ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS
WORKING PAPER SERIES

State Government Effects on the Spatial Distribution of Inward Foreign Direct Investment

STATE GOVERNMENT EFFECTS ON THE
SPATIAL DISTRIBUTION OF INWARD
FOREIGN DIRECT INVESTMENT

Cletus C. Coughlin,* Joseph V. Terza,**
and Vachira Arromdee***

Federal Reserve Bank of St. Louis

87-007

  * Senior Economist, Federal Reserve Bank of St. Louis

 ** Department of Economics, Pennsylvania State University,
     University Park, Pennsylvania

*** Department of Economics, University of Georgia, Athens,
     Georgia

ABSTRACT

In a recent review of the literature, Wasylenko (1981) concluded that taxes have very little effect on interregional business location decisions. The present study examines the impact of state taxes and incentive programs on the spatial distribution of inward foreign direct investment in manufacturing. The results reveal that taxes, which were measured in various ways, deter foreign direct investment. Conversely, states providing tax incentives, financial assistance, and employment assistance tended to have larger numbers of foreign direct investments.

State Government Effects on the Spatial Distribution of Inward
Foreign Direct Investment

I. Introduction

In a recent review of the literature, Wasylenko (1981) concluded that

taxes have very little effect on interregional business location decisions.

Nonetheless, the continued use of tax and fiscal inducements by governments

suggests that policymakers believe otherwise. As a result economists have

continued to explore this issue. For example, Carlton (1983) concluded that

taxes and state incentive programs did not have major effects on the location

of new branch plants across standard metropolitan statistical areas. On the

other hand, Bartik (1985) found that state taxes deterred the location of new

branch plants at the state level.[1]

The present study examines the impact of taxes and incentive programs in

the context of the spatial distribution of inward foreign direct investment.

The stock of foreign direct investment in U.S. manufacturing increased by a

factor of five from $11.4 billion in 1975 to $60.8 billion in 1985.[2]

Hartman (1984) demonstrated theoretically and empirically that federal tax

policy has a substantial effect on foreign direct investment in the United

States; however, there has been virtually no research attempting to identify

state government effects on the spatial distribution of foreign direct

investment. In fact, despite the increasing importance of foreign direct

investment for economic development at the state level, literature reviews by

Ó hUallacháin (1986) and Arpan et al. (1981) revealed that there has been

virtually no economic analysis of the spatial distribution of foreign direct

investment across all states.

A recent paper by Coughlin et al. (1987) began to address this void by

developing and estimating a linear conditional logit model in order to

identify the location determinants of foreign direct investment. Their findings suggested that state government actions have influenced the distribution of foreign direct investment. Specifically, the general level of taxation and the use of unitary taxation deterred foreign direct investment, while there was a positive association between foreign direct investment and reverse-investment promotion expenditures.

The basic model developed in Coughlin et al. is used as a foundation for the present research. Additional tax and incentive variables are examined in a systematic manner in hopes of producing a generalization concerning the impact of taxes and fiscal incentives on the location of foreign direct investment. A summary of the previously developed model is presented in the next section. Following the summary of the model, previous results are discussed and the tax and incentive variables are identified. Next, the empirical analysis of the impact of tax and incentive variables is presented. A final discussion highlights the basic findings concerning the impact of state government effects on the location of foreign direct investment.

II. Summary of Model

A linear conditional logit model is used to examine statistically the potential determinants of foreign direct investment.$\underline{3/}$ After a review of the linear conditional logit model, the primary findings of Coughlin et al. (1987) are summarized. This section concludes with an identification of the tax and incentive variables that are the focus of the present study.

We assume that a foreign firm will choose to invest in a particular state if and only if doing so will maximize profit. Formally, the jth state is chosen by the ith firm if and only if

$$\Pi_{ij}^* = \max (\Pi_{im}^*; \ m = 1, \ \ldots, \ 50) \qquad (1)$$

where Π_{ij}^* denotes the profit of the ith firm given that it locates in the jth state ($j = 1, \ \ldots, \ 50$).

