# Lag-Length Selection and Tests of Granger Causality Between Money and Income

LAG-LENGTH SELECTION AND GRANGER CAUSALITY

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I.   Introduction

It is generally acknowledged that the results of
Granger-Sims causality tests may be sensitive to the
specification of the lag length, and recent studies by Hsiao
(1981) and Choudhri (1983) and Fackler and McMillin (1983) have
given lag length selection a more prominent role in their
investigations of Granger causality.  Nevertheless, the
importance of subjective criteria for model specification in
lag length selection has been neither recognized nor
appreciated.  It is the purpose of this paper to show that the
results of causality tests are extremely sensitive to the lag
specification and, ipso facto, to the implicit weight that
individuals assign to various factors in selecting the order of
the finite lags of the typical Granger causality test.

It will also be shown that the often used procedure of
arbitrarily selecting lag specifications, such as 4-4 and 8-8,
can produce misleading results.  Furthermore, the arbitrary
choice of lags ignores the important role that model
specification plays in Granger causality tests and gives the
erroneous impression that lag specification can be controlled
for adequately by considering a couple of arbitrarily chosen
lag lengths.

## II. Motivation

In order to illustrate how sensitive these tests can be to lag length specification, the significance levels (i.e., the probabilities of rejecting a true hypothesis) corresponding to calculated F-statistics of Granger tests for various lag specifications are presented in Tables 1 and 2. The outcomes that yield a rejection of the null hypothesis at the 5 percent level are shaded. (The reader may wish to note how this region would change had a 10 percent significance level been chosen). Tables 1 and 2 present results of regressions of M2 and M1, respectively, on nominal GNP (Y). The data are in compounded annual growth rates. Twelve lags were considered on both variables, and the equations were estimated over the period II/1962 and III/1982, regardless of the lag specification. The significance levels are presented rather than their corresponding F-statistics because the latter are not degree-of-freedom invariant.

The results suggest that one can substantiate either the presence or absence of Granger-causality by the judicious choice of lag length. It is interesting to note that had the commonly chosen lags of 4-4 and 8-8 been used on the regression of M2 on Y, one would have obtained conflicting results. A lag structure of 4-4 suggests that income does not Granger-cause M2, while 8-8 yields the opposite conclusion. Furthermore, had slightly longer lags been used, say 5-5 and 9-9, the above conclusion would have been reversed.

Thus, lag length selection is extremely important, and Hsiao (1981) is correct in suggesting that the lag should be chosen on the basis of some statistical criterion.[1] However, a number of such criteria exist and the choice of a particular one depends on one's subjective evaluation of the importance of various factors in model selection. These issues will now be dealt with in a more systematic manner, starting with a bivariate structural model.

## III. The Model

A standard bivariate dynamic structural model on which the Granger test is based can be expressed as

$$(1) \quad \begin{aligned} y_t + \alpha x_t &= L(\beta)^H y_{t-1} + L(\delta)^K x_{t-1} + \epsilon_{1t} \\ x_t + \delta y_t &= L(\lambda)^I x_{t-1} + L(\mu)^N y_{t-1} + \epsilon_{2t}. \end{aligned}$$

Here, the maintained structure has x and y jointly determined endogenous variables, $\epsilon_1$ and $\epsilon_2$ are assumed to be IID $(0, \sigma_i^2)$, $i = 1, 2$, such that--for the sake of simplicity --$E(\epsilon_{1t} \epsilon_{2t'}) = 0$ for all t and t', and $L(\cdot)^J$ is the polynomial lag operator of order J, e.g., $L(\beta)^J y_{t-1} = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \dots + \beta_J y_{t-J}$. The reduced form of this model is

$$(2) \quad \begin{aligned} y_t &= L(\pi_{11})^G y_{t-1} + L(\pi_{12})^P x_{t-1} + u_{1t} \\ x_t &= L(\pi_{21})^G y_{t-1} + L(\pi_{22})^P x_{t-1} + u_{2t}, \end{aligned}$$

where G and P are the larger of H or N and K or I, respectively. The $\pi$'s are nonlinear functions of the structural parameters in (1). The equations in (2) are the standard equations for the Granger variant of the test for causality.

