Do Real Exchange Rates Have Autoregressive Unit Roots? A test under the Alternative of Long Memory and Breaks

Authors	Michael J. Dueker, and Apostolos Serletis
Working Paper Number	2000-016A
Creation Date	July 2000
Citable Link	https://doi.org/10.20955/wp.2000.016
Suggested Citation	Dueker, M.J., Serletis, A., 2000; Do Real Exchange Rates Have Autoregressive Unit Roots? A test under the Alternative of Long Memory and Breaks, Federal Reserve Bank of St. Louis Working Paper 2000-016. URL https://doi.org/10.20955/wp.2000.016

DO REAL EXCHANGE RATES HAVE AUTOREGRESSIVE UNIT ROOTS? A TEST UNDER THE ALTERNATIVE OF LONG MEMORY AND BREAKS

June 2000

Abstract

In this paper, we estimate (by maximum likelihood) the parameters of univariate fractionally integrated real exchange rate time series models, and test for autoregressive unit roots on the alternative of a covariance stationary long-memory process. We use quarterly dollar-based real exchange rates (since 1957) for seventeen OECD countries, and that the finding of unit autoregressive roots does not go away even with this more sophisticated alternative.

KEYWORDS: Fractional integration, Long memory processes, Real exchange rates

JEL CLASSIFICATIONS: C22

Michael Dueker
Federal Reserve Bank of St. Louis
Research Department
P.O. Box 442
St. Louis, MO 63102
(314) 444-8594
dueker@stls.frb.org

Apostolos Serletis
Department of Economics
The University of Calgary
Calgary, Alberta T2N 1N4
(403) 220-4091
(403) 282-5262—fax
serletis@acs.ucalgary.ca

I. INTRODUCTION

The theory of purchasing power parity (PPP) has attracted a great deal of attention and has been explored extensively in the recent literature using recent advances in the field of applied econometrics. Based on the law of one price, PPP asserts that relative goods prices are not affected by exchange rates -- or, equivalently, that exchange rate changes will be proportional to relative inflation. The relationship is important not only because it has been a cornerstone of exchange rate models in international economics, but also because of its policy implications -- it provides a benchmark exchange rate and hence has some practical appeal for policymakers and exchange rate arbitragers.

Although purchasing power parity has been studied extensively, empirical studies generally fail to find support for long-run PPP, especially during the recent floating exchange rate period. In fact, the empirical consensus is that PPP does not hold over this period -- see for example, Adler and Lehman (1983), Mark (1990), Grilli and Kaminski (1991), Flynn and Boucher (1993), Serletis (1994), and Serletis and Zimonopoulos (1997). But there are also studies covering different groups of countries [see Phylaktis and Kassimatis (1994)] as well as studies covering periods of long duration [see Lothian and Taylor (1996) and Perron and Vogelsang (1992)] or country pairs experiencing large differentials in price movements [see Frenkel (1980) and Taylor and McMahon (1988)] that report evidence of mean reversion towards PPP.

A sufficient condition for a violation of purchasing power parity is that the real exchange rate is characterized by a unit root. A number of approaches have been developed to test for unit roots. Nelson and Plosser (1982), using augmented Dickey-Fuller (ADF) type regressions [see

Dickey and Fuller (1981)], argue that most macroeconomic time series (including real exchange rates) have a unit root. Perron (1989), however, has shown that conventional unit root tests are biased against rejecting a unit root where there is a break in a trend stationary process. More recently, Serletis and Zimonopoulos (1997), using the methodology suggested by Perron and Vogelsang (1992) and quarterly (from 1957:1 to 1995:4) dollar-based and DM-based real exchange rates for seventeen OECD countries, show that the unit root hypothesis cannot be rejected even if allowance is made for the possibility of a one-time change in the mean of the series at an unknown point in time.

Given that integration tests are sensitive to the class of models considered (and may be misleading because of misspecification), in this paper we consider a more general model. We test for fractional integration (a series is fractionally integrated if it is integrated of order zero only after fractional differencing), using the fractional ARIMA model. Fractional integration is a popular way to parameterize long-memory processes (whose autocorrelation structure decays slowly to zero or, equivalently, whose spectral density is highly concentrated at frequencies close to zero).

