Stochastic Capital Depreciation and the Comovement of Hours and Productivity?

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Abstract

An unresolved question concerning stochastic depreciation shocks is whether they have to be unrealistically large to have any useful role in a dynamic general equilibrium model economy, as Ambler and Paquet (1994) first suggested. We first consider implied depreciation rates from sectoral data from the Bureau of Economic Analysis. These depreciation rates vary across time solely due to compositional changes within each sector. Hence, they tend to understate the range of fluctuation that would hold if the economic shelf life of capital varied endogenously as in Cooley, Greenwood and Yorukoglu (1997). We find, however, that if depreciation rates follow a Markov switching process, a low variance of the depreciation rate can generate the low correlation between hours worked and productivity in a simple model economy. White noise and autoregressive depreciation shocks, in contrast, require a counterfactually large variance in the depreciation rate to reduce the hours-productivity correlation. We also illustrate the level effects implied by nonlinear decision rules in simulations of dynamic general equilibrium models that include Markov switching parameters. Linear decision rules, in contrast, imply certainty equivalence and ignore the aversion that agents have to the skewed shock distributions that characterize Markov switching.

1 Introduction

Numerous dynamic stochastic general equilibrium (DSGE) macroeconomic models now allow for variation in the depreciation rate of capital. The most common approach treats the depreciation rate as an endogenous variable such that the choice to use capital intensively or to spend little on maintenance and repair results in high depreciation [Greenwood, Hercowitz and Huffman (1988); Burnside, Eichenbaum and Rebelo (1996); King and Rebelo (2000) for the former; McGrattan and Schmitz (1999), Collard and Kollintzas (2000) and Licandro and Puch (2000) for the latter]. Procyclical variation in capital utilization amplifies the effect of a technology shock on output. When the depreciation rate is a function of maintenance and repair, the assumption is that each unit of capital is matched with labor input that is geared toward either production or capital maintenance and repair.

In both of these scenarios, variation in the depreciation rate is a means and not an end. Endogenous depreciation equates margins at less than full capital utilization or introduces a role for large, countercyclical expenditures on maintenance and repair. In this way, endogenous depreciation serves to amplify and augment the persistence of the effects of technology shocks on output. But, fully endogenous depreciation only amplifies technology shocks and does not allow for random changes in the depreciation rate as an independent source of economic fluctuations.

Stochastic depreciation, on the other hand, allows depreciation shocks to serve as an additional driving force behind macroeconomic fluctuations, along with technology shocks. The motivation for the stochastic process rests with the observation that for many types of capital—particularly intangible assets, such as music, film, or software—the economic lifespan does not necessarily obey a decay function of time and intensity of use. Along these lines, Cooley, Greenwood and Yorukoglu (1997) present a vintage capital model in which capital is scrapped because of economic obsolescence rather than physical breakdown.[1] Even tangible assets such as machinery and structures have uncertain and variable shelf lives after which they are scrapped as a result of economic obsolescence. Of course, tangible assets are also subject to physical shocks, such as fire and storm. At the same time, compositional shifts between sectors can also lead to time variation in the aggregate depreciation rate.

These properties of depreciation suggest that capital is often destroyed or scrapped for reasons other than intense capital utilization. With this in mind,

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[1]See Chapter 7 in OECD (2001) for a discussion on the complexities of measuring economic depreciation for intangible assets.

Ambler and Paquet (1994) introduced stochastic depreciation to a DSGE model and showed that a white noise shock to the depreciation rate could account for the low hours-productivity correlation observed in the data.[2] The intuition is that a random increase in the depreciation rate causes hours worked and labor productivity to respond in opposite directions. When the depreciation rate rises, the capital-output ratio begins to fall, so labor hours are substituted for capital. Consequently, hours and labor productivity move in opposite directions. This source of negative correlation between hours and labor productivity can counteract the positive correlation implied by technology shocks to result in a low correlation between hours and productivity that matches the data. The results for Ambler and Paquet's linearized model show that depreciation shocks can bring down the hours-productivity correlation to levels consistent with the data.[3]

Our objective in this article is to extend the results of Ambler and Paquet (1994) in two ways. First, we seek to introduce greater realism to the shock process. To do this, we provide empirical evidence that the depreciation rates for many types of capital follow a process that is more general than the white noise assumption in Ambler and Paquet (1994). More specifically, we find that Markov switching behavior (i.e., nonlinearity and asymmetry) provides a better description of depreciation rates than a white noise process. Second, we demonstrate that stochastic fluctuations of the depreciation rate within a narrow band at low and moderate levels is by itself able to generate a low hours-productivity correlation in a general equilibrium model. A Markov switching process is a natural way to impose a narrow range of fluctuation on the depreciation rate. In this way, we reinforce Ambler and Paquet's (1994) point that stochastic depreciation induces a substantial reduction in the co-movement of hours and labor productivity. The difference is that the Markov process generates a low hours-productivity correlation at a lower variance of depreciation than Ambler and Paquet's (1994) white noise process. Hence, this approach is less prone to the "Where are the shocks?" critique.

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[2]Bernanke, Gertler, and Gilchrist (1999) and Carlstrom and Fuerst (2001) consider shocks to the relative price of capital goods in terms of foregone consumption.

[3]It should be noted that stochastic shocks to depreciation represent only one way to reduce the hour-productivity correlation. Aiyagari (1994) has shown that if an additional shock (i.e. to preferences, technology, or government spending) is added to the model, then this correlation also is reduced. Further, two sector models by Hornstein and Krusell (1996) and Greenwood *et al.* (2000) and others, where one sector produces investment goods and the other consumer goods are also able to generate similar effects. When the two sectors have the same production function, the setup is comparable to a model with shocks to the depreciation rate.

