Identification of Dynamic Economic Models from Reduced Form VECM Structures: An Application of Covariance

Authors	Robert H. Rasche
Working Paper Number	2000-011B
Revision Date	January 2000
Citable Link	https://doi.org/10.20955/wp.2000.011
Suggested Citation	Rasche, R.H., 2000; Identification of Dynamic Economic Models from Reduced Form VECM Structures: An Application of Covariance, Federal Reserve Bank of St. Louis Working Paper 2000-011. URL https://doi.org/10.20955/wp.2000.011

Identification of Dynamic Economic Models from Reduced Form VECM Structures:
An Application of Covariance Restrictions*

Robert H. Rasche

Senior Vice President and Director of Research
Federal Reserve Bank of St. Louis

Revised
May 2001

ABSTRACT

This analysis is a straightforward implementation of both long-run and short-run identifying or overidentifying restrictions on a vector error correction model in the "structural VAR" framework. The framework utilizes covariance restrictions, long-run multiplier restrictions, error correction coefficient restrictions, and restrictions on slope coefficients of the stimultaneous interactions in the "economic model". The framework is general enough to incorporate restrictions on impact multipliers. Two examples are provided. The first example is a dynamic M2 demand specification with a comparison to previous results that are constructed using restrictions on distributed lag coefficients to achieve identification. The second example illustrates the identification of equilibrium short-term and long-term interest responses of money demand using error-correction coefficient restrictions when the individual coefficients are underidentified in the cointegrating vectors in the presence of stationary interest rate spreads.

JEL Codes: C3,E4

Keywords: Identification, VAR Models, Vector Error Correction Models, Short-run Money Demand Functions, M2.

*The views expressed here are mine and do not necessarily reflect official positions of the Federal Reserve System or the Federal Reserve Bank of St. Louis. I wish to thank Richard G. Anderson and Hailong Qian for helpful comments.

Identification of Dynamic Economic Models from Reduced Form VECM Structures: An Application of Covariance Restrictions

Robert H. Rasche*

Sims (1980) publication of *Macroeconomics and Reality* provoked a revolution in applied macroeconomic modeling in particular and econometric time series analysis in general. Until that point in time, large-scale macroeconometric model construction for forecasting and policy analysis was a prominent academic activity in the United States and was exported widely to the rest of the world. Since that time, with a few notable exceptions, the type of macroeconometrics practiced in the 60s and 70s has disappeared as an academic activity in the US. It now survives in North America almost exclusively in private for-profit forecasting firms and in the research staffs of government agencies such as the Congressional Budget Office and the Board of Governors of the Federal Reserve System.[1] The VAR analysis as proposed by Sims, and variants developed since, have become predominant in the applied macroeconometrics literature.

Sims' (1980) principal criticism of large-scale macroeconometric modeling as it was practiced in the US in the late 60s and 70s was that "the style in which 'identification' is achieved for these models – is inappropriate, to the point at which claims for identification in these models cannot be taken seriously."[2] He argued that the

* Senior Vice President and Director of Research, Federal Reserve Bank of St. Louis. The views expressed here do not necessarily reflect official positions of the Federal Reserve System.

[1] In the United States, two remaining academic practitioners of the traditional type macroeconometric modeling trade are Ray Fair and John Taylor (1993) though their models are strongly influenced by the rational expectations hypothesis. Fair has made his forecasts and model available over the internet <http://fairmodel.econ.yale.edu>. Another ongoing macroeconometric modeling project with an academic home is the Michigan Model of the US economy <http://rsqe.econ.lsa.umich.edu/forecast/table.html>, though this venture has commercial aspects. Recently the staff of the Board of Governors of the Federal Reserve System has developed a new vintage of macroeconometric models that are used for internal analyses (Levin, Rogers and Tyron, 1997; Braydon, *et. al.,* 1997) The structure of these models appear to have more in common with the VAR tradition than with the earlier large-scale macroeconometric modeling practice represented by the MPS model.

[2] Sims (1980), p 1.

equation-by-equation approach to identification relying on exclusion restrictions on lagged values of endogenous variables was “incredible”.

Since the publication of Sims’ 1980 article, the literature on short-run demand for money functions has largely ignored the identification problem. (e.g. Rasche (1987), Hetzel (1989), Small and Porter (1989), Moore, Porter and Small (1990), Rasche (1990), Hallman, Porter and Small, (1991), Mehra (1991)). One analysis that does implicitly mention the identification problem is Mehra (1993). Mehra states:

> “The IV regression of (3) included only contemporaneous values of real income and opportunity cost and two lagged values of real M2 ($n_1 = n_2 = 0$ and $n_3 = 2$). The instruments are a constant, four lagged values of $\Delta rGNP_t$ and $\Delta(R\text{-}RM2)_t$, and a lagged value of $rM2_t$, $rGNP_t$, and $(R\text{-}RM2)_t$.”[3]

In other words, Mehra applies the identifying restrictions that Sims had decried 13 years earlier as “incredible”.

Sims proposed an alternative form for macroeconometric models, the vector autoregression (VAR). He certainly was well aware of the importance of the identification problem for inference from VAR structures, though his original description of his identification scheme appears to have left subsequent practitioners with less than a full understanding of the technique they were using:

> “The best descriptive device appears to be analysis of the system’s response to typical random shocks. Except for scaling, this is equivalent to tracing out the system’s moving average representation by matrix polynomial division. As well be seen below, the resulting system responses are fairly smooth, in contrast to the autoregressive lag structures, and tend to be subject to reasonable economic interpretation.
>
> The ‘typical shocks’ whose effects we are about to discuss are positive residuals on one standard deviation unit in each equation of the system. The residual in the money equation, for example, is sometimes referred to as the ‘money innovation’ since it is that component of money

[3] Mehra (1993), footnote 4, p. 457. RM2 is the log of real M2 balances; rGNP is the log of real GNP; RM2 is the log of the difference between the commerical paper rate and the “own” rate on M2.

which is 'new' in the sense of not being predicted from past values of variables in the system. The residuals are correlated across equations. In order to be able to see the distinct patterns of movement the system may display it is therefore useful to transform them to orthogonal form. There is no unique way to do this. What I have done is to triangularize the system with variables ordered as M, Y, U, W, P, PM. Thus the residuals whose effects are being tracked are the residuals from a system in which contemporaneous values of other variables enter the right-hand-sides of the regression with a triangular array of coefficients."[4]

Early in the development of the VAR literature there appears to be a widely held misconception that the new form of modeling made the identification problem obsolete. This misconception appears more recently in the vector error correction model (VECM) literature.

The current analysis focuses on the fundamental problem of identification that was central to Sims' criticism of the established practice and is shared by all of the modeling techniques subsequently developed. In section 1 the identifying restrictions implicit in standard VAR models are reviewed. In section 2 the identification of "common trends" or permanent shocks from reduced vector error correction models is presented in the standard structural VAR identification framework. In section 3 the structural VAR identification for permanent shocks is augmented with additional restrictions on error correction terms to identify transitory shocks (short-run specifications) from reduced form vector error correction models. This approach is illustrated with a comparison to the Mehra (1993) analysis of the demand for real M2.

