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| **Working Paper Number** | 1997-012B |
| **Citable Link** | https://doi.org/10.20955/wp.1997.012 |
| **Suggested Citation** | Bullard, J., Russell, S.H.; How Costly is Sustained Low Inflation for the U.S. Economy?, Federal Reserve Bank of St. Louis Working Paper 1997-012. URL https://doi.org/10.20955/wp.1997.012 |

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| **Published In** | Federal Reserve Bank of St. Louis Review |
| **Publisher Link** | https://doi.org/10.20955/r.86.35-67 |

# How Costly is Sustained Low Inflation for the U.S. Economy?

**March 1998**

**Abstract**

We study the welfare cost of inflation in a general equilibrium life cycle model with growth, costly financial intermediation, and taxes on nominal quantities. We find a stationary equilibrium of the model matches a wide variety of facts about the postwar U.S. economy. We then calculate that the inflation policy of the monetary authority has welfare consequences for agents that are an order of magnitude larger than existing estimates in the literature. These effects are large even at very low inflation rates. The bulk of the welfare cost of inflation can be attributed to the fact that inflation increases the effective tax rate on capital income.

**Keywords:** inflation, welfare, taxation

**JEL Classification:** E4, E5

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For useful discussant comments, we thank Bruce Smith at the October 1997 Penn/FRB-Philadelphia conference *Macroeconomic Theory and Monetary Policy*, Roger Farmer at the October 1997 Rocheser/UCLA/FRB-Minneapolis conference *Dynamic Models of Economic Policy*, Wilbur John Coleman II at the Summer 1997 North American Econometric Society meetings, and, on a very preliminary version, Frank Gong at the November 1995 Federal Reserve System Committee on Macroeconomics. We also thank seminar participants at the Bank of England, Cornell, Indiana, Midwest Macroeconomics, Rutgers, Simon Frazer, Texas, and Western Ontario for helpful feedback on preliminary versions.

## 1. Introduction

**1.1. Overview.** In this paper we study a quantitative-theoretic general equilibrium model in which changes in monetary policy can have important welfare consequences for economic agents. Our main finding is an estimate of the welfare cost of inflation which is an order of magnitude larger than estimates found in the literature. The most important reason for this finding is that, in the economy we study, higher inflation reduces real returns to all assets in a quantitatively important way, and therefore forces agents to alter their life cycle consumption, work and savings plans. This creates a large distortion, because real interest rates are the central allocation mechanism in the neoclassical growth framework we use.

Economic growth in the model is driven by exogenous technological change and labor force growth. We use standard specifications for preferences and technology, and we employ standard rational expectations, market-clearing assumptions throughout. We include costly financial intermediation, transactions money demand, and tax elements, including taxation of nominal quantities, but we maintain the principle that these features complicate the analysis no more than strictly necessary. Agents gain utility by consuming a single nonstorable good and by taking leisure; they have finite lives and save for life-cycle reasons by holding government bonds, renting physical capital to firms, and making consumption loans to other households. Monetary policy is a growth rate for the monetary base which translates into a steady state rate of inflation. In this paper we study steady states, mostly to allow direct comparison to other research in this area. We estimate a stationary equilibrium of the model that matches a variety of important facts about the postwar U.S. economy.

From this brief description of our model it is already clear that it includes a number of distortions other than inflation. These distortions include costly financial intermediation, direct proportional taxation of labor and capital income, and government debt that returns relatively low real interest rates. The large welfare cost of inflation that we report is largely due to [1] the interaction of inflation with taxation of nominal income from capital and [2] the effect of this interaction on the before- and after-tax real interest rates on government debt. Although our intermediation-cost assumptions play an important role in allowing our model to replicate key features of postwar-U.S. data, they have little effect on the magnitude of our welfare-cost estimates.

From a policy perspective, the fact that inflation derives most of its welfare effects from interaction with the tax system need not be a concern, since it is total welfare cost of inflation, taking other features of the economy as given, that is the crucial yardstick by which judgements must be made. But from a theoretical perspective, as well as for purposes of comparison to other research in this area, one would like to know how much of the total cost of inflation is a "purely monetary" cost—a cost of the type studied in the literature on the optimum quantity of money—and how much of it is due to inflation interacting with the tax system. For this reason, we devote some effort to estimating the size of various welfare effects of inflation and attributing them to the appropriate sources. The bottom line of this exercise is that in our model the purely monetary welfare cost of inflation is about as large as previous estimates in the literature. Thus, the large welfare cost of inflation that we report is due to the interaction of inflation with the tax system.

We are able to go further and decompose this tax interaction into two components, each of which involves the effect of inflation on real rates of return. The first component, a *direct tax effect,* occurs because higher inflation increases effective capital income tax rates, reducing the after-tax real rate of return associated with any before-tax real rate of return. This type of effect has been studied by Feldstein (1996), among others. The second component, a *general equilibrium effect,* involves the tendency of higher inflation to reduce *before-tax* real interest rates. It is attributable to the fact that, all else equal, increases in the inflation rate produce significant increases in government revenues. These revenue increases result in an endogenous reduction in the stock of debt the government must roll over and the pretax real interest rate on this smaller stock of debt. Effects of this type have not been described previously, even by other researchers who have studied the welfare cost of inflation in general equilibrium models.

Decomposing the overall welfare effect of inflation into these two components leads to an important finding: locally, in the neighborhood of our baseline equilibrium, the general equilibrium effect of a change in the inflation rate is about $3\frac{1}{2}$ times as large as the direct tax effect. Thus, the general equilibrium nature of our analysis is essential to our findings, as is the ability of the particular general equilibrium model we employ to give government policy the power to have a significant impact on pretax real interest rates.

We also report results on the nature of disinflation policies. The government in our model, like the U.S. government, relies on seigniorage for a relatively small portion of its financing. It can reduce inflation from moderate levels without increasing existing tax rates or reducing the ratio of government expenditures to real output. However, our analysis suggests that beyond a certain critical low rate of inflation this is no longer possible. Further disinflation must then be accompanied by a reduction in government expenditures relative to output, an increase in other distortionary taxes, or both. In other words, government budget problems become more severe once inflation moves below the critical rate. We estimate that the U.S. economy is currently near the critical rate, and so we look at this problem in some detail. We conclude that there are still benefits from moving to lower inflation rates, even if other distortionary taxes are increased in order to maintain government spending at its pre-disinflation share of real output. In particular, if the lost revenues are replaced by increasing all direct taxes equally in percentage point terms, then the welfare gain produced by moving to zero inflation from our baseline inflation rate of four percent is about 2.86 percent of the baseline level of output, or about \$217 billion in 1996 dollars. However, our estimates also indicate that once the critical inflation rate is reached, the marginal benefit from lower inflation may be much smaller if the fiscal authority reacts to a loss of revenue by increasing other, more distortionary taxes. Thus, our analysis is somewhat sobering on the prospects for disinflation beyond certain limits, since it implies that reducing the inflation rate below the critical rate requires coordination (tacit or otherwise) between the fiscal and monetary authorities—coordination that is not necessary when the monetary authority disinflates from higher levels.

**1.2. Recent related literature.** The welfare cost of inflation literature is large, and we cannot summarize it effectively here. Recent quantitative estimates of the welfare cost of inflation in general equilibrium models include Cooley and Hansen (1989), Dotsey and Ireland (1996), Gomme (1993), Haslag (1995), İmrohoroğlu and Prescott (1991), Jones and Manuelli (1993), Lacker and Schreft (1996), and Lucas (1994). This list is representative, though not exhaustive, and we will use the results of several of these studies as a benchmark comparison for our results. A recent paper on the interaction of inflation with the tax code is Feldstein (1996). Feldstein (1996) does not attempt a full general equilibrium analysis, but nearly all of the discussion, including the effect of inflation in reducing real rates of return received by savers, as well as the implications for government finance, is consistent with the general equilibrium of our model. A conference commentary on Feldstein (1996) by Abel (1996) analyzes tax interaction effects in a general equilibrium framework considerably different from ours, and finds a significant welfare cost, but one that is about half as large as the welfare cost isolated in this paper.[1] Important work on banking in computable general equilibrium models which we view as related to ours is by Díaz-Giménez, Prescott, Fitzgerald and Alvarez (1992) and Prescott (1993).

[1]Similarly, Black, *et al.*, (1994), isolate a welfare cost of inflation coming mostly from inflation interacting with nominal taxation in a general equilibrium model calibrated to Canadian data. But their comparable effects are less than one-fifth the size of the effects isolated in this paper (see their Table 3, second column, last two entries). Interestingly, the endogenous growth and open economy versions of their model suggest that such extensions enlarge the calculated welfare cost of inflation significantly. We think this is an important area for future research.

## 2. A GENERAL EQUILIBRIUM LIFE CYCLE MODEL

**2.1. Overview.** Our model can be succinctly described as "Sargent and Wallace meet Auerbach and Kotlikoff." Auerbach and Kotlikoff (1987) pioneered the study of general equilibrium life cycle models for the study of public finance questions, but the framework has rarely been applied to questions in monetary theory.[2] Our model differs from Auerbach and Kotlikoff (1987) mainly in that we have monetary elements, explicit growth and depreciation, and a production technology compatible with real business cycle models.[3] Sargent and Wallace (1981, 1982, 1985) used small-scale overlapping generations models to study fundamental questions in monetary theory and policy. We have adopted aspects of their approach, such as careful attention to the government budget constraint in analyzing monetary policy issues. In addition, the source of money demand in our model is that intermediaries hold reserves, which is an application of the legal restrictions theory studied by Sargent and Wallace. However, we interpret our money demand specification as a proxy for a transactions demand for money, not as a legal restriction. Our money demand specification can be viewed in the same light as the cash-in-advance specifications often employed in the welfare cost of inflation literature.[4]

In constructing the model, we label parameters in two ways. For parameters that are not very controversial, in the sense that they map more or less directly into available data, we will use conventional symbols. In the quantitative exercise, we will deduce and assign values for these parameters directly from available data—this will be the *fixed parameter vector*, $F$, of our model. The remaining parameters—deep parameters in preferences and technology, as well as our intermediation cost, money demand, and two tax parameters; nine in all—will be denoted $\delta_i$, $i = 1, ..., 9$ and together constitute the *deep parameter vector*, $\Delta$, of our model. Much less is known about these parameters in the abstract and so it is less clear what values should be assigned to them. A further complication is that a change in one of the deep parameters often helps to fit the data on one dimension while being detrimental on other dimensions. One way to express this situation is that the map from the parameters to the data defines a rugged surface in a space of endogenous outcomes, and so it is not clear that estimating or calibrating parameter values using standard techniques will deliver globally optimal values. We adopt the following approach

[2] An exception is Altig and Carlstrom (1991).

