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Limited Stock Market Participation and Asset Prices in a Dynamic Economy*

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Abstract

We present a consumption-based model that explains the equity premium puzzle through two channels. First, because of borrowing constraints, the shareholder cannot completely diversify his income risk and requires a sizable risk premium on stocks. Second, because of limited stock market participation, the precautionary saving demand lowers the risk-free rate but not stock return and generates a substantial liquidity premium. Our model also replicates many other salient features of the data, including the first two moments of the risk-free rate, excess stock volatility, stock return predictability, and the unstable relation between stock volatility and the dividend yield.

Keywords: limited stock market participation, borrowing constraints, uninsurable income risk, equity premium puzzle, excess volatility, stock return predictability, leverage effect.

JEL number: C68, E21, G10.

1 Introduction

Empirical evidence documented in the past two decades has challenged the conventional wisdom about financial markets. Fama and French (1989) find that stock return is predictable. Shiller (1981) shows that stock prices are too volatile to be justified by the subsequent movement in dividends; Schwert (1989) also claims that large variations in stock volatility cannot be accounted for by stock valuation models. Mehra and Prescott (1985) argue that the consumption-based capital asset pricing model (CAPM) cannot explain the large observed equity premium. These puzzles or anomalies seem to suggest that stocks are not priced by the fundamentals stressed in the frictionless neoclassical models.

In this paper, we show that adding three market frictions—(1) limited stock market participation, (2) uninsurable income risk, and (3) borrowing constraints—to an otherwise standard model explains these puzzling phenomena in a coherent way.[1] Specifically, we analyze an infinite horizon economy inhabited by two (types of) agents: Only one agent holds stocks and receives dividends, while both agents receive labor income. Agents trade one-period discount bonds with each other to diversify income risk; however, such insurance is imperfect because of borrowing constraints. The model is calibrated using the income process estimated by Heaton and Lucas (1996), and the simulation matches the data well under reasonable parameterization. First, we replicate the first two moments of the risk-free rate, stock return, the equity premium, the long-term bond return, and the price-dividend ratio, as well as their autocorrelations and crosscorrelations obtained from the data. Second, consistent with Fama and French (1989), the price-dividend ratio and the term premium forecast stock return in simulated data. Third, we duplicate Cochrane's (1991) volatility test, which shows that most variations in the price-dividend ratio are explained by movements in

[1] These frictions have been well documented in the empirical literature, e.g., see Mankiw and Zeldes (1991) and Vissing-Jorgensen (1998) for limited stock market participation and Hayashi, Altonji, and Kotlikoff (1996) for uninsurable income risk and borrowing constraints.

expected stock return, but not by movements in dividends.[2]

We generate a large equity premium through two channels. First, because of borrowing constraints, the shareholder cannot completely diversify his income risk and his consumption is more volatile and more positively related to stock return than aggregate consumption. As a result, the shareholder requires a sizable risk premium on stocks. This mechanism, which has been emphasized in the empirical literature, e.g., Mankiw and Zeldes (1991) and Vissing-Jorgensen (1998), is similar to the limited stock market participation model by Basak and Cuoco (1998). Second, uninsurable income risk and borrowing constraints—as shown by the early authors, e.g., Telmer (1993) and Heaton and Lucas (1996)—generate a precautionary saving demand for tradable assets such as one-period discount bonds and thus lower the risk-free rate. However, the precautionary saving demand does not lower stock return because of limited stock market participation. Such an asymmetry between stocks and bonds generates a substantial liquidity premium, which allows us to adopt a reasonable calibration for the shareholder's consumption.[3] To our best knowledge, the second mechanism is innovative and warrants further discussion below.

In our model, we generate a liquidity premium because stocks and bonds are not always priced by the same pricing kernel. In particular, while stocks are priced by the shareholder's intertemporal marginal rates of substitution (IMRS), bonds are determined by the IMRS of the unconstrained agent(s) or the maximum of the two agents' IMRS. Given that the former is lower and more volatile than the latter if borrowing constraints are occasionally

[2] Campbell and Cochrane (1999) show that a habit formation model can also replicate these features of the data. However, in their model, there is a monotonic relation between stock volatility and the price-dividend ratio, which is at odds with empirical evidence by Schwert (1989), who finds an unstable relation between the two variables. As a result, the habit formation model implies a leverage effect much stronger than that in the data. In contrast, stock volatility is a U-shaped function of the price-dividend ratio and the leverage effect is moderate in our model.

[3] The volatility of the shareholder's consumption growth is 6.6 percent at an annual frequency in our baseline model, which is consistent with that reported by Vissing-Jorgensen (1998) using the Consumer Expenditure Survey (CEX). However, it should be noted that, as argued by Brav, Constantinides, and Geczy (2002), a large portion of the consumption volatility in CEX might be due to measurement error. Nevertheless, our number is much smaller than the 11.2 percent used by Basak and Cuoco (1998).

binding, stock return is high and volatile while the risk-free rate is low and smooth, as observed in the data. This mechanism distinguishes our model from the early literature.[4] Intuitively, given that dividends are smooth in the data, if stocks and bonds are priced by the same pricing kernel, their returns should have similar mean and variance. For example, if both agents hold stocks, Heaton and Lucas (1996) show that uninsurable income risk and borrowing constraints cannot produce a sizable equity premium because they lower both stock return and the risk-free rate. Similarly, Basak and Cuoco (1998) find that limited stock market participation can generate a large risk price if the shareholder's consumption is volatile because of high leverage; however, their model also implies a volatile risk-free rate because it is always determined by the shareholder's IMRS.

Allen and Gale (1994) and Aiyagari and Gertler (1999) have emphasized the important effect of liquidity on asset prices. Constantinides, Donaldson, and Mehra (2002) and Storesletten, Telmer, and Yaron (2001) have shown that the lack of intergeneration risk sharing might lead to limited stock market participation and thus helps explain the equity premium puzzle. However, these authors do not fully characterize the liquidity effect in a dynamic setting, as in this paper.

The remainder of the paper is organized as follows. We present a heterogeneous agent model in section 2 and discuss numerical solutions in section 3. The simulation results from the baseline model are presented in section 4, and we conduct the robustness check in section 5. Section 6 offers some concluding remarks.

2 A Limited Stock Market Participation Model

In an exchange economy, there is one perishable consumption good and there are two types of agents of infinite life horizons. We use index $i = 1, 2$ to indicate the representative agent

[4] However, this approach has been (implicitly) widely adopted in the empirical literature; for example, the risk factors for stocks are different from the risk factors for bonds.

of each type. These agents receive stochastic labor income $L_{i,t}, i = 1, 2$ and $t \in [0, \infty)$ by supplying labor inelastically; the total labor income is $L_t = L_{1,t} + L_{2,t}$. Because of moral hazard, they cannot write contracts contingent on the realization of their labor income; thus, labor income is uninsurable. There is also a tree that produces a stochastic dividend D_t, $t \in [0, \infty)$. The tree is endowed to agent 1 (shareholder) at time $t = 0$, and he is not allowed to sell it. The aggregate endowment Y_t is the sum of total labor income and dividend income, or $Y_t = L_{1,t} + L_{2,t} + D_t$. Vector $X_t = [\log(\frac{Y_t}{Y_{t-1}}), \log(\frac{D_t}{Y_t}), \log(\frac{L_{1,t}}{L_t}) - \log(\frac{L_1}{L})]$ describes the income process of the model economy, where $\log(\frac{Y_t}{Y_{t-1}})$ is the growth rate of aggregate income, $\frac{D_t}{Y_t}$ is the dividend share, and $\frac{L_{1,t}}{L_t}$ is the shareholder's labor income share, the mean of which is $\frac{L_1}{L}$. We assume that X_t follows a stationary Markov process, which will be discussed in the next section. In the absence of insurance markets, both agents hedge income risk only through borrowing or lending against each other in a one-period discount bond market. Such a risk-sharing scheme, however, is limited by borrowing constraints: $B_{i,t} \geq \overline{B}_{i,t}$, where $B_{i,t}$ is the outstanding debt of agent i and $\overline{B}_{i,t}$ is his borrowing limit. $B_{i,t}$ is positive (negative) if agent i has a long (short) position in the bond market and $\overline{B}_{i,t}$ is always negative. We assume that there is no outside bond supply and the net bond supply is zero:

$$B_{1,t} + B_{2,t} = 0. \tag{1}$$

The intertemporal budget constraints of agents 1 and 2 are described by equations (2) and (3), respectively. P_t is the equilibrium price of the one-period discount bond at time t that pays one unit of consumption good at time $t+1$, P_t^s is the stock price at time t, $C_{i,t}$ is the consumption of agent i at time t, and S_{t+1}^1(S_t^1) is the stockholding of agent 1 at time $t+1$ (t). Because of limited stock market participation, stocks do not enter the budget constraints of agent 2 (nonshareholder). It should also be noted that, in equilibrium, because shareholders can trade stocks only among themselves, they always hold the same amount of stocks as in Lucas (1978) or $S^1_{t+1} = S^1_t$ for $t \in [0, \infty)$.

