R&D Spending and Cyclical Fluctuations: Putting the 'Technology' in Technology Shocks

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Abstract

We examine the dynamic properties of an endogenous growth model with an explicit R&D sector in order to evaluate its ability to propagate temporary disturbances into persistent fluctuations in macroeconomic variables. We demonstrate that a large proportion of the variability and persistence of measured Solow residuals can be thought of as reflecting the endogenous accumulation and adaptation of technical knowledge rather than simply exogenous processes. By explicitly modeling R&D, we use a framework in which it is possible to explicitly consider the role of technology in "technology shocks."

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1. Introduction

In recent years, the use of general equilibrium models of economic growth has given rise to a new growth theory paradigm where growth is an endogenous process generated from investment in human capital or research and development (R&D). At the same time, stochastic general equilibrium models have been utilized to examine cyclical fluctuations as optimal responses to unanticipated shocks (the real business cycle literature). In this literature, while technology plays a crucial role in generating business cycles, technological change is treated as exogenous. In this paper, we use an endogenous growth framework to examine the effect of endogenous technological change on the properties of a real business cycle model.

In standard real business cycle models, exogenous shocks to the production technology are often presumed to be the source of business cycle impulses, with these shocks propagated endogenously. Although the underlying disturbances are commonly referred to as productivity or technology shocks, they are usually understood to represent a combination of other exogenous factors as well. This interpretation has lead to criticism that the process generating these technological shocks is not modeled. Moreover, models relying on this type of exogenous shock require a high degree of autocorrelation in the underlying disturbances to generate the persistence in macroeconomic variables that are observed in the data [e.g. King, Plosser and Rebelo (1988a), Cogley and Nason (1995), Rotemberg and Woodford (1996)].
Although capital accumulation and intertemporal labor substitution serve as endogenous propagation mechanisms, they are not sufficient to generate cycles from transitory productivity shocks consistent with the data.

Several approaches have been taken to introduce stronger propagation mechanisms. Some have introduced multiple sectors, as in Beaudry and Devereux (1996), Benhabib, Perli and Sakellaris (1997) and Perli and Sakellaris (1997). Others have considered variable factor utilization rates or adjustment costs; e.g., Burnside and Eichenbaum (1996), DeJong, et al (1996), and Cogley and Nason (1995). Wen (1998) assumes present and past leisure are complements, which serves to slow down the response of workers to a productivity shock. The ultimate objective of each of these efforts is to find endogenous channels of shock propagation, reducing the reliance on imposed persistence in “exogenous” Solow residuals.¹

We differ from these papers in two ways: First, we explicitly model the process generating technology and therefore the process underlying these technology shocks. In addition, we distinguish between a productivity shock, which we treat as a shock to the production function, and a technology shock, which here is a shock to the production of technology itself. In this way we can demonstrate that a large proportion of the variability and persistence of measured Solow residuals can be thought of as reflecting the endogenous accumulation and adaptation of technical knowledge rather than simply an exogenous process. To look at this issue, we consider the role of research and development activities in generating and propagating shocks to production. By explicitly modeling R&D, we use a

¹ See Benhabib et al (1997) for additional discussion of recent work on the issue of persistence in business cycle models.
framework in which it is possible to explicitly consider the role of technology in "technology shocks."

Our approach is similar in spirit to other papers that examine the relationship between endogenous growth models and business cycle fluctuation. For example, L. Jones et al (1997) examine the implication of productivity shocks on endogenous growth trends. Similarly, Bean (1990) and Williams (1994) analyze the long-run effects of temporary shocks. Whereas these papers focus on the growth implications of cycles, we are interested in the cyclical implications of shocks to the long-run process of technological accumulation and growth.³

The R&D model we use is a discrete-time variant of the endogenous growth model developed by C. Jones (1995). One important feature of the model for analyzing dynamics is its semi-endogenous growth aspect: Although economic growth originates within the model structure, in the steady state it is a function of constant parameters. This provides a baseline growth trend around which the model's short-run dynamics can be evaluated. The use of an endogenous growth model also has the attractive feature of allowing us to consider growth and fluctuations as being generated by the same process. Consequently, we are able to calibrate the model's parameters by exploiting its growth implications.

The decomposition of the Solow residual allows us to distinguish between a shock directly to the production sector (a productivity shock) and a shock to the process generating new knowledge itself (a technology shock). We find that a transitory shock to the final output sector generates dynamics that are typical of RBC models, with the persistence problem intact.
The R&D sector does act as a propagation mechanism, but its quantitative significance in that respect is marginal. However, we find that transitory shocks to the R&D sector—a true "technology shock"—gives rise to protracted responses in productivity and output.

Subsequent sections of this paper describe the model environment, calibration, and dynamic simulations. A conclusion briefly summarizes our findings.

