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A Warning on the Use of the Cochrane-Orcutt Procedure Based on A Money Demand Equation for the United States

by

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ABSTRACT

We show that estimates of the elasticity of demand for money in the United States depend crucially on which of the three minima of the residual sum of squares is selected by the Cochrane-Orcutt procedure applied to a model which contains a lagged endogenous variable. The model constitutes the first *real example* of multiple minima obtainable by the Cochrane-Orcutt procedure — with or without a lagged endogenous variable — and is used to caution against routine use of this procedure.

I. INTRODUCTION

Economists frequently consider the multiple regression model with a lagged endogenous variable and autocorrelated residuals

$$y_t = \beta_1 + \beta_2 y_{t-1} + \sum_{k=3}^{K} \beta_k X_{t,k} + u_t , \qquad t = 1,\ldots,N, \qquad (1)$$

$$u_t = \rho u_{t-1} + e_t , \qquad t = 1,\ldots,N, \qquad (2)$$

where y_t is the t^{th} observation of the *endogenous* variable, $X_{t,k}$ is the t^{th} observation of the k^{th} *exogenous* regressor ($3 \le k \le K$), u_t is the t^{th} value of the disturbance term and $\underline{\beta} = (\beta_1,\ldots,\beta_K)'$ and ρ are parameters. It is assumed that the e_t are independent, have mean zero and constant variance σ^2; further, y_o is usually taken as given.

A widely used approach to estimating model parameters consists of combining (1) and (2) and of considering the transformed model

$$(y_t-\rho y_{t-1}) = \beta_1(1-\rho) + \beta_2(y_{t-1}-\rho y_{t-2}) + \sum_{k=3}^{K} \beta_k(X_{t,k}-\rho X_{t-1,k}) + e_t , \qquad t = 2,\ldots,N, \qquad (3)$$

with y_0 and y_1 taken as fixed. Scanning or iterative procedures are then used to choose the values of ρ and $\underline{\beta}$ which yield the smallest sum of squared residuals.

The Cochrane-Orcutt [2] iterative technique, which alternately minimizes the sum of squared errors with respect to $\underline{\beta}$ conditionally on ρ and then with respect to ρ conditionally on $\underline{\beta}$ until successive estimates differ by a given small amount, is probably the most widely used of the iterative techniques. Convergence of this algorithm was proved by Sargan [8] and Oberhofer and Kmenta [7]. An important question which arises here is whether the algorithm necessarily converges to the minimum of the residual sum of squares. In theory, because of its arbitrary starting point, the procedure can also converge to a local minimum of the objective function (if multiple minima are possible) or to a saddle point. On this issue, Sargan has noted that, although convergence to a saddle point is possible, it is an event which "occurs with probability zero" : the question of practical importance is therefore whether the residual sum of squares minimized by the algorithm can have several minima.[1]

In practice, the procedure has been widely presumed to yield the smallest sum of squared errors, probably because of the experience of practitioners that multiple points of convergence do not occur, at least with typical economic data, or have not been detected in practice; indeed, we have not found in the literature a single well established example of multiple points of convergence based on real data even in models which contain only exogenous regressors.[2]

In this paper, we intend to fill this gap and show that multiple minima can occur in the presence of lagged endogenous variables by reporting on a real example of multiple minima obtained from a standard money demand

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1. This issue is addressed at length in Dufour and Gaudry [4].

2. The only systematic search of multiple solutions was apparently performed by Sargan [8], whose students did not find multiple minima in a set of 53 cases examined. Some of the present authors (Dufour, Gaudry and Liem [5]) have recently reported on two *numerical* examples containing only exogenous regressors, but these are based on artificial data. Similarly, the numerical illustration given by Betancourt and Kelejian [1] is a *constructed* example of a structure which yields, without recourse to actual or artificial data, three fixed points asymptotically.

equation for the United States. We thus present the first well documented real case of multiple minima of the objective function minimized by the Cochrane-Orcutt algorithm, with or without lagged endogenous regressors. We show that the estimates of elasticity of demand for money depend crucially on which of the three minima are selected by the procedure. Our results considerably strengthen the warning of Betancourt and Kelejian [1] concerning routine use of the Cochrane-Orcutt procedure in the presence of lagged endogenous variables without having otherwise verified that the algorithm has converged to the true minimum in the parameter region considered of interest.

II. MULTIPLE MINIMA: A REAL EXAMPLE

Consider the following U.S. money demand model studied by Hafer and Hein [6] :

$$\ln (M_t/P_t) = \beta_0 + \beta_1 \ln (M_{t-1}/P_{t-1}) + \beta_2 \ln (CPR_t) + \beta_3 \ln (RTD_t) + \beta_4 (GNPR_t) + u_t \quad , \tag{4}$$

where M denotes old M1 (currency + demand deposits) balances, P is the implicit GNP price deflator (1972 = 100), CPR is the commercial paper rate, RTD is the commercial bank passbook rate, GNPR is real GNP (1972 dollars) and u_t, the error term, is assumed to follow a first order autoregressive process as in (2). The data are quarterly and cover the period 1955-I to 1978-IV.[3] Because of the presence of a lagged endogenous variable

3. See Appendix 1 for a listing of the data.

and the use of a first order autoregressive scheme on the errors, the effective sample contains 94 observations (1955-III - 1978-IV).

