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DERIVATION OF THE SET OF EXACT HEDGES FOR THE FINANCIAL PORTFOLIO

by Michael T. Belongia and G. J. Santoni

I. Introduction

Research on hedging the interest rate exposure of financial portfolios generally has evaluated the performance of hedge ratios based on one of two criterion. Cicchetti, Dale and Vignola [4], Ederington [6], Franckle [8], Franckle and Senchack [9] and Koppenhaver [16] have developed hedge ratios for particular assets (liabilities) that adjust for basis risk--the risk that the spread between the prices of the futures and cash instrument may change during the period of the hedge. A second group of studies has developed "optimum" micro hedging strategies for individual assets or liabilities of different duration. These studies assumed (at least implicitly) that exposure to interest rate risk is minimized with a hedging strategy that minimizes the variance of the net cash flow that would otherwise result from changes in the interest rate. The work of Hill, Liro and Schneeweis [12], Jacobs [13], Koch, Steinhauser and Whigham [15], Parker and Daigler [17], Pitts and Kopprasch [18] and Toevs [23] discuss hedging strategies designed to accomplish this objective.

In contrast to these practical guides to hedging, a number of studies acknowledge that the ultimate objective of a hedging strategy is to minimize the variance in the market value of the firm's equity.[1/] The literature offers few insights, however, concerning the principles of implementing so-called macro hedges. This shortcoming not only reveals a sizable gap between hedging in theory and practice but, from a regulatory point of view, may impinge upon the use of futures markets by financial institutions. Drabenstott and McDonely [5, p. 26], for example, have pointed out that an "overriding" guideline employed by the regulatory agencies of financial institutions is that "financial futures should reduce the net interest rate exposure for <u>the balance sheet as a whole</u> (emphasis added)."

The purpose of this article is to derive conditions for the set of theoretically exact hedges that eliminate the interest rate risk of a balance sheet (portfolio) composed of financial assets and liabilities of unmatched durations. In the process, we argue that a constant net cash flow (net interest margin) is neither a necessary nor sufficient condition for the market value of the portfolio to be invariant to interest rate changes. In fact, there are many different streams of cash that will do the trick but the original stream of cash is not one of these.

Of course, real as well as financial assets and liabilities are included in the balance sheets of most firms. A hedging strategy designed to protect the market value of the firm necessarily involves consideration of the interest elasticities of both types of assets and liabilities. This requires an analysis that distinguishes between changes in market interest rates that are the result of changes in the *ex ante* real rate vs. those that result from changes in expected inflation.

Since the balance sheets of financial firms are mainly composed of financial assets and liabilities, the following analysis is restricted to consideration of the financial portfolio. Thus, the hedge ratios we derive protect the net present value of the financial portfolio but ignore the effect of changes in the *ex ante* real interest rate on the present values of the firm's real assets and liabilities.

II. Interest Rate Risk and the Financial Portfolio

The market prices of existing financial assets (liabilities) that yield given streams of future dollar receipts (payments) are sensitive to interest rate changes. By interest rate risk we mean changes in the market prices of financial instruments that are related to unexpected changes in the interest rate.[2/] The relationship between these changes is given by the interest rate elasticities of the financial instruments.[3/] Thus, for given variation in the interest rate, interest elasticity measures the interest rate risk inherent in financial instruments. Similarly, the interest rate risk of a portfolio (firm) is measured by the elasticity of its market price with respect to the interest rate.[4/]

Following Samuelson [21, p. 19], let R_t and P_t, respectively, be the dollar value of the firm's receipts and payments in period t that are generated by the financial portfolio. If the life of the firm's financial investment plan is n years, its market value in period t, E_t, is:

$$E_t = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t} - \sum^{n} \frac{P_t}{(1+i)^t} = A_t - L_t \ . \qquad (1)$$

The interest rate, i, is the opportunity cost of capital.[5/] The terms A_t and L_t are the present values of the stream of receipts and payments at time t.