2. The Model

The specification of technological change and output growth used is a modified discrete-time version of the R&D model of C. Jones (1995). This framework is placed into a standard RBC environment with labor/leisure choice and capital accumulation with depreciation. The central feature is an R&D production sector, which combines effort and previously accumulated knowledge to produce technological innovations. Letting $A_t$ represent the current level of technology, the R&D production function is of the form:

$$A_{t+1} - A_t = z_{A_t} \eta L_{A_t} A_t^{\theta} \bar{L}_{A_t}^{1-\theta},$$

where $L_{A_t}$ is labor applied to R&D activities, $\bar{L}_{A_t}$ captures the externalities associated with duplication in the R&D process (which are external to the firm) and $z_{A_t}$ is a stationary exogenous shock variable. In equilibrium, $L_{A_t} = \bar{L}_{A_t}$. The parameter $\eta$ governs the rate at which new innovations arise. Note that the stock of previously accumulated knowledge

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3 King, Plosser and Rebelo (1988b) also studied the stochastic properties of an endogenous growth model, although that is not the emphasis of their work.

4 As Jones points out, this model is a variant of those exposited by Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and others.

5 We adopt a convention that new technologies affect total factor productivity one period later.
(A_t) enters the production function with the exponential parameter \( \phi \), which determines whether the rate of technological innovation decreases with the level of knowledge \( (\phi<0) \) or increases with the level of knowledge \( (\phi>0) \). The parameter \( 0<\lambda<1 \) is a measure of external diminishing returns.

The level of accumulated knowledge, \( A_t \), enters the production function for final output as a form of labor-augmenting technological change:

\[
Y_t = z_t (A_t L_t)^{\alpha} K_t^{l-a},
\]

with \( z_t \) representing an exogenous technology shock that is independent of \( A_t \). Thus we can decompose the Solow residual, represented by \( z_t A_t^{\alpha} \), to look at two possible sources of disturbances to productivity.

In the initial version of the model considered, labor is perfectly substitutable across sectors, implying that marginal product will be equated to the real wage:

\[
W_t = \alpha z_t A_t^{\alpha} L_t^{a-l} K_t^{l-a} = P_{A_t} z_t \eta A_t^{\phi} L_t^{l-1},
\]

where \( P_{A_t} \) is the relative price of purchasing new patents from the R&D sector.

The R&D sector produces designs that are then purchased by an intermediate goods sector and rented to firms. The R&D sector earns monopolistically competitive profits such that their rental rate includes a markup above the rental rate for capital:

\[
r_{A_t} = \frac{r_t}{1-\alpha} = (1-\alpha) z_t (A_t L_t)^{\alpha} K_t^{l-a} - \delta,
\]

where \( r_t \) is the return on capital and \( \delta \) is the depreciation rate of capital. This yields profits of:

\[
\Pi_t = \alpha (1-\alpha) \left( \frac{Y_t}{A_t} \right).
\]
The price of patents is determined by the following arbitrage condition:

$$r_{t+1} = \frac{\Pi_{t+1}}{P_{At}} + \frac{P_{At+1} - P_{At}}{P_{At}}.$$

Households consume the output of the final goods sector and face a standard optimization problem of maximizing (infinite horizon) lifetime utility over consumption (Ct) and leisure (Lt), subject to budget and time constraints:

$$\max \sum_{t=0}^{\infty} \beta U(C_t, L_t),$$

subject to:

$$K_{t+1} = (1 - \delta) K_t + W_t (L_y + L_a) + \Pi_t - P_{At} (A_{t+1} - A_t) - C_t, \quad \text{and}$$

$$L_y + L_a + L_t = L.$$

Equation (8) shows that consumption plus capital accumulation (net of depreciation, \(\delta\)) is constrained by total household income, which is the sum of capital rental returns, wages, and profits and returns from the R&D. Equation (9) constrains the sum of all uses of time to the normalized value of one. First order conditions for the households problem are standard and are summarized by:

$$\frac{U_C(C_t, L_t)}{\beta U_C(C_{t+1}, L_{t+1})} = 1 + r_t.$$  

$$\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} = W_t.$$  

Equations (1)-(11) provide the system of equations determining the model's dynamic solution.
3. Calibration and Simulation

In order to examine the model's dynamic behavior, we derive a calibrated log-linear approximation around the steady state. The specific equations used are shown in the Appendix. The model's long-run growth implications are exploited to generate a stationary representation of the steady state and to derive the parameters of the dynamic approximation.\(^6\)

As described in C. Jones (1995), the model implies a steady-state growth path that depends only on the parameters of the R&D production technology and the population growth rate. In particular,

\[
(\gamma_A - 1) = \frac{\lambda}{1 - \phi} (\gamma_L - 1),
\]

where \(\gamma_A\) is the steady-state (gross) growth rate of the level of knowledge, \(A\), and \(\gamma_L\) is the population growth rate. From equations (1)-(11), we can solve for the growth rates of all of the model's variables. Specifically, the growth rates of per capita output, consumption, investment and capital are all equal to \(\gamma_A\), while the price of patents and profits grow at the same rate as the population, \(\gamma_L\). (That is, the price and profitability of innovation \textit{per capita} is constant.)

Letting \(X_t\) and \(L_t\) represent indices of the steady state level of technology and population, respectively, dividing equations (1)-(11) by appropriate combinations of these growth indices generates a stationary representation of the steady state; in which, e.g., \(y_t = Y_t / X_t\) is the stationary measure of output.