[3] Ríos-Rull (1994) studies a general equilibrium life cycle model with aggregate technology shocks. The model has no source of money demand and no role for monetary policy. Our model includes money and permits the study of monetary policy, but it is not stochastic. Aggregate uncertainty will not matter for the types of steady state comparisons undertaken in this paper. Ríos-Rull's findings suggest that if a technology shock was added to our model, the implied business cycle features would be similar to those of real business cycle models. See also Ríos-Rull (1995).

[4] See Haslag (undated) for a discussion of some fairly general conditions under which cash-in-advance and reserve requirement economies are allocationally equivalent.

to this problem. A steady state of our model will have many implications for observable quantities we can estimate from the data, such as the aggregate capital-to-output ratio. We will use a nonlinear optimizer (a genetic algorithm) to choose a vector of values for the deep parameters to come as close as possible to a vector of targets on observable quantities, following the principle of an equal number of targets and deep parameters. The targets on observable quantities we select are natural ones from the perspective of our model. We will return to a more detailed explanation of this strategy following our presentation of the model.

**2.2. Preferences.** A generation of identical agents (*a.k.a.* households) is born at each discrete date $t = ..., -2, -1, 0, 1, 2, ...,$ and lives for $n \equiv 55$ periods. We interpret the time period in the model as a year. Successive generations of agents are identified by their birthdates and differ from each other only in their populations, which grow at a gross rate of $\psi \geq 1$ per period. Each agent is endowed with a single unit of time per period, as well as an effective labor productivity coefficient $e_i$ at each period of life $i$, $i = 1, ..., n$. Preferences are defined over intertemporal bundles that include the quantity of the single good consumed at each date and the quantity of leisure enjoyed at each date. The consumption and leisure choices of a member of generation $t$ at date $t+j$ are denoted $c_t(t+j)$ and $\ell_t(t+j)$, $j = 0, ..., n-1$, respectively, where subscripts denote birthdates and parentheses denote real time.

The preferences of the agents are described by the standard utility function

$$U\left(\{c_t(t+j), \ell_t(t+j)\}_{j=0}^{n-1}\right) = \sum_{j=0}^{n-1} \frac{\delta_0^j}{1-\delta_2}\left[c_t(t+j)^{\delta_3}\ell_t(t+j)^{1-\delta_3}\right]^{1-\delta_2} \tag{1}$$

where $\delta_2 > 0$, $\delta_3 > 0$, and $\delta_0 \equiv 1/(1+\delta_1)$, with $\delta_1 > -1$. We require $c_t(t+j) \geq 0$ and $\ell_t(t+j) \in [0,1]$ $\forall t, j$. These preferences imply that the elasticity of intertemporal substitution in consumption, $\sigma$, is the inverse of the coefficient of relative risk aversion, $\nu$, where $\nu \equiv 1 - \delta_3(1-\delta_2)$. Because this is an overlapping generations model, no restrictions are placed on the value of $\delta_1$, other than $\delta_1 > -1$. We will therefore use this parameter to help us hit our target for the average growth rate of household consumption over a lifetime. We define an effective rate of time preference, $\varphi \equiv 1 - (1+\delta_1)^{-1/\delta_2}$, for this model based on the rate of consumption growth agents would choose when faced with a zero net real interest rate.[5] The parameter $\delta_3$ is the elasticity of intratemporal consumption-leisure

[5]This is easiest to see by considering a two period endowment model with inelastic labor supply and preferences given by $u = \sum_{i=0}^{1}(1-\delta_2)^{-1}\delta_0^i c_t(t+i)^{1-\delta_2}$. The first order conditions imply $c_t(t+1) = (\delta_0 R(t))^{1/\delta_2} c_t(t)$, where $R(t)$ is the gross real interest rate between date $t$ and $t+1$ that the agent faces. The effective rate of time preference we define, $\varphi$, is the arithmetic opposite of the consumption growth rate that occurs when the agent faces a zero net real interest rate. Thus, $\varphi = -(c_t(t+1)/c_t(t)$

substitution, and is the dominant parameter governing the share of available time the agent devotes to market activity. We require households to retire from the labor force at an age $n^{\star} \leq n$; that is, $\ell_t(t+j) = 1$ for $j = n^{\star} - 1, ..., n-1$.

**2.3. Technology.** Each period there are an arbitrary number of competitive firms that have access to a technology that uses labor and capital to produce the consumption good. The aggregate stock of capital goods available for use in production at the beginning of date $t$ is denoted $K(t)$. Since the technology exhibits constant returns it suffices to describe an aggregate production function

$$Y(t) = \lambda^{(1-\delta_4)(t-1)} K(t)^{\delta_4} L(t)^{1-\delta_4}, \tag{2}$$

where $L(t)$ is the aggregate supply of effective labor and $k(t) \equiv K(t)/L(t)$ is the ratio of capital to effective labor. The parameter $\lambda \geq 1$ is the gross rate of growth of labor productivity, and the parameter $\delta_4 \in (0,1)$ governs the capital share of output. Capital goods depreciate at a *net* rate of $\delta_5 \in [0,1]$ per period. The firms rent capital and hire effective labor at rental and wage rates equal to these inputs' respective marginal products: the rental rate is $r(t) = \lambda^{(1-\delta_4)(t-1)} \delta_4 k(t)^{\delta_4 - 1}$ and the wage rate is $w(t) = \lambda^{(1-\delta_4)(t-1)}(1-\delta_4)k(t)^{\delta_4}$.

**2.4. Asset market structure.** There are four assets in the model: physical capital, consumption loans, government bonds, and fiat currency. We assume that all borrowing and lending in the economy takes place through perfectly competitive intermediaries. Households that rent capital to the firms or make consumption loans to other households are paid the real returns net of a constant cost per unit of capital intermediated; we denote the cost $\delta_6 \in (0,1)$. Households that save by holding government bonds are paid the real returns net of a smaller cost; for purposes of simplicity, we push this cost all the way to zero. We will estimate the value of the cost parameter $\delta_6$ with the help of estimates of the size of the financial intermediation sector in the postwar U.S. economy. These estimates, which were constructed by Díaz-Giménez, et al. (1992), are displayed in Figure 1. They suggest that the quantity of resources devoted to financial intermediation in the U.S. is large—roughly 5 to 7 percent of GDP in the early 1980s.

Our asset market structure is simple and stark, in keeping with our principle that the financial intermediation, tax and monetary features of the analysis should not complicate the model any more than absolutely necessary. The rationale behind our intermediation cost assumption is simple. In actual economies there are a great many firms and households, so that it is difficult to match borrowers and lenders and to make sure that

$-1) \,|_{R(t)=1}$, which, from the first order condition, is $\varphi = 1 - \delta_0^{1/\delta_2}$, or $1 - (1+\delta_1)^{-1/\delta_2}$. This result carries over to the multi-period model with elastic labor supply that we study.

Figure 1. Financial intermediation in the U.S.

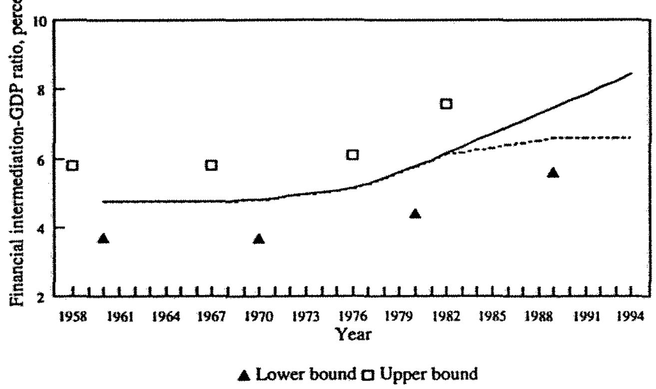



Figure 1: The value of financial intermediation services provided in the U.S. economy is large. The boxes and triangles represent upper (total product basis) and lower (value added basis) bounds, respectively, as calculated by Díaz-Giménez, et al., (1992). The lines are simple midpoints based on linear interpolation between data points and extrapolation of existing trends (solid line) and no trend (dotted line) at the end of the sample.

borrowers have the ability and inclination to repay their loans. Lending to the government, on the other hand, is a relatively simple matter since there is only one borrower (or a small number of borrowers) whose characteristics are well known.[6]

We complete our asset market specification by assuming that financial intermediaries hold a constant fraction $\delta_7 \in (0,1)$ of their liabilities in the form of fiat currency reserves. As we have indicated, we use reserve demand as a proxy for base money demand from all sources—most of which are presumably associated with the need to hold money to facilitate transactions. Reserve demand is a simple way of ensuring that in the equilibria of our model, money demand will be closely related to national income. We will use monetary and income data for the postwar U.S. economy to help estimate the value of the $\delta_7$ parameter.

[6]Recent work on financial intermediation in macroeconomic models includes Boyd and Smith (forthcoming), Bernanke and Gertler (1989), Greenwood and Williamson (1989), Williamson (1987), and Díaz-Giménez, et al. (1992). Our approach is most closely related to the latter paper. Luttmer (1996) analyzes asset pricing in economies related to ours with frictions in the form of transactions costs.

Strictly speaking, the agents in our model would prefer to avoid the financial intermediation technology altogether by purchasing government bonds directly or by lending directly to other households or to firms. However, we interpret our environment as one in which households could not avoid the transactions costs associated with lending by attempting to provide their own intermediation services.

**2.5. The government.** The government in the model is a consolidated federal, state and local entity. We will target the size of the government in our model using data from the national income and product accounts. At date $t$ the government must finance real expenditures of $G(t)$ by a combination of direct taxation and seignorage. The expenditures grow at gross rate $\lambda\psi$, the aggregate growth rate of the economy, so that $G(t+1) = \lambda\psi G(t)$. Government expenditures are assumed to leave the economy.

The government collects the bulk of its revenues using three direct proportional taxes: a tax on labor income levied at rate $\delta_8 \in [0, 1)$, a tax on the nominal interest income of households levied at rate $\tau^i \in [0, 1)$, and a "corporate profits tax" on the firms' nominal returns to capital levied at rate $\delta_9 \in [0, 1)$. Our tax structure is intended to be a parsimonious representation of the current U.S. tax system. Our representation reflects two important features of the U.S. system for taxing capital income: double taxation of dividend income and the fact that household interest income and corporate earnings are taxed on a nominal basis. Lump-sum taxation is assumed to be unavailable.

**2.6. Arbitrage conditions.** Our assumptions about money demand, intermediation and tax structure determine the interest rate structure of the economy. The lowest gross real rate of return in the economy will be the return to real currency balances, which is $R^h(t) = P(t)/P(t+1)$, where $P(t)$ is the nominal price of the consumption good at time $t$. The gross inflation rate is $1/R^h(t) = P(t+1)/P(t)$. The highest gross real rate of return will be the total return to capital, which is $R^k(t) \equiv 1 + r(t+1)$, where $r(t+1)$ is the marginal product of capital defined above. The gross real return to capital net of depreciation is then $R^{kn}(t) \equiv R^k(t) - \delta_5$. In our specification, firms are taxed on their nominal, net-of-depreciation returns to capital, so the after-tax gross real rate of return they pay to intermediaries is $R^{ka}(t) \equiv (1 - \delta_9)R^{kn}(t) + \delta_9 R^h(t)$.[7] This is also the gross real rate of return on consumption loans, $R^{cl}(t)$, since arbitrage forces consumption loans and capital to pay equal returns.