In order to test for Granger causality between x and y, one tests the hypotheses that either $L(\pi_{12})^P = 0$ or $L(\pi_{21})^G = 0$, or both. If neither can be rejected, then x and y are independent series. If both are rejected, then there is "feedback" between x and y. If the former hypothesis is rejected but the latter is not, there is unidirectional causality running from x to y, while if the latter is rejected and the former is not, the reverse is true.

The test of the null hypothesis is critically dependent on the values of the unknown values of the parameters P and G. It is always possible to fail to reject either of the above hypotheses by selecting large values for these parameters, and, it is reasonable to assume, the same may be true by choosing small values for these parameters as well. (Note that this was true for both of the above examples). Thus, one could argue that one should accept the hypothesis of unidirectional causality (or independence) only if the hypotheses $L(\pi_{12})^P = 0$ or $L(\pi_{21})^G = 0$ (or both) are not rejected for all values of P and G over some reasonable range of values of these parameters.[2] While such a search can be conducted in a reasonably efficient manner for the bivariate case, it may be considered too burdensome for some.[3] Furthermore, there may exist some lag specifications for which the null hypothesis is not rejected, at say the 5 percent level, by chance alone.

An alternative approach is to identify the

"appropriate" lag structure for the equations in (2). One possible way to do this is to use a statistical criterion to establish the parameters G and P. This is the approach taken by Hsiao. There is an infinite set of statistical criteria, however, on which the selection of these parameters could be based.[4/] As a result, different causality test results from the same data can be based on different statistical criteria for choosing values of P and G. Individuals could arrive at different, but equally legitimate, conclusions concerning the Granger-causal relation between time series due solely to differences in their model selection criteria.

A question arises naturally: why should different criteria be used for determining the lag length and for the tests of Granger causality? Certainly the same classical hypothesis testing framework which is normally employed in tests of Granger causality could be employed in lag length selection. Likewise, if one preferred an alternative criterion, say a mean square error (MSE) norm such as those suggested by Toro-Vizcarrondo and Wallace (1968) and Wallace (1972), for selecting the order of the model, the same criterion could be applied to tests of Granger causality. It may be, however, that Granger-causality is only one of several questions to be addressed. Furthermore, many of the commonly used criteria for identifying the order of a model do not have a hypothesis testing counterpart. Whatever the reason, the case where the objective functions differ is considered below.

## IV. Model Specification Criteria

The researcher interested in selecting the order of the previous model will choose a lower order model which is a subset of the highest order model believed to be reasonable. If the correct values of these parameters are selected, the estimates will be best linear unbiased. If either P or G (or both) is too large, the estimates will be unbiased but inefficient. If either P or G (or both) is too small, the estimates will be biased but will have a smaller variance.[5/] If one is too large and the other too small, the estimates will be biased and may be inefficient. The typically used selection criteria trade off the bias associated with a parsimonious parameterization against the inefficiency associated with overparameterization.

Frequently cited criteria used here are: Mallows' (1973) Cp-statistic ($C_p$), Akaike's (1969) Final Prediction Error (FPE), Schwartz' (1978) Bayesian Information Criterion (SBIC), the Bayesian Estimation Criterion (BEC) suggested by Geweke and Meese (1981) and a technique suggested by Pagano and Hartley (P-H).[6/] In applications, here and elsewhere [Batten and Thornton (1983a)], the P-H technique is similar to using a standard F-test. For a more thorough discussion of these criteria, see Batten and Thornton (1983b).[7/]

## V. Lag-Length Selection Results

The above criteria are applied to bivariate Granger-type equations involving nominal income (Y), M1, M2, M2 net of M1 (NM2), the adjusted monetary base (MB) and the Treasury bill rate (TBR). All dollar denominated variables are expressed in compounded annual rates of growth, and the TBR is first differenced. The equations are estimated for the period II/1962-III/1982.[8/] The autocorrelation functions suggest that all of the univariate time series, except MB, are covariance stationary; nevertheless, as commonly suggested, a time trend was included in all regressions.

