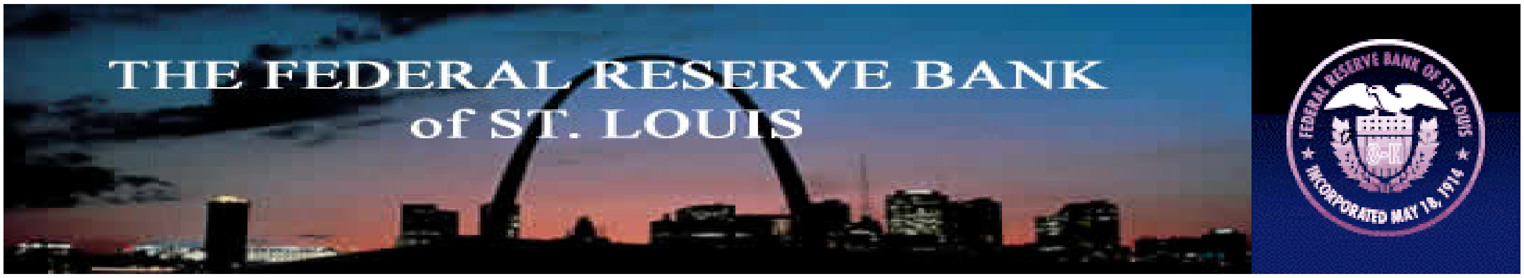

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Committing and Reneging: A Dynamic Model of Policy Regimes.

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Abstract

Actual policy decisions are made in real time and are not irrevocable. These observations are mundane, but most policy modeling has neglected them. We show that when policy is made in an environment of uncertainty, costs of switching policies give the option to wait positive value. This insight has several implications: First, the option to wait itself makes the incumbent regime relatively more attractive (compared to the traditional once-and-for-all analysis). Second, the option to wait means that increased uncertainty makes the incumbent regime more attractive. Third, because the commitment decision takes place in real time, policy choice displays hysteresis.

Economists interested in policy have long focused on two kinds of questions. The first, "Should we have this policy?" is the normative counterpart of the second, "Why do we have this policy rather than that policy?" Both are static questions. Despite the importance of dynamic modeling in recent years, our analysis has rarely asked the dynamic analogues of these questions: "*When* should we commit to this policy?" and "When do we switch to a *different* policy?" In this paper we propose a simple framework for analyzing these dynamic policy choice problems. Our analysis is limited to binary choices, but many examples fall under this rubric: Should Country A sign an international trade treaty now? What determines Country B's decisions first to commit to then, later, to dismantle a currency board system? Should Country C discontinue bank deposit insurance this year? Often policy choices that are, strictly speaking, more continuous are formulated in discrete terms: Should financial regulators in Country D switch from "tough" to laissez-faire policies toward banks?

The previous literature on commitment has centered on time consistency issues, and we continue this tradition; the common theme in the examples we describe is the desirability of tying policymakers' hands, at least in some states of the world. Thus a country "commits" to a free-trade agreement because of protectionist temptations in its absence, to a currency board because of the temptation to inflate in a less restrictive regime, and so forth.

The decision to commit to a policy is like the decision to make an irreversible investment; the policymaker and/or society bears some cost generated in the political system or by the economic costs associated with adjustment to the new policy, but once the policy is in place, this is a sunk cost, and reversal is also costly. Our intent in this paper is to draw out this analogy to show how techniques associated with valuing real options can shed light on policy choices.

We consider the decision problem of a policymaker or government who chooses repeatedly between a restrictive regime ("rule") and a less restrictive regime ("discretion"). By contrast, previous literature for the most part considers only static, once-and-for-all choices between rules and discretion. It is not useful in this context to define a "rule" as a fully state-contingent plan, since actual implementation of a policy rule requires limiting the number of contingencies for the same reasons that private contracts limit the list of contingencies. A government committed to an incomplete or simple rule may find its rule undesirable in some states and regret its commitment. Since it is manifestly true that governments can rarely, if ever, make fully irrevocable commitments, the question of when a government will "renege" on a commitment is an integral part of the commitment question.

When policy decisions are made in real time—rather than at some mythical time 0—policymakers have the option to wait. The key insight of our approach is this: When policy is made in an environment of uncertainty, the costs of switching policies give this option to wait positive value. This insight has several implications: First, the option to wait itself makes the incumbent regime relatively more attractive (compared to the traditional once-and-for-all analysis). Second, the option to wait means that increased uncertainty makes the incumbent regime more attractive. Third, because the commitment decision takes place in real time, policy choice displays hysteresis.

1. Policy games

Policy choices of the sort described above are typically Prisoners' Dilemmas (we describe an exception below). Before turning to the specifics of our dynamic model of policy commitment and reversal, we describe some examples that motivate our modeling choices.

Our first example is drawn from Argentina's history, and we find it especially thought-provoking because it involves the use of a policy regime—currency boards—

that is generally regarded as a strong commitment mechanism. Argentina used a currency board structure with specie backing from 1899 to 1935. Raúl Prebish, then the finance minister, convinced the government of the need for a more flexible monetary institution (a central bank) to respond to the Great Depression (della Paolera and Taylor [1998]). Though the most recent currency board, established in 1991, successfully tamed Argentina's hyperinflation, it did not disappear with the hyperinflation. Instead, it was not until recent financial crises, when there was considerable pressure on the banking system, that speculation arose that the Menem government might be forced to abandon the currency board. Thus, even a mechanism that is widely regarded as a strong commitment device comes and goes with current needs.[1] Commitment to a currency board can be thought of as an application of the standard rules vs. discretion analysis of monetary economics (Kydland and Prescott [1977], Barro and Gordon [1985]). In another paper (Haubrich and Ritter [forthcoming]), we have reformulated the standard question in dynamic terms.[2]

Second, consider the following stylized description of the interaction between a regulator and a bank (or the banking system), abstracted from relatively static discussions of closure policy (for example, Mailath and Mester [1994] or Kane [1989]). The regulator may choose to be either tough or weak. Tough regulators do not bail out insolvent banks; weak regulators do. A bank chooses to be safe or risky. If banks are truly safe, the regulator prefers to relax vigilance, take it easy, and be weak. This has the effect of minimizing regulatory burden while bailing out "unlucky" banks. If banks are risky, however, the regulator prefers to be tough: If the regulator is tough, banks

[1] This point has been made informally by Zarazaga [1995].