Differentiating (1) with respect to i and converting to elasticities yields:[6/]

$$\frac{dE_t}{di}\frac{i}{E_t} = [-\frac{1}{(1+i)} \Sigma \frac{t \cdot R_t}{(1+i)^t}\frac{i}{A_t}]\frac{A_t}{E_t} - [-\frac{1}{(1+i)} \Sigma \frac{t \cdot P_t}{(1+i)^t}\frac{i}{L_t}]\frac{L_t}{E_t} \ . (2)$$

The interest rate elasticities of the present values of the receipt and payment streams are given by the bracketed terms in (2). Below, these elasticities are represented by η_R and η_P, respectively, as in equation (3).

$$\eta_E = \eta_R A/E - \eta_P L/E. \tag{3}$$

The interest elasticity of the portfolio is the difference between the weighted interest elasticities of the receipt and payment streams where the weights are the ratios of the present values of each stream to the portfolio's market price.

Since η_R and η_P both are negative, η_E may be positive, negative or zero. Any nonzero value for elasticity, however, means that the portfolio is subject to interest rate risk. For given variation in the interest rate, the magnitude of the interest rate risk is measured by the absolute value of η_E.

The portfolio of an unhedged firm is insulated against interest rate risk if η_E is zero. By manipulation of equation (3), the condition for risk elimination is:

$$\eta_R = \eta_P L/A \tag{4}$$

That is, the interest rate elasticity of the stream of receipts, η_R, must equal the interest rate elasticity of the stream of payments, η_P, weighted by the ratio of liabilities to assets (in present value terms).

III. Risk, Duration and the Financial Portfolio

Interest rate elasticity is simply duration multiplied by $-i/(1+i)$, so equation (4) can be rewritten as:

$$D_R = D_P L/A \tag{5}$$

where D_R and D_P are the durations of the streams of receipts and payments, respectively.[7/]

Frequently, the literature refers to the matching of asset and liability durations as a sufficient condition in insulating the value of the firm's portfolio. It is clear from equation (5) that this is only true in the limit as L/A approaches one. With the exception of this special case, a portfolio of spot assets and liabilities of matched durations will be exposed to interest rate risk. Moreover, the condition expressed in (5) generally does not hold for the typical financial institution. For these firms, the present value of financial assets exceeds the present value of financial liabilities. In addition, the duration of the receipt stream, D_R, generally exceeds the duration of the payment stream, D_P (particularly in the case of savings and loan associations). This suggests that the portfolios of these firms are particularly sensitive to interest rate changes.[8/]

IV. Futures Contracts

A financial futures contract is an agreement between a seller and buyer to trade some well-defined financial instrument (Treasury bills, GNMA passthrough certificate contracts, 90-day CDs, Treasury bond or Treasury notes) at some specified future date at a price now agreed to but paid in the future at time of delivery.

The futures contract represents both an asset and liability to the seller and buyer. From the seller's point of view, the futures asset is the present value of the expected future receipt of fixed amount. The futures liability, on the other hand, is the present value of the expected cost of covering the futures contract given the present structure of interest rates. At the time the contract is made, these two present values are equal so the net present wealth of the seller is not affected by the exchange.

From the buyer's point of view, the asset is the present value of the expected price of the financial instrument upon delivery while the liability is the present value of the future payment of a fixed amount that must be made on the delivery date. Again, these two present values are equal at the time the contract is made so the buyer's wealth is unaffected by the exchange.

If the interest rate should rise during the interval of the contract, the seller gains because the cost of covering the contract falls while the expected future receipt remains unchanged. As a result, the present value of the futures asset rises relative to the present value of the futures liability, causing an increase in the seller's net present wealth, other things unchanged. The reverse occurs should the interest rate fall. The changes in present values from the buyer's point of view are the mirror image of those for the seller.

The greater the duration of the futures instrument being traded the greater is the change in the expected cost of covering the contract for a given change in the interest rate and the larger is the wealth transfer between the seller and buyer. For example, trading in 91-day Treasury bills results in a relatively small wealth transfer for a given change in the interest rate in comparison to trading in Treasury bonds.

Of course, it is precisely because these wealth transfers occur that futures market transactions are effective in offsetting the interest rate risk inherent in a firm's cash (spot) portfolio.

