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Dynamic Shoe-Leather Costs in a Shopping-Time Model of Money

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Abstract

A general-equilibrium shopping-time model of money demand is used to obtain estimates of some dynamic costs of inflation under alternative monetary policy rules. After examining the welfare implications of steady-state inflation, dynamic welfare costs are evaluated for inflation-targeting and price-level targeting regimes in a stochastic setting in which agents are uncertain about the underlying inflation trend. The regimes are distinguished by the presence or absence of a unit root in the money supply and the price level. Uncertainty about the underlying inflation rate is introduced as a mechanism for modeling the role of policy credibility.

*The views expressed in this paper are those of the author, and do not necessarily represent official positions of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

Dynamic Shoe-Leather Costs in a Shopping-Time Model of Money

1. Introduction

General equilibrium models of the shopping time motivation for money demand, as described by McCallum and Goodfriend (1987), capture the essence of the transactions motive for money demand in the same spirit as the models of Baumol (1952) and Tobin (1956). In both settings, individuals trade off the convenience of using money to conduct transactions -- reflected in "shoe leather costs" of holding high balances -- against the opportunity cost of doing so, the nominal interest rate. In the Baumol-Tobin framework the cost of managing money balances is a brokerage fee, while in the shopping time model it is reflected by a time-cost of conducting transactions.[1]

As a framework for thinking about the welfare costs of inflation, the shopping time model provides a natural setting. Higher rates of inflation induce agents in the model to economize on real-money balances, requiring higher costs in terms of time spent conducting transactions. Indeed, Lucas (1994) showed that a shopping time model represents a general equilibrium relationship among real money balances, interest rates and spending that is analogous to a traditional money demand function, so that welfare costs of inflation can be measured using the traditional area under the demand curve approach.

In this paper I calibrate a shopping time model of money demand by thinking about the shoe-leather costs of inflation as being proxied by resources allocated to the

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[1]Karni (1974) discusses the role of the opportunity cost of time in a Baumol-Tobin framework. The shopping-time model of money is, in a sense, a general equilibrium version of his analysis.

financial sector. With this calibrated model in hand, I first demonstrate that the steady-state welfare costs of inflation it implies are on the same order of magnitude as estimates using other models or approaches. I then evaluate some *dynamic* costs of inflation and inflation uncertainty using the shopping time model. In the dynamic model, the money supply process is subject to two types of shocks. One shock, affecting the growth rate of money, represent uncertainty about the trend rate of inflation. The second shock affects the *level* of the money stock (and hence the price level) from path.

The time series properties of the level-shock are used to represent two types of policy regimes. In an inflation targeting regime, the level shock has a unit root. Deviations of money and prices from trend are not offset, but are fully accommodated by the monetary authority. On the other hand, if shocks to the level of the money stock are mean-reverting, deviations of money and prices from the inflation path are subsequently corrected.

The welfare effects of these policies are evaluated in two settings for the agents' information sets. In the first case, agents are able to distinguish the two types of shocks, and respond to each appropriately. In the second setting, agents observe only the current level of the money stock, and must estimate the impact of the two types of shocks using a signal extraction process. Hence, there is fundamental uncertainty about the path of money and inflation as agent learn about the nature of accumulated shocks to the money stock and its growth rate.

I find that the comparison of welfare costs for inflation targeting and price level targeting regimes differ depending on the information structure. Inflation targets are preferred under the full-information assumption because once-and-for-all shocks to the level of the money stock are neutral in the model. However, when agents are uncertain about whether an observed deviation of the money stock from it's path represents a level-shock or a growth shock, a price-level target can be preferable. This is so because the uncertainty prevents agents from fully reacting to either type of shock. The learning problem faced by agents results in delayed responses to persistent changes in the inflation rate and sharply dampened responses to transitory disturbances, so that monetary shocks in general result in less variability.

The limited-information feature of the model is similar to the analysis of "regime shifts" by, e.g., Andolfatto and Gromme (1997) and Dueker and Fisher (1998). However, because shocks to the inflation trend are always assumed to be mean-reverting, the uncertainty about policy modeled in this paper is of a more "routine" type. It might be thought of as uncertainty associated with an imperfectly credible inflation policy in which the monetary authority might tolerate persistent, if not permanent changes in the inflation rate. In a sense, this is a plausible characterization of present policy setting in the U.S. and other countries, in which monetary policy is considered to be on a generally successful course, but with people always harboring concerns about an outbreak of future inflation (or deflation, for that matter).

In the limited information setting of the model, the agents' (possibly subjective) knowledge of the relative magnitude and importance of the two type of shocks can be thought of as a measure of the credibility of policy. That is, increasing the perceived importance of the money growth shock relative to the level shock in the agents' information problem provides a means to model the welfare effects of the erosion of credibility that occurs when individuals perceive a greater likelihood that inflation will persistently deviate from its current trend. The model demonstrates that changes in credibility affect the magnitude of welfare costs and comparisons of different regimes.

In the following sections, I describe the set-up of the model, discuss its calibration and steady-state welfare implications, examine the dynamic costs of inflation policies in both full and incomplete information settings, and conclude with some comparisons illustrating the role of policy credibility in dynamic welfare comparisons.

2. A Shopping-Time Model

Preferences and Technology

A single representative agent maximizes a discounted stream of utility derived from consumption and leisure:

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{1}$$

where the utility function defines a composite good using a Cobb-Douglas function in C_t and L_t, and is CRRA with respect to the composite:

$$U(C_t, L_t) = \frac{1}{1-\sigma} (C_t^{\theta} L_t^{1-\theta})^{1-\sigma}$$

The total amount of time endowed to the agent each period is normalized to one, which can be allocated to leisure, work effort, and shopping time:

$$L_t + N_t + S_t = 1 \tag{2}$$

Shopping time depends on the quantity of real money balances, including both beginning of period money balances M_t' current monetary transfers, T_t.

$$S_t = S(\frac{M_t'/P_t + T_t/P_t}{C_t})$$

with S′(·)<0 and S″(·)>0. In particular, the calibrated version of the shopping-time function takes the form

$$S(\cdot_t) = \mu_1(\frac{M_t'/P_t + T_t/P_t}{C_t})^{-\mu_2} . \tag{3}$$

The parameters μ_1 and μ_2 determine the level and elasticity of the shopping time function. The agent faces a sequence of budget constraints given by:

$$Y_t + \frac{M_t'}{P_t} + \frac{T_t}{P_t} = C_t + I_t + \frac{M_{t+1}'}{P_t} \tag{4}$$

where investment, I_t, is gross capital accumulation:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5}$$

Output is produced using capital and labor via a constant returns to scale, Cobb-Douglas function:

$$Y_t = F(K_t, X_t N_t) = K_t^{\alpha}(X_t L_t)^{1-\alpha}. \tag{6}$$

where X_t represents labor agumenting technical progress, which is assumed to grow at a constant rate γ.

