The Government Budget Constraint with Endogenous Money

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THE BUDGET CONSTRAINT, ENDOGENOUS MONEY AND THE RELATIVE IMPORTANCE OF FISCAL POLICY UNDER ALTERNATIVE FINANCING SCHEMES

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I. INTRODUCTION

Blinder and Solow's (1973) use of the budget constraint to argue for the long-run effectiveness of bond financed government spending has engendered considerable interest in the long-run effectiveness and stability of governmental fiscal policy. Much of this research has been directed at analyzing the relative effectiveness and stability of money or bond financed expenditures for alternative model specifications and for different exogenous policy variables, (e.g., Infante and Stein (1976), Christ (1978, 1979), Holmes and Smyth (1979), Hayakawa (1979), Smith (1980) and Cohen and de Leeuw (1980)).

Unfortunately, these models are deficient in two respects. First, they treat only two polar cases; pure bond financing and pure money financing. Neither of these extremes is particularly appropriate given the structure of the monetary system and the way that the Federal Reserve has historically conducted monetary policy. In the real world economy the money supply is controlled largely by the actions of the Federal Reserve.[1/] Monetary expansions are usually achieved through open market operations and, to a lesser extent, through bank borrowing and changes in reserve requirements. Money created in this way cannot be used to directly finance government spending without the simultaneous issuance of new debt by the Treasury. Thus, there can be no pure money financed deficit if the money supply is changed through monetary policy operations of the Federal Reserve.

Furthermore, it is generally accepted that until very recently the Federal Reserve has focused on interest rates as its primary operating target for monetary policy.[2/] As a result, monetary policy has tended to accommodate treasury debt financing operations. If this were the case, there would be no pure bond financed deficit. Thus, it is unlikely that one would encounter either pure bond or pure money financing in the "real world". The most likely case would be a mixed bond/money financed deficit where the stocks of bonds and money are changed simultaneously.

Second, most of these models treat money as being exogenous fiat currency, and hence the budget constraint makes no allowance for the structure of the monetary system. Three recent studies, (Christ (1978, 1979) and Cohen and de Leeuw (1980), treat money as endogenous; however, they employ a budget constraint that does not adequately differentiate between the tools of monetary policy. Their budget constraint contains the stock of high-powered money. This suggests that deficits can be financed by increasing the stock of high-powered money. Of course, this is only true if the stock of high-powered money is increased by currency issue. Increases in the stock of high-powered money through bank borrowing or open market operations cannot be used directly for this purpose. Thus, the principle tool of monetary policy, open market operations, is not adequately treated.[3/]

It is the purpose of this paper to correct these deficiencies by considering the case of a mixed bond/money financed expenditure in the context of a macroeconomic model with an endogenous money stock and a government budget constraint that differentiates between alternative instruments of monetary policy. It will be shown that the long-run multiplier for a mixed bond/money financed expenditure cannot be larger than the long-run multiplier for a pure bond financed expenditure. Indeed, it may be smaller depending on how the monetary expansion is achieved. The mixed bond/money financed deficit is more likely to be stable, however, depending on the instrument of monetary policy employed.[4/]

II. THE MODEL

$$Y = C(Y+B_p/P-T(PY-B_p)/P,W) + I(r) + G \quad C_Y>0,\ C_W>0,\ I'<0 \tag{1}$$

$$M^s = M(r,H,\alpha) \quad M_r>0,\ M_H>0,\ M_\alpha<0 \tag{2}$$

$$M^d = P \cdot l(r,Y,W) \quad l_r<0,\ l_Y>0,\ l_W>0 \tag{3}$$

$$M^s = M^d \tag{4}$$

$$W = K + B_f/Pr + B_p/Pr + CC/P \tag{5}$$

$$B_p/rP = (B-B_f)/rP \tag{6}$$

$$H = B_f/r + BB + CC \tag{7}$$

$$Y = Y(N,K) \quad Y_N>0,\ Y_K>0 \tag{8}$$

$$w = P \cdot f(N,K) = Y_N(N,K),\ f_N<0 \tag{9}$$

$$w = g(N,P,W) \quad g_N>0,\ g_P>0,\ g_W>0 \tag{10}$$

$$\dot{CC} + \dot{B}/r = PG + B_p - T(PY+B_p) \tag{11}$$

List of Notation

Y = real income

C = real consumption

I = real investment

G = real government expenditures

W = real wealth

P = the price level

B_p = nominal interest payments on debt held by the public

H = nominal stock of high-powered money, assumed to be Federal Reserve holdings of government debt.