Our solution method for the general equilibrium model with Markov switching makes use of Judd's (1998) projections that yield nonlinear decision rules. With Markov switching, it is important to have nonlinear decision rules because linear decision rules do not allow agents to recognize that the errors in predicting the Markov state have a discrete distribution over just a few values—two for a two-state Markov process. In addition, we present quantitative results on how the departure from certainty equivalence under our nonlinear decision rules creates a level effect, whereby capital investment is riskier with stochastic shocks to the depreciation rate and this risk leads to a lower steady state capital stock and output.

The article is organized as follows. Section 2 motivates our use of a Markov switching process for a time-varying depreciation rate. We present Markov switching estimates for various sectoral data series from the Bureau of Economic Analysis (BEA), U.S. Department of Commerce. Section 3 presents the baseline RBC model. The same section discusses calibration strategy. Section 4 presents the main results in the form of sensitivity analysis and impulse responses. Section 5 concludes.

2 Time-Varying Depreciation and Markov Switching

In this article, the stochastic shock to the depreciation rate is motivated by economic obsolescence. Our heuristic description of capital depreciation is not a machine that wears out physically. Instead, we are viewing depreciation in terms of the shelf life of the average microprocessor or textbook, which can vary across time.[4]

However, evidence from Fraumeni (2001) shows that this effect on the aggregate capital stock is small. The BEA estimates disaster damage when damage is at least 0.25% of consumption of fixed capital, i.e., $2.6 billion in 2000. Varying economic shelf lives are perhaps a more important source of fluctuation in depreciation, due in part to consumer expectations regarding comfort, safety, or environmental standards. Note that we remove any secular upward trend from depreciation rates in our empirical work. Hence, we view our treatment of the depreciation rate as a stochastic process as a shorthand approach to the Cooley, Greenwood and Yorukoglu (1997) vintage capital model with an endogenously varying depreciation rate. The prospect of eco-

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[4]Stochastic depreciation rates have been motivated in a multitude of ways. These include the role of hazards, war, or calamities other than ordinary wear and tear on fixed capital.

nomic obsolescence is especially pronounced for firms making buy versus lease investment decisions in high-tech equipment.[5] Similarly in the service sector, which is heavily reliant on computers, the rate of technological obsolescence of microprocessors is closely related to rate of technical improvements—Moore's law—and is independent of how intensively the CPU cycles are used.

Technical obsolescence is not sector specific. Even in a localized setting, stochastic shocks that render capital obsolete can take alternative forms. A court ruling favoring stricter environmental standards for forest conversation can render a network of logging roads economically obsolete. Similarly, the economic life of a Hollywood film is largely determined by the public's reaction to the initial screenings. The outcome of a clinical test for a pharmaceutical company can influence positively or negatively the economic lifespan of a drug. In this arena, the concept of capital includes the portfolio of intellectual property and patents that drug companies possess.
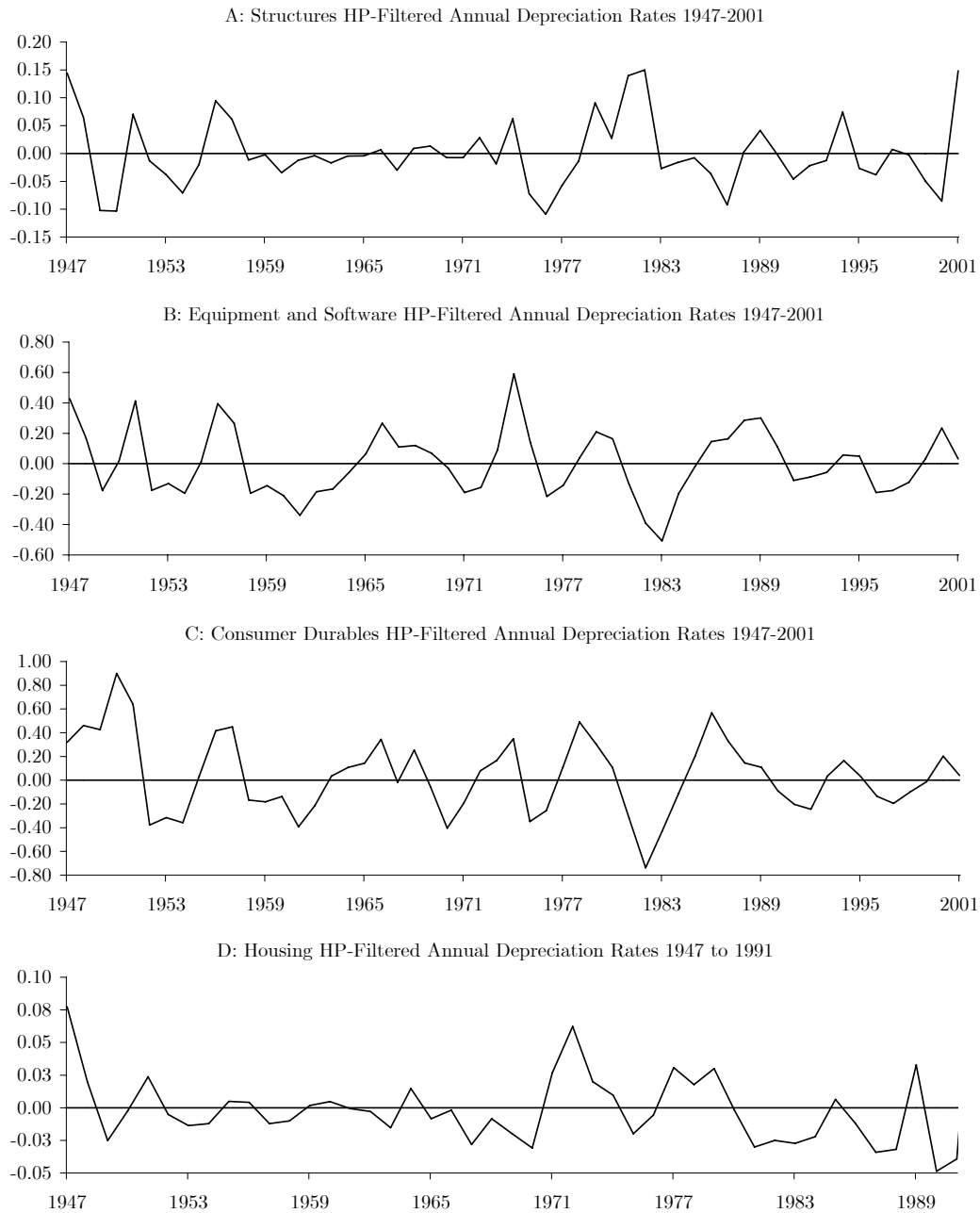
Clearly not all forms of capital are affected by economic obsolescence. Therefore, when trying to understand the economic implications of time-varying depreciation on the comovement of hours and productivity, for example, in a one sector model economy, the size of the fluctuations to depreciation becomes the focus. A priori, we want the calibrated variance of the depreciation rate to be small enough to be consistent with the depreciation data yet still engender a low hours-productivity correlation. One of the few means, however, to study the time-series properties of depreciation rates is to examine sectoral data from the BEA. Figure 1 plots the H-P filtered depreciation rates of four sectors: private fixed nonresidential equipment and software, consumer durables, private fixed nonresidential structures, and private residential fixed assets. The annual data from 1947 to 2001 show that the depreciation rates differ substantially across sectors as do their fluctuations: the sectors with high depreciation rates (equipment and software and consumer durables) fluctuate more than the sectors with low depreciation rates (structures and housing).[6] The BEA estimates should be seen as a lower bound. Fraumeni (1997) argues that the BEA approximation of geometric decay of each type of capital works well as long as one does not condition on discards and survival rates. This suggests that the role of obsolescence is underestimated. The BEA uses (infinite) geometric depreciation without applying a mortality function.