$$\begin{array}{l} P_t B_{1,t+1} + P^s_t S^1_{t+1} + C_{1,t} + \leq B_{1,t} + P^s_t S^1_t + L_{1,t} + D_{1,t} \\ B_{1,t+1} \geq \overline{B}_{1,t+1} \end{array} \quad , \quad 0 \leq t < \infty \tag{2}$$

$$\begin{array}{l} P_t B_{2,t+1} + C_{2,t} \leq B_{2,t} + L_{2,t} \\ B_{2,t+1} \geq \overline{B}_{2,t+1} \end{array} \quad , \quad 0 \leq t < \infty \qquad . \tag{3}$$

Agents maximize their objective functions, which are defined in equation (4):

$$Max \quad E[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) | \Omega_0], i = 1, 2 \qquad , \tag{4}$$

where E is an expectation operator conditional on information set Ω_0, which includes all information available at time $t = 0$. β is the time preference and $U(\cdot)$ is the instantaneous utility function. In this paper, we use a power utility function as defined in equation (5), in which the relative risk aversion coefficient is constant and is equal to γ:

$$U(C) = \begin{array}{ll} \dfrac{C^{1-\gamma} - 1}{1 - \gamma}, & if \quad \gamma > 0 \quad and \quad \gamma \neq 1 \\ \log(C), & if \quad \gamma = 1 \qquad . \end{array} \tag{5}$$

The first-order necessary conditions, which determine the equilibrium bond price, are described in equation (6). $\beta \dfrac{U\prime(C_{i,t+1})}{U\prime(C_{i,t})}$ is the IMRS, which is also the pricing kernel.

$$\begin{array}{l} P_t \geq E[\beta \dfrac{U\prime(C_{i,t+1})}{U\prime(C_{i,t})} | \Omega_t], \quad B_{i,t+1} \geq \overline{B}_{i,t+1} \\ (P_t - E[\beta \dfrac{U\prime(C_{i,t+1})}{U\prime(C_{i,t})} | \Omega_t])(B_{i,t+1} - \overline{B}_{i,t+1}) = 0 \\ 0 \leq t < \infty, \quad i = 1, 2 \qquad . \end{array} \tag{6}$$

Because of limited stock market participation, the stock price, P^s_t, is determined only by the shareholder's IMRS as in equation (7):

$$P_t^s = E[\beta(P_{t+1}^s + D_{t+1})\frac{U\prime(C_{1,t+1})}{U\prime(C_{1,t})}|\Omega_t], \quad 0 \leq t < \infty \qquad . \tag{7}$$

Equation (6) indicates that the IMRS are not equalized across two agents when borrowing constraints are binding. More importantly, bonds are priced by the IMRS of the non-shareholder and stocks are priced by the IMRS of the shareholder when the shareholder is constrained. As shown in log-linear approximation below, this mechanism is important in explaining the equity premium puzzle.

The simple risk-free rate from time t to $t+1$ is $R_{t+1}^f = \frac{1}{P_t} - 1$ and the log risk-free rate r_{t+1}^f is $log(\frac{1}{P_t})$. Similarly, the simple stock return from time t to $t+1$ is $R_{t+1} = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} - 1$ and the log stock return is $r_{t+1} = \log(\frac{P_{t+1}^s + D_{t+1}}{P_t^s})$. We use lowercase letters to denote log variables throughout. If the conditional joint distribution of consumption growth and asset returns is log-normal, the conditional risk-free rate defined by equation (6) and the conditional stock return defined by equation (7) can be rewritten as equations (8) and (9), respectively.[5] $r_{i,t+1}^f$ is the shadow risk-free rate priced by agent i's IMRS, $g_{i,t+1}$ is the rate of consumption growth, and $\sigma_{i,t+1}^2$ is its variance. $\sigma_{s,t+1}^2$ is the variance of stock return and $\sigma_{s1,t+1}$ is the covariance between the shareholder's consumption growth and stock return. Equation (8) shows that the equilibrium risk-free rate r_{t+1}^f should be low and smooth because it is the minimum of the two shadow risk-free rates. On the other hand, equation (9) shows that stock return is determined only by the shareholder's IMRS and, therefore, is relatively high and volatile. Substitution of equation (8) into equation (9) gives the equity premium as in equation (10), which has two components. The first component, $\gamma\sigma_{s1,t+1}$, is the risk premium in the standard consumption-based CAPM. The second component,

[5] It should be noted that, although log-linear approximation is helpful for illustration purposes, the approximation error can be large in a model with borrowing constraints as analyzed in this paper.

$r^f_{1,t+1} - \min\{r^f_{1,t+1}, r^f_{2,t+1}\}$, which is non-negative and is strictly positive when the shareholder is constrained, can be thought of as a liquidity premium because it reflects the fact that the shareholder cannot use stocks to buffer income shocks:

$$\begin{aligned} r^f_{i,t+1} &= -\log(\beta) + \gamma E[g_{i,t+1}|\Omega_t] - \frac{\gamma^2 \sigma^2_{i,t+1}}{2}, i = 1, 2 \qquad (8) \\ r^f_{t+1} &= \min(r^f_{1,t+1}, r^f_{2,t+1}) \end{aligned}$$

$$E[r_{t+1} - r^f_{1,t+1}|\Omega_t] + \frac{\sigma^2_{s,t+1}}{2} = \gamma \sigma_{s1,t+1} \qquad (9)$$

$$E[r_{t+1} - r^f_{t+1}|\Omega_t] + \frac{\sigma^2_{s,t+1}}{2} = \gamma \sigma_{s1,t+1} + r^f_{1,t+1} - \min\{r^f_{1,t+1}, r^f_{2,t+1}\} \quad . \qquad (10)$$

We want to emphasize that limited stock market participation plays an important role in resolving the equity premium puzzle. If both agents hold stocks as in Heaton and Lucas (1996), the agent who is constrained in the bond market should also be constrained in the stock market. As a result, income risk and borrowing constraints lower both the risk-free rate and stock return through a precautionary saving demand and their model is unable to produce a sizable equity premium. However, limited stock market participation alone cannot explain all the asset pricing phenomena either. For example, in the model by Basak and Cuoco (1998), there are no borrowing constraints and stocks and bonds are priced by the shareholder's IMRS, in particular, $r^f_{1,t+1}$ is always equal to $r^f_{2,t+1}$. Because of limited stock market participation, the shareholder's consumption could be much more volatile than aggregate consumption, which generates a large risk premium, $\gamma \sigma s_{1,t+1}$. However, volatile consumption also implies a volatile risk-free rate, which is at odds with the data. In contrast, because the liquidity premium $r^f_{1,t+1} - \min\{r^f_{1,t+1}, r^f_{2,t+1}\}$ accounts for a significant portion of the equity premium, the risk premium $\gamma \sigma s_{1,t+1}$ need not be very large or the shareholder's consumption need not be extremely volatile in our model. Moreover, the risk-free rate is priced by the IMRS of the nonconstrained agent(s) and is thus relatively smooth in our model, even though the shareholder's consumption is relatively volatile.