CC = currency in circulation

BB = bank borrowing from the Federal Reserve

B = the nominal interest payments on the total government debt

r = the interest rate

B_f = nominal interest payments on debt held by Federal Reserve

T = nominal tax revenue

K = the stock of real capital

M^s = the nominal money supply

M^d = the nominal money demand

N = the quantity of labor

α = a shift parameter reflecting changes in reserve requirements

w = the money wage rate

$\dot{CC}$ = dCC/dt

$\dot{B}$ = dB/dt

t = time

The above is a basic macroeconomic model with endogenous prices. The first seven equations taken together represent the basic IS-LM model. Equations (1)-(5) are the standard model with a wealth effect in the consumption function and the demand for money. Equation (6) is an identity defining the real value of government debt held by the nonbank public as being equal to the total real stock of government debt outstanding less that held by the Federal Reserve. Equation (7) defines the nominal stock of high-powered money (monetary base) as being equal to government debt held by the Federal Reserve plus bank indebtedness to the

Federal Reserve plus currency in circulation. All other sources of the monetary base are assumed to be zero. Equation (6) and (7) allow the money supply to be changed through open market operations (B_f), changing reserve requirements (α), bank borrowing (BB), or by the Treasury issuance of currency (CC). Equation (5) is the definition of wealth. It takes explicit recognition of the fact that while open market operations transfer bond holdings from the public to the Federal Reserve or vice versa -- they do not alter the public's holdings of wealth. The purchase of government debt by the Federal Reserve merely replaces government debt with deposits at the Federal Reserve.[5/]

Equations (8)-(10) represent the supply side of the model. The labor supply function allows for a possible money illusion and for a wealth effect in labor supply.[6/] The wealth effect in the labor supply has been considered recently by Hayakawa (1979) and Smith (1980).

The model is closed out with the budget constraint. The budget constraint differs from the standard one. It takes explicit recognition of the fact budget deficits can only be financed by issuing currency or bonds. Monetary expansions through open market operations, bank borrowing or changes in the reserve requirements will not provide the government with the money to finance the deficit. Thus, there are three possible ways that deficits can be financed; (1) pure bond financing where the stock of high-powered money is fixed, (2) mixed bond/money financing where monetary policy is accommodating, but the increases in the money supply cannot be used directly to finance deficits without the simultaneous issue of debt by the Treasury, and (3) a pure money financed deficit where the deficit is financed by the treasury issuing currency.[7/] The last possibility is perhaps the least likely, and is the only money financed deficit compatible with the budget constraint of Christ (1978, 1979) and Cohen and de Leeuw (1980).

Finally, we should note that only interest payments on that part of the debt held by the public are included in the budget constraint. This is done on the assumption that interest payments on debt held by the Federal Reserve are rebated back to the Treasury.[8/]

The remainder of this paper is devoted to analyzing the long-run policy effects and stability of pure bond and mixed bond/money financed government expenditures. This will be done in the context of two versions of the model. The first version ignores the supply side, equations (8)-(11), and holds the price level constant; following Blinder and Solow, P is set equal to one. The second version is the complete model with endogenous prices. These versions will be referenced BS1 and BS2, respectively.

II. THE LONG-RUN EFFECTIVENESS OF FISCAL POLICY

First, consider the long-run effects of government expenditures on aggregate demand in BS1. It will be assumed that mixed bond/money financing is achieved via open market operations unless it is explicitly stated otherwise.

The long-run multiplier can be obtained by substituting equation (6) into equation (11), setting the l.h.s. of the resulting equation equal to zero and differentiating with respect to G. The long-run multiplier thus obtained is

$$\frac{dY}{dG} = \frac{1 + (1-T')\,(1-\lambda)\,{}^{dB}/dG}{T'}\,. \qquad (12)$$

This general result was obtained by letting $dB_f = \lambda dB$, $\lambda \geq 0$. Thus, λ

represents the ratio of debt purchased by the Federal Reserve to debt issued by the Treasury at a given interest rate. The value of λ is less than, equal to, or greater than one depending on whether the Federal Reserve purchases debt in an amount less than, equal to, or greater than the amount issued by the Treasury. If $\lambda < 0$ the Federal Reserve would be pursuing a restrictive monetary policy, however, only positive values of λ will be considered here.[9/]

The various long-run multipliers can be obtained as special cases of equation (12). In the case of a pure bond financed expenditure, $\lambda=0$, the long-run multiplier is identical to the one obtained by Blinder and Solow. If there is a currency financed expenditure, $\lambda=0$ and $dB/dG=0$, the long-run multiplier is simply the reciprocal of the marginal tax rate. If there is mixed bond/money financing, $\lambda>0$, the long-run multiplier will be between the pure bond and pure money multiplier for $0<\lambda<1$, and less than or equal to the long-run money multiplier for $\lambda \geq 1$.[10/] The reason for this is easily understood. A deficit which is mixed bond/money financed requires that the stock of debt held by the pulic increase less than the total stock of government debt (or in the case of $\lambda>1$, actually decrease) because of open market purchases by the Federal Reserve. Hence, the larger the value of λ the larger the proportion of an increase in the interest on the public debt is paid directly to the Federal Reserve and rebated back to the Treasury. This reduces the tax burden on income required to reach budgetary balance, given the marginal tax rate on income. Thus, the mixed bond/money multiplier is smaller than the pure bond multiplier, and may be smaller than the pure money multiplier if open market operation are pursued aggressively enough. If the mixed bond/money financing is achieved through bank borrowing or through a change in reserve requirements (λ=0), the long-run multiplier is identical with the pure bond financing case. Therefore, the long-run effectiveness of a bond/money financed expenditure is not invariant to the policy instrument used to expand the money supply.