Table 1 offers summary statistics for the detrended depreciation rates. We pay closest attention to the standard deviations, which range from 0.0075 for

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[5]One advantage of lease contracts is that they are seen as an insurance against technological obsolescence. In 2002, the nominal value of leased equipment was four times larger than equipment purchased through loans.

[6]The sample for housing has been shortened because of two outliers in the early 1990s.

Figure 1: Depreciation Rates for Four Sectors

A: Structures HP-Filtered Annual Depreciation Rates 1947-2001

B: Equipment and Software HP-Filtered Annual Depreciation Rates 1947-2001

C: Consumer Durables HP-Filtered Annual Depreciation Rates 1947-2001

D: Housing HP-Filtered Annual Depreciation Rates 1947 to 1991

time-detrended equipment and software to 0.0005 for time-detrended housing. The same table also provides information on the first-order autocorrelation coefficient and deviations from normality using the Jarque-Bera test. The former finds weak evidence of autocorrelation in the case of equipment and software and housing, whereas the latter test presents strong evidence of non normality.[7] The rejection of normality is also confirmed by the BDS test for nonlinearity, see Brock, Dechert, and Scheinkman (1996). The results, given in Table 2, suggest that a mixing distribution may be responsible for the observed fluctuations in the depreciation rates.

Table 1: Statistics for Annual U.S. Depreciation Rates 1947-2001

	Equipment and Software	Structures	Consumption Durables	Housing
Deviations from HP Trend				
$std(\delta_t)$	0.0040	0.0010	0.0033	0.0009
$\rho_{\delta_t}(1)$	0.3220*	−0.1450	0.1430	−0.5660*
$JB(\delta_t)$	0.0030	0.0000	0.5560	0.0020
$max(\delta_t)$	0.0078	0.0028	0.0069	0.0016
$min(\delta_t)$	−0.0098	−0.0015	−0.0066	−0.0008
Deviations from Time Trend				
$std(\delta_t)$	0.0075	0.0010	0.0035	0.0005
$\rho_{\delta_t}(1)$	0.9213	0.6829	0.6030	0.4190
$JB(\delta_t)$	0.4399	0.0334	0.2929	0.0000
$max(\delta_t)$	0.0178	0.0029	0.0069	0.0016
$min(\delta_t)$	−0.0128	−0.0017	−0.0068	−0.0008

Note: JB gives the p-value of the Jacque-Bera test.

To understand whether the properties of nonlinearity or persistence can be captured in a more general model than the white noise assumption of Ambler and Paquet (1994), a two-state Markov model is fitted to the detrended depreciation rates. The mean parameters are denoted μ_1 and μ_2 and the respective persistence parameters for the states are denoted p_δ and q_δ. Note that the means can be negative because the data are expressed as deviations from either a time trend or HP trend. The parameter estimates, given in Tables 3 and 4, show that a Markov switching model works well for most depreciation

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[7]One of the few observable measures for technical obsolescence is discussed in Lee (1978). His study shows that the technical efficiency of Japanese fishing boats exhibits evidence of autocorrelation.

Table 2: Nonlinearity (BDS) Test Results for Sectoral Depreciation Rates

ϵ	m	Equipment and Software	Structures	Consumption Durables	Housing
0.5	2	13.2000**	10.9000**	10.5000**	5.1274**
0.5	3	19.5000**	14.4000**	15.6000**	7.5942**
0.5	4	29.8000**	20.7000**	24.7000**	10.9358**
1.0	2	7.3755*	6.5780*	8.8055*	2.9603*
1.0	3	9.3428*	7.5819*	11.0239*	3.2979*
1.0	4	11.7455*	8.8329*	12.7682*	3.6226*
1.5	2	5.4015*	4.7249*	2.7023	3.4221*
1.5	3	5.7163*	4.7562*	3.9193*	3.7779*
1.5	4	6.2008*	5.0888**	3.5364*	3.8260*
2.0	2	5.8903*	2.1738	0.6764	3.3057*
2.0	3	5.8237*	2.2470	1.5896	3.8593*
2.0	4	5.5628*	2.6164	1.3564	3.9686*

Notes: The results of the BDS tests of the pre-whitened sectoral depreciation rates are marked by * and ** to denote significance at the 5% and 10% levels. The BDS test is described in Brock, Dechert, and Scheinkman (1996), ϵ is the sup norm on the m-histories and m is the embedding dimension.

series in that the sum of the transition probabilities is greater than 1.0, which indicates that the high/low depreciation states are serially correlated.