[2] The monetary-misperceptions models that underlie most of the time-consistency literature in monetary economics rely on information lags that blur the analogy with irreversibly investment and are more easily handled in discrete time models (which we use in the other paper).

have an incentive to stay safe, but if the regulator is weak, higher risk increases profits, but there are more bailouts. Similar terms could describe the interaction between between an environmental regulator and the public, with periods of bad macroeconomic performance (low productivity growth, for example) possibly inducing the regulator and the public to prefer weaker standards.

Dixit [1996] sketches the Prisoners' Dilemma nature of international trade politics in some detail. The core idea is this:

> Liberal or open trade regimes are in the mutual interest of all countries jointly, but each has a private incentive to deviate in various ways: (1) to levy import tariffs to exploit its monopoly power in trade and improve its terms of trade, (2) to restrict imports or promote exports to give its firms a strategic advantage in industries characterized by international oligopoly, and (3) to satisfy the demands of its domestic protectionist pressure groups. (Dixit [1996], p. 73)

Most examples that come to mind have the character of a Prisoners' Dilemma, and the role of commitment is to allow the policymaker to avoid the Nash equilibrium. But the real world is not so static. At certain times the payoffs may not resemble a Prisoners' Dilemma. The prospect of financial crisis may increase the value of discretion relative to a currency board. In some circumstances, the possibility of systemic collapse might cause regulators to prefer weak oversight in spite of moral hazard problems. This has led to a literature on "opt-out" clauses (Flood and Isard [1989], Lohmann [1992]). Our objective here is to explore the rich dynamics involved in the entire process of opting in and opting out.

To admit the possibility that policymakers will regret their commitment, we allow payoffs to the policymaker and the public to change over time, driven by a stochastic state variable u_t.[3] Our stylized policy game is:

(G1) TIME t PAYOFFS FOR POLICYMAKER AND PUBLIC

	Policymaker	
Public	$P^R(u_t)$, $P^R(u_t)$	$T(u_t)$, $L(u_t)$
	$L(u_t)$, $T(u_t)$	$P^D(u_t)$, $P^D(u_t)$

We assume the payoff functions are differentiable with respect to u_t and are related to one another in such a way that: (i) Down–Right is the unique Nash equilibrium. (ii) Up-Left is Pareto superior to the Nash equilibrium for small values of the state variable u_t, but Pareto inferior to the Nash equilibrium during extreme times.[4] Specifically, letting $\pm u_0$ be the points at which $P^D(u)$ crosses $P^R(u)$, we require

$$P^R(u_0) - P^D(u_0) = P^R(-u_0) - P^D(-u_0) = 0,$$

$$P^R(u) - P^D(u) > 0, \quad u \in (-u_0, u_0),$$

$$P^R(u) - P^D(u) < 0, \quad u \notin [-u_0, u_0],$$

$$T(u) > P^R(u), \quad \text{for all } u,$$

$$L(u) < P^D(u), \quad \text{for all } u.$$

The assumption that $P^D(u)$ and $P^R(u)$ cross at $\pm u_0$ is merely a convenience, which comes at no cost since it can be always be achieved by rescaling u_t. Obviously, the

[3] "The public" is not ideal terminology for all applications. In the analysis of trade regimes mentioned above, the row player could, for example, be the existing members of the World Trade Organization, who will lower trade barriers if the country joins, but not otherwise.

[4] Saying that the Nash equilibrium is inferior for "small" shocks is really just normalizing u_t; if, for a given application, Up-Left is preferred when u_t is large, we could simply use $v_t = 1/u_t$ instead. There is, however, another idea lurking in the background: If u_t displays some reversion to a mean of zero, the statement that Up-Left is better for small values of u_t is closely related to (but not quite identical to) the statement that Up-Left is preferred "most of the time." We do not pursue this interpretation until a later section.

crossing-point assumption embeds the assumption that u_t takes on both positive and negative values. In the present context, it is not natural to restrict the stochastic process to positive values, but the assumption is not essential to our argument. Apart from ensuring that Down–Right is always the unique Nash equilibrium, the functions $T(u)$ and $L(u)$ play no role in our subsequent analysis. The public's payoffs play a similarly unobtrusive role in the sequel.

The payoff functions should be interpreted as a reduced form that embodies the relevant structure of the economy, but they contain a simplification that is not immediately apparent. As they appear in (G1), the payoffs depend only on the value of u_t and on the current policy regime. In general, however, one might expect payoffs to depend on the entire process that generates the policy regime. That is, current payoffs may depend on the proximity (in terms of u) of a switch. This dependence seems most reasonable when the application involves investments that have different values under the two policy regimes: If trade liberalization appears to be imminent, an importer's inventory policy (and, therefore, current profits) will be different than if liberalization is a remote prospect. The specifics of this kind of behavior are tightly tied to the particular application, so we abstract from them in the interest of simplicity. Although we assume that current payoffs do not depend on the proximity of a regime switch, when we solve a simple regime-switching model in section 3, value functions *do* depend on this proximity.

