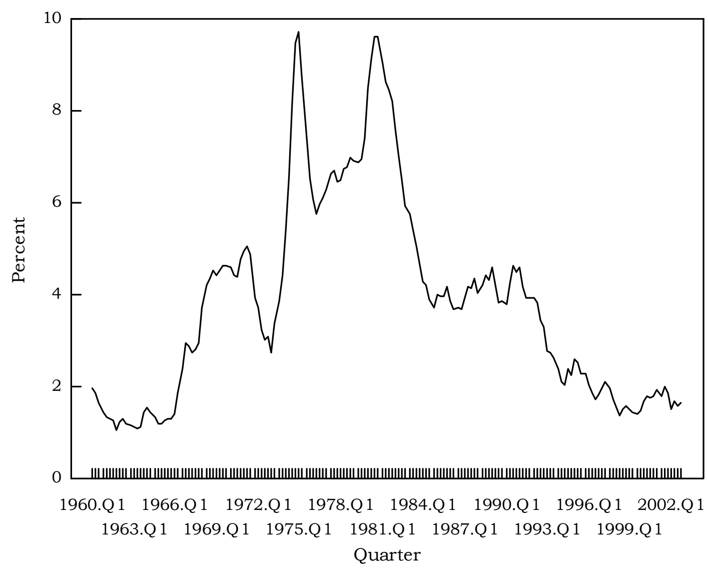
The expression (53) is a matrix of coefficients that is independent of the learning process of the central bank. On the other side, the matrix of constants, equation (54) depends on the central bank's estimates of the long run growth rate of output. In fact rational expectations of the private sector implies not only perfect information about the productivity change but also perfect information about the mistakes of the central bank. The actual law of motion of the economy under real time learning is described by the following equation

$$V_t = D_1 + D_2 E^{cb.}_{t-1} g_y + \\ D_3 \left(\Omega^{ps,\star}_{0,t-1} + \Omega^{ps,\star}_1 \Omega^{ps,\star}_{0,t-1} + \left(\Omega^{ps,\star}_1\right)^2 V_{t-1} \right) + D_4 V_{t-1} + D_5 \epsilon_t. \tag{55}$$

The remainder of the analysis then follows as above.

Figure 1. U.S. inflation, 1960-2002

Core PCE inflation from one year earlier.



Personal consumption expenditures inflation rate from the previous year, less food, energy, and insurance payments.

FIG. 1 The U.S. inflation experience includes a sharp increase in observed inflation following the onset of the productivity slowdown in the early 1970s. Excluding volatile components and smoothing the data slightly provides one indication of what might be called medium-run inflation movements. Our analysis is designed to address movements at this frequency.

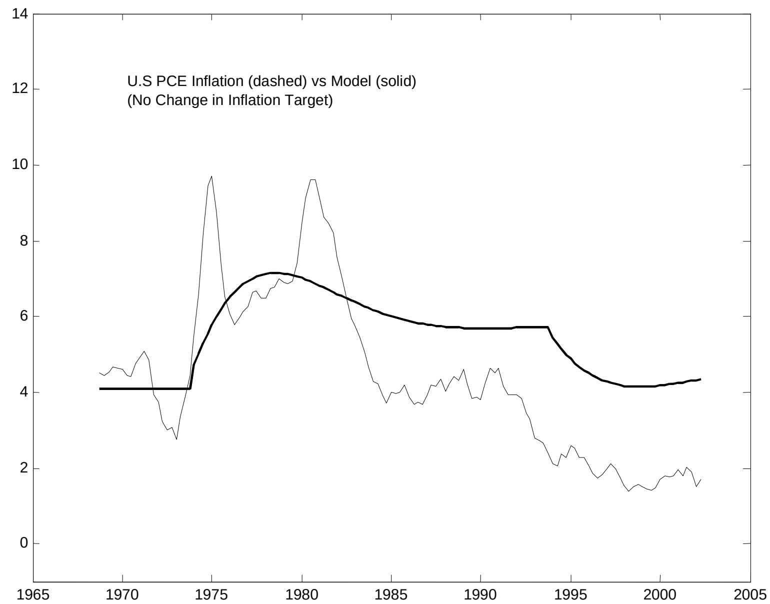


FIG. 2 The inflation dynamics in the baseline model versus the U.S. data. Inflation increases significantly in response to the productivity slowdown, but remains persistently high.