Based on the estimates in the Tables 3 and 4 and the desire to heed the standard deviations of the depreciation rates from Table 1, we will calibrate a Markov switching process in the model economy that implies a standard deviation (when converted from quarterly to annual data) of 0.0041, which is not counterfactually large.

The quarterly Markov process that will achieve this has $\mu_1 = 0.017, \mu_2 = 0.023, p_\delta = 0.90, q_\delta = 0.75$, where $p_\delta = Prob(\mu_t = \mu_1 | \mu_{t-1} = \mu_1)$ and $q_\delta = Prob(\mu_t = \mu_2 | \mu_{t-1} = \mu_2)$. These persistence parameters are broadly consistent with the estimates from Tables 3 and 4.

The evidence of nonlinearity and persistence in Tables 3 and 4 is a strong departure from the white noise assumption in Ambler and Paquet (1994). The strategy for the remainder of the paper is to model time-varying depreciation as a Markov process that puts strict bounds on the lower and upper ranges of the depreciation rate. A natural question to ask is what does Markov switching

Table 3: Estimates Fit to Sectoral Depreciation Rates (x 100)
Expressed as Deviation from Time Trend

	Equipment and Software	Structures	Consumption Durables	Housing
μ_1	−0.5790	−0.0356	−0.2492	−0.0232
	(0.0747)	(0.0104)	(0.0395)	(0.0048)
μ_2	0.7112	0.1624	0.3219	0.0785
	(0.0823)	(0.0247)	(0.0527)	(0.0108)
p_δ	0.9550	0.9575	0.8136	0.9189
	(0.0355)	(0.0306)	(0.0735)	(0.0407)
q_δ	0.9730	0.8548	0.7400	0.7392
	(0.0295)	(0.1662)	(0.0983)	(0.1467)
σ^2	0.1453	0.0043	0.0366	0.0009
	(0.0286)	(0.0009)	(0.0090)	(0.0002)

Note: Standard errors are in parentheses.

Table 4: Estimates Fit to Sectoral Depreciation Rates (x 100)
Expressed as Deviation from HP Trend

	Equipment and Software	Structures	Consumption Durables	Housing
μ_1	−0.1295	−0.0203	−0.1180	−0.0099
	(0.0377)	(0.0078)	(0.0936)	(0.0039)
μ_2	0.1940	0.1045	0.3432	0.0367
	(0.0461)	(0.0183)	(0.1333)	(0.0093)
p_δ	0.7463	0.8717	0.8316	0.9003
	(0.0923)	(0.0730)	(0.1443)	(0.0621)
q_δ	0.6591	0.4068	0.6772	0.5527
	(0.1277)	(0.2776)	(0.1395)	(0.2347)
σ^2	0.0210	0.0016	0.0505	0.0004
	(0.0055)	(0.0005)	(0.0143)	(0.0001)

Note: Standard errors are in parentheses.

deliver beyond a simpler AR(1) process. It will be shown in section 4 that the linear AR(1) process that matches the hours-productivity correlation implies a much higher standard deviation of the depreciation rate than is observed in the data, whereas the Markov switching process does not.

3   Model Structure and Calibration

The model is a standard DSGE model with indivisible labor. The model economy is populated by a large number of infinitely lived agents whose expected utility is defined by

$$E_0[\sum_{t=0}^{\infty} \beta^t (ln(c_t) + \theta \frac{N_t}{\hat{N}} [ln(1 - \hat{N})])], \tag{1}$$

where β is the time discount factor; c_t is private consumption; θ is a positive scalar that determines the relative disutility of non-leisure activities; N_t is expected work time under a Rogerson (1988) employment lottery; $\hat{N}$ is the indivisible time spent at work for those working [Hansen (1985)].[8]

Aggregate output, Y_t, is assumed to depend on the total amount of capital, K_t, and on total hours of work, N_t, with labor-augmenting technological progress at the gross rate λ

$$Y_t = e^{z_t} K_t^{\alpha} (\lambda^t N_t)^{1-\alpha}. \tag{2}$$

A time constraint restricts leisure and work to sum to one:

$$l_t + n_t = 1. \tag{3}$$

The technology shock, z_t, is assumed to follow an AR(1) process with the following law of motion:

$$z_t = \rho z_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma_\epsilon^2). \tag{4}$$

The technology shock, ϵ_t, is drawn from a normal distribution with mean zero and standard deviation σ_ϵ.

The capital stock evolves according to

$$k_{t+1} = (1 - \delta_{S_t})k_t + i_t, \tag{5}$$

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[8]We follow the general practice, where lower case letters are used to denote individual choices and upper case letters denote economy-wide per-capita quantities.

where i_t is the chosen level of investment and δ_{S_t} is the rate of depreciation of capital, which is assumed to follow a two-state Markov process that is independent of z_t.[9] The installation of capital takes one period, making the time $t + 1$ capital stock predetermined at time t, but there are otherwise no installation or adjustment costs. Lastly, the aggregate resource constraint of the economy is given by

$$y_t = c_t + i_t. \tag{6}$$

Calibration for the Baseline Model

The model is calibrated to the parameter values listed in Table 5. The rate of time preference and the Cobb-Douglas production function coefficients are standard. The values for indivisible labor are taken from Li (1999). The autoregressive coefficient for the technology process, z, is set to 0.95. The standard deviation of the technology shocks is set at $\sigma_\rho = 0.0045$ to match the variance of output in the data when depreciation switches as follows. The quarterly depreciation rate switches between 0.017 in the low state and 0.023 in the high state. These values imply an annual depreciation rate between roughly 6.8 and 9.2 percent. The low state value is consistent with Stokey and Rebelo (1995) and the high state value with King and Rebelo (2000). Much of our analysis that works with a constant depreciation rate sets the quarterly depreciation rate set to 0.020, which is consistent with values used by Gilchrist and Williams (2000). The persistence for the two states is defined by

$$p_\delta = P(\delta_t = \delta_0 \mid \delta_{t-1} = \delta_0) = 0.90$$

$$q_\delta = P(\delta_t = \delta_1 \mid \delta_{t-1} = \delta_1) = 0.75.$$

With this setup, we assume that there is low persistence in the high depreciation state.