We proceed with the assumptions that make (G1) a Prisoners' Dilemma for small u, but this is not the only useful structure. A game of "Chicken" or "Battle of the Sexes," in which both Up-Left and Down-Right are Nash equilibria, fits our framework as well. For different values of u_t, the players may wish to coordinate on different equilibria. They key thing for our framework is that different policies are preferred in different states and that switches are costly. One example of this kind of policy game is used by Sargent [1986]. He outlines a game between the monetary authority (Fed) and fiscal authority (Treasury) and argues that payoffs are such that tight monetary policy is compatible with tight fiscal policy, but not with easy fiscal policy. Easy monetary policy, however, works best with easy fiscal policy. There are two equilibria in the policy game he lays out, Tight/Tight and Easy/Easy, the first preferred by the Fed and the second by the Treasury. Several other two-by-two games of the sort Brams [1983] identifies as having "staying power" will also work.

2. Entering and Exiting Commitment

In previous literature, *commitment* has meant that the policymaker's hands are tied; in terms of game (G1) she must play Left. The game is structured so that if the policymaker is committed to playing Left, the public will always play Up. However, mechanisms to commit a government irrevocably are almost impossible to imagine.

It is not difficult, however, to think of mechanisms that make it costly for a government to alter its policy. In the United States for example, a constitutional amendment is difficult to put into place and difficult to repeal. Ordinary legislation has lower costs at both ends. Governments can, in effect, tie their hands loosely or tightly, but can always escape, if they have the will to bear the corresponding levels of pain. The pain can come from direct political costs and the real cost to the economy of adjusting to a different regime. In this section we allow the policymaker to enter and exit commitment, but impose costs at both ends. We stop short of modeling the policymaker's choice of commitment mechanism, which would take us far afield.

It is important to understand that the possibility of reneging on commitment does not make commitment a vacuous concept. Reneging is costly, both because of the direct cost we impose, and because the direct cost of *committing*—should that become desirable once again—imparts an option value to continuing the rule.

We maintain the traditional semantics of commitment and discretion, but we wish to highlight a bias in tone that creeps into the discussion when commitment is not irrevocable. This innovation forces us into the use of words like "renege" and "weasel" with clear negative connotations that we regard as unfortunate. (The latter term proves notationally convenient, as we have already reserved 'R' to denote rules.) We interpret the results of this section as a model of optimal behavior and tolerate the terminology only to maintain consistency with the literature.

Although some additions and modifications are necessary to fit the ideas into a policy framework, one purpose of this paper is to highlight the similarity between irreversible investment and policy commitment issues. A world in which policymakers can, at a cost, enter and exit commitment (or, more generally, any policy regime) bears a close similarity to Dixit's [1989] model of the entry and exit problem faced by competitive firms.

2.1 The model

We implement these ideas as follows. Let u be a continuous time stochastic process (we suppress time subscripts where it causes no ambiguity). We assume throughout that u is Markov, but at this stage we do not need to restrict it further. In particular, u may have continuous sample paths or may be a jump process. This description does not preclude a process that jumps at regular intervals; thus, some of our analysis (the "Option to Wait" section, in particular) applies directly to discrete-time models as well. At each date t, the policymaker and public's flow payoffs will be determined by (G1), conditional on two state variables: u_t and the incumbent policy regime—either rules or discretion (R or D). The policymaker makes two choices at t, a regime choice and which column to play in (G1). The *discretion* regime is defined by the policymaker's ability to choose either Left or Right. In the *rules* regime, however, the policymaker can play only Left. The policymaker also chooses a policy regime for the future, paying a lump sum cost $C > 0$ to commit to the rules regime, or a cost $W > 0$ to renege on (or weasel out of) a previous commitment. Continuing the incumbent regime is costless. We assume that both parties discount future payoffs and costs at rate r. In order to ensure that the regime-choice problem is well-defined, we assume that $|P^j(u)|,\ j \in \{D, R\}$, is bounded above by a polynomial so that the value of discretion forever or rules forever is finite:

$$\mathrm{E}\left[\int_{-\infty}^{\infty} P^j(u_s)e^{-rs}\,ds\right]$$

is finite for $j = R, D$.

2.2 The option to wait

It is possible to see the basic principles—option values—that drive specific solutions without imposing any more structure on the payoffs or stochastic process u. Though it is not difficult to find degenerate cases (extremely large switching costs, for example), the optimal regime choice will, in general, respond to u by switching between the two regimes. Option values drive a wedge between the values of u at which optimal commitment and reneging occur. This section explores that claim in depth.

Recall that the payoff functions $P^D(u)$ and $P^R(u)$ are assumed to cross only at $\pm u_0$, with flow payoffs from rules exceeding those from discretion between $-u_0$ and u_0. Now, suppose first that the status quo is discretion. Then there may be a nonempty set CR, such that the policymaker optimally Commits to Rules if $u \in$ CR.[5] Let $\underline{u}_c = \inf \mathrm{CR}$ and $\overline{u}_c = \sup \mathrm{CR}$. The assumption that u is Markov ensures that CR is

[5] Ordinarily CR will be an interval, but if the process u is badly behaved, it might not be. For example, suppose that there is a small interval A around 0 from which, with high probability, u jumps to points outside $(-u_0, u_0)$. Suppose further that u can jump into A. After such a jump, the policymaker would not commit because of the high probability that discretion will again be preferred in the near future. This would put a hole in CR. Various assumptions (for example, continuous sample paths) could rule this out, but our subsequent argument does not depend on CR being an interval.

not time-dependent, but the results of this section would still be meaningful if it were.

Our first result is that for positive values of C and W, $\mathrm{CR} \subset (-u_0, u_0)$. Since the policymaker does not commit as soon as the current payoff from rules exceeds that from discretion, the option to wait to commit has positive value. To see this, consider choosing to commit at $u_t \notin (-u_0, u_0)$. For such a value of u, the current payoff from discretion is no lower than that for rules ($P^D(u_t) \geq P^R(u_t)$), so it is better (or not detrimental, at least) to wait to commit on that score. Furthermore, moving the payment of C farther into the future is desirable because of discounting. This value of waiting is the essence of option values in our problem.