Because financial intermediation is costly, intermediaries are not willing to pay the gross real rate $R^{ka}(t)$ to household savers. Intermediation costs are $\delta_6$ per unit of capital intermediated, so the net-of-intermediation-costs gross real rate of return is $R^{kc}(t) \equiv R^{ka}(t) - \delta_6$. In addition, intermediaries cannot lend the entire amount of their deposits because they hold reserves. They therefore pay a weighted average of the real return on fiat currency and the net-of-intermediation costs real return to capital: this defines the gross real rate of return on deposits as $R^d(t) \equiv (1 - \delta_7) R^{kc}(t) + \delta_7 R^h(t)$. Since

[7]The gross nominal return to capital employed at date $t-1$ is $R^{kn}(t-1)/R^h(t-1)$, so at date $t$ the firms have to make a nominal tax payment to the government of $\delta_9\left(\left(R^{kn}(t-1)/R^h(t-1)\right) - 1\right) K(t-1)P(t-1)$. Firms' total real net-of-depreciation earnings are $R^{kn}(t-1)K(t-1)$. Dividing the former expression by $P(t)$ to put it into real terms and then subtracting it from the latter expression produces the gross rate of return expression given in the text. A similar calculation defines $R^{da}$ below.

bonds are intermediated costlessly, arbitrage implies that this is also the gross real rate of return to bonds, which we denote $R^b(t)$. Households have to pay taxes on nominal interest income at rate $\tau^i$, so the after-tax gross real rate of return on deposits is $R^{da}(t) \equiv \left(1-\tau^i\right) R^d(t) + \tau^i R^h(t)$. This is also the gross real after-tax rate of return on bonds, which we denote $R^{ba}(t)$.

Thus, our steady state asset return structure will obey the following:

$$R^h < R^{da} = R^{ba} < R^d = R^b < R^{kc} < R^{cl} = R^{ka} < R^{kn} < R^k. \tag{3}$$

The gross nominal interest rates are these gross real rates divided by $R^h$. The monetary authority determines $R^h$ through its conduct of monetary policy. The equilibrium conditions of the model can be thought of as determining the equilibrium value of $R^k$, and the tax, intermediation cost and money demand parameters then determine the remainder of the interest rate structure.

**2.7. Household decisions.** Households maximize (1) subject to a lifetime budget constraint, which we now define. We denote the date $t+j$ demand for assets of an agent born at date $t$ by $a_t(t+j)$, $j = 0, ..., n-1$. Agents can borrow or lend in any period of life. If they borrow at date $t$ then they pay the gross real rate $R^{ka}(t)$. If they lend by holding deposits with the financial intermediary then they earn the gross real after-tax return $R^{da}(t)$. The budget constraints of an agent born at date $t$ are

$$\tag{4}$$

$$c_t(t) + a_t(t) = (1-\delta_8) w(t) e_1 \left(1 - \ell_t(t)\right),$$

$$\begin{aligned} c_t(t+j) + a_t(t+j) &= (1-\delta_8) w(t+j)\, e_{j+1} \left(1 - \ell_t(t+j)\right) + \\ & \quad \widehat{R}(t+j-1)\, a_t(t+j-1), \end{aligned}$$

for $j = 1, ..., n-2$, and

$$\begin{aligned} c_t(t+n-1) &= (1-\delta_8) w(t+n-1)\, e_n \left(1 - \ell_t(t+n-1)\right) + \\ & \quad \widehat{R}(t+n-2)\, a_t(t+n-2), \end{aligned}$$

where $w(t)$ is the before-tax real wage at date $t$, and

$$\widehat{R}(t+j) = \begin{cases} R^{ka}(t+j) & \text{if} \quad a_t(t+j) < 0, \\ R^{da}(t+j) & \text{if} \quad a_t(t+j) \geq 0. \end{cases}$$

Aggregate assets at date $t$ are

$$A(t) \equiv \sum_{j=0}^{n-2} \psi^{t-j} a_{t-j}(t),$$

where we are normalizing the date $t$ population to unity. We can decompose $A(t)$ into $A^+(t)$, the aggregate asset demand of the agents whose asset demand is non-negative at date $t$, and $A^-(t)$, the aggregate asset demand of the agents whose asset demand is negative at time $t$. The first group earns a gross rate of return $R^{da}(t) = R^{ba}(t)$, while the second group must pay interest at gross rate $R^{ka}(t)$. We will define $A^-(t)$ as a positive number, so that $A(t) = A^+(t) - A^-(t)$. The liabilities of the financial intermediaries are then $K(t) + A^-(t)$. Agents can choose corner solutions for a particular period of life $j$, in which case they set $a_t(t+j) = 0$.

**2.8. The government budget constraint.** The government taxes labor income at rate $\delta_8$, so aggregate labor income tax revenues at date $t$ are $T^l(t) \equiv \delta_8 w(t)L(t)$. The government also taxes household net nominal interest income at rate $\tau^i$, producing nominal tax revenues at date $t$ of $\tau^i[R^d(t-1)/R^h(t-1) - 1]A^+(t-1)P(t-1)$ and real revenues of $T^i(t) \equiv \tau^i[R^d(t-1) - R^h(t-1)]\, A^+(t-1)$. The corporate profits tax produces nominal revenues of $\delta_9 \left(R^{kn}(t-1)/R^h(t-1) - 1\right) K(t-1)P(t-1)$, so that real revenues are $T^k(t) \equiv \delta_9 \left(R^{kn}(t-1) - R^h(t-1)\right) K(t-1)$. Government revenues from currency and bond seignorage at date $t$ are $C^s(t) \equiv H(t) - R^h(t)H(t-1)$ and $B^s(t) \equiv B(t) - R^b(t-1)B(t-1)$, respectively.[8] The government budget constraint is therefore

$$G(t) = T^l(t) + T^i(t) + T^k(t) + C^s(t) + B^s(t) \tag{5}$$

where $G(t) \geq 0\ \forall t$.

**2.9. Equilibria.** Agents hold aggregate deposits $D(t)$ with intermediaries. The intermediaries use a fraction of these deposits to acquire fiat currency reserves: $\delta_7 D(t) = H(t)/P(t)$, where $H(t)/P(t)$ denotes aggregate real balances. The remainder of the deposits are lent to firms and households: $(1-\delta_7)D(t) = A^-(t) + K(t+1)$. The money market clearing condition is

$$\frac{H(t)}{P(t)} = \frac{\delta_7}{1-\delta_7}\left[A^-(t) + K(t+1)\right]. \tag{6}$$

The credit-market clearing condition is

$$A(t) = \frac{H(t)}{P(t)} + K(t+1) + B(t), \tag{7}$$

[8] Bond seigniorage, which is conceptually similar to currency seigniorage, is the revenue the government can earn from maintaining a real stock of outstanding debt when the real interest rate on its bonds is lower than the output growth rate. See Miller and Sargent (1984).

where $B(t)$ denotes the aggregate real market value of the government debt. These conditions can be combined, producing

$$B(t) = A^{+}(t) - \frac{1}{1-\delta_7}\left[A^{-}(t) + K(t+1)\right]. \tag{8}$$

Equation (5), the government budget constraint, involves $B(t)$. If we substitute equation (8) into the government budget constraint, then the right hand side of the resulting equation can be written entirely as a function of $R^h(t)$, $R^k(t)$, and the parameters of the model, given the solution of the agents' decision problem:

$$G(t) = g\left[R^h(t), R^k(t), F, \Delta\right]. \tag{9}$$

If we take the level of government revenue $G(t) = G$ to be determined exogenously by the fiscal authority, and the rate of return on fiat currency $R^h(t) = R^h$—the inverse of the gross inflation rate—to be determined by the monetary authority, then equation (9) is a complicated difference equation in the total returns to capital $R^k(t)$. Steady state perfect foresight equilibria are sequences $\left\{R^k(t)\right\}_{t=-\infty}^{\infty}$ that solve equation (9) and have the property that $R^k(t) = R^k\ \forall t$. We will study equilibria of this type. The total revenue Laffer curve defined by equation (9) will normally produce two such equilibria in the quantitative cases we study. In this paper, we will confine ourselves to studying equilibria on the left side of the Laffer curve—the side along which increasing the rate of inflation produces lower real rates of return on all assets. We think this is the relevant case for industrialized economies.[9]

It is possible for the government revenue setting $G$ to be inconsistent with the settings of the tax parameters and the inflation rate, in which case equation (9) will have no solution. When we try to fit the model to the data, we will want to solve both the agents' decision problem and equation (9) repeatedly for a wide variety of parameter values. For this reason we take a slightly different approach to using equation (9). We leave $R^h$ fixed, but we allow the amount of government revenue $G(t)$ to be endogenous. We then fix a second real interest rate, $R^b$, by matching its value to the data, and let the elements of the parameter vectors $F$ and $\Delta$ determine the rest of the interest rate structure. This approach ensures that equation (9) always has a solution. Of course, the value of $G(t)$ produced may not be very consistent with the data, but we will make the ratio of government consumption to output one of the targets of our system. Later in the paper, when we report welfare comparisons across steady state equilibria with different

[9]The other equilibrium seems less promising empirically. For most of the quantitative economies we study, it involves counterfactually high rates of return to government debt, a much larger (five-fold in our baseline case) stock of bonds relative to GDP, and implies that higher inflation raises the real rates of return on bonds and capital when government spending is held constant. For recent empirical evidence on related questions, see Weber (1994).

inflation rates, we hold the ratio of $G$ to total output fixed at a baseline level, because in those calculations we will want to hold fiscal policy constant.

In any steady state equilibrium the capital-labor ratio grows at gross rate $\lambda$, as does the real wage. The levels of all real aggregates—such as real output, asset demand, the capital stock, and real balances—grow at gross rate $\lambda\psi$. The steady states we study are quantity-theoretic in the sense that the rate of money growth dictates the inflation rate through a standard quantity theory equation.

## 3. Confronting the data

**3.1. Strategy.** Our goal is to find a plausible parameterization of the model that can produce a convincing match to the data. We divide the parameter set into two parts. The first is a fixed parameter vector $F$, which we will set element by element based on available data. The second is our deep parameter vector $\Delta$, whose elements we will estimate jointly based on the data.