The money stock, M_t', is subject to both growth rate shocks and level shocks. It follows a process given by:

$$M_{t+1}' = G_{t+1}v_{t+1}, \quad with \quad v_{t+1} = v^{(1-\rho_v)}v_t^{\rho_v}\epsilon_{vt+1}$$

$$and \quad G_{t+1} = g_{t+1}G_t \quad with \quad g_{t+1} = g^{(1-\rho_g)}g_t^{\rho_g}\epsilon_{gt+1}$$

G_t is a money growth component which increases at (gross) rate g_t, and v_t represents a stochastic process driving deviations of the money stock from its growth trend. Both disturbances follow a first order autoregressive process with the exogenous shocks, ϵ_g and ϵ_v independently lognormally distributed.

Stationary Transformations

In order to examine the model's dynamics, the problem is first transformed to achieve stationarity. This involves adjusting the real variables for trend productivity growth (γ) and the nominal variables for trend money growth rate (g).

To adjust for productivity growth, divide all quantity variables by X_t. This implies two modifications to the model.[2] First, the capital accumulation equation becomes:

$$\gamma k_{t+1} = (1-\delta)k_t + i_t \,. \tag{5'}$$

[2]See King, Plosser and Rebelo (1988).

where lower case is used to represent the transformed stationary variables. Second, the transformation of consumption alters the effective rate of time preference. The new discount factor, β, is given by:

$$\beta = \beta\gamma^{\theta(1-\sigma)} .$$

The growth rate of nominal variables is determined by the growth rate of G_t, g. Dividing M_t' and P_t by beginning of period money balances G_t (yielding transformed variables m_t' and p_t), the nominal side of the model is rendered stationary. This modifies the budget constraint to be:

$$y_t + \frac{m_t'}{p_t} + \frac{t_t}{p_t} = c_t + i_t + g_t\frac{m_{t+1}'}{p_t} \tag{4'}$$

After transforming the model to achieve stationarity, the first-order conditions on the optimization of the representative agent define a stationary equilibrium. The full optimization problem and first-order conditions are described in Appendix A. The role of shopping time in the model can be highlighted with two equations derived from those conditions.

First, the representative agent's trade-off of consumption and leisure is distorted:

$$\frac{U_L(\cdot_t)}{U_C(\cdot_t)} = \frac{w_t}{1 + w_t S_C(\cdot_t)} \tag{8}$$

where $w_t = F_N(\cdot_t)$ is the real wage rate (the marginal product of labor). The marginal cost of consumption in terms of shopping time, $S_C(\cdot_t)$, serves as a wedge distorting the usual equalization of the agent's marginal rate of substitution between consumption and leisure with the wage rate.

The first-order condition for the agent's choice of money balances to carry forward, which can be expressed as

$$(1+i_t) = E_t\{1 + w_{t+1}S_M(\cdot_{t+1})\} , \tag{9}$$

reflects the trade-off of the opportunity cost of holding a dollar, the nominal interest rate, against the marginal benefit of lower future shopping time. For the assumed functional form of the shopping time function, (9) implies a "money demand" relationship:

$$\frac{m'_{t+1}}{p_{t+1}} = \left(\frac{\mu_1 \mu_2 w_{t+1} c_{t+1}^{\mu_2}}{i_t}\right)^{\frac{1}{1+\mu_2}}. \tag{9a}$$

3. Steady State Calibration and Welfare Costs

In the absence of shocks, the transformed model defines a stationary steady-state. The first-order conditions yield a set of relationships among steady-state variables which provide the basis for calibrating the model and examining the steady-state welfare costs of inflation.

Calibration

Parameters of the dynamic system are calibrated by matching long-run characteristics of the U.S. economy to the model's steady state solutions. Table 1 lists the key model parameters. Most have been selected to be consistent with previous calibrations of equilibrium business cycle models.[3] The steady state per-capita growth rate and the inflation rate are set at their long-run average values of 1.6% and 5% annually. Capital's share in production is set to 0.3 and the capital depreciation rate is 10% per year. The discount factor is .99, and the coefficient of relative risk aversion is set to equal 2. Leisure's share in overall utility, $(1-\theta)$, is selected to yield steady-state work effort as a fraction of the total time endowment at 0.3.

To calibrate the shopping-time function, I use the fraction of the labor force employed in the Finance, Insurance, and Real Estate (FIRE) sector -- plotted in Figure 1 -- as a point of departure. In modern, developed economies periods of high and variable inflation are associated with financial innovation and increased financial sector activity as individuals seek to minimize losses in the purchasing power of nominal assets. Increased employment in the financial sector therefore detracts from other productive activities and leisure, analogous to the "shopping time" paradigm of the model.[4] Figure 1 illustrates that the fraction of employment in the FIRE sector did, in fact, rise along with the rate of inflation over the period from the mid 1960s to mid 1980s.

The average value of the ratio over the sample period was approximately 6%. Obviously, not all activity in the FIRE sector is associated with shoe-leather costs of inflation, neither are all shoe-leather costs associated with activity in that sector (or in the market, in general). In an attempt not to overstate the share of "shopping-time"

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[3]E.g. Kydland and Prescott (1982); King, Plosser and Rebelo (1988); etc.