1. Identification in VAR Models

The above quotation from Sims is the origin of the subsequent practice of "reordering and orthogonalizing" of VAR models.[5] The process described there can be

[4] Sims (1980), p.21

[5] In retrospect, there are two unfortunate aspects to this statement 1) the characterization of this procedure

described algebraically as follows: Let X_t be a p x 1 vector of data series and define the reduced form VAR data generating process for X_t as:

$$(1) \qquad X_t - \sum_{i=1}^{k} \Gamma_i X_{t-i} = [I_p - \Gamma(L)]X_t = \varepsilon_t,$$

where Γ(L) is a polynomial matrix in the lag operator L. Let W be a nonsingular pxp permutation matrix with the property that $W'W = I_p$. Then the "reordered" VAR can be written as:

$$(2) \qquad W[I_p - \Gamma(L)]X_t = [I_p - W\Gamma(L)W']WX_t = W\varepsilon_t = \varepsilon_t^*.$$

"Orthogonalize" the "reordered" VAR structure by decomposing the covariance matrix of the $\varepsilon_{t,}^*$ Σ_{ε^*} as $\Sigma_{\varepsilon^*} = TDT'$, where T is a pxp lower triangular matrix normalized to 1.0 on the principal diagonal and D is a diagonal matrix.[6] Then premultiply the "reordered" VAR structure by T^{-1} to get the identified "economic model"[7]:

$$(3) \qquad T^{-1}[I_p - W\Gamma(L)W']WX_t = T^{-1}\varepsilon_t^* = u_t.$$

Note that the covariance matrix of the identified u_t "economic shocks" is $T^{-1}\Sigma_{\varepsilon^*}(T^{-1})' = T^{-1}(TDT')(T^{-1})' = D$, so by construction these residuals are uncorrelated in the sample. This identification scheme in equation (3) is not new to the econometrics literature. It was proposed by Wold (1954) as a "causal chain" structure and was criticized intensely in the literature of the 1950s and early 60s (e.g. Basmann, 1963). This identification scheme was strongly defended as appropriate for economic structures in a series of articles by Wold (1954, 1960) and Strotz and Wold (1960). Nevertheless,

as a "descriptive device" and 2) that lack of any citations in the final sentence of the quotation to the established econometrics literature.

[6] Alternatively, assume $D=I_p$ and place no restrictions on the principal diagonal of T.

[7] T^{-1} is lower triangular since it is the inverse of a lower triangular matrix. The T matrix defines the

the approach was never accepted as a "credible" representation of an economic structure by mainstream econometricians in the 60s and 70s.[8]

Much is known about the economic model described by (3). First, the restrictions that the covariance matrix of the u_t is diagonal and that the T^{-1} matrix is triangular exactly identify the causal chain model. Second, the matrix $T^{-1}W$ that defines the economic model is not invariant to the choice of *W*. Third, the T^{-1} matrix of the "economic model" in (3) can be estimated consistently by single equation OLS, and this OLS estimator is the full information maximum likelihood (FIML) estimator of this system (e.g. Theil, 1971, pp. 460-3, 525). The Cholesky decomposition of the "reordered" covariance matrix is just an application of "indirect least squares" to the unrestricted reduced form estimates of an exactly identified model (e.g. Goldberger, 1964, pp. 526-28).

Identification in Structural VARs

The first modifications to the original causal chain VARs appeared in separate articles by Sims (1986) and Bernanke (1986). Let the VAR be defined as in (2) above and assume that the relationship between the shocks to the "economic model" and the reduced form shocks in the VAR are of the form:

(4) $Bu_t = A\varepsilon_t$, or $\varepsilon_t = A^{-1}Bu_t$.

The traditional VAR imposed the identifying restrictions that Σ_u is diagonal, $A = W$ and B is lower triangular. The "structural VAR" models that Sims proposed maintain the assumptions that Σ_u is diagonal, assume that $B = I_p$, and impose sufficient zero

"impact multipliers" of the "economic shocks".

[8] For example: "Identification based on restrictions on the disturbance distribution is attractive only when there exists sufficient knowledge of the process which generates the disturbances. This is usually not the

(exclusion restrictions) on the A matrix to exactly identify the "economic model". His various assumed restrictions do not satisfy a complete lower triangular pattern.[9] Note that this imposes identifying (or overidentifying) restrictions on the slope coefficients of the "economic model", consistent with the identification in traditional simultaneous equation models. This can be seen by substituting for ε_t in (1) from (4) to get:

(5) $$[I_p - \Gamma(L)]X_t = A^{-1}u_t .$$

Multiply (5) by A to get the "economic model":

(6) $$A[I_p - \Gamma(L)]X_t = u_t .$$

Hence the only difference between the identification in this type "structural VARs" and the practice in large-scale dynamic macromodels is that covariance restrictions are utilized for identification rather than restrictions on the lag structures of the model.

2. *Identification of Permanent Shocks in Vector Error Correction Models*

The King, Plosser, Stock and Watson (1991) "common trends" model identifies the permanent shocks in a VECM by assuming a block triangular structure of an economic model specified in terms of transformed data that are generated by (p-r) permanent and (r) transitory economic shocks, with the permanent shocks ordered in the first block in this structure. The permanent shocks are assumed to be uncorrelated with each other, and uncorrelated with all of the transitory shocks. Each of the permanent shocks is assumed to have a particular long-run impact on a specific element of the

---

case." Theil (1971), p. 494.

[9] Giannini (1992) defines this as the "K" class of models. Since there are only p(p+1)/2 independent elements in Σ_ε, and there are p parameters to be estimated in Σ_u, there are at most p(p-1)/2 free parameters that can be identified in A. Conditions for identification of the parameters of such models are discussed in Giannini (1992), Chapter 2.

VECM vector, in effect imposing overidentifying restrictions on the steady-state multipliers for each of the permanent shocks.[10]

Define the reduced form VECM as:

(7) $[I_p - \Gamma(L)]\Delta X_t = \alpha\beta' L X_t + \varepsilon_t$

and let the MA representation of the (p x 1) vector X_t be:

(8) $\Delta X_t = C(L)\varepsilon_t = [C(1) + (1-L)C^*(L)]\varepsilon_t$,

with Σ_ε as the covariance matrix of the reduced form error vector ε_t. By construction $C(0) = I_p$. Rank $C(1)$ =p-r and $C(1)\alpha = 0$.

Let W_1 be a (p-r) x p permutation matrix, such that $W_1 C(1)\varepsilon_t = Bu_t^P$ defines the (nonorthogonalized) permanent shocks. Choose W_1 such that rank $[W_1 C(1)]$= p-r. The independent rows of $C(1)$ selected by W_1 determine the elements of X_t to which the long-run effects of the elements of u_t^P are transmitted. In terms of Sims "structural VAR" approach $Bu_t = A\varepsilon_t$, $A = W_1 C(1)$. Assume that B is lower triangular. Transform the reduced form VECM as:

(9) $W_1 C(1)[I_p - \Gamma(l)]\Delta X_t = W_1 C(1)\alpha\beta' L X_t + W_1 C(1)\varepsilon_t$

so that

(10) $W_1 C(1)[I_p - \Gamma(L)]\Delta X_t = Bu_t^P$.

Note that the(p-r) equations in (10) are first differenced specifications so that the u_t^P shocks transmit permanent effects to the elements of X_t. Under the assumed structure, B can be estimated without identifying the transitory shocks.

---

[10] Rasche (1981, pp. 267-9) argues that the iterative estimation and simulation process used by large-scale macroeconometric modelers effectively placed restrictions on the long-run multipliers of those models.

Decompose the covariance matrix of the errors in (9):

$$[W_1C(1)]\Sigma_e[W_1C(1)]' = BDB'$$

with the restrictions that B is lower triangular with 1.0s on the principal diagonal and D is a (p-r)x(p-r) diagonal matrix. These restrictions are sufficient to exactly identify the orthogonalized permanent shocks, the B and D matricies as a Wold causal chain structure.