Suppose now that the status quo is rules. For finite values of C and W, there will be a set RR, such that the policymaker Retains Rules for $u \in \mathrm{RR}$ and reneges for $u \notin \mathrm{RR}$. (However, RR may be the entire real line, given our assumptions thus far.) If RR is finite, let $\underline{u}_w = \inf \mathrm{RR}$ and $\overline{u}_w = \sup \mathrm{RR}$.[6] Essentially the same argument that gives $\mathrm{CR} \subset (-u_0, u_0)$ also ensures that $(-u_0, u_0) \subset \mathrm{RR}$: Consider reneging when $u_t \in (-u_0, u_0)$. At this instant t, the payoff from rules still exceeds that from discretion. Thus delay allows both higher current payoffs and delay in the payment of cost W. Combining both results we have a general result about option values for a dynamic policy commitment and reneging problem:

Proposition: Let $U_0 = (-u_0, u_0) = \{u | P^R(u) > P^D(u)\}$. *Let* CR *be the set of values of* u *for which the policymaker commits, and let* RR *be the set for which she reneges. Let* $C > 0$ *and* $W > 0$. *Then*

$$\mathrm{CR} \subset U_0 \subset \mathrm{RR}. \tag{1}$$

A formal proof works along the following lines.[7] Suppose, contrary to the proposition, that $u_t = \overline{u}_c \geq u_0$. Since $P^R(u)$ and $P^D(u)$ are continuous, we can find $\epsilon > 0$

[6] Like CR, RR is normally an interval.

[7] In fact, the principles behind the proposition are not related to our assumption that the payoff functions do not depend on CR and RR. The only complication from relaxing the assumption would be that the set of points for which $P^R > P^D$ must be treated as a function of CR and RR: $U_0(\mathrm{CR}, \mathrm{RR})$. The variational argument used here would remain

such that the following inequalities are satisfied for $\delta \in (0, \epsilon)$:

$$rC > P^R(u_0 - \delta) - P^D(u_0 - \delta)$$

$$rC > P^R(-u_0 + \delta) - P^D(-u_0 + \delta).$$

The first inequality says that for any $u \geq u_0 - \epsilon$, the flow benefit from delaying payment of C is greater than the flow benefit from rules relative to discretion. Consider the following feasible alternative for the policymaker: Do not commit until u enters the interval $A = [-u_0 + \epsilon, u_0 - \epsilon]$. Then return to the original policy (which would be using rules at any point in the interval A). Denote the first date at which $u \in A$ by T_A. Note that T_A depends on the sample path of u. Let $P(u_t)$ be the flow of payoffs under the original policy. ($P(u)$ is initially $P^R(u)$, but may switch to $P^D(u)$ and back if the sample path of u induces the policymaker to renege.) Let $\hat{P}(u_t)$ be the flow under the alternative. By assumption, $P(u_t) - \hat{P}(u_t) = 0$ for $t \geq T_A$. The alternate plan is constructed so that $P(u_t) - \hat{P}(u_t) = P(u_t) - P^D(u_t) < rC$ for $t < T_A$. Thus, for *any* sample path of u between t and T_A, the flow of losses from not committing is always less than the value of delaying payment of C. From T_A onward, the plans are identical, so the alternative plan is preferable to the original, contradicting the premise that $\overline{u}_c \geq u_0$ is an optimal switch point. The same argument can be applied from $u_t = \underline{u}_c \leq -u_0$. A minor variation accommodates reneging from $u_t = \overline{u}_w \leq u_0$ and $u_t = \underline{u}_w \geq -u_0$.

The economic interpretations of the simple mathematical statement in the proposition are significant. First, the option to wait makes the incumbent regime relatively more attractive. Second, allowing the regime choice to take place in real time introduces hysteresis into regime choice; when $u \in \mathrm{RR} - \mathrm{CR}$ (or, if CR and RR are intervals,

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valid, and (1) would be

$$\mathrm{CR} \subset U_0(\mathrm{CR}, \mathrm{RR}) \subset \mathrm{RR}.$$

$u \in [\underline{u}_w, \underline{u}_c) \cup (\overline{u}_c, \overline{u}_w])$, the prevailing regime depends on history. This result is driven by the economics of waiting; it has nothing to do with characteristics of the stochastic state variable u (other than the fact that it *is* stochastic). It is also worth mentioning that it does not depend on the possibility of future switches; CR $\in (-u_0, u_0)$ would be true if commitment decisions were made in real time, but were—unrealistically—irrevocable.

3. Optimal Switching

In this section we draw out the analogy with irreversible investment using a parametric example of optimal regime switching. While a discrete time approach can sometimes handle particular versions of the problems (as Lambson [1992] does for entry-exit decisions), the continuous time approach highlights the analogy with irreversible investment problems (exposited in Dixit and Pindyck [1994]).

3.1 A simple switching model

We assume that the payoff functions are quadratic and that u is a simple Ito process

$$du = \sigma\, dz, \tag{2}$$

where dz is a Wiener process. We assume for simplicity that neither u nor the payoff functions exhibits drift.[8] It is not difficult to show that since u will have continuous sample paths, CR and RR will be intervals. As (1) indicates, the policymaker operates in one of three overlapping regions: a high discretion region ($u > \overline{u}_c$), a low discretion region ($u < \underline{u}_c$), and a rules region ($u \in RR = [\underline{u}_w, \overline{u}_w]$). Thus, there are four unknown boundaries, $\underline{u}_c$, $\overline{u}_c$, $\underline{u}_w$, and $\overline{u}_w$.

[8] If the rates of drift of u and the payoffs differed, u would ultimately drift so far into one of the regions where $P^D(u) > P^R(u)$ that for all practical purposes, discretion would be permanent.