The lag-length selection results are reported in Table 3. While there were considerable differences across criteria, the Bayesian criteria, SBIC and BEC, tended to pick similar lags, as did the $C_p$ and FPE criteria. In no instance, however, did any of the criteria select identical lag structures for both reduced form equations, as suggested by the simple bivariate structural model.[9/]

The P-H technique did not succeed in selecting a unique lag structure when M1 and the TBR were considered. Unlike the other criteria considered, the P-H technique does not necessarily lead to a definitive lag specification when two or more lags are being chosen simultaneously. Indeed, in every instance when $\Delta$TBR is included, the P-H chosen lags were difficult to identify.

## VI. Granger Causality Test Results

In this section the results of a standard F-test for Granger causality are presented for the lag length specifications of the previous sections.[10/] The causality test results for the SBIC- and $C_p$-selected lag structures are not presented due to their close correspondence to the lag structures chosen by BEC and FPE, respectively. The results for the lag specifications selected by the remaining three criteria, along with commonly used 4-4 and 8-8 specifications, are presented in Table 4. The double dash indicates that this criterion chose no lag for the independent variable.

There are several interesting aspects of Table 4. First, in many instances, the test results based on alternative lag specifications conflict. There were three instances when 4-4 and 8-8 produced contradictory results. Furthermore, there were three instances in which the null hypothesis was not rejected for the arbitrarily chosen lag structures but was rejected for at least one of the other lag structures.

Second, there were two instances when the FPE criterion chose a positive lag for the independent variable that was not significant in the subsequent F-test. This illustrates the problem of using Hsiao's suggestion that Granger causality exists if $FPE(G, P) < FPE(G, 0)$, i.e., this procedure lacks a mechanism for determining significant differences in FPE levels. Also, there was one instance where the null hypothesis could not be rejected at the 5 percent

level for the FPE chosen lag, but could be rejected for the P-H chosen lag. Although not shown here, there was a fairly large portion of the lag space considered over which the hypothesis could be rejected. It is important that these limitations of the Hsiao procedure be recognized since it is becoming more widely used.[11]/

Third, lack of Granger causality is indicated frequently by the Bayesian criteria because they choose extremely short lag specifications.

Finally, there are instances when all the specifications of Table 4 show or fail to show causality. While not all of these instances were investigated further, five of these cases are presented in Tables 5-9. Of these five, there were two instances where the qualitative result is consistent over the entire lag space. In both instances (Y on MB, and NM2 on TBR) strong support for the lack of Granger causality was obtained. Thus, there is fairly strong evidence of the exogeneity of the monetary base with respect to income and of the Treasury bill rate with respect to net M2.[12]/ This conclusion is independent of the lag specification.[13]/

## VII. A Comparison of the Criteria

The above results suggest that the FPE criterion appears to do a good job of finding a significant lag structure which will reject the null hypothesis if such a lag structure exists. The question naturally arises as to how well does this

specification compare with those selected with the other criteria and with the arbitrarily chosen lag structures. It appears that the FPE criterion fares well in this regard. To illustrate this, the FPE-selected models are compared with the other models of Table 4 using a F-test. The results are reported in Table 10. The models which are nonnested are identified by N.N. If the FPE structure is of a lower order than the alternative, it is denoted by (L); if it is of a higher order, it is denoted by (H). If the FPE criterion always picked the model which was preferred by this classical hypothesis testing norm at the 5 percent level, all the F's designated H would be significant, while those designated L would not. While this is generally the case, it is not always. Thus, the FPE criterion performs well by this norm, but its performance is not uniform. (These results are consistent with previous research, see Batten and Thornton (1983b)). Thus, while the FPE criteria appears to be useful, there is no good substitute for determining the model specification criteria ex ante or for an extensive search of the lag space if one wishes to insure that the causality test results are not critically dependent on the judicious (or fortuitous) choice of the lag structure.