Premultiply the transformed VECM model (10) by the B^{-1} matrix to obtain the implied identified "economic model" of the permanent shocks as:

$$B^{-1}W_1C(1)[I_p - \Gamma(L)]\Delta X_t = u_t^P. \tag{11}$$

The B matrix defines the steady-state multipliers of the permanent shocks on the p-r elements of X_t selected by W_1. Premultiply (8) by $B^{-1}W_1$:

$$B^{-1}W_1\Delta X_t = [B^{-1}W_1C(1) + (1-L)B^{-1}W_1C^*(L)]\varepsilon_t = u_t^P + (1-L)B^{-1}W_1C^*(L)\varepsilon_t, \text{ so}$$

$$W_1\Delta X_t = Bu_t^P + (1-L)W_1C^*(L)\varepsilon_t, \text{ and}$$

$$W_1X_t = B\sum_{j=0}^{t} u_j^P + (1-L)W_1\sum_{j=0}^{t} C^*(t-j)\varepsilon_j$$

If B is diagonal then each element of u_t^P has a long-run impact on a unique element of X_t; i.e. the "common trends" model implies overidentifying restrictions. Such overidentifying restrictions can be tested.[11]

3. Identification of transitory shocks by Exclusion Restrictions on Cointegrating Vectors

---

The "common trends" structure applies such restrictions directly.

[11] e.g. Hoffman and Rasche (1996)

The "common trends" hypothesis places no restrictions on the remaining r equations in the model, so the transitory shocks remain underidentified. Boswijk (1995) suggests that individual equations with transitory shocks can be identified from a reduced form VECM by imposing exclusion restrictions on the matrix of error correction coefficients in the "economic model".[12] By appropriate transformation of the reduced form VECM this approach also can be implemented as an extension of the "common trends" model. Consider an alternative transformation of the residuals of the reduced form VECM of the form $Bu_t = A\varepsilon_t$:

where $B = \begin{bmatrix} T_{11} & 0 \\ B_{21} & I_r \end{bmatrix}$ and $A = \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2 (\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$.

Assume that T_{11} and T_{22} are lower triangular matricies normalized to unity on the principal diagonal. These restrictions and the restriction that $\Sigma_u = D$ is a diagonal matrix exactly identify the "economic model". Premultiply the reduced form VECM by $\begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2 (\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$ and substitute $Bu_t = A\varepsilon_t$:

$$\begin{aligned} \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2 (\alpha'\alpha)^{-1}\alpha' \end{bmatrix} [I_p - \Gamma(L)]\Delta X_t &= \begin{bmatrix} I_{p-r} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2 (\alpha'\alpha)^{-1}\alpha' \end{bmatrix} \alpha\beta' X_{t-1} \\ &\quad + \begin{bmatrix} T_{11} & 0 \\ B_{21} & I_r \end{bmatrix} \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix} \end{aligned} \tag{12}$$

[12] An alternative approach to imposing both short-run and long-run restrictions is used by Gali (1992). Gali's identification follows the structural VAR approach in that he estimates an unrestricted reduced form VAR and assumes that the 'economic shocks' are uncorrelated. In addition, he simultaneously imposes restrictions on long-run multipliers, impact multipliers and the contemporaneous interactions of the elements of the data vector. His restrictions define one permanent and three transitory shocks in his four variable system. These restrictions do not appear consistent with the conclusion of his unit root analysis that argues for two cointegrating vectors among four nonstationary variables.

Mellander, Vredin and Warne (1992), Hoffman and Rasche (1996) and Crowder, Hoffman and Rasche (1999) identify transitory shocks by imposing restrictions on the impact multiplier matrix.

Note that: $\begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)\alpha' \end{bmatrix} \alpha\beta' X_{t-1} = \begin{bmatrix} 0 \\ W_2 \end{bmatrix} \beta' X_{t-1}$. With this transformation of the model, the permutation matrix W_2 "reorders" the cointegrating vectors $\beta' X_{t-1}$ in the last r equations of the transformed model.

The "economic model" identified by the restrictions on this transformed reduced form model is:

$$(13) \qquad \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \left[\{I_p - \Gamma(L)\}\Delta X_t - \alpha\beta' L X_t \right] = \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix}$$

where $\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} T_{11}^{-1} & 0 \\ -B_{21}T_{11}^{-1} & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$.

In terms of the transformed data vector, $\begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)\alpha' \end{bmatrix} \Delta X_t$, the model has a Wold causal chain structure, so the FIML estimator of the model is just OLS applied to each of the equations. Only the first cointegrating vector selected by W_2 appears in the $(p-r+1)^{th}$ equation of the "economic model". Additional equations can be restricted to contain only a single cointegrating vector by applying the overidentifying restrictions that a submatrix of T_{22} is a diagonal matrix.

Note that the final r equations in (11) are normalized on individual cointegrating vectors. Conventional interpretations of such error correction equations normalize on an element of the appropriate row of the matrix of contemporaneous (simultaneous) interactions of the elements of ΔX_t. Define T_{22} as a lower triangular matrix *without* normalization of the elements on the principal diagonal. The economic model is then underidentified by r restrictions. The matrix of the contemporaneous interactions of the elements of ΔX_t in the equations of the "economic model" with the transitory shocks is:

$H_2 = [-B_{21}T_{11}^{-1} \quad T_{22}]\begin{bmatrix} W_1C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}$. Let $V = \begin{bmatrix} W_1C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}^{-1}$. Then $H_2V = \left[-B_{21}T_{11}^{-1} \quad T_{22}\right]$ defines r(r-1)/2 linear restrictions on the elements of H_2. The transitory shocks and the dynamic structure of the last r equations in the "economic model" can be exactly identified by imposing r additional linear restrictions (normalizations) of the form:

$$R * vec[H_2] = r_0 .$$

Again, multiple equations can be constrained to contain a single cointegrating vector by applying the overidentifying restrictions that a submatrix of T_{22} is a diagonal matrix. Alternately additional restrictions on contemporaneous interactions and/or impact multipliers could be specified.

Example: Demand for Real M2 Balances

Mehra (1993) estimates a single equation error correction model on quarterly data for the log of real M2 ($rM2_t$), the log of real GNP ($rGNP_t$), and the log of the spread between the commercial paper rate (R_t) and an estimated "own" interest rate on M2 ($RM2_t$). He uses both OLS and an IV estimator with lags of the regressors and the dependent variable as instruments. Mehra interprets that equation as a dynamic demand function for real M2. Prior to the estimation he concluded that real M2 and real GNP are nonstationary [I(1)], but that the interest rate spread is stationary [I(0)]. His equilibrium demand for real M2 (equation 1) can be written as:

(14) $rM2_t - \beta_1 - \beta_2 rGNP_t = -\beta_3(R_t - RM2_t) + u_t .$

With u_t a stationary disturbance, the left hand side of (12) must be stationary [I(0)]. Hence, he implicitly specifies a cointegrating vector: $\left(1.0 \quad -\beta_2\right)$ between $rM2_t$ and Y_t.

From the Granger Representation Theorem, (Johansen, (1991)) there exists a reduced form VECM that describes the data generating process for Mehra's specification. The question is how a short-run demand function for real balances, comparable to Mehra's specification, can be identified from the reduced form VECM. Let $X_t' = (rM2_t, R_t - RM2_t, rGNP_t)$. The reduced form VECM is:

(15) $[I_p - \Gamma(L)]\Delta X_t - \alpha\beta' L X_t = \varepsilon_t$

where $\beta' = \begin{bmatrix} 1.0 & 0 & -\beta_2 \\ 0 & 1.0 & 0 \end{bmatrix}$. Here r = rank($\beta$) = 2 so there is (p-r) = 1 permanent shock implied by his specification. Let the selection vector for the permanent shock be W_1 = (0, 0, 1.0) so that the single common trend is identified as having a unitary long-run multiplier on real output and specify the selection matrix for the cointegrating vectors as $W_2 = \begin{bmatrix} 0 & 1.0 \\ 1.0 & 0 \end{bmatrix}$. Let $B_{21} = \begin{bmatrix} b_{21} \\ b_{22} \end{bmatrix}$ and $T_{22} = \begin{bmatrix} t_{11} & 0 \\ t_{21} & t_{22} \end{bmatrix}$. Since p-r = 1, $T_{11} = 1.0$.