[4]Dotsey and Ireland (1996) cite a finding by Yoshino (1993) that inflation and employment in banking have been positively correlated over time in several countries.

represented by this admittedly crude measure I cut the estimate in half, setting the scale parameter of the shopping-time function, μ_1, to yield a value of 3% of total work effort.

The steady-state conditions of the model imply that shopping time is inversely related to the inflation rate. This can be used to pin down the curvature parameter of the shopping-time function, μ_2 (i.e., the elasticity of the shopping-time function). As shown in Figure 1, the FIRE employment ratio has varied from about 4.5 to 6.5 percent between 1964 and 1996. In order for the model to replicate this magnitude of variation in response to movements in trend inflation over the same period (e.g., inflation rates of between 2 and 10 percent), a curvature parameter of about 0.8 to 1.0 is appropriate. I have used a value of 1.0 so that the implied interest elasticity of money "demand" is equal to its conventionally measured value of one-half (see equation 9a).

The parameters of the stochastic processes for the two money shocks are described below in the dynamic section of the paper.

Steady-state Welfare Costs of Inflation

With a calibrated, steady-state version of the model in hand it is straightforward to calculate welfare costs of trend inflation. Table 2 provides a comparison of steady states for various inflation rates, where welfare costs are measured as a percentage of steady-steady state consumption which agents would agree to give up in order to make them indifferent between the given inflation rate and zero inflation.[5]

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[5]Specifically, the required compensation in terms of steady-state zero-inflation

$$[(1-\kappa)c_0]^{\upsilon}L_0^{1-\upsilon}=c^{\upsilon}L^{1-\upsilon}$$

The estimates shown in Table 2 are broadly consistent with previous studies. They are somewhat higher than the partial-equilibrium estimates of Fisher (1981) and Lucas (1981), which measure the area under conventional money demand functions [following Bailey (1956)] , but of the same order of magnitude as other general-equilibrium models examined in the recent literature [e.g. Cooley and Hansen (1989), Imrohoroglu (1992), Dotsey and Ireland (1996) etc.].[6] The relatively large welfare gains implied for moving from zero inflation to the optimal Friedman rule are consistent with the findings of Lucas (1994) and Dotsey and Ireland (1996).[7]

4. Dynamics

Log-linear Approximations and Dynamics

A log-linear approximation is used to evaluate the dynamic properties of the model. Expressing variables as proportional deviations from their deterministic steady-state values ($\hat{x}_t = \partial \ln(x_t) = \partial x/\bar{x}$), the first order conditions (7) yield a linear first-order

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consumption, κ, is defined by the relationship:
where the subscript 0 refers to steady state values at zero inflation.

[6]On the other hand, the welfare costs are generally much smaller than estimates which consider distortionary interactions of inflation with the tax code, as in Bullard and Russel (1997).

[7]Mulligan and Sala-I-Martin (1997) examine modifications to the shopping-time function to reflect a satiation level of real money balances as the optimal inflation rate is approached, which would considerably lower the estimates of welfare gains from moving to the optimal rate. The steady-state welfare costs reported here are not intended to provide new evidence, but to demonstrate that the calibrated model is consistent with welfare costs previously found in the literature.

difference equation system. Solving this system by standard techniques yields a set of decision rules for consumption, leisure, work effort, etc., in terms of the underlying state variables and exogenous variables of the system.[8]

Figures 2a and 2b illustrate impulse response functions for some of the key model variables following a one percent level shock and growth shock, respectively. For the purposes of illustration, the autoregressive parameter of each shock is set to 0.9. The first notable feature of the two figures is that the responses to the two shocks are mirror images of each other, in a constant proportion. The responses of model variables to an growth shock are ten times as large as the responses to a level shock.

This observation highlights the role of inflation expectations in generating the non-neutralities of the shopping-time model. A positive money growth shock of one percent is associated with the expectation that M_{t+1} will be one percent higher than M_t. A one percent level-shock implies that the deviation of money from its growth path will be one-tenth of one percent lower in the subsequent period because $\rho_v = 0.9$. Hence, inflation expectations are similarly symmetrical.

Just as Cooley and Hansen (1989) illustrated for a basic cash-in-advance model, an increase in expected inflation induces agents to substitute away from market activity in favor of leisure. However, the inclusion of a shopping-time function adds a new margin of substitutiblity. As illustrated in Figure 2b, the desire to economize on money balances results in an increase in shopping time. The combination of this substitution of shopping-time for leisure and labor in the goods-producing sector with the associated negative wealth effect results in a decline in leisure rather than an increase as in a cash-in-advance

[8]The approach used to solve the system follows King, Plosser and Rebelo (1989).

model. Work effort in the final output sector also declines by more than it would in a cash-in-advance environment. The increase in desired shopping-time is also associated with an increase of the real wage rate. The persistence of the decline in work effort lowers the marginal product of capital so that investment demand decreases. Hence, unlike the cash-in-advance model, a positive money growth shock results in a drop in consumption, output *and* investment.

Although the real effects of monetary policy in the shopping-time model are generated exclusively by responses to expected inflation, the qualitative nature of the impulse response functions illustrated in Figure 2 is consistent with a wide range of models in which monetary injections can have short run effects that increase economic activity, but in which long-run inflation is costly. It is precisely this property which turns out to be crucial in comparing the welfare costs of price-level and inflation targeting regimes under full-information versus limited information.

Measuring Dynamic Welfare Costs

The decision rules of the linearized system used to generate impulse-response functions for consumption and leisure are of the form:

$$\hat{c}_t = a_1 \hat{k}_t + a_2 \hat{v}_t + a_3 \hat{g}_t \tag{8a}$$

$$\hat{l}_t = b_1 \hat{k}_t + b_2 \hat{v}_t + b_3 \hat{g}_t \tag{8b}$$

These decision rules provide the basis for calculating dynamic welfare costs of stochastic policy regimes. Fluctuations in the money stock due to the two exogenous shocks give rise to variability in consumption and leisure, lowering expected utility. These welfare costs can be approximated by a transformation of the variances of consumption and leisure by exploiting the model's underlying log-normal distribution.[9]

Letting $D_t = C_t^{\theta} L_t^{1-\theta}$ represent the composite utility-producing commodity, the welfare costs of variability are approximated by the equation

$$\hat{\kappa}_d = \frac{\sigma}{2} var(D)$$

which expresses the welfare cost in terms of a fraction of steady-state D that an agent would give up to be compensated for living in the stochastic world. It is then a straightforward matter to convert this measure to represent a fraction of steady state consumption.