\(^6\)The simulation procedure is outlined in King, Plosser and Rebelo (1988a).
A log-linear approximation around the steady state of the stationary system is used for dynamic analysis, in which the key parameters are functions of the underlying model parameters. Table 1 lists the parameters and their baseline values.

Several parameters are selected to be consistent with typical RBC calibrations. Labor’s share in the final production sector, $\alpha$, is set equal to 0.64. The parameter defining utility shares of consumption and leisure, $\theta$, is set to 0.34, in order to imply a steady-state share of total work-effort ($L_Y + L_A$) equal to 0.30. Taking the frequency of the model to be quarterly, the discount factor, $\beta$, is set to 0.99 and the capital depreciation rate, $\delta$, is set to 0.025.

The parameters of the R&D sector technology are derived from the model’s growth implications. Using growth trends as a baseline (1948:Q1-1997:Q4), we set $\gamma_A=1.0046$ and $\gamma_L=1.0035$, reflecting annual growth rates of output per-capita and population of 1.84% and 1.40%, respectively. Equation (12) then defines the relationship between the two key parameters of the R&D production function, $\lambda$ and $\phi$. Given that $\lambda$ is constrained to be in the interval (0,1] and the population and per-capital output growth rate are pinned down by the data, $\phi$ is constrained to be in the interval (0,1). Thus the data supports the idea that there are increasing returns to the discovery of knowledge; however, not by as much as suggested by the initial endogenous growth literature. For our baseline parameterization, we set $\phi = 0.5$, implying that $\lambda = 0.664$.

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7 See, for example, Romer (1986) and Aghion and Howitt (1992).
The sizes of the final output and R&D sectors are related by:

\[
\frac{L_A}{L_T} = \frac{1}{1-\alpha} \left( \frac{1+r}{\gamma_L} - 1 \right) \left( \frac{1}{\gamma_A} - 1 \right). \tag{13}
\]

For the given parameterization, this implies that about 11.5% of total work effort is devoted to R&D activities. Although this is significantly higher than a literal measure of R&D employment (e.g. approximately 0.75% of the labor force are scientists and engineers\(^8\)), 11% does not seem unrealistic for a somewhat broader view of R&D activities which, for example, includes both organizational and technical workers.

### 4. Business Cycle Persistence, Productivity Shocks, and Technology Shocks

Some of the basic properties of U.S. business cycles with which we are concerned are reported in Table 2. Because the growth model under consideration has important low-frequency dynamics, the data in Table 2 have been detrended using a wide high-pass filter (rather than being smoothed by first-differencing or H-P filtering), which leaves most of the lower-frequency fluctuations intact.\(^9\) The business cycle characteristics of the data displayed are standard: consumption and productivity are much less variable than output while investment is much more variable. All of the variables listed are positively correlated with

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\(^8\) National Science Board (1996).

\(^9\) Williams (1995) shows that the transition dynamics of a very similar model display such long adjustment periods that level shifts may show up empirically as changes in growth trends. By using the wide-band high-pass filter, we seek to leave much of those long-run dynamics in the summary statistics as possible. The actual filtering technique used is that described in Baxter and King (1995). The high-pass filter passes fluctuations with frequencies corresponding to cycle lengths of 2 to 120 quarters, with a moving-average truncation of 12 periods.
output, but productivity is less so than consumption, investment or work effort. Most importantly for our purposes, the first-order autocorrelations of the variables show a high degree of persistence.

**Productivity Shocks:**

To compare these properties of the data with model simulations, we turn first to a typical RBC model. The fundamental lack of persistence encountered in such models is illustrated in the first panel of Table 3, which reports some implied second-moments from the model of King, Plosser and Rebelo (1988) when it is subjected to purely temporary productivity shocks.\(^\text{10}\) Although the convexity of utility implies that consumption exhibits a high degree of autocorrelation, other model variables—particularly output and investment—display much lower autocorrelations than are found in the data.

In principle, the addition of an R&D sector to an otherwise basic business cycle model should provide an additional mechanism for propagating shocks to the production function for final goods. As cited in the introduction, the strategy of introducing multiple sectors has been one common way of augmenting endogenous propagation of shocks. As shown in the second panel of Table 3, however, the introduction of an R&D sector does little to enhance the persistence of responses to temporary shocks in our model.

Figure 1 illustrates impulse-responses of some of the key model variables to a productivity shock. The patterns of responses to this type of shock shows the source of the additional persistence: the increase in the capital stock is larger and more protracted than it is

\(^{10}\) In all of the model simulation results reported below, the magnitude of the underlying disturbances is selected so as to generate a standard deviation of output equal to the statistic reported in Table 2.
in a basic RBC model. Knowledge and capital serve as substitutes in final production so that a productivity shock leads to a slowdown in knowledge accumulation (since the R&D sector is relatively less productive) and an increase in the accumulation of capital. The presence of an R&D sector enhances the capital-stock propagation mechanism that is already present in RBC models. Nevertheless, the magnitude of the effect is still so small that it makes little quantitative difference in terms of autocorrelations. Hence, if we are to rely on this type of “productivity” shock alone to drive the model’s dynamics, we would still require a high degree of persistence in the shocks themselves.

