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The Remarkable Stability of Monetary Base Velocity in the United States, 1919-1999

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Abstract

This analysis examines the long-run demand for the adjusted monetary base in the United States, 1919–1999. When the "price" of the base is measured by the inverse of the yield on long-term, high-quality corporate bonds and an appropriate functional form is selected, the quantity of base money demanded is found to be a stable function "…of a small number of variables."

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In a fiat-money economy, the price level is anchored by the "outside money," or monetary base, provided by the central bank. In such an economy, the *monetary base* consists of (1) those liabilities of the monetary authorities that are used as media of exchange by the nonbank public (households and firms other than depository institutions) and (2) those liabilities that are used by depository institutions to settle interbank payments.[1] In most cases, payment in such central-bank money constitutes final settlement of debts. To be useful for monetary policy analysis, the monetary base must be adjusted for the effect that changes in statutory reserve requirements have on the amount of base money held by banks. The *adjusted monetary base* removes such effects. The resulting series satisfies the conditions, proposed by a number of authors, for a variable to be a policy index: Up to a linear transformation, the variable must provide a time-invariant, monotone scale for measuring whether a change in the policy instrument results in a more or less expansionary policy, with respect to aggregate economic activity.[2] To the best of our knowledge, the idea of using the adjusted monetary base as an index of the stance of monetary policy dates to an analysis published in 1961 by Karl Brunner.[3]

The Challenge: Does A Stable Demand Relationship Exist?

It is the current "conventional wisdom" among (many) economic scholars that stable, long-run relationships involving monetary aggregates and economic activity in the United States do not exist. John Taylor's initial exploration in 1987 of interest rate–based monetary policy rules, for example, grew from a frustration with the apparent instability of U.S. money demand.[4] Later, Friedman and Kuttner (1992, p. 490) asserted that all extant stable money-demand relationships from the 1970s are sample specific and "disintegrate when the sample extends into the 1980s."

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[1] Balbach and Burger (1976).
[2] See, for example, Brunner (1969), Blinder and Solow (1976), and Friedman (1990).
[3] Brunner (1961). See also Brunner and Meltzer (1964).
[4] Taylor (1985).

Friedman set down the challenge for researchers:

> "What matters is whether it is possible to identify before the event a set of regularities of sufficient centrality and robustness to provide the qualitative and quantitative basis for sound policymaking." (1993, page 152)
>
> "The well-known reason for this dramatic reversal [of fortune] is simply that the empirical relationships that once connected money growth to the growth of either income or prices in the United States [during the 1970s] have utterly collapsed. As Kenneth Kuttner and I have shown (1992), data for the most recent quarter-century of U.S. experience provide no evidence of any predictive content of money growth with respect to subsequent movements of either income or prices—or, for that matter, any other macroeconomic variables commonly taken to be of interest for purposes of monetary policy." (1994, page 124)

Despite such skepticism, subsequent research has suggested the existence of some stable relationships. Empirical modeling is always beset by compromises and choices, and inauspicious choices may lead to incorrect inferences. In the case of money demand, one of these seems to be the choice of functional form. Previous studies have demonstrated that Friedman and Kuttner's criticism is not robust to the use of reasonable, alternative functional forms.[5] In their work, Friedman and Kuttner examine only semi-log functional forms, that is, functions that include the *log-levels* of real balances and real income and the *levels* of nominal interest rates. This functional form, based on early work by Cagan (1956) and others, is popular because it preserves linear relationships among the model's endogenous variables. It is problematic in monetary policy–oriented empirical research, however, for two reasons.

- First, there is no *a priori* way to assess its closeness to the true, unknown data–generating process. It is essential to test robustness to the choice of functional form. Below, for the adjusted monetary base, we show stable long-run relationships using functional forms that include logs and reciprocals of interest rates.

- Second, the preservation of aggregate portfolio relationships—the major motivation for the form—may be irrelevant for monetary policy analysis. The currently most popular model of monetary policymaking assumes that policy consists of setting the near-term level of a short-term interest or exchange rate.[6] The data–generating process in such models includes three variables—real output, the inflation rate, and a short-term nominal interest rate—and three equations: (1) a Hicks aggregate demand curve (equivalent to an IS curve under the assumption of no wealth effects in consumption and intertemporal separability of utility), (2) a Lucas expectations–augmented Phillips curve, and (3) a Taylor monetary policy "rule" specifying the decision process followed to determine the desired level of the target interest rate. The model is closed by invoking rational expectations. In such a model, aggregate portfolio relationships play no role; hence, there is little motivation to examine only money demand functions that are linear in interest rates.

If we are to question the appropriateness of Cagan's semi-logarithmic functional form, what are the alternatives? Slightly before Cagan, Latané (1954) proposed an alternative equation using the reciprocal of a long-term interest rate. In this analysis, we examine functional forms containing two nonlinear transformations of interest rates: natural logs and reciprocals (inverses). A priori, we believe that a specification (functional form) is preferred over other forms if the implied relationship between monetary–base velocity (or its log) and the interest rate (or its transformation) is more linear (or less nonlinear). Why? Because extrapolation to out–of–sample experience is more likely to remain valid. Our criteria for choosing between them are explored further below.

The Historical Behavior of Monetary Base Velocity

For more than 30 years, the Federal Reserve Bank of St. Louis has constructed and published figures on the *adjusted monetary base*. Recently, St. Louis Fed has developed historical figures

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[5] For results concerning M1, see Rasche (1987) and Hoffman and Rasche (1996). For results concerning M2, see Rasche (1990) and Carlson et.al. (2000).

[6] See Taylor (1999) and McCallum (1999).

for 1919–1935 and constructed new estimates for 1994–1999.[7] This long, consistent time-series allows us to re-examine the stability of the demand for the adjusted monetary base.

Perhaps the most elementary analysis of the demand for a monetary aggregate, *M*, is a scatter plot of its GDP velocity, *Y/M*, and a nominal interest rate, analogous to Lucas's (1988) discussion of M1 demand. Figure 1 displays, for 1919–1999, such a diagram for the adjusted monetary base. Annual-average values of the log of monetary–base velocity, ln*(Y/M)*, are plotted against the vertical axis, and the Aaa corporate bond rate is plotted against the horizontal axis. The graph has two striking features: the relatively tight scatter of the points and the apparently highly nonlinear relationship between the two variables. Although the tight fit is attractive, we find the nonlinearity less so. In addition, when interpreted as a demand curve, the Lucas scatter diagram suggests that the interest elasticity of money demand is relatively larger at high interest rates and relatively smaller at low rates.[8] In the absence of knowledge of the true data–generating process, there is no agreed-upon way to assess, a priori, the reasonableness of this result. Ceteris paribus, however, we would prefer a relationship that is more linear and displays an interest-rate elasticity that is non-increasing in the level of nominal interest rates.

Figure 2 is scatter plot of a constant-elasticity specification: the log of monetary-base velocity is plotted on the vertical axis and the log of the Aaa bond rate on the horizontal axis.[9] The relationship is distinctly more linear than that shown in Figure 1 (and, by design, displays a constant interest elasticity)—but a distinct curvature remains. In Figure 3, the log of monetary-base velocity is plotted against the *inverse* of the Aaa bond rate. A striking aspect of this Figure is that, with the exception of an outlier in 1921 and several outliers during the 1930s, the relationship is remarkably linear over the 80-year span of the data. Some heteroscedasticity is

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[7] Anderson and Rasche (1999, 2001).
[8] The interest-rate semi-elasticity is constant. The elasticity varies directly (positively) with the level of the interest rate.
[9] The functional form in Figure 2 is comparable to the double-log, constant-elasticity regression specifications for M1 and M2 in Meltzer (1963) and Chow (1966).

apparent—at lower interest rates, the points are more widely dispersed than at higher rates—but there is no evident curvature in the relationship. This specification satisfies our prior belief that the interest elasticity should be a decreasing (non-increasing) function of the level of the interest rate—that is, as firms and households confronted by rising interest rates reduce their holdings of base money, each successive reduction becomes more difficult.

We do not find the apparent relationship between velocity and the Aaa bond rate surprising. A number of previous authors have suggested that long–term, rather than short–term, rates are more appropriate in money demand functions, particularly if longer-run relationships are of primary interest.[10] Other authors, such as Friedman and Schwartz (1982), have argued that the entire spectrum of the yield curve should be included because money is held for a number of reasons, including as a precaution against uncertain future income and expenses. Unfortunately, attempts in empirical work to include both short- and long-term rates have suffered from severe parameter instability. Usually, a choice must be made to include either a short– or long–term rate, but not both.

One might interpret the monetary base held by the public as a zero-coupon government perpetuity, that is, as a consol. If so, the opportunity cost of holding base money might be well measured by the yield on long-term corporate bonds with a very low default risk. In their comparison of money demand in the United States and the United Kingdom, Friedman and Schwartz (1982) chose the yield on British consols for the United Kingdom and the yield on long-term high-grade corporate bonds for the United States, no consol yield being available.[11] In this respect, we follow Friedman and Schwartz and interpret the bond yield as a reasonable proxy for a consol yield. Additional support for our choice is provided by a comparison of the Aaa bond yield with the "American consol" series of Amsler (1984). The insight of Amsler (1984) is that certain issues of corporate preferred stock may be interpreted as consols after an adjustment for

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[10] See, for example, Hamburger (1977).

default risk. Amsler examines Pacific Gas and Electric's 6% cumulative preferred stock, first issued in 1914, which trades on the American Stock Exchange as PCGA. Under the U.S. tax code, dividends paid on stock issued by public utilities prior to October 1, 1942, are a deductible expense for the payer and taxable income for the recipient, similar to bonds. To estimate the unobservable yield on a default risk–free Treasury consol, Amsler adjusts the PCGA current yield for default risk by subtracting two items: the difference between Moody's Public Utility AAA Bond Index and Moody's Long Term Government Bond Index, and the difference between Moody's AA Public Utility Bond Index and Moody's AAA Public Utility Bond Index. During her sample period of 1953–1983, the first adjustment varies in size from 20 basis points during the early 1950s and early 1960s to more than 200 basis points in 1971 and 1974–1975. The second adjustment is near zero during the early 1950s and 1960s but, during the 1970s, begins a steady increase to reach 80 to 90 basis points during 1981–1982. For this analysis, we have extended Amsler's series through 1999. The upper panel in Figure 4 shows the current yield on PCGA, Amsler's consol yield (plus our extension for 1984–99), and the Aaa corporate bond yield; the lower panel shows their inverses.[12] The PCGA current yield and the Aaa bond yield move together closely. The consol yield and the Aaa bond yield have approximately the same trend except during the late 1960s and early 1970s when the risk premium for AAA utilities increased sharply over Treasuries.

We conclude from the above analysis that an index of Aaa-rated corporate-bond yields—which is implicitly adjusted for default risk by the inclusion of only Aaa-rated firms—is, for studies of monetary-base demand, an adequate measure of the unobservable yield on a Treasury consol.

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[11] Friedman and Schwartz (1982, p. 500).

[12] PG&E's senior 6% cumulative preferred issue were first issued at $100 in 1914 and split to $25 in 1927. Virtually all of the 4.2 million shares outstanding were issued prior to 1933. Table 1 in Amsler (1984) shows current yields (equal to the annual dividend divided by the market price) as-of the first business day

Outliers and the Great Depression

A well-formed economic model should fit the data reasonably well during all periods, albeit perhaps with a small number of necessary intervention and/or dummy variables. The scatter plot in Figure 3 suggests three groups of outliers. The first is 1921, a single year that falls outside the cluster of points. The second group is a "loop" that begins with Great Britain's departure from the gold standard in 1931 and ends with 1936's tentative economic recovery. The third group is a loop that begins with the economic slowdown in 1938 and ends with the war-related economic expansion of 1941. If our scatter diagram is to be interpreted as a monetary-base demand curve, what are we to make of these outliers?

There is no clear explanation for the outlier in 1921. The business cycle of 1920–1921 was one of the sharpest and briefest on record. Bank failures rose tenfold between 1919 and 1921, but there were neither bank runs nor an apparent loss of confidence by the public in banks that would have sharply increased holdings of base money.[13] Although monetary policy clearly was concerned with gold flows, interest rates were not at unusual levels. Friedman and Schwartz (1963, p. 241) note that the rapid rebound in 1921 makes annual–average income data misleading as a measure of the economic situation: But is it more leading for income than for the monetary base? To test this hypothesis, we replaced the numerator of the 1921 observation in Figure 3 with the 1921–1923 average of GNP while retaining the 1921 monetary base as the denominator. This new point falls within the main scatter of points, albeit at the bottom edge. Nevertheless, we conclude that we have been unable to uncover any compelling evidence to explain the 1921 outlier.

Unlike 1921, the events of the Great Depression, including a sharp increase in the demand for currency, are compelling and well known. During the 1930s, systemic,

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of the quarter from 1953:Q1 to 1983:Q2. We have calculated annual yields as geometric averages of Amsler's quarterly yields.

[13] Friedman and Schwartz (1963) note that the currency-deposit ratio changed very little during the period.

nondiversifiable risk in the economy increased sharply. This, in turn, increased the demand for the monetary base, both as currency and as bank reserves. The increase in perceived risk likely wasn't uniform during the decade. It can be argued that up to mid-1931 households believed (or hoped) that recovery would begin soon.[14] After mid-1931, the economy began its slide into the abyss and perceptions of default risk on assets other than U.S. government debt likely increased rapidly. The 1936–1937 recovery likely raised hopes (and earnings) until the economy slowed into the 1937 recession.

Certainly, the riskiness of investments has fluctuated at other times, but the 1930s are unique in our sample because the increases collided headlong with the deterioration in household balance sheets that began in 1930.[15] The stock market crash, itself, wiped out significant household liquid wealth. In addition, deflation led to sharp increases in the real value of household liabilities because faltering economic activity made it more difficult to service such debts. The real value of liabilities, relative to assets, increased sharply. In this environment, both the ability and willingness of households to bear risk were reduced. In addition, banks increased their holdings of vault cash as a precaution against bank runs, even though vault cash was not eligible to satisfy statutory required reserves.[16]

Temin (1976) documents the increase in perceived riskiness of corporate bonds. By construction, the increase in perceived risk is not measurable from observations of yields on constant-quality Aaa– and Baa–rated bonds. Temin discusses two other measures. First, down-ratings of bonds by rating agencies increased sharply, from approximately 1 to 2 percent of the

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[14] Temin (1976) provides a chronology of discussion and opinion in the business and financial press.
[15] Mishkin (1978) and Olney (1999)
[16] During the Depression, only deposits at Federal Reserve Banks could be used to satisfy statutory reserve requirements. To the extent that banks, after 1932, held larger stocks of vault cash as a precaution against bank runs, demand for the monetary base was higher. Note that these precautionary amounts of bank vault cash are not included in Board of Governors' statistics on bank reserves because vault cash was not eligible to satisfy statutory reserve requirements. They are, however, included in the Federal Reserve Bank of St. Louis figures on bank reserves and the monetary base (Anderson and Rasche, 2001).

outstanding stock in 1929, to 5 percent in 1930, and to 12 percent in 1931 and 1932.[17] Second, Temin develops a new measure of the increase in the risk premium by comparing the changes in the spread between Aaa and Baa yields to changes in the spread between Aaa yields and yields on a fixed-company sample of bonds. Both measures somewhat more than double between December 1929 and June 1931. The former measure increases from 128 basis points during December 1929 to 300 basis points during June 1931, and the latter increases from 308 to 699 basis points. Clearly, the perceived riskiness of corporate debt had increased. Temin (1976) documents that, in many cases, bank runs and failures during 1930 were rational actions by the public: many banks had suffered bond defaults and were insolvent when the bonds they held were marked to their market values.

Our hypothesis is that these events during 1930 and 1931 sensitized the public to financial market problems. The economic outlook of households and firms, which was guardedly optimistic during most of 1930, turned sharply pessimistic during the fourth quarter when the expected recovery failed to appear. Although some modest return to financial stability was evident during early 1931, the departure of Great Britain from the gold standard likely shattered the last confidence in recovery.