4 Quantitative Analysis of the Model

We present results on the comovement of hours and productivity in the form of sensitivity analysis and impulse responses. The sensitivity analysis high-

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[9]Without imposing a positive correlation between technology and the depreciation rate, this model does not significantly reduce the probability of technical regress, as time-varying capital utilization can, according to Burnside, Eichenbaum, and Rebelo (1996) and King and Rebelo (2000).

Table 5: Calibration of Markov Switching Economy

Rate of Time Preference	β	0.9917
Production Function Coefficient	α	0.33
Leisure Coefficient	θ	2.9474
Labor Productivity Growth	λ	1.00373
Indivisible Labor	$\hat{L}$	0.25
Depreciation Rates	δ	0.017, 0.023
Depreciation Transition Probability	p_δ, q_δ	0.90, 0.75
AR Technology	ρ	0.95
Standard Deviation of Technology Error	σ_ρ	0.0045

lights several findings concerning the influence of depreciation on the hours-productivity correlation. Impulse responses illustrate why the model attains a low correlation between hours and productivity. We begin the discussion with some baseline results. Details on the solution procedure are presented in the appendix.

Main Results

Table 6 presents correlations and standard deviations of model-implied data compared with actual data taken from Li (1999). For models with a fixed depreciation rate, the rate is set to 0.020—two percent per quarter. The first column gives information of the model of section 3 with only the technology shock (i.e., the Markov switching has been shut down). A principal weakness of this version of the model is that it has difficulty matching the hours-productivity correlation. The model's simulated correlation is 0.86 opposed the actual data correlation of 0.25.

The next column presents statistics from a log linearized model of section 3 with time-varying depreciation from the class of AR(1) autoregressive processes. We find that, within the AR(1) class of models, the value of the AR coefficient that can match the hours-productivity correlation with the lowest implied variance of the depreciation rate is a white noise process, as Ambler and Paquet (1994) studied. Nevertheless, the second column of Table 6 shows that the variance of depreciation must be at least three times as large as in the data to match the hours-productivity correlation. Persistent AR processes can also match the hours-productivity correlation but only with an even larger (and unrealistic) unconditional variance of the depreciation rate, relative to white noise depreciation shocks. Nevertheless, the white noise process with shocks that are large enough to reduce the hours-productivity correla-

Table 6: Business Cycle Statistics for the Markov-Switching Model

	Technology Shock Only	δ_t Defined by White Noise Process	δ_t Defined by Markov Process	Actual Data
σ_y	1.01	1.21	1.08	1.24
σ_c/σ_y	0.40	0.35	0.45	0.52
σ_i/σ_y	3.03	3.88	3.37	2.50
σ_n/σ_y	0.61	0.86	0.79	0.77
$\sigma_{y/n}/\sigma_y$	0.39	0.34	0.46	0.46
$\sigma_n/\sigma_{y/n}$	1.67	2.54	1.72	1.64
$annualized\ \sigma_\delta$	0.00	0.013	0.0046	0.003-0.004
$corr(c,y)$	0.96	0.39	0.68	0.84
$corr(i,y)$	0.99	0.96	0.95	0.92
$corr(n,y)$	0.98	0.94	0.89	0.88
$corr(y/n,y)$	0.96	0.56	0.63	0.66
$corr(y/n,n)$	0.89	0.25	0.22	0.25

Notes: Actual data for σ_δ based on Equipment and Software and Durables.
All series were run through the Hodrick-Prescott filter.
Actual data, except for depreciation, are taken from Li (1999).

tion also detracts in other dimensions, such as making output too volatile relative to consumption. The results from AR(1) depreciation processes suggest that a persistent but bounded AR process would be able to match the hours-productivity correlation without inducing such an unrealistically large variance in depreciation. Implementing decision rules for agents that would be consistent with a bounded AR process would be difficult, however. For this reason, we turn to a Markov switching process, which allows for persistence yet puts bounds on the depreciation rate. This model will require nonlinear decision rules, but it is more straightforward to implement than a bounded AR process with a reflecting barrier.

The statistics from the nonlinear Markov switching model, shown in the third column, demonstrate that Markov switching in the depreciation rate is able to replicate the low hours-productivity correlation and resolve some of the deficiencies of the AR(1) class of models. The Markov switching parameters are $\delta_0 = 0.017$ and $\delta_1 = 0.023$ and the transition probabilities are $p_\delta = 0.90$ and $q_\delta = 0.75$. Depreciation shocks of this magnitude raise the standard deviation of output by about seven percent, so the addition of depreciation shocks leave the volatility of output at a level that is still consistent with the data. The introduction of switching in the depreciation rate still generates an investment-output ratio that is somewhat too high, but offers a way to reduce the hours-productivity correlation with a much lower variance in the

depreciation rate that does not exceed the variances found in the sectoral BEA data. The nonlinear model also matches the data in terms of the correlation between productivity and output.

Nonlinear Decision Rules and Non − Certainty Equivalence

The reduction in the hours-productivity correlation brought by Markov switching in the depreciation rate also holds in a linearized version of the model that is simulated with linear decision rules. In the linearized model with Markov switching depreciation, agents treat the depreciation rate as an AR(1) process:

$$\delta_t - \delta_0 = (1 - p_\delta)(\delta_1 - \delta_0) + (p_\delta + q_\delta - 1)(\delta_{t-1} - \delta_0) + e_t, \qquad (7)$$

where agents using linear decision rules do not take account of either the variance of depreciation or the fact that the mean-zero error term, e_t, only takes on only two possible values when $\delta_{t-1} = \delta_0$ and two possible values when $\delta_{t-1} = \delta_1$. In other words, linear decision rules connote certainty equivalence so that agents ignore both the variance and skewness in the depreciation shocks.