Then the "economic model" is:[13]

$$\begin{bmatrix} 1.0 & 0 \\ -B_{21} & T_{22} \end{bmatrix} \begin{bmatrix} W_1 C(1) \\ H_2 \end{bmatrix} [I_p - \Gamma(L)]\Delta X_t - \begin{bmatrix} 0 & 0 \\ 0 & t_{11} \\ t_{22} & t_{21} \end{bmatrix} \beta' L X_t = \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix}$$

Only the second cointegrating vector appears in second equation in the identified "economic model", and both cointegrating vectors appear in the third equation of the

[13] The identifying restrictions that define the permanent shock here are identical to those used in Gali (1992) to identify his supply shock. Casual examination of this structure suggests that the coefficients on the contemporaneous values of ΔX_t are sensitive to the choice of W_1. In this model this is not true. Since the rank[$C(1)$] = 1, the rows of $C(1)$ differ only by a scale factor.

identified "economic model". Both equations require a normalizing restriction. When the second equation is normalized on $(R_t - RM2_t)$ it specifies the data generating process for the stationary interest rate spread. The third equation has the same form as Mehra's equation (2). When this equation is normalized on $rM2_t$ it specifies the short-run money demand function. For this model, let $\begin{bmatrix} W_1 C(1) \\ W_2(\alpha'\alpha)^{-1}\alpha' \end{bmatrix}^{-1} = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{bmatrix}$. Then

$$H_2 V = \begin{bmatrix} h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} V = \begin{bmatrix} -b_{21} & t_{11} & 0 \\ -b_{22} & t_{21} & t_{22} \end{bmatrix}, \text{ so } h_{21}v_{13} + h_{22}v_{23} + h_{23}v_{33} = 0.0 .$$

To normalize the second equation on the contemporaneous change in the opportunity cost variable requires the linear restriction $h_{22} = 1.0$. To normalize the third equation on the contemporaneous change in the log of real balances requires the linear restriction $h_{31} = 1.0$. These two normalizations, in addition to the restriction stated previously, exactly identify the dynamic money demand function.

Estimation

The model can be estimated in several ways. OLS can be applied to the transformed data to obtain the FIML estimates of the parameters of the specification normalized on the cointegrating vectors. The estimated coefficients of the third equation can be renormalized for the contemporaneous change in real balances. Alternatively, the same estimates can be obtained directly using either FIML estimation, or as an application of instrumental variables.[14] To estimate the model by IV, first note that u_t^P is assumed to be orthogonal to u_t^T and appears prior to u_t^T in a Wold causal chain structure.

[14] For an interpretation of FIML as an instrumental variable estimator in the presence of covariance

Thus u_t^P can be estimated prior to u_t^T and the elements of this vector are valid instruments for both the equations with the transitory shocks. The regression specification for the second equation (the first transitory shock) in the model can be determined by solving $h_{21}v_{13} + v_{23} + h_{23}v_{33} = 1.0$ for h_{23}. Substitute this expression into the second equation to get:

(16) $$(\Delta X_{2t} - \frac{v_{23}}{v_{33}}\Delta X_{3t}) + h_{21}(\Delta X_{1t} - \frac{v_{13}}{v_{33}}\Delta X_{3t}) = \Pi_2(L)\Delta X_t + t_{22}\beta_2' LX_t + u_{1t}^T$$

Estimation of this equation requires only a single instrument.

The residual vector from the estimation of u_{1t}^T is assumed orthogonal to u_{2t}^T and hence is a valid instrument for the estimation of the third equation in the system:

(17) $$\left[1.0, -\beta_{opp\cos t}, -\beta_{rgnp}\right]\Delta X_t = \Pi_3(L)\Delta X_t + \phi\beta' LX_1 + u_{2t}^T$$

where $\beta_{opp\cos t}$ and β_{rgnp} are the short-run elasticities of the demand for real M2 with respect to the opportunity cost and real GNP respectively. $\Pi_3(L) = \left[1.0, -\beta_{opp\cos t}, -\beta_{rgnp}\right]\Gamma(L)$ and $\phi = \left[1.0, -\beta_{opp\cos t}, -\beta_{rgnp}\right]\alpha$

Comparison with Results from Mehra's Identification Assumptions

Mehra is not specific about the sources of the data that he used.[15] Since his sample period ends with 91:2, it is assumed here that the vintage of the data is late 1991, and series for real GNP and the GNP deflator vintage November 1991 from the Federal Reserve Bank of Philadelphia real time data bank have been utilized. M2 data from 1959 through 91:2 are also taken from the Federal Reserve Bank of Philadelphia real time data bank, vintage November 1991. Mehra cites Hetzel (1989). Table AV of Hetzel's paper

restrictions see Hausman and Taylor (1983).

[15] The article indicates that the data are available at the *JMCB* editorial office. This is no longer the case.

contains a quarterly data series for the own rate on M2 from 1946:1 through 1989:2. This series is supplemented with eight observations from 89:3 through 91:2 that reflect current measurement of the own rate on M2.[16] Hetzel also indicated that he used data from Friedman and Schwartz (1970) on M4 to measure the post-1980 concept of M2 for the years prior to 1959 (p. 25-6).[17] Finally, data on the commercial paper rate is from *Banking and Monetary Statistics, 1941-70* and various issues of the *Annual Statistical Digest*. The published estimates from Mehra (1993) Table 1 and estimates from the reconstructed data set are shown in Table 1. It is apparent that while the original data set has not been replicated exactly, the reestimations do not differ in any significant respect from the published numbers.

In Table 2 the Mehra specification has been modified to impose a common lag length of one on all of the variables. In the IV estimation observations at lags 2-4 of changes in real M2, real GNP and the opportunity cost variable in addition to the constant, the three dummy variables and the lagged levels of real M2, real GNP and the opportunity cost variable have been used as instruments. The purpose of this is respecification is to facilitate comparisons with the VAR based estimates. A comparison of the IV estimates in Tables 1 and 2 reveals that the differences between the OLS and IV estimates result from the exclusion of the lagged observations on $\Delta rGNP$ and $\Delta(R - RM2)$ from the IV specification. When these lags are included (Table 2) then the estimated coefficients from the IV regression are almost the same as those from the OLS regression. Further, when a uniform lag length is established for all variables the

[16] We checked the values of the own rate on M2 as currently measured for 88:1-89:2 against those in the Hetzel table. The largest discrepancies are no more than several basis points.

[17] Hetzel apparently did not chain the Friedman and Schwartz M4 series to the post-1980 M2 series at 59:1. The two series have been concatenated without chaining in this analysis.

estimated coefficient on the velocity error correction term becomes insignificant in both the OLS and IV regressions.

With the exception of the estimated distributed lag coefficients on $\Delta rGNP$, the same similarities exist between the two estimators when the lag length is extended to two on all variables (Table 3). This suggests that the goodness of fit is very high in the first stage regressions that use the lagged changes as instruments. When a uniform lag length of two is set on all three variables and lags of three and four periods of $\Delta rGNP$ and $\Delta(R - RM2)$ are used as instruments, the sign of the coefficient on the velocity error correction term is reversed.

The results of the estimations using the structural VAR identifying restrictions are considerably different, both statistically and economically. This can be seen by comparing the third column of Tables 2 and 3 with either the first or second columns of those tables. First, the estimated error correction term on M2 velocity is roughly an order of magnitude bigger using the IV estimator with covariance restrictions than the estimate of this term with either OLS or IV with lagged change instruments. Second, the error correction coefficient on the opportunity cost variable is not significantly different from zero using the covariance IV estimator. This implies that the elasticity of the long-run demand for real M2 with respect to the opportunity cost variable is not significantly different from zero. Third, the covariance IV estimates of the elasticities of the demand for real M2 with respect to contemporaneous real GNP are substantially greater than unity. In contrast to the estimates from IV with lagged change instruments that range from 0.15 to 0.35 in the examples in Tables 1 and 2. Finally a comparison of the estimates in Tables 2 and 3 suggests that the results from covariance IV estimator do not appear to be very sensitive to the choice the lag length in the models. In contrast, when the lag length is increased from one to two, estimates of critical coefficients in the equation change sign using the IV estimator with lagged change instruments.