We apply the fractional ARIMA model to quarterly dollar-based real exchange rates, covering the period from 1957:1 to 1995:4, for seventeen OECD countries. The countries involved are Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, Norway, Spain, Switzerland, and the United Kingdom -- see Serletis and Zimonopoulos (1997) for details regarding the calculation and time plots of the real exchange rates.

The remainder of the paper consists of three sections. The first provides a brief discussion of the methodology, the second discusses estimation issues and presents the results, and the last summarizes the paper.

II. LONG-MEMORY REAL EXCHANGE RATE MODELS

The general fractionally integrated time series model can be written as [see Sowell (1992) or Baillie (1996) for more details]

$$\phi(L)\ (1-L)^d y_t = \theta(L)\varepsilon_t$$

where $\varepsilon_t \sim \text{IIDN}(0, \sigma^2)$, $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, and $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$. All the roots of $\phi(L)$ and $\theta(L)$ are assumed to lie outside the unit circle (thus satisfying the stationarity and invertibility conditions) and the fractional difference operator, $(1 - L)^d$, can be expanded as a Taylor series about $L = 0$ to give

$$\begin{aligned}(1-L)^d &= \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(-d)\ \Gamma(j+1)} L^j \\ &= 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)\ (d-2)}{3!} L^3 + \ldots\end{aligned}$$

Hence, the fractional difference operator provides an infinite-order lag-operator polynomial with slowly and monotonically declining weights, meaning that autocorrelations in the fractionally integrated time series model decay at a hyperbolic rate, rather than the geometric rate at which ARMA autocorrelations decline.

If long-memory processes are estimated with the usual autoregressive moving-average [ARMA(p,q)] model, without considering fractional orders of integration, the estimated autoregressive process can exhibit spuriously high persistence close to a unit root. Since real exchange rates might depart from their means with long memory, we condition our tests for autoregressive unit roots in real exchange rates on the alternative of a covariance stationary long-memory [fractionally integrated I(d)] process, rather than the usual alternative of the series being I(0). In this case, if we fail to reject an autoregressive unit root, we know it is not a spurious finding due to neglect of the relevant alternative of fractional integration and long memory.

Omitted breaks in the mean of the series represent a second source of possibly spurious findings of autoregressive unit roots. In fact, as Perron (1989) shows, conventional unit root tests (such as, for example, the Dickey-Fuller and Phillips-Perron tests) are biased against rejecting a unit root where there is a break in a trend stationary process. For this reason, we parameterize the long-memory alternative by assuming a fractionally integrated time series model

$$\phi(L)\ (1 - L)^d\ (y_t - \mu_t) = \theta(L)\,\varepsilon_t$$

but we allow for a break point in the mean, μ_t, of the real exchange rate series we study, by replacing μ_t by sub-sample means in the estimation, where the break points are taken from Serletis and Zimonopoulos (1997).

III. MAXIMUM LIKELIHOOD ESTIMATES

To facilitate estimation, note that in univariate models, such as ours, we are free to reverse the ordering of the AR polynomial, $\phi(L)$, and the fractional integration polynomial, $(1 - L)^d$. At each function evaluation, we can then quasi-difference the data, using the latest values of the AR parameters

$$\tilde{y}_t = y_t - \phi_1 y_{t-1} - \dots - \phi_p y_{t-p}.$$

This simplification is useful because the quasi-differenced data have the autocovariance structure of a fractional ARIMA (0, d, q) process for which closed-form expressions exist. Sowell (1992) shows that the autocovariances for the general fractional ARIMA (p, d, q) model do not have a closed-form expression and involve infinite sums. From Sowell (1992), we repeat the expression for the autocovariances at lag s for a fractional ARIMA (0, d, q) process, where $k = 1$ for univariate time series

$$\Sigma_{i,j}(s) = \sum_{n=1}^{k} \sum_{r=1}^{k} \sigma_{nr} \sum_{m=0}^{q} \sum_{l=0}^{q} \theta_{i,n}(m)\theta_{j,r}(l) \cdot \frac{\Gamma(1 - d_n - d_r)\Gamma(d_r + s + m - \ell)}{\Gamma(d_r)\Gamma(1 - d_r)\Gamma(1 - d_n + s + m - \ell)}$$

for element (i,j), where σ is the variance matrix of ε, and $\theta(m)$ is the matrix in the moving-average polynomial corresponding with lag m, and Γ is the gamma function.