With nonlinear decision rules, in contrast, agents take account of the true two-point nature of the error process and consider the fact that the depreciation rate has a positively skewed distribution when $p_\delta + q_\delta > 1$. Nonlinear decision rules allow agents to act on their risk averse preferences so that they invest less than they would under certainty equivalence because they dislike the variance and positive skewness in depreciation. Thus, while many simulated correlations and second moments from models with Markov switching parameters will be virtually identical whether linear or nonlinear decision rules are used to simulate the model, nonlinear decision rules capture important level effects due to non-certainty equivalence.

To illustrate the level effect that nonlinear decision rules permit, we investigate the effect of a mean-preserving spread of the depreciation rate. For this illustration, the baseline case has $\delta_0 = 0.017, \delta_1 = 0.023, p_\delta = 0.90, q_\delta = 0.75$.

One mean-preserving spread that increases both the variance and the positive skewness of depreciation is to set $\delta_0 = 0.01625, \delta_1 = 0.022, p_\delta = 0.85, q_\delta = 0.80$. With linear decision rules and certainty equivalence, this mean-preserving spread would have no effect on the average level of investment and output. Since higher moments matter in the nonlinear decision rules, it is interesting to note what effect risk terms related to depreciation switching have on the level of the path of output. We simulate the model 400 times with and without the mean-preserving spread and find that the level effect on output

lowers output by 0.25 percent (or about 25 billion dollars in the U.S. economy). Given that the model has time-separable log utility, this level effect is perhaps surprisingly large. This level effect would be even larger if the preferences included habit persistence, for example.

Sensitivity Analysis

We begin our analysis by investigating the influence of switching between states with a low and a high depreciation rate. The correlation results are given in Figure 2 for three different processes defined by $\delta + / - \mu$, where $\delta = (0.017, 0.020, \text{and } 0.022)$ and $\mu = (0, 0.0005, ..., 0.005)$. The transition matrix for the two state process is set to $p_\delta = 0.9$ and $q_\delta = 0.75$. The results find that independent of δ relatively small deviations from the conditional mean between $+/ - 0.003$ and $+/ - 0.0035$ generate hours-productivity correlations that are consistent with the empirical data of 0.25 (see Table 6). Our estimated Markov switching models from Tables 3 and 4 suggest that the fluctuations of $\delta + / - 0.003$ are not counterfactually large.

As an alternative to the Hodrick-Prescott filter that is applied here and elsewhere in the literature prior to calculating correlations, Cogley (1997) uses a consumption-based measure of the business cycle from Cochrane (1994). The idea of this nonstructural measure of the trend is that consumption provides a good estimate of the trend level of output, since consumers try to distinguish between permanent and transitory movements in income. With this measure of the cyclical components, Cogley (1997) finds that hours and productivity are negatively correlated. Figure 2 shows that, even with the HP filter applied to the model-generated data, negative correlations readily attained from our model with depreciation switching, with switching, for example, with $\delta = 0.017 + / - .005$.

Impulse Responses

The precise meaning of impulse responses to a switch in a Markov process requires explanation. In particular, we have to define the 'shock' behind an impulse response. We run parallel simulations of the DSGE model for the 'switch' and 'no switch' scenarios. Both simulations share the same technology shocks and a depreciation rate that randomly follows the Markov switching process until 20 periods before the end of the sample of length T. At that point, the first simulated series puts depreciation into the low state for the next six periods; the second series puts depreciation into the low state for the next period, the high state for the next four periods, and then the low

Figure 2: Hours-productivity Correlation and Depreciation Switching
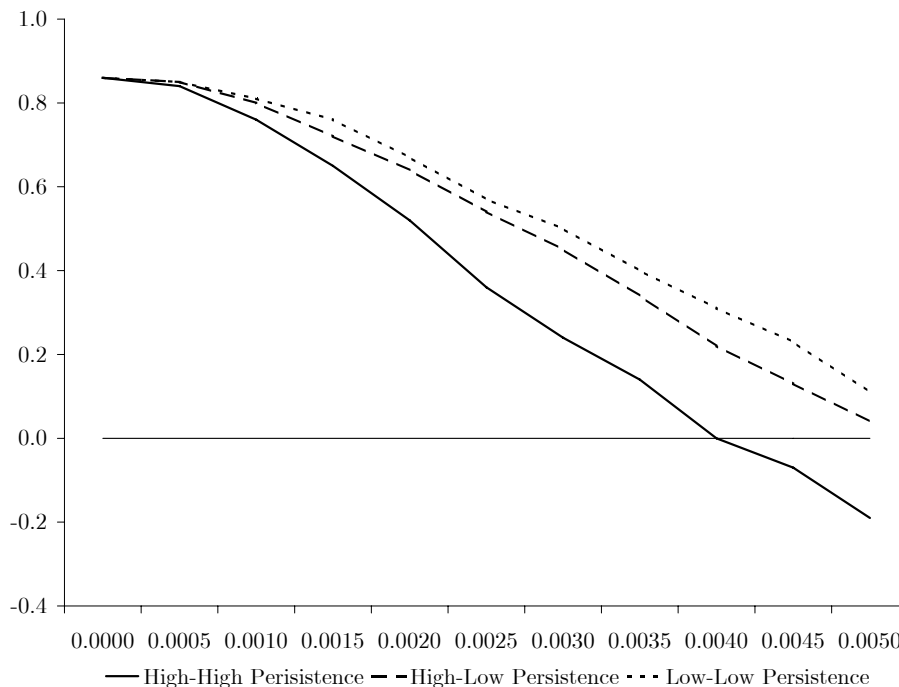
state again in the sixth period. A switch in the depreciation rate becomes known in the same period and the endogenous variables reflect this new information, such that the variables should respond in the impulse before the capital stock is affected. After the specified return to the low state in the sixth period, the two series again share a common set of realizations of the Markov switching process for depreciation. The four-period duration of the high-depreciation state roughly reflects the half life of a spell in that state according to its transition probability, $q_\delta = 0.85$ ($0.85^4 = 0.51$). The difference between the paths of variables with and without the four period sojourn to the high-depreciation state serves as a measure of the impulse response to a switch to the high-depreciation state. The reported impulse response is the average response from 400 simulations of the model.