Following Carlton (1983) we assume that

$$\Pi_{ij} = C + X_j \beta + \varepsilon_{ij} \qquad (2)$$

where $\Pi_{ij} = \ln \Pi_{ij}^*/\theta$, C is an unknown constant, $X_j = [\ln X_{j1}^*, \ \ldots, \ \ln X_{jK}^*]$, $X_j^* = [X_{j1}^*, \ \ldots, \ X_{jK}^*]$ is a vector of observable characteristics for the jth state, β is a vector of unknown coefficients to be estimated, ε_{ij} is the random term denoting the unobservable (by the researcher) unique profit advantages to the ith firm from locating in the jth state, and θ is the exponent of the random term in the untransformed version of the profit function.[4/] Assuming that the ε_{ij}'s are independent log-Weibull distributed McFadden (1974) shows that

$$P_j = \exp(X_j \beta) / \sum_{k=1}^{50} \exp(X_k \beta) \qquad (3)$$

where P_j denotes the population relative frequency of locating in state j. From (3) it follows that

$$\log(P_j/P_1) = Z_j \beta \qquad (4)$$

where $Z_j = X_j - X_1$.

Since we cannot observe P_j, the population relative frequency, we make equation (4) operational by replacing P_j with p_j, the observed investment frequencies for the 50 states for a particular year. This substitution, however, brings with it the possibility that the left-hand side of (4) will be undefined since some states may be observed with zero investment frequencies. One's first inclination is to correct this problem by simply substituting some arbitrarily small constant for the null observed frequencies. Unfortunately small changes in any constant added to, or subtracted from, the observed frequencies can cause large differences in the estimation results. Therefore such a constant must be chosen judiciously. It is shown in Coughlin et al. (1987) that the theoretically correct value is 1/2n and that

$$\log \left(\frac{P_j + (1/2n)}{P_1 + (1/2n)} \right) = \log \left(\frac{P_j}{P_1} \right) + e_j = Z_j \beta + e_j \tag{5}$$

$$(j = 1, \ldots, 49)$$

where n denotes the total number of observed investments in the sample, and e_j is the stochastic term such that $E[e_j] = \emptyset$ except for terms of order smaller than n^{-1} in probability.

Rewriting (5) in matrix notation we have

$$y = Z\beta + e \tag{6}$$

where y is the 49x1 vector whose j^{th} element is $\log \left(\frac{P_j + (1/2n)}{P_1 + (1/2n)} \right)$, Z is

the 49xK matrix whose j^{th} row is Z_j, and e is the 49x1 vector whose j^{th} element is e_j. In order to exploit efficiency gains we apply the following feasible GLS estimator to (6)

$$\tilde{\beta} = (Z'\hat{\Omega}^{-1}Z)^{-1}Z'\hat{\Omega}^{-1}y \tag{7}$$

where the explicit form of $\hat{\Omega}$ is given in the appendix. $\tilde{\beta}$ is consistent and asymptotically normal with covariance matrix $(Z'\Omega^{-1}Z)^{-1}$, where Ω denotes the covariance matrix of e.[5/] Furthermore $(Z'\hat{\Omega}^{-1}Z)^{-1}$ is a consistent estimator for $(Z'\Omega^{-1}Z)^{-1}$.

III. Previous Results and Additional Tax and Incentive Variables

The probability of selecting a specific state for a foreign direct investment transaction depends on the levels of its characteristics that affect profits relative to the levels of these characteristics in other states. In addition, the probability of a manufacturing foreign direct investment transaction in a state depends on the number of potential sites for locating the investment. Coughlin et al. (1987) identified a number of statistically significant determinants of the number of foreign direct investments by state. Aside from taxes, the transportation infrastructure, and the labor-management legal environment, six determinants were the number of potential site (LAND), state per capita income (PINC), average state wage of production workers in manufacturing (WAGE), state unemployment rate (UNEM), state expenditures on reverse-investment promotion (PROM), and energy costs (EN). The definitions of these variables, as well as all other variables used in the present study, are provided in Table 1, while the empirical results associated with aforementioned six variables are listed at the beginning of Table 2.

The results are consistent with expectations. The proxy for the number of sites, per capita income, unemployment, and reverse-investment promotion expenditures are positive, statistically significant determinants of the location of foreign direct investment, while wage rates and energy costs are negative, statistically significant determinants of the location of foreign direct investment. Given the preceding determinants, seven tax and three incentive variables are examined to see if there is any evidence that state tax and incentive policies affect the location decisions of foreign investors. In other words, is there any systematic evidence that taxes deter foreign direct investments or that state incentives attract foreign direct investments?

