A Warning on the Use of the Cochrane-Orcutt Procedure Based on a Money Demand Equation for the United States

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by
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April 1981].
ABSTRACT

We show that estimates of the elasticity of demand for money in the United States depend crucially on which of the three minima of the residual sum of squares is selected by the Cochrane-Orcutt procedure applied to a model which contains a lagged endogenous variable. The model constitutes the first real example of multiple minima obtainable by the Cochrane-Orcutt procedure — with or without a lagged endogenous variable — and is used to caution against routine use of this procedure.
I. INTRODUCTION

Economists frequently consider the multiple regression model with a lagged endogenous variable and autocorrelated residuals

\[ y_t = \beta_1 y_{t-1} + \sum_{k=3}^{K} \beta_k x_{t,k} + u_t, \quad t = 1, \ldots, N, \]  

(1)

\[ u_t = \rho u_{t-1} + e_t, \quad t = 1, \ldots, N, \]  

(2)

where \( y_t \) is the \( t \text{th} \) observation of the endogenous variable, \( x_{t,k} \) is the \( t \text{th} \) observation of the \( k \text{th} \) exogenous regressor \((3 \leq k \leq K)\), \( u_t \) is the \( t \text{th} \) value of the disturbance term and \( \beta = (\beta_1, \ldots, \beta_K)' \) and \( \rho \) are parameters. It is assumed that the \( e_t \) are independent, have mean zero and constant variance \( \sigma^2 \); further, \( y_0 \) is usually taken as given.

A widely used approach to estimating model parameters consists of combining (1) and (2) and of considering the transformed model

\[ (y_t - \rho y_{t-1}) = \beta_1 (1-\rho) + \beta_2 (y_{t-1} - \rho y_{t-2}) + \sum_{k=3}^{K} \beta_k (x_{t,k} - \rho x_{t-1,k}) + e_t, \quad t = 2, \ldots, N, \]  

(3)

with \( y_0 \) and \( y_1 \) taken as fixed. Scanning or iterative procedures are then used to choose the values of \( \rho \) and \( \beta \) which yield the smallest sum of squared residuals.

The Cochrane-Orcutt [2] iterative technique, which alternately minimizes the sum of squared errors with respect to \( \beta \) conditionally on \( \rho \) and then with respect to \( \rho \) conditionally on \( \beta \) until successive estimates differ by a given small amount, is probably the most widely used of the iterative techniques. Convergence of this algorithm was proved by Sargan [8] and Oberhofer and Kmenta [7]. An important question which arises here
is whether the algorithm necessarily converges to the minimum of the residual sum of squares. In theory, because of its arbitrary starting point, the procedure can also converge to a local minimum of the objective function (if multiple minima are possible) or to a saddle point. On this issue, Sargan has noted that, although convergence to a saddle point is possible, it is an event which "occurs with probability zero": the question of practical importance is therefore whether the residual sum of squares minimized by the algorithm can have several minima.¹

In practice, the procedure has been widely presumed to yield the smallest sum of squared errors, probably because of the experience of practitioners that multiple points of convergence do not occur, at least with typical economic data, or have not been detected in practice; indeed, we have not found in the literature a single well established example of multiple points of convergence based on real data even in models which contain only exogenous regressors.²

In this paper, we intend to fill this gap and show that multiple minima can occur in the presence of lagged endogenous variables by reporting on a real example of multiple minima obtained from a standard money demand

¹. This issue is addressed at length in Dufour and Gaudry [4].
². The only systematic search of multiple solutions was apparently performed by Sargan [8], whose students did not find multiple minima in a set of 53 cases examined. Some of the present authors (Dufour, Gaudry and Liem [5]) have recently reported on two numerical examples containing only exogenous regressors, but these are based on artificial data. Similarly, the numerical illustration given by Betancourt and Kelejian [1] is a constructed example of a structure which yields, without recourse to actual or artificial data, three fixed points asymptotically.
equation for the United States. We thus present the first well documented
real case of multiple minima of the objective function minimized by the
Cochrane-Orcutt algorithm, with or without lagged endogenous regressors.
We show that the estimates of elasticity of demand for money depend
crucially on which of the three minima are selected by the procedure. Our
results considerably strengthen the warning of Betancourt and Kelejian
[1] concerning routine use of the Cochrane-Orcutt procedure in the presen-
tce of lagged endogenous variables without having otherwise verified
that the algorithm has converged to the true minimum in the parameter
region considered of interest.