The Stability Conditions

Now turn to the question of stability. The dynamic system for BS1 can be reduced to:

$$Y(t) = F(B_f, B, BB, K, \alpha, CC; G)$$

$$r(t) = R(B_f, B, BB, K, \alpha, CC; G)$$

$$\dot{CC} + \dot{B}/r = G + (B-B_f) - T(Y+(B-B_f))$$

The stability condition for a bond and a mixed bond/money financed deficit can be obtained by setting $\dot{CC}$=0, and differentiating the third equation with respect to B. Evaluating the result within a neighborhood of equilibrium the necessary and sufficient condition for local stability becomes,

$$F_B + F_{Bf}\lambda > \frac{(1-\lambda)\ (1-T')}{T'} . \qquad (13)$$

If λ=0, this condition is identical to the one obtained by Blinder and Solow. If λ=1, the stability condition becomes $F_B + F_{Bf} > 0$. It can easily be shown that this condition will hold if the money multiplier, M_H, is larger than the wealth effect on money demand. Since this condition seems likely to hold, a mixed bond/money financed deficit is likely to be stable if the Treasury's bond issue is matched dollar for dollar by open market purchases.[11/] The reason for this is straightforward. When λ=1, the net effect on aggregate demand through the impact of these activities on disposable income is nil since there will be no change in the public's holding of Treasury debt. Thus, the effect on aggregate demand will come from the positive wealth effect in consumption due to an increase in B, and from interest induced spending which will be positive under the conditions stated above. The stability conditions need not hold, however, if $0<\lambda<1$. In this case, the l.h.s. of (13) will be positive, but not necessarily larger than the r.h.s. Nevertheless, the mixed bond/money financed expenditure is more likely to be stable than the pure bond financed expenditure, since $(1-\lambda)\ (1-T')/T' < (1-T')/T'$ for $0 < \lambda \leq 1$.

If $\lambda > 1$, the impact of open market operations on aggregate demand through their effect on disposable income is negative. If this effect is large enough, $F_B + F_{Bf}\lambda$ will be negative. Thus, a fiscal deficit will result in a larger budgetary gap as income induced tax revenue declines. If the impact of reduced tax revenue on the deficit is larger than the direct reduction in the deficit due to reduced flow of interest payments to the public, the system may be unstable. While this situation seems unlikely, stability is not guaranteed. Furthermore, it is difficult to assess whether mixed bond/money financing is more or less likely to be stable than pure bond financing. The fact that $FB_f\lambda$ could be negative tends to reduce the prospects for stability, all other things constant. The r.h.s. of (13) would be negative, however, if $\lambda > 1$; thus enhancing the prospects for stability.

One cannot conclude, therefore, that a mixed bond/money financed deficit is more likely to be stable than a pure bond financed expenditure, if the monetary expansion is accomplished through open-market operations. It depends, in part, on the extent to which open-market operations are pursued. Nevertheless, it is possible to have long-run stability even if the instantaneous bond multiplier, F_B, is negative. Thus, mixed bond/money stability does not rule out the possibility of third-level crowding out.

If the mixed bond/money financed deficit is accomplished through bank borrowing or changes in reserve requirements, the stability conditions become

$$F_B + F_{BB} \frac{dBB}{dB} > \frac{1 - T'}{T'}$$

and

$$F_B + F_\alpha \frac{d\alpha}{dB} > \frac{1 - T'}{T'} \quad ,$$

respectively. Since it can easily be shown that $F_{BB}>0$ and $F_\alpha<0$, a mixed bond/money financed expenditure is more likely to be stable than a pure bond financed expenditure, if the money supply is increased through bank borrowing or through changes in reserve requirements. Thus, not only is this type of deficit financing as expansionary as a pure bond financed deficit, as was shown above, but it is more likely to be stable. Furthermore, stability is not inconsistent with a negative instantaneous bond multiplier.

Stability With Alternative Policy Variables

Cohen and de Leeuw (1980) have shown that bond financing which is unstable when government expenditures is the policy variable is stable when other exogenous policy variables are specified. The purpose of this section is to investigate whether mixed bond/money financing stability is more likely with alternative exogenous policy variables. Two alternative policy variables are considered; $G' = G+B_p$ and $G'' = G+B_p-T(B_p)$. The stability conditions corresponding to G' and G" are, respectively,

$$F_B + F_{Bf}\lambda > (\lambda-1) \tag{14}$$

and

$$F_B + F_{Bf}\lambda > 0. \tag{15}$$

Comparing the r.h.s of equations (14) and (15) with (13), it is easy to see that stability is more likely if $0 \leq \lambda < 1$. This is due to the facts that $F_B + F_{Bf}\lambda$ is strictly positive under this condition and that the r.h.s. of (14) and (15) are smaller than the r.h.s. of (13) in this instance.[12/] If $\lambda=1$, then the stability conditions given by these three equations are identical. The alternative fiscal policy variables will not increase the likelihood of the system being stable as Cohen and de Leeuw's results suggest. The reason for this is simple. If $\lambda=1$, neither bond financing nor open market operations have any direct effect on the budget deficit. Therefore, it makes no difference if G, G' or G" is the policy variable.