**3.2. The fixed parameter vector.** The fixed parameter vector is given by

$$F = \left[\{e_i\}_{i=1}^{n}, n^{\star}, \lambda, \psi, R^h, R^b, \tau^i\right]. \tag{10}$$

We set $n^{\star}$, the retirement age, to agent age 44 (figurative age 65). Based on postwar U.S. averages, we set the gross rate of technological change at $\lambda = 1.015$, the gross rate of labor force growth at $\psi = 1.017$, the gross real rate of return to currency holding at $R^h = 0.9615$ (implying a 4 percent inflation rate), and the before-tax gross rate of return on bonds at $R^b = 1.01$. Our estimate of the average after-tax gross real return on federal government bonds is $R^{ba} = 1$.[10] We use this information and the equilibrium relationship between $R^b$ and $R^{ba}$ to set $\tau^i = 0.2$. We set the productivity profile according to data from Hansen (1993).[11]

**3.3. The deep parameter vector.**

**Overview.** The deep parameter vector for our model has elements $\delta_i$, $i = 1, ..., 9$. For these parameters, much less is known in the abstract. However, stationary equilibria of our model will have many endogenously generated implications for observable quantities

[10] We constructed this estimate using marginal tax rate data provided by Joseph Peek of Boston College. We thank him for his cooperation.

[11] The Hansen data is collected from samples taken in 1979 and 1987. The data separate males from females. We average the data from the two years, and we also average the data across males and females using weights of 0.6 and 0.4. The resulting profile is a step function, because the data are collected for age groupings. We fit a fifth-order polynomial to this step function. This yields the smooth profile $e_{i-20} = m_0 + m_1 i + m_2 i^2 + m_3 i^3 + m_4 i^4 + m_5 i^5$ for $i = 21, ..., 76$, with the vector of coefficients $m = [-4.34, 0.613, -0.0274, 0.0063, -0.717 \times 10^{-5}, 0.314 \times 10^{-7}]$. This profile peaks at agent age 28 (figurative age 48), when productivity is about 1.6 times its level at agent age 1 (figurative age 21). Productivity in the final year of life is virtually the same as in the first year of life.

| Table 1. Targets on observable quantities. | | | |
|---|---|---|---|
| Observable Quantity | Target | Range | Source |
| $K/Y$ | 3.32 | [2.32,4.32] | Cooley-Prescott (1994). |
| $I/K$ | 0.076 | [0.066,0.086] | Cooley-Prescott (1994). |
| $B/Y$ | 0.47 | [0.255,0.686] | U.S. data, 1959-94. |
| $icg$ | 0.015 | [0.01,0.03] | Laitner (1992). |
| $alt$ | 0.154 | [0.075,0.33] | Authors' calculations. |
| $I^m/Y$ | 0.06 | [0.05,0.07] | Díaz-Giménez, et al., (1992). |
| $H/Y$ | 0.0592 | [0.041,0.078] | U.S. data, 1959-94. |
| $G/Y$ | 0.151 | [0.121,0.18] | U.S. data, 1959-94. |
| $T^k/G$ | 0.119 | [0.036,0.201] | U.S. data, 1959-94. |

Table 1: Nine targets on observable quantities. U.S. data source: *1996 Economic Report of the President.*

which can be estimated from the data. We choose a set of nine target quantities of this type, on the principle of the same number of targets and parameters. The map from the deep parameters to the targets describes a rugged surface in the nine dimensional space of endogenous outcomes. For this reason, we employ a genetic algorithm to find a best fit vector $\Delta$ based on a criterion defined over the space of target quantities. We now turn to a detailed description of this procedure.

**Targets on observable quantities.** The observable quantities we target are listed in Table 1. Most of the target values are established estimates of postwar averages, so we will not discuss them in much detail. The estimates of the average capital-output ratio $K/Y$ and the average investment-capital ratio $I/K$ are due to Cooley and Prescott (1994). These estimates are based on a broad definition of capital that includes consumer durables and government capital. We target the equilibrium bonds to output ratio $B/Y$ at the average value of the ratio of gross federal debt to output in postwar data. The money-output ratio, $H/Y$, is the average ratio of the monetary base to output in postwar data. The average size of the financial intermediation sector, $I^m/Y$, is targeted based on the data presented in Figure 1.

Laitner (1992) has argued that, on average, individual consumption growth rates are not too different from the aggregate consumption growth rate. Consequently, we set the target for $icg$, an individual's lifetime consumption growth rate, at 0.015, which is the rate of technological progress (from $F$) and thus the net rate of aggregate consumption growth per capita. In overlapping generations models the lifetime consumption growth rate can be very different from the aggregate consumption growth rate, so this target imposes a significant constraint on our parameter choices. On the individual level we also target $alt$, the average share of household time devoted to the labor market.[12]

We view the government in the model as a consolidated federal, state and local entity. We target $G/Y$, the ratio of government expenditures to output, at the average value of consolidated government revenue, net of transfers and government investment, relative to GDP. In the model, the revenues of the government come from five sources: taxes on labor income, individual interest income, and corporate profits, plus currency and bond seignorage. The personal interest income tax rate is an element of $F$. The levels of currency and bond seignorage will be determined by the interest rates $R^m$ and $R^b$, which are part of $F$, and by the ratios of money and bonds to output, both of which we have targeted. We target $T^k/G$, the ratio of corporate profits tax revenues to total government expenditures, to match the average value of this ratio in postwar data.

We also define plausible ranges around the targets as given in Table 1. The target ranges will affect the operation of our algorithm, but final outcomes will be very close to the targets listed.

**Nonlinear optimization.** We use a genetic algorithm to learn about the irregular nonlinear map between $\Delta$ and the targets.[13] Given a candidate vector $j$ at algorithm time $s$, $\Delta_{js}$, we can calculate the solution to the agents' decision problem, and based on that information, we can find the implied steady state equilibrium values for the targets associated with candidate vector $j$. We define a fitness criterion for a candidate vector $\Delta_{js}$ based on deviations of these implied values from targets. We use a genetic algorithm with real-valued coding, and operators providing tournament reproduction, three types of crossover, and non-uniform mutation, as explained below. Because our non-uniform mutation procedure slowly reduces the mutation rate to zero by time $T$, separate genetic algorithm searches can yield different best fit candidate vectors $\Delta_T^\star$. We conduct ten such searches and report the best fit vectors.

We begin by defining a fitness criterion across the nine targets of our system. We want to consider a criterion on the order of sum of squared deviations from target, but we also want the genetic algorithm to consider the fact that some targets are tighter than others in that the plausible deviation from them is smaller. Accordingly, we think of the target ranges as defining the space of plausible outcomes, and we design our fitness criterion to penalize candidate vectors $\Delta_{js}$ more severely if they deliver values outside the target range. This will prevent the genetic algorithm from spending a lot of time searching areas of the parameter space which are good on many dimensions but bad on a few dimensions.

[12] This is the authors' calculation based on a 24 hour day, a five day workweek, ten vacation days, ten holidays and a 70 percent labor force participation rate.

[13] It would take us too far astray to discuss the principles of genetic algorithms here in great detail. For an introduction, as well as detailed discussion of the real-valued approach we use and the associated genetic operators, see Michalewicz (1994).

We assign penalty points based on deviations from target on each dimension. The penalty points are assigned linearly up to the boundary of the target, such that a candidate vector is penalized one point if a particular value is at the boundary of the target range. Outside the target range, an appropriately scaled quadratic penalty in the difference between the value and target boundary is added to the linear penalty. If we denote implied values of a candidate vector $\Delta_{js}$ by $\theta_{ijs}$, target values by $\theta_i^\star$, and upper and lower target bounds by $\bar{\theta}_i$ and $\underline{\theta}_i$, respectively, where $i = 1, ..., 9$, then the fitness of the candidate vector, $\mathcal{F}[\Delta_{js}]$, is given by

$$\mathcal{F}[\Delta_{js}] = \sum_{i=1}^{9} \mathcal{P}_{ijs} \tag{11}$$

where

$$\mathcal{P}_{ijs} = \begin{cases} (\theta_{ijs} - \theta_i^\star) / (\bar{\theta}_i - \theta_i^\star) + (\theta_{ijs} - \bar{\theta}_i)^2 / \bar{\theta}_i (\bar{\theta}_i - \theta_i^\star) & \text{if} \quad \theta_{ijs} > \bar{\theta}_i \\ (\theta_{ijs} - \theta_i^\star) / (\bar{\theta}_i - \theta_i^\star) & \text{if} \quad \theta_{ijs} \in [\theta_i^\star, \bar{\theta}_i] \\ (\theta_i^\star - \theta_{ijs}) / (\theta_i^\star - \underline{\theta}_i) & \text{if} \quad \theta_{ijs} \in [\underline{\theta}_i, \theta_i^\star] \\ (\theta_i^\star - \theta_{ijs}) / (\theta_i^\star - \underline{\theta}_i) + (\underline{\theta}_i - \theta_{ijs})^2 / \underline{\theta}_i (\underline{\theta}_i - \theta_i^\star) & \text{if} \quad \theta_{ijs} < \underline{\theta}_i \end{cases} \tag{12}$$

This definition means that the better fit vectors will have lower fitness values, and a vector that delivers an exact fit on all targets will have a fitness of zero.

The genetic algorithm is an iterative directed search procedure acting on a population of $j$ candidate vectors at algorithm time $s$. At time $s$, the fitness of all candidate vectors in the population is calculated. To obtain the next set of candidate vectors, $\Delta_{js+1}$, we apply genetic operators. The first operator is tournament reproduction. We select two vectors at random with replacement from the time $s$ population. The vector with the better fitness value is copied into the time $s+1$ population. This operator is repeated enough times to produce a time $s+1$ population equal in size to the time $s$ population. Reproduction provides most of the evolutionary pressure in the search algorithm, but we need other operators to allow the system to experiment with new, untried candidate vectors. Crossover and mutation provide the experimentation, and operate on the time $s+1$ population before the fitness values for that population are calculated.

To implement our crossover operators, we consider the time $s+1$ population two vectors, $j$ and $j+1$, at a time, and we implement the crossover operator with probability $p^c$. If crossover is to be performed on the two vectors, we use one of three methods with equal probability. In single-point crossover, we choose a random integer $i_{cross} \in [1, ..., 9]$ and swap the elements of $\Delta_{j,s+1}$ and $\Delta_{j+1,s+1}$ where $i \geq i_{cross}$. In arithmetic crossover, we choose a random real $a \in [0, 1]$ and create post-crossover vectors $a\Delta_{j,s+1} + (1-a)\Delta_{j+1,s+1}$ and $(1-a)\Delta_{j,s+1} + a\Delta_{j+1,s+1}$. In shuffle crossover, we exchange elements of $\Delta_{j,s}$ and $\Delta_{j+1,s}$ based on draws from a binomial distribution, such that if the $i^{th}$ draw is unity, the $i^{th}$ elements are swapped, otherwise the $i^{th}$ elements are not swapped. Each of these operators has been shown to have strengths in the artificial intelligence literature in certain types of difficult search problems, and we use them all here in order to improve the prospects for success.