Long-term bonds do not enter the model directly. As an approximation, we assume that there is a consol paying one unit of consumption good in each period and that its price is determined through auction. If the supply is zero, the price of the consol P_t^c is given by equation (11) below, where $P_{i,t}^c$ is the shadow consol price determined by agent i's IMRS. The simple return on the consol from time t to $t+1$ is $R_{t+1}^c = \frac{1+P_{t+1}^c}{P_t^c} - 1$ and the log return is $r_{t+1}^c = \log(\frac{1+P_{t+1}^c}{P_t^c})$. The yield is $y_{t+1}^c = \frac{1}{P_{t+1}^c}$. Equation (11) shows that, like the risk-free rate, the long-term bond return should also be small and smooth:

$$\begin{aligned} P_{i,t}^c &= E[\beta(P_{t+1}^c + 1)\frac{U\prime(C_{i,t+1})}{U\prime(C_{i,t})}|\Omega_t], \quad 0 \le t < \infty \text{ and } i = 1, 2 \\ P_t^c &= \max(P_{1,t}^c, P_{2,t}^c) \quad . \end{aligned} \tag{11}$$

Finally, equations (1), (2), (3), and (6), along with the goods market clearing condition equation (12) below, define the equilibrium of our model economy:

$$C_{1,t} + C_{2,t} = L_{1,t} + L_{2,t} + D_{1,t} \quad . \tag{12}$$

3 Numerical Solutions and Calibration

The model does not have an analytical solution because the bond holdings are an endogenous state variable, which changes over time. We solve the model numerically using the method developed by Telmer (1993). First, we discretize the exogenous state variables of the vector X_t and the endogenous state variable $B_{1,t}$ to approximate the continuous state spaces by finite grids.[6] Then we calculate the policy functions of each state by iterating the Euler

[6] $B_{1,t}$ is a sufficient statistic for the bond market because the net bond supply is zero, or $B_{1,t} + B_{2,t} = 0$.

equation (6) along with the equations (1), (2), (3), and (12), recursively. Last, we feed the model with simulated income processes to generate artificial time series of asset prices.

Heaton and Lucas (1996) assume that the vector X_t follows a first-order VAR process as in equation (13), where μ is a vector of intercepts, Λ is a matrix of slopes, Θ is a matrix of coefficients, and ε_t is a vector of i.i.d. shocks that have standard normal distributions and are orthogonal to each other.

$$X_t = \mu + \Lambda X_{t-1} + \Theta \varepsilon_t \qquad . \tag{13}$$

Heaton and Lucas estimate equation (13) using the annual National Income and Product Account (NIPA) data and the Panel Study of Income Dynamic (PSID) data and then use Tauchen and Hussey's (1991) quadrature method to approximate the estimated income process with an eight-state (two grids for each state variable) Markov process.

In this paper, we adopt Heaton and Lucas' (1996) eight-state income process. Moreover, Mankiw and Zeldes (1991) report that only 25 percent of US households own stocks and that they receive higher labor income than nonshareholders. Therefore, we assume that the shareholder on average receives 30 percent of the total labor income and the nonshareholder gets the remainder, 70 percent, in the baseline model. The simulated income process of the baseline model is reported in equation (14)[7]:

$$\mu = \begin{bmatrix} 0.11 \\ -0.52 \\ 0.00 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0.15 & 0.05 & 0.00 \\ -0.34 & 0.72 & 0.00 \\ 0.00 & 0.00 & 0.49 \end{bmatrix}, \quad \Theta = \begin{bmatrix} 0.03 & 0.00 & 0.00 \\ 0.01 & 0.04 & 0.00 \\ 0.00 & 0.00 & 0.22 \end{bmatrix} \quad . \tag{14}$$

[7] We report only the shareholder's labor income in equation (14). The nonshareholder's labor income process is similar to, but less volatile than that of the shareholder in the baseline model. In particular, the volatility of annual income growth is 16.1 percent and 11.0 percent for the shareholder and the nonshareholder, respectively.

Finally, we assume that the borrowing limit $\overline{B}_{i,t}$ is proportional to aggregate income Y_t and the ratio $BC = -\frac{\overline{B}_{i,t}}{Y_t}$ is constant over time. This is a standard assumption in the literature, e.g., Telmer (1993). Since $B_{1,t}$ is equal to $-B_{2,t}$ in the equilibrium, the normalized bond holding $\frac{B_{i,t}}{Y_t}$ should fall into the interval $[-BC, BC]$. In the calibration, $\frac{B_{1,t}}{Y_t}$ can take the value of $-BC$, BC, or 1240 grids evenly spaced over the interval $[-BC, BC]$.

4 Baseline Model

The parameterization of the baseline model is listed in the table below. It should be noted that the frequency is annual in the simulation. We assume that, on average, dividends account for 15 percent of aggregate income as in Heaton and Lucas (1996), and the shareholder receives 30 percent of total labor income. The relative risk aversion coefficient, γ, is equal to 3, and the time preference, β, is set to be 0.99. Finally, we assume that each agent can borrow up to 10 percent of aggregate income. We want to stress that the assumption about borrowing constraints is not unrealistic: In the baseline model, both the shareholder and the nonshareholder can still diversify most income risk by borrowing and the volatility of their consumption growth is close to those reported by Vissing-Jorgensen (1998). It also should be noted that one shareholder can borrow three times as much as what one nonshareholder can borrow if shareholders account for 25 percent of the population, as in the baseline model.

$\frac{D}{Y}$	$\frac{L_1}{L}$	γ	β	BC
15%	30%	3	0.99	10%

We simulate the income process 50,000 times and use the last 20,000 realizations to calculate the relevant statistics, which are then compared with their empirical counterparts. Given its prominent role in the asset pricing literature, we first show that our model helps explain the equity premium puzzle. We then provide some economic intuitions by illustrating the dynamic of the model, and also show that our model is able to replicate many other salient features of financial market data. Finally, we discuss the social welfare loss associated with the frictions incorporated in our model.

A. Means and Standard Deviations

The first column of Table 1 reports the mean and the standard deviation of asset returns in simulated data. The risk-free rate r^f is 2.1 percent and the consol return r^c is 2.0 percent, compared with stock return r of 6.2 percent. Our model thus generates a large equity premium $r - r^f$ of 4.2 percent but a small term premium $r^c - r^f$ of –0.1 percent. Also, the standard deviation of stock return is 15.4 percent, which is much higher than 4.7 percent for the risk-free rate and 8.3 percent for the consol return. These numbers match their empirical counterparts of various sample periods, which are reported in columns 2 through 4 of Table 1. The mean and the standard deviation of the price-dividend ratio match the data as well.

[Insert Table 1 here]

In the frictionless consumption-based CAPM, the unconditional equity premium is approximately equal to $\gamma\sigma_{cs} - \frac{\sigma_s^2}{2}$, where $\frac{\sigma_s^2}{2}$ is Jensen's inequality term and σ_{cs} is the covariance between excess stock return and aggregate consumption. In simulated data, the covariance between excess stock return and aggregate consumption growth is 6.1E-04 and implies a relative risk aversion coefficient of 88, compared with 3 used in the calibration.[8] This is the equity premium puzzle argued by Mehra and Prescott (1985). Equation (10) shows that their formula is inappropriate for two reasons. First, we should use the shareholder's consumption growth, which implies a risk premium $\gamma\sigma_{s1}$ of 2.8 percent. Second, the liquidity premium $r^f_{1,t+1} - \min\{r^f_{1,t+1}, r^f_{2,t+1}\}$ is about 2.6 percent, which accounts for almost half of the equity

[8]It should be noted that, in our exchange economy, aggregate consumption is equal to aggregate income, of which the growth rate has a mean of 1.8 percent and a standard deviation of 2.8 percent.

premium in simulated data. As mentioned above, Heaton and Lucas (1996) cannot generate a sizeable equity premium because both agents can hold stocks and thus there is no liquidity premium in their model.

B. Dynamics of the Baseline Model

It helps to understand the economic intuitions of our model by looking at statistics conditional on the normalized shareholder's bond holding $\frac{B_{1,t}}{Y_t} \in [-BC, BC]$. As mentioned above, we discretize $\frac{B_{1,t}}{Y_t}$ and it can take the value of $-BC$, BC, or any of 1240 grids evenly spaced over the interval $[-BC, BC]$. Figure 1 shows the distribution of $\frac{B_{i,t}}{Y_t}$ in simulated data. Point 1 (12) is the case $\frac{B_{1,t}}{Y_t} = -BC$ ($\frac{B_{1,t}}{Y_t} = BC$) or that the shareholder (nonshareholder) is constrained. Point 3 corresponds to the first 124 grids immediate to $-BC$, point 4 is the next 124 grids, and so forth. The vertical axis is the fraction of the time that $\frac{B_{1,t}}{Y_t}$ falls into these corners or subintervals. We find that both agents hit the borrowing limit quite frequently: 17 percent for the shareholder and 20 percent for the nonshareholder.