V. Hedging the Portfolio with Futures

In deriving the set of exact hedges for the financial firm, let E represent the present (market) price of the financial portfolio. The present value of the expected stream of receipts generated by the firm's collection of assets purchased on the spot market is A^s, while the present value of the stream of payments obligated by the firm's spot liabilities is L^s. Let A^f be the present value of the expected future receipt that results from a futures contract while L^f is the present value of the expected future cost of covering the contract.[10/] At the time the futures contract is made, $A^f = L^f = F$. As before, the interest rate, i, is the opportunity cost of capital.

The value of a portfolio containing both spot and futures assets and liabilities is given in equation (6).

$$E = A^s - L^s + A^f - L^f \tag{6}$$

The interest elasticity of the portfolio is given by differentiating (6) with respect to i and expressing the result in terms of elasticities as in equation (7):

$$\frac{dE}{di}\frac{i}{E} = \eta_E = \eta_R^s \frac{A^s}{E} - \eta_P^s \frac{L^s}{E} + \eta_R^f \frac{A^f}{E} - \eta_P^f \frac{L^f}{E} \tag{7}$$

Recall that the hedging objective is to drive η_E to zero. To determine the futures market position (long or short) the firm should take to reduce the exposure of its portfolio to interest rate risk, set (7) equal to zero and solve to obtain:

$$\eta_R^s A^s - \eta_P^s L^s = -(\eta_R^f A^f - \eta_P^f L^f) \tag{8}$$

As before, equation (8) can be rewritten in terms of the durations of the streams of receipts and payments.

$$D_R^s A^s - D_P^s L^s = -(D_R^f A^f - D_P^f L^f) \tag{9}$$

The term on the left side of equation (9) is the difference between the duration's of the firm's stream of spot receipts and payments weighted by their respective present values. This difference may be positive, negative or zero. The right side of (9) is the difference between the duration of the firm's stream of futures receipts and payments weighted by their respective present values.

If the firm is short futures (has sold contracts),its receipt on the delivery date does not vary with the interest rate but its cost of covering the contract does. As a result, A^f has a shorter duration (is less interest elastic) than L^f, so $D_R^f < D_P^f$. Since $A^f = L^f$ when the contract is initiated, the right side of (9) is positive for a short futures position ($D_R^f A^f - D_P^f L^f < 0$ and this difference is premultiplied by a negative sign). The reverse is true if the firm is long futures and it is, of course, zero if the firm has no futures position.

Equation (9) is consistent with intuition. It indicates that a hedge requires a futures market position that is opposite the firm's position in the spot market. Thus, if the weighted duration of the firm's receipt stream exceeds the weighted duration of its payment stream, a hedge requires the firm to short futures. A long position in futures would imply that this firm is speculating on a fall in interest rates.

Equation (9) can be solved for the dollar value of the hedge that will drive η_E to zero. Note that $A^f = L^f =$ F when the futures contract is formed. Substituting F into (9) and solving yields:

$$F = -A^s(D_R^s - D_P^s L^s/A^s)/(D_R^f - D_P^f) \qquad (10)$$

It is clear from (10) that the size of an exact hedge depends on a variety of factors including the durations of both spot assets and liabilities, the present values of spot assets and liabilities and the duration of the futures contract used in constructing the hedge.

<u>Some Implications</u>

In the limit, as L^s approaches A^s, the dollar value of the futures position that insulates the firm's portfolio approaches the negative of the present value of its spot assets adjusted by the ratio of the "net" duration of its spot receipts $(D_R^s - D_P^s)$ to the net duration of its future receipts $(D_R^f - D_P^f)$. For example, suppose the duration of the stream of spot receipts, D_R^s, is 360 days while the duration of the stream of spot payments, D_P^s, is 180 days. If the firm chooses to hedge the portfolio with a stack of 3-month Treasury bill futures, the dollar value of the position that insulates the firm's portfolio is twice spot assets, i.e., a hedge ratio of two. Had the firm chosen to hedge with a stack of 6-month Treasury bills, the optimum hedge ratio falls to one, etc.[11/]

In the more realistic case, when $L^s < A^s$, the above optimum hedge ratios would be larger. For example, if spot assets are twice spot liabilities, the optimum hedge ratio is three if the firm hedges with a stack of 3-month Treasury bill futures and 1.5 if it hedges with 6-month Treasury bill futures. Clearly, the optimum hedge depends upon the durations of the spot and futures assets and liabilities of the firm as well as the firm's leverage, the ratio of spot liabilities to spot assets.