Technology Shocks

Introducing shocks affecting the production of knowledge makes significant progress towards finding an endogenous propagation mechanism for persistent shocks. Technology is modeled such that new technology is developed as a result of investment in research and development with a given probability \( \eta \). Thus a temporary shock to the production of knowledge could be, for example, an increase (or decrease) in the amount of successful innovation in any given time period. Table 4 reports statistics summarizing the model’s dynamics when it is subject to these types of “technology shocks.” In response to these shocks, output and productivity show a much higher degree of persistence. The first panel of Figure 4 shows that purely temporary shocks to the R&D sector imply that output has an autocorrelation coefficient of 0.60. The second panel shows that only a slight degree of persistence in the shocks \( (\rho_A = .5) \) is sufficient to give output roughly the same persistence as observed in the data. In addition, the model replicates other standard stylized facts, such as the co-movement between investment, consumption, output and productivity.
Some of the other features of the model’s dynamic responses to “technology” shocks are counterfactual, however. Even with some persistence in the shocks, investment shows a much lower degree of persistence than shown in the data. Work effort or employment similarly shows a low degree of persistence, and is negatively correlated with output. Moreover, the model predicts that work effort is less variable than output and productivity is more variable, while the reverse is true in the data.

The inability of the model to generate persistence on both output and factors of production stem from its implications for very different short-run versus long-run dynamics. Figure 2 illustrates these properties using impulse-response functions. A shock to the R&D sector sets in motion slow-moving transition dynamics back to the steady state for both capital and the level of knowledge. These transition dynamics are also transmitted to the adjustment of output, as well as the other model variables. In the case of the labor and investment variables, however, short-run dynamics dominate the patterns of responses. In the initial response to a shock, labor is drawn into the R&D sector from both final output production and leisure. Because the decline in the number of workers producing final goods lowers the marginal product of capital, investment also falls sharply. For a period of time, then, output, consumption and investment fall below the steady state in response to a positive technology shock, while total work effort rises as resources are channeled into the R&D sector. Thus the initial negative effect dominates the positive correlation that occurs as the economy moves to a new steady state.

An initial stage of contraction followed by a protracted rise in economic activity is atypical of the type of dynamics found in models subject solely to productivity shocks. It is,
However, consistent with the adjustment processes described in the literature “general purpose technologies” (Aghion and Howitt, Chapter 8, 1998). Technological breakthroughs are theorized to require a period of adjustment in which the technology must be integrated into the capital stock and applications are developed. In our model, this process is modeled through the dependence of new innovations on the existing stock of knowledge. It takes time to fully exploit new technologies because they give rise to further innovation.

Technology Shocks with Labor Adjustment Frictions

Although the movement of labor into R&D in response to a positive technology shock is economically sensible, the magnitude and speed of the labor adjustment is unrealistic given training and adjustment costs which surely exist. Table 5 shows the effects of some simple modifications to the model which are intended to dampen the flow of factors into R&D. The first modification is the introduction of a simple specification for labor adjustment costs in the technology sector. We consider a quadratic adjustment cost formulation modifying the time constraint:

\[(9a) \quad L_{\gamma t} + L_{\alpha t} + L_{\alpha t} + \frac{v}{2}(L_{\alpha t} - L_{\alpha t-1})^2 = 1\]

This specification implies that a small fraction of the time allocated away from leisure and final-goods production is lost in training and adjustment. Beginning from the steady state growth path, the time cost as a fraction of total time allocated toward R&D is:

\[\tau = \frac{v(\beta - 1)}{\beta + v(\beta - 1)} .\]

The first panel of Table 5 shows the results of introducing a cost of \(v=10\), implying that roughly 3% of time diverted away from leisure and final production is lost in training.
costs. The second panel shows higher costs, corresponding to $v=100$ (a 25% time loss). The third panel of Table 5 shows the effect of a more restrictive assumption regarding R&D work-effort (although not necessarily unrealistic): in this specification time allocated to the technology sector is not productive until one period later (that is, $L_{A,1}$ enters the R&D production function). In each case of labor adjustment restrictions, the longer run dynamics become relatively more important, imparting a stronger autocorrelation to the model’s variables.

While we are able to better replicate dynamics reflecting longer run phenomena, the modifications tend to move the model’s implications further from the data along the dimensions of increasing the variability of consumption, lowering the variability of investment, and increasing the correlation of productivity and output. These features stem from the reduced importance of short-run fluctuations in the restricted versions of the model. Moreover, the negative correlation between output and hours remains.