Let $V^D(u)$ be the value function for the problem when the state variables are u and policy regime $= D$ (discretion). While discretion prevails, the value function obeys

$$rV^D(u) = P^D(u) + \frac{1}{dt}\mathrm{E}[dV^D(u)].$$

Applying Ito's Lemma gives us a differential equation for $V^D(u)$:

$$\frac{1}{2}\sigma^2 V^D_{uu}(u) - rV^D(u) = -P^D(u). \tag{3}$$

(With no drift, there is no $V^D_u(u)$ term.) Similarly, for the interior of the region where rules continue, the value function for rules, $V^R(u)$, obeys

$$\frac{1}{2}\sigma^2 V^R_{uu}(u) - rV^R(u) = -P^R(u). \tag{4}$$

The solutions for the high discretion, low discretion, and rules regions are, respectively,

$$V^{Dh}(u) = A_{1h}e^{\beta_1 u} + A_{2h}e^{\beta_2 u} + S^D(u)$$

$$V^{D\ell}(u) = A_{1\ell}e^{\beta_1 u} + A_{2\ell}e^{\beta_2 u} + S^D(u)$$

$$V^R(u) = B_1 e^{\beta_1 u} + B_2 e^{\beta_2 u} + S^R(u),$$

where $S^D(u)$ and $S^R(u)$ are quadratic particular solutions. The values of β_1 and β_2 are roots of the characteristic equation

$$Q(\beta) = \frac{1}{2}\sigma^2\beta^2 - r = 0.$$

This is a version of what Dixit and Pindyck [1994] refer to as the "fundamental quadratic." Since $Q(0) < 0$ and $Q''(\beta) > 0$, the equation has one negative root, which we denote β_1, and one positive root, β_2.

In the high discretion region, as u gets very large, any return to the rules regime recedes farther and farther into the future. Therefore, $V^D(u_t)$ must converge to the expected present value of an infinite stream of payoffs $P^D(u_s),\ s \geq t$ (that is, the value of discretion forever $V^{DF}(u_t)$), which is finite. In fact, it turns out that the particular solution equals the value of discretion forever, that is, $S^D(u_t) = V^{DF}(u_t)$.[9] Thus, since $e^{\beta_2 u}$ explodes relative to a quadratic as u grows, we must set $A_{2h} = 0$ in order to ensure that $\lim_{u\to\infty}[V^D(u) - V^{DF}(u)] = 0$. Employing a similar argument in the low discretion region, we set $A_{1\ell} = 0$.

This leaves us with four undetermined coefficients and four unknown switching boundaries. A value-matching condition and a smooth-pasting condition must be met at each boundary:

$$\begin{aligned}
V^R(\underline{u}_c) - C &= V^{D\ell}(\underline{u}_c) \qquad & V^R_u(\underline{u}_c) &= V^{D\ell}_u(\underline{u}_c)\\
V^R(\overline{u}_c) - C &= V^{Dh}(\overline{u}_c) \qquad & V^R_u(\overline{u}_c) &= V^{Dh}_u(\overline{u}_c)\\
V^{D\ell}(\underline{u}_w) - W &= V^R(\underline{u}_w) \qquad & V^R_u(\underline{u}_w) &= V^{D\ell}_u(\underline{u}_w)\\
V^{Dh}(\overline{u}_w) - W &= V^R(\overline{u}_w) \qquad & V^R_u(\overline{u}_w) &= V^{Dh}_u(\overline{u}_w).
\end{aligned}$$

Thus we have eight equations in eight unknowns.

For numerical solutions, we start from a baseline of $C = W = \sigma = 1$ and $r = 0.02$, with payoff functions $P^D(u) = -u^2$ and $P^R(u) = 8 - 3x^2$, which cross at ± 2. (That is, $u_0 = 2$.)

Figure 1 highlights the importance of history and the option to wait. It depicts the optimal commitment and reneging boundaries for this baseline and one sample path of u. The time spent committed to rules is shaded. Shading starts when the sample path hits $\overline{u}_c$ but does not end until the path hits $\overline{u}_w$. For anything between $\overline{u}_c$ and $\overline{u}_w$, a policymaker committed to rules sticks with rules and a policymaker using discretion sticks with discretion. In these states, therefore, current policy depends on past policy. Quite apparently, then, it is incorrect to judge policy simply on the current state of the economy, and particularly inappropriate to naively contrast current policy with past policies at a similar state of the economy (stage of the business cycle, for example).

[9] For details, see appendix.

This shifting reemphasizes a point stressed by Flood and Garber (1984) in their work on the gold standard: to evaluate a policy rule, the entire dynamic policy sequence must be analyzed, including those periods where discretion reigns.

Figure 2 plots the commitment and reneging boundaries as the commitment cost C changes, keeping W fixed at 0. Apart from what is implied by the earlier proposition—that $|\overline{u}_c| < |\overline{u}_w|$—there are three features of Figure 2 that add some insight. First, the commitment region (CR) shrinks and ultimately vanishes as C rises. Second, the policymaker is more reluctant to *renege* as the *commitment* cost increases. This phenomenon reflects a slightly different kind of option value than emphasized in the previous section. When u is near $\overline{u}_w$, the policymaker knows that discretion will be desirable in the immediate future, but is concerned with the next time rules will become advantageous. If C is higher, it will be more costly to restore rules, should that become desirable. Therefore, the policymaker prefers to delay reneging until u is higher, thus pushing the expected (re-)commitment farther into the future. Third, it is clear that the $\overline{u}_c$ and $\overline{u}_w$ move quickly away from u_0 as C increases. This is a general phenomenon in real-options type models; option values rise quickly as small amounts of irreversibility are introduced.

A chart that instead varied W, holding C fixed, would look quite similar to Figure 2, except that the commitment region would vanish at a value of W higher than the value of C that does the trick in Figure 2. This is simply because of the relative remoteness of the prospect of having to pay W to (re-)renege.