II. MULTIPLE MINIMA: A REAL EXAMPLE

Consider the following U.S. money demand model studied by Hafer and
Hein [6]:

$$
\ln \left( \frac{M_t}{P_t} \right) = \beta_0 + \beta_1 \ln \left( \frac{M_{t-1}}{P_{t-1}} \right) + \beta_2 \ln (\text{CPR}_t) + \beta_3 \ln (\text{RTD}_t) + \beta_4 (\text{GNPR}_t) + u_t
$$

(4)

where \( M \) denotes old M1 (currency + demand deposits) balances, \( P \) is the
implicit GNP price deflator (1972 = 100), CPR is the commercial paper rate,
RTD is the commercial bank passbook rate, GNPR is real GNP (1972 dollars)
and \( u_t \), the error term, is assumed to follow a first order autoregressive
process as in (2). The data are quarterly and cover the period 1955-I
to 1978-IV.\textsuperscript{3} Because of the presence of a lagged endogenous variable

\textsuperscript{3} See Appendix 1 for a listing of the data.
and the use of a first order autoregressive scheme on the errors, the effective sample contains 94 observations (1955-III - 1978-IV).

To obtain parameter estimates for this model, we performed a grid search over the range $-2 \leq \rho \leq 2$, minimizing the residual sum of squares (or the standard error of the regression) for each value of $\rho$ examined. In this range, we found three local minima of the standard error of the regression$^4$, at $\hat{\rho} = 0.4676$, $\tilde{\rho} = 0.9448$ and $\hat{\rho} = 1.0118$, with two local maxima, at $\hat{\rho} = 0.7334$ and $\tilde{\rho} = 0.9665$. The results of this grid search are depicted on Figure 1. It is of interest to note here that, while two of the minima occur at values of $\rho$ between 0 and 1, the lowest of the three minima is at a value of $\rho$ greater than, but very close to, 1. In order to make sure that no numerical problem had occurred, we carefully checked for multicollinearity in the neighborhood of the five above values. For each of these values of $\rho$, we regressed in turn every transformed explanatory variable $X^*_t,k = X_{t,k} - \hat{\rho} X_{t-1,k}$, $(1 \leq k \leq K$, where $X_{t,1} = 1$ and $X_{t,2} = y_{t-1}$) on the others and verified that there was no case of perfect or almost perfect fit; we then computed $(X^* X^*)^{-1}$ $(X^* X^*)$, where $X^*$ is the matrix of transformed regressors, and verified that the result was the identity matrix. These exercises were performed successfully for 21 values of $\rho$ spaced equally (by steps of .0001) within each of the 5 relevant ranges.$^5$

All computations were made in single precision on a CDC Cyber 197 computer.

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4. Defined as $\left[ \sum_{t=2}^{N} \hat{\sigma}_t^2 / (n-K) \right]^{1/2}$, where $N = 96$, $n = 94$ and $K = 5$.

5. The ranges were: $0.4666 \leq \rho \leq 0.4686$, $0.7324 \leq \rho \leq 0.7344$, $0.9438 \leq \rho \leq 0.9458$, $0.9655 \leq \rho \leq 0.9675$ and $1.0108 \leq \rho \leq 1.0128$. 
III. CONCLUSION

As can be seen from Table 1, the issue about the unimodality of the sum of squared residuals of the transformed model (3) is not merely of theoretical import: coefficients (or elasticities of demand in this case) and their conditional t-statistics differ considerably among the three minimum solutions.
<table>
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<th></th>
<th>$\rho = .4676$</th>
<th>$\rho = .9448$</th>
<th>$\rho = 1.018$</th>
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<td><strong>Constant</strong></td>
<td>-0.10</td>
<td>-0.60</td>
<td>-1.18</td>
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<td></td>
<td>26.69</td>
<td>7.35</td>
<td>6.55</td>
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<tr>
<td><strong>Commercial paper rate</strong></td>
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<td>-0.013</td>
<td>-0.014</td>
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<td></td>
<td>-5.06</td>
<td>-3.13</td>
<td>-3.34</td>
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<tr>
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<td>-0.044</td>
</tr>
<tr>
<td></td>
<td>0.71</td>
<td>-2.55</td>
<td>-2.64</td>
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<tr>
<td><strong>Real GNP</strong></td>
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<td>0.144</td>
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<table>
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<td></td>
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</table>

* The t-statistics shown are obtained with the usual conditional procedure and do not incorporate the correction suggested by Cooper [3].

Note that the sign of the coefficient of the Commercial bank passbook rate varies across minima; note also that the estimated income elasticity of demand for money is 8 times larger at $\hat{\rho} = 0.94$ than at $\hat{\rho} = 0.46$ and 18 times larger at $\hat{\rho} = 1.01$ than at $\hat{\rho} = 0.46$. These results show the practical importance of making sure that the Cochrane-Orcutt procedure yields the global minimum in the relevant region.
### APPENDIX 1

#### U.S. QUARTERLY DATA

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<td>588.3</td>
<td>3.94</td>
<td>30</td>
<td>90.93</td>
</tr>
<tr>
<td>1950-I</td>
<td>106,400</td>
<td>586.7</td>
<td>3.96</td>
<td>0</td>
<td>91.43</td>
</tr>
</tbody>
</table>

**Note:** The data represents quarterly economic figures from 1935 to 1950, with each year showing a decrease in the N (number) and an increase in the economic indicators such as GNP, ESTD, CFFB, and P.
REFERENCES