## VIII. Summary and Conclusions

The purpose of this paper was to show the extent to which tests of Granger causality are dependent upon the

selected order of the model. We have shown that for many of the bivariate comparisons that are usually made, Granger causality may or may not be rejected simply by choosing alternative orders of the model. Because of this, we argue individuals can arrive at different, but equally legitimate, conclusions concerning the Granger-causal relation between time series based solely on differences in their model selection criteria. Consequently, the order of these models cannot be specified arbitrarily.

A comparison of several commonly used lag-length selection criteria suggests that Akaike's FPE criteria performs well relative to the others considered for the model, based on a standard classical hypothesis testing norm. Furthermore, it appears that the FPE criterion does a reasonably good job of finding an order of the model which will give evidence of Granger causality, if such a lag structure exists. Nevertheless, there is no substitute for determining the criteria for model specification ex ante or for an extensive search of the lag space if one wishes to insure that the causality test results are not critically dependent on the judicious or fortuitous choice of the lag structure.

# FOOTNOTES

[1] For an interesting statistical criterion for choosing the lag length, see Noble and Fields (1983).

[2] Of course, this criterion may be too stringent since the hypothesis may not be rejected for some P or G simply by chance.

[3] The F-statistics on which Tables 1 and 2 are based can be calculated in an efficient manner using orthogonal regressions. See Batten and Thornton (1983a).

[4] For a discussion of some of these and references to others, see Judge, et. al., (1980), chapter 11.

[5] Assuming, of course, that the included and excluded variables are not orthogonal.

[6] All of these criteria are based on the residual sum of squares. Leamer (1978) suggests that such procedures give way to a Bayesian approach. Furthermore, nearly all of these techniques give rise to the problem of preliminary test estimation. Hence, a Stein rule might be desirable.

[7] The application of these criteria is complicated by the fact that there are two lags in the equation. This is particularly true for the P-H technique.

[8] It appears that there may have been a change in the usual income-money relationship in IV/1982 and I/1983; see Batten and Thornton (1983c). The sample ended with III/1982 so the results would not be affected by this development. The beginning of the sample was determined by the availability of the M2 series.

[9/] This could result from other types of misspecification of these bivariate reduced form equations, such as the omission of other relevant variables. See Cooley and LeRoy (1982) and Conway, et. al. (1983) for other criticisms of tests of Granger causality.

[10/] There are a number of other ways this test can be performed--most notably the Sims (1972) version. Results by Geweke, et. al. (1983) and Guilkey and Salemi (1982) indicate that the Granger variant is preferred.

[11/] Furthermore, results (not reported here) suggest that Hsiao is correct in stating that his sequential search procedure need not lead to a global minimum for the FPE.

[12/] Of course, endogeneity is not completely ruled out because of the possibility of "spurious exogeneity." See Jacobs, et. al (1979) and Cooley and LeRoy (1982) for details.

[13/] In order to investigate the sensitivity of these results to the stationarity of the time series and to prefiltering of the data, all of the tests reported here were repeated using the first difference of the growth rates of dollar-denominated variables and the second difference of the TBR. The qualitative results were essentially the same as those reported here. The only exceptions involved the adjusted monetary base. In particular, the conclusion that Y does not Granger-cause MB was not independent of the lag specification.