[Insert Figures 1-9 here]

Figure 2 shows the growth rates of consumption and income from time t to $t+1$, conditional on $\frac{B_{1,t}}{Y_t}$. dy1 (dy2) and dc1 (dc2) are the shareholder's (nonshareholder's) income and consumption growth rates, respectively. At point 1, the shareholder has to reduce his consumption at time t because he cannot borrow any more after a string of bad income shocks. However, his expected consumption growth rate from time t to $t+1$ is high because he anticipates a high income growth rate given that income shocks are transitory. Meanwhile, the nonshareholder's consumption at time t is high because he cannot save any more. His expected consumption growth rate from time t to $t+1$, however, is low because his expected income growth rate is low. Conversely, the expected consumption growth is low (high) for the shareholder (nonshareholder) at point 12, when the nonshareholder's borrowing constraints are binding. For the other points, two agents can completely diversify income risk and, therefore, have the same consumption growth rates. Although borrowing constraints are relatively stringent, both agents can still diversify most income risk by borrowings. As shown in Figure 3, conditional income is much more volatile than conditional consumption for both agents. Also, the unconditional volatility of the consumption growth rate is 6.6 percent for the shareholder and is 4.9 percent for the nonshareholder, compared with the income volatility of 16.1 percent for the shareholder and 11.0 percent for the nonshareholder. As mentioned in the introduction, the shareholder's consumption volatility is close to its empirical counterpart reported by Vissing-Jorgensen (1998). It should also be noted that the conditional consumption volatility is a U-shaped function of the normalized shareholder's bond holding $\frac{B_{1,t}}{Y_t}$ because neither agents can diversify income shocks at point 1 and point 12. As we show below, this also generates a U-shaped stock volatility.

In Figure 4, r_1^f (r_2^f) is the shareholder's (nonshareholder's) shadow risk-free rate as defined in equation (8) and r^f is the equilibrium risk-free rate obtained from simulated data. Consistent with equation (8), r^f is approximately equal to $\min(r_1^f, r_2^f)$. The risk-free rate is low at point 1 (12) because it is determined by the nonshareholder's (shareholder's) IMRS, which is low, as shown in Figure 2. For the other points, the risk-free rate is flat and relatively high. The conditional consol return displays a similar pattern to that of the risk-free rate and is not reported here. In Figure 5, r_1 is the shareholder's shadow stock return as defined in equation (9) and r is stock return obtained from simulated data. These two variables are approximately equal to each other because stocks are always priced by the shareholder's IMRS. Stock return is high at point 1 because the shareholder's consumption is expected to rise and he has little motivation to save. Conversely, stock return is low at point 12 because the shareholder wants to save for the future decline in his income. Stock return is flat and moderate at the other points when the shareholder can perfectly diversify income risk.

Overall, unlike the risk-free rate, borrowing constraints do not lower stock return on average and our model can thus generate a sizable equity premium. This point is further illustrated in Figure 6: *premium* is the equity premium obtained from simulated data, and *premium*1 is the shadow equity premium defined as the difference between the shareholder's shadow risk-free rate and the shareholder's shadow stock return, which is equal to $\gamma\sigma_{s1} - \frac{\sigma_{s,t+1}^2}{2}$ as in equation (9). *premium* is approximately equal to *premium*1 except at point 1, where the IMRS are not equalized across two agents and the risk-free rate is determined by the nonshareholder's IMRS. The difference is the liquidity premium $r_{1,t+1}^f - \min\{r_{1,t+1}^f, r_{2,t+1}^f\}$ as shown in equation (10). It should be noted that, although the liquidity premium is important, it accounts for only about half of the equity premium in simulated data and the remainder is explained by the risk premium. Similarly, Figure 7 shows that the Sharpe ratio $\frac{E(r-r^f)}{\sigma(r-r^f)}$ spikes at point 1 and is flat at the other points.

Figure 8 shows that the conditional stock volatility is a U-shaped function of $\frac{B_{1,t}}{Y_t}$ and is skewed to the left, as expected. On the other hand, the price-dividend ratio is a monotonically increasing function of $\frac{B_{1,t}}{Y_t}$, as shown in Figure 9. Together, our model predicts an unstable relation between stock volatility and the price-dividend ratio: The two variables are negatively (positively) correlated when the price-dividend ratio is high (low). This pattern is consistent with empirical evidence documented by Schwert (1989) and below we further discuss its implication for the leverage effect. The conditional risk-free rate volatility is also a U-shaped function of $\frac{B_{1,t}}{Y_t}$, although much smaller than stock volatility.

C. Autocorrelation

The lower panel of Table 2 reports the autocorrelation in the data. While the risk-free rate r^f is somewhat persistent, stock return r and excess stock return $r - r^f$ show small and usually negative autocorrelation. Fama and French (1988) and Poterba and Summers (1988) also document a slow univariate mean-reversion in stock prices. This is demonstrated by the partial sum of the autocorrelation coefficient of excess stock return $\sum_{i=1}^{j} \rho(r_t - r_t^f, r_{t+i} - r_{t+i}^f)$, which is negative and decreases with the horizon j. Moreover, Fama and French (1989) find that the price-dividend ratio $\frac{P}{D}$ and the default premium DEF are more persistent than the term premium $TERM$.[9]

[Insert Table 2 here]

The default premium DEF is not directly defined in our model, and we use the shareholder's outstanding debts $B_{1,t}$ as an approximation for it.[10] In simulated data shown in the upper panel of Table 2, both the price-dividend ratio $\frac{P}{D}$ and the default premium DEF are more persistent than the term premium $TERM$, which is defined as the yield spread between the consol and the risk-free rate. Also, the autocorrelation of the other variables displays a similar pattern to that in the data.

D. Leverage Effect

Christie (1982), among others, argues for a leverage effect that stock prices are negatively correlated with stock volatility.[11] Following Campbell and Cochrane (1999), we use the absolute value of excess stock return $|r_{t+j} - r_{t+j}^f|$ as a measure of stock volatility and report its coefficient of correlation with the log price-dividend ratio $p_t - d_t$ in Table 3. The coefficient is indeed negative; however, the magnitude is rather small in both simulated and actual data. In contrast, Campbell and Cochrane (1999) predict a much larger and much more persistent

[9] In Fama and French (1989), the default premium is defined as the yield spread between Aaa bonds and a bond portfolio; the term premium is defined as the yield spread between long-term Aaa bonds and the risk-free rate.

[10] Fama and French (1989) find that the price-dividend ratio and the default premium are highly correlated, with a coefficient of correlation of 0.61 for the period 1927-1987 and 0.75 for the period 1941-1987. Interestingly, these two variables have a coefficient of correlation of 0.93 in simulated data.

[11] Guo and Whitelaw (2001), among others, suggest that a volatility feedback effect may also explain the negative relation between stock prices and volatility.

leverage effect, which is reproduced in the last row of Table 3. The reason for the difference between the two models is as follows. Stock volatility decreases monotonically with the price-dividend ratio in Campbell and Cochrane (1999); however, as shown in Figure 8, it is an asymmetric U-shaped function of the price-dividend ratio in our model.

[Insert Table 3 here]

E. Long Horizon Predictability

Fama and French (1989) find that the price-dividend ratio, the default premium, and the term premium forecast stock return and the first two variables have longer forecasting horizons than the last one. The left column of Table 4 shows that our simulation replicates Fama and French's results. The price-dividend ratio $\frac{P}{D}$ and the term premium $TERM$ both predict stock return.[12] Also, while R^2 increases with horizons for the price-dividend ratio, it peaks after 2-3 years for the term premium.[13] The default premium DEF, which is approximated by the shareholder's liquidity conditions $B_{1,t}$ in simulated data, exhibits the same pattern as the price-dividend ratio in forecasting stock return and is not reported here.

R^2 increases with horizons for the price-dividend ratio because the price-dividend ratio tracks the liquidity component in conditional excess return, which is relatively persistent. For example, when the shareholder is constrained, expected excess return is high. However, because the labor income shock is persistent, the shareholder is likely to be constrained again in the next period, and the realized excess return thus might be low. On the other hand, the labor income shock is not permanent and the shareholder's consumption eventually reverts

[12] Lettau and Ludvigson (2001) show that the consumption-wealth ratio, which is the error term of the cointegration relation among aggregate consumption, labor income, and wealth, is a strong predictor of stock return. In our simulated data, the coefficient of correlation between the consumption-wealth ratio and the price-dividend ratio is -0.99, and the two variables have very similar forecasting abilities for stock return.

[13] We do not report the empirical counterpart for the term premium in Table 4 because Fama and French (1989) show that the term premium has predictive power for only 1-2 quarters, while the frequency of our simulation is annual.

to the trend level over a long horizon; accordingly, the realized excess return is high over the long horizon.