The implication that employment and hours move opposite to one another in response to a technology shock should not necessarily be taken as evidence against the importance of such shocks, however. This is particularly true if we consider that technology shocks are but one of the exogenous processes driving economic fluctuations. Indeed, Gali (1996) found that impulse response functions from a structural VAR model indicate a decline in employment following a positive technology shock, and that the conditional correlation
between employment and productivity is negative for technology shocks. These are also features of the initial responses to a technology shock in our model.¹¹

A well-known problem with typical RBC models is that they have exactly the opposite implication: hours and productivity are predicted to be strongly positively correlated, whereas in the data the correlation is near zero. In the data used to generate Table 2, for example, this correlation is only 0.034. This discrepancy has led some to suggest that the productivity shocks driving RBC models must be augmented by other shocks that offset the positive comovement between employment and productivity that is implied by productivity shocks alone. We now turn to the possibility that our specification of “technology” shocks can serve such a role.

Productivity and Technology Shocks Combined

To consider the importance of both types of shocks jointly, we exploit the hours-productivity correlation. In the basic version of the model with technology shocks alone, the correlation between hours and productivity is -0.902 for the case of white noise disturbances and -0.778 when the shocks have an autocorrelation coefficient of 0.5. As is typical of RBC models, productivity shocks imply a strongly positive correlation. If the model’s dynamics are assumed to be driven by a combination of technology and productivity shocks, the opposite co-movements generated by each shock separately can be combined to create a

¹¹ Gali’s key identifying restriction was that technology shocks are the sole source of permanent movements in output and productivity. Because the technology shocks in our model result in extremely protracted responses (near unit-roots), the conditional responses to technology shocks measured by Gali reflect the same type of disturbances we have postulated as “technology” shocks in our model.
correlation near zero, as observed in the data. Table 6 reports the implied fraction of the model’s variability attributable to technology shocks using this identification scheme. For each specification of the model, three pieces of information from Table 1 were used to calibrate the shocks: the overall variance of the two shocks was chosen to yield a standard deviation of output equal to 2.039 percent, the autocorrelation of the productivity shocks \( z_{yt} \) was selected to yield an output autocorrelation of 0.875, and the relative magnitude of the shocks was calibrated to the hours-productivity correlation of .034.

The first column of Table 6 shows the contribution of technology shocks to the overall variance of the Solow residual, \( A_t z_{yt} \). The second column shows the contribution of technology shocks to the variance of output. The estimates of the two contributions to model variance are similar, and range from about 7 percent for the baseline specification to about 80% for the variant of the model with lagged labor adjustment in the R&D sector. Two estimates are shown for each model specification, one in which the technology shocks are white noise, the other in which there is some persistence to innovations in the R&D sector. For the baseline model and the variant with small labor adjustment costs, the introduction of some persistence in the technology shocks contributes to a larger fraction of overall variance being accounted for by R&D shocks. For the specification in which labor adjustment is more costly, the persistence of technology shocks makes little difference.

\[12\] This is the same general approach used by Christiano and Eichenbaum (1992) to address the issue of the hours-productivity correlation. In their specification, demand shocks (government spending) give rise to negative comovement between productivity and hours, while technology shocks generate the typical positive comovement. The combination of the two shocks are then shown to be able to generate correlations that are consistent with the data. Aiyagari (1994) uses a similar technique to show that productivity shocks alone can only account for a fraction of output variance.
As an illustration of the overall behavior of the model when it is subjected to both types of shocks, Table 7 lists second moments for the model for the specification with large labor adjustment costs and no persistence in technology shocks. Combining the two shocks provides the most realistic dynamics results. Persistence in work effort and investment increased significantly, although they remain below that found in the data. Autocorrelation in consumption moved closer to that found in the data, and work effort is positively correlated with output. Taking this particular example as representing the closest correspondence between our theoretical model and the data, we conclude that a reasonable estimate of the relative importance of "technology" shocks is that they explain about one half of the overall variance of Solow residuals affecting final-goods productivity.

5. Conclusion

In this paper we examine the role of technology change in real business cycle dynamics by explicitly modeling a process of endogenous technological innovation. In this way, we distinguish between two types of supply shocks: a productivity shock, which is a shock directly to the production function, and a technology shock, which is a shock to the production of knowledge subsequently used in the production process.

We find that shocks to technology are associated with a propagation mechanism that imparts considerable persistence to the dynamic responses of model variables. Even under our basic specification where technology shocks are white noise and labor market adjustment
is instantaneous, we find that our approach is able to generate a significant amount of autocorrelation in output and productivity. To make the model more realistic, we introduce labor adjustment costs, further enhancing the role of technology shocks in generating persistence.

When we combine both productivity and technology shocks, model performance is further enhanced. Using the hours-productivity correlation found in the data as a benchmark for calibrating the relative magnitudes of the two shocks, our results suggest that one-half or more of the variance of measured Solow residuals might be reasonably attributed to disturbances associated with the technology of innovation.

One criticism of real business cycle models has been their reliance on persistence in exogenous productivity shocks to generate persistence in macroeconomic fluctuations. Moreover, little has been done to try to formally examine how technological improvements – which are presumed to underlie economic growth – simultaneously affect business cycle dynamics. By modeling the dynamic adjustment of the economy to endogenous technological innovation, we have addressed the persistence problem by considering the extent to which both growth and cycles might be affected by the dynamics of "technology".
References


National Science Board, Science and Engineering Indicators 1996.