The forward-looking variable "down-rating of corporate bonds" was not significant in our empirical regression models. One reason, perhaps, is that the variable may not be a good measure of future expectations during the 1930s: the fates of corporations likely deteriorated much faster than financial analysts could capture it in a bond-rating index. Our alternative is to assume a rough type of "perfect foresight" and include in regressions the actual, observed default rate on investment-grade corporate bonds. Annual new defaults and the default rate (relative to beginning–of–year outstandings) are shown in Figure 5. The 1930s are clearly unusual, with

[17] Temin (1976, p. 81, Table 10). The figures in the text are our calculations from Hickman (1960), which is the source cited by Temin. The figures reported in Temin appear to be in error; for details, see the Appendix.

respect to both the level of defaults and the speed with which the default rate accelerated. We report regressions in Table 1 with and without the default rate on corporate bonds.[18]

Estimates of linear regressions based on the inverse-rate functional form are shown in Table 1. In the upper panel, the dependent variable is the log of the GDP velocity of the adjusted monetary base. In the lower panel, the velocity specification is relaxed such that the dependent variable is the log of the deflated adjusted monetary base and real GDP is included as a regressor. Note that the coefficients on real GDP reported in the lower panel are neither statistically nor economically different from unity. As a result, the estimated coefficients on the Aaa bond rate in the upper and lower panels are approximately the same, equal to –0.03.

The effect of removing the regression-estimated effect of defaults is shown in Figure 6. In the figure, "default-adjusted" velocity is calculated by subtracting from monetary-base velocity the product of the corporate-bond default rate and its estimated regression coefficient. The "adjustment" clearly brings the outliers (except 1921) back into the main constellation of points. After this adjustment, some curvature seems apparent for observations during the bond-pegging period, 1941–1951 (lower right-hand corner of Figure 6).

As a further check on robustness, Figures 7 and 8 display scatter plots using the yield on Baa-rated bonds. Our conclusions are unchanged. In Figure 7, the same overall linearity and outliers are evident as in Figure 4; in Figure 8, removing the estimated effect of bond defaults moves the outliers back into the main constellation of points. The relationships shown in the Figures also appear in quarterly data.

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[18] The data on default rates of investment-quality corporate bonds through 1965 are from Hickman (1960) and Atkinson (1967). These data end in 1965. However, during the 1950s and 1960s the volume of defaults relative to outstanding corporate debt is extremely small. Since there have been no default epidemics on investment-grade bonds since that time, we have assumed that the value of the series is zero for all years since 1965. This assumption is supported by publicly available data from Moody's Investor Service, available on Moody's Internet web site (www.moodys.com).

Statistical Tests for Functional Form

The above analysis suggests that the inverse-rate functional form is highly consistent with the data. In this section, we test this hypothesis. In models of money demand, we prefer one functional form relative to another if it (1) is more nearly linear (perhaps after transformation of the variables) and (2) displays an interest elasticity that is a decreasing function of the level of interest rates. Extrapolation and counterfactual policy analysis is hazardous at best—nonlinear functional forms only make it more so.

The compatibility of our preferences regarding functional form with the observed data may be statistically tested with a generalization of the Box-Cox transformation.[19] Let $y_t^{(\lambda)}$ denote $\left((y_t^{\lambda}-1)/\lambda\right)$ for $\lambda \neq 0$ and denote $\ln(y_t)$ for $\lambda = 0$. Consider the general transformation-of-variables function

$$m^{(\lambda_0)} = \alpha + \beta\, i^{(\lambda_1)} + \gamma\; y^{(\lambda_2)} , \tag{1}$$

where *m* denotes the real monetary base, *i* the interest rate, and *y* real income. If we set $\lambda_0 = \lambda_2 = 0$ and $\gamma = -1$ and multiply the equation by minus unity (that is, –1), we obtain (ignoring the sign of the intercept) the velocity specification

$$\ln(v) = \alpha - \beta\, i^{(\lambda_1)}, \tag{2}$$

where $v = (y/m)$ is monetary-base velocity. When $\lambda_1 = 0$, the log-log form $(\ln v = \alpha - \beta \ln i)$ arises; when $\lambda_1 = 1$, the log-level (semi-log) form $(\ln v = \alpha - \beta\, i)$ arises; and when $\lambda_1 = -1$, the log-inverse form $(\ln\ v\ =\ a\ +\ a_1\ /\ i)$ arises. If the velocity restriction ($\gamma = -1$) is relaxed such that γ is unrestricted, the general transformation is

[19] See, for example, Judge et al. (1988) and Zarembka (1974). The usual Box-Cox transformation is a special case of the above with only a single parameter, $\lambda_0 = \lambda_1 = \lambda_2 = \lambda$. Although various analyses suggest that estimates of the Box-Cox transformation parameter λ are robust to departures from normality, they also suggest that estimates may be biased toward values of λ that imply linearity if the data generating process of the disturbance is heteroskedastic (Zarembka, 1974).

(3) $$\ln(m) = a + \beta\, i^{\lambda_1} + \gamma \ln(y)\,.$$

If $\lambda_1 = -1$, then (1) becomes

(4) $$\ln(m) = a + \frac{\beta}{i} + \gamma \ln(y)\,.$$

Recursive estimates of β for the velocity-restricted model (2) are shown in Figures 9, 10 and 11 for three values of λ_1: –1, 0, and 1; similar estimates of β and γ in (3), without the velocity restriction, are shown in Figures 12, 13, and 14. In each figure, the dashed line denotes a recursion that begins with 10 years of data, 1919 to 1929, and adds one year of data to the end of the period until achieving a span of 1919 to 1999. The solid line denotes a recursion that begins with 1989–1999 and adds one year of data to the beginning of the period until achieving a span of 1919 to 1999. Comparing results, the successive estimated values of β and γ for the inverse-rate specification, $\lambda_1 = -1$, settle down near their final values somewhat sooner than do estimates for the other values of λ_1. CUSUM and CUSUMSQ diagnostic statistics, with 95 percent confidence bounds, also are shown in Figures 9 through 14.[20] The CUSUM statistics suggest that the inverse-rate specification is superior to the semi-log and log-log specifications, both with and without the velocity restriction. The CUSUMSQ statistics are inconclusive, with no model superior to the others.

We conclude that the inverse-rate, velocity-restricted specification is at least as compatible with the data as the other two alternatives.

Beyond recursive estimation for alternative (but fixed) values of λ_1, equations (2) and (3) suggest that β and γ may be considered as continuous functions of λ_1, that is, as $\beta(\lambda_1)$ and $\gamma(\lambda_1)$. Estimates of $\beta(\lambda_1)$ and $\gamma(\lambda_1)$ for the range ($-1 \le \lambda_1 \le 1$) are shown in Figure 15; the estimated

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[20] Harvey (1981, pp. 151–2). The CUSUM statistic consists of partial sums of standardized recursive residuals, where each residual is divided by the standard error of the regression. The CUSUMSQ statistic consists of partial sums of the squares of recursive residuals, where each residual is divided by the aggregate sum of squared residuals.

coefficient on the corporate-bond default rate and the regression standard error also are shown. Values of γ, the income-elasticity coefficient, remain near 1 for all values of λ_1; values of the other coefficients are quite sensitive to λ_1. The residual standard error of the regression is smallest for the log-inverse specification ($\lambda_1 = -1$), as suggested by Figures 1, 2 and 3.

Overall, we conclude once more that the inverse-rate specification appears to be the best fit to the data.[21]

Besides $\beta(\lambda_1)$ and $\gamma(\lambda_1)$, the interest elasticity of monetary-base demand is a (nonlinear) function of both the sample data (especially the interest rate) and λ_1, denoted as $\eta(\lambda_1, i)$. The three lines shown in Figure 16 examine interactions among the values of λ_1 and the value of the Aaa rate. Let i_{min}, i_{mean}, and i_{max} denote the minimum, mean, and maximum values of the Aaa bond rate during the sample period. Then, the three lines are: $\eta(\lambda_1, \mathrm{i}_{min})$, labeled "1"; $\eta(\lambda_1, \mathrm{i}_{mean})$, labeled "2"; and $\eta(\lambda_1, \mathrm{i}_{max})$, labeled "3." The value $\lambda_1 = 0$ implies a log-log constant-elasticity specification: $\eta(\lambda_1, i)$ does not vary with i and, the three lines intersect at $\eta(\lambda_1, \mathrm{i}_{mean}) \cong -0.64$. Note that when $\lambda_1 \neq 0$ the cross-partial derivatives of $\eta(\lambda_1, i)$ are strongly non-zero, as illustrated by the difference between the lines labeled "1" and "3."

Below, to simplify notation, we denote as $\eta(i)$ the case where $\lambda_1 = -1$, that is, the VECM specified with the inverse of the interest rate.

Implications of Functional Form for Monetary Policy

Beyond pragmatic concerns regarding linearity and statistical stability, the choice of functional form has implications for the formulation and conduct of monetary policy. During the

[21] A reader asked why we did not examine more general tests for (non)linearity of regression functions, in the context of money demand. We offer two defenses. First, such tests most often consider the null hypothesis that f( ) is linear in the relationship $w = \mathrm{f}(x,y,z)$. In our analysis, we consider the narrower hypothesis that false rejection of the linearity of f( ), when f( ) is linear, may be due to an incorrect choice of an interior, nested function. That is, we consider the null hypothesis that there exists a function g(y), possibly nonlinear, such that the linearity of f(x, g(y), z) is not rejected. In our analysis, y is a market interest rate or yield. Second, the practice of testing for functional form does not remove inference

1970s, when inflation and interest rates were "high," many central banks embraced monetary aggregates as intermediate policy guidelines. During the 1990s, when inflation and interest rates were relatively "low," most of the same central banks abandoned monetary aggregates, citing the "instability" of money demand. Could there be a connection among the level of interest rates, the functional form of the money demand curve, and the information content of monetary aggregates? Is it possible that the growth of monetary aggregates contains a relatively large amount of information useful for monetary policy when interest rates are high and a relatively small amount when interest rates are low?

Our functional form analysis suggests that this may be the case. Consider the familiar Cagan semi-log (that is, log-level) form. In this model, the interest elasticity of money demand increases with the level of interest rates: As money's opportunity cost increases, households and firms reduce their money holdings relative to income and spending—but the functional form says more. It asserts that the *ease* with which agents substitute away from money i*ncreases* as they reduce their money holdings, that is, that the interest elasticity of money demand is an increasing function of the level of interest rates. From a microeconomic perspective, and assuming that at least some money is held by households in general equilibrium, this seems contrary to the diminishing marginal rate of substitution implicit in Inada conditions. Similarly, when money's opportunity cost decreases because the central bank has achieved price stability, households and firms likely will increase their money holdings relative to income and expenditure; but the Cagan form asserts that this substitution against other assets becomes successively *more difficult* as money's opportunity cost decreases. These implications of the Cagan functional form seem unattractive.

In contrast, the inverse-rate functional form asserts that the ability of households and firms to further reduce money holdings relative to income—that is, the interest elasticity of money

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problems regarding, for example, in what way the model might be nonlinear; nor does it solve issues demand—decreases as they reduce their money holdings. Some monetary-policy models suggest an inverse correlation between the monetary-policy information content of a monetary aggregate and its elasticity relative to market interest rates, for two reasons.[22] First, a sustained change in the growth of the aggregate is more likely to be indicative of a change in economic activity rather than an adjustment to movements in market interest rates. Second, market interventions by the central bank that are intended to change short-term interests actions can, for a given desired change in market rates, be smaller. For these reasons, the underlying properties of a functional form are important to any assessment of the monetary-policy role of monetary aggregates (including the adjusted monetary base) regardless of any putative "instability" in the function.

A Structural VAR Interpretation

In this section, we provide a structural interpretation of our long-run monetary-base velocity equation as a demand curve. The analysis follows Rasche (2001), who extends the identification schemes proposed by Bernanke (1986) and Sims (1986) to provide a simple, consistent IV estimator of both the permanent and transitory shocks in an identified VECM model. For a VAR model of size p with r cointegrating vectors, the procedure may be interpreted in three steps. First, reorder the VECM such that the (p-r) equations that contain permanent shocks are ordered above the r equations that contain transitory shocks. Identify the (p-r) equations and their permanent shocks by imposing a Wold-like recursive structure and interpreting the equations as a King, Plosser, Stock, and Watson (1991) common-trends model. Next, impose normalization and exclusion restrictions on the r equations that contain the transitory shocks (although other restrictions are feasible) to identify these equations. The exclusion restrictions exclude some of the cointegrating vectors from the last r equations, and the normalization restrictions specify a priori some of the coefficients in the matrix of

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regarding hypothesis testing with a fixed sample size (Hamilton, 2001).
[22] Mishkin (1996).

contemporaneous (simultaneous) interactions among the variables. Finally, use the estimated identified permanent shocks as instrumental variables in estimation of the last r equations containing the transitory shocks.

Let X_t be a $(p \times 1)$ vector of data series and define the reduced form VECM data generating process for X_t as

(1) $$[I_p - \Gamma(L)]\Delta X_t - \alpha\beta' L X_t = \varepsilon_t,$$

where the matricies α, β are $(p \times r)$ and $\Gamma(L)$ is a matrix polynomial in the lag operator L. Let the moving average representation of the reduced-form data generating process be

(2) $$\Delta X_t = C(1)\varepsilon_t + (1-L)C^*(L)\varepsilon_t$$

with rank $C(1) = p - r$.

First, following Sims (1986) and Bernanke (1986), assume that the shocks in the underlying structural model are contemporaneously uncorrelated, $\Sigma_u = I_p$, and that the relationship between the shocks in the structural ("economic") and reduced form (VECM) models is of the form

(3) $$Bu_t = A\varepsilon_t.$$

Define the transformation $B = \begin{bmatrix} T_{11} & 0 \\ B_{21} & I_r \end{bmatrix}$ and $A = \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$, where W_1 is a $(p\text{–}r \times p)$ permutation matrix that selects $(p\text{–}r)$ independent rows of $C(1)$ and W_2 is a $(r \times r)$ permutation matrix chosen such that rank $\begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix} = p$. Then, the yet-to-be-identified economic model may be written as in equation (12) of Rasche (2001):

$$(4)\qquad \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix} \left[\mathrm{I}_p - \Gamma(\mathrm{L}) \right] \Delta X_t$$
$$= \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix} \alpha\beta'\mathrm{L}X_t + \begin{bmatrix} T_{11} & 0 \\ B_{21} & I_r \end{bmatrix} \begin{bmatrix} u_t^p \\ u_t^T \end{bmatrix}.$$

Note that $\begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix} \alpha\beta'\mathrm{L}X_t = \begin{bmatrix} 0 \\ W_2 \end{bmatrix} \beta'\mathrm{L}X_t$, that is, that the function of W_2 is to re-order the model's equations such that the cointegrating vectors appear in the last r equations of the transformed model.

The identification problem stated in (3) cannot be resolved solely by covariance restrictions; some restrictions must be placed on the parameters of the "economic" model.[23] Further, such restrictions always, in some part, are arbitrary because a minimally sufficient set of restrictions (that is, a set which gives exact identification) cannot be statistically tested. With that caveat in mind, impose the familiar reduced-form restrictions that T_{11} and T_{22} are lower-triangular. Then, (4) becomes

$$(5)\qquad \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \left[\{ I_p - \Gamma(L) \} \Delta X_t - \alpha\beta' L X_t \right] = \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix}$$

where $\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} T_{11}^{-1} & 0 \\ -B_{21}T_{11}^{-1} & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$ defines the contemporaneous (simultaneous) relationships among the ΔX_t .

The restrictions imposed to this point are too few to identify the model. Numerous alternative restrictions are possible. Normalizing each equation, for example, would be sufficient to exactly identify the model. To discuss normalization, it is useful to consider separately the first (p-r) and last r equations. For the former, normalizing the principal diagonal of T_{11} to unity is

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[23] Identification via a Wold causal-chain structure requires both that the covariance matrix of the structural shocks be diagonal and that the transformation matrix be lower-triangular (Fisher, 1966, ch. 4). Fisher (1966) shows that identification of a model, except for the case of a Wold causal chain, cannot be

sufficient to identify the first *(p-r)* equations. With this restriction, the first (*p-r*) equations form a King, Plosser, Stock and Watson (1991) "common trends" model. Hence, this normalization seems (to us) a very acceptable assumption.