The measurement of state tax burdens is complex. Identifying the incidence of a tax, the possibility that taxes are financing goods and services desired by business, and the use of tax incentives are some of the numerous difficulties of deciding whether taxes affect business location decisions and then constructing measures of these taxes. A number of tax measures are examined. Two standard measures are state and local taxes per capita (PTAX) and state and local taxes as a percentage of personal income (TAXSPI). Future tax liabilities might also be a deterrent. State long term debt per capita (PDEBT) is used as a proxy for future tax liabilities.

In addition to the preceding general measures of state tax burdens, four measures of taxes related directly to business are examined. Wheaton (1983) constructed two measures of taxes restricted to the legal liabilities of manufacturers. These measures are state and local taxes paid by manufacturers as a percentage of income (TBI) and state and local taxes paid by manufacturers as a percentage of capital stock (TCS).

The two remaining measures of business taxes are dummy variables. The first is the existence of a state corporate income tax (TCORP). The second business tax measure pertains to a tax issue associated with foreign direct investment. The use of unitary taxation (TUNIT) has generated numerous objections from multinational corporations (Tannenwald, 1984). The controversy stems from the division of a company's taxable income among different tax jurisdictions. A state with unitary taxation taxes a fraction of a multinational corporation's worldwide income rather than just the income earned in the state. Multinational corporations have objected to unitary taxation on the grounds that they become exposed to double taxation and are forced to bear additional accounting costs. On the other hand, state tax officials argue that unitary taxation is the only method that prevents multinational corporations from reallocating profits from high-tax areas to low-tax areas via transfer pricing.

In addition to the tax variables, the impacts of three incentive variables are examined. Comprehensive information on the use of reverse-investment incentives is limited. A survey by Berry and Mussen (1980) generated useful data on whether or not particular states used various reverse-investment incentives in 1980. The survey revealed the use of numerous programs and services that could be characterized as incentives. In the present study the impacts of tax incentives (TAXASS), financial assistance (FINASS), and employment assistance (EMPASS) are examined. Tax incentives (e.g., property tax reductions) were used by seventeen states, financial assistance (e.g., low interest loans) was provided by twenty-three states, and employment assistance (eg., training and recruitment of employees) was available in thirty-eight states. Due to the difficulty of quantifying these incentives a dummy variable is used for each type of incentive.

IV. Results

The linear conditional logit results using the tax and incentive variables are summarized in Table 2. In order to avoid repetition and conserve space the results concerning the number of potential sites, per capita income, wage rates, unemployment rates, reverse-investment promotion, and energy costs are listed only one time.[6/] It is sufficient to note that the signs of the estimated coefficients remain unchanged across all variants and that the statistical significance is generally unaffected.[7/]

The eighteen reported variants can be divided into four groups for discussion purposes. Variants #1-#6 show the results of appending the tax variables to what may be called the basic model. Variant #1 reveals that per capita state and local taxes is a negative, statistically significant determinant of the location of foreign direct investment. An identical comment, although the result is not reported, is appropriate when taxes as a percentage of personal income is used as the measure of state tax burden. Variant #2 shows that per capita state long term debt is related negatively to foreign direct investment, but it is not statistically significant.

Variants #3-#6 reveal that the tax measures related directly to business taxation perform as expected. Variants #3 and #4 show that Wheaton's (1983) estimates of state and local taxes paid by manufacturers as a percentage of income and of capital stock for 1977 are related negatively to the spatial distribution of foreign direct investment. Similar statements can be made concerning the impact of the existence of a corporate income tax in variant in #5 and the use of unitary taxation in variant #6.

The second group of reported models, variants #7-#9, highlight the results of appending each of the three incentive variables to the basic model. The

coefficient estimates for tax incentives (#5), financial assistance (#6), and employment assistance (#7) reveal a positive impact, but only the variables for financial assistance and employment assistance are statistically significant at the .10 level.

Variants #10-#12 show the results of appending two tax measures to the basic model. In each reported case there are separable deterrent effects associated with the different tax measures. In variant #10 per capita state and local taxes and unitary taxation deter foreign direct investment, but unitary taxation is not statistically significant at the .10 level. In variant #11 per capita state and local taxes and the existence of a corporate income tax are negative, statistically significant determinants of foreign direct investment. In variant #12 the preceding statement can be made for state and local taxes paid by manufacturers as a percentage of income and the use of unitary taxation.