To obtain parameter estimates for this model, we performed a grid search over the range $-2 \le \rho \le 2$, minimizing the residual sum of squares (or the standard error of the regression) for each value of ρ examined. In this range, we found three local minima of the standard error of the regression[4], at $\hat{\rho} = 0.4676$, $\hat{\rho} = 0.9448$ and $\hat{\rho} = 1.0118$, with two local maxima, at $\hat{\rho} = 0.7334$ and $\hat{\rho} = 0.9665$. The results of this grid search are depicted on Figure 1. It is of interest to note here that, while two of the minima occur at values of ρ between 0 and 1, the lowest of the three minima is at a value of ρ greater than, but very close to, 1. In order to make sure that no numerical problem had occurred, we carefully checked for multicollinearity in the neighborhood of the five above values. For each of these values of ρ, we regressed in turn every transformed explanatory variable $X^*_{t,k} = X_{t,k} - \hat{\rho}\, X_{t-1,k}$, ($1 \le k \le K$, where $X_{t,1} = 1$ and $X_{t,2} = y_{t-1}$) on the others and verified that there was no case of perfect or almost perfect fit; we then computed $(X^{*\prime}X^*)^{-1}\,(X^{*\prime}X^*)$, where X^* is the matrix of transformed regressors, and verified that the result was the identity matrix. These exercises were performed successfully for 21 values of ρ spaced equally (by steps of .0001) within each of the 5 relevant ranges.[5] All computations were made in single precision on a CDC Cyber 197 computer.

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4. Defined as $\left[\sum_{t=2}^{N} \hat{e}_t^2/(n-K)\right]^{1/2}$, where $N = 96$, $n = 94$ and $K = 5$.

5. The ranges were: $0.4666 \le \rho \le 0.4686$, $0.7324 \le \rho \le 0.7344$, $0.9438 \le \rho \le 0.9458$, $0.9655 \le \rho \le 0.9675$ and $1.0108 \le \rho \le 1.0128$.

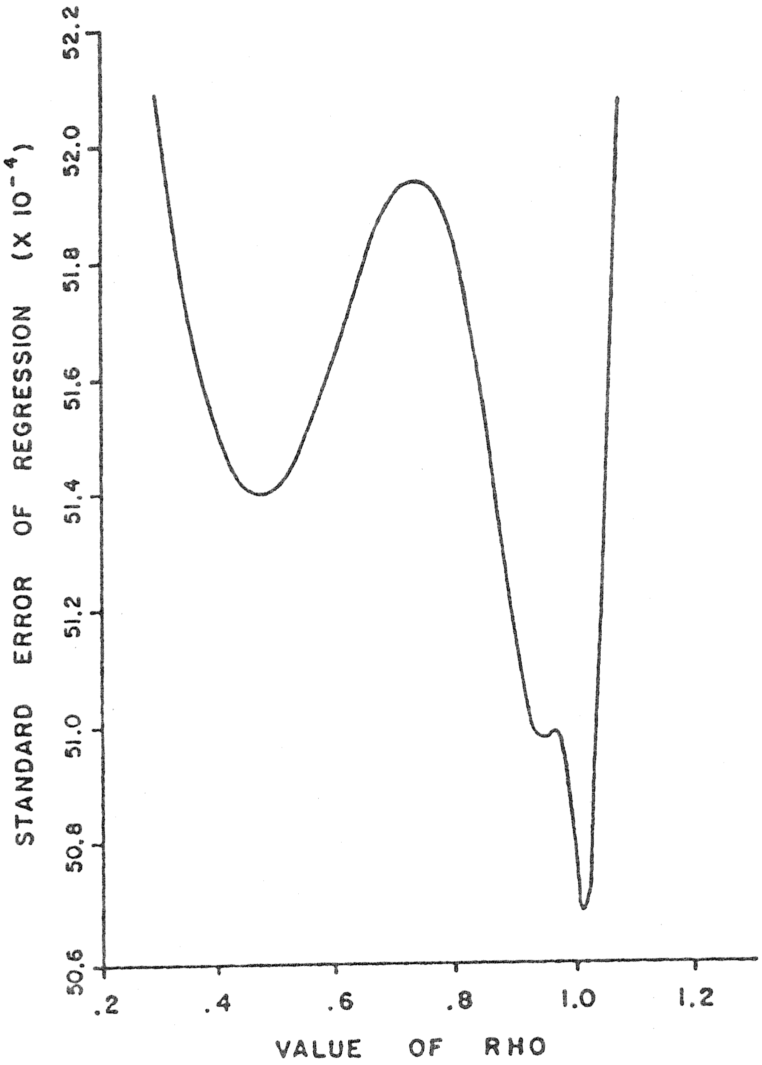

FIGURE 1

III. CONCLUSION

As can be seen from Table 1, the issue about the unimodality of the sum of squared residuals of the transformed model (3) is not merely of theoretical import: coefficients (or elasticities of demand in this case) and their conditional t-statistics differ considerably among the three minimum solutions.