Appendix

The stationary version of the model consists of the following equations. Equations (1)-(7) describe technology, factor demands and the market structure of the R&D sector. Equations (8)-(13) are associated with the representative household’s optimization problem.

(A1) \[ \gamma_k a_{t+1} - a_t = z_{at} \eta_t a_t^\alpha \tilde{y}_{at}^{\lambda-1} \]

(A2) \[ y_t = z_{yt} (a_t l_{yt})^\alpha k_{t}^{1-\alpha} \]

(A3) \[ w_t = \alpha \left( \frac{y_t}{l_{yt}} \right) \]

(A4) \[ w_t = p_{at} z_{at} \eta_t a_t^\alpha \tilde{y}_{at}^{\lambda-1} \]

(A5) \[ \frac{r_t}{1-\alpha} = (1-\alpha) z_{yt} (a_t l_{yt})^\alpha k_{t}^{1-\alpha} - \delta \]

(A6) \[ \pi_t = \alpha (1-\alpha) \left( \frac{y_t}{a_t} \right) \]

(A7) \[ 1 + r_{t+1} = \frac{\pi_{t+1}}{p_{at}} + \frac{p_{at} \pi_{t+1} - p_{at}}{p_{at}}. \]

(A8) \[ \gamma_k k_{t+1} - (1-\delta) k_t = r_t k_t + w_t (l_{at} + l_{yt}) + a_t \pi_t - p_{at} (\gamma_o a_{t+1} - a_t) - c_t \]

(A9) \[ l_{at} + l_{yt} + l_{ut} = 1. \]

(A10) \[ u_t (c_t) = q_t \]

(A11) \[ u_t (c_t) = \omega_t \]

(A12) \[ \omega_t = w_t q_t \]

(A13) \[ 1 + r_{t+1} = \frac{\gamma_k}{\beta} \left( \frac{q_t}{q_{t+1}} \right) \]

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A Condensed Version:

Before solving and simulating the dynamic system, it is helpful to make some substitutions and reorganize. Using the utility-denominated wage in (A12) and defining a similar transformation for the price of patents, \( \varphi_t = pAq_t \), equations (A9)-(A11) and (A3)-(A4) can be written as:

\[
\begin{align*}
  u_c(\cdot, \cdot) &= q_t \\
  u_L(\cdot, \cdot) &= \omega_t \\
  \omega_t &= \varphi_t z_A \eta a_t^{-\alpha} l_{at}^{\alpha-1} \\
  \omega_t &= q_t \alpha z_y a_t^{\alpha} l_{yt}^{\alpha-1} k_t^{1-\alpha} \\
  l_{at} + l_{yt} + l_{ot} &= 1
\end{align*}
\]

This set of equations can be solved for expressions relating the variables \( c_t, l_{at}, l_{yt}, l_{ot}, \) and \( \omega_t \) to the state variables of the system, \( k_t \) and \( a_t \), and their shadow prices, \( q_t \) and \( \varphi_t \). The remaining equations can be combined to create the fundamental difference equation system determining the dynamics of the state variables:

\[
\begin{align*}
  \gamma_k \varphi_t &= \beta \{ q_{t+1} \alpha (1-\alpha) z_{yt+1} a_{t+1}^{\alpha-1} l_{yt+1}^{\alpha} k_{t+1}^{1-\alpha} + \varphi_{t+1} \} \\
  q_t &= \beta q_{t+1} \{ (1-\alpha)^2 z_{yt} (a_t l_t)^{-\alpha} k_t^{1-\alpha} + [1-(1-\alpha)\delta] \} \\
  z_{yt} (q_t l_{yt})^\alpha k_t^{1-\alpha} &= \gamma_k k_{t+1} - (1-\delta) k_t + c_t \\
  \gamma_k a_{t+1} - a_t &= z_{at} \eta a_t^{\delta} l_{at}
\end{align*}
\]
Table 1: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Intertemporal substitution</td>
<td>$\sigma$</td>
<td>1.0</td>
</tr>
<tr>
<td>Consumption share</td>
<td>$\theta$</td>
<td>0.34</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital’s share</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Labor returns in R&amp;D</td>
<td>$\lambda$</td>
<td>0.664</td>
</tr>
<tr>
<td>Growth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population Growth</td>
<td>$\gamma_L$</td>
<td>1.00345</td>
</tr>
<tr>
<td>Output Per Capita Growth</td>
<td>$\gamma_A$</td>
<td>1.00458</td>
</tr>
</tbody>
</table>
Table 2:  
Cyclical Properties of U.S. Time Series*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to Y_t</th>
<th>Autocorrelation</th>
<th>Cross Correlations With Y_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2.039</td>
<td>1.000</td>
<td>0.875</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>0.973</td>
<td>0.477</td>
<td>0.875</td>
<td>0.933</td>
</tr>
<tr>
<td>I</td>
<td>5.896</td>
<td>2.892</td>
<td>0.900</td>
<td>0.849</td>
</tr>
<tr>
<td>N</td>
<td>2.120</td>
<td>1.040</td>
<td>0.900</td>
<td>0.607</td>
</tr>
<tr>
<td>Y/N</td>
<td>1.204</td>
<td>0.591</td>
<td>0.727</td>
<td>0.617</td>
</tr>
</tbody>
</table>