Fractionally integrated processes are covariance stationary only if $d \leq 0.5$ and we imposed this restriction. To estimate the model for $d > 0.5$ it is necessary to difference the data, in which case we would have lost all inference concerning autoregressive unit roots. Thus, in practice, we test for autoregressive unit roots under the alternative of "covariance-stationary" long memory. Moreover, because these models are still fairly computationally intensive and the number of series in our data set is large, we did not conduct a search for optimal orders, (p, q), of the AR and MA processes. Instead, we estimated a fractional ARIMA (2, d, 2) model in all cases, knowing that in some cases the model might be overparameterized. We felt, however, that it was better to err on the side of overparameterization than to find spurious evidence of long memory due to understating the order of the ARMA process.

The results of the fractional ARIMA (2, d, 2) model are presented in Tables 1-2. In Table 1 we present results without a break in the mean of the real exchange rates. In Table 2 we allow for a break in the mean and we choose the break point so as to minimize (or maximize) Perron and Vogelsang's (1992) $t_{\hat{\alpha}}(\mathrm{I0}, T_b, k)$ statistic -- see Serletis and Zimonopoulos (1997) for more details. On average the standard errors are not particularly small due perhaps to the relatively short post-1957 sample period. Therefore, the long-memory parameter d is generally not significantly different from zero. Nevertheless, we generally find some degree of long-memory positive autocorrelation in the real exchange rates, but the autoregressive unit roots appear to be present anyway. Conditional on fractional integration parameters in the range of 0.1 to 0.2, the real exchange rates still have autoregressive roots large enough that a unit root cannot be rejected -- the results are very consistent with the sum of the two AR coefficients being (generally) above 0.9.

One final issue concerns the non-standard distribution of the t-statistics. The Dickey-Fuller distribution and critical values will not hold for the test for autoregressive unit roots in the presence of long memory. To our knowledge, no one has tabulated this distribution, so we do not have critical values for our t-tests. Nevertheless, given that our t-statistics would almost uniformly fail to reject under the classical t distribution and critical values, they are certain not to reject under a non-standard distribution. This reasoning parallels the fact that the Dickey-Fuller critical values are always larger than the classical ones, so failure to reject at the classical critical values implies failure to reject under a fatter-tailed non-standard distribution. Clearly, the low *t*-statistics we generate leave little room for ambiguity.

IV. CONCLUSION

We have estimated (by maximum likelihood) the parameters of univariate fractionally integrated real exchange rate time series models and tested for autoregressive unit root on the alternative of a covariance stationary fractionally integrated process. Our main contribution is that the previous tests for autoregressive unit roots have I(0) as the alternative, whereas we have a much more general alternative of I(d) fractional order of integration with d taking on any value less than 0.5. We show that the finding of unit autoregressive roots does not go away even with this more sophisticated alternative.

An area for potentially productive future research would be to assume conditional heteroscedasticity in the disturbances. To date, however, no methodology exists for handling conditional heteroscedasticity in a long-memory model without compromising the fractional integration in an ad-hoc manner by truncating the fractional differencing operator at the number of points in the sample. In this paper, it seemed preferable to accept the loss of efficiency from not addressing conditional heteroscedasticity, rather than deviate from maximum-likelihood estimation of the long-memory process.