If $\lambda>1$, then stability is less likely if either G' or G" is the policy variable. In these cases, vigorous pursuit of open market operations does not work to reduce the size of the deficit directly as it does in the case where G is the policy variable. Thus, stability is less likely. It should also be noted that F_{Bf} is more likely to be negative if $\lambda>1$. If this were the case, then stability under G' or G" would be impossible if the instantaneous bond multiplier is negative. Mixed bond/money stability would rule out the possibility of short-run monetarist crowding out if either G' or G" were the policy variable and if $F_{Bf}<0$.[13/]

IV. THE EXTENDED MODEL

Now consider the complete model. The reduced-form can be obtained first solving the labor market equilibrium condition given by equations (9) and (10) for the implicit employment function below

$N = N(P,r,B,CC,\bar{K}) \quad N_P>, \; N_r>0, \; N_B<0, \; N_{CC}<0,$

where N denotes the equilibrium quantity of labor. N is then substituted into the production function. The reduced-form of the entire system can be obtained by combining the resulting equation with equations (1)-(7). The dynamic model represented by equations (1)-(11) can be written as:[14/]

$$Y(t) = F(B_f, B, BB, \bar{K}, \alpha, CC; G)$$

$$R(t) = R(B_f, B, BB, \bar{K}, \alpha, CC; G)$$

$$P(t) = P(B_f, B, BB, \bar{K}, \alpha, CC; G)$$

$$\dot{CC} + \dot{B}/r = P \cdot G + (B-B_f) - T(P \cdot Y + (B-B_f))$$

The long-run government expenditure multiplier now becomes

$$\frac{dY}{dG} = \frac{P + (1-T')\,(1-\lambda)\; dB/dG + (G-T'Y)\; dP/dG}{T'} \quad . \qquad (16)$$

The long-run multipliers for the various financing schemes can be seen as special cases of equation (16). The pure bond, mixed bond/money and pure money multipliers can be obtained by letting $\lambda=0$, $\lambda>0$, and $\lambda=0$ and $dB/dG=0$, respectively (however, it should be emphasized that the expression for dP/dG will be different in each of these cases).

Since it can be argued that the term $(G-T'Y)$ will be negative with a progressive tax structure, (Hayakawa, 1979), the relative strength of fiscal policy depends on the magnitude of λ and the sign of dP/dG. The sign of dP/dG in turn depends on the sign of P_G, and the signs of P_B, P_{Bf} and P_{CC} depending on the financing scheme used.[15/] It can be shown that $P_G>0$, and that the signs of P_B, P_{Bf} and P_{CC} are indeterminate. Thus, it is possible for the long-run mixed bond/money multiplier to be larger than the pure bond multiplier, however, not only would dP/dG have to be negative, but the gain from the last term on the r.h.s. of equation (16) would have to off-set the smaller value of the middle term due to the fact that $\lambda>0$. In the final analysis, this is an empirical question.

If the mixed bond/money financing is achieved through bank borrowing or changing reserve requirements, the long-run multiplier will be smaller than for pure bond financing. This is true because P_α and P_{BB} are strictly negative and positive, respectively, if the aggregate supply curve is upward sloping. Thus, dP/dG must be larger for both bank borrowing and changes in reserve requirements, and the long-run multipliers are therefore smaller.[16/] A simultaneous increase in both G and BB, for example, exert additional upward pressure on prices which, given the nonindexed tax structure, increases the flow of tax revenue relative to real income. Hence, budgetary balance can be obtained with a smaller increase in real output. In all other respects the multipliers obtained here are essentially the same as those obtained by Hayakawa, and hence require no further elaboration.[17/]

The Stability Conditions

Consider again the question of stability. The general expression for stability within a neighborhood of equilibrium is,

$$F_B + F_{Bf}\lambda > \frac{(G-T'Y)\ (P_B+P_{Bf}\lambda) + (1-\lambda)\ (1-T')}{T'P} .$$

Note that this expression is identical to Hayakawa's if $\lambda=0$. The stability of mixed bond/money financing depends critically on the signs of F_B, F_{Bf}, P_B and P_{Bf}. The signs of all of these are indeterminate. Thus, it is not possible to say if mixed bond/money financing is stable when prices are endogenous, nor is it possible to say whether mixed bond/money financing is more likely to be stable than pure bond financing. If we restrict ourselves to the case where the direct effect of open market operations on aggregate demand through disposable income is less than the indirect effect through interest rates, and if the aggregate supply curve is upward sloping, then $P_{Bf}>0$. Under these circumstances, r.h.s. of (17) will be smaller for mixed bond/money financing than for pure bond financing. This suggests that mixed bond/money financing is more likely to be stable than pure bond financing. Unfortunately, the sign of F_{Bf} is still indeterminate. It depends on whether or not the aggregate demand schedule shifts rightward by more or less than the aggregate supply schedule shifts leftward. If it shifts more, mixed bond/money financing will be more stable than pure bond financing. If it shifts less, this conclusion need not hold.