* Reported statistics are for data which have been detrended using a high-pass filter as described by Baxter and King (1995). The band width captures cycles of duration 2-120 quarters; a moving-average truncation of 12 periods was used. Output (Real GDP), consumption (non-durables plus services), and investment (fixed, private nonresidential) are from the BEA National Income and Product Accounts (chain-weighted 1993 dollars). Employment (business sector weekly hours) and productivity (business sector output per hour) data are from the BLS Productivity and Costs report. The sample period is 1952:1-1995:1 (after 12 quarters have been eliminated at each end of the sample as a result of the filtering technique).
Table 3: Productivity Shocks

**Panel A – Standard RBC Model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to $Y_t$</th>
<th>Autocorrelation</th>
<th>Cross Correlation With $Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>2.039</td>
<td>1.000</td>
<td>0.025</td>
<td>1.000</td>
</tr>
<tr>
<td>$C$</td>
<td>0.403</td>
<td>0.198</td>
<td>0.945</td>
<td>0.387</td>
</tr>
<tr>
<td>$I$</td>
<td>7.525</td>
<td>3.691</td>
<td>-0.020</td>
<td>0.990</td>
</tr>
<tr>
<td>$N$</td>
<td>1.343</td>
<td>0.659</td>
<td>-0.026</td>
<td>0.981</td>
</tr>
<tr>
<td>$Y/N$</td>
<td>0.766</td>
<td>0.376</td>
<td>0.271</td>
<td>0.941</td>
</tr>
</tbody>
</table>

**Panel B – The R&D Model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to $Y_t$</th>
<th>Autocorrelation</th>
<th>Cross Correlation With $Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>2.039</td>
<td>1.000</td>
<td>0.069</td>
<td>1.000</td>
</tr>
<tr>
<td>$C$</td>
<td>0.486</td>
<td>0.238</td>
<td>0.959</td>
<td>0.433</td>
</tr>
<tr>
<td>$I$</td>
<td>9.665</td>
<td>4.741</td>
<td>-0.006</td>
<td>0.983</td>
</tr>
<tr>
<td>$N$</td>
<td>1.090</td>
<td>0.535</td>
<td>-0.016</td>
<td>0.971</td>
</tr>
<tr>
<td>$Y/N$</td>
<td>1.014</td>
<td>0.497</td>
<td>0.265</td>
<td>0.966</td>
</tr>
</tbody>
</table>
### Table 4: Technology Shocks

**Panel A – No Persistence (\(\rho_A = 0\))**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to (Y_t)</th>
<th>Autocorrelation</th>
<th>Cross Correlation With (Y_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2.039</td>
<td>1.000</td>
<td>0.577</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>1.497</td>
<td>0.735</td>
<td>0.999</td>
<td>0.770</td>
</tr>
<tr>
<td>I</td>
<td>6.864</td>
<td>3.367</td>
<td>0.050</td>
<td>0.823</td>
</tr>
<tr>
<td>N</td>
<td>1.752</td>
<td>0.859</td>
<td>0.017</td>
<td>-0.654</td>
</tr>
<tr>
<td>Y/N</td>
<td>3.449</td>
<td>1.692</td>
<td>0.211</td>
<td>0.923</td>
</tr>
</tbody>
</table>

**Panel B – Some Persistence (\(\rho_A = 0.5\))**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to (Y_t)</th>
<th>Autocorrelation</th>
<th>Cross Correlation With (Y_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2.039</td>
<td>1.000</td>
<td>0.900</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>1.765</td>
<td>0.866</td>
<td>0.999</td>
<td>0.901</td>
</tr>
<tr>
<td>I</td>
<td>4.966</td>
<td>2.436</td>
<td>0.566</td>
<td>0.778</td>
</tr>
<tr>
<td>N</td>
<td>1.197</td>
<td>0.587</td>
<td>0.524</td>
<td>-0.453</td>
</tr>
<tr>
<td>Y/N</td>
<td>2.793</td>
<td>1.370</td>
<td>0.723</td>
<td>0.924</td>
</tr>
</tbody>
</table>
Table 5:  
Technology Shocks in the Presence of Labor Market Frictions

*Panel A – Small Quadratic Adjustment Costs*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to $Y_t$</th>
<th>Autocorrelation</th>
<th>Cross Correlation With $Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>2.039</td>
<td>1.000</td>
<td>0.865</td>
<td>1.000</td>
</tr>
<tr>
<td>$C$</td>
<td>1.711</td>
<td>0.869</td>
<td>0.999</td>
<td>0.899</td>
</tr>
<tr>
<td>$I$</td>
<td>4.989</td>
<td>2.447</td>
<td>0.413</td>
<td>0.774</td>
</tr>
<tr>
<td>$N$</td>
<td>1.162</td>
<td>0.570</td>
<td>0.382</td>
<td>-0.450</td>
</tr>
<tr>
<td>$Y/N$</td>
<td>2.763</td>
<td>1.355</td>
<td>0.639</td>
<td>0.927</td>
</tr>
</tbody>
</table>