What about the last *r* equations, the error-correction equations of the model that define the transitory shocks? Consider the matrix T_{22}. Above, we defined T_{22} as lower-triangular but placed no restrictions on the *r* elements of its principal diagonal. Absent these restrictions, the economic model is underidentified by *r* restrictions. Often, the last *r* equations are normalized on an element of the appropriate row of the matrix of contemporaneous (simultaneous) interactions of the elements of ΔX_t. Here, this matrix is

$$H_2 = [-B_{21}T_{11}^{-1} \quad T_{22}]\begin{bmatrix} W_1C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}.$$

Define $V = \begin{bmatrix} W_1C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}^{-1}$. Then, $H_2V = \begin{bmatrix} -B_{21}T_{11}^{-1} & T_{22} \end{bmatrix}$ defines *r*(*r*–1)/2 linear restrictions on the elements of H_2. To complete exact identification of both the transitory shocks and the dynamic structure of the last *r* equations in the "economic model," we can impose *r* additional linear restrictions (normalizations) of the form

$$R * vec[H_2] = r_0 .$$

This analysis focuses on a three-variable VECM model with a single cointegrating vector. Let X' = {rAMB, rGNP, InverseBondRate, DefaultRate}, where rAMB and rGNP denote the real adjusted monetary base and real gross national product, respectively. (Note that the dimension of the VECM is three because the default rate is treated as exogenous.) Johansen's (1991) maximum-likelihood estimation procedure suggests one cointegrating vector, and hence

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accomplished solely with covariance restrictions. Identification always requires some, perhaps "incredible," restrictions on the structural coefficients of the model.

the model has (p–r) = 2 permanent shocks.[24] The selection matrix for the permanent shocks is

$$W_1 = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix},$$ so that the common trends have unitary long-run (reduced form)

multipliers on real output and the inverse interest rate. The selection matrix for the cointegrating vector is a scalar, $W_2 = [1.0]$. The matrix $B_{21} = [b_{21}, b_{22}]$ and $T_{11} = \begin{bmatrix} 1 & 0 \\ t_{21} & 1 \end{bmatrix}$. Identification of the third equation of the model is accomplished with a normalization restriction on the coefficient of ΔrAMB in H_2 (setting the coefficient to 1).

Instrumental variables estimates of the third equation of the VECM—the dynamic short-run monetary-base demand curve—are shown in Table 2 both with and without the velocity restriction in the cointegrating vector.[25] In statistical tests, the velocity restriction in the cointegrating vector is not rejected; the Table makes clear that model estimates are robust with respect to whether or not it is imposed. (In the cointegrating vector, the unrestricted coefficient estimate on real GDP is 1.081.) The error-correction effect is significant, with a coefficient of –0.5. Strong simultaneity among current-period changes in the real adjusted monetary base, real GNP, and the inverse bond yield also is apparent.

The distributed lags between the real adjusted monetary base and, in turn, real GNP and the inverse bond rate are shown in Table 3. In each case, the one-period lagged effect has the opposite sign relative to the current period, with the sign again reversed in the second lagged period.

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[24] For the hypotheses that $r = 0$, $r = 1$, and $r = 2$, the values of the λ-max and trace (rank) statistics, respectively, are 28.1, 15.7 and 0.44, and 44.1, 16.1, and 0.44. The corresponding ninety five–percent critical values for the trace statistic are 34.8, 20.0, and 9.1.

[25] Instruments used in the estimation include the estimated permanent shock from the first equation of the model; the part of the second permanent shock that is orthogonal to the first permanent shock; one-period lags of the change in the real adjusted monetary base, the change in real GNP, and the change in the inverse bond rate; one-period lagged levels of the real adjusted monetary base, real GNP, and the inverse bond rate; and, current and one-period lagged values of the change in the bond default rate.

Impulse response functions (IRFs) are shown in Figures 17, 18, and 19 for the responses of real output, the real monetary base, monetary-base velocity, and the level (not inverse) of the interest rate to shocks in the inverse interest rate, the real monetary base, and real output. Only IRFs for the velocity-restricted model are shown; IRFs for the model without the velocity restriction are similar.

For each shock, the interest-rate IRF is a nonlinear transformation of the *inverse* interest-rate IRF (which is the variable that appears in the model) and is calculated from the inverse interest-rate IRF by the formulas shown in appendix B. As previously discussed, the interest elasticity of money demand is a nonlinear function of the level of the interest rate. Hence, the calculations necessarily depend on the initial conditions, that is, the initial level of the interest rate. To illustrate the potential range of outcomes, for each of the three shocks we conducted three calculations of the interest-rate IRF. One calculation sets the interest rate at the lowest value observed in our sample, that is, the highest interest elasticity of monetary-base demand. Another sets the rate at the highest observed interest rate (lowest interest elasticity), and the third sets the interest rate at its sample mean. For the case of the (inverse) interest rate shock, the three interest-rate IRFs are normalized such that the long-run change in the interest rate (after 45 periods) is the same.

Overall, the IRFs suggest that the VECM is sensible and well behaved. In Figure 17, a permanent positive unit shock to the *inverse* of the interest-rate induces a rapid fall in the level of the interest rate, an increase in the real monetary base, and a decrease in real output. The impact effect on the interest rate is an inverse function of the initial level of the interest rate; that is, the initial decrease in the interest rate is relatively larger if the initial interest rate is relatively low (or equivalently the interest-elasticity of monetary-base demand is relatively large). For real output and the real monetary base, the sizes of the impact effects are positively related to $\eta(i)$ with the change for the lowest elasticity being several times greater than the change for the largest. Overall, the money-demand response is that a sharp, permanent decrease in the interest rate leads to an increase in monetary-base demand and lower velocity, with no long-run effect on real output. The short-run decrease in real output is puzzling, although output fairly promptly returns to its previous level.[26]

Figure 18 shows the effects of a permanent positive unit-size real-output shock: The real monetary base increases and velocity falls, but the effect on the interest rate is ambiguous. As expected, the long-run interest-rate change depends on the initial conditions. If the interest rate initially is relatively high (and, hence, the interest elasticity of monetary-base demand relatively small), then the interest rate response (IRF) is near zero throughout. If the interest rate initially is relatively low (and the interest elasticity of monetary-base demand relatively large), the long-run level of the interest rate may be lower than the initial level; if the initial level of the interest rate is near its sample mean, the long-run level of the interest rate may be higher than the initial level.

Figure 19 shows the effect of a unit shock to the real monetary base. The impact effects on real output and velocity are negative, and both are unchanged in the long run. The impact on the interest rate varies sharply with the initial interest elasticity, but the effect diminishes quickly and rates are unchanged in the long run.

Summary and Conclusion

The putative nonexistence of stable money-demand functions is often cited as an established fact in monetary economics. For 80 years of observations on the Federal Reserve Bank of St. Louis adjusted monetary base, we find a well-defined stable demand function. Our analysis suggests that demand for the monetary base may be modeled as demand for a zero-coupon, default risk–free consol.

We find that a demand curve specified in terms of the inverse of a long-term low default-risk interest rate is found to be more consistent with the data than log-log or semi-log

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[26] Given the typically wide width of confidence intervals for IRFs, it is likely that these responses are not statistically different.

specifications. An important aspect of the inverse-rate specification is that the interest elasticity of money demand is a decreasing function of the level of interest rates; that is, successive substitutions away from money become increasingly difficult as the level of interest rates increases. Such a specification is consistent with popular general equilibrium models that include cash-in-advance constraints and/or suggest a demand for money in general equilibrium.

In the context of an identified VECM model, we estimated a dynamic short-run demand function for the monetary base. Strong, statistically significant, positive interactions are estimated among changes in the real monetary base, real output, and the inverse interest rate. Lagged variables are not statistically significant except for the cointegrating vector, suggesting a strong "pull" from any short-run disequilibrium toward the long-run equilibrium defined by the cointegrating vector.

Our results suggest that growth of the monetary base, at least at the relatively low frequency of annual data, can provide guidance for monetary policymakers, particularly when inflation or the level of nominal interest rates is high.

Appendix A: Data

The data used in this analysis are annual figures from 1919 to 1999.[27] The adjusted monetary base data for 1919–1999 are from Anderson and Rasche (1999, 2001). Real GNP and GNP price-index data for 1919–1946 are from Balke and Gordon (1986). These data are chained to the Bureau of Economic Analysis quarterly GDP data at the first quarter of 1947. The nominal interest rate data are the long-term corporate Aaa rate or the Baa rate. Both are available at a monthly frequency for the entire sample period.

Data on bond down-ratings are from Hickman (1960) for 1919–1943, extended to 1965 by Atkinson (1967), and are the same as used by Temin (1976, p. 81, Table 10). Data on bond defaults for 1919–1943 are from Hickman (1960) and Atkinson (1967, p. 44, Table 17, column 1). Data on outstanding amounts of bonds are from Hickman (1960) prior to 1944 and from Atkinson (1967, p. 49, Table 21) for 1944–1965. Default rates in these data, as a percentage of beginning-of-year stocks, averaged 1.0 percent per year during the 1920s, 3.2 percent during the 1930s, 0.4 percent during the 1940s, 0.04 during the 1950s, and 0.03 percent for 1960–1965. We assume an annual default rate of zero from 1966 to 1999, consistent with data available on the Internet from Moody's Investor Services.

For bond down-ratings, our calculations suggest that Temin's Table 10 is in error: the "Net Upgrading" amounts he reports apparently are too small by a factor of 10. Consider as an example the figures for 1932. Temin's table shows beginning-of-year "Total Bonds Outstanding (billions of dollars)" as \$29.1 billion. This agrees with Hickman. But, Temin reports net upgrading of category I and II bonds as –\$339 and –\$305 million, respectively, or –1.17 and –1.05 percent of the initial outstanding stock. Our calculations show –\$3.39 and –\$3.05 billion, or –11.7 and –10.5 percent of the initial outstanding stock.

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[27] We have performed the same analysis with quarterly data and obtained closely similar results. Bond default data are available only annually, however.

Appendix B: The IRF for the Level of the Interest Rate

The VECM model is specified in terms of the inverse of the interest rate, or $z_t = \left[\frac{1}{i_t}\right]$. Denote the estimated impulse response function of z_t as $\left(z_t - z_o\right) = \sum_{k=1}^{t} \overbrace{\Psi_k u_t}^{\wedge}$ and the unknown impulse response function for i_t as $\left(i_t - i_o\right) = \sum_{k=1}^{t} \Theta_k v_t$. Then, since $\left(i_t - i_o\right) = -\left(i_o\, i_t\right)\left(z_t - z_o\right)$; $i_o = \left[\frac{1}{z_o}\right]$; and $i_t = \left[1 \Big/ \left(z_o + \sum_{k=1}^{t} \overbrace{\Psi_k u_t}^{\wedge}\right)\right]$, an estimator of the IRF for the level of the interest rate is

$$\left(i_t - i_o\right) = \sum_{k=1}^{t} \overbrace{\Theta_k v_k}^{\wedge} = \left\{ \frac{-\sum_{k=1}^{t} \overbrace{\Psi_k u_k}^{\wedge}}{z_o\left(z_o + \sum_{k=1}^{t} \overbrace{\Psi_k u_k}^{\wedge}\right)} \right\}.$$

Appendix C: Effect of Foreign Holdings of U.S. Currency

In recent years, a great deal of U.S. currency has been exported to foreign countries. Because the monetary base currently is approximately 90 percent currency, changes in foreign-held currency are potentially important to our modeling. A number of authors have presented estimates of the amount of currency abroad, including Porter and Judson (1996), Bach (1997), and Anderson and Rasche (1997, 2000). Each author estimates the amount of currency held broad by assuming an initial benchmark amount and accumulating subsequent flows. Lacking adequate data for years before the mid-1970s, early studies (Porter and Judson, Bach) assumed as an initial benchmark that approximately half of U.S. currency was held abroad as of the mid-1970s. Based on data beginning in 1965, Anderson and Rasche (1997) argued that these initial benchmarks were far too large.

In this Appendix, we take estimates of foreign-held U.S. currency from the Federal Reserve's Flow of Funds accounts. These data are constructed from an initial benchmark that no U.S. currency was held abroad as of year-end 1963. This is very similar to the benchmark of Anderson and Rasche (1997) who assume that no currency was held abroad as of year-end 1964. The Flow of Funds figures increase from $3.8 billion held abroad at year-end 1964 to $254.6 billion at year-end 1999. Subtracting these amounts from the monetary base yields a rough estimate of the "domestic" monetary base. Figures A1, A2, and A3 show the log velocity of the domestic monetary base versus, respectively, the inverse of the Aaa bond rate, the Aaa bond rate, and the log of the Aaa bond rate. Note that the subtraction affects only points in the left-hand third of Figure A1; points to the right of this area reflect the relatively lower interest rates before 1964. In Figure A1, the scatter for the inverse-rate model clearly differs less, relative to Figure 3, than the other models differ relative to Figures 1 and 2. We conclude that our analysis of 80 years of data likely is not significantly affected by currency exports in recent years.

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Table 1: Estimated Linear Regressions using the Adjusted Monetary Base and the Aaa bond rate (1919-1999)

Dependent variable: GDP velocity of the adjusted monetary base = log(GDP/adjusted monetary base)				
	Coefficient estimates			Standard error of estimate
estimation method ↓	constant	Inverse of Aaa bond rate, times 100	Rate of new bond defaults, percent of outstanding stock	
OLS	–3.631	–0.032	--	0.144
DOLS (2 leads, 2 lags)	–3.606	–0.034	--	0.115
OLS	–3.622	–0.030	–0.0004	0.090
DOLS (1 lead, 1 lag)	–3.606	–0.031	–0.0003	0.082
FIML	--	–0.031	–0.0000[H]	

Dependent variable: Deflated adjusted monetary base = log(adjusted monetary base/GDP chain-type price index)					
	Coefficient estimates				Standard error of estimate
estimation method ↓	constant	Real GDP (chained 1996$)	Inverse of Aaa bond rate, times 100	Rate of new bond defaults, percent of outstanding stock	
OLS	4.490	0.903	0.027	--	0.131
DOLS (2 leads, 2 lags)	3.892	0.970	0.032	--	0.091
OLS	3.452	1.019	0.031	0.0004	0.090
DOLS (1 lead, 1 lag)	3.227	1.044	0.034	0.0003	0.073
FIML	--	1.069	0.033	0.0000[H]	

[H] Coefficient estimates rounds to this value.