TABLE 1

FRACTIONAL ARIMA (2, *d*, 2) MODELS OF DOLLAR-BASED REAL EXCHANGE RATES WITH NO BREAKS IN THE MEAN

Country	Log Likelihood	d	ρ_1	ρ_2	θ_1	θ_2	σ^2	$\rho_1 + \rho_2$	"classical" t-statistic
Austria	-303.7	.067 (.251)	.887 (.297)	.036 (.266)	.219 (.358)	-.152 (.155)	19.5 (2.23)	.923	1.22
Belgium	-322.5	.096 (.205)	.827 (.272)	.147 (.262)	.153 (.310)	-.159 (.117)	24.9 (2.85)	.974	.882
Canada	-320.0	.216 (.186)	.726 (.235)	.208 (.214)	.204 (.257)	-.229 (.116)	24.0 (2.75)	.934	1.36
Denmark	-320.0	.112 (.199)	.828 (.229)	.141 (.271)	.226 (.128)	.199 (.128)	22.8 (2.61)	.969	1.07
Finland	-321.2	.063 (.153)	.135 (.089)	.773 (.089)	1.07 (.191)	.198 (.164)	24.3 (2.77)	.907	1.25
France	-323.7	.108 (.205)	.804 (.263)	.152 (.248)	.200 (.297)	.181 (.123)	25.3 (2.89)	.956	1.14
Germany	-302.6	0.27 (.221)	0.99 (.643)	-.056 (.592)	0.96 (.626)	0.43 (.128)	19.2 (2.20)	.934	1.01
Greece	-322.6	.082 (.198)	.877 (.251)	.094 (.241)	.164 (.289)	-.174 (.119)	25.0 (2.86)	.970	1.13
Ireland	-293.4	.060 (.196)	.817 (.256)	.156 (.250)	.211 (.316)	.202 (.137)	17.0 (1.95)	.972	1.28
Italy	-340.8	.019 (.202)	.910 (.343)	.079 (.337)	.130 (.376)	.106 (.115)	31.6 (3.62)	.988	0.60
Japan	-169.2	.300 (.157)	.425 (.396)	.487 (.369)	.334 (.408)	.199 (.136)	3.38 (.382)	.912	1.40
Netherlands	-320.7	-.436 (.195)	1.76 (.101)	-.758 (.100)	-.199 (.200)	.119 (.090)	24.1 (2.76)	1.00	0.80
Norway	-300.7	.191 (.195)	.901 (.140)	.026 (.123)	.047 (.214)	.107 (.094)	18.7 (2.14)	.927	1.58
Spain	-299.9	.196 (.193)	.836 (.461)	.086 (.424)	-.103 (.478)	-.054 (.118)	18.5 (2.19)	.921	1.28
Switzerland	-311.5	.122 (.227)	.781 (.301)	.188 (.290)	.425 (.353)	.110 (.175)	21.5 (2.46)	.969	0.85
U.K.	-299.9	.196 (.193)	.836 (.461)	.085 (.424)	-.103 (.478)	-.054 (.118)	18.5 (2.19)	.921	1.28

NOTES: Sample period, quarterly data, 1957:1 - 1995:4. Numbers in parentheses are standard errors.

TABLE 2

FRACTIONAL ARIMA (2, d, 2) MODELS OF DOLLAR-BASED REAL EXCHANGE RATES WITH THE SERLETIS AND ZIMONOPOULOS (1997) BREAK POINTS, BASED ON THE PERRON AND VOGELSANG (1992) $t_{\hat{\alpha}}(\text{IO}, T_b, k)$ STATISTIC