[Insert Table 4 here]

Fama (1990) argues that variations in long-term rates are less extreme because the risk-free rate is a mean-reverting process. In the baseline model, the standard deviation of the yield on the consol is only 0.2 percent, compared with 4.7 percent for the risk-free rate. Most variations in the term premium, therefore, come from innovations in the risk-free rate, which in turn are primarily caused by innovations in aggregate income.[14] In contrast, as discussed above, the price-dividend ratio forecasts stock return because it tracks closely the shareholder's liquidity conditions, of which movements are explained by idiosyncratic income shocks. Therefore, idiosyncratic income shocks and aggregate income shocks are the two major economic forces that influence expected stock return in our model economy; however, the former has much larger and much more persistent effects on stock prices than the latter does.

F. Volatility Test

Cochrane (1991) decomposes the variance of the price-dividend ratio into two parts—shocks to expected stock return $-\sum_{j=1}^{\infty}\rho^j cov(p_t - d_t, r_{t+j})$ and shocks to the dividend growth $\sum_{j=1}^{\infty}\rho^j cov(p_t - d_t, \Delta d_{t+j})$, as in equation (15) below, where $\rho = \dfrac{P/D}{1+P/D}$ and P/D is the unconditional price-dividend ratio.

$$var(p_t - d_t) \approx \sum_{j=1}^{\infty}\rho^j cov(p_t - d_t, \Delta d_{t+j}) - \sum_{j=1}^{\infty}\rho^j cov(p_t - d_t, r_{t+j}) \tag{15}$$

[14] As shown in equation (13), our model economy is perturbed by three shocks: dividend shocks, aggregate income shocks, and idiosyncratic income shocks. Because dividends move closely with aggregate income, the model dynamic is primarily driven by aggregate income shocks and idiosyncratic income shocks. In simulated data, while the former explains 10 percent of the variations in the risk-free rate, the latter accounts for only 1 percent; a large fraction is explained by its own lags.

The volatility test mirrors Shiller's (1981) excess volatility puzzle: As shown in Table 5, shocks to expected return account for most variations of the price-dividend ratio in the data. Our model replicates this feature well. While shocks to expected stock return account for 95 percent of variations in the price-dividend ratio, shocks to dividends explain only 1 percent in the simulation.

[Insert Table 5 here]

G. Social Welfare Loss

In this subsection, we define and calculate the social welfare loss associated with the market frictions analyzed in this paper from the perspective of a social planner, who cares about the shareholder and the nonshareholder equally. It should be noted that both agents can completely diversify income risk through trading on the stock market if there is no limited stock market participation. In this case, the outcome of the decentralized economy is Pareto optimal and is the same as that of the social planner economy; the associated expected social utility is $E\sum[\beta^t U(\alpha C_t) + \beta^t U((1-\alpha)C_t)]$, where α $(1-\alpha)$ is the average ratio of the shareholder's (nonshareholder's) income to aggregate income and C_t is aggregate consumption, which is equal to aggregate income in our exchange economy. In the decentralized economy of the baseline model, the expected social utility is $E\sum[\beta^t U(C_{1,t}) + \beta^t U(C_{2,t})]$, where $C_t = C_{1,t} + C_{2,t}$. To calculate the social welfare loss, we define $E\sum[\beta^t U(\alpha(1-\lambda)C_t) + \beta^t U((1-\alpha)(1-\lambda)C_t)] = E\sum[\beta^t U(C_{1,t}) + \beta^t U(C_{2,t})]$, where λ is the fraction of aggregate consumption that the social planner is willing to spend on eliminating the market frictions. Also, we define the welfare gain λ_1 of the nonshareholder for becoming a shareholder as $E\sum[\beta^t U((1-\alpha)C_t)] = E\sum[\beta^t U((1+\lambda_1)C_{2,t})]$. The shareholder also benefits from the removal of the stock market participation restriction: The welfare gain λ_2 for the shareholder is defined as $E\sum[\beta^t U(\alpha C_t)] = E\sum[\beta^t U((1+\lambda_2)C_{1,t})]$.

We calculate the welfare loss or gain through a grid search using simulated data and find that, in the baseline model, λ is about 1.24 percent. Also, λ_1 is about 0.47 percent for the nonshareholder and λ_2 is about 1.61 percent for the shareholder. The shareholder benefits more than the nonshareholder does because the income of the former is more volatile than that of the latter in the baseline model.[15] Admittedly, the social welfare loss is relatively large. However, it is at least qualitatively consistent with empirical evidence, e.g., Hayashi, Altonji, and Kotlikoff (1996) and many others. More importantly, some recent research provides direct support to our model. Using shareholders' consumption from PSID data, Jacobs (1999) reports overwhelming rejection of the Euler equation for the risk-free rate, but not for stock market return. Also, Heaton and Lucas (2000) find that proprietary income risk, which is borne mostly by shareholders, has significant effects on stock prices.

5 Alternative Specifications

In this section, we calibrate the model using different parameters and income processes. In general, our results are robust to reasonable variations in parameterization.

A. Borrowing Constraints

Our model predicts a large equity premium because the precautionary saving demand lowers only the risk-free rate, not stock return. The more stringent the borrowing constraints are, the larger the effect of the precautionary saving demand on the equity premium is. This is clearly demonstrated in Table 6. As BC increases, the percentage of the time that the shareholder ($F1$) and the nonshareholder ($F2$) are constrained decreases or their abilities to

[15] In Basak and Cuoco (1998), while the nonshareholder's welfare improves, the shareholder's welfare actually deteriorates if the restriction of limited stock market participation is removed. The difference between their model and ours is explained by the fact that Basak and Cuoco (1998) do not consider idiosyncratic income risk and borrowing constraints.

diversify income risk increases. As a result, the risk-free rate R^f increases and stock return R decreases; the equity premium $R - R^f$ disappears when borrowing constraints become so loose that both agents can perfectly diversify income risk.

[Insert Table 6 here]

B. Relative Risk Aversion Coefficient

Changing γ has two opposite effects on the risk-free rate. First, as shown in Table 7, the frequency of binding constraints goes up for both agents when the relative risk aversion coefficient γ increases because, with higher γ, agents prefer smoother consumption. In other words, higher γ leads to a stronger precautionary saving motive and thus a lower risk-free rate. Second, in the power utility function, γ is the reciprocal of the elasticity of intertemporal substitution. Therefore, higher γ implies a higher risk-free rate if the consumption growth rate is fixed. Overall, the risk-free rate first increases then decreases with γ in Table 7. Because the precautionary saving motive does not affect stock return directly, stock return increases monotonically with γ. The equity premium also increases with γ.

We also allow the shareholder to be more risk averse than the nonshareholder, as in Basak and Cuoco (1998). We find that such an asymmetry helps explain the equity premium puzzle. For example, with the other parameters the same as in the baseline model, if γ is equal to 3 (1) for the shareholder (nonshareholder), our model generates an equity premium of 4.5 percent for $BC = 30\%$, compared with 1.5 percent reported in Table 6. The intuition is as follows. When the shareholder is constrained, the nonshareholder, who has a lower γ and thus a higher elasticity of intertemporal substitution, is more willing to substitute consumption intertemporally, accepts a lower risk-free rate relative to the baseline model. Specifically, the asymmetry in preference amplifies the liquidity premium and thus the equity premium. Similarly, our model generates a smaller equity premium relative to the baseline if the shareholder is less risk averse than the nonshareholder. In this case, however, we can always restore the equity premium by assuming more stringent borrowing constraints. Therefore, heterogeneous risk preference does not qualitatively affect our results.

[Insert Table 7 here]

C. Dividend and Labor Income

In Table 8, we allow the dividend share $\frac{D}{Y}$ and the shareholder's labor income share $\frac{L_1}{L}$ to deviate from those in the baseline model and find that these modifications have no qualitative effects. These results should not be a surprise because our model generates a sizable equity premium as long as the shareholder is occasionally constrained. Mankiw (1986) points out that if labor income shocks are concentrated in the troughs of business cycles, the risk-free rate should be lower than would be the case if these shocks are acyclical. To address this issue, we calibrate the model using the cyclical labor income estimated by Heaton and Lucas (1996) and the results are reported under the column *Cyclical* of Table 8. As expected, the risk-free rate is lower in the cyclical model than in the baseline model; stock return and the equity premium are also slightly higher. There is, however, no significant difference between the baseline model and the cyclical model.