One method of evaluating the marginal effect of the alternative choice of instruments is to construct yet a third IV estimate that combines the two sets of instruments. If the additional lagged variables are to be valid instruments, then they must appear in at least one of the other equations in the data generating process (DGP). Hence they should appear in the reduced form DGP. For purposes of using both lagged variables and covariance restrictions as instruments, the reduced form VECM is extended to four lagged changes in all three variables. When estimating the short-run demand for real M2 equation, all lag distributions are truncated at lag = 2. Thus the equation is overidentified. The comparison of the three estimations is shown in Table 4. The results with the combined sets of instruments resemble closely those with the covariance instruments. The impact income elasticity is large (indeed it is almost identical to the long-run elasticity) and the impact elasticity of the opportunity cost variable is small in absolute value, but significantly different from zero. Finally, the long-run opportunity cost elasticity in these regressions is also not significantly different from zero.

These results suggest that on the margin, the omitted variables are not particularly useful instruments. This can be examined by considering the marginal contribution of the additional lags to the reduced form VECM. The computed Chi-squared statistic for the exclusion of the third and fourth lagged changes of all three variables from all three reduced form equations is 25.79, which with 3 x 2 x 3 = 18 degrees of freedom has a p value of .105. Thus the hypothesis that all of the additional lagged changes can be excluded from the reduced form is not rejected. One possible inference from this is that none of the additional lagged changes enter significantly into any equation of the "economic model". This hypothesis is consistent with the weak marginal contribution of these variables to the instrument list.

It is interesting to consider the implied rational polynomial distributed lag structure for the dynamic demand for real M2 implied by the various estimators. The estimated distributed lag coefficients from three estimations are plotted in Figure 1.[18] The estimated distributed lag coefficients for both right hand side variables in the short-run demand for real M2 are almost identical for the two sets of IV estimates. In contrast, the OLS estimates are considerably different. In the case of real GNP, the OLS coefficients start our relatively small and reach a peak at a two quarter lag. The IV coefficients start large and decline monotonically, and become slightly negative after two quarters. The opportunity cost coefficients start out essentially the same for all three estimates, but they approach zero much more rapidly with the IV estimates compared to the OLS estimates. The IV coefficients also overshoot zero, while the OLS coefficients go monotonically to zero after one lag.

Additional dynamic analysis can be constructed to determine the reaction of the three variables in the system to the permanent (supply) shock, the transitory opportunity cost shock, and the transitory shock to real balances. The impulse response functions for the three variables in the model plus M2 velocity with respect to the three shocks are plotted in Figures 2-4. Each graph shows the response function for three specifications of the dynamic structure of the reduced form VECM. For the most part there is little

[18] Mehra's IV estimation in Table 2, column 2 is not shown because the lag structure is essentially the same as that of OLS. The IV estimates with lagged instruments in Table 3 are not shown since the estimated AR

difference between the estimated response functions for the structure with two lagged differences and the structure with three lagged differences. In some cases the estimated response functions from the structure with only a single lagged change is substantially different. This appears to result from an inadequate parameterization of the dynamic structure of the data generating process.

The response of real output to the permanent (supply) shock is characteristic of results in the literature. The response starts out small (in this case much closer to zero than has been found in other studies), and builds to the steady-state response over a span of two to four years. The response of real M2 is similar to that of real output, with the possibility of some "overshooting" of the steady-state response. The short-run effect of this shock on the opportunity cost variable is strongly negative, but dies out over a horizon of approximately two years. Finally, since the short-run effect of this shock on real M2 is larger in absolute value than the short-run impact on real output, the transitory response of M2 velocity to the "supply" shock is negative.

The impact effect of the opportunity cost shock on opportunity cost is large and positive and dies out quite slowly. A close examination of the scale in the response functions for the other variables in Figure 3 reveals that the opportunity cost shock has little effect on any of these variables.

The responses to the shock to the demand for real balances are shown in Figure 4. The response of real M2 starts out positive, builds to a peak after about five quarters and then dies out very slowly. The initial response of real output is negative, but smaller in absolute value than that of real M2. The response of real output overshoots zero slightly after about two years, then gradually dies out. The net effect of these two responses is a

---

polynomial for this equation has a explosive root and is not invertible.

strong negative response of M2 velocity that dies out after three to four years. Since the elasticity of the demand for real balances is very close to zero, the "LM curve" implied by these estimates is very close to vertical. The responses of real M2 and real output are consistent with a shift to the left of such a "LM curve" in response to a positive transitory shock to the demand for real balances (or equivalently a negative transitory shock to the supply of real balances). The impact effect on the opportunity cost variable is almost zero in the specifications with two and three lagged changes. The subsequent short-run effect on this variable is quite negative. It does not seem reasonable to presume that the own rate on M2 responds more strongly or more quickly than the commercial paper rate to this type of shock. Hence this pattern does not seem consistent with the prediction from a shift to the left of an "LM curve" along a negatively sloped "IS curve". However, it is possible that the response of the opportunity cost variable to this shock is measured very imprecisely. In any event, there is no evidence here of transitory "liquidity effects" on the spread of market rates over the own rate on M2.

4. Conclusions

This analysis has demonstrated a straightforward framework for identifying transitory shocks through exclusion restrictions on error correction coefficients in a vector error correction model. The identifying restrictions are from the Bernanke-Sims "structural VAR" class. An example of the identification of a short-run demand function for M2 is presented. These results are compared the results from identification using traditional exclusion restrictions on lagged variables. The conclusion from this analysis is that the choice of identifying restrictions can be an important factor in the economic interpretation of the estimates of an 'economic model'. In the example shown here, Sims' criticism of the use of exclusion restrictions on distributed lag structures to achieve identification appears justified.

Table 1

Alternative Estimations of Mehra Specification 1953:1 – 1991:2
Original Estimates and Reestimated Coefficients

	Original Estimates		Reestimated Coefficients	
Regressor	OLS	IV Lagged Inst.[a]	OLS	IV Lagged Inst.
Constant	-0.161 (-1.3)	-0.452 (-2.6)	-0.163 (-1.3)	-0.416 (-2.5)
$rM2_{t-1}$	-0.033 (-1.4)	-0.087 (-2.6)	-0.034 (-1.5)	-0.080 (-2.5)
$rGNP_{t-1}$	0.033 (1.4)	0.082 (2.6)	0.033 (1.4)	0.082 (2.5)
$(R-RM2)_{t-1}$	-0.004 (-2.3)	-0.007 (-4.2)	-0.004 (-2.4)	-0.007 (-4.3)
$\Delta rGNP_t$	0.103 (1.8)	0.39 (1.9)	0.095 (1.7)	0.339 (1.7)
$\Delta rGNP_{t-1}$	0.130 (2.2)	na	0.110 (1.9)	na
$\Delta rM2_{t-1}$	0.342 (4.9)	0.407 (5.1)	0.376 (5.3)	0.438 (5.6)
$\Delta rM2_{t-2}$	0.120 (1.7)	-0.007 (-0.1)	0.101 (1.4)	-0.022 (-0.3)
$\Delta(R-RM2)_t$	-0.011 (-5.6)	-0.017 (-4.3)	-0.011 (-5.5)	-0.017 (-4.2)
$\Delta(R-RM2)_{t-1}$	-0.011 (-4.8)	na	-0.010 (-4.5)	na
CC_1	-0.013 (-2.2)	-0.009 (-1.2)	-0.014 (-2.4)	-0.011 (-1.5)
CC_2	0.011 (1.8)	0.011 (1.7)	0.011 (1.9)	0.012 (1.9)
$D83Q1$	0.026 (4.7)	0.027 (4.2)	0.026 (4.7)	0.027 (4.3)
ser	0.00554	0.00641	0.0055	0.0062
F	0.13	0.95	0.07	0.98

[a] Instruments are the lagged (log) levels of real M2, real GNP and the opportunity cost measure; 1 to 4 lagged differences in the logs of real GNP and the opportunity cost measure, 1 to 2 lags on the difference in the log of real M2, the constant and the three dummy variables.