Figure 3 illustrates the exercise of changing the variance parameter σ. As σ rises, the commitment interval (CR) narrows and the retention interval (RR) widens. This is a consequence of the options component of the decision. As the variance rises, so too does the option value of "not switching;" the "don't commit" and the "don't renege" sets both get larger. This is the bad news principle from the irreversible investment literature in a policy context: If you end up deep in the region where discretion is better, you will greatly regret the commitment to rules, but if you end up in the rules region, you can commit when you get there, so committing today doesn't help much. The former scenario becomes more likely with higher variance, so high variance makes commitment less likely. With a high enough variance, the policymaker never commits; the likelihood of near-term regret ($|u| > u_0$) is just too high to justify paying C and, later, W.

It is worth mentioning that the discount rate r has little influence on the boundaries in these numerical examples: Changing r from the baseline value of 0.02 to 0.2 nudges the commitment boundaries from ± 1.396 to ± 1.385 and the reneging boundaries from ± 2.548 to ± 2.556.

3.2 Expected time in regime

Merely looking at the size of $\overline{u}_w$ can give a distorted view of how likely we are to stay in rules. The variability of the stochastic process matters: The higher is σ, the more likely the exit from a given interval. Fortunately, probability has a set of tools ideally equipped to address this issue; this is the question of first passage, first exit, or hitting times. For the Brownian motion (2), the expected time to exit from the interval (a, b) (starting from 0,with $a < 0 < b$) is $|a||b|/\sigma^2$ (Breiman [1992], section 13.2).

In our example, the rules region is symmetric: $\underline{u}_w = -\overline{u}_w$. This means that starting from $u = 0$ the expected time of exit from rules into discretion is $\overline{u}_w^2/\sigma^2$. As Figure 3 illustrates, when σ increases, the reneging boundaries widen, but at nowhere near a quadratic rate, so although the two effects run in opposite directions, the expected time in rules decreases with σ.

Starting from a position other than 0, such as x, yields an expected exit time of $\overline{u}_w^2 - x^2$, which exhibits substantial nonlinearity in the starting value. In particular, the expected exit time decreases more and more rapidly as x approaches $\overline{u}_w$. Thus, expectations about policy can change quickly as a regime boundary nears.

3.3 Mean reversion

The Brownian motion described by (2) has the advantage of simplicity, but mean reversion introduces important effects (and may also seem more natural for many policy applications). To illustrate, we also explored a version of the model in which u follows a zero mean Ornstein-Uhlenbeck process:

$$du = -\eta\, u\, dt + \sigma\, dz,$$

where $\eta > 0$ governs the strength of mean reversion. With this process (3) and (4) become

$$\frac{1}{2}\sigma^2 V^j_{uu}(u) - \eta\, u V^j_u(u) - rV^j(u) = -P^j(u), \quad j \in \{D, R\}.$$

The solutions to this equation behave in much the same way as the corresponding solutions using simple Brownian motion. They are, however, more complex and add little insight, so we relegate them to an appendix, along with derivation of the boundary conditions that supplement the value-matching and smooth-pasting conditions.

Overall, the behavior of the model is similar to that shown in Figures 2 and 3. For any given parameter vector, however, the policymaker is more willing to commit (CR is bigger) and less willing to renege (RR is bigger). Combining these results with the fact that the mean-reverting process will spend more time near zero, it is easy to see that, overall, more time will be spent in the rules regime.

There are two effects at work to produce the result that mean reversion makes commitment more attractive. First, mean reversion means that at t the policymaker sees the conditional distribution of u_{t+s} shifted toward zero—toward the region where she likes rules. Second, increasing mean reversion reduces the conditional variance of the process at all horizons greater than 0.[10] Both effects make it less likely the policymaker will regret commitment. Turning to the decision to renege, the tendency of the u process to wander back toward zero means the policymaker is more likely to regret reneging with more mean reversion. Figure 3 also indicates, however, that reduced variance makes reneging *more* attractive, but in our numerical experiments the location-shift effect dominated. (Mean reversion also increases the time in rules, for a given $\overline{u}_w$.)

4. Conclusion

The key message of our paper is that the option to wait has value in a policy context, and that this value has important implications for the dynamics of committing and reneging whenever the economic environment is uncertain. This message has both positive and normative interpretations and implications. From the positive point of view, economists often exhibit frustration about policymakers' procrastination propensities; losses mount until the evidence in favor of a regime change appears overwhelming. Our perspective generates insight into this inertia in the policy process. It emphasizes the key role of regret in optimal policy switching when there is uncertainty. Thus, we argue that the policy commitment and reversal process cannot be adequately understood without careful analysis of the stochastic environment, the unrecoverable costs of switching, and the potential costs of reversing course.

The analogy to irreversible investment is relevant from the normative point of view as well. Addressing practitioners in the *Harvard Business Review,* Dixit and Pindyck [1995] conclude by saying, "To make intelligent investment choices, managers need to consider the value of keeping their options open." Economists who offer policy advice

[10] It is not possible to change σ and η simultaneously to keep the conditional variance constant at all horizons. Thus it is impossible to completely uncouple the location-shift and variance effects of mean reversion.

should be similarly aware of the value of doing nothing—at least for a while. We have drawn attention to the close similarity to irreversible investment problems in order to emphasize that the tools to extend policy analysis in this direction are already at hand.