Finally, consider the stability conditions for mixed bond/money financing when the money supply is expanded through bank borrowing of changes in reserve requirements. The stability conditions are,

$$F_B + F_{BB}\frac{dBB}{dB} > \frac{(G-T'Y)\ (P_B+P_{BB}\frac{dBB}{dB}) + (1-T')}{PT'},$$

and

$$F_B + F_\alpha\ \frac{d\alpha}{dB} > \frac{(G-T'Y)\ (P_B+P_\alpha\frac{d\alpha}{dB}) + (1-T')}{PT'}.$$

It can easily be shown that P_{BB} and P_α are strictly positive and negative, respectively, if aggregate supply is upward sloping. The signs of F_{BB} and F_α depend on whether the shift in aggregate demand is greater or less than the shift in aggregate supply. If the shift is greater, those terms will be positive and negative respectively, and mixed bond/money financing is more likely to be stable than pure bond financing. If the shift is less, then mixed bond/money financing is not necessarily more stable. The latter case requires a relatively strong interest induced wealth effect on labor supply.

V. THE CONCLUSIONS

The purpose of this paper was to review the long-run impact and stability of discretionary fiscal policy for the case of mixed bond/money financing, where the money supply is endogenous. The following conclusions emerge. The long-run mixed bond/money multiplier may be larger, equal to or smaller than the pure bond multiplier depending on the model specification and on the monetary policy tools used. If one considers only aggregate demand and if open market operations are used, then the multiplier will be smaller. Indeed, if the Federal Reserve purchases securities at a faster rate than the Treasury issues new bonds, the long-run mixed multiplier will be smaller than the pure money multiplier that is commonly reported. If the money supply is expanded through bank borrowing or by changing reserve requirements, the mixed bond/money and pure bond multipliers are identical. If price is endogenous, these conclusions need not hold. However, if we restrict ourselves to the case where aggregate supply is positively sloped, they will.

With respect to stability, it cannot be shown that mixed bond/financing is stable. However, mixed financing is more likely to be stable than pure bond financing for the fixed-price model, under fiarly reasonable conditions. The same conclusion holds for the variable price model if the wealth effect on labor supply is weak.

Furthermore, if the traditional government expenditure variable, G, is replaced by government expenditures plus interest payments on the debt, or by government expenditures plus interest payments on the debt net of taxes, the chances of stability are enhanced only if monetary policy is partially accommodative. If monetary policy is exactly accommodative, stability is invariant to the government expenditure variable. If open market operations are used to acquire bonds in excess of the new-issue of government debt, then the prospects for stability are reduced. Thus, if open market operations are pursued aggressively, a bond/money financial deficit is more likely to be stable if G is the policy variable.

Finally, mixed bond money stability does not necessarily rule out the possibility of a negative instantaneous bond multiplier. Thus, long-run effectiveness of fiscal policy is not necessarily inconsistant with third-level, monetarist crowding out if G is the relevant fiscal policy variable.

FOOTNOTES

1/ Christ (1978, 1979) treats the stock of high powered money as his policy variable. It is clear from his expression for the budget constraint that he is implicitly only considering the case where high-powered money and currency are synonymous.

2/ The Federal Reserve moved from a federal funds rate targeting procedure to a reserve aggregate targeting procedure on October 6, 1979. For a discussion of this move see, Lang (1980) and Axilrod and Lindsey (1981).

3/ This budget constraint is of the form $H/P+B/Pr = G+B-T(P\cdot Y+B)$, where H denotes the stock of high-powered money. It should be noted that the r.h.s. of this budget constraint does not allow for the direct impact of open market operations on the deficit.

4/ Sargent (1979) has recently questioned whether or not the budget constraint is redundant for models where there are no direct wealth effects in the consumption or money demand functions. Since our model does not fall in this category, this objection is not considered further.

5/ Patinkin (1949) has argued that Federal Reserve open market operations simply replace the public's holdings of wealth in the form of government bonds with wealth in the form of deposits at the Federal Reserve. He defines wealth to be exclusive of government debt held by the Federal Reserve, but inclusive of deposits at the Federal Reserve. Borrowings alter the monetary base, but have no effect on society's wealth.