For these exercises, the stochastic processes for the monetary disturbances are based on estimated time series properties of M1 and M2 for the period 1959 to 1996. Table 3a reports parameter values estimated for the two aggregates. The firsts two rows of Table 3a show estimated autocorrelation coefficients and shock variances under alternative assumptions that only the growth shock or the level shock drives money stock fluctuations. The parameters for the former are estimated using logged first-differences, while those for the latter are based on log deviations from a Hodrick-Prescott filter.

Table 3a also reports estimates of the parameters for a set of measures decomposing the two shocks. The growth shock is proxied by a moving average of money growth -- three year, four year, and five year moving averages are considered. Deviations of quarterly monetary growth rates from this moving average trend are then

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[9]The approach taken here follows Lucas (1987).

used to construct a series for the level-shock, v.

Table 3b shows the parameter values used in the dynamic analysis of the model. They have been selected to be generally consistent with the estimated parameters for the monetary aggregates shown in Table 3a.

Dynamic Welfare Costs with Perfect Information

Table 4a reports the welfare costs of consumption and leisure variability when the sources of disturbances to the money stock are known by agents. The first column shows the baseline parameterization. The first notable features of these estimates is that they are tiny. This is consistent with Lucas' (1987) observation that the welfare costs of consumption fluctuations are generally quite small relative to the costs of lower growth. Moreover, the real effects of money shocks in the shopping-time model are small.[10]

When the money supply process is assumed to be generated by one or the other of the shocks alone, the welfare cost measures are of comparable magnitude. However, the estimate for the combination of the two shocks is much higher. As shown in the final two rows of Table 4a, the increased variability is attributable primarily to the money growth shock. Even though its variance represents a small fraction of total money stock variability, the high autocorrelation coefficient implies very persistent movements in the "trend" rate of inflation. As a result, the shoe-leather costs of avoiding expected inflation following a positive innovation are sharply higher. The shoe-leather benefits of a negative

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[10]Plausible modifications to the calibration of the model (e.g. increasing the steady-state shopping time) might result in measured costs of a greater magnitude, but the interest here is to compare the welfare costs of various policy settings, setting aside the issue of scale.

innovation are large as well, so consumption and leisure are more variable.

Due to the method used to decompose money growth shocks and level shocks, the calibrated parameters of the model implicitly define a price-level targeting regime. With $\rho_v < 1$, the money stock and price level are trend-stationary: past deviations of the money stock from its target path are corrected over time. The second column of Table 4 shows the welfare cost of monetary variability for an alternative hypothesis of $\rho_v = 1$.[11] In this type of regime, deviations of the money stock from its path are not offset, so that there is base-drift in the money stock and price level. This type of time-series process is tantamount to an inflation targeting regime.[12]

Comparing the welfare costs of a price-level targeting regime in the second column of Table 4a with those of an inflation targeting regime in first column, it is clear that the inflation target is preferred. Under the assumption that $\rho_v = 1$, the costs of shocks to the level of the money stock are nil. This result is due to the monetary neutrality that exists when shocks to the money stock are permanent. When this is the case, increases in the level of the money supply contain no information about future expected inflation, so they give rise to a simple quantity theory outcome: all nominal magnitudes rise in proportion to the money shock, with all real quantities unchanged.

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[11]The standard deviation of the underlying disturbance, $\sigma_{\epsilon v}$, is adjusted for the unit-root case so that the variance of the *growth rate* of v is equal for both assumptions about the regime.

[12]This method of parameterizing the difference between inflation and price-level rules has been used in other settings, e.g., Black, et al (1997).

A Limited Information Setting

Uncertainty about the policy regime is introduced to the model by restricting information about the particular origin of shocks to the money stock. Agents observe only the current and past levels of the money stock, which is subject to the two independent disturbances, ϵ_{vt} and ϵ_{gt}. Consequently, agents must solve a signal extraction problem in order to estimate a decomposition of the two shocks. It is this estimate which determines expected future money stock and price level changes in the limited-information setting. The agents are initially assumed to correctly know the true parameters of the underlying time-series processes, including the relative variances of the two disturbances. With this information in hand, agents estimate the proportion of money stock fluctuations which are due to the two disturbances from a simple signal-extraction problem. Perceived shocks to the money supply are attributed to growth-shock and level-shock components in proportion to their relative variances, and subsequent money supply forecasts are generated from the first-order autocorrelation coefficients on the two shock processes. Subsequent perceived shocks to the money supply process are calculated as forecast errors, and the process continues recursively. Appendix B describes the signal extraction problem in more detail.

Dynamic Welfare Costs with Imperfect Information

The first row of Table 4b shows the welfare costs of the two alternative inflation policies in the incomplete information setting. Notice that the welfare costs are much *lower* than for the full information case. This is due to the fact that agents are uncertain about the source of shocks to the money supply process, so in general they under-react.

To the extent that a perceived shock is presumed to be a level shock, agents expect either a subsequent correction of the deviation or a permanent shift in the level of the money stock, depending on the type of regime. The responses to a level shock under a price level targeting regime are in the opposite direction to responses to growth shock. Hence, they tend to *offset* responses to the portion of a shock that is presumed to represent a deviation of the growth trend.

As in the complete information setting, the inflation targeting regime is associated with lower welfare costs than the price-level targeting regime for the baseline parameterization. The reasons for this outcome more complex in this case, however. The representative agent's responses to perceived level-shocks so overwhelms the responses to growth shocks with a price-level target that the elimination of responses to level-shocks from moving to an inflation-targeting regime leaves only the modest responses to growth shocks.