Figure 3 plots the impulse responses of output, capital, hours and productivity to a switch to the high-depreciation state that lasts four periods. In the low state the depreciation rate δ is set to 0.018 with transition probability 0.9, whereas in the high state $\delta = 0.022$ with transition probability 0.85.

Figure 3: Impulse Responses to a High-depreciation Episode
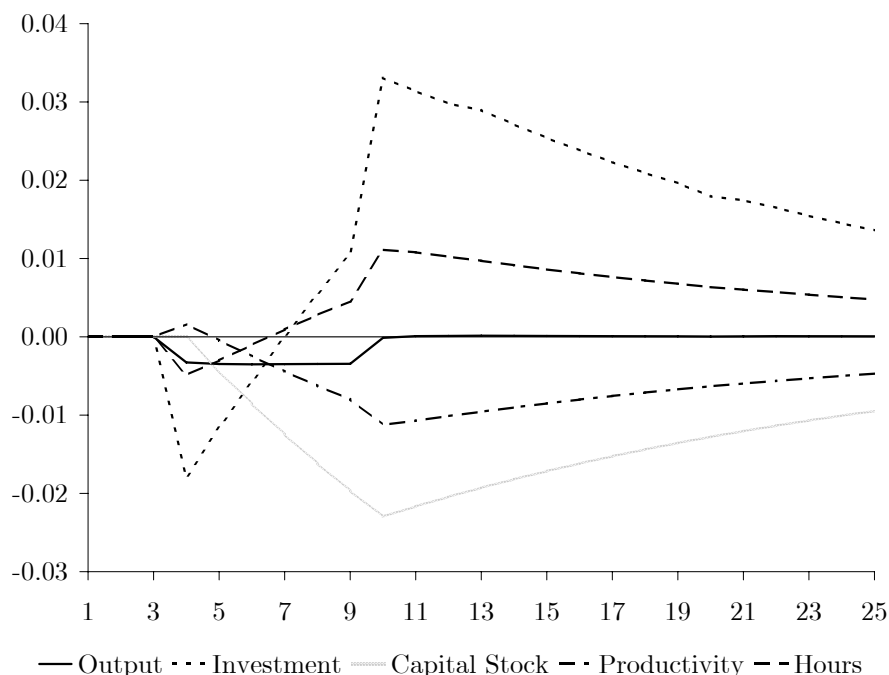
The shock to capital generates the typical non-humped shaped dynamics common among RBC models (see Gilchrist and Williams, 2000). More important for us is the observation that depreciation switching tends to push the hours-productivity correlation downward or even makes it negative, depending on the degree of switching. With enough variation in the depreciation rate, the negative effect on the hours-productivity correlation can overcome the tendency of technology shocks to induce a positive correlation.

The depreciation shock causes a negative level shift in output. Hours fall initially because the capital stock is relatively high at first in relation to the lower level of output. As the capital stock falls, however, labor hours must be substituted for capital to maintain steady output. When the depreciation rate switches back to the low state, output returns immediately to its initial level. To achieve this level of output, hours must jump above their initial level because the capital-output ratio is lower than it was initially. Gradually hours decline as the rebuilding of the capital stock allows for substitution of capital for labor. Throughout this process, the capital-output ratio moves in the opposite direction from hours. Thus, labor productivity moves in the opposite

direction from hours. If depreciation switching is of sufficient magnitude and frequency, the negative correlation it imparts between hours and productivity can overcome the positive correlation implied by technology shocks, as shown in Figure 2.

5   Summary and Conclusions

A big question mark concerning stochastic depreciation shocks is whether they have to be unrealistically large to have any useful role in a dynamic general equilibrium model economy. We first consider implied depreciation rates from sectoral data from the Bureau of Economic Analysis. These depreciation rates vary across time solely due to compositional changes within each sector. Hence, they tend to understate the range of fluctuation that would hold if the economic shelf life of capital varied endogenously as in Cooley, Greenwood and Yorukoglu (1997). Even with this understated variance from the data, however, we find that if depreciation rates follow a Markov switching process, the variance does not have to exceed the variance found in the data to have economically significant effects on the behavior of a model economy.

In this way, we re-investigate the question posed by Ambler and Paquet (1994): can a stochastic depreciation rate for capital account for a low correlation between hours worked and labor productivity? We find that the answer is yes, given a Markov switching process for depreciation that is similar to what we estimate from the data. Several results emerge from a simple RBC model augmented with Markov-switching depreciation. First, the model is able to replicate the low hours-productivity correlation found in the data using shocks that have a lower variance than the white-noise shocks from Ambler and Paquet (1994) would require. Second, although the data on depreciation rates exhibit large differences across sectors, the aggregate depreciation rate can be held within a relatively narrow range and still engender a low correlation between hours and productivity. Third, linear decision rules imply certainty equivalence and ignore the aversion that agents have to skewed parameter distributions. We illustrate the level effect implied by nonlinear decision rules in simulations of dynamic general equilibrium models that include Markov switching parameters.

Appendix: Numerical Implementation

A feature of the dynamic general equilibrium for the model described in Section 3 makes the use of traditional solution techniques problematic. Since Markov switching is present in the model's parameters, there is no model steady state to serve as a center of approximation. In addition, the values of these switching parameters in different states are of crucial importance in determining the decisions of agents. Thus, the switching cannot simply be "turned-off" to provide a deterministic steady state.

In light of these problems, we use a solution technique first discussed by Judd (1998), called the projection method (see Gong (1995) and Andolfatto and Gomme (2003) for alternative solution procedures). The idea is to approximate the agent's decision rules by polynomials that "nearly" solve the agent's optimization problem in a way made formal below. Using polynomials allows us to represent the approximate rules in a compact form.