Table 2
Alternative Estimations of Mehra Specification 1953:1 – 1991:2
Lag Length = 1

Regressor	OLS	IV Lagged Inst.[a]	*IV* Covariance[b]
Constant	-0.135 (-1.1)	-0.162 (-0.9)	-1.170 (1.3)
$rM2_{t-1}$	-0.028 (-1.2)	-0.033 (-1.0)	-.221 (-3.0)
$rGNP_{t-1}$	0.028 (1.2)	0.033 (0.9)	.226 (3.0)
$(R-RM2)_{t-1}$	-0.004 (-2.8)	-0.004 (-2.2)	-.005 (-1.1)
$\Delta rGNP_t$	0.101 (1.8)	0.152 (0.6)	1.687 (4.4)
$\Delta rGNP_{t-1}$	0.120 (2.1)	0.104 (1.0)	-.086 (-0.5)
$\Delta rM2_{t-1}$	0.415 (6.4)	0.410 (6.0)	.259 (1.5)
$\Delta(R-RM2)_t$	-0.011 (-5.8)	-0.011 (-2.0)	-.033 (-4.6)
$\Delta(R-RM2)_{t-1}$	-0.010 (-4.3)	-0.009 (-3.8)	-.004 (-0.6)
CC_1	-0.014 (-2.4)	-0.013 (-1.5)	.010 (0.6)
CC_2	0.012 (2.1)	0.012 (1.8)	.002 (0.1)
$D83Q1$	0.026 (4.7)	0.027 (4.7)	.020 (1.3)
ser	0.0055	0.0055	.0146
F	0.08	0.05	na

[a] Instruments are the lagged (log) levels of real M2, real GNP and the opportunity cost measure; 1 to 4 lagged differences in the logs of real GNP and the opportunity cost measure, 1 to 2 lags on the difference in the log of real M2, the constant and the three dummy variables.
[b] From the reduced form VECM estimated using Johansen's (1991) FIML estimator,

$$\beta'=\begin{bmatrix}1.0 & 0 & -1.022\\ 0 & 1.0 & 0\end{bmatrix},\ \alpha'=\begin{bmatrix}-0.011 & -0.809 & 0.109\\ -0.003 & -0.121 & -0.002\end{bmatrix},\text{ and } C(1)=\begin{bmatrix}1.387 & -0.037 & -.0130\\ 0.0 & 0.0 & 0.0\\ 1.358 & -0.036 & -0.127\end{bmatrix}.$$

Table 3
Alternative Estimations of Mehra Specification 1953:1 – 1991:2
Lag Length = 2

Regressor	OLS	IV Lagged Inst.[a]	IV Covariance/ FIML[b]
Constant	-0.116 (-0.9)	0.153 (0.5)	-0.877 (-2.7)
$rM2_{t-1}$	-0.025 (-1.1)	0.026 (0.5)	-0.168 (-2.8)
$rGNP_{t-1}$	0.024 (1.0)	-0.028 (-0.5)	0.170 (2.8)
$(R-RM2)_{t-1}$	-0.003 (-1.9)	-0.005 (-1.6)	-0.002 (-0.5)
$\Delta rGNP_t$	0.085 (1.6)	-0.368 (-0.9)	1.402 (4.9)
$\Delta rGNP_{t-1}$	0.095 (1.6)	0.350 (1.3)	-0.205 (-1.4)
$\Delta rGNP_{t-2}$	0.173 (3.1)	0.241 (2.3)	0.077 (0.6)
$\Delta rM2_{t-1}$	0.340 (4.5)	0.328 (2.9)	0.263 (1.5)
$\Delta rM2_{t-2}$	0.083 (1.2)	0.037 (0.3)	0.014 (0.1)
$\Delta(R-RM2)_t$	-0.012 (-5.7)	-0.026 (-1.4)	-0.012 (-2.6)
$\Delta(R-RM2)_{t-1}$	-0.013 (-5.4)	-0.011 (-2.3)	-0.009 (-1.7)
$\Delta(R-RM2)_{t-2}$	-0.002 (-1.1)	-0.008 (-1.1)	-0.001 (-0.2)
CC_1	-0.013 (-2.3)	-0.028 (-1.7)	0.014 (1.0)
CC_2	0.008 (1.4)	0.016 (1.4)	-0.002 (-0.2)
$D83Q1$	0.028 (5.1)	0.028 (3.5)	0.023 (1.9)
ser	0.0053	0.0075	.0122
F	0.38	0.44	na

[a] Instruments are the lagged (log) levels of real M2, real GNP and the opportunity cost measure; 1 to 4 lagged differences in the logs of these three variables, the constant and the three dummy variables.
[b] From the reduced form VECM estimated using Johansen's (1991) FIML estimator,

$$\beta'=\begin{bmatrix}1.0 & 0 & -1.013\\ 0 & 1.0 & 0\end{bmatrix},\ \alpha'=\begin{bmatrix}-0.027 & 0.312 & 0.103\\ -0.003 & -0.084 & -0.002\end{bmatrix}\text{and}\ C(1)=\begin{bmatrix}1.303 & -0.0518 & 0.495\\ 0.0 & 0.0 & 0.0\\ 1.274 & -0.051 & .484\end{bmatrix}.$$

Table 4
Alternative Estimations of Mehra Specification 1953:1 – 1991:2
Lag Length = 2

Regressor	IV Covariance[a]	IV Combined Instrument sets
Constant	-0.877 (-2.7)	-0.699 (-3.1)
$rM2_{t-1}$	-0.168 (-2.8)	-0.134 (-3.1)
$rGNP_{t-1}$	0.170 (2.8)	0.134 (3.9)
$(R-RM2)_{t-1}$	-0.002 (-0.5)	-0.002 (-0.8)
$\Delta rGNP_t$	1.402 (4.9)	1.073 (5.34)
$\Delta rGNP_{t-1}$	-0.205 (-1.4)	-0.131 (-1.2)
$\Delta rGNP_{t-2}$	0.077 (0.6)	0.100 (1.0)
$\Delta rM2_{t-1}$	0.263 (1.5)	0.283 (2.0)
$\Delta rM2_{t-2}$	0.014 (0.1)	0.031 (0.2)
$\Delta(R-RM2)_t$	-0.012 (-2.6)	-0.012 (-3.2)
$\Delta(R-RM2)_{t-1}$	-0.009 (-1.7)	-0.010 (-2.4)
$\Delta(R-RM2)_{t-2}$	-0.001 (-0.2)	-0.001 (-0.3)
CC_1	0.014 (1.0)	0.007 (0.7)
CC_2	-0.002 (-0.2)	0.001 (.04)
$D83Q1$	0.023 (1.9)	0.024 (2.4)
ser	.0122	.0098
F	na	na

[a] From the reduced form VECM estimated using Johansen's (1991) FIML estimator,

$$\beta' = \begin{bmatrix} 1.0 & 0 & -1.009 \\ 0 & 1.0 & 0 \end{bmatrix},\ \alpha' = \begin{bmatrix} -0.027 & 0.312 & 0.103 \\ -0.003 & -0.084 & -0.002 \end{bmatrix} \text{and } C(1) = \begin{bmatrix} 1.303 & -0.0518 & 0.495 \\ 0.0 & 0.0 & 0.0 \\ 1.274 & -0.051 & .484 \end{bmatrix}.$$

Bibliography

Basmann, R.L. (1963), "The Causal Interpretation of Non-Triangular Systems of Economic Relations," *Econometrica*, 31: 439-448.