However, the observation that responses to perceived level-shocks and growth shocks tend to offset one another in a price-level targeting regime raises the possibility that the welfare rankings of the two regimes might be reversed in the incomplete information setting. A stark example of this possibility can be shown for the special case associated with the parameter values used for the impulse-response functions in Figure 2, $\rho_v = \rho_g = 0.9$. Because the two autoregressive parameters are equal, agents allocate forecasts of future money stock movements equally to the two types of disturbances. Recall that the responses to growth shocks and level shocks were in the proportion of 10 to 1. When the ratio of the variance of level-shocks to that of growth shocks is also assumed to be 10 to 1, responses to perceived shocks under a price level regime exactly offset one another. The welfare cost of monetary fluctuations is literally zero. In an inflation regime, however, agents respond to disturbances which are perceived to be growth shocks but not to level shocks, so that there is a positive cost in terms of consumption and leisure variability.

As shown in the second and third rows of Table 4b, a slight modification to the calibrated standard deviations of growth shocks and level shocks is sufficient to reverse the relative welfare costs of the two regimes. When the standard deviation of innovations to the growth-shock process are increased relative to that of the level-shock process, the price-level targeting regime yields *lower* welfare costs. The absence of responses to level-shocks under an inflation targeting regime results in higher overall variability of consumption and leisure.

Credibility

In the signal-extraction problem solved by agents in the limited information setting, the ratio of variances of the two disturbances is a fundamental parameter. If we allow for the possibility that this ratio is not known, but evaluated subjectively, we can represent a measure of credibility possessed by the monetary authority. The higher is the perceived variance of growth shocks relative to level shocks, the less confidence agents have in their expectations of money growth and inflation.

The last two rows of Table 4b report the welfare costs of the two regimes when the actual variances of disturbances is the baseline case, but in which the perceived variance of the money growth shock are higher. These welfare comparisons show that it is the *perception* of relative variances which is crucial in generating the result that a price level target can be preferable to an inflation target. When agents consider a change in the underlying growth rate of money and inflation to be a possibility, the allocation of observed money stock deviations to potential changes in the growth trend makes the real effect of those shocks more pronounced when they are not offset by responses to temporary level shocks, as is the case under inflation targeting.

6. Summary and Conclusions

This paper has examined the issue of price-level versus inflation targeting in the context of a simple shopping-time model of money in which the nature and persistence of shocks to the money supply process are ambiguous and uncertain. In so doing, the analysis has demonstrated the importance of uncertainty in characterizing the tradeoff of the two regimes.

As pointed out by Svensson (1996), analyses of price-level versus inflation targeting have often focussed on the tradeoff between short-term and long-term variability. Because a price level target inherently implies less uncertainty about deviations of prices from expectations far into the future, a price-level targeting regime is preferable if the costs of that uncertainty are important relative to the short-term variability implied by central bank efforts to offset temporary deviations of the price level from its target path. In the model framework utilized in this paper, such long-run considerations are absent. Nevertheless, a price level target can be preferable when agents have incomplete information about the nature and persistence of shocks to the money supply process. In this type environment, reactions to observed shocks give rise to smaller real effects, generating lower short-term variability of output and consumption.

Appendix A: The Transformed Optimization Problem and First-Order Conditions

After transforming the model to induce stationarity, the first-order conditions describe a stationary equilibrium. Substituting (5) and (6) into the resource constraint (4), and substituting the shopping-time specification (3) into the time constraint (2), yields the following dynamic optimization problem for a hypothetical social planner:

$$\max \sum_{t=0}^{\infty} \beta^t (c_t^{\theta} L_t^{1-\theta})^{1-\sigma}/(1-\sigma)$$

Subject to:

$$L_t + N_t + w_1(m_t^{/}/p_t + t_t/p_t)^{-w_2} = 1 \tag{A1}$$

$$F(k_t,N_t) + \frac{m_t^{/}}{p_t} + \frac{t_t}{p_t} = c_t + \gamma k_{t+1} - (1-\delta)k_t + g_t\frac{m_{t+1}^{/}}{p_t} \tag{A2}$$

Each period, the representative agent chooses c_t, l_t, N_t, k_{t+1}, and m_{t+1}'. Letting ω_t and λ_t be the multipliers on constraints (2') and (4') [the shadow values of leisure time and capital], the maximization problem yields the following first-order conditions:

$$U_c(c_t,L_t) = \lambda_t + [w_1 w_2 (\frac{m_t/p_t}{c_t})^{-w_2} c_t^{-1}]\omega_t \tag{A3a}$$

$$U_L(c_t,L_t) = \omega_t \tag{A3b}$$

$$\omega_t = \lambda_t F_N(k_t, N_t) \tag{A3c}$$

$$\gamma\lambda_t = \beta E_t\{\lambda_{t+1}[F_k(k_{t+1}, N_{t+1}) + (1-\delta)]\} \tag{A3d}$$

$$g_t \frac{\lambda_t}{p_t} = \beta E_t\{\frac{\lambda_{t+1}}{p_{t+1}} + \frac{\omega_{t+1}}{p_{t+1}}[w_1 w_2 (\frac{m_{t+1}/p_{t+1}}{c_{t+1}})^{-w_2-1} c_{t+1}^{-1}]\} \tag{A3e}$$

$$L_t + N_t + w_1(\frac{m_t/p_t}{c_t})^{-w_2} = 1 \tag{A3f}$$

$$F(k_t, N_t) = c_t + \gamma k_{t+1} - (1-\delta)k_t \tag{A3g}$$

where the post-transfer money stock, $m_t = m_t' + t_t$, has been substituted into equations (A3).

Note that the marginal utility of consumption in (A3a) is subject to a shopping-time wedge that incorporates the opportunity cost of time (ω). Equation (A3e) reflects the choice of money balances carried into the subsequent period, which determines the equilibrium price level.[13] The solution for p_t, along with the solution for λ_t from equation (7d), will be forward-recursive, representing co-states of they system. Equations (7g) is the fundamental difference equation for the state variable (k). Hence, equations (A3d), (A3e) and (A3g) comprise the fundamental first-order difference equation system will determine the models' dynamic solution.