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Appendix

Solutions with Mean Reversion

1. General solutions

Let u follow an Ornstein-Uhlenbeck process:

$$du = -\eta u\, dt + \sigma\, dz.$$

With this process the differential equations (3) and (4) become

$$\frac{1}{2}\sigma^2 V^j_{uu}(u) - \eta u V^j_u(u) - rV^j(u) = -P^j(u), \quad j \in \{D, R\}$$

Again we need separate discretion solutions for high and low values of u:

$$V^{Dh}(u) = A_{1h}\, {}_1F_1\left(\frac{r}{2\eta}, \frac{1}{2}, \frac{\eta u^2}{\sigma^2}\right) + A_{2h}\left(\frac{\eta u^2}{\sigma^2}\right)^{\frac{1}{2}} {}_1F_1\left(\frac{r}{2\eta} + \frac{1}{2}, \frac{3}{2}, \frac{\eta u^2}{\sigma^2}\right) + S^D(u)$$

$$V^{D\ell}(u) = A_{1\ell}\, {}_1F_1\left(\frac{r}{2\eta}, \frac{1}{2}, \frac{\eta u^2}{\sigma^2}\right) + A_{2\ell}\left(\frac{\eta u^2}{\sigma^2}\right)^{\frac{1}{2}} {}_1F_1\left(\frac{r}{2\eta} + \frac{1}{2}, \frac{3}{2}, \frac{\eta u^2}{\sigma^2}\right) + S^D(u)$$

$$V^{R}(u) = B_{1}\, {}_1F_1\left(\frac{r}{2\eta}, \frac{1}{2}, \frac{\eta u^2}{\sigma^2}\right) + B_{2}\left(\frac{\eta u^2}{\sigma^2}\right)^{\frac{1}{2}} {}_1F_1\left(\frac{r}{2\eta} + \frac{1}{2}, \frac{3}{2}, \frac{\eta u^2}{\sigma^2}\right) + S^R(u),$$

where $S^D(u)$ and $S^R(u)$ are quadratic particular solutions (different from those used when u has no mean reversion), and ${}_1F_1(\cdot)$ is the confluent hypergeometric or Kummer function.

2. Lemma needed to derive boundary conditions

In order to derive restrictions on the undetermined coefficients that are analogues of $A_{2h} = 0$ and $A_{1\ell} = 0$ used for the simple Brownian motion case, we need the following lemma:

Lemma: $S^D(u_t) = V^{DF}(u_t)$, where $V^{DF}(u_t)$ is the value of discretion forever given that the current state is u_t.

Proof: First use some standard results for an Ornstein-Uhlenbeck process to derive the expectation of a quadratic function of the process.[11] For all $s \geq 0$:

[11] Necessary formulae can be found on page 106 of C.W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences,* second edition.

$$
\begin{aligned}
\mathrm{E}[u_{t+s}|u_t] &= u_t e^{-\eta s} \\
\mathrm{E}[u_{t+s}^2|u_t] &= \mathrm{E}\left[u_t e^{-\eta s} + \sigma \int_0^s e^{-\eta(s-\tau)}\, dz(\tau)\right]^2 \\
&= \mathrm{E}\Bigg[u_t^2\, e^{-2\eta s} + 2u_t e^{-\eta s}\sigma \int_0^s e^{-\eta(s-\tau)}\, dz(\tau) \\
&\qquad + \left(\sigma \int_0^s e^{-\eta(s-\tau)}\, dz(\tau)\right)^2\Bigg] \\
&= u_t^2\, e^{-2\eta s} + \sigma^2 \int_0^s e^{-2\eta(s-\tau)}\, d\tau \\
&= u_t^2\, e^{-2\eta s} + \frac{\sigma^2}{2\eta}\left[1 - e^{-2\eta s}\right]
\end{aligned}
$$

Now apply Fubini's theorem and substitute, using $P^D(u) = p_0 + p_1 u + p_2 u^2$:

$$
\begin{aligned}
V^{DF}(u_t) &= \mathrm{E}\left[\int_0^\infty P^D(u_{t+s}) e^{-rs}\, ds\right] \\
&= \int_0^\infty \mathrm{E}[P^D(u_{t+s})|u_t]\, e^{-rs}\, ds \\
&= p_0 \int_0^\infty e^{-rs}\, ds + p_1 u_t \int_0^\infty e^{-\eta s} e^{-rs}\, ds \\
&\quad + p_2 u_t^2 \int_0^\infty e^{-2\eta s} e^{-rs}\, ds + \frac{p_2 \sigma^2}{2\eta} \int_0^\infty e^{-rs}\, ds + \frac{p_2\sigma^2}{2\eta} \int_0^\infty e^{-2\eta s} e^{-rs}\, ds \\
&= \left[\frac{p_0}{r} + \frac{p_2\sigma^2}{2\eta}\left(\frac{1}{r} - \frac{1}{r+2\eta}\right)\right] + \frac{p_1}{r+\eta}\, u_t + \frac{p_2}{r+2\eta}\, u_t^2
\end{aligned}
$$

The last line is easily verified to equal the quadratic particular solution $S^D(u_t)$. To use the result for simple Brownian motion ($\eta = 0$), apply L'Hospital's rule to the second term in brackets.

3. Boundary conditions

As $u \to \pm\infty$, $V^D(u)$ must converge to the value of discretion forever, that is, for large u_t,

$$
V^D(u_t) \approx V^{DF}(u_t) \equiv \mathrm{E}\left[\int_0^\infty e^{-rs} P^D(u_{t+s})\, ds \,\middle|\, u_t\right]
$$

As the lemma above notes, the particular solution $S^D(u)$ turns out to equal the value of discretion forever, so the terms involving ${}_1F_1(\cdot)$ must converge to 0 as u gets large.