Country	T_b	Log Likelihood	d	ρ_1	ρ_2	θ_1	θ_2	σ^2	$\rho_1 + \rho_2$	"classical" t-statistic
Austria	1970:4	-307.9	.036 (.205)	.816 (.334)	.075 (.292)	.272 (.376)	-.123 (.152)	20.5 (2.35)	.891	1.56
Belgium	1971:1	-347.6	.102 (.209)	.774 (.378)	.129 (.337)	.116 (.399)	-.106 (.107)	34.6 (3.96)	.903	1.36
Canada	1976:1	-327.4	.195 (.174)	.583 (.221)	.325 (.197)	.328 (.240)	-.243 (.122)	26.5 (3.03)	.908	1.58
Denmark	1971:1	-345.7	.077 (.201)	.751 (.324)	.152 (.289)	.226 (.352)	.127 (.122)	33.7 (3.86)	.903	1.51
Finland	1972:3	-325.7	.002 (.131)	.095 (.087)	.731 (.065)	.949 (.050)	.047 (.068)	25.6 (2.94)	.826	3.19
France	1971:2	-336.5	.071 (.199)	.701 (.302)	.193 (.266)	.027 (.319)	.149 (.119)	29.9 (3.42)	.893	1.51
Germany	1970:3	-306.4	.038 (.226)	.965 (.848)	-.054 (.764)	.089 (.825)	.032 (.141)	20.2 (2.31)	.911	0.92
Greece	1991:1	-338.4	.032 (.203)	.886 (.249)	.075 (.237)	.222 (.305)	.184 (.140)	30.6 (3.50)	.961	1.37
Ireland	1984:3	-313.1	.010 (.147)	1.07 (.446)	-.099 (.428)	.026 (.474)	.102 (.118)	22.0 (2.52)	.971	1.21
Italy	1984:4	-365.1	.026 (.200)	.792 (.369)	.165 (.354)	.236 (.413)	.116 (.133)	43.5 (4.97)	.957	1.15
Japan	1971:1	-171.0	.317 (.146)	.447 (.386)	.458 (.357)	.308 (.394)	.198 (.130)	3.41 (.391)	.905	1.53
Netherlands	1970:4	-392.7	.000 (.076)	.972 (.600)	.029 (.545)	.045 (.486)	.029 (.078)	62.3 (7.13)	.943	0.97
Norway	1971:1	-311.0	.161 (.181)	.864 (.158)	.035 (.137)	.055 (.219)	.075 (.091)	21.4 (2.44)	.899	1.93
Spain	1971:2	-300.6	.193 (.195)	.836 (.493)	.831 (.452)	-.103 (.496)	-.051 (.125)	18.7 (2.14)	.919	1.25
Switzerland	1972:3	-325.0	.022 (.207)	.635 (.329)	.223 (.278)	.316 (.340)	.122 (.121)	25.7 (2.94)	.859	1.48
U.K.	1976:2	-340.3	.265 (.134)	.262 (.134)	.596 (.114)	.593 (.166)	-.256 (.124)	31.1 (3.56)	.859	1.87

NOTES: Sample period, quarterly data, 1957:1 - 1995:4. Numbers in parentheses are standard errors.

REFERENCES

Adler, Michael, and Bruce Lehman. "Deviations from Purchasing Power Parity in the Long Run." *Journal of Finance* 38 (1983): 1471-87.

Baillie, Richard T. "Long Memory Processes and Fractional Integration in Econometrics."" *Journal of Econometrics* 73 (1996), 5-59.

Dickey, David A., and Wayne A. Fuller. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica* 49 (1981): 1057-72.

Flynn, N. Alston, and Janice L. Boucher. "Tests of Long-Run Purchasing Power Parity Using Alternative Methodologies." *Journal of Macroeconomics* 15 (1993): 109-22.

Frenkel, Jacob A. "Exchange Rates, Prices and Money: Lessons from the 1920s." *American Economic Review* (Papers and Proceedings) 70 (1980): 235-242.

Grilli, Vittorio, and Graciela Kaminsky. "Nominal Exchange Rate Regimes and the Real Exchange Rate: Evidence from the United States and Great Britain, 1885-1986." *Journal of Monetary Economics* 27 (1991): 191-212.

Lothian, James R., and Mark P. Taylor. "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries." *Journal of Political Economy* 104 (1996): 488-509.

Mark, Nelson C. "Real and Nominal Exchange Rates in the Long Run: An Empirical Investigation." *Journal of International Economics* 28 (1990): 115-136.

Nelson, Charles R., and Charles I. Plosser. "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications." *Journal of Monetary Economics* 10 (1982): 139-162.

Perron, Pierre. "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57 (1989): 1361-1401.

Perron, Pierre, and Timothy J. Vogelsang. "Nonstationarity and Level Shifts With an Application to Purchasing Power Parity." *Journal of Business and Economic Statistics* 10 (1992a): 301-320.