6/ This model, like most of these models, implicitly assumes that government bonds are part of society's net wealth. For an opposing viewpoint, see Barro (1974). This model could easily be modified to accommodate various assumptions about the net wealth character of government debt, e.g., Holmes and Smyth (1979).

7/ Actually, there are other possibilities where both currency and bonds are used. These alternatives are not believed to be important, and are not considered here.

8/ This analysis implicitly assumes that all interest payments received by the Federal Reserve are returned to the treasury. This assumption is unquestionably false. Federal Reserve operations require real resources, and hence contribute themselves to real income. However, since everyone seems so willing to ignore these costs, this assumption will be maintained here as well.

9/ There is a sense of arbitrariness about λ. There is nothing the model that determines its value for a given policy objective. One way to overcome this difficulty would be to assume the Federal Reserve "targets" some level for the interest rate and then allow λ to become endogenous, obtaining whatever level necessary to maintain the target interest rate. This may be too restrictive, however, in that only one policy objective is considered. Furthermore, such a modification would go well beyond the scope of this paper.

10/ It is easy to show that the long-run multiplier for open market operations is negative, just as Blinder and Solow (1976) conjectured.

11/ A sufficient condition for this to hold is that the monetary base multiplier be greater than one and the wealth elasticity of money demand be less than or equal to one. If the latter condition holds, then $l_W\, W/M \leq 1$. Since W/M will most assuredly be greater than one, this implies that $l_W < 1$. Since, it can be shown that $F_B + F_{Bf}\, \lambda > 0$ if $(M_H - l_W) > 0$, we have the desired result.

12/ Assuming that $(M_H - l_W) > 0$, see footnote 11 above.

13/ The condition necessary for $F_{Bf} < 0$ is $|\lambda C_y\,(1-T')| > |\lambda C_W l_r + (M_H/r)\,(C_W B/r^2 - I')|$.

14/ The first two equations are functionally different from their counterparts in the previous section. Nevertheless, the same functional notation was used for ease of comparison.

15/ In general, $dP/dG = P_G + P_B\, dB/dG + P_{Bf} dBf/dG + P_{CC} dCC/dG$, evaluated at the equilibrium. The specific form of dP/dG depends on which of the exogenous variables are being held constant.

16/ The long-run multiplier would be

$P + (1-T')\, dB/dG + (G-T'Y)\,(P_G + P_{BB} dBB/dG)$ and

$P + (1-T')\, dB/dG + (T-T'Y)\,(P_G + P_\alpha d\alpha/dG)$, respectively.

17/ Hayakawa (1979) has obtained the basic results for a Blinder-Solow model with endogenous prices. The reader should note, however, that Hayakawa's model is fundamentally different from the one presented here because in his model <u>both</u> money and bonds are exogenous and both are part of net wealth. Thus, <u>both</u> bond and money financed expenditures have a direct impact on demand and supply through their impact on wealth.

REFERENCES

Axilrod, S. and D.E. Lindsey. "Federal Reserve Implementatin of Monetary Policy: Analytical Foundations of the New-Approach." American Economic Review 71 (May 1981): 246-52.

Barro, R. "Are Government Bonds Net Wealth?" Journal of Political Economy 82 (November 1974): 1095-1118.

Blinder, A. S. and R. M. Solow. "Does Fiscal Policy Matter?" Journal of Public Economics 2 (November 1973): 319-37.

______________. "Does Fiscal Policy Still Matter?: A Reply." Journal of Monetary Economics 2 (April, 1976): 501-10.

Christ, C. F. "A Simple Macroeconomic Model With a Government Budget Restraint." Journal of Political Economy 76 (January/February 1968): 53-67.

______________. "Some Dynamic Theory of Macroeconomic Policy Effects on Income and Prices Under the Government Budget Restraint." Journal of Monetary Economics 4 (January 1978): 45-70.

__________. "On Fiscal and Monetary Policies and the Government Budget Restraint." American Economic Review 69 (September 1979): 526-38.

Cohen, D. and F. de Leeuw. "A Note on the Government Budget Constraint." Journal of Monetary Economics 6 (October 1980): 547-60.

Hayakawa, H. "Does Fiscal Policy Really Matter in the Context of Variable Prices?" Journal of Macroeconomics 1 (Fall 1979): 321-46.

Holmes, J. M., and D. J. Smyth "Deficit Financing, Liquidity and the Government Budget Constraint," Journal of Macroeconomics 1 (Winter 1979): 83-106.

Infante, E.F. and J.L. Stein. "Does Fiscal Policy Matter?" Journal of Monetary Economics 2 (November 1976): 473-500.

Lang, R.W. "The FOMC in 1979: Introducing Reserve Targeting." Review Federal Reserve Bank of St. Louis 62 (March 1980): 2-25.

Patinkin, D. "Price Flexibility and Full-Employment: Reply." American Economic Review 39 (June 1949): 726-8.

Sargent, T.J. Macroeconomic Theory New York: Academic Press, 1979.