Table 2: Parameter Estimates of the Identified VECM

Sample: 1921-1997, annual data

(standard errors in parentheses)

	Model without velocity restriction in cointegrating vector[a]	Model with velocity restriction in cointegrating vector[b]
Constant	1.60 (0.335)	1.82 (0.442)
ΔrAMB $_{-1}$	0.146 (0.138)	0.213 (0.153)
ΔrGNP $_{-1}$	-0.299 (0.180)	-0.301 (0.197)
ΔInverseRate $_{-1}$	-0.862 (0.530)	-0.922 (0.585)
ΔDefaultRate	5.37 (1.06)	5.02 (1.17)
ΔDefaultRate $_{-1}$	-0.463 (1.26)	0.430 (1.31)
ΔCIV $_{-1}$	-0.548 (0.115)	-0.502 (0.122)
ΔrGNP	0.761 (0.185)	0.889 (0.213)
ΔInverseRate	4.03 (0.650)	4.13 (0.718)
Standard error of regression	0.06126	0.06679

Notes:

a:) from the reduced-form VECM estimated using Johansen's FIML estimator,

$\beta = [1.0 \quad -1.081 \quad -3.397 \quad -11.857]$, $\alpha' = [-0.179 \quad 0.136 \quad 0.066]$,

and $C(1) = \begin{bmatrix} 1.448 & 0.778 & 2.337 \\ 0.577 & 0.976 & -0.442 \\ 0.243 & -0.082 & 0.829 \end{bmatrix}$

b:) from the reduced-form VECM estimated using Johansen's FIML estimator,

$\beta = [1.0 \quad -1.000 \quad -3.089 \quad -8.313]$, $\alpha' = [-0.146 \quad 0.137 \quad 0.057]$,

and $C(1) = \begin{bmatrix} 1.460 & 0.707 & 2.050 \\ 0.681 & 0.950 & -0.547 \\ 0.252 & -0.079 & 0.841 \end{bmatrix}$

Table 3: Implicit Distributed Lag Coefficients in Identified VECM
(including error-correction term)

	Without velocity restriction		With velocity restriction	
	Real GNP	InverseRate	Real GNP	InverseRate
Period *t*	0.761	4.03	0.889	4.13
Period *t-1*	-0.468	-3.03	-0.688	-3.50
Period *t-2*	0.299	0.862	0.301	0.922

Figure 1
Log Monetary-Base Velocity vs. Aaa Rate

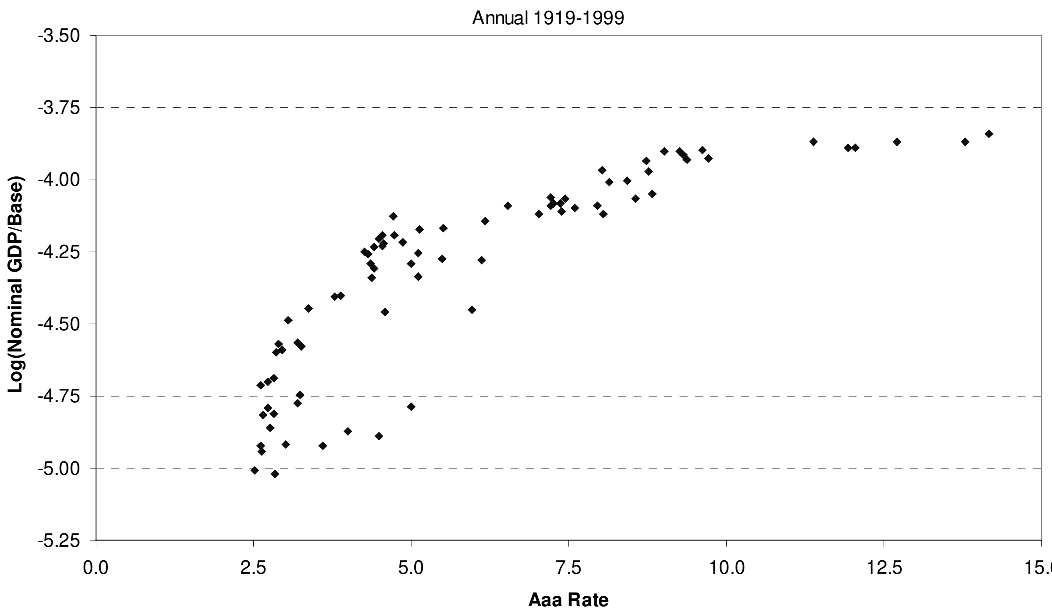
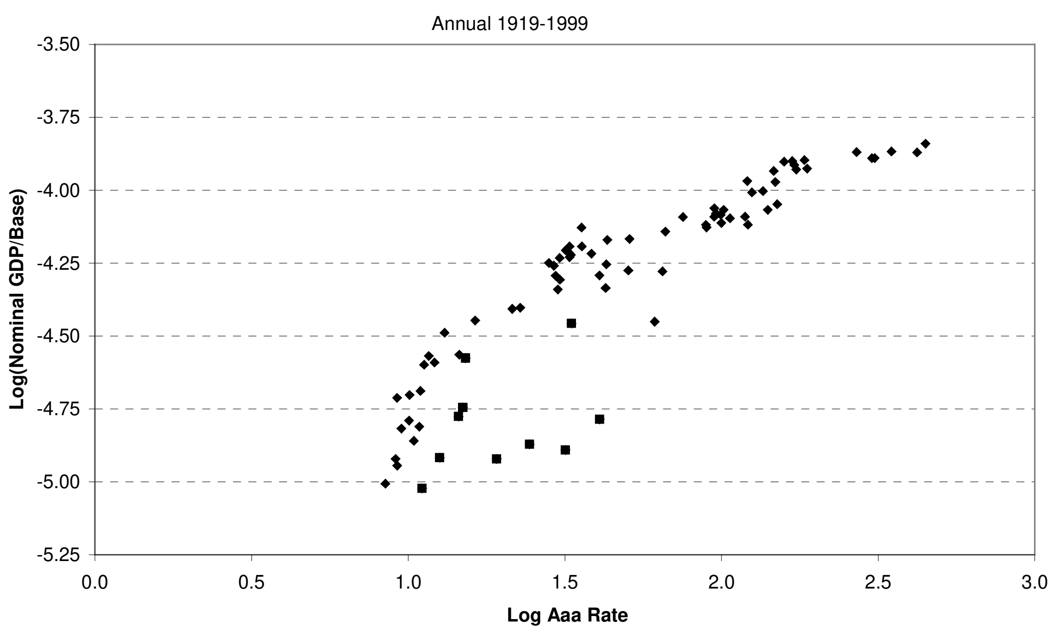


Figure 2
Log Monetary-Base Velocity vs. Log Aaa Bond Rate



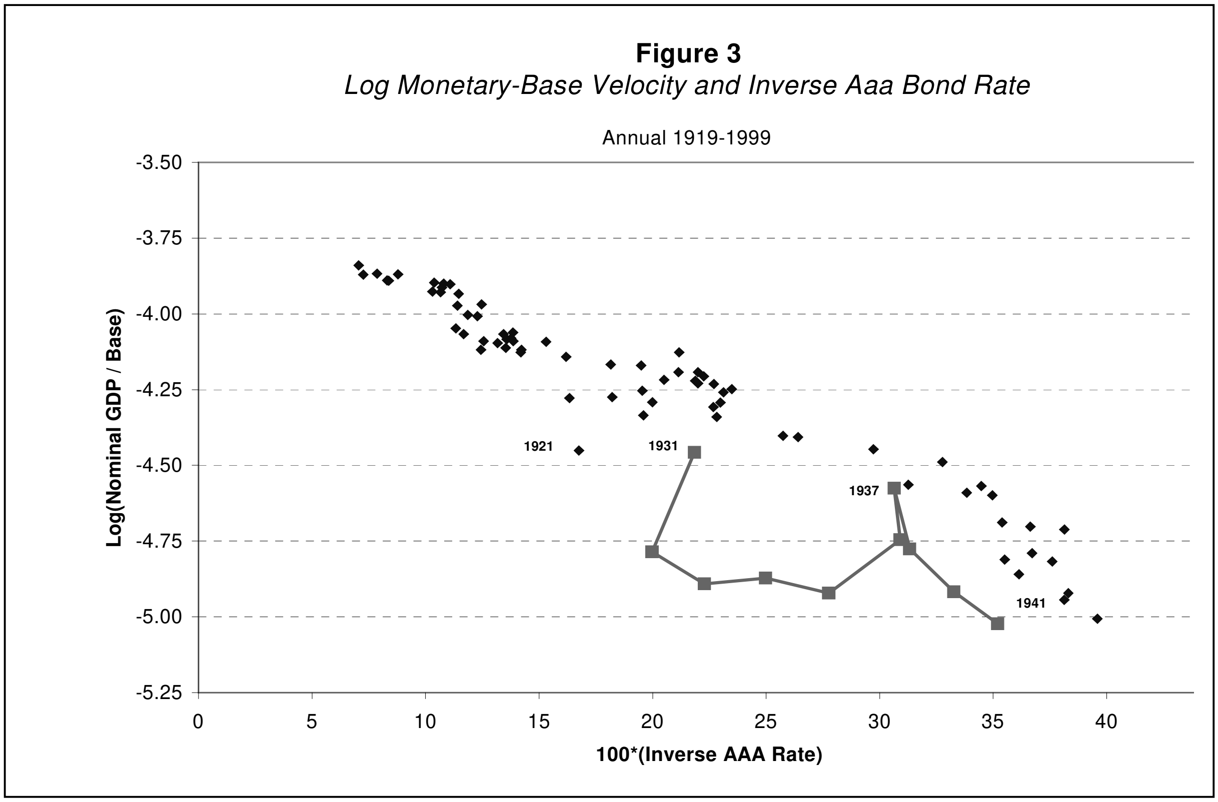


Figure 3
Log Monetary-Base Velocity and Inverse Aaa Bond Rate
Annual 1919-1999
Log(Nominal GDP / Base)
100*(Inverse AAA Rate)
1921
1931
1937
1941
-3.50
-3.75
-4.00
-4.25
-4.50
-4.75
-5.00
-5.25
0
5
10
15
20
25
30
35
40

Figure 4

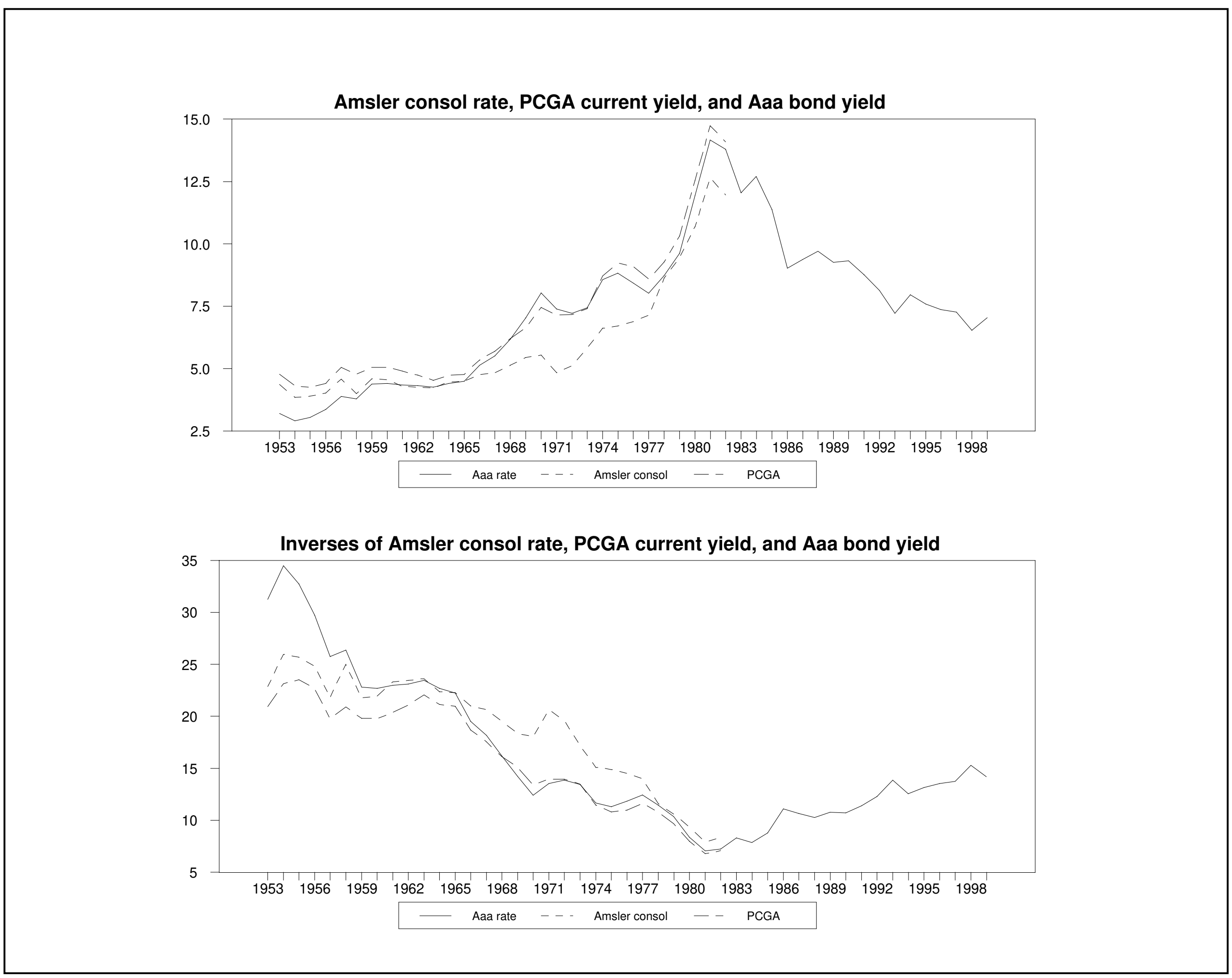

Amsler consol rate, PCGA current yield, and Aaa bond yield
15.0
12.5
10.0
7.5
5.0
2.5
1953 1956 1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998
Aaa rate
Amsler consol
PCGA
Inverses of Amsler consol rate, PCGA current yield, and Aaa bond yield
35
30
25
20
15
10
5
1953 1956 1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998
Aaa rate
Amsler consol
PCGA

Figure 5

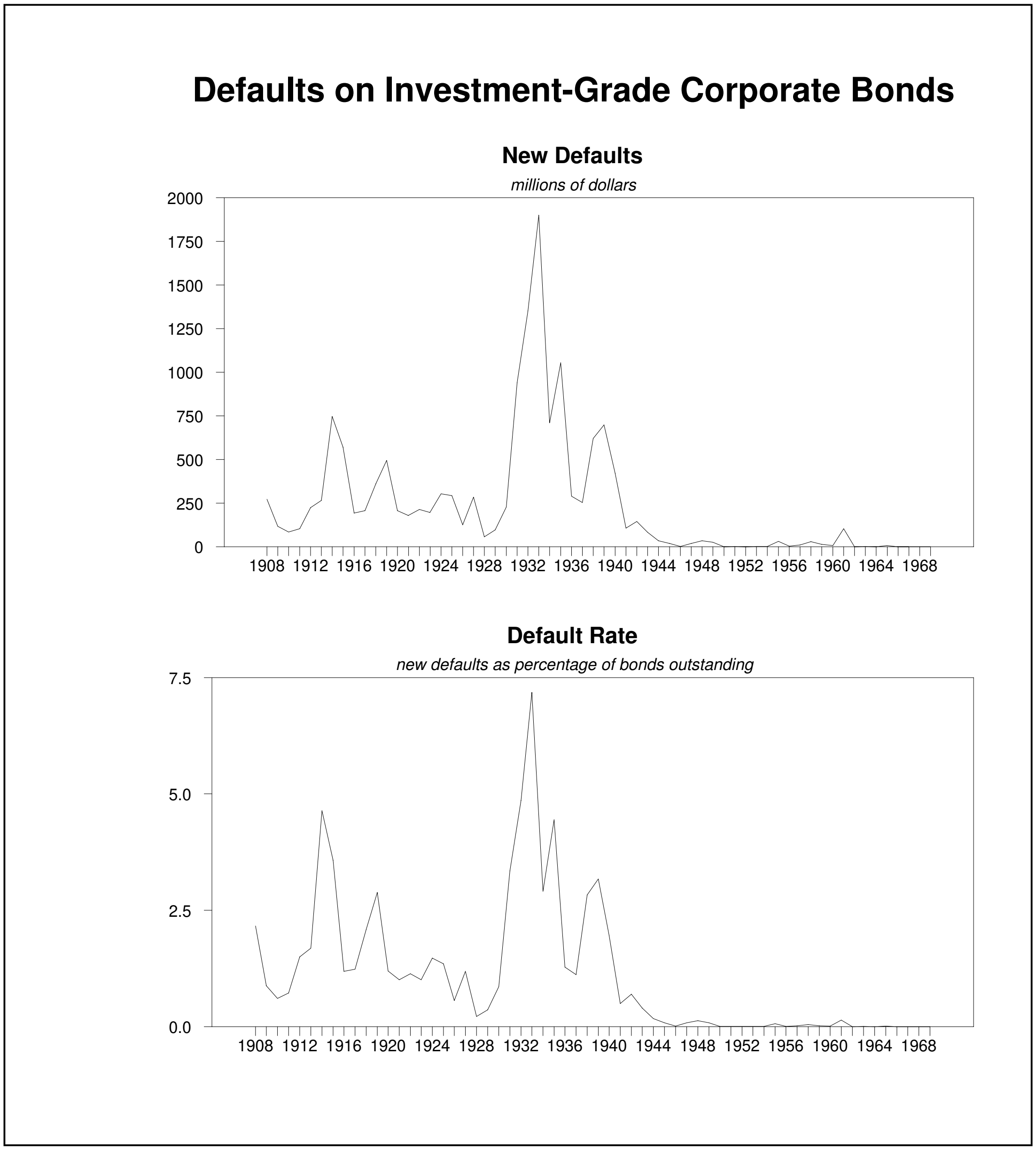

Defaults on Investment-Grade Corporate Bonds
New Defaults
millions of dollars
2000
1750
1500
1250
1000
750
500
250
0
1908 1912 1916 1920 1924 1928 1932 1936 1940 1944 1948 1952 1956 1960 1964 1968
Default Rate
new defaults as percentage of bonds outstanding
7.5
5.0
2.5
0.0
1908 1912 1916 1920 1924 1928 1932 1936 1940 1944 1948 1952 1956 1960 1964 1968