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[13]Substituting for the interest rate on nominal bonds,

$$(1+i_t) = \frac{g_t P_{t+1}}{P_t} \frac{\lambda_t}{\beta\lambda_{t+1}}$$

equation (A3e) yeilds equation (9) in the text.

Appendix B: The Signal-Extraction Problem Under Incomplete Information

Letting a tilde (~) represent perceived values of the components of the money stock process, next period's growth component can be calculated recursively to be

$$\tilde{G}_{t+1} = \bar{g}^{1-\rho_g}\,\tilde{g}_t^{\rho_g}\,\tilde{G}_t \quad with \quad \tilde{G}_t = \prod_{j=0}^{t} \tilde{g}_{t-j}$$

The forecasted value for next period's money stock is then

$$\tilde{M}_{t+1} = \tilde{G}_{t+1}\tilde{v}_t^{\rho_v} .$$

Forecasted values for the exogenous variables in subsequent periods are extracted from the forecast error:

$$\tilde{g}_{t+1} = \phi e_{t+1} \quad and \quad \tilde{v}_{t+1} = (1-\phi)e_{t+1} ,$$

where the forecast error is

$$e_{t+1} = M_{t+1} / \tilde{M}_{t+1} .$$

The parameter ϕ is the probability that observed shock is an innovation to the growth process,

$$\phi = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_v^2} .$$

Figure B shows paths of the money stock following reduction in the trend rate of inflation from three percent to two percent, along with one step ahead forecasts using the procedure outlined above. Forecast errors, upon which are based the perceived shocks the system, are shown in the lower panel of Figure B (generated under the assumption that ϕ=0.10).

For the given parameterization, it about 10 quarters from the time of the shock for agents to fully recognize the change in trend. Initially, the deviation is attributed partly to both shocks, with corrections made gradually over time.

This signal extraction problem can be transformed to implement in the log-linearized version of the model economy by providing agents with a signal $\hat{S}_t$ that consists of the cumulative sum of deviations from the steady-state growth path,

$$\hat{S}_t = \hat{v}_t + \sum_{j=0}^{t} \hat{g}_{t-j}.$$

The optimal forecast for next periods money stock, the forecast error, and the formation of beliefs about shocks to the money supply process become

$$\hat{g}_{t+1} = \phi \hat{e}_{t+1} \quad and \quad \hat{v}_{t+1} = (1-\phi)\hat{e}_{t+1} \ ,$$

with

$$\hat{e}_{t+1} = \hat{S}_{t+1} - \tilde{S}_{t+1}, \quad and \quad \tilde{S}_{t+1} = \rho_g \hat{g}_t + \rho_v \hat{v}_t + \sum_{j=0}^{t} \hat{g}_{t-j} \ ,$$

where variables with the circumflex (^) are to be interpreted here as *perceived* deviations of the underlying variables from their steady-state values.

The linearly approximated version of the signal extraction problem of this form yields paths for e_t which differ from the underlying nonlinear paths by only hundredths of a percentage point for the example shown in Figure B. The fact that linearly approximated version of the signal extraction problem accurately tracks true forecast errors for this example of a large shock demonstrates the accuracy of the approximation.

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Table 1: Basline Parameter Values

	Description	Symbol	Value
Preferences	Discount Factor	β	0.99
	Intertemporal Substitution	σ	2
	Consumption Share	θ	0.322
Technology	Capital's Share	α	0.3
	Capital Depreciation Rate	δ	0.025
Shopping Time	Scale parameter	μ_1	0.0111
	Curvature parameter	μ_2	1
Growth Trends	Technology growth	γ	1.004
	Money growth	g	1.123

Table 2: Effects and Welfare Costs of Inflation in the Steady-State

	Relative to Zero Inflation (%)			
			Welfare Cost as a percentage of	
Inflation Rate (%)	Consumption	Leisure	Consumption	Output
(β-1)/β	1.90	0.06	-2.05	-1.59
1	-0.22	-0.01	0.24	0.19
3	-0.61	-0.02	0.65	0.50
5	-0.93	-0.03	0.99	0.77
10	-1.61	-0.05	1.70	1.32

Table 3a:
Estimated Parameters of the Supply Money Process

	Autoregressive Parameters		Standard Deviations (Percent)	
	ρ_v	ρ_g	$\sigma_{\varepsilon v}$	$\sigma_{\varepsilon g}$
One shock				
v-shocks only				
M1	.926	—	.916	—
M2	.881		.669	
g-shocks only				
M1	—	.688	—	.824
M2		.730		.602
Two shocks				
4 year moving avg.				
M1	.898	.932	.768	.200
M2	.877	.987	.612	.096
5 year moving avg.				
M1	.920	.939	.891	.154
M2	.898	.985	.689	.099
6 year moving avg.				
M1	.923	.954	1.056	.116
M2	.911	.982	.734	.104

Table 3b:
Baseline Calibration of the Money Supply Process

	Autoregressive Parameters		Standard Deviations (Percent)	
	ρ_v	ρ_g	$\sigma_{\varepsilon v}$	$\sigma_{\varepsilon g}$
One shock				
v-shocks only	.90	—	.80	—
g-shocks only	—	.70	—	.70
Two shocks	.90	.96	.75	.12

Table 4a:
Welfare Costs of Monetary Fluctuations with Perfect Information
(as a percent of steady-state consumption – in *basis points*)

	$\rho_v = \hat{\rho}_v$	$\rho_v = 1$
One shock		
v-shock only	.000290	.000000
g-shock only	.001427	.001427
Both shocks	.005545	.005289
Due to v-shocks	.000256	.000000
Due to g-shocks	.005289	.005289

Table 4b:
Welfare Costs of Monetary Fluctuations with Limited Information
(as a percent of steady-state consumption – in *basis points*)