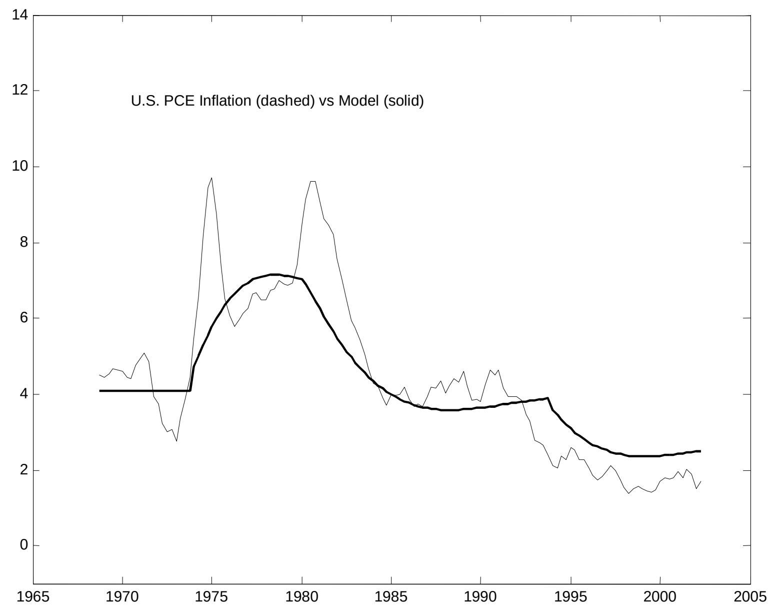


FIG. 3 If the central bank unexpectedly lowers the inflation target to two percent in 1980, the inflation dynamics begin to approximate the data quite well.

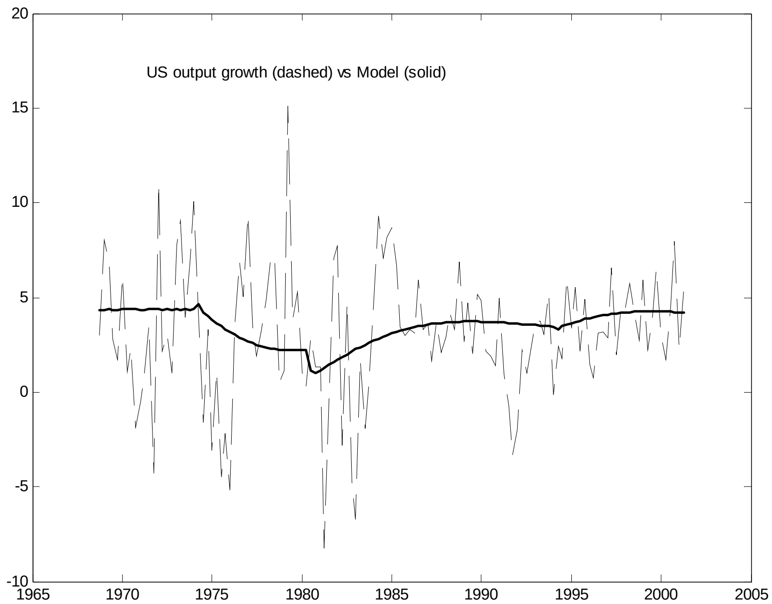


FIG. 4 The output growth trend from the benchmark model with an inflation target change in 1980, as compared to output growth rates in the data. Not surprisingly, output growth rates are highly variable in the data relative to the model.

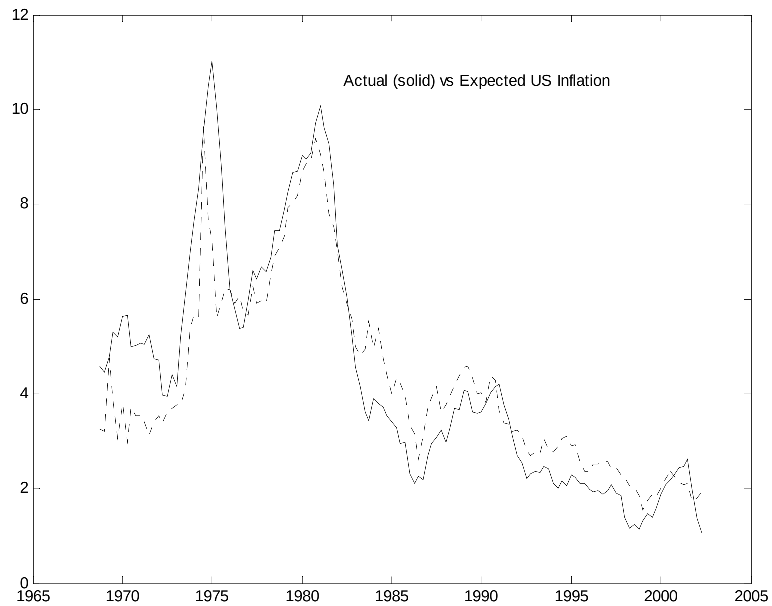


FIG. 5 Actual versus expected inflation in the U.S. data. As is well-known, expectations appear to "lag behind" inflation.