To apply the projection method, we express the agent's optimization problem in the following form:

$$\underset{u_t}{Max}\, E_0 \left[\sum_{t=0}^{\infty} \beta^t r(x_t, u_t, D_t) \right]$$

subject to the constraints

$$x_{t+1} = g(x_t, u_t, D_t, \varepsilon_{t+1})$$

with x_0 a given. In this formulation, x_t is an $n \times 1$ vector of state variables, known by the agent at time t; u_t is an $m \times 1$ vector of the agent's decision variables. The $d \times 1$ vector D_t is a vector of variables referred to as economy-wide variables (The transition function, D_t is not needed for our analysis, but it is included to keep the discussion general). These are variables that the agent assumes are unaffected by his decisions and which lack fixed transition equations. They will be determined by a set of equilibrium conditions. Finally, ε_{t+1} is an $e \times 1$ vector of random shocks and $r(x_t, u_t, D_t)$ is the agent's utility $U(c_t, l_t)$.

State variables can be further sub-divided into two groups. One group consists of variables that are purely exogenous to the agent. Their transitions will depend only on factors outside of the agent's control, namely themselves and economy-wide variables. These variables represent things like the technology level. State variables that are under the control of the agent do not have transitions dependent on their current values. For such a variable, x_t^i, controlled by the agent, x_{t+1}^i depends only on u_t, D_t, and possibly ε_{t+1}. These

variables are things specific to the agent. Our assumption as to transitions for these variables are not really restrictive in that we have yet to come across a dynamic general equilibrium model that cannot be put in a form that satisfies this assumption. The primary reason for this restriction is to enable us to write the agent's first-order conditions as described below without having to deal with a value function as well.

In what follows, we denote differentiation with respect to a decision variable with a Greek subscript, ψ. Differentiation with respect to a state variable is denoted with a Roman subscript. The first-order conditions of the agent's optimization problem then are:

$$r_\psi(x_t, u_t, D_t) + \beta E_t \left[\sum_{i \in C} r_i(x_{t+1}, u_{t+1}, D_{t+1}) g^i_\psi(x_t, u_t, D_t, \varepsilon_{t+1}) \,|x_t \right] = 0,$$

for $\psi = 1, \ldots, m$. The sum of products of derivatives of the return function and the transitions functions is taken over those variables that are under the agent's direct control, hence the shorthand notation, $i \in C$. The equilibrium can be characterized as a set of functions, $u_t \equiv u(x_t)$ and $D_t \equiv D(x_t)$, that determine the agent's decisions and the values of the economy-wide variables as functions of today's states. The agent's first-order conditions are m functional equations in these unknown functions. To complete the determination of these unknown functions, we need d additional functional equations. These will be equilibrium conditions for the economy and can take a wide range of forms. The usual case is for them to take a simple form such as $e(x_t, u(x_t), D(x_t)) \equiv 0$.

In the concrete case discussed above, we can express the vectors above as follows:

$$x_t = (z_t, \delta_t, k_t).$$

Note that agent takes the level of technology, beliefs about the current state of the Markov switching parameters as given. The decision vector is:

$$u_t = (k_{t+1}, l_t).$$

Since it is impossible to derive an analytic expression for these unknown functions, they are approximated by sums of polynomials of the following form:

$$u(x_t) \approx \sum_{w_1 + w_2 + \cdots + w_n \leq W} c_{(w_1, w_2, \ldots, w_n)} \varphi_{(w_1, w_2, \ldots, w_n)}(x_t),$$

where W is the upper bound on the degree of the polynomial and $c_{(w_1, w_2, \ldots, w_n)}$ are scalar weights for the basis polynominals φ. The φ functions are polynomials that take the form:

$$\varphi_{(w_1, w_2, \ldots, w_n)}(x_t) = T^{w_1}(x_t^1) T^{w_2}(x_t^2) \cdots T^{w_n}(x_t^n),$$

where $T^{w_i}(x)$ is a polynomial in x with degree w_i. $T^{w_i}(x)$ could, technically, be simply x^{w_i}, but these polynomials are notorious for their extremely poor approximation properties. Consequently, we use suitably scaled and translated Tchebychev polynomials for the $T^{w_i}(x)$ functions. These not only have excellent approximation properties, but are also easy to evaluate with the intrinsic functions that come with most standard software packages. Usually defined on the interval $[-1, +1]$ as $T^n(x) = \cos(n \arccos(x))$, they can be easily evaluated by any package that has an intrinsic cosine and arccosine function.

In general, $m + d$ functional equations implicitly determine both $u(x_t)$ and $D(x_t)$. Let us denote them as

$$R_\psi(x_t, u(x_t), D(x_t)) = 0,$$

for $\psi = 1, \ldots, (m + d)$. We need to find coefficients for the polynomial approximations so that the approximations "nearly" solve the set of functional equations above. There are numerous ways to do this, as described in detail in Judd(1998). The most natural choice is a set of coefficients that sets

$$\int R_\alpha(x, u(x), D(x)) \varphi_{(w_1, \ldots, w_n)}(x) dx = 0,$$

where these integrals are taken over some pre-determined region of space thought to capture most of the dynamic behavior of the economy and the $\varphi(x)$ functions have been translated to center on this region. This approach is appealing since it transparently gives one equation for each unknown coefficient in each functional approximation. It is also eminently reasonable in that, if for some reason, the equations $R_\alpha(x, u(x), D(x)) = 0$ were satisfied exactly by polynomials u and D, these conditions would determine their coefficients exactly.

As long as the R_α functions are set to zero in some average sense over a region, we should have reasonable approximations to the agent's decision rules over this region. Consequently, we use a pseudo-random Monte Carlo method to calculate the integrals above. Thus, derivation of the coefficients in our polynomial approximations has been reduced to the solution of a set of nonlinear equations in these coefficients. While evaluation of the equations

themselves can be slow, Broyden's method for solving nonlinear equations works well.

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