We implement a non-uniform mutation operator that makes use of upper and lower bounds, $\bar{\delta}_i$ and $\underline{\delta}_i$, respectively, on the elements of a candidate vector $\Delta_{j,s+1}$. This operator is implemented with probability $p^m$ on element $\delta_{ij,s+1}$. If mutation is to be performed on the element, we choose a pair of random reals $r_1, r_2 \sim U[0,1]$. The new, perturbed value of the element is then set according to

$$\delta_{ij,s+1}^{new} = \begin{cases} \delta_{ij,s+1} + \left(\bar{\delta}_i - \delta_{ij,s+1}\right)\left(1 - r_2^{\left(\left(1-\frac{s}{T}\right)^b\right)}\right) & \text{if } r_1 > .5, \\ \delta_{ij,s+1} - \left(\delta_{ij,s+1} - \underline{\delta}_i\right)\left(1 - r_2^{\left(\left(1-\frac{s}{T}\right)^b\right)}\right) & \text{if } r_1 < .5, \end{cases} \tag{13}$$

where $b$ is a parameter. With this mutation operator, the probability of choosing a new element far from the existing element diminishes as algorithm time $s \to T$, where $T$ is the maximum algorithm time. This operator is especially useful in allowing the genetic algorithm to more intensively sample in the neighborhood of the algorithm time $s$ estimate of the best fit vector in the latter stages of the search.

We conducted ten genetic algorithm searches in order to identify a best fit deep parameter vector $\Delta_T^{\star}$ according to our set of targets defined in Table 1.[14] The results are reported in Table 2.

We find that the algorithm time $T$ population of parameter vectors $\Delta_{jT}^{\star}$ provide a close fit on our target data. The only quantitatively significant discrepancies from targets occur on individual consumption growth and individual time devoted to market, and then the implied values are typically only .2 to .35 of a penalty point from target, meaning that implied values on these dimensions lie away from the target only 20 to 35 percent of the distance between the target and a target bound. We found little or no variation among individual parameters within algorithm time $s = T$ populations. Across searches, we found some variance, almost all of it in the preference parameters. The estimates of the elasticity of intertemporal substitution, for instance, ranged from a low of .114 to a high of .185. Search number 5 provided the best overall fit, and we now turn to a discussion

[14] We set the parameters in the genetic algorithm, $\{population,\ p^c,\ p^m,\ T,\ b\}$, as $\{30, .95, .11, 1000, 2\}$ based on standards in the artificial intelligence literature. In our final search, we set $T = 2500$, but we did not observe a commensurate inprovement in performance, and so we did not pursue higher values of $T$ any further. We set the bounds on elements $\delta_i$, $i = 1, ..., 9$, according to $[-0.3, 0.1]$, $[1.1, 40]$, $[0.075, 0.33]$, $[0.25, 0.4]$, $[0.025, 0.075]$, $[0.01, 0.04]$, $[0.01, 0.08]$, $[0.01, 0.4]$, $[0.01, 0.25]$. This amounts to a set of constraints on the search to values that are typically viewed as economically plausible. We intialize the system by choosing elements of an initial population of vectors $\Delta$ randomly from uniform intervals defined by these bounds.

| Table 2. Results of nonlinear optimization. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Search | $\frac{K}{Y}$ | $\frac{I}{K}$ | $\frac{B}{Y}$ | $icg$ | $alt$ | $\frac{I^m}{Y}$ | $\frac{H}{Y}$ | $\frac{G}{Y}$ | $\frac{T^k}{G}$ | $\mathcal{F}$ |
| 1 | 0.02 | 0.14 | 0.00 | 0.24 | 0.32 | 0.01 | 0.00 | 0.00 | 0.05 | 0.78 |
| 2 | 0.05 | 0.04 | 0.00 | 0.31 | 0.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.69 |
| 3 | 0.01 | 0.00 | 0.00 | 0.29 | 0.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.57 |
| 4 | 0.00 | 0.06 | 0.00 | 0.29 | 0.28 | 0.01 | 0.00 | 0.00 | 0.01 | 0.65 |
| 5 | 0.01 | 0.02 | 0.00 | 0.25 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 |
| 6 | 0.02 | 0.08 | 0.00 | 0.31 | 0.33 | 0.00 | 0.00 | 0.00 | 0.02 | 0.76 |
| 7 | 0.01 | 0.01 | 0.00 | 0.29 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.61 |
| 8 | 0.08 | 0.06 | 0.00 | 0.33 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.78 |
| 9 | 0.04 | 0.04 | 0.00 | 0.33 | 0.46 | 0.00 | 0.00 | 0.00 | 0.03 | 0.91 |
| 10 | 0.06 | 0.02 | 0.00 | 0.30 | 0.23 | 0.00 | 0.00 | 0.00 | 0.01 | 0.62 |

Table 2: The fit to the data. The entries are deviations from target, by target and total, in penalty points, for each of the ten searches we conducted. Columns 2 through 10 are averages across algorithm time $s = T$ populations. Within algorithm time $s = T$ populations, we found little or no variation across fitness components.

of the economy described by the associated best fit deep parameter vector.[15]

## 4. Characteristics of the baseline economy

**4.1. A quantitative match to the data.** Our baseline economy has the features summarized in Table 3. The main features of the artificial economy match the data on the dimensions we have selected. The average growth rate of household consumption over the lifetime is a little higher than our target, but this is also the target on which we have the least evidence. In including this target, we were mainly concerned that we avoid ending up with a baseline economy in which individual consumption growth was considerably different from the aggregate consumption growth rate, as can and has happened in research on general equilibrium life cycle models, and which would probably be viewed as a defect of the model. We think we have succeeded on this goal, even though the fit is not exact.

The effective rate of time preference is $-0.0067$, which is close to the $-0.0098$ point estimate that can be calculated from Hurd's (1989, p. 801) estimates of our $\delta_1$ and $\delta_2$ parameters. Similar values have been used by Ríos-Rull (1994) and others. The baseline economy has an elasticity of intertemporal substitution (EIS) of 0.151, and thus a coefficient of relative risk aversion (CRRA) of 6.6.[16] These values are well within

[15] For search number 5, the average value of the algorithm time $s = T$ population of candidate vectors is given by $\{-.223,\ 37.4,\ .154,\ .26,\ .0439,\ .018,\ .0169,\ .11,\ .0742\}$. The economic interpretation is summarized in Table 3.

[16] While the elasticity of intertemporal substitution is an important parameter in our model, risk aversion is not relevant in our nonstochastic framework. We mention the CRRA only because we want to

**Table 3. Baseline steady state characteristics.**

| Aggregate performance | | Model | Target |
|---|---|---|---|
| Real output growth | $\lambda\psi$ | 1.032 | fixed |
| Inflation | $1/R^h$ | 1.04 | fixed |
| Technological progress | $\lambda$ | 1.015 | fixed |
| Labor force growth | $\psi$ | 1.017 | fixed |
| **Preferences** | | | |
| Effective rate of time preference | $\varphi$ | -0.0067 | open |
| CRRA | $\nu$ | 6.6 | open |
| EIS | $\sigma$ | 0.151 | open |
| Individual consumption growth | $icg$ | 0.0188 | 0.015 |
| Lifetime average agent time devoted to labor | $alt$ | 0.1539 | 0.154 |
| **Asset holdings** | | | |
| Capital-output ratio | $K/Y$ | 3.33 | 3.32 |
| Bonds-output ratio | $B/Y$ | 0.48 | 0.47 |
| Money-output ratio | $H/Y$ | 0.0591 | 0.0592 |
| **Technology** | | | |
| Capital share | | 0.26 | open |
| Depreciation rate | | 0.0439 | open |
| Investment-capital ratio | $I/K$ | 0.0762 | 0.076 |
| **Intermediation** | | | |
| Intermediation-output ratio | $I^m/Y$ | 0.0599 | 0.06 |
| **Government size** | | | |
| Government-output ratio | $G/Y$ | 0.151 | 0.151 |
| Revenue from firms | $T^k/G$ | 0.119 | 0.119 |
| **Government revenue sources** | | | |
| Household labor tax | $T^w/Y$ | 0.081 | open |
| Household interest tax | $T^i/Y$ | 0.037 | open |
| Corporate profits tax | $T^k/Y$ | 0.018 | open |
| Bond seignorage | $B^s/Y$ | 0.010 | open |
| Currency seignorage | $C^s/Y$ | 0.004 | open |

| **Rates of return** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $R^h$ | $R^{da}, R^{ba}$ | $R^d, R^b$ | $R^{kc}$ | $R^{ka}, R^{cl}$ | $R^{kn}$ | $R^k$ |
| Target | fixed | fixed | fixed | open | open | open | open |
| Real | 0.9615 | 1.0003 | 1.0100 | 1.0108 | 1.0288 | 1.0342 | 1.0781 |
| Nominal | 1.0000 | 1.0403 | 1.0504 | 1.0513 | 1.0700 | 1.0756 | 1.1213 |

Table 3: Baseline steady state characteristics. Our endogenous quantities are close to target on the nine dimensions we consider. The term "fixed" in a target entry means we set these quantities directly based on U.S. data. The term "open" in a target entry means we did not fix or target these quantities directly.

conventional ranges, and are consistent with evidence presented by Hall (1988). The question of the amount of curvature in preferences is controversial, however. Attanasio and Weber (1995), for example, report a point estimate for the EIS of 0.56. Other researchers have used high coefficients of relative risk aversion in an effort to explain rate of return differentials. Campbell and Cochrane (1994), for instance, use a local risk aversion coefficient of 48.4 in their model with habit formation preferences. Later in this paper, when we obtain estimates of the welfare cost of inflation, we will consider economies with both higher and lower EIS values. We will show that within the range defined by these alternative estimates, the value of EIS is qualitatively unimportant to our conclusions concerning the size of the cost of inflation: our estimates of the cost will always be an order of magnitude larger than those in the existing literature.[17]

Since our model has rate of return differentials, we can compare them to the data. In our model the counterpart to the return on a basket of stocks is $R^{ka}$, since corporate profits taxes are deducted from firms' earnings before they pay dividends, and the capital gains on firms' stock presumably reflects market adjustments for depreciation. The equity premium comparable to the literature in our model is then $R^{ka}$ less the before-tax return to bonds, $R^b$, which is 188 basis points in our baseline equilibrium. In this differential, 8 basis points are due to money demand and the remaining 180 basis points are due to the cost of financial intermediation. Campbell, Lo, and MacKinlay (1996, Table 8.1) report that the comparable equity premium in the U.S. data, measured as an average of annual excess returns over a long time horizon, is 410 basis points with an approximate 95% confidence band of $[64, 756]$. Our baseline equity premium thus accounts for about half of the differential based on the point estimate but is well within the 95% confidence band.