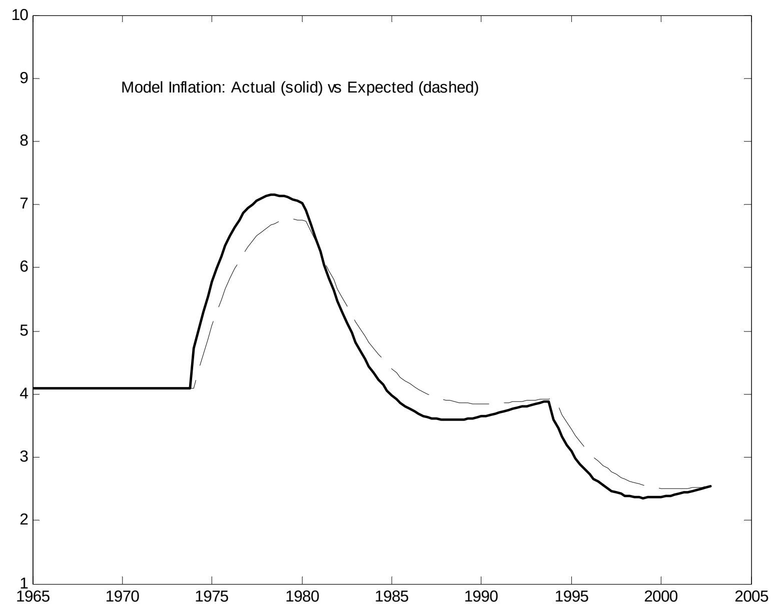


FIG. 6 Actual versus expected inflation in the benchmark model with an inflation target change in 1980. Expectations tend to be too low in the 1970s and too high later in the sample, consistent with the data as shown in Figure 5. The model suggests that this is what one should expect to observe when households are learning about structural productivity changes.

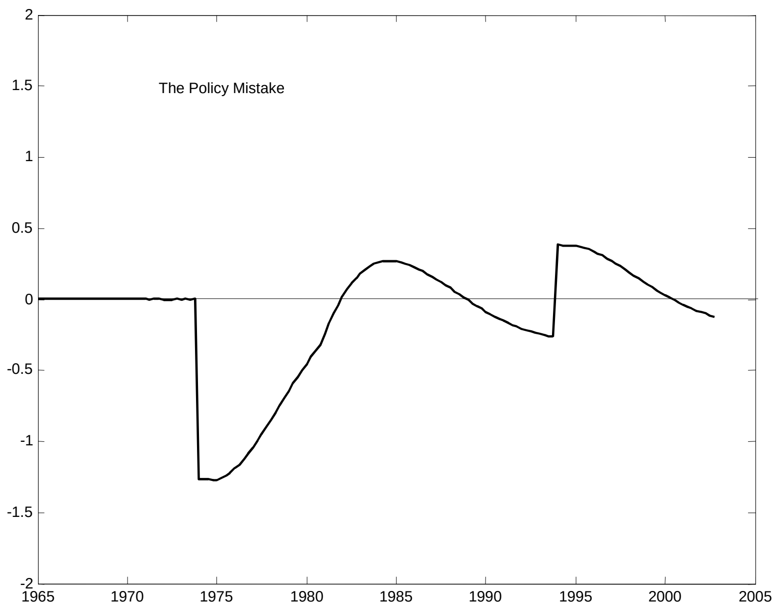


FIG. 7 The policy "mistakes" of the central bank are based on misperceptions of the output growth gap.

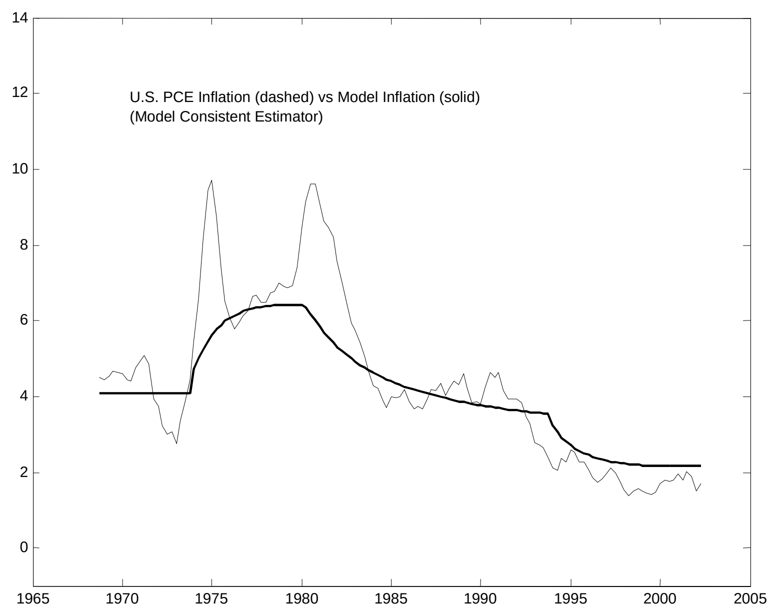


FIG. 8 The behavior of inflation in the model versus the U.S. data in the benchmark economy when the central bank uses a model-consistent estimator of the output growth rate. This example includes the reduction in the inflation target in 1980. The inflation performance is largely the same even when the central bank uses more information.