Bernanke, B. (1986), "Alternative Explanations of the Money-Income Correlation," *Carnegie-Rochester Conference Series on Public Policy,* 25:49-99.

Boswijk, H.P. (1995), "Efficient Inference on Cointegration Parameters in Structural Error Correction Models," *Journal of Econometrics*, 69:133-158.

Braydon, F., E. Mauskopf, D. Reifschneider, P. Tinsley and J. Williams (1997), "The Role of Expectations in the FRB/US Macroeconomic Model," *Federal Reserve Bulletin*, April, 83:227-245.

Crowder, W.J., D.L. Hoffman and R.H. Rasche, "Identification, Long-Run Relations, and Fundamental Innovations in a Simple Cointegrated System," *Review of Economics and Statistics*, 81:109-121.

Friedman, M. and A.J. Schwartz (1970), *Monetary Statistics of the United States*, New York: Columbia University Press.

Giannini, C. (1992), *Topics in Structural VAR Econometrics,* Berlin: Springer-Verlag.

Gali, J. (1992), "How well does the IS/LM Model fit Postwar U.S. Data?" *Quarterly Journal of Economics*, 709-738.

Goldberger, A. S. (1964), *Econometric Theory*, New York: John Wiley & Sons, Inc.

Hallman J.J., R.D. Porter and D.H. Small (1991), "Is the Price Level Tied to the M2 Monetary Aggregate in the Long Run?" *American Economic Review*, 81:841-858.

Hausman, J.A. and W.E. Taylor (1983), "Identification in Linear Simultaneous Equation Models with Covariance Restrictions: An Instrumental Variables Interpretation," *Econometrica*, 51:1527-1550.

Hetzel, R.L. (1989) "M2 and Monetary Policy," Federal Reserve Bank of Richmond *Economic Review,* September/October, 14-29.

Hoffman, D.L. and R.H. Rasche (1996), *Aggregate Money Demand Functions,* Boston: Kluwer Academic Publishers.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59:1551-1580.

King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 81:819-840.

Levin, A., J. Rogers, and R. Tyron (1997), "Evaluating International Economic Policy with the Federal Reserve's Global Model," *Federal Reserve Bulletin*, October, 83:797-817.

Mehra, Y.P. (1991), "An Error-Correction Model of U.S. M2 Demand," Federal Reserve Bank of Richmond *Economic Review*, May/June, 3-12.

Mehra, Y.P. (1993), "The Stability of the M2 Demand Function: Evidence from an Error Correction Model," *Journal of Money, Credit and Banking*, 25:455-460.

Mellander, E., A.Vredin and A. Warne, "Stochastic Trends and Economic Fluctuations in a Small Open Economy," *Journal of Applied Econometrics*, 7:369-394.

Moore, G.R., R.D. Porter and D.H. Small (1990), "Modeling the Disaggregated Demands for M2 and M1: The U.S. Experience in the 1980s," *Financial Sectors in Open Economics: Empirical Analysis and Policy Issues,* Washington DC: Board of Governors of the Federal Reserve System, 113-161.

Rasche, R.H. (1981), "Comments on the Size of Macroeconomic Models," in J. Kmenta and J.B. Ramsey (eds.), *Large-Scale Macro-econometric Models,* Amsterdam: North-Holland Publishing Company.

Rasche, R.H. (1987), "M1 Velocity and Interest Rates: Do Stable Relationships Exist?" *Carnegie-Rochester Conference Series on Public Policy*, 27:9-88.

Rasche, R.H. (1990), "Demand Functions for Measures of U.S. Money and Debt," *Financial Sectors in Open Economics: Empirical Analysis and Policy Issues,* Washington DC: Board of Governors of the Federal Reserve System, 113-161.

Sims, C.A. (1980), "Macroeconomics and Reality," *Econometrica*, 48:1-48.

Sims, C.A. (1986), "Are Forecasting Models Usable for Policy Analysis?" Federal Reserve Bank of Minneapolis *Quarterly Review*, Winter, 2-16.

Small, D.H. and R.D. Porter (1989), "Understanding the Behavior of M2 and V2," *Federal Reserve Bulletin*, April, 75:244-254.

Strotz, R.H. and H.O.A. Wold (1960), "Recursive vs Nonrecursive Systems", *Econometrica*, 28:417-427.

Taylor, J.B. (1993), *Macroeconomic Policy in a World Economy*, New York: W.W. Norton and Co.

Theil, H. (1971), *Principles of Econometrics*, New York: John Wiley and Sons.

Wold, H.O.A. (1954), “Causality and Econometrics,” *Econometrica*, 22:162-177.

Wold, H.O.A. (1960), “A Generalization of Causal Chain Models,” *Econometrica*, 28:443-462.