Figure 6

Base Velocity (adjusted for new defaults) vs. Inverse Aaa Bond Rate

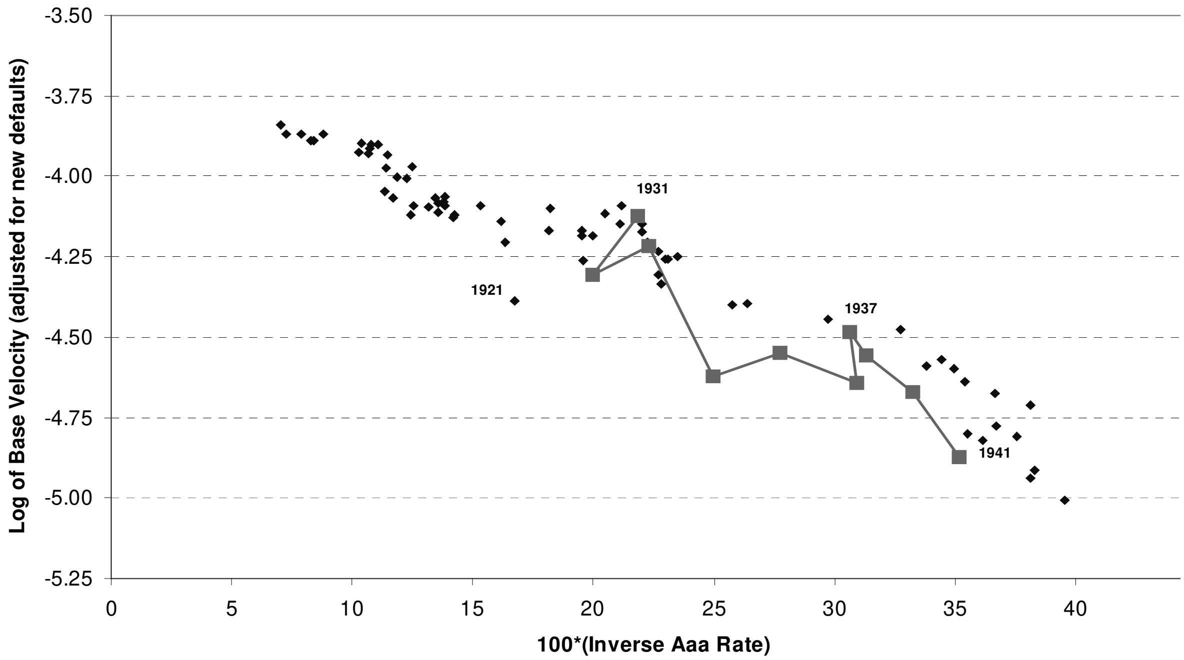


Figure 7

Log of Monetary-Base Velocity vs. Inverse of Baa Rate

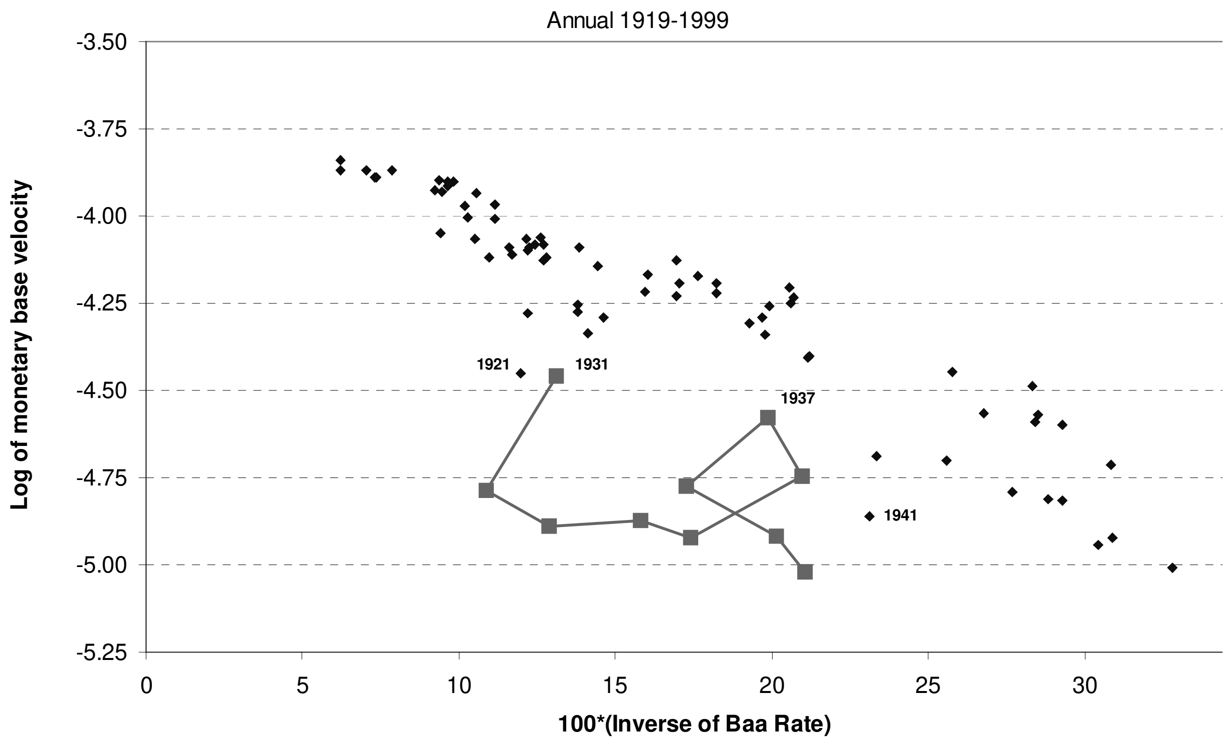


Figure 8

Log of Monetary Base Velocity (adjusted for defaults) vs. Inverse Baa Rate

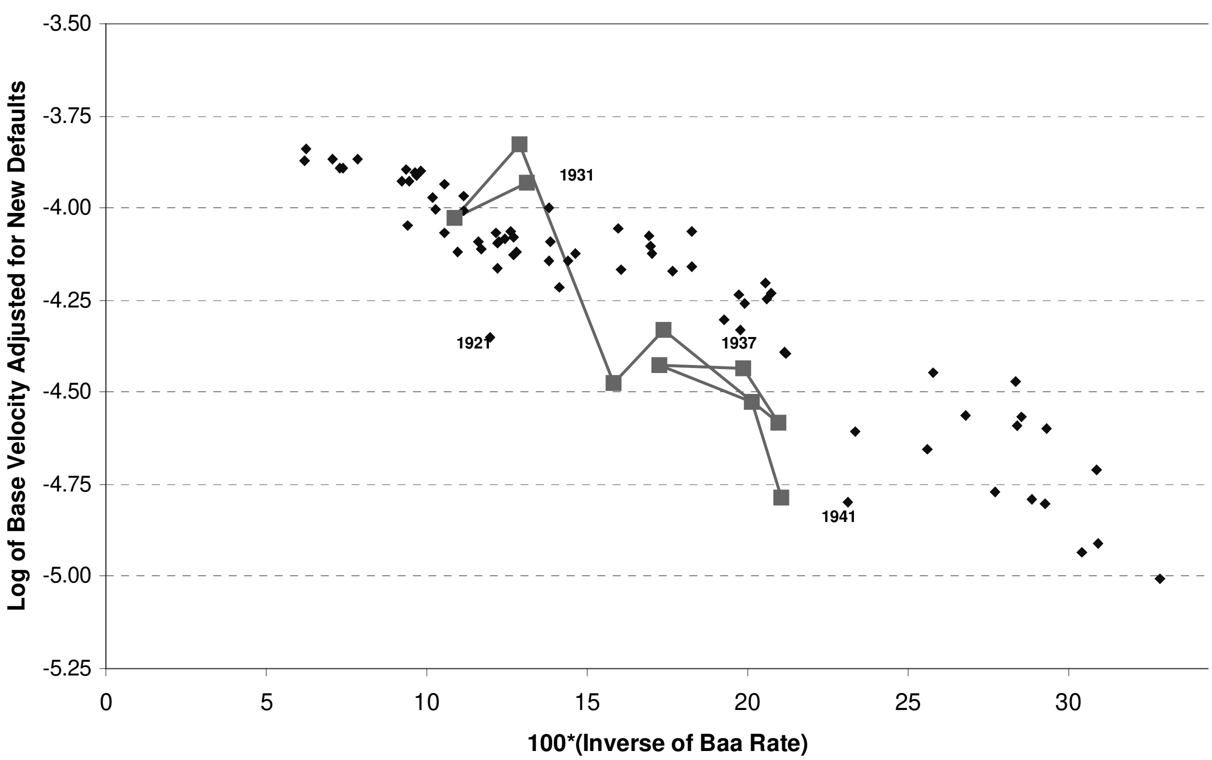
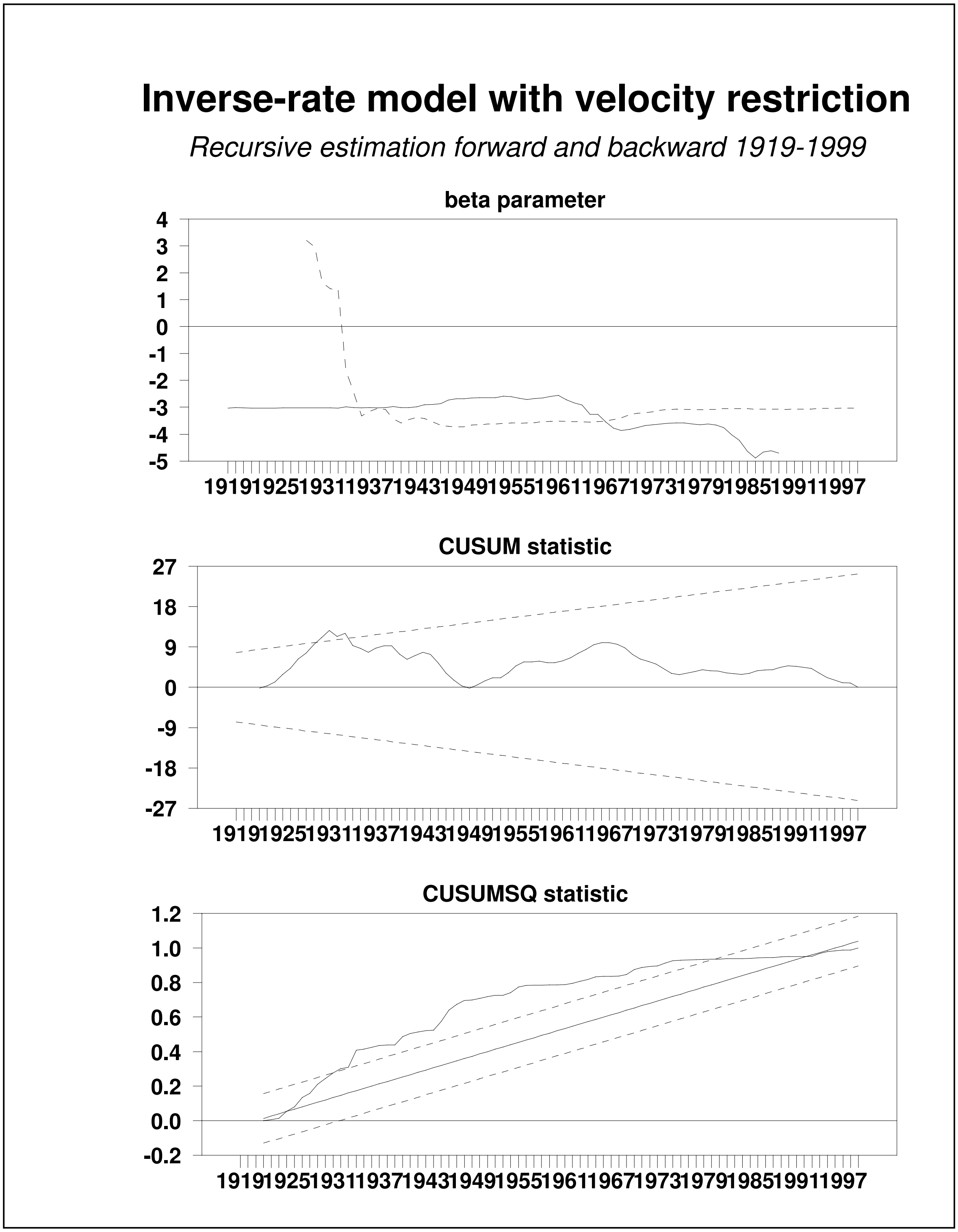


Figure 9



Inverse-rate model with velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
4
3
2
1
0
-1
-2
-3
-4
-5
19191925193119371943194919551961196719731979198519911997
CUSUM statistic
27
18
9
0
-9
-18
-27
19191925193119371943194919551961196719731979198519911997
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
19191925193119371943194919551961196719731979198519911997