[Insert Table 8 here]

6 Conclusion

We find that a combination of some well-documented market frictions explains the equity premium puzzle. Our main innovation is that, in addition to the risk premium in the standard model, shareholders also require a liquidity premium on stocks because of limited stock market participation. Interestingly, the liquidity premium also sheds light on some ongoing controversies in the asset pricing literature. For example, because the liquidity premium can be negatively related to the risk premium, we might find a negative risk-return relation in the data, which contradicts the CAPM. Nevertheless, our model suggests a positive risk-return tradeoff once we control for the liquidity premium; Guo (2002a) finds that these implications are supported by the post-World War II data. Also, like Merton's (1973) intertemporal CAPM, our model highlights the inadequacy of the CAPM because investment opportunities, e.g., conditional stock return and volatility, change over time. In particular, given that past volatility and the price-dividend ratio forecast stock return and volatility in our model, they should be included as risk factors in addition to market return (Campbell (1993)). Indeed, Guo (2002b) finds that these factors help explain the cross section of stock returns. We believe that the market frictions analyzed in this paper, given their success in explaining the asset pricing phenomena, are important to understanding many other related economic issues and warrant attention in future research.

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Table 1: Means and Standard Deviations: Baseline Model and Data

	Baseline Model	1871-1998	1871-1945	1946-1998
		Mean		
r^f	2.1	2.4	2.9	1.6
r	6.2	6.9	6.4	7.6
$r - r^f$	4.2	4.5	3.5	5.9
r^c	2.0	2.2	2.9	1.2
$r^c - r^f$	-0.1	-0.2	-0.0	-0.5
$\frac{P}{D}$	23.7	23.5	20.1	28.1
		Standard Deviation		
r^f	4.7	9.4	10.6	8.0
r	15.4	18.4	17.9	19.1
$r - r^f$	15.5	17.6	19.0	15.6
r^c	8.3	11.7	10.9	12.7
$r^c - r^f$	7.2	5.8	3.7	8.0
$\frac{P}{D}$	6.0	7.9	4.8	8.9

r^f is the risk-free rate; r is stock return; r^c is the consol return in the model as well as the long-term government bond return in data; $\frac{P}{D}$ is the price-dividend ratio. In columns 2-4, data of the risk-free rate, stock return, and the price-dividend ratio were obtained from Robert Shiller at Yale University; data of the long-term government bond were provided to us by Jack Wilson at University of North Carolina. The price-dividend ratio is reported in level and all other variables are reported in percentage. The frequency is annual.

Table 2: Autocorrelation: Baseline Model and Data

	lead				
	1	2	3	5	7
Baseline Model					
r^f	0.61	0.31	0.12	-0.01	0.01
r	0.00	-0.05	-0.07	-0.08	-0.04
$r - r^f$	0.02	-0.02	-0.06	-0.07	-0.04
$\frac{P}{D}$	0.80	0.61	0.45	0.21	0.08
DEF	0.87	0.67	0.47	0.31	0.12
$TERM$	0.60	0.31	0.12	-0.02	0.01
$\sum_{i=1}^{j} \rho(r_t - r_t^f, r_{t+i} - r_{t+i}^f)$	0.02	0.00	-0.06	-0.18	-0.30
Data 1871-1998					
r^f	0.28	0.08	0.08	0.05	0.14
r	-0.07	-0.15	0.09	-0.01	0.12
$r - r^f$	0.03	-0.23	0.11	-0.14	0.15
$\frac{P}{D}$	0.75	0.56	0.55	0.40	0.40
DEF	0.74	0.61	0.34	0.42	0.46
$TERM$	0.46	0.24	0.04	0.20	0.06
$\sum_{i=1}^{j} \rho(r_t - r_t^f, r_{t+i} - r_{t+i}^f)$	0.03	-0.20	-0.09	-0.29	-0.10

DEF is the default premium and is approximated by shareholder's bond holding $B_{1,t}$ in simulated data. $TERM$ is the term premium and is defined as the yield spread between the consol and the one-period discount bond in the model. $\rho(r_t - r_t^f, r_{t+i} - r_{t+i}^f)$ is the coefficient of correlation between $r_t - r_t^f$ and $r_{t+i} - r_{t+i}^f$. All other variables are defined in the note of Table 1. In the lower panel, the default premium and the term premium are reproduced from Table 1 of Fama and French (1989); all other variables are calculated from Shiller's data. The frequency is annual.

Table 3: Leverage Effect: Baseline Model and Data

	Lead				
	1	2	3	5	7
Baseline Model					
$p_t - d_t, \lvert r_{t+j} - r^f_{t+j} \rvert$	-0.10	-0.12	-0.12	-0.09	-0.05
Data 1871-1998					
$p_t - d_t, \lvert r_{t+j} - r^f_{t+j} \rvert$	-0.04	-0.01	-0.14	-0.03	0.03
Campbell and Cochrane (1999)					
$p_t - d_t, \lvert r_{t+j} - r^f_{t+j} \rvert$	-0.49	-0.42	-0.37	-0.28	-0.21

$p_t - d_t$ is the log price-dividend ratio and $|r_{t+j} - r^f_{t+j}|$ is the absolute value of excess stock return. Shiller's data are used to calculate the empirical counterparts in the middle panel. Table 4 of Campbell and Cochrane (1999) is also reproduced for comparison. The frequency is annual.

Table 4: Long-Horizon Predictability: Baseline Model and Data

Horizon	Baseline Model		Data 1871-1998	
	$\sum_1^j (r_{t+j} - r^f_{t+j}) = a + b * (p_t - d_t)$			
	slope	R^2	slope	R^2
1	-0.17	0.07	-0.09	0.02
2	-0.32	0.14	-0.25	0.08
3	-0.46	0.19	-0.31	0.09
5	-0.66	0.25	-0.54	0.17
7	-0.76	0.27	-0.58	0.16
	$\sum_1^j (r_{t+j} - r^f_{t+j}) = a + b * Term$			
	slope	R^2		
1	0.63	0.03		
2	1.01	0.04		
3	1.19	0.04		
5	1.11	0.02		
7	0.85	0.01		

$p_t - d_t$ is the log price-dividend ratio and $Term$ is the term premium, defined as the yield spread between the consol and the one-period discount bond in simulated data. We use overlapped data in the regressions. Shiller's data are used for the empirical counterparts. The frequency is annual.

Table 5: Volatility Test: Baseline Model and Data

	Expected Return	Dividend
Baseline Model	95	1
Data	101	-10

Cochrane (1991) shows that the variance of the price-dividend ratio can be decomposed into two parts,

$$var(p_t - d_t) \approx \sum_{j=1}^{\infty} \rho^j cov(p_t - d_t, \Delta d_{t+j}) - \sum_{j=1}^{\infty} \rho^j cov(p_t - d_t, r_{t+j}).$$

The first component is the variation caused by dividend shocks and the second component is due to expected return shocks. Following Campbell and Cochrane (1999), we use the first 15 leads to calculate these statistics in simulated data. The empirical counterparts are taken from Campbell and Cochrane (1999). All the numbers are reported in percentage.

Table 6: Changing Borrowing Constraints

BC	R^f	$\sigma(R^f)$	R	$\sigma(R)$	$R-R^f$	$\sigma(R-R^f)$	F1	F2
0.07	0.6	5.5	8.3	20.8	7.7	21.0	21.9	25.1
0.10	2.2	4.7	7.7	17.0	5.5	17.2	16.9	20.3
0.15	3.7	3.9	7.2	13.1	3.5	13.3	12.0	15.3
0.20	4.5	3.4	6.9	10.6	2.5	10.8	9.1	11.2
0.30	5.2	2.7	6.7	7.8	1.5	7.8	5.8	8.2
0.50	5.8	2.2	6.6	5.2	0.7	5.1	3.0	4.9
0.80	6.1	1.9	6.5	3.9	0.4	3.5	1.6	2.7
1.00	6.2	1.8	6.5	3.5	0.3	3.1	1.1	2.1
1.20	6.2	1.8	6.5	3.4	0.3	2.9	0.8	1.6
1.50	6.2	1.8	6.5	3.1	0.2	2.6	0.5	1.2
2.00	6.3	1.8	6.5	3.1	0.2	2.6	0.1	0.4
3.00	6.2	1.7	6.5	3.2	0.3	2.7	0.0	0.2
4.00	6.2	1.7	6.5	3.5	0.3	3.0	0.0	0.0

The specifications are the same as the baseline model, except the borrowing constraints BC. All the numbers are reported in percentage and the frequency is annual. F1 (F2) is the percentage of the time that the shareholder's (non-shareholder's) borrowing constraints are binding.