In an influential paper, Abel, et al., (1989) noted two stylized facts about the U.S. economy that are seemingly contradictory and would be difficult to reconcile in general equilibrium models. The first fact is that real rates of return on short-term government debt are consistently lower than output growth rates. The second fact is that gross capital income has consistently exceeded gross investment. Our baseline steady state reconciles these observations. By construction, this steady state matches long-run averages from U.S. data on the real pretax rate of return on short-term government debt and the real output growth rate. To investigate the second stylized fact, we can use the features of our steady state to reconstruct the calculation that Abel, et al. (1989) performed using U.S. data.

provide intuition for the model with aggregate uncertainty. However, adding uncertainty would have little impact on the steady state comparisons conducted in this paper—the same sorts of comparisons that are studied in most papers in the literature on the welfare cost of inflation.

[17]Our capital share estimate is consistent with most standard interpretations of the national income and product accounts. However, Cooley and Prescott (1994) use an estimate of 0.4. We will examine an alternative economy that has this capital share value, and we will show that the particular value we use is not essential to our argument.

In our model, the ratio of gross investment to output is $(K(t+1) - K(t) + \delta_5 K(t))/Y(t)$, which is $(\lambda\psi - 1 + \delta_5)(K/Y)$ in a steady state. Our baseline value of this ratio is 0.253, a value that is consistent with calculations presented by Cooley and Prescott (1994). The ratio of gross capital income to output in our model is $(R^k(t)-1)K(t)/Y(t)$, which is 0.26, and thus exceeds our baseline ratio of investment to output.[18] The key to the ability of our model to reconcile these two stylized facts is that the model includes intermediation costs that are calibrated to duplicate the observed size of the U.S. financial services sector. Total capital income in our model is relatively high because it includes income spent on these services, while the returns households receive on safe assets are relatively low because the cost of providing these services has been deducted.

Our baseline steady state matches postwar-U.S. data on many other dimensions, but a complete discussion of its features would take us too far astray from the question at hand.

**4.2. Optimality.** There is a respecification of our baseline model that has a Pareto optimal steady state. In this respecification, the direct tax rates are set at zero. The monetary authority sets the inflation rate according to the Friedman rule: the net nominal interest rate is zero, and the real rates of return on both currency and government bonds are equal to the output growth rate. If government expenditures are positive, they are financed through lump-sum taxation. This respecification is conceptually interesting because we can view our baseline steady state as a distorted version of the Pareto optimum, where the distortions are due to direct proportional taxes and inflation. Unfortunately, the Pareto optimum is not quantitatively interesting because it requires either that government expenditures are set at zero or that they are financed through lump-sum taxation. Neither of these alternatives is empirically plausible. As a result, the Pareto optimum cannot provide the basis for a useful analysis of the welfare cost of inflation. Our alternative approach to calculating the welfare cost of inflation takes the level of government expenditures, the structure of the direct tax system and the structure of the intermediation system as given. To determine the welfare cost of inflation, we compare the level of welfare associated with a benchmark inflation rate to the welfare levels associated with feasible alternative inflation rates. We now turn to calculations of this type.

[18] Although our baseline steady state thus matches the second stylized fact in a qualitative sense, Abel, et al. (1989) obtained a much lower estimate of the ratio of investment to ouput. The reason for this is that in performing their calculations they used data on gross investment in private business capital, as opposed to the broader concept of capital used by Cooley and Prescott (1994), which includes government capital and consumer durables. Using this narrower concept, the gross investment figure in the data declines to 0.16, and so the comparison ends up as 0.16 versus 0.26.

## 5. The welfare cost of inflation

**5.1. Welfare comparisons.** We compare two economies that share exactly the same environment, preferences and technology, including common values of all parameters except $R^h$. One economy has a higher inflation rate (a lower value of $R^h$) than the other, and a typical agent in the high inflation economy will be worse off, in a welfare sense, than a typical agent in the low inflation economy. We want to calculate the amount of consumption-good compensation necessary, at each date, to make the agents in the high-inflation economy indifferent between staying in that economy or moving to the low-inflation economy. This measure of the welfare cost of inflation is conceptually similar to measures used by Cooley and Hansen (1989) and others. However, the agent heterogeneity in our model makes our calculations slightly more complicated.

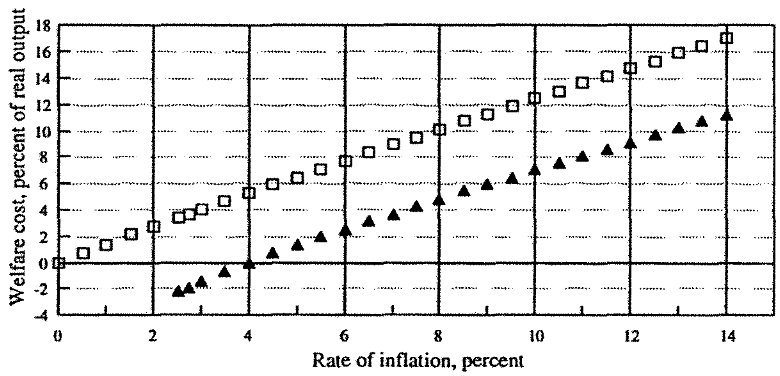
To calculate the welfare cost of a particular rate of inflation relative to a benchmark rate of inflation, we first calculate the lifetime utility of a representative member of an arbitrary generation $t$ in the steady state associated with the benchmark inflation rate. Next, we solve the model for new steady state associated with the inflation rate whose cost we are calculating, and we save the consumption and leisure choices of a representative member of the same generation $t$ in that steady state. We then fix the leisure choices of these agents and imagine giving them compensation, in units of the consumption good, until they are indifferent between the two steady states. We distribute the compensation as follows. Let $x_t(t)$ denote the amount of compensation given to a member of generation $t$ at date $t$. We allow this value to grow at gross rate $\lambda$ per year over these agents' lifetimes, so that $x_t(t+i) = \lambda^i x_t(t)$, $i = 1, \ldots, n-1$. We then determine the value $x_t(t)$ that will restore the members of generation $t$ to their original level of utility.

Each member of generation $t-k$ will receive the same amount of date $t$ compensation as each member of generation $t$: that is, $x_{t-k}(t) = x_t(t)$. The reason for this is that these agents are in the $k$+1st year of their lives, which tends to increase their compensation by a factor of $\lambda^k$, but they are also from the $k$th previous generation, so they are poorer over their lifetimes by a factor of $\lambda^k$ and thus need proportionally less compensation to reach to their original level of utility. If the population of generation $t$ is normalized to unity then the total compensation that must be given to all the agents alive at date $t$ is $\sum_{i=1}^{n} x_t(t)/\psi^{i-1}$. Our measure of the welfare cost of inflation is the ratio of this amount to total date $t$ real output in the steady state associated with the benchmark inflation rate.

### 5.2. Results.

**Welfare cost relative to baseline cases.** We will report the welfare cost of inflation relative to a benchmark inflation rate of four percent, and also relative to a benchmark

Figure 2. The welfare cost of inflation.
Relative to baseline cases of four percent and zero inflation.




▲ Relative to a baseline economy with 4 percent inflation.
□ Relative to a baseline economy with zero inflation.

Welfare cost is compensating consumption as a fraction of baseline real output.

Figure 2: The welfare cost of a percentage point on the inflation rate is a distortion which has a size of at least a percentage point of real output. The size of the distortion is approximately linear in the inflation rate for the range considered in the figure.

inflation rate of zero percent. These results, which constitute our main findings, are summarized in Figure 2. The triangles in the figure give the welfare cost of the indicated inflation rates relative to a reference inflation rate of four percent per year. Since an inflation rate of four percent yields no cost or benefit, the triangle at the point $(4, 0)$ represents our baseline steady state. Lower inflation rates yield welfare benefits to agents relative to the reference inflation rate, so the welfare cost is negative moving left from the point $(4, 0)$. Higher rates of inflation impose higher welfare costs on agents, so the line formed by the triangles is upward-sloping. As the inflation rate increases, the rate of increase in the welfare cost is astoundingly high. Over the range of inflation rates considered, a one percent increase in the annual inflation rate increases the welfare cost of inflation by more than one percent of real output per year. These costs are an order of magnitude larger than comparable estimates from the previous literature.[19]

. There are no triangles in the figure for inflation rates of less than 2.5 percent. The reason for this is that once inflation moves below this level, we cannot hold $G/Y$ constant without adjusting some of the direct tax rates in the model. We use two different methods to address this situation. The simplest method involves modifying our baseline steady state by holding all the parameters of the model except the inflation rate fixed and accepting whatever level of government expenditures is consistent with a benchmark inflation rate of zero. We can then calculate the welfare cost of inflation relative to a zero inflation steady state.[20] The results of these calculations are indicated by the boxes in Figure 2. Again, over the range of inflation rates considered the increase in the welfare cost as inflation rises is better than one percent of real output per percent increase in the inflation rate. We think this is the best comparison of our results to those in the previous literature, which abstracts from the issue of the level of government revenue produced by calibrated tax rates. It illustrates the fact that at low rates of inflation, the size of the marginal distortion from inflation remains large. In particular, based on Figure 2 one would not advise an economy enjoying price stability to allow the inflation rate to increase.

The steady state at zero inflation inherits most of the quantitative properties of the steady state at four percent inflation, so we do not report these properties here. The principal exception is government revenue as a fraction of real output, which is about 11 percent in the zero inflation case versus 15.1 percent in the baseline case. Table 4 summarizes recent comparable estimates of the welfare cost of inflation in the literature.[21]

**Confronting the revenue shortage directly.** A second way to confront the loss of revenue when inflation is lowered is to directly raise other taxes to make up for lost revenue. Using this approach, we lose comparability to the situation considered by most

[19]Some of the data points from Figure 2 are listed in the following table:

| Inflation | 0 | 1 | $2\frac{1}{2}$ | 3 | 4 | 5 | $7\frac{1}{2}$ | 10 | 14 |
|---|---|---|---|---|---|---|---|---|---|
| Base 0 | 0 | 1.37 | 3.33 | 3.97 | 5.22 | 6.46 | 9.48 | 12.42 | 17.03 |
| Base 4 | – | – | −2.26 | −1.48 | 0 | 1.29 | 4.23 | 6.99 | 11.98 |

[20]At the cost of some complications, we could use a lower, Friedman-rule rate of inflation—a deflation rate equal to the output growth rate—as a benchmark rate without changing our qualitative conclusions. However, zero inflation has often been used as a benchmark in the literature.