Figure 10

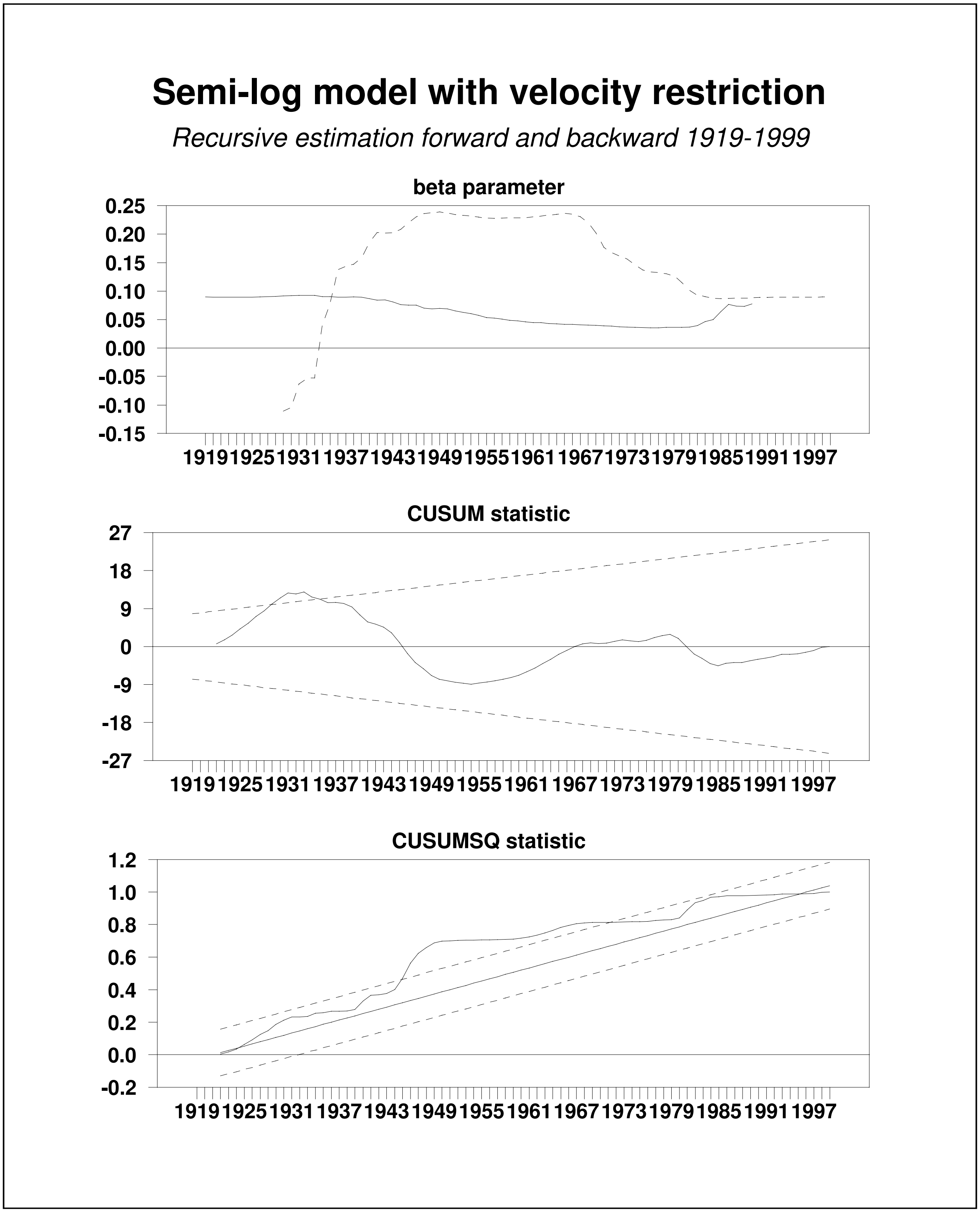
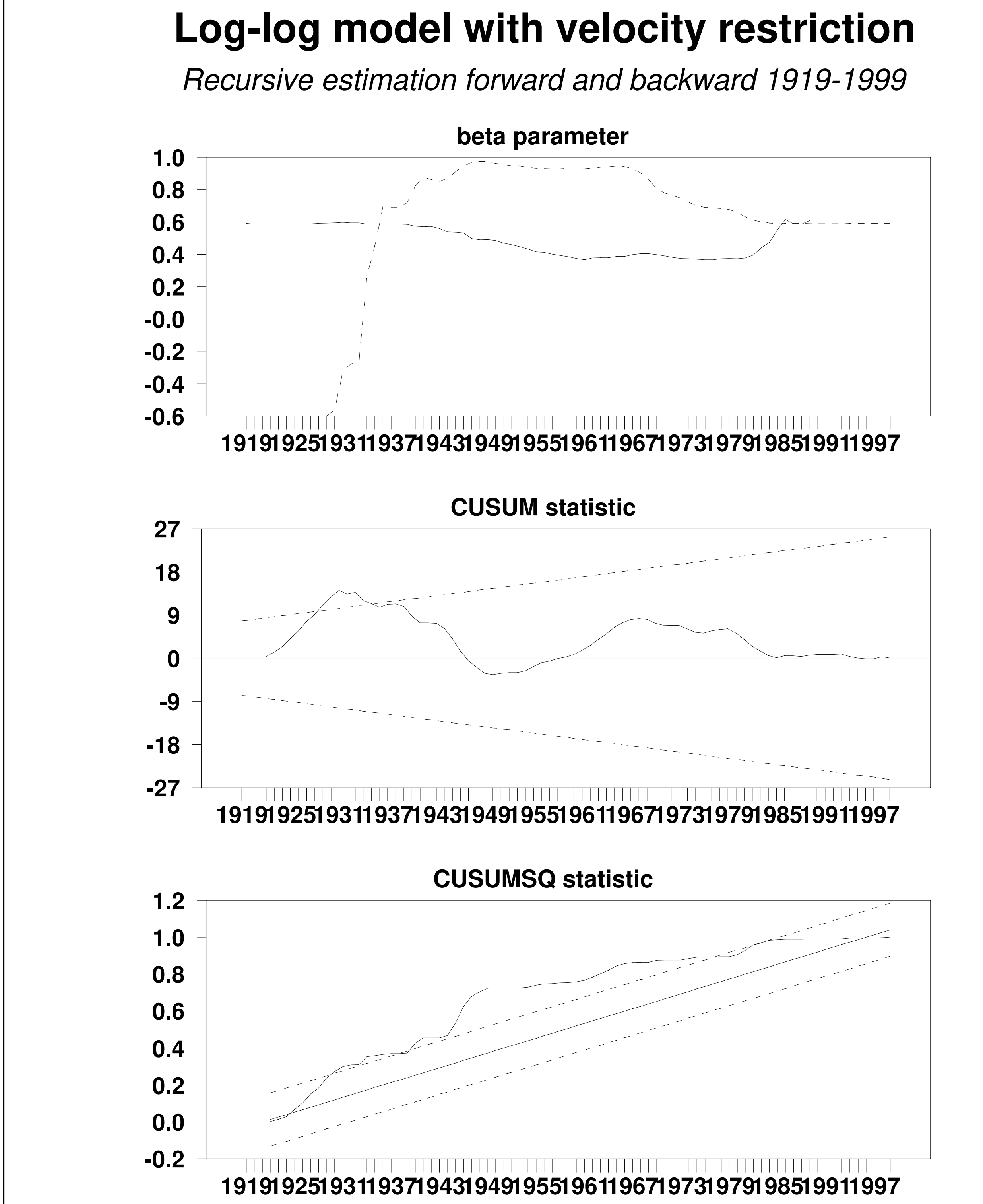

Semi-log model with velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUM statistic
27
18
9
0
-9
-18
-27
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997

Figure 11



Log-log model with velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
1.0
0.8
0.6
0.4
0.2
-0.0
-0.2
-0.4
-0.6
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUM statistic
27
18
9
0
-9
-18
-27
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997

Figure 12

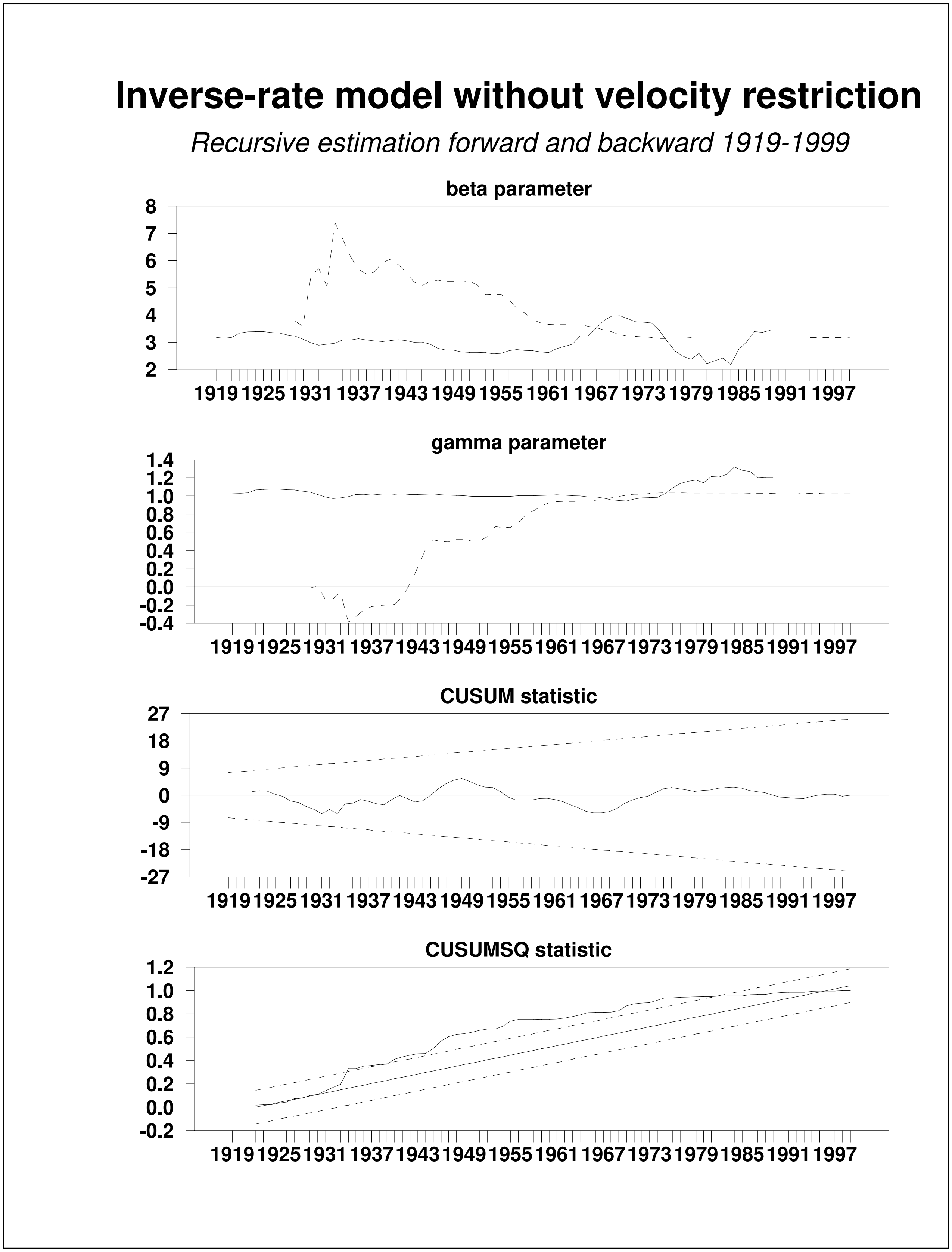
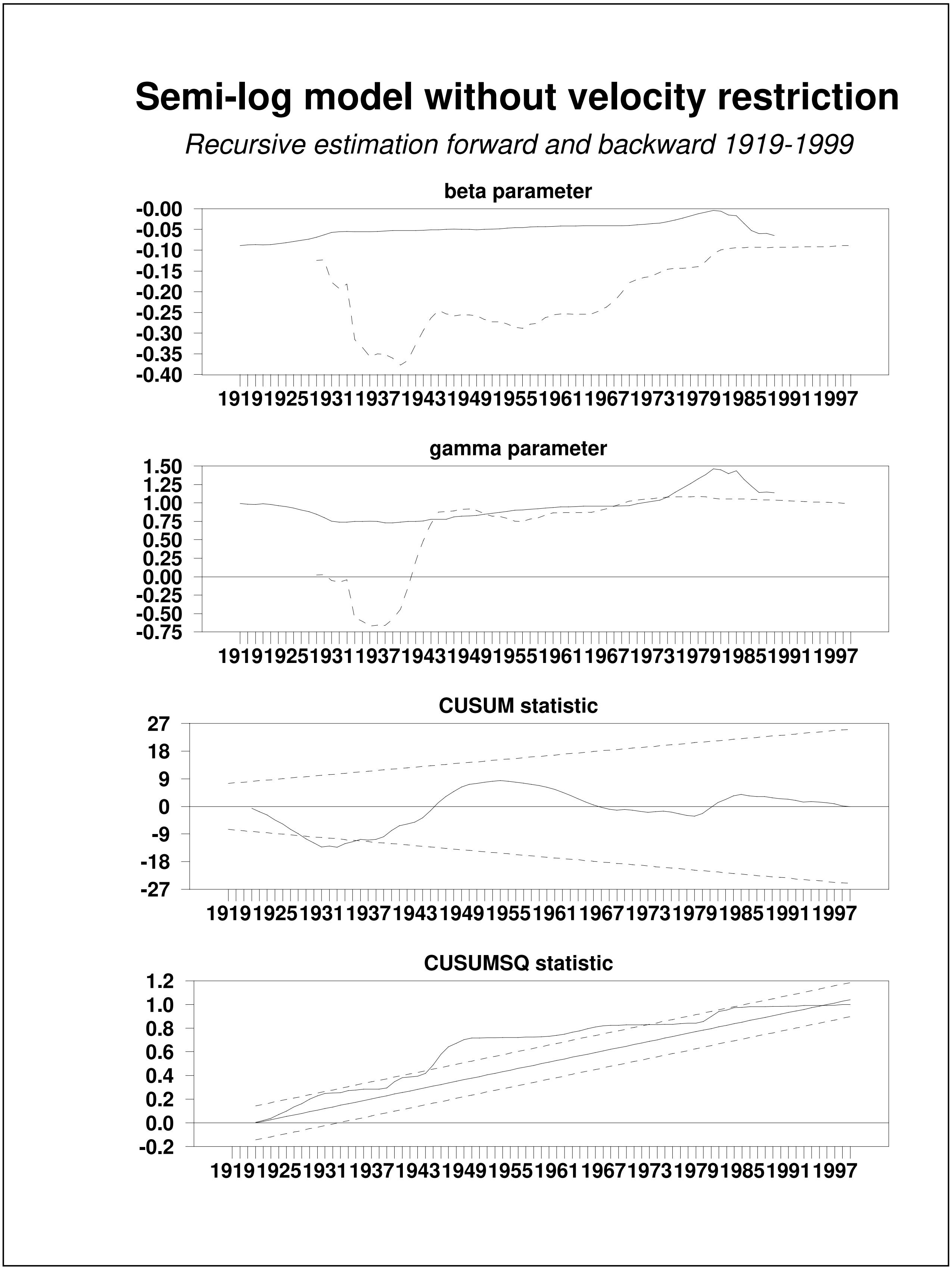
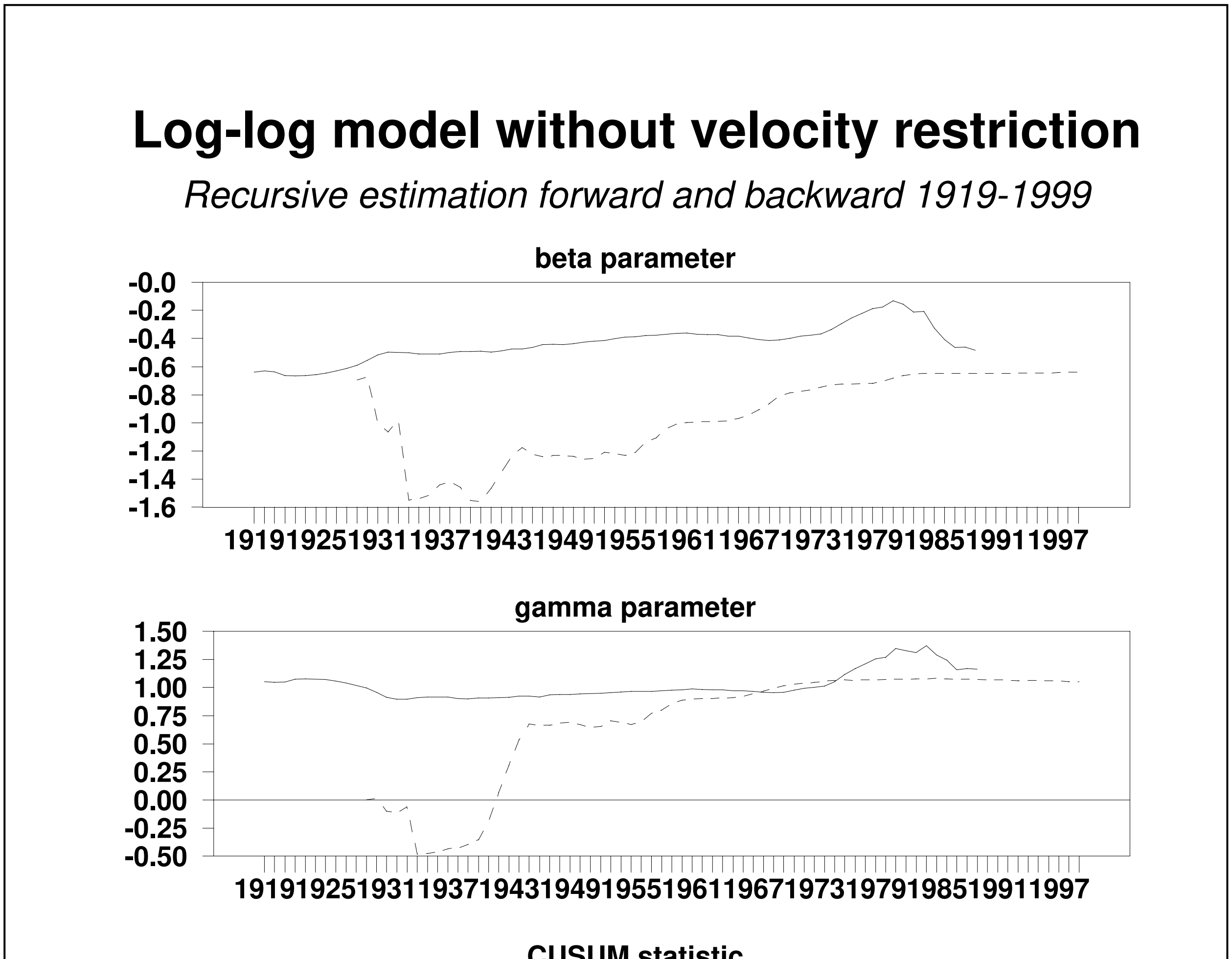

Inverse-rate model without velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
8
7
6
5
4
3
2
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
gamma parameter
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUM statistic
27
18
9
0
-9
-18
-27
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997