Table 1: Significance Levels for Granger Causality Tests of M2 on Y

Lags of M2     Lags of Y

| Lags of M2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .803 | .777 | .604 | .565 | .065 | | | | | | | |
| 1 | | .103 | .130 | .201 | | | | | | | | |
| 2 | | .130 | .158 | .239 | | | | | | | | |
| 3 | | .120 | .117 | .187 | | | | | | | | |
| 4 | | .116 | .131 | .213 | | | | | | | | |
| 5 | | .119 | .130 | .204 | | | | | | | | |
| 6 | | .130 | .120 | .204 | | | | | | | | |
| 7 | .085 | .216 | .186 | .273 | | | .055 | | | | | |
| 8 | .108 | .267 | .223 | .317 | | | .056 | | | | | |
| 9 | .148 | .350 | .336 | .452 | .056 | .063 | .078 | | .071 | .065 | .054 | |
| 10 | .149 | .352 | .343 | .458 | .055 | .064 | .080 | | .073 | | | |
| 11 | .136 | .332 | .344 | .455 | .061 | .073 | .093 | .059 | .085 | .060 | | |
| 12 | .175 | .399 | .408 | .525 | .074 | .089 | .109 | .064 | .090 | .070 | .051 | |

Table 2:  Significance Levels for Granger Causality Tests of M1 on Y

| Lags of M1 | Lags of Y | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | .967 | .977 | .526 | .455 | .546 | .177 | .203 | .213 | .198 | .149 | | .066 |
| 1 | .882 | .959 | .515 | .404 | .465 | .156 | .150 | .146 | .126 | .111 | | |
| 2 | .936 | .954 | .499 | .395 | .444 | .164 | .158 | .149 | .120 | .112 | | |
| 3 | .947 | .946 | .473 | .370 | .418 | .156 | .159 | .152 | .112 | .116 | | |
| 4 | .813 | .700 | .616 | .233 | .257 | .105 | .106 | .124 | .099 | .128 | | .073 |
| 5 | .736 | .532 | .283 | .223 | .093 | | | .053 | .055 | .070 | | .066 |
| 6 | .929 | .588 | .377 | .342 | .114 | | | | | | | |
| 7 | .828 | .810 | .454 | .490 | .280 | .059 | | | | | | |
| 8 | .812 | .850 | .343 | .414 | .188 | .063 | | | | | | .054 |
| 9 | .812 | .852 | .348 | .410 | .194 | .067 | | | | | | |
| 10 | .767 | .825 | .454 | .500 | .268 | .103 | | | | .059 | | .072 |
| 11 | .802 | .844 | .468 | .506 | .280 | .107 | | .054 | | .064 | | .074 |
| 12 | .803 | .921 | .508 | .546 | .238 | .062 | | | | | | |

Table 3: Lag Specifications for the Various Criteria

| Dependent Variable/ Independent Variable | Cp | SBIC | BEC | FPE | P-H |
|---|---|---|---|---|---|
| Y/M1 | 0/2 | 0/2 | 0/1 | 0/2 | 7/10 |
| M1/Y | 8/7 | 0/0 | 0/0 | 8/7 | 6/6 |
| Y/M2 | 0/1 | 0/1 | 0/1 | 0/1 | 11/6 |
| M2/Y | 1/12 | 1/0 | 1/0 | 1/12 | 1/12 |
| Y/NM2 | 0/1 | 0/1 | 0/1 | 0/1 | 11/1 |
| NM2/Y | 1/12 | 1/1 | 1/0 | 1/12 | 1/12 |
| Y/MB | 0/3 | 0/1 | 0/1 | 0/3 | 0/3 |
| MB/Y | 1/0 | 1/0 | 1/0 | 1/0 | 1/11 |
| Y/TBR | 0/6 | 0/0 | 0/0 | 0/6 | 11/9 |
| TBR/Y | 5/11 | 5/2 | 5/2 | 5/11 | 5/10 |
| M1/TBR | 2/1 | 1/1 | 0/1 | 2/1 | I.D. |
| TBR/M1 | 3/7 | 0/1 | 0/1 | 3/12 | I.D. |
| M2/TBR | 1/5 | 1/1 | 1/1 | 1/5 | 5/8 |
| TBR/M2 | 6/0 | 6/0 | 6/0 | 11/1 | 5/8 |
| NM2/TBR | 1/5 | 1/1 | 1/1 | 2/5 | 1/5 |
| TBR/NM2 | 6/0 | 6/0 | 6/0 | 11/0 | 6/8 |
| MB/TBR | 1/12 | 1/1 | 1/1 | 2/12 | 1/12 |
| TBR/MB | 6/1 | 6/0 | 6/0 | 6/1 | 6/1 |

I.D. - indeterminant.