Figure 1

Alternative Estimators

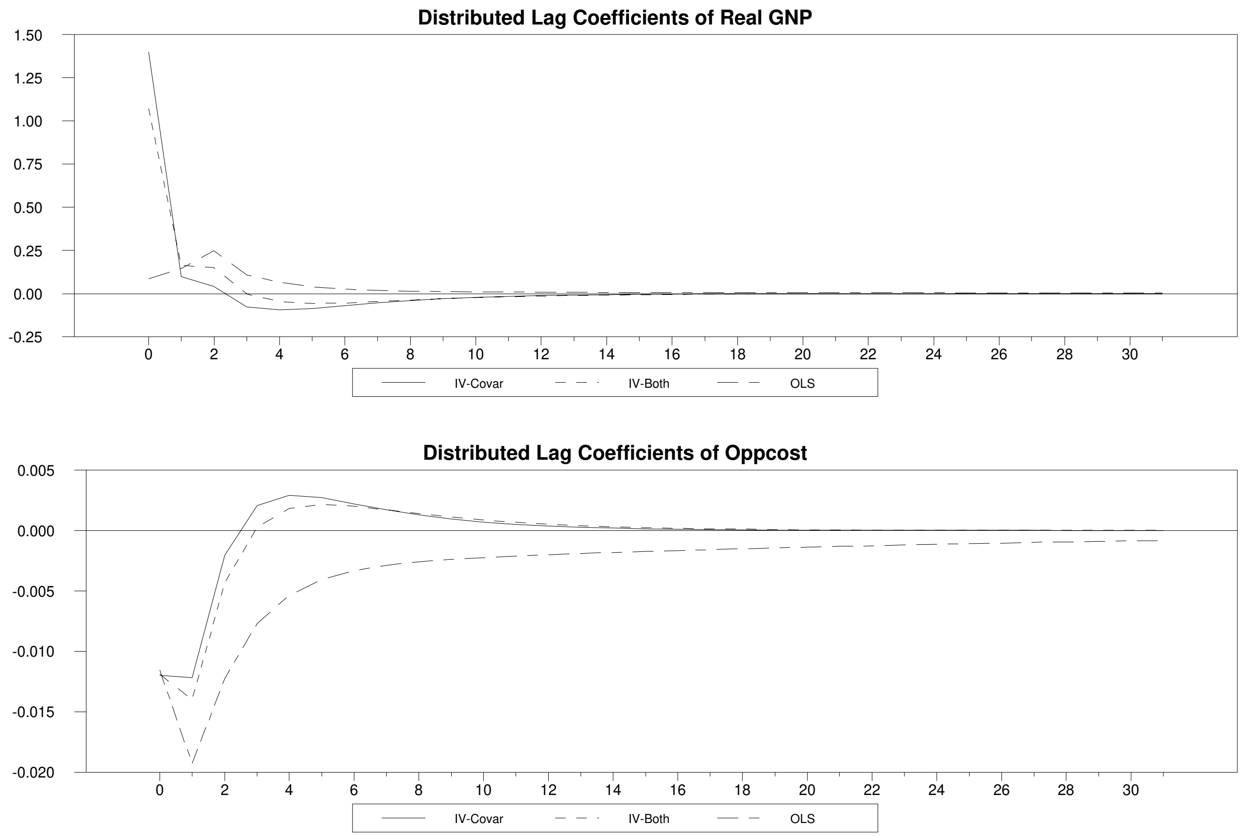

Figure 2
Impulse Response Functions

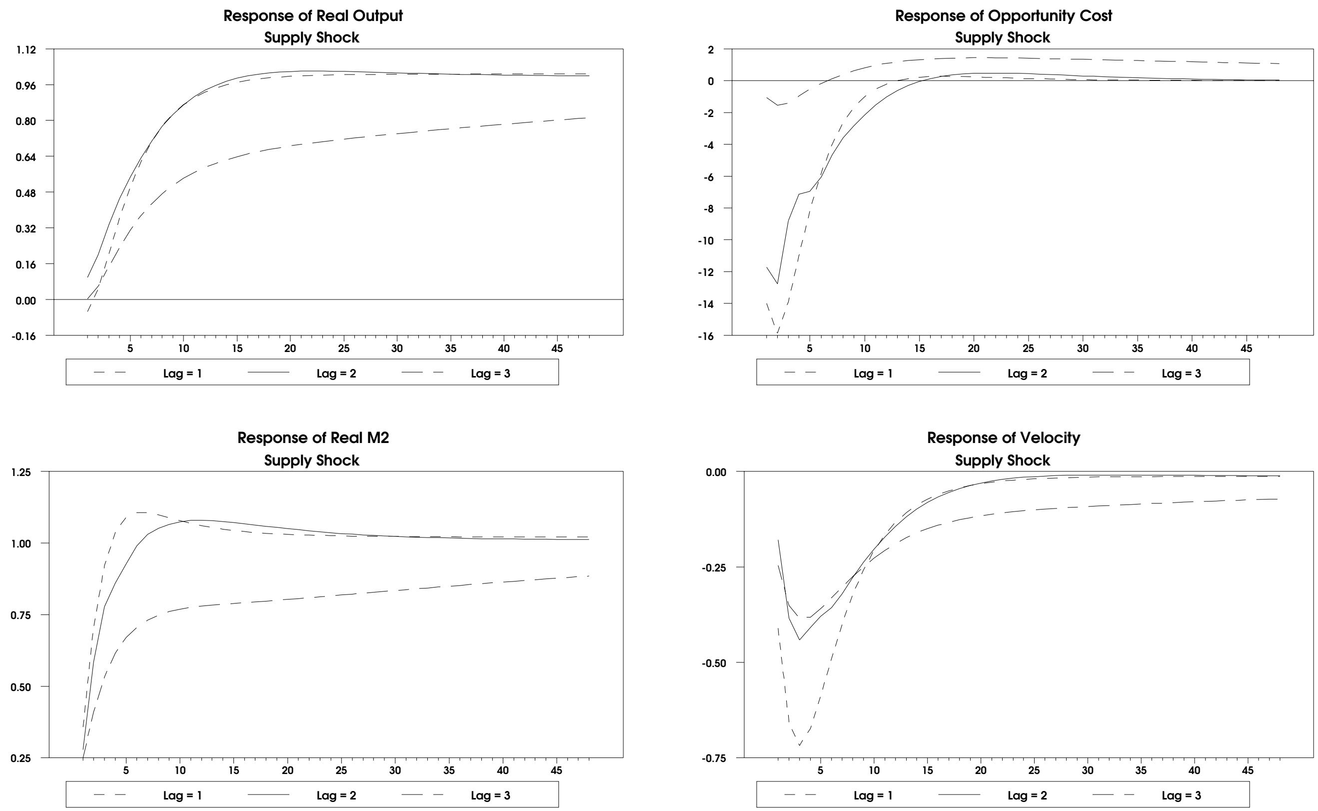

Figure 3
Impulse Response Functions

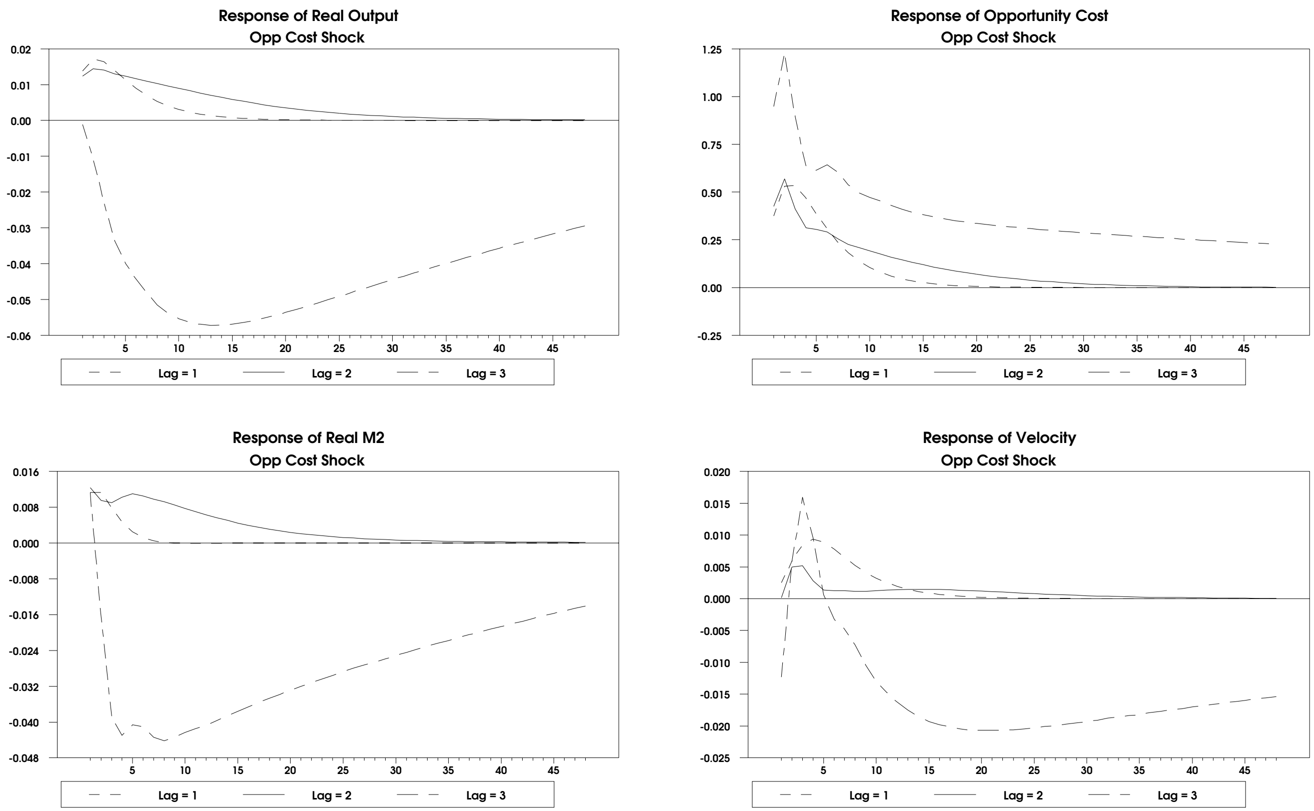

Figure 4
Impulse Response Functions