VI. Hedging and Cash Flow

As mentioned above, previous literature has concentrated upon hedging the firm's net cash flow (net interest margin). Protection of the firm's cash flow, of course, may be a management (owner) objective. However, any hedge that fulfills this objective necessarily does so at the expense of subjecting the market value of the portfolio to interest rate risk. A hedge that results in the maintenance of net cash flow is equivalent to converting the portfolio into a bond that yields a constant stream of cash over its life. Clearly, the market price of this bond will vary with changes in the interest rate, and the greater the bond's duration the greater the effect of an interest rate change on its price.

In contrast, hedges that protect the present value of the portfolio necessarily imply net cash flows that vary with changes in the interest rate. In principle, there are an infinite number of different streams of cash (in both total volume and duration) that are consistent with maintaining the present value of the portfolio in the face of a given change in the interest rate but the original stream of cash is not one of these.

There are, of course, various reasons why a firm's managers may be interested in cash flow. However, if these concerns permit some degree of substitution with respect to either the total volume of the cash flow, its duration or both, they will not necessarily conflict with the objective of hedging the present (market) value of the portfolio.

VII. Conclusions

From a theoretical point of view, the ultimate objective of any hedging strategy is the protection of wealth. Furthermore, regulatory agencies require that the hedging strategies of financial institutions be guided by this objective. While current hedging literature provides valuable insights concerning basis risk as well as problems associated with hedging individual assets and liabilities, the problem of hedging the market value of a firm's portfolio has received relatively little attention.

We return to the issue of macro hedging and derive a theoretically exact hedge ratio for a portfolio composed of spot assets and liabilities of different duration. The expression holds in the absence of basis risk and for a flat yield curve that shifts in parallel fashion.

In addition, we argue that hedging a portfolio's net cash flow is neither a necessary nor sufficient condition for the market value of the portfolio to be invariant to interest rate changes. In principle, there are many different streams of cash that will equate the present value of the portfolio at the new interest rate to its original value at the old rate. However, the original stream of cash is not one of these.

FOOTNOTES

1/ See Asay, et. al. [2, p. 12] for a discussion of the importance of macro hedging strategies.

2/ See Alchian and Allen [1, pp. 445-6]. In general, the market price of any asset or liability will change with changes in the interest rate. The discussion emphasizes financial instruments because it is focused on financial institutions.

3/ See Hicks [11, p. 186].

4/ For estimates of the interest elasticity of the portfolios of financial firms, see Flannery and James [7] and Santoni [21].

5/ The spread between the firm's receipts and payments is a return to the firm's specialized capital employed in intermediating financial transactions. Further, since we are primarily concerned with financial institutions whose balance sheets are composed almost exclusively of nominal assets and liabilities, i is a nominal interest rate and R_t and P_t are fixed in nominal terms.

6/ We assume that all shifts in the yield curve are parallel shifts and that the yield curve is flat.

7/ See Santoni [22, p. 15] and Toevs [23, p. 30].

8/ The evidence appears to support this implication. See Flannery and James [7, pp. 1151-52] and Santoni [22, pp. 18-20].

9/ There is, of course, the risk that the buyer may default. However, this risk is very small since the exchange backs all transactions. See Kane [14].

10/ If the firm has sold (bought) a strip of futures contracts, A^f and L^f are the present values of streams of receipts and payments.

11/ If futures market transactions were costless, the firm would be indifferent between any hedge (stacks or strips with nearby delivery dates or distant delivery dates) that drives η_E to zero. Given costly transactions, however, the firm will choose from amongst this set of hedges the one that minimizes transaction costs.

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