Table 4: Granger Causality F-statistics for Various Lag
Specifications

| Dependent Variable/ | | | Model chosen by: | | |
|---|---|---|---|---|---|
| Independent Variable | 4-4 | 8-8 | BEC | FPE | P-H |
| Y/M1 | 4.10* | 2.91* | 14.46* | 10.52* | 3.06* |
| M1/Y | 1.43 | 2.32* | -- | 2.63* | 2.88* |
| Y/M2 | 2.54* | 2.03 | 11.68* | 11.68* | 4.72* |
| M2/Y | 1.50 | 2.31* | -- | 3.04* | 3.04* |
| Y/NM2 | 1.46 | 1.24 | 7.04* | 7.04* | 13.34* |
| NM2/Y | 1.84 | 1.85 | -- | 3.06* | 3.06* |
| Y/MB | 4.87* | 3.06* | 16.95* | 7.79* | 7.79* |
| MB/Y | 0.95 | 0.81 | -- | -- | 1.40 |
| Y/TBR | 3.57* | 2.61* | -- | 3.65* | 3.30* |
| TBR/Y | 3.93* | 2.74* | 10.87* | 4.64* | 4.57* |
| M1/TBR | 7.67* | 3.87* | 20.86* | 33.39* | N.A. |
| TBR/M1 | 6.02* | 3.47* | 20.30* | 5.72* | N.A. |
| M2/TBR | 12.25* | 6.87* | 32.03* | 11.50* | 7.50* |
| TBR/M2 | 1.32 | 1.79 | -- | 2.25 | 2.44* |
| NM2/TBR | 5.52* | 3.72* | 16.74* | 6.94* | 7.08* |
| TBR/NM2 | 0.32 | 1.26 | -- | -- | 1.14 |
| MB/TBR | 8.32* | 5.29* | 28.63* | 5.98* | 5.14* |
| TBR/MB | 1.53 | 1.01 | -- | 3.96 | 3.96 |

N.A. - not available.
*     - significant at the 5 percent level.
--    - no lag chosen for the independent variable.

# Table 5: Significance Levels for Granger Causality Tests of ΔTBR on Y

| Lags of TBR | Lags of Y | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | .112 | .067 | .133 | .222 | .082 | .123 | .190 | .255 | .286 | | | |
| 1 | .151 | .088 | .168 | .270 | .102 | .148 | .223 | .294 | .327 | | | |
| 2 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | |
| 10 | | | | | | | .057 | .054 | .057 | | | |
| 11 | | | | | | | .083 | .065 | .079 | | | |
| 12 | | | | | | | .079 | .039 | .055 | | | |

Table 6:  Significance Levels for Granger Causality Tests of MB on Y

| Lags of MB | Lags of Y | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | .944 | .930 | .581 | .721 | .812 | .788 | .867 | .758 | .774 | .539 | .483 | .500 |
| 1 | .569 | .718 | .437 | .441 | .583 | .511 | .599 | .513 | .611 | .428 | .193 | .260 |
| 2 | .468 | .678 | .431 | .447 | .586 | .498 | .592 | .488 | .589 | .439 | .205 | .273 |
| 3 | .379 | .678 | .314 | .344 | .483 | .449 | .558 | .484 | .576 | .461 | .164 | .224 |
| 4 | .461 | .756 | .473 | .438 | .583 | .572 | .678 | .547 | .633 | .558 | .237 | .311 |
| 5 | .523 | .815 | .521 | .567 | .709 | .668 | .776 | .638 | .731 | .642 | .339 | .426 |
| 6 | .486 | .783 | .482 | .539 | .674 | .578 | .696 | .610 | .707 | .646 | .361 | .450 |
| 7 | .473 | .770 | .439 | .496 | .636 | .578 | .693 | .596 | .690 | .645 | .381 | .470 |
| 8 | .466 | .765 | .446 | .508 | .648 | .589 | .703 | .597 | .691 | .626 | .363 | .452 |
| 9 | .438 | .733 | .465 | .534 | .679 | .606 | .721 | .635 | .732 | .670 | .356 | .445 |
| 10 | .499 | .794 | .636 | .621 | .750 | .758 | .847 | .801 | .871 | .861 | .507 | .595 |
| 11 | .503 | .795 | .636 | .590 | .728 | .732 | .828 | .803 | .873 | .866 | .452 | .535 |
| 12 | .472 | .768 | .607 | .562 | .692 | .666 | .774 | .784 | .858 | .834 | .430 | .523 |