Table 7: Changing Relative Risk Aversion Coefficient

γ	R^f	$\sigma(R^f)$	R	$\sigma(R)$	$R-R^f$	$\sigma(R-R^f)$	F1	F2
1	1.7	1.6	3.5	6.8	1.9	6.9	16.7	18.4
2	2.1	3.2	5.5	12.0	3.4	12.2	16.8	19.4
3	2.2	4.7	7.7	17.2	5.5	17.2	16.9	20.3
4	2.1	6.3	10.0	21.8	7.9	21.8	17.0	21.7
5	1.9	7.7	12.2	26.0	10.3	26.0	17.2	22.6

The specifications are the same as in the baseline model except the relative risk aversion coefficient γ. All the numbers are reported in percentage and the frequency is annual. F1 (F2) is the percentage of the time that the shareholder's (nonshareholder's) borrowing constraints are binding.

Table 8: Means and Standard Deviations: Alternative Specifications

	Baseline	$\frac{D}{Y}$=5%	$\frac{D}{Y}$=10%	$\frac{L_1}{L}$=20%	$\frac{L_1}{L}$=40%	Cyclical
			Mean			
R^f	2.2	.6	1.7	4.2	-1.0	1.9
R	7.7	8.9	8.1	7.1	8.5	8.3
$R - R^f$	5.5	8.3	6.4	2.9	9.5	6.4
$\frac{P}{D}$	23.7	25.2	24.3	23.1	24.4	24.0
			Standard Deviation			
R^f	4.7	6.2	5.2	3.7	5.9	7.0
R	17.1	24.6	19.8	12.1	22.3	20.8
$R - R^f$	17.3	24.3	19.7	12.1	23.2	21.4
$\frac{P}{D}$	6.0	8.4	6.9	4.4	7.3	6.5

The specifications are the same as the baseline model except the dividend share in columns 2-3, the shareholder's labor income share in columns 4-5, and cyclical income shocks in column 6. The price-dividend ratio is reported in level and all other variables are reported in percentage. The frequency is annual.