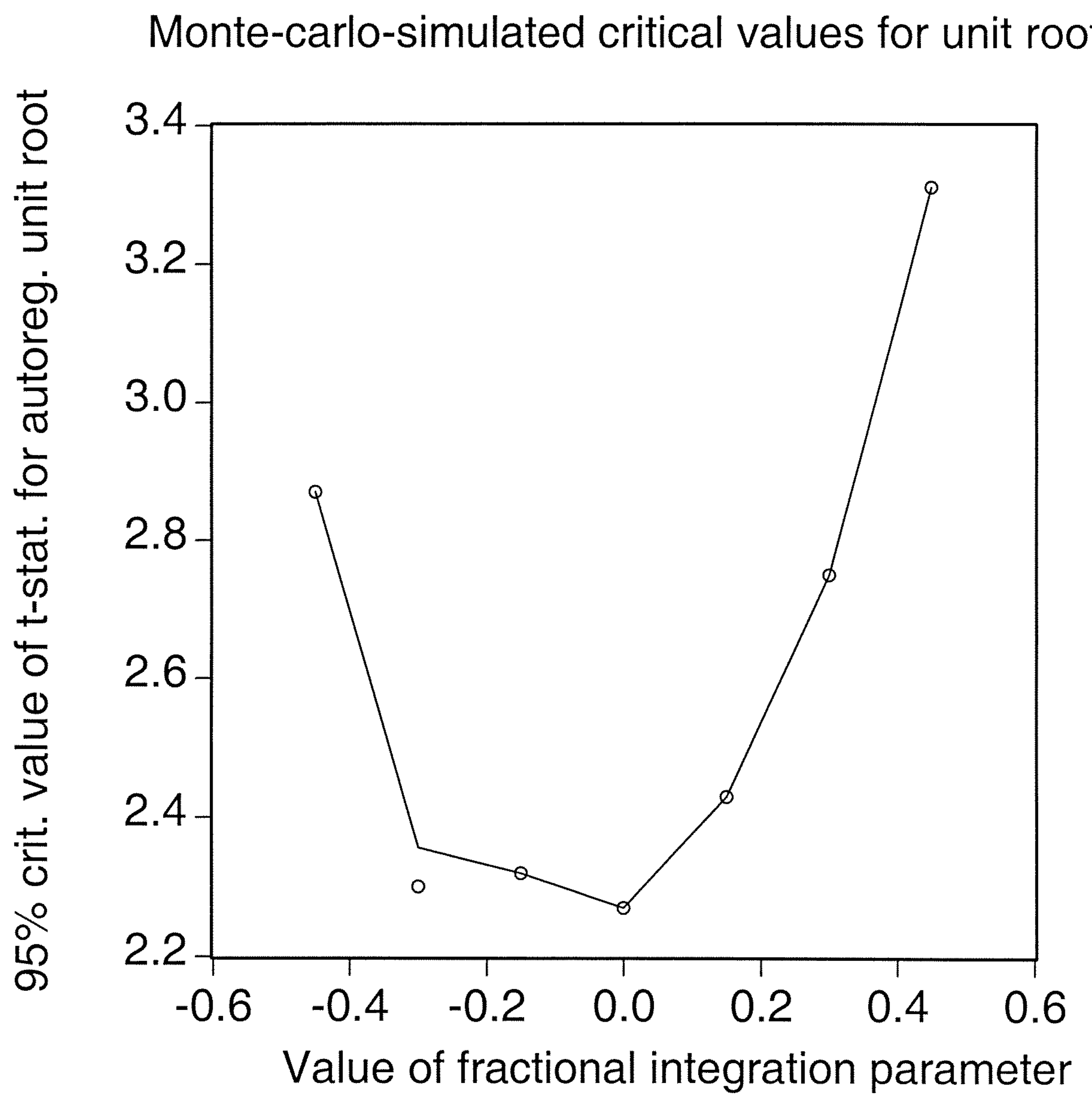
Phylaktis, Kate, and Yiannis Kassimatis. "Does the Real Exchange Rate Follow a Random Walk? The Pacific Basin Perspective." *Journal of International Money and Finance* 13 (1994): 476-495.

Serletis, Apostolos. "Maximum Likelihood Cointegration Tests of Purchasing Power Parity: Evidence from Seventeen OECD Countries." *Weltwirtschaftliches Archiv*, 130 (1994): 476-493.

Serletis, Apostolos, and Grigorios Zimonopoulos. "Breaking Trend Functions in Real Exchange Rates: Evidence from Seventeen OECD Countries." *Journal of Macroeconomics*, forthcoming (1997).

Sowell, Fallaw. "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models." *Journal of Econometrics* 53 (1992), 165-188.

Taylor, Mark P., and Patrick C. MacMahon. "Long-Run Purchasing Power Parity in the 1920s." *European Economic Review* 32 (1988): 179-197.


Monte-carlo-simulated critical values for unit root
95% crit. value of t-stat. for autoreg. unit root
3.4
3.2
3.0
2.8
2.6
2.4
2.2
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
Value of fractional integration parameter