Smith, P.E. "Supply, Wealth Effects, Negative Multipliers, and Stability." Journal of Macroeconomics 2 (Winter 1980): 41-62

APPENDIX

COMPARATIVE STATICS FOR THE REDUCED-FORM EQUATIONS FOR BS1.

Setting P=1, the first seven questions of the model can be reduced to

$$Y \quad = \quad C(Y+B-B_f-T(Y+B+B_f), K+CC+B/r) + I(r) + G \tag{A.1}$$

$$M(r, BB+CC+B_f/r, \alpha) \quad = \quad l(r, Y, K+CC+B/r) \tag{A.2}$$

Totally differentiating equations (A.1) and (A.2) yields,

$$\begin{bmatrix} 1-C_Y(1-T') & C_W B/r^2 - I' \\ -l_Y & \delta \end{bmatrix} \begin{bmatrix} dY \\ dr \end{bmatrix} = \begin{bmatrix} 1 & C_Y(1-T')+C_W/r & C_Y(T'-1) & 0 & 0 & C_W \\ 0 & l_W/r & -M_H/r & -M_H & -M_\alpha & M_H + l_W \end{bmatrix} \begin{bmatrix} dG \\ dB \\ dB_f \\ dBB \\ d\alpha \\ dCC \end{bmatrix}$$

Where $\delta = M_r - M_H B_f/r^2 - l_r + l_W B/r^2$. Now $\delta > 0$, if $M_H B_f/r^2 = 0$. This term represents the interest induced change in the base which is assumed to be zero. The determinant of the matrix of coefficients on the endogenous variables is $\Delta = \delta(1-C_y(1-T') + l_y(C_W B/r^2 - I') > 0$. Thus,

$$F_B = \frac{\delta(C_y(1-T')+C_W/r) - l_W/r(C_W B/r^2 - I')}{\Delta}$$

Note that $F_B < 0$ only if the wealth effect in the demand for money is sufficiently strong. Likewise,

$$F_{Bf} = \frac{\delta \; C_y(T'-1)\lambda + (M_H/r)(C_W B/r^2 - I')}{\Delta}$$

Note that $F_{Bf}<0$ if the direct effect of open-market operations on disposable income is larger than the interest induced changes in consumption and investment. Thus, it is easy to see that

$$F_B + F_{Bf}\lambda = \frac{\delta(C_w/r + C_y(1-T')(1-\lambda)) + \frac{1}{r}(M_H - 1_w)(C_w B/r^2 - I')}{\Delta}$$

If $\lambda=1$, this equation reduces to

$$F_B + F_{Bf} = \frac{\delta C_w/r + \frac{1}{r}(M_H - 1_w)(C_w B/r^2 - I')}{\Delta}$$

The term on the r.h.s. will be positive whenever, $(M_H - 1_w)>0$. This condition will hold if the monetary base multiplier is greater than one and $0 \leq 1_w < 1$.

If $\lambda>1$, then $F_B + F_{Bf}\lambda$ could be $\lesseqgtr 0$ depending on the magnitude of $\delta C_y(1-T')(1-\lambda)$ relative to the other terms in the numerator of the above expression, even if $(M_H - 1_w)>0$.

COMPARATIVE STATICS FOR BS2

The employment equilibrium is given by:

$P \cdot f(N) = g(N,P,B/rP + CC/P + K)$

From this we obtain,

$$\frac{\partial N}{\partial P} = \frac{g_P - f - g_w B/rP^2 - g_w CC/P^2}{Pf_N - g_N} > 0$$

$$\frac{\partial N}{\partial P} = \frac{-g_w B/r^2 P}{Pf_N - g_N} > 0$$

$$\frac{\partial N}{\partial B} = \frac{g_w/rP}{Pf_N - g_N} < 0$$

$$\frac{\partial N}{\partial CC} = \frac{g_w/P}{Pf_N - g_N} < 0$$

When the employment function is substituted into the production function, equations (1)-(10) can be reduced to:

$$Y = C(Y+B/P-B_f/P-T(Y\cdot P+B/P-B_f/P)\ (P,\ B/Pr+CC/P+\overline{K}) + I(r) + G$$

$$M(r,\ B_f/r+BB+CC,\alpha) = P \cdot l(r,Y,B/rP+CC/P+\overline{K})$$

$$Y = Y(N(P,r,b,CC,\overline{K}),\overline{K}) = Y(P,r,B,CC,\overline{K})$$

The aggregate demand and supply functions can be obtained by totally differentiating the above system. Solving the second equation for dr and substituting into the first and third equations, we obtain expressions for aggregate demand and supply, respectively. Substituting dr in the first equation, we get

$$dY = \frac{1}{\delta_1}\ \{(C_Y \frac{(P-T')}{P^2} + \frac{C_w}{Pr} + \frac{\delta_5}{\delta_2}\ \frac{l_w}{rP}\ )\ dB$$