# Table 7: Significance Levels for Granger Causality Tests of Y on MB

| Lags of Y | Lags of MB | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | .000 | .000 | .000 | .000 | .001 | .002 | .003 | .006 | .012 | | | |
| 1 | .000 | .001 | .000 | .001 | .003 | .006 | .009 | .016 | .020 | | .065 | .090 |
| 2 | .000 | .001 | .000 | .001 | .003 | .006 | .010 | .017 | .026 | | .067 | .093 |
| 3 | .000 | .001 | .000 | .001 | .002 | .005 | .008 | .014 | .024 | | .057 | .080 |
| 4 | .000 | .001 | .001 | .002 | .003 | .006 | | | | | .068 | .094 |
| 5 | .000 | .001 | .000 | .001 | .002 | .004 | .006 | .011 | .016 | | | .052 |
| 6 | .000 | .001 | .000 | .001 | .002 | .002 | .006 | .017 | .020 | | | .057 |
| 7 | .000 | .001 | .000 | .001 | .002 | .005 | .008 | .014 | .022 | | | .062 |
| 8 | .000 | .001 | .000 | .001 | .001 | .002 | .004 | .006 | .010 | | | |
| 9 | .000 | .001 | .000 | .001 | .002 | .003 | .006 | .007 | .012 | | | |
| 10 | .000 | .002 | .001 | .001 | .002 | .004 | .006 | .010 | .012 | | | |
| 11 | .001 | .004 | .002 | .006 | .007 | .012 | .019 | .023 | .034 | .056 | .072 | .068 |
| 12 | .004 | | | | .014 | | | | | .056 | .078 | .098 |

Table 8:  Significance Levels for Granger Causality Tests of TBR on NM2

| Lags of TBR | Lags of NM2 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | .168 | .388 | .569 | .728 | .793 | .171 | .246 | .287 | .302 | .371 | .455 | .365 |
| 1 | .130 | .297 | .476 | .643 | .720 | .158 | .235 | .272 | .306 | .372 | .458 | .371 |
| 2 | .766 | .936 | .868 | .949 | .956 | .261 | .363 | .293 | .344 | .354 | .445 | .384 |
| 3 | .319 | .391 | .590 | .559 | .612 | .095 | .152 | .118 | .176 | .157 | .211 | .200 |
| 4 | .667 | .668 | .834 | .864 | .638 | .134 | .201 | .137 | .196 | .171 | .214 | .224 |
| 5 | .076 | .105 | .212 | .306 | .401 | .376 | .493 | .193 | .271 | .302 | .345 | .390 |
| 6 | .523 | .643 | .808 | .860 | .935 | .669 | .691 | .348 | .436 | .447 | .526 | .582 |
| 7 | .549 | .659 | .822 | .861 | .935 | .616 | .675 | .350 | .435 | .456 | .537 | .597 |
| 8 | .522 | .560 | .764 | .671 | .704 | .361 | .329 | .281 | .314 | .347 | .436 | .472 |
| 9 | .577 | .625 | .777 | .743 | .845 | .692 | .634 | .578 | .604 | .632 | .718 | .738 |
| 10 | .552 | .692 | .800 | .782 | .867 | .845 | .860 | .816 | .797 | .760 | .831 | .838 |
| 11 | .470 | .633 | .722 | .788 | .884 | .830 | .895 | .917 | .853 | .864 | .901 | .925 |
| 12 | .454 | .606 | .706 | .791 | .890 | .846 | .905 | .932 | .831 | .862 | .906 | .936 |