The last group, variants #13-#18, reveals the results of appending a tax measure and an incentive measure to the basic model. Variants #13-#15 use the existence of a corporate income tax with the three incentive measures, respectively. In each case the tax measure is negative, statistically significant determinant of foreign direct investment, while the incentive measures are positive, statistically significant determinants. Variants #16-#18 use state and local taxes paid by manufacturers as a percentage of income with the three incentive measures, respectively. In each case the tax measure is a negative, statistically significant determinant. While all the incentive measures are related positively to foreign direct investment, only tax incentive and employment assistance are statistically significant. Unreported combinations of tax measures and incentive measures revealed that taxes were consistently statistically significant, while only employment

assistance of the incentive variables was consistently statistically significant.

IV. Conclusion

Despite the empirical regularity that taxes have very little effect on interregional business location decisions, it is clear that this finding cannot be generalized to the location of foreign direct investment at the state level. The measurement of taxes in a general manner such as per capita taxation or as a percentage of income and the measurement of business-related taxes such as state and local taxes paid by manufacturers as a percentage of either income or capital stock, the existence of a corporate income tax, or the use of unitary taxation generated same conclusion. Similar to Bartik's (1985) recent finding contradicting the conventional wisdom that taxes do not affect the location of domestic investment, it is clear that taxes deter foreign direct investment.

It is also clear that Carlton's (1983) finding that incentive programs did not have a major effect on the location of new branch plants cannot be generalized to the location of foreign direct investment at the state level. States providing tax incentives, financial assistance, and employment assistance tended to receive larger numbers of foreign direct investments.

In conjunction with the previous finding by Coughlin, Terza, and Arromdee (1987) that state expenditures on reverse-investment promotion were related positively to foreign direct investment, the current findings indicate that state government fiscal policies have a significant impact on the location of foreign direct investment. These findings can be used as the foundation to explore the many issues surrounding state government and foreign direct

investment. A possible reason for the empirical regularity that taxes have little effect on interregional business location decisions is that the taxes are financing the provision of goods and services valued by business. In view of the statistical significance of the tax variables, the current study reveals the deterrent effects of taxes without controlling for public expenditures (other than reverse-investment promotion expenditures). Nonetheless, future studies could control for different types of public expenditures.[8/] There are also numerous efficiency questions arising from the involvement by state governments. In addition, there are numerous questions concerning the impacts of these fiscal policies upon different industries and different source countries.

FOOTNOTES

1. Recent studies by Steinnes (1984), Helms (1985), Wasylenko and McGuire (1985), and Benson and Johnson (1986) have examined the effect of fiscal policy at the state and local level on economic growth. A standard criticism of studies that conclude taxes do not matter is that because they are cross-section studies they are unable to estimate the impact of a tax change on a particular region over time. The aforementioned studies address this criticism and, excluding Steinnes, conclude that taxes deter growth.

2. The figure for 1975 was taken from Selected Data on Foreign Direct Investment in the United States, 1950-79 and is based on 1974 benchmark data. The figure for 1985 was taken from an article in the Survey of Current Business, "Foreign Direct Investment in the United States: Detail for Position and Balance of Payments Flows, 1985," and is based on 1980 benchmark data. The estimates are sensitive to the benchmarks, but for present purposes the figures are not so sensitive as to raise doubts about the rapid increase in foreign direct investment in the United States.

3. The Department of Commerce (1982) defines foreign direct investment as the direct or indirect ownership by a foreign entity of 10% or more of the voting securities of an incorporated business enterprise or an equivalent interest in an unincorporated business enterprise. A foreign direct investment transaction in manufacturing could involve an acquisition, a merger, an equity increase, a joint venture, a new plant or a plant extension. In 1981 there were 274 manufacturing foreign direct investment transactions in the U.S.

4. See Carlton (1983) p. 441.

5. The explicit form of Ω is given in the appendix.

6. Complete results are available upon request from Coughlin.

7. The only noteworthy exception concerning statistical significance is that the inclusion of the two general measures of taxation, PTAX and TAXSPI, cause energy costs to be statistically insignificant.

8. Bartik (1985) shows that improved public services can affect business location decisions. A related finding by Helms (1985) is that increases in state and local taxes to fund transfer payments retard state economic growth, however, when the revenues are used to finance public services, the positive growth effects of these public services may more than offset the disincentive effects of the increased taxes. The finding that taxes, to the extent they are redistributive, deter economic growth has also been demonstrated by Romans and Sabrahmanyan (1979). Wasylenko and McGuire (1985) have also found that increased spending on a public service such as education can mitigate the adverse growth consequences of higher taxes.