For large values of u,

$$ {}_1F_1(a,b,u) = \frac{\Gamma(b)}{\Gamma(a)}\, e^u u^{a-b}[1 + o(1/u)], $$

and for large negative u,

$$ {}_1F_1(a,b,u) = \frac{\Gamma(b)}{\Gamma(b-a)}\, u^{-a}[1 + o(1/u)], $$

where $o(1/u)$ converges to zero with $1/u$. For the part of the solution involving ${}_1F_1(\cdot)$ to vanish for large positive values of u, we must have

$$ A_{1h}\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{r}{2\eta}\right)} + A_{2h}\frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{r}{2\eta}+\frac{1}{2}\right)} = 0. $$

For large negative values of u we need

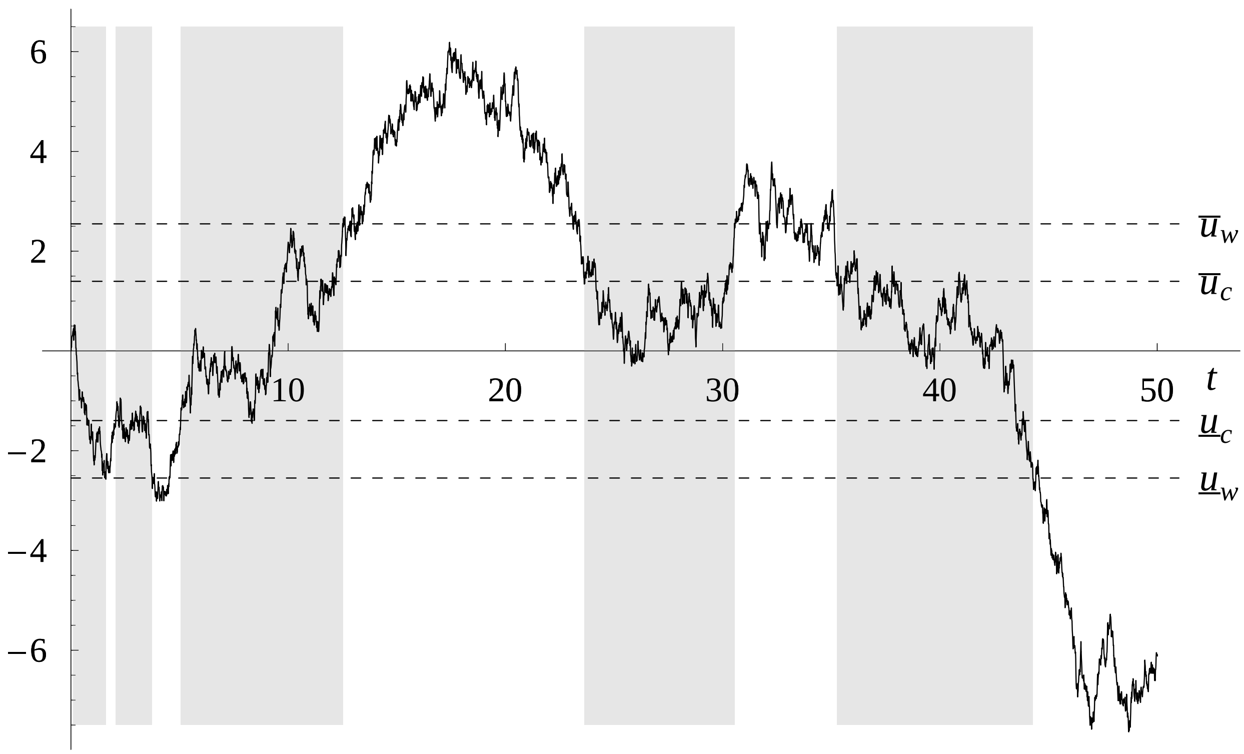
$$ A_{1\ell}\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}-\frac{r}{2\eta}\right)} - A_{2\ell}\frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(1-\frac{r}{2\eta}\right)} = 0. $$

These are the analogues of $A_{2h} = 0$ and $A_{1\ell} = 0$ in the Brownian case discussed in the text. (The equations do not reduce to $A_{2h} = 0$ and $A_{1\ell} = 0$ when $\eta = 0$ because coefficients are normalized in a different way here.)

We now have eight value-matching and smooth-pasting conditions plus these two equations to determine four commitment and reneging boundaries and six undetermined coefficients.

Figure 1

Entering and Exiting Commitment



Note: Shaded areas indicate time spent in rules regime.

Figure 2

CHANGING COMMITMENT COST C

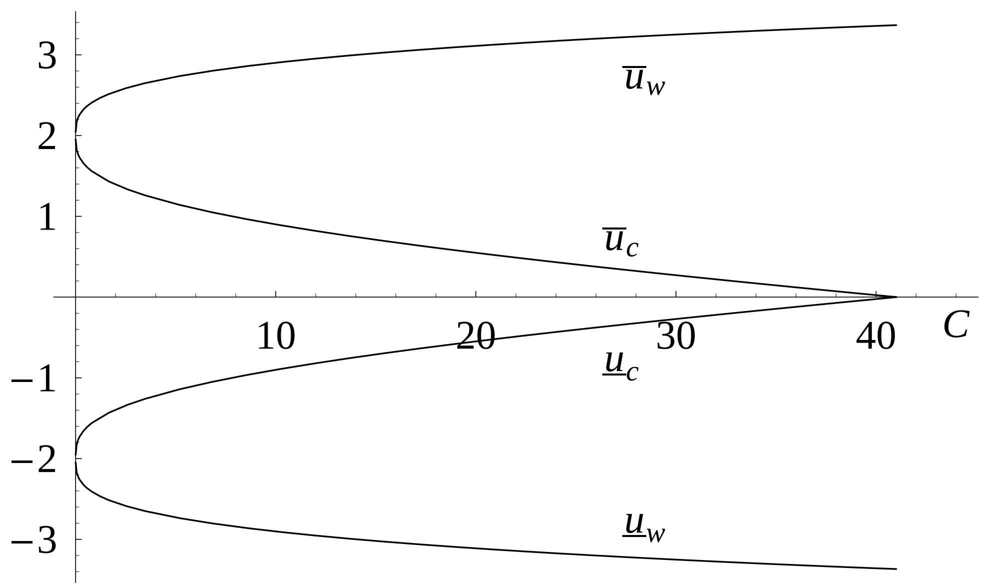


Figure 3

Changing Diffusion Parameter σ