| Lags of TBR | Lags of MB | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | .078 | .111 | .149 | .166 | .174 |
| 1 | ▓ | .058 | ▓ | ▓ | ▓ | ▓ | .058 | .092 | .129 | .167 | .186 | .198 |
| 2 | .305 | .570 | ▓ | ▓ | ▓ | ▓ | .061 | .097 | .128 | .157 | .160 | .185 |
| 3 | .178 | .174 | .070 | .069 | ▓ | ▓ | .056 | .083 | .111 | .126 | .110 | .138 |
| 4 | .265 | .260 | .192 | .203 | ▓ | .057 | .094 | .138 | .186 | .212 | .160 | .205 |
| 5 | .088 | .076 | .053 | .105 | ▓ | .080 | .129 | .086 | .224 | .288 | .225 | .298 |
| 6 | .050 | .107 | .119 | .205 | .200 | .286 | .341 | .417 | .467 | .534 | .462 | .555 |
| 7 | .051 | .109 | .126 | .215 | .210 | .298 | .351 | .432 | .481 | .547 | .479 | .573 |
| 8 | ▓ | .091 | .139 | .233 | .241 | .340 | .372 | .435 | .432 | .495 | .379 | .465 |
| 9 | .068 | .149 | .217 | .353 | .360 | .486 | .544 | .559 | .594 | .596 | .430 | .522 |
| 10 | .115 | .236 | .352 | .510 | .458 | .591 | .632 | .565 | .526 | .474 | .418 | .511 |
| 11 | .193 | .360 | .480 | .625 | .613 | .738 | .743 | .686 | .552 | .554 | .543 | .632 |
| 12 | .215 | .398 | .520 | .663 | .655 | .774 | .772 | .719 | .586 | .582 | .577 | .665 |

Table 10: Diagnostic Tests of the FPE Lag Specifications

| Dependent Variable/ Independent Variable | 4-4 | 8-8 | BEC | P-H |
|---|---|---|---|---|
| Y/M1 | 0.09 (L) | 0.71 (L) | 5.72*(H) | 0.97 (L) |
| M1/Y | 3.46*(H) | 0.33 (L) | 2.33*(H) | 2.38 (L) |
| Y/M2 | 0.37 (L) | 0.75 (L) | -- | 1.80 (L) |
| M2/Y | N.N. | N.N. | 3.04*(H) | -- |
| Y/NM2 | 0.35 (L) | 0.60 (L) | -- | 1.66 (L) |
| NM2/Y | N.N. | N.N. | 3.06*(H) | -- |
| Y/MB | 0.22 (L) | 0.70 (L) | 2.82 (H) | -- |
| MB/Y | 0.93 (L) | 0.84 (L) | -- | 1.40 (L) |
| Y/TBR | N.N. | 0.65 (L) | 3.65*(H) | 1.42 (L) |
| TBR/Y | 5.42*(H) | N.N. | 2.74*(H) | 3.58 (H) |
| M1/TBR | 0.40 (L) | 1.16 (L) | 5.39*(H) | N.A. |
| TBR/M1 | N.N. | N.N. | 4.79*(H) | N.A. |
| M2/TBR | N.N. | 0.91 (L) | 4.81*(H) | 0.73 (L) |
| TBR/M2 | N.N. | N.N. | 1.97 (H) | N.N. |
| NM2/TBR | N.N. | 0.54 (L) | 3.68*(H) | 2.08 (H) |
| TBR/NM2 | N.N. | N.N. | 1.87 (H) | N.N. |
| MB/TBR | N.N. | N.N. | 2.91*(H) | 1.84 (H) |
| TBR/MB | N.N. | 0.59 (L) | 3.96 (H) | -- |

 *   - significant at the 5 percent level.
N.A. - not available.
N.N. - not nested.
 --  - lag same as that chosen by the FPE.

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