Figure 13



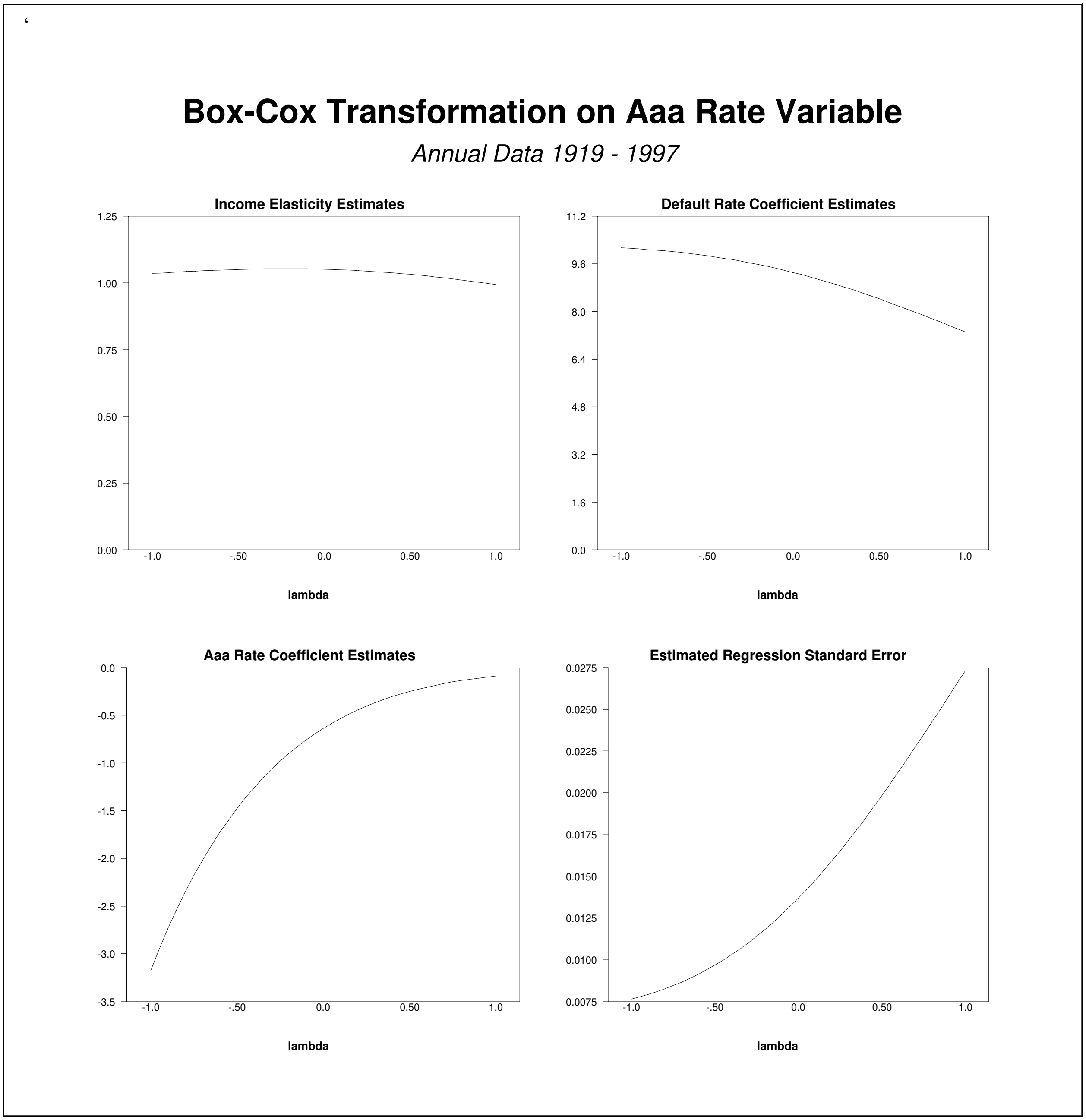
Semi-log model without velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
-0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
gamma parameter
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUM statistic
27
18
9
0
-9
-18
-27
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997

Figure 14



Log-log model without velocity restriction
Recursive estimation forward and backward 1919-1999
beta parameter
-0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
1919192519311937194319491955196119671973197919851991199 7
gamma parameter
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
1919192519311937194319491955196119671973197919851991199 7
CUSUM statistic
27
18
9
0
-9
-18
-27
1919192519311937194319491955196119671973197919851991199 7
CUSUMSQ statistic
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1919192519311937194319491955196119671973197919851991199 7

Figure 15



Box-Cox Transformation on Aaa Rate Variable
Annual Data 1919 - 1997
Income Elasticity Estimates
1.25
1.00
0.75
0.50
0.25
0.00
-1.0
-.50
0.0
0.50
1.0
lambda
Default Rate Coefficient Estimates
11.2
9.6
8.0
6.4
4.8
3.2
1.6
0.0
-1.0
-.50
0.0
0.50
1.0
lambda
Aaa Rate Coefficient Estimates
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-1.0
-.50
0.0
0.50
1.0
lambda
Estimated Regression Standard Error
0.0275
0.0250
0.0225
0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
-1.0
-.50
0.0
0.50
1.0
lambda

Figure 16

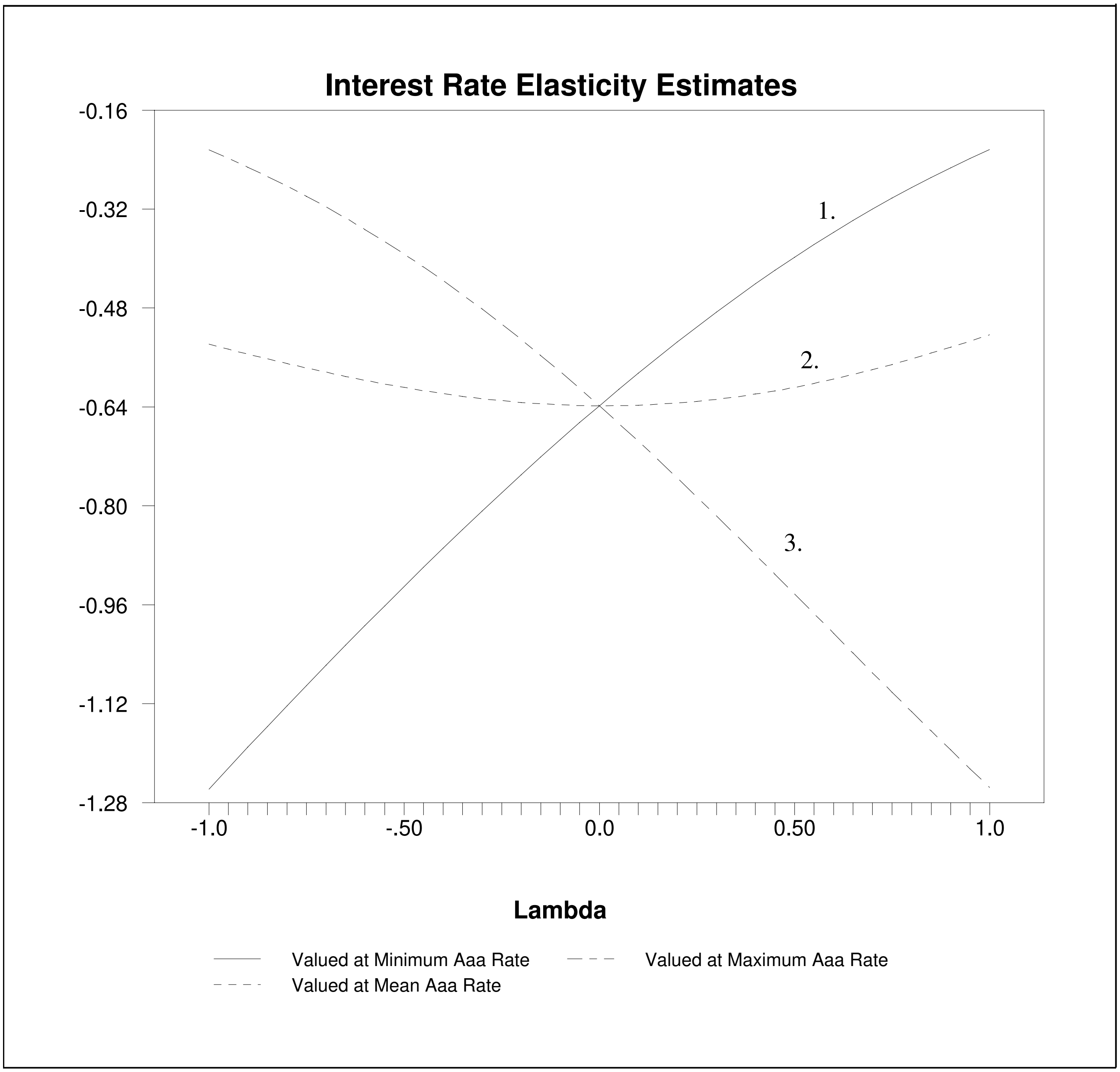

Interest Rate Elasticity Estimates
-0.16
-0.32
-0.48
-0.64
-0.80
-0.96
-1.12
-1.28
1.
2.
3.
-1.0
-.50
0.0
0.50
1.0
Lambda
Valued at Minimum Aaa Rate
Valued at Maximum Aaa Rate
Valued at Mean Aaa Rate

Figure 17

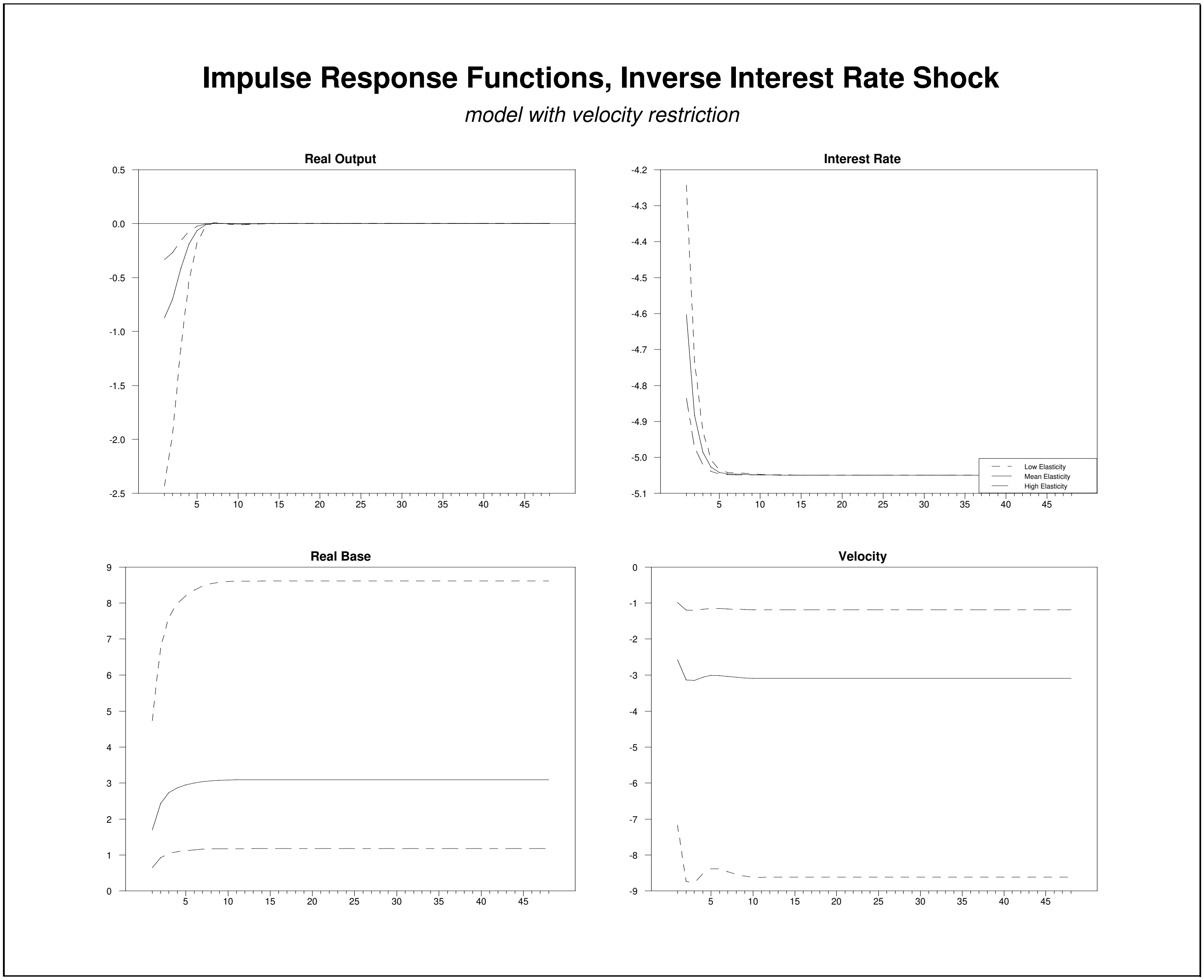
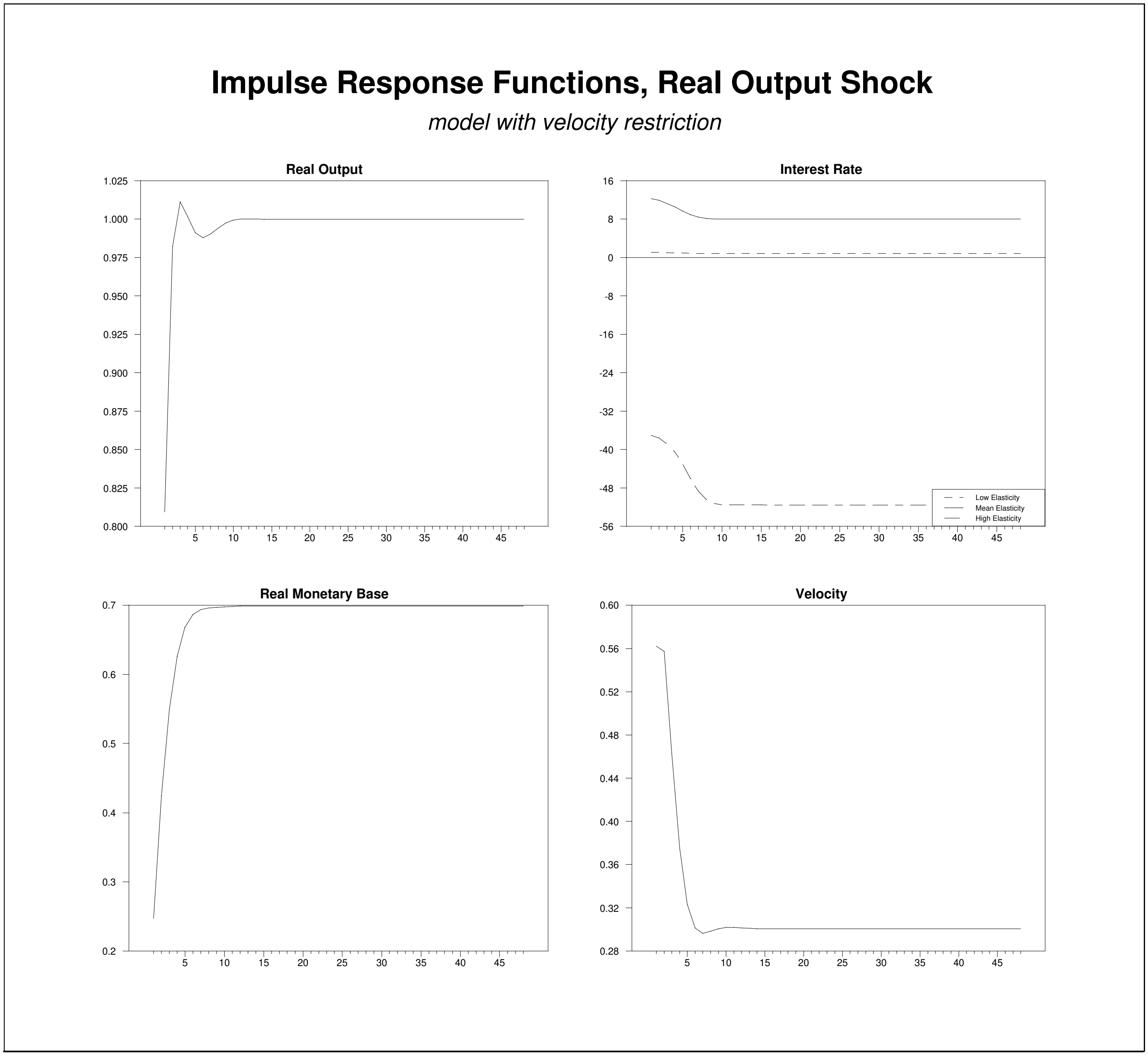