*Panel B – Larger Quadratic Adjustment Costs*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to $Y_t$</th>
<th>Autocorrelation</th>
<th>Cross Correlation With $Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>2.039</td>
<td>1.000</td>
<td>0.974</td>
<td>1.000</td>
</tr>
<tr>
<td>$C$</td>
<td>1.939</td>
<td>0.951</td>
<td>0.999</td>
<td>0.976</td>
</tr>
<tr>
<td>$I$</td>
<td>3.148</td>
<td>1.544</td>
<td>0.681</td>
<td>0.837</td>
</tr>
<tr>
<td>$N$</td>
<td>0.495</td>
<td>0.243</td>
<td>0.676</td>
<td>-0.232</td>
</tr>
<tr>
<td>$Y/N$</td>
<td>2.208</td>
<td>1.083</td>
<td>0.925</td>
<td>0.976</td>
</tr>
</tbody>
</table>
Table 5 (Cont.):  
Technology Shocks in the Presence of Labor Market Frictions

*Panel C – Lagged Adjustment to R&D Labor*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to $Y_t$</th>
<th>Autocorrelation</th>
<th>Cross Correlation With $Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2.039</td>
<td>1.000</td>
<td>0.995</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>1.975</td>
<td>0.968</td>
<td>0.999</td>
<td>0.995</td>
</tr>
<tr>
<td>I</td>
<td>2.463</td>
<td>1.208</td>
<td>0.873</td>
<td>0.949</td>
</tr>
<tr>
<td>N</td>
<td>0.130</td>
<td>0.064</td>
<td>0.311</td>
<td>-0.440</td>
</tr>
<tr>
<td>Y/N</td>
<td>2.100</td>
<td>1.030</td>
<td>0.999</td>
<td>0.998</td>
</tr>
</tbody>
</table>
### Table 6:
The Contribution of Technology Shocks to Model Variance

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Fraction of variance attributable to “Technology” Shocks (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solow Residual</td>
</tr>
<tr>
<td>Baseline R&amp;D Model</td>
<td></td>
</tr>
<tr>
<td>( \rho_A = 0.0 )</td>
<td>9.9</td>
</tr>
<tr>
<td>( \rho_A = 0.5 )</td>
<td>25.6</td>
</tr>
<tr>
<td>Small Labor Adjustment Costs</td>
<td></td>
</tr>
<tr>
<td>( \rho_A = 0.0 )</td>
<td>26.0</td>
</tr>
<tr>
<td>( \rho_A = 0.5 )</td>
<td>38.4</td>
</tr>
<tr>
<td>Large Labor Adjustment Costs</td>
<td></td>
</tr>
<tr>
<td>( \rho_A = 0.0 )</td>
<td>67.1</td>
</tr>
<tr>
<td>( \rho_A = 0.5 )</td>
<td>65.4</td>
</tr>
<tr>
<td>Lagged Labor Adjustment</td>
<td></td>
</tr>
<tr>
<td>( \rho_A = 0.0 )</td>
<td>87.4</td>
</tr>
<tr>
<td>( \rho_A = 0.5 )</td>
<td>86.9</td>
</tr>
</tbody>
</table>

### Table 7:
Technology Shocks and Productivity Shocks

*(Large Labor Adjustment Cost Specification)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev. (Percent)</th>
<th>Std. Dev. Relative to ( Y_t )</th>
<th>Autocorrelation</th>
<th>Cross Correlation With ( Y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>2.039</td>
<td>1.000</td>
<td>0.875</td>
<td>1.000</td>
</tr>
<tr>
<td>( C )</td>
<td>1.613</td>
<td>0.791</td>
<td>0.997</td>
<td>0.867</td>
</tr>
<tr>
<td>( I )</td>
<td>5.698</td>
<td>2.794</td>
<td>0.642</td>
<td>0.817</td>
</tr>
<tr>
<td>( N )</td>
<td>0.717</td>
<td>0.352</td>
<td>0.628</td>
<td>0.383</td>
</tr>
<tr>
<td>( Y/N )</td>
<td>1.885</td>
<td>0.924</td>
<td>0.913</td>
<td>0.936</td>
</tr>
</tbody>
</table>
Figure 1:
Impulse-Responses to a Transitory Productivity Shock (zy)

- Capital (K)
- Knowledge (A)
- Labor in Final Output Sector (Ly)
- Labor in R&D Sector (La)
- Consumption (C)
- Leisure (Lu)
- Output (Y)
- Investment (I)
Figure 2:
Impulse-Responses to a Transitory Technology Shock (za)

- Capital (K)
- Knowledge (A)
- Labor in the Final Output Sector (Ly)
- Labor in the R&D Sector (La)
- Consumption (C)
- Leisure (Lu)
- Output (Y)
- Investment (I)