REFERENCES

Arpan, Jeffrey S., Edward B. Flowers, and David A. Ricks. "Foreign Direct Investment in the United States: The State of Knowledge in Research," Journal of International Business Studies, XII (Spring/Summer 1981), 137-153.

Bartik, Timothy J. "Business Location Decisions in the United States: Estimates of the Effects of Unionization, Taxes, and Other Characteristics of States," Journal of Business and Economic Statistics, III (January 1985), 14-22.

Benson, Bruce L. and Ronald N. Johnson. "The Lagged Impact of State and Local Taxes on Economic Activity and Political Behavior," Economic Inquiry, XXIV (July 1986), 389-401

Berry, Willard M. and William A. Mussen. "Part I - A Report," in Export Development and Foreign Investment: The Role of the States and Its Linkage to Federal Action, Committee on International Trade and Foreign Relations, Washington, D.C.: National Governors' Association, 1980.

Carlton, Dennis W. "The Location and Employment Choices of New Firms: An Econometric Model with Discrete and Continuous Endogeneous Variables," Review of Economics and Statistics, XV (August 1983), 440-449.

Coughlin, Cletus C., Joseph V. Terza, and Vachira Arromdee. "State Characteristics and the Location of Foreign Direct Investment within the United States: A Linear Conditional Logit Model," Federal Reserve Bank of St. Louis, unpublished manuscript, August 1987.

Hartman, David G. "Tax Policy and Foreign Direct Investment in the United States," National Tax Journal, XXXVII (December 1984), 475-487.

Helms, L. Jay. "The Effect of State and Local Taxes on Economic Growth: A Time Series - Cross Section Approach," Review of Economics and Statistics, LXVII (November 1985), 574-582.

McFadden, Daniel. "Conditional Logit Analysis of Qualitative Choice Behavior," in Paul Zarembka (ed.), Frontiers in Econometrics, New York: Academic Press, 1974, 105-142.

O hUallachain, Breandan. "The Role of Foreign Direct Investment in the Development of Regional Industrial Systems: Current Knowledge and Suggestions for a Future American Research Agenda," Regional Studies, XX (April 1986), 151-162.

Romans, Thomas, and Ganti Sabrahmanyan. "State and Local Taxes, Transfers and Regional Economic Growth," Southern Economic Journal, XLVI (October 1979), 435-444.

Steinnes, Donald N. "Business Climate, Tax Incentives, and Regional Economic Development," Growth and Change, XV (April 1984), 38-47.

Steinnes, Donald N. "Business Climate, Tax Incentives, and Regional Economic Development," *Growth and Change*, XV (April 1984), 38-47.

Tannenwald, Robert. "The Pros and Cons of Worldwide Unitary Taxation," *New England Economic Review*, (July/August 1984), 17-28.

U.S. Department of Commerce, Bureau of Economic Analysis. "Foreign Direct Investment in the United States: Detail for Position and Balance of Payments Flows, 1985," *Survey of Current Business*, LXVI (August 1986), 74-88.

U.S. Department of Commerce, Bureau of Economic Analysis. *Selected Data on Foreign Direct Investment in the United States, 1950-79*, Washington, DC: U.S. Government Printing Office, 1984.

U.S. Department of Commerce, International Trade Administration. *Foreign Direct Investment in the United States -- 1981 Transactions*, Washington, DC: U.S. Government Printing Office, 1982.

Wasylenko, Michael and Therese McGuire. "Jobs and Taxes: The Effect of Business Climate on States' Employment Growth Rates," *National Tax Journal*, XXXVIII (December 1985), 497-511.

Wheaton, William C. "Interstate Differences in the Level of Business Taxation," *National Tax Journal*, XXXVI (March 1983), 83-94.

Zellner, Arnold and Tong Hun Lee. "Joint Estimation of Relationships Involving Discrete Random Variables," *Econometrica*, XXXIII (April 1965), 382-394.