Impulse Response Functions, Inverse Interest Rate Shock
model with velocity restriction
Real Output
Interest Rate
Real Base
Velocity
Low Elasticity
Mean Elasticity
High Elasticity

Figure 18



Impulse Response Functions, Real Output Shock
model with velocity restriction
Real Output
1.025
1.000
0.975
0.950
0.925
0.900
0.875
0.850
0.825
0.800
5 10 15 20 25 30 35 40 45
Interest Rate
16
8
0
-8
-16
-24
-32
-40
-48
-56
5 10 15 20 25 30 35 40 45
Low Elasticity
Mean Elasticity
High Elasticity
Real Monetary Base
0.7
0.6
0.5
0.4
0.3
0.2
5 10 15 20 25 30 35 40 45
Velocity
0.60
0.56
0.52
0.48
0.44
0.40
0.36
0.32
0.28
5 10 15 20 25 30 35 40 45

Figure 19

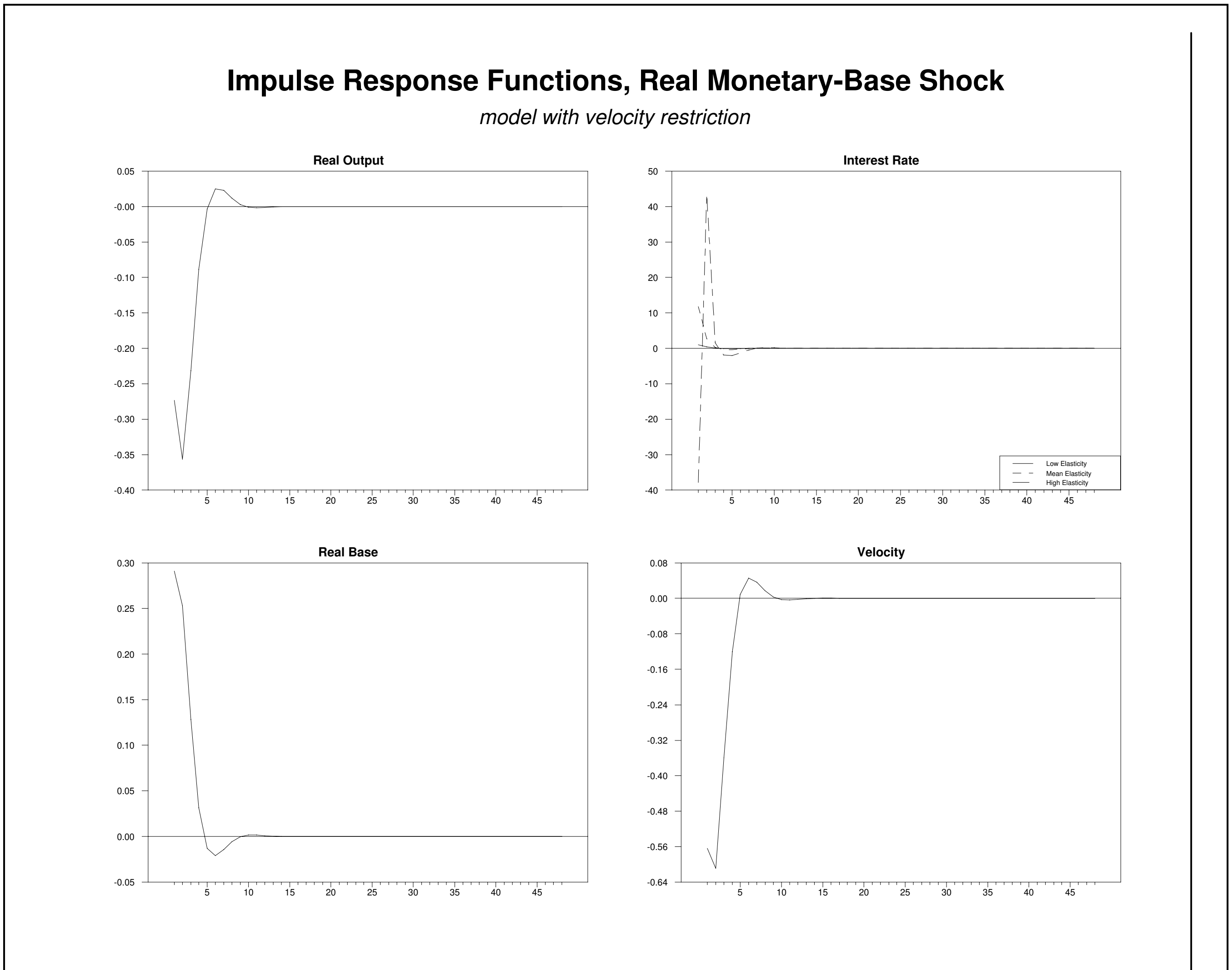

Impulse Response Functions, Real Monetary-Base Shock
model with velocity restriction
Real Output
Interest Rate
Real Base
Velocity
Low Elasticity
Mean Elasticity
High Elasticity

Figure A1

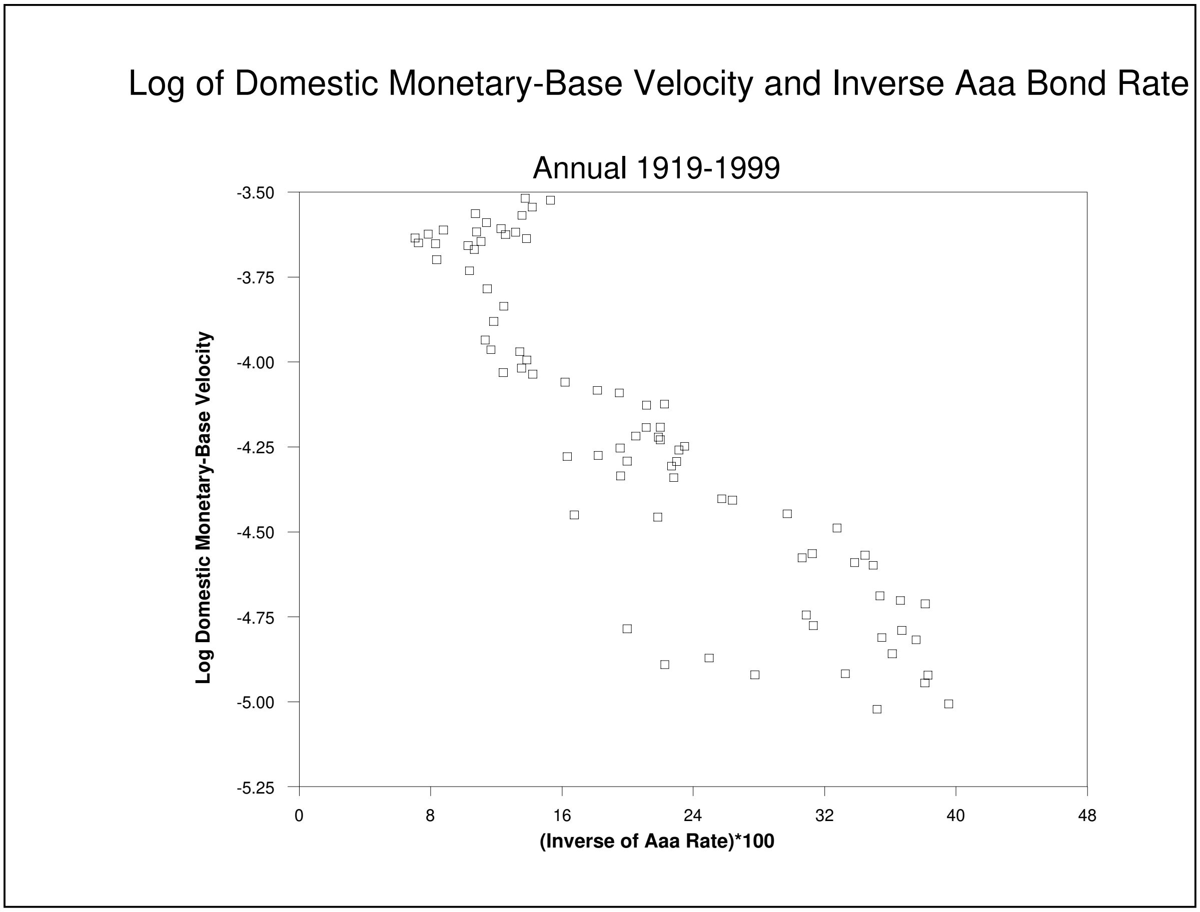


Figure A2

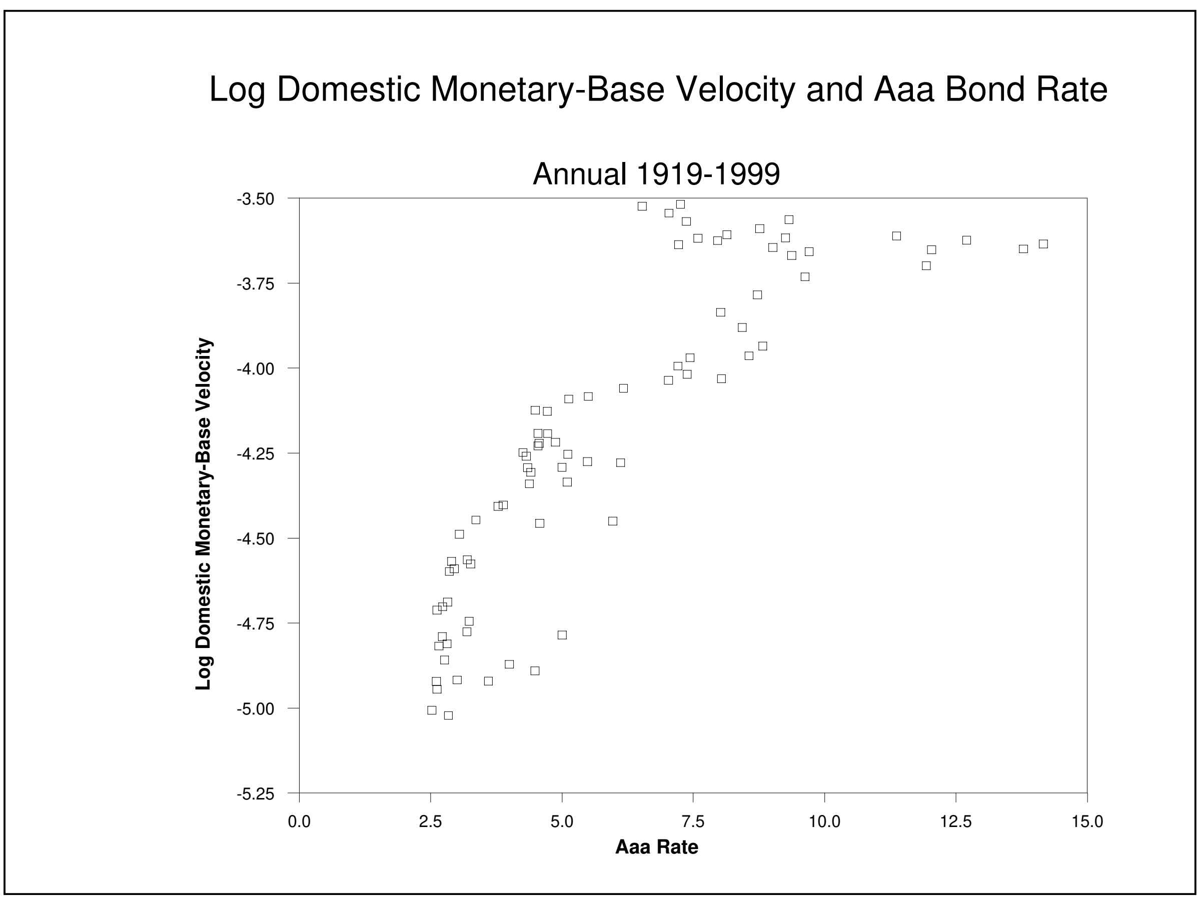
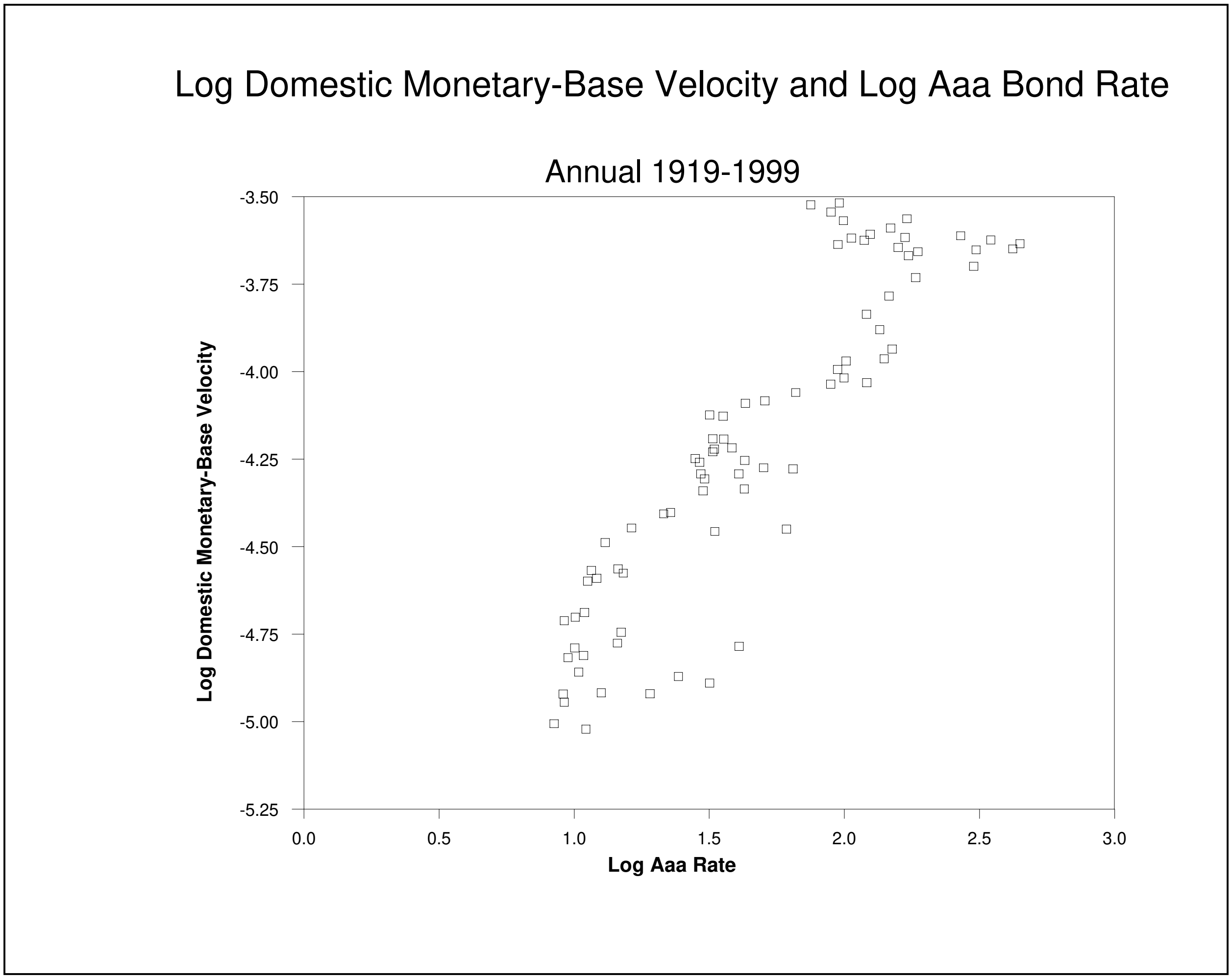


Figure A3


Log Domestic Monetary-Base Velocity and Log Aaa Bond Rate
Annual 1919-1999
Log Domestic Monetary-Base Velocity
-3.50
-3.75
-4.00
-4.25
-4.50
-4.75
-5.00
-5.25
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Log Aaa Rate