[21]One interpretation of Lacker and Schreft (1996) would put the welfare cost of 10 percent versus zero inflation at a fairly large 4.27 percent of output. Other interpretations, however, are consistent with the lower cost estimates reported in the papers listed in the Table. Lacker and Schreft (1996) emphasize resource-costly credit and the impact of inflation on real returns. Feldstein (1996) estimates that a two percent reduction in inflation would produce a welfare gain of about one percent of GDP, which is about half the magnitude of the gains we report. Feldstein's analysis is similar to ours in emphasizing tax code interaction, but he does not use general equilibrium methodology. Abel (1996), commenting on Feldstein, does compute a general equilibrium. The total effect is still less than half our baseline effect, however.

| Table 4. Recent estimates of the welfare cost of inflation. | | | |
|---|---|---|---|
| Study | Features | Inflation Comparison | Welfare Cost |
| Cooley/Hansen (1989) | RBC model with cash-in-advance | 10% vs. optimal | 0.387 % |
| Gomme (1993) | Endogenous growth with cash-in-advance | 8.5% vs. optimal | 0.0273 % |
| İmrohoroğlu/ Prescott (1991) | Idiosyncratic labor income risk | 10% vs. 0% | 0.9 % |
| Dotsey/Ireland (1996) | Endogenous growth with cash-in-advance | 10% vs. 0% | 1.73 % |
| Lucas (1994) | Representative agent with shopping time | 10% vs. optimal | 1.3 % |
| Bullard/Russell (1997) | Life cycle economy with financial intermediation | 10% vs. 0% | 12.4 % |

Table 4: Some recent studies of the welfare cost of inflation. Costs are expressed as compensating consumption necessary to make agents indifferent between the two inflation regimes. The cost calculated in this paper is an order of magnitude larger than those from the earlier literature.

of the other research considered in Table 4, but we perhaps gain some insight into the problems of actual disinflation from moderate levels. The government in the model funds a portion of its activities with seignorage revenue and taxes on nominal quantities. These revenue sources are going to be less lucrative for the government at lower rates of inflation, and so, if the monetary authority moves inflation to less than 2.5 percent in our baseline economy, either the government is going to have to spend less relative to output, or raise taxes. Raising taxes to maintain revenue could in principle cause a larger distortion on net when inflation is reduced. We consider a tax increase scenario in which all the lost revenue is made up by increasing all tax rates equally in percentage point terms.[22]

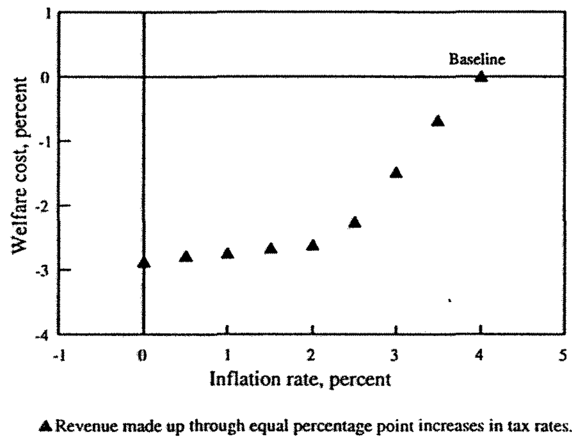
Figure 3 illustrates how the need to raise other distortionary taxes affects the welfare benefits of disinflation from low inflation rates.[23] In the figure, the welfare benefits from further reductions in inflation decline markedly as the inflation rate is reduced below 2.5 percent. The total net welfare benefit achieved by moving from 2.5 percent inflation to zero inflation is less than 0.2 percent of baseline real output. While this is still large by the standards of the papers listed in Table 4, it is quite small by the standards established

[22] An alternative tax increase scenario, in which revenue is made up only through increases in the tax rate on labor income, produced similar results.

[23] Some of the data points from Figure 3 are listed in the following table:

| | | | | | | |
|---|---|---|---|---|---|---|
| Inflation | 0 | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ |
| Welfare cost | −2.86 | −2.80 | −2.74 | −2.67 | −2.60 | −2.26 |

Figure 3
Disinflation with distorting taxes.




Welfare cost is compensating consumption as a percent of baseline real output.

Figure 3: Disinflation with distorting taxes. Declines in inflation from a baseline of four percent reduce government revenue. At low inflation rates, direct tax rates must be increased in order to maintain the baseline level of government spending relative to output. The marginal benefit of lower inflation is accordingly smaller in this region.

in this paper. The bottom line of this analysis is that the welfare gains from lower inflation can be largely offset by the welfare losses produced by more severe direct-tax distortions. Thus, while our model indicates that there are large potential welfare benefits from moving to inflation rates lower than 2.5 percent, it also suggests that capturing these benefits requires a degree of coordination between fiscal and monetary authorities that is not necessary when disinflating from higher levels of inflation.

**A purely monetary welfare cost.** Much of the literature on the welfare cost of inflation has concentrated on "purely monetary" welfare costs. In our analysis, however, inflation produces several distortions that are not directly connected to money demand. From a policy perspective, we think our approach provides a useful contribution, since the total distortion caused by inflation, taking other features of the economy as given, is the crucial yardstick by which judgements must be made. In this section of the paper, however, we would like to apportion our welfare cost estimates by source in order to allow easier comparisons to other studies and to provide intuition for our results.

We use the following approach. We shut off various sources of distortion one by one

| Table 5. Sources of welfare cost by type of distortion. | | | | |
|---|---|---|---|---|
| Case | Inter-mediation cost | Tax on interest income | Tax on corporate profits | Welfare cost of 5% vs. 4% inflation |
| 1 | 0 | 0 | 0 | 0.08 % |
| 2 | 0 | 0 | baseline | 0.42 % |
| 3 | 0 | baseline | 0 | 0.80 % |
| 4 | 0 | baseline | baseline | 1.01 % |
| 5 | baseline | 0 | 0 | 0.14 % |
| 6 | baseline | 0 | baseline | 0.77 % |
| 7 | baseline | baseline | 0 | 1.00 % |
| 8 | baseline | baseline | baseline | 1.29 % |

Table 5: Sources of welfare cost by type of distortion.

and in combination, leaving all other parameters set as in our baseline case and retaining our baseline inflation rate of 4 percent. Each time we shut off a source of distortion the government loses revenue, so the baseline ratio of government revenue to output declines. We hold the ratio of government revenue to output ratio fixed at this adjusted level and report the welfare cost of increasing the inflation rate to 5 percent. This approach is simple and illuminating, although we emphasize that the baseline steady states produced by shutting off various sources of distortion will not match the data as well as the baseline economy described in Table 3. Table 5 summarizes our findings. In the Table, an entry of "baseline" indicates that a parameter has been left at its baseline value, while an entry of zero indicates that the value of a parameter has been set at zero for the given case. Case 8 is our baseline equilibrium.

Case 1 in Table 5 gives our estimate of the purely monetary welfare cost of inflation: When intermediation costs and both capital income tax rates are set at zero, then a one percentage point increase in the rate of inflation produces a welfare cost of only 0.08 percent of the baseline level of real output. This cost estimate is of the same order of magnitude as the estimates from the previous literature that are summarized in Table 4. Including the cost of financial intermediation has not influenced our estimates very much, according to Table 5, which shows that that feature of our model is mainly useful in helping to obtain a reasonable fit to the data. Table 5 also makes it clear that including taxes that are levied on nominal quantities—either interest income taxes or corporate profits taxes—greatly increases the estimated welfare cost relative to the purely monetary case. We conclude that abstracting from inflation's interaction with the tax code would give a misleading picture of the welfare cost of inflation in our model.

**A decomposition.** In our model, an increase in the inflation rate reduces welfare primarily because it reduces the steady state real rate of return on household assets. We can gain insight into this effect by decomposing it into two parts. First, there is a *direct tax effect* by which increases in inflation increase the effective tax rate on capital income. In our model, this effect arises mainly because direct taxes on capital income are levied on nominal rather than real income, and to a lesser extent because inflation increases the cost to intermediaries of holding base money reserves. As a result of the direct tax effect, inflation reduces the after-tax real rate of return associated with any before-tax real rate of return.

But before-tax real rates of return also fall in the face of higher inflation in our model. This is the *general equilibrium effect* of an increase in the inflation rate. The general equilibrium effect stems from the fact that any increase in the inflation rate increases the amount of government revenue associated with each real interest rate. To see this, consider the non-labor-tax (the tax on labor is levied on a real quantity) real revenue of the government in steady state. The terms in the sum $T^i + T^k + C^s + B^s$ in steady state can, according to our earlier discussion of the equilibria of the model, be written as

$$\begin{aligned} T^i &= \left[-.012 - .182R^h + .182R^k\right]\left[B + 1.05K\right], \\ T^k &= \left[-.003 - .074R^h + .074R^k\right]\left[K\right], \\ C^s &= \left[.969 - .969R^h\right]\left[.018K\right], \\ B^s &= \left[1.008 - .087R^h - .882R^k\right]\left[B\right], \end{aligned} \tag{14}$$

where we have used our baseline parameter values and rearranged in order to reduce clutter.[24] In these expressions, the first term in brackets is the real tax rate for a given revenue source, and the second term in brackets is the tax base for that revenue source. The aggregate levels of capital, $K$, and bonds, $B$, can also be written entirely as (complicated) functions of $R^h$ and $R^k$, a fact we want to keep in mind but do not explicitly note in these expressions. If we consider a steady state with a higher level of inflation relative to baseline (a lower value of $R^h$), then the tax rate portions of all these expressions will increase. In our model, the levels of $B$ and $K$ will also change, even if $R^k$ is held fixed, because the real rates of return intermediate between $R^{kn}$ and $R^h$ will all fall when $R^h$ falls, since they are all essentially convex combinations of $R^h$ and $R^{kn}$. However, in our baseline economy, these changes in asset demand are by themselves insufficient to offset the direct increase in tax rates. Thus, government revenue from all of these sources will increase. Since we want to maintain the ratio of government spending to output at a constant, this cannot be a new steady state equilibrium. To create a new equilibrium,

[24] In producing these expressions, we have taken advantage of the fact that $A^-$ is zero in our baseline equilibrium.

Figure 4. A shifting Laffer curve

Baseline economy

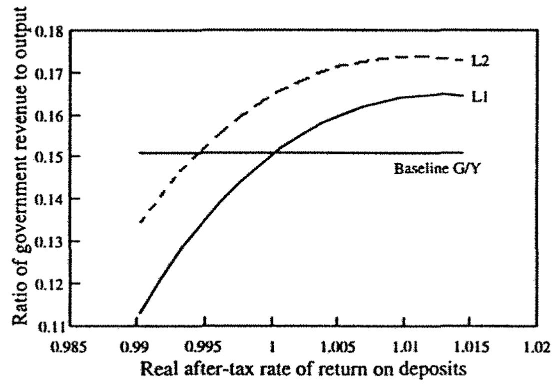



Figure 4: When inflation rises, government revenue relative to output rises for any given interest rate. Here, the curve labelled L1 is the relevant portion of the total revenue Laffer curve for four percent inflation, our baseline case, and L2 is the same curve for five percent inflation.

the before-tax real rate of return $R^k$ falls and crowds in private credit. As $R^k$ falls, the expressions above show that the effective tax rates on capital income are reduced, which helps to move the economy closer to an equilibrium. The effective tax rate in the bond seignorage equation, however, is increased further, and sharply. The crowding in of private credit offsets this effect and allows the economy to reach an equilibrium by causing the level of bonds to fall. In our quantitative comparisons of steady states with four versus five percent inflation, real government revenues relative to output from capital income taxes and base money seniorage rise with inflation, while bond seniorage revenues relative to output decline because of the fall in bonds. The fall in bonds is the key element that allows the government budget constraint to be satisfied at the new, higher level of inflation.[25]

The effect of an increase in the inflation rate on the real after-tax rate of return facing households is illustrated in Figure 4. The figure displays the relevant portions (the left sides) of a pair of total revenue Laffer curves—curves that indicate the ratio of total government revenue to output that is associated with each after-tax real interest rate on deposits, with the inflation rate and all the direct tax rates held fixed. In the figure, the curve labelled $L1$ is the total revenue Laffer curve for our baseline inflation rate of four

[25] For a more detailed analysis of this effect see Espinosa and Russell (1996). Their analysis is conducted using a two-period overlapping generations model that does not include a direct tax system.

percent. This curve intersects a horizontal line representing the baseline $G/Y$ ratio at the baseline value of the real after-tax deposit rate. The curve labelled $L2$ is the total revenue Laffer curve for an inflation rate of five percent, which is shifted up relative to $L1$ by the increase in government revenues produced by the increased inflation rate. This curve intersects the baseline $G/Y$ ratio at a new, lower real rate of return.