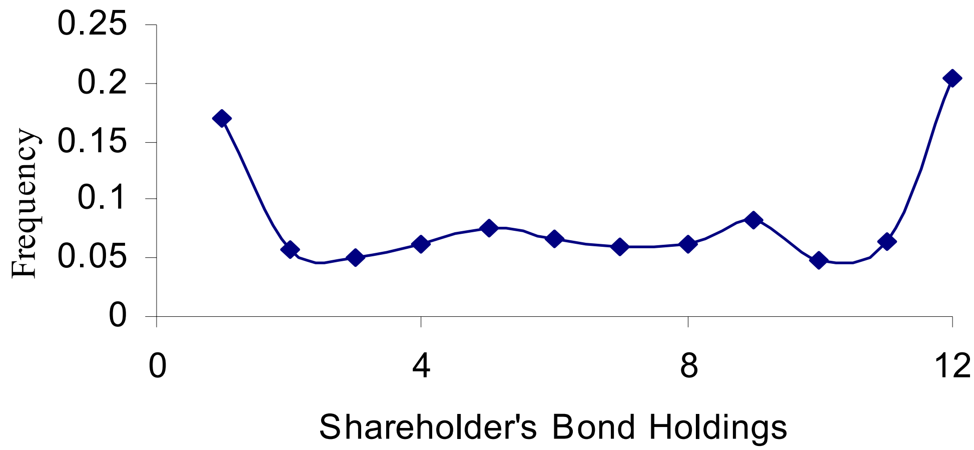

Figure 1: Distribution of Shareholder's Bond Holdings

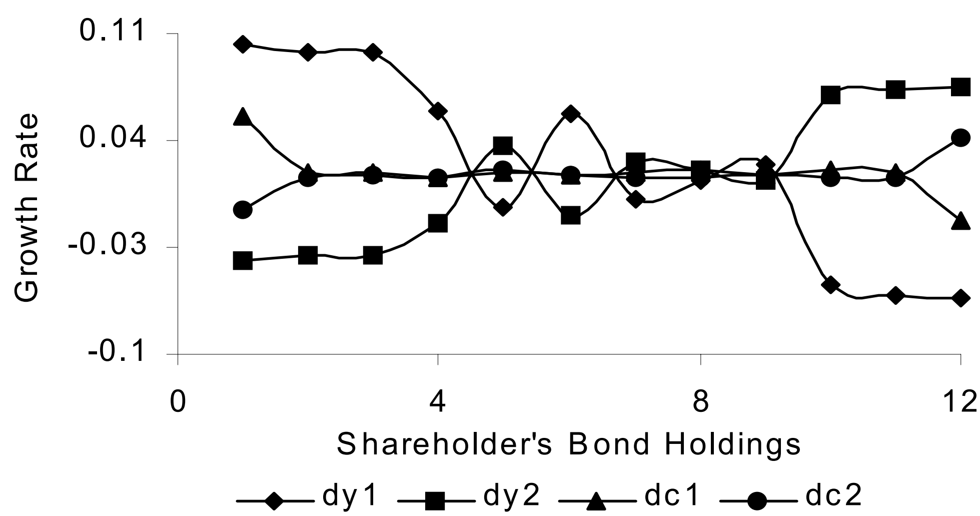

Figure 2: Conditional Income and Consumption Growth

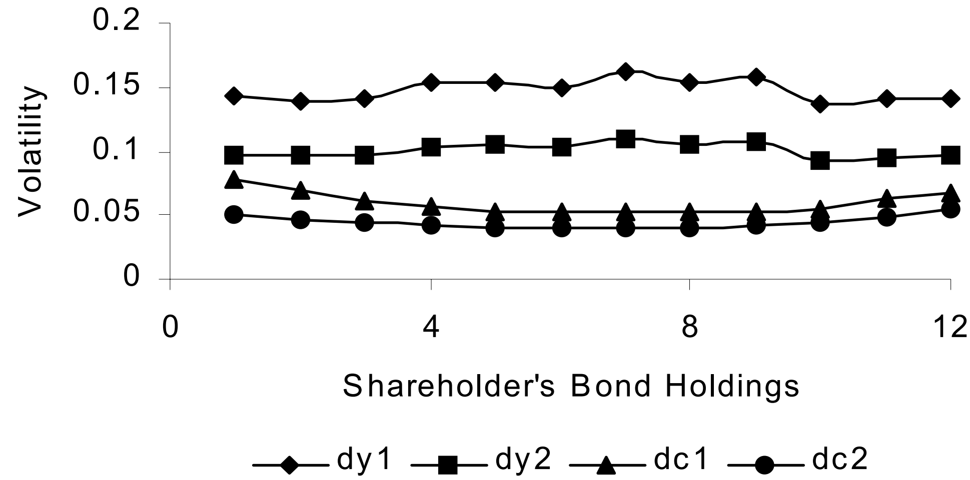

Figure 3: Conditional Income and Consumption Volatility

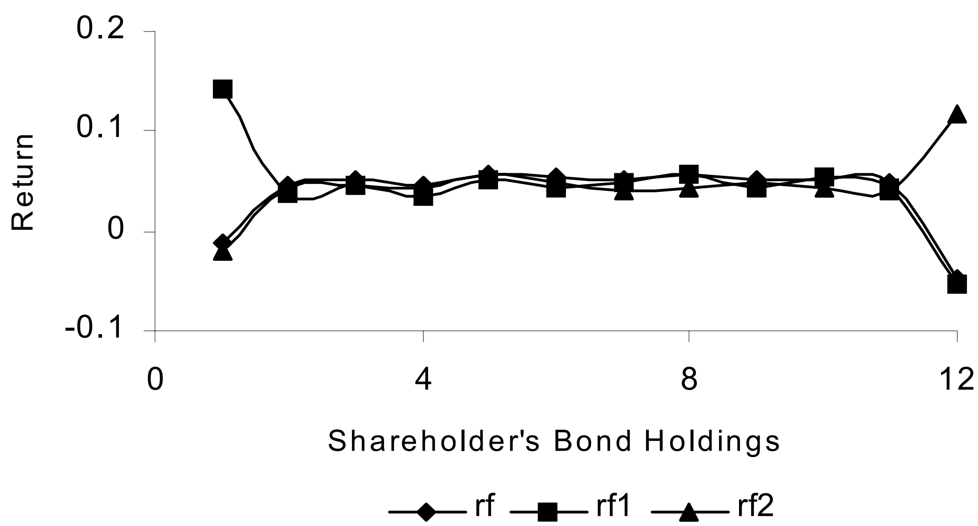

Figure 4: Conditional Risk-Free Rate

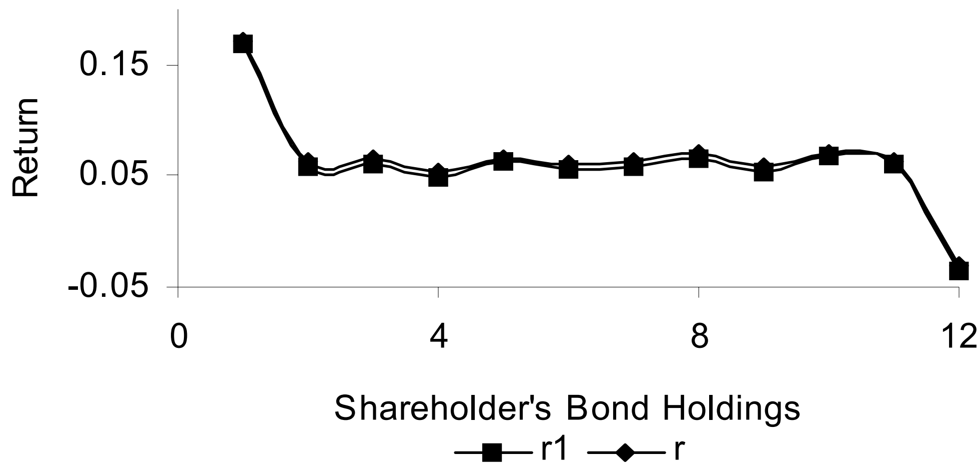

Figure 5: Conditional Stock Return

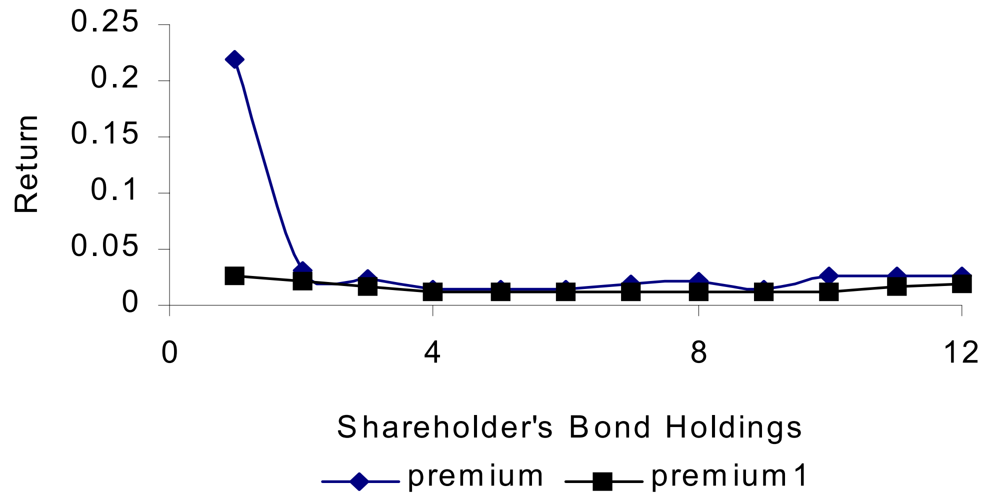

Figure 6: Conditional Equity Premium

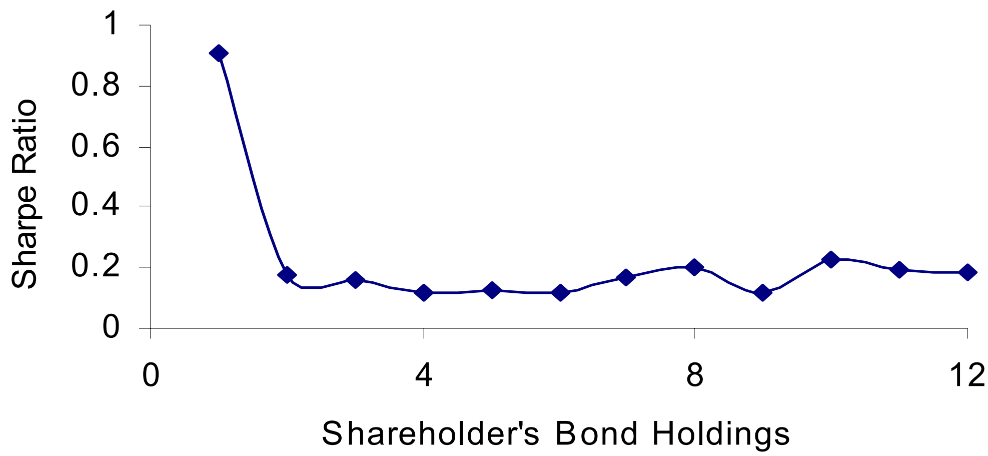

Figure 7: Conditional Sharpe Ratio

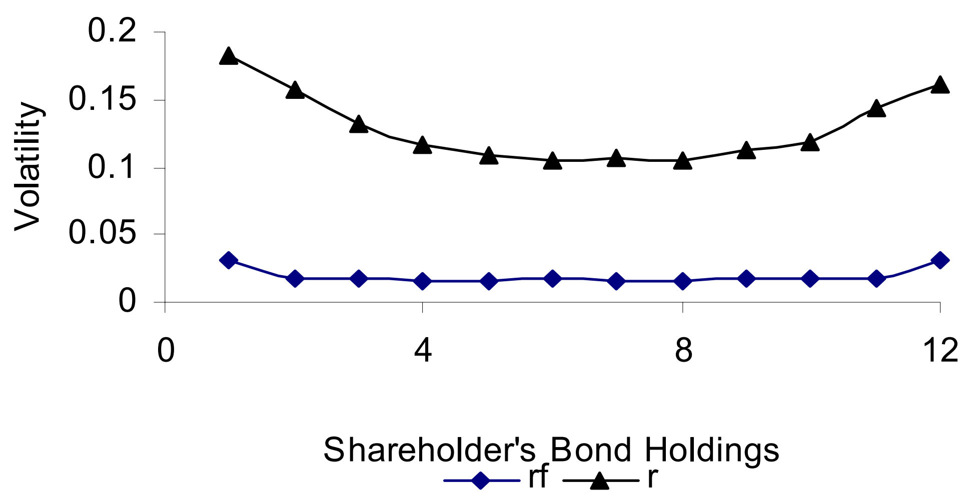

Figure 8: Conditional Return Volatility

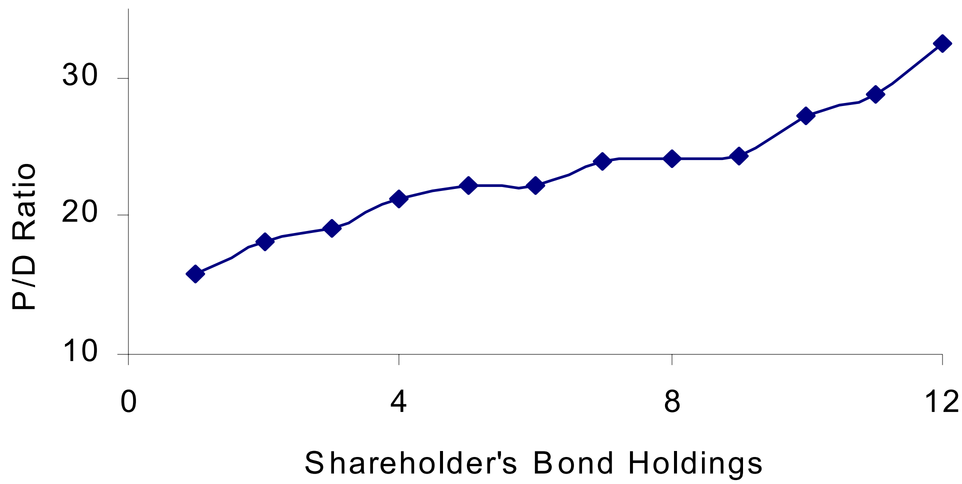

Figure 9: Conditional Price-Dividend Ratio