	$\rho_v = \hat{\rho}_v$	$\rho_v = 1$
Baseline	.000769	.000615
$\sigma_{\varepsilon g} = .120$		
Modified Variances		
$\sigma_{\varepsilon g} = .130$	.000831	.000877
$\sigma_{\varepsilon g} = .140$	.001077	.001207
Low Credibility		
$\tilde{\sigma}_{\varepsilon g} = .130$	.000715	.000759
$\tilde{\sigma}_{\varepsilon g} = .140$	.000811	.000917

Figure 1:

Ratio of FIRE to Aggregate Weekly Hours

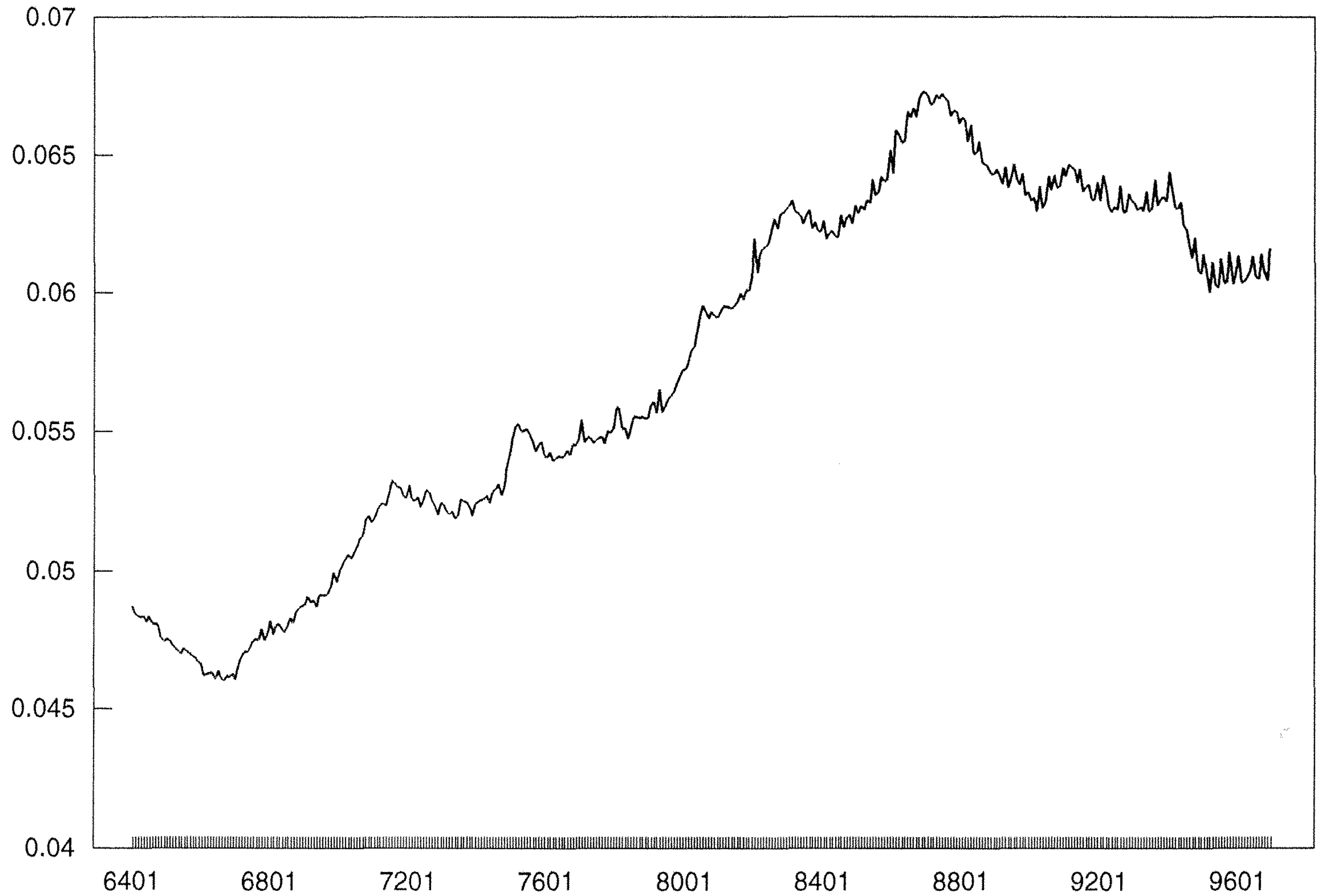

Figure 2A: Response to a Positive Money Shock

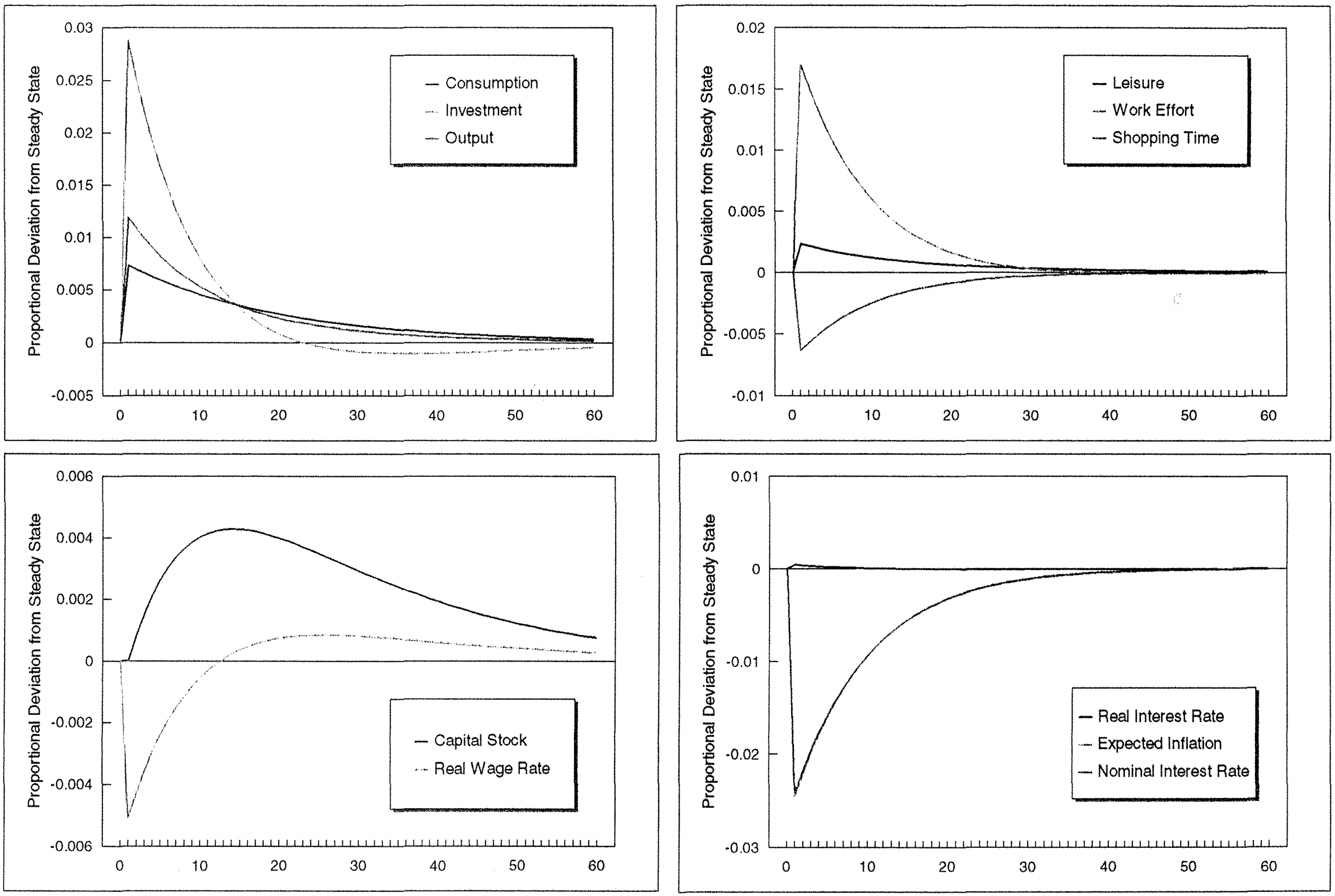

Figure 2B: Response to a Positive Money Growth Shock

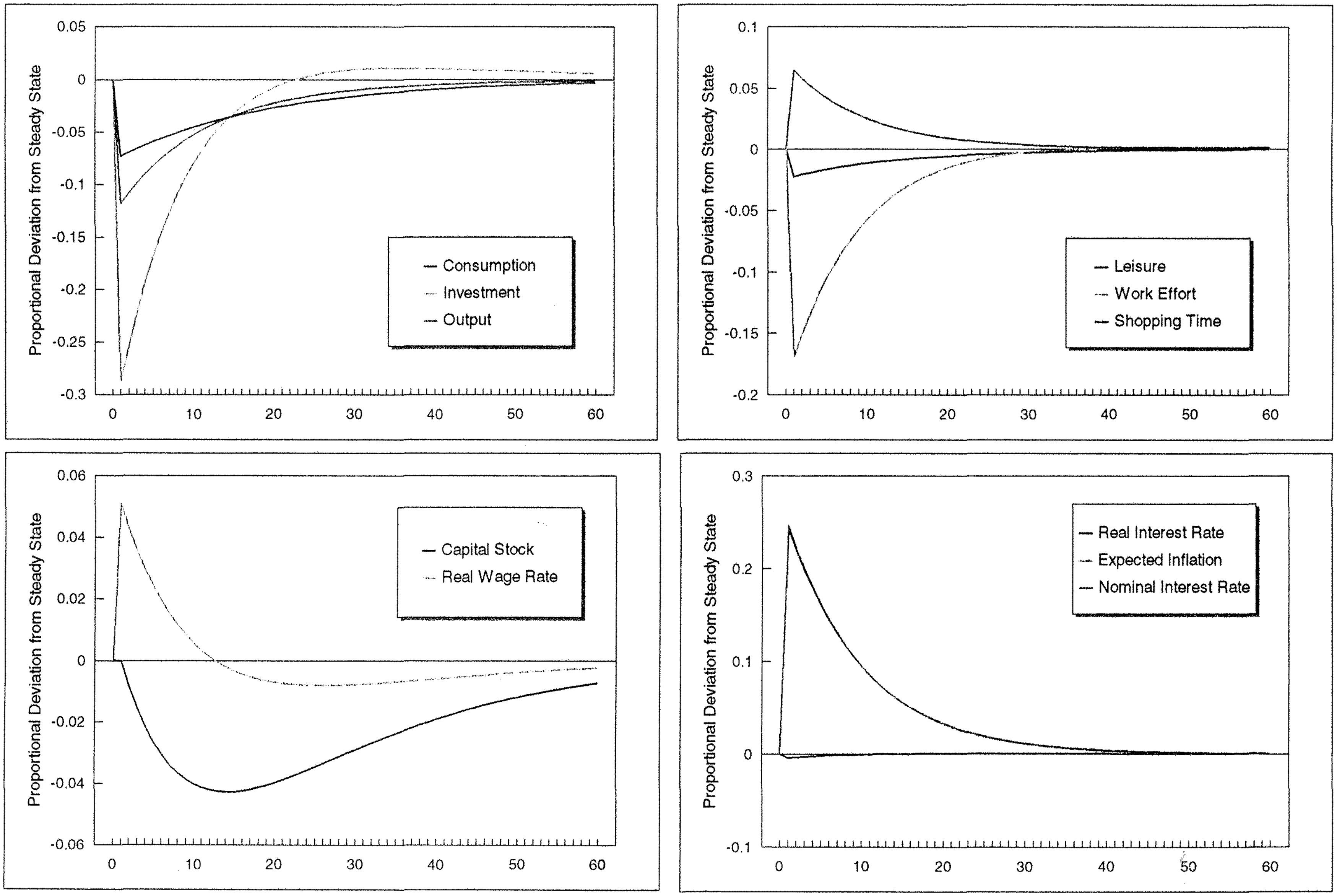

Figure B:
Learning About a Change in the Growth Trend