We can measure the relative sizes of the general equilibrium effect and direct tax effect by performing the following decomposition. Let us suppose that the fiscal authority in our model attempts to offset the increase in government revenue produced by moving from four to five percent inflation by reducing the only direct tax that is levied on a real quantity, which is the tax on labor income. Suppose they reduce this tax rate just until the point where the total real return to capital is unchanged from the baseline, four percent inflation case. Although the before-tax real return on capital is thus unchanged by construction, after-tax real rates of return will be reduced because higher inflation has increased the effective tax rate on capital income. In Figure 4, this amounts to drawing a third total revenue Laffer curve intermediate between $L1$ and $L2$, a curve we have omitted in the figure only to reduce clutter, and determines a third equilibrium real after-tax return to deposits intermediate between the two in the figure. This calculation effectively decomposes the total effect of higher inflation on the real after-tax rate of return to deposits into (1) the general equilibrium effect, the reduction in real returns that comes from a shift up in the total revenue Laffer curve that occurs because of a tax increase, and (2) the direct tax effect, the change in the shape of the total revenue Laffer curve that occurs because an increase in inflation directly increases effective tax rates on nominal quantities in the model.

This decomposition yields the following important finding: in our baseline economy, the general equilibrium component of the welfare cost of inflation is about $3\frac{1}{2}$ times larger than the direct tax component. Of the total welfare cost of five versus four percent inflation of 1.29 percent reported above, our decomposition has it as a general equilibrium effect of about 1.01 percent and a direct tax effect of about .28 percent. Both effects are important—our estimate of the cost from the direct tax effect alone is roughly three times larger than the typical total welfare cost estimate in the literature. However, the general equilibrium effect is clearly the dominant one. This dominance is also indicated in Table 6, which reports the effects of a one percent increase in inflation on the entire structure of real interest rates in our baseline case.

**Alternative parameterizations.** In this subsection, we report the results from a few alternative parameterizations of our baseline economy. Some economists might argue that we have too much curvature in our preference map—an objection that raises the

| Table 6. General equilibrium and direct tax effects. | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $R^h$ | $R^{ba}$ | $R^b$ | $R^{kc}$ | $R^{ka}$ | $R^{kn}$ | $R^k$ |
| Baseline | 0.9615 | 1.0003 | 1.0100 | 1.0108 | 1.0288 | 1.0342 | 1.0781 |
| Direct tax effect | 0.9524 | 0.9978 | 1.0092 | 1.0101 | 1.0281 | 1.0342 | 1.0781 |
| GE effect | 0.9524 | 0.9950 | 1.0057 | 1.0066 | 1.0246 | 1.0304 | 1.0743 |

Table 6: Direct tax and general equilibrium effects for a movement from four to five percent inflation. The total effect is the difference from the first to the third line, and the general equilibrium effect is the difference between the second and third lines. The change in the after-tax real return to bonds is approximately split between the two effects, but the general equilibrium effect has a larger impact on the rest of the return structure.

question of whether fairly strong resistance to intertemporal substitution is an important source of our findings. In fact, our results do display some sensitivity to different intertemporal substitution elasticities. We can illustrate this sensitivity by resetting the curvature parameter $\delta_2$ to a value that produces an EIS of .56, a point estimate due to Attanasio and Weber (1995).[26] The welfare cost of five versus four percent inflation is then 0.89 percent of the baseline level of real output. While this cost estimate is lower than our baseline estimate of 1.29 percent, it remains large by the standards of the previous literature. This alternative baseline steady state does not match the data very well on the dimensions we used in Table 3.

Other economists might argue that more curvature in the preference map might be necessary in order to attribute the observed equity premium to the pricing of risk. Campbell and Cochrane (1994) use a model with a local relative risk aversion coefficient of 48.4 to duplicate the equity premium. While this figure is not strictly comparable to ours because the authors use habit formation preferences, the idea is just to consider a high risk aversion case. With a CRRA of 48.4 (an EIS of 0.021), the implied welfare cost of a one percent increase in the inflation rate is 1.46 percent of the baseline level of real output, which is larger than our preferred estimate. This alternative steady state matches our targets about as closely as our baseline parametrization.

Finally, our best fit capital share parameter is lower than values sometimes used in the literature. Cooley and Prescott (1994) recommend a value of 0.4 for this parameter. If we make this alteration to our baseline case, the welfare cost of five versus four percent inflation falls to 0.67 percent of the baseline level of real output, which is smaller than our preferred estimate but not enough to change our qualitative conclusions. This alternative baseline steady state is not, however, a good match for the data.

[26]When we make this alteration and the one below, we also change $\delta_1$ in a way that keeps the effective rate of time preference unchanged.

**5.3. A caution on interpretation.** We have followed most of the previous literature on the welfare cost of inflation by confining our analysis to steady state comparisons. Conducting comparisons of this sort is generally regarded as a plausible strategy for obtaining order-of-magnitude estimates of the welfare cost of inflation in actual economies. Of course, these comparisons involve permanent and perfectly credible changes in monetary policy, and policy changes of this type have not actually been observed in the postwar U.S. A more complete analysis of the welfare implications of changes in inflation would also study costs (or benefits) incurred during the transition path from one steady state to another. Our preliminary work on this question indicates that in our model these transitions take significant amounts of time. Thus, our results should not be interpreted as suggesting that welfare gains from lower inflation can be realized overnight, and it is conceivable that a transition analysis might alter some of our basic findings.

We can, however, cite one interesting piece of evidence which suggests that this probably would not be the case. Suppose we imagine that a shift from the old steady state to a new steady state occurs instantly at some date $T$. We can use our utility function to conduct across-steady-state comparisons of the remaining welfare of the members of generations $T, T-1, T-2, \ldots, T-54$—that is, of cohorts of agents with 55, 54, ... , 1 years left to live at date $T$—from date $T$ to the end of their lives. In our baseline economy, the remaining welfare of the members of each of these cohorts is higher in a steady state with 3 percent inflation than in the baseline steady state with 4 percent inflation. Thus, across steady states all age cohorts benefit from lower inflation. Consequently, if the transition path between the steady states is close to linear then the benefits of a disinflation undertaken at a date $T$ should start accruing immediately to all members of the society.

We recognize that our assumptions about the tax system are at best a crude approximation of the complex and nonlinear array of taxes imposed by U.S. federal, state and local governments. We have adopted the conservative approach of allowing a large fraction of government expenditures to be financed by a tax on real labor income — a tax whose effective rate does not depend on the rate of inflation. This decision probably causes us to understate the historical welfare cost of inflation: actual income taxes are levied on nominal income in a progressive manner, and prior to the 1980s "bracket creep" allowed increases in inflation to increase both effective labor income tax rates and government labor income tax revenues. We also ignore the historical effect of "bracket creep" on income from interest and capital gains. (In the case of capital gains, the tax reforms of the 1980s reduced this effect but did not eliminate it entirely.) On the other hand, our assumption that the corporate profits tax acts analogously to the interest income tax as a tax on nominal returns to capital is at least partly counterfactual: under the U.S. tax system, the effective corporate profits tax rate is not directly increased by inflation. However, we think this assumption is reasonable, as a first approximation, for two reasons. First, our corporate profits tax is intended partly as a proxy for a tax on capital gains, which is absent from our model: the effective tax rate on capital gains does increase with higher inflation. Second, the fact that the U.S. tax system uses historic cost depreciation allows inflation to increase the effective tax rate on corporate profits indirectly, by reducing the real value of depreciation allowances.[27]

Our model also abstracts from another important feature of the U.S. tax system, which is that households may deduct mortgage interest payments (and before the 1980s, other interest payments) from their taxable incomes. But our tax assumptions account for this effect by taxing households on their *net* interest income. In our general equilibrium environment, the net asset position of the households is essentially the capital stock of the economy. Similarly, our model does not distinguish returns paid by firms as dividends from returns paid as interest: under the U.S. tax system, firms are taxed on the former but not the latter. As a result, it may seem that we are overstating the extent of double taxation of capital. We address this problem by choosing a relatively low corporate profits tax rate—a rate that allows us to duplicate the observed ratio of corporate profits tax revenue to government expenditures in our baseline equilibrium.

In sum, we think our tax assumptions provide an approximation of the U.S. tax system that is adequate for our purposes. However, further research on the nature of the interaction between inflation and the tax code in general equilibrium models is certainly warranted.

## 6. Concluding remarks

In this paper, we use a dynamic general equilibrium model to estimate the welfare cost of inflation in the U.S. economy. Our analysis is based on the hypothesis that one of the principal costs of inflation is its tendency to reduce real rates of return—a view that is often expressed in both policy circles and academic discussions. According to our estimates, the welfare cost of inflation is far larger than most of the literature to date has indicated. However, our estimate of the purely monetary component of the welfare cost—the component studied in most previous work on this topic—is of the same order of magnitude as previous estimates. In our model, the bulk of the welfare cost of inflation can be attributed to the fact that it increases the effective tax rate on capital income. Interestingly, only a relatively small portion of this cost component grows out of the fact that inflation-induced increases in the effective tax rate reduce the after-tax real rate of

[27] Both these effects are discussed by Feldstein (1996), who concludes that inflation does indeed increase the effective tax rate on corporate profits and that overall, the effect of inflation on the effective tax rate on income produced by firms in the form of dividends and capital gains is actually somewhat larger than its effect on the effective tax rate on interest paid by firms.

return on capital, a distortion whose impact has been studied previously. Instead, the lion's share of the welfare cost of inflation is due to the general equilibrium impact of the increased effective tax rate on the before-tax real rate of return. Thus, our results suggest that abstracting from general equilibrium considerations may seriously understate the welfare cost of inflation, a conclusion we share with Dotsey and Ireland (1996). We conclude that the impact of inflation on the macroeconomy may be far more corrosive than most economists have previously believed.

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