$$+ \ (C_Y \frac{(T'-P)}{P^2} - \frac{\delta_5}{\delta_2}\ \frac{M_H}{r}\ )\ dB_f$$

$$+ \ (\frac{C_w}{P} - \frac{\delta_5}{\delta_2}\ (M_H+l_w))\ dCC$$

$$- \ \frac{\delta_5}{\delta_2} M_\alpha\ d_\alpha - \frac{\delta_5}{\delta_2} M_H\ dBB$$

$$+ \quad (\delta_4 + \frac{\delta_5}{\delta_2} (l(\cdot) - l_w(B/rP + CC/P)) \; dP$$

$$+ \quad dG\}$$

where,

$$\delta_2 = (M_r - Pl_r + \frac{Pl_w B}{r^2 P}) > 0$$

$$\delta_5 = (I' - \frac{C_w B}{r^2 P}) < 0$$

$$\delta_4 = C_Y \frac{(P-T')\,(B_f - B)}{P^3} + C_Y \frac{(T-PYT')}{P^2} - \frac{C_w(B/rP+CC/P)}{P} \lesseqgtr 0$$

$$\delta_1 = (1 - C_Y(1-T') - \frac{\delta_5}{\delta_2} Pl_Y) > 0,$$

Since $(1 - C_Y(1-T')) > 0$ and $\delta_5/\delta_2 < 0$,

the shift in aggregate demand schedule for each of the exogenous variables is:

$$\frac{dAD}{dBB} > 0 \qquad \frac{dAD}{da} < 0 \qquad \frac{dAD}{dCC} > 0$$

$$\frac{dAD}{dB} \lesseqgtr 0 \qquad \frac{dAD}{dB_f} \gtreqless 0 \qquad \frac{dAD}{dG} > 0.$$

The slope of the aggregate demand schedule is given by:

$$\left.\frac{dP}{dY}\right|_{AD} = \frac{\delta_1}{\delta_4 + \delta_5/\delta_2(l(\cdot) - l_w(B/rP+CC/P))}.$$

In general, $\delta_4<0$. (T-PYT') will be negative as long as the tax system is progressive (the marginal tax rate is greater than the average tax rate). The term $(C_Y(P-T')\ (B_f-B))/P3$ will be negative if $T' < P$, because $B\geq B_f$ But even if $T'>P$, it is likely δ_4 will remain negative. Thus, the aggregate demand curve will be downward sloping unless the wealth effect in the dmand for money (1_w) is sufficiently strong. Substituting (dr) into the supply equation we get

$$dY = \frac{1}{\delta_3}\{((Y_P + (Y_r/\delta_2)\ (1(\cdot) - 1_w(B/rP+CC/P))\ dP$$

$$- (Y_r/\delta_2)\ (M_H/r)\ dB_f - (Y_r/\delta_2)\ M_H\ dBB$$

$$+ (Y_{CC} - (Y_r/\delta_2)\ (1_w - M_H))\ dCC$$

$$- (Y_r/\delta_2)\ M_\alpha\ d_\alpha + (Y_B + (Y_r/\delta_2)\ (1_w/r))\ dB\}$$

where $\delta_3 = (1 - (Y_r/\delta_2)\ P1_Y)$, $\delta_3\neq 0$.

The slope of the aggregate supply schedule will be

$$\left.\frac{dP}{dY}\right|_{AS} = \frac{1 - (Y_r/\delta_2)\ P1_Y}{Y_P + (Y_r/\delta_2)\ (1(\cdot) - 1_w(B/rP+CC/P))}$$

The aggregate supply schedule will be positively sloped unless the wealth effect in the demand for money is sufficiently strong, or unless $(Y_r/\delta_2)\ P1_Y>1$. This would, however, require a strong interest induced wealth effect in the labor supply schedule. The aggregate supply schedule will shift in the following ways for the exogenous variables.

$$\frac{dAS}{dB_f} \lesseqgtr 0 \quad \text{if} \quad (Y_r/w_2)\ P1_Y \lesseqgtr 1$$

$$\frac{dAS}{dBB} \lesseqgtr 0 \quad \text{if} \quad (Y_r/w_2)\ P1_Y \lesseqgtr 1$$

$$\frac{dAS}{dCC} \gtrless 0 \quad \text{if} \quad (Y_r/w_2)\, Pl_Y \gtrless 1$$

$$\frac{dAS}{dB} \lesseqgtr 0 \quad Y_B > (Y_r/\delta_2)\,(l_w/r) \gtrless 0 \ \text{and} \ (Y_r/\delta_2)\, Pl_Y \lesseqgtr 1$$

$$\frac{dAS}{dG} = 0.$$

Now from these relationships we can infer the signs of the comparative static derivatives of F(.) and P(.).

$$F_G > \quad \text{if} \quad |dP/dYAS| < |dP/dYAD|,$$

and the condition will hold whenever the wealth effect in the demand for money is not sufficiently strong, or the interest induced wealth effect in the labor supply isn't too strong.