Table 1

Definitions and Expected Impacts of Independent Variables

LAND   — natural logarithm of 1981 state land area excluding federal
land (+)

PINC   — natural logarithm of 1981 state per capita income (+)

WAGE   — natural logarithm of 1981 average state wage of production
workers in manufacturing (−)

UNEM   — natural logarithm of 1981 state unemployment rate (+)

PROM   — natural logarithm of 1980 state expenditure on
reverse-investment promotion (+)

EN   — natural logarithm of 1981 energy costs per dollar of
value-added in manufacturing (−)

PTAX   — natural logarithm of 1981 state and local taxes per capita (−)

TAXSPI — natural logarithm of 1981 state and local taxes as a percentage
of state personal income (−)

PDEBT   — natural logarithm of 1981 state long term debt per capita (−)

TBI   — natural logarithm of 1977 state and local taxes paid by
manufacturers as a percentage of income (−)

TCS   — natural logarithm of 1977 state and local taxes paid by
manufacturers as a percentage of capital stock (−)

TCORP — dummy variable equal to one if a state has a corporate income
tax in 1981 and zero otherwise (−)

TUNIT   — dummy variable equal to one if a state has "total worldwide
combination" unitary taxation in 1981 and zero otherwise (−)

TAXASS — dummy variable equal to one if a state provides tax incentives
for reverse-investment in 1980 and zero otherwise (+)

FINASS — dummy variable equal to one if a state provides financial
assistance for reverse-investment in 1980 and zero otherwise (+)

EMPASS — dummy variable equal to one if a single state provides
employment assistance for reverse-investment in 1980 and zero
otherwise (+)

Table 2

Estimation Results

Independent Variables	LAND	PINC	WAGE	UNEM	PROM	EN
Coefficient Estimates (t-ratios)	0.304^a (3.11)	5.305^a (4.68)	-4.442^a (-4.55)	2.254^a (4.35)	0.124^a (2.85)	-0.699^b (-2.67)

Variant	Independent Variables	Coefficient Estimates (t-ratios)
#1	PTAX	-1.416^a (-4.77)
#2	PDEBT	-0.071 (-0.57)
#3	TBI	-0.662^a (-3.53)
#4	TCS	-0.718^a (-4.82)
#5	TCORP	-0.836^b (-2.16)
#6	TUNIT	-1.523^a (-3.83)
#7	TAXASS	0.262 (1.66)
#8	FINASS	0.338^c (1.73)
#9	EMPASS	0.623^c (1.85)
#10	PTAX	-1.100^a (-3.44)
	TUNIT	-0.692 (-1.67)

Table 2, continued

#11	PTAX	-1.384^{a} (-4.65)
	TCORP	-0.707^{c} (-1.84)
#12	TBI	-0.468^{b} (-2.44)
	TUNIT	-1.143^{a} (-2.80)
#13	TCORP	-0.940^{b} (-2.42)
	TAXASS	0.315^{c} (1.97)
#14	TCORP	-0.896^{b} (-2.31)
	FINASS	0.387^{c} (1.97)
#15	TCORP	-0.843^{b} (-2.08)
	EMPASS	0.713^{b} (2.07)
#16	TBI	-0.687^{a} (-3.67)
	TAXASS	0.269^{c} (1.72)
#17	TBI	-0.639^{a} (-3.38)
	FINASS	0.177 (0.90)
#18	TBI	-0.663^{a} (-3.38)
	EMPASS	0.632^{b} (1.95)

[a]statistically significant at the .01 level (two-sided)
[b]statistically significant at the .05 level (two-sided)
[c]statistically significant at the .10 level (two-sided)

Appendix

The typical element of Ω, the covariance matrix of e is

$$\omega_{ji} = \frac{(P_1 - P_j)^2 + 4n(P_jP_j^2 + P_j^2P_1)}{4n^1P_j^2P_1^2} \quad \text{if } j = i$$

and

$$\omega_{ji} = \frac{(P_1 - P_j)(P_1 - P_i)^2}{4n^2P_jP_iP_1^2} + \frac{1^2}{nP_1} \quad \text{if } j \neq i.$$

The estimated covariance matrix $\hat{\Omega}$ is the same as Ω with $\hat{P}_j$ substituted for

P_j, where $\hat{P}_j = \exp\left(X_j\hat{\beta}\right) / \sum\limits_{k=1}^{50} \exp\left(X_k\hat{\beta}\right)$ and $\hat{\beta}$ is the OLS estimator of β.

See Coughlin et al. (1987) for the derivations of the above expressions.