TABLE 1

Hafer-Hein U.S. Quarterly Money Demand Model *

	ρ = .4676		ρ = .9448		ρ = 1.018	
	β	t	β	t	β	t
Constant	-0.10	-1.42	-0.60	-2.50	-1.18	-3.47
Lagged real balances	1.00	26.69	0.62	7.35	0.55	6.55
Commercial paper rate	-0.020	-5.06	-0.013	-3.13	-0.014	-3.34
Bank passbook rate	0.005	0.71	-0.049	-2.55	-0.044	-2.64
Real GNP	0.017	1.26	0.144	3.70	0.250	4.29
Standard error of regression	.005140		.005098		.005068	

* The t-statistics shown are obtained with the usual conditional procedure and do not incorporate the correction suggested by Cooper [3].

Note that the sign of the coefficient of the Commercial bank passbook rate varies across minima; note also that the estimated income elasticity of demand for money is 8 times larger at $\hat{\rho} = 0.94$ than at $\hat{\rho} = 0.46$ and 18 times larger at $\hat{\rho} = 1.01$ than at $\hat{\rho} = 0.46$. These results show the practical importance of making sure that the Cochrane-Orcutt procedure yields the global minimum in the relevant region.

APPENDIX 1

U.S. QUARTERLY DATA

Quarter	M	CNPE	RTD	CPR	P
1955-I	133,500	641.1	1.35	1,613	60,66
1955-II	134,300	650.8	1.39	1,967	60,76
1955-III	134,867	660.3	1.42	2,327	61,18
1955-IV	135,100	667.0	1.47	2,833	61,50
1956-I	135,367	664.1	1.50	3,000	62,03
1956-II	135,933	667.5	1.55	3,263	62,54
1956-III	135,967	667.9	1.60	3,350	63,25
1956-IV	136,600	675.7	1.65	3,630	63,77
1957-I	136,867	680.4	1.99	3,630	64,51
1957-II	136,933	680.9	2.07	3,683	64,77
1957-III	136,967	685.6	2.12	3,953	65,37
1957-IV	136,233	676.7	2.14	3,993	65,64
1958-I	136,067	661.4	2.17	2,817	65,69
1958-II	137,633	668.2	2.20	1,717	65,83
1958-III	139,300	684.4	2.23	2,130	66,21
1958-IV	140,700	702.1	2.26	3,213	66,61
1959-I	142,600	710.7	2.29	3,303	66,98
1959-II	143,800	724.3	2.33	3,603	67,65
1959-III	144,533	718.6	2.38	4,193	67,70
1959-IV	143,633	724.2	2.43	4,760	67,95
1960-I	143,300	740.7	2.48	4,687	68,42
1960-II	142,767	736.9	2.54	4,073	68,55
1960-III	143,933	735.7	2.59	3,373	68,81
1960-IV	144,233	731.9	2.63	3,270	68,94
1961-I	144,833	734.6	2.66	3,013	68,85
1961-II	146,033	744.0	2.68	2,960	69,18
1961-III	146,867	758.7	2.80	2,997	69,48
1961-IV	148,300	775.9	2.90	3,057	69,59
1962-I	149,167	788.1	3.50	3,263	70,17
1962-II	149,833	798.3	3.60	3,203	70,41
1962-III	149,533	804.3	3.60	3,333	70,60
1962-IV	150,433	805.8	3.60	3,263	71,03
1963-I	151,833	812.5	3.60	3,310	71,32
1963-II	153,333	823.7	3.60	3,317	71,37
1963-III	154,800	838.8	3.70	3,697	71,58
1963-IV	156,400	846.9	3.70	3,907	72,07
1964-I	157,333	861.1	3.70	3,950	72,28
1964-II	158,833	872.0	3.70	3,933	72,53
1964-III	161,433	880.5	3.70	3,910	72,93
1964-IV	163,400	883.9	3.70	4,063	73,38
1965-I	164,500	903.0	3.80	4,300	73,68
1965-II	165,733	916.4	3.80	4,380	74,06
1965-III	167,633	932.3	3.80	4,380	74,56
1965-IV	170,333	952.0	3.90	4,470	74,92
1966-I	173,333	969.6	3.90	4,970	75,68
1966-II	175,633	976.3	3.90	5,427	76,57
1966-III	175,233	985.4	3.90	5,790	77,02
1966-IV	175,500	992.8	3.90	6,000	77,73
1967-I	177,167	994.4	3.90	5,450	78,19
1967-II	179,633	1 001.3	3.90	4,717	78,48
1967-III	183,967	1 013.6	3.90	4,973	79,24
1967-IV	186,700	1 021.5	3.90	5,303	80,15
1968-I	189,167	1 031.4	3.90	5,580	81,18
1968-II	192,667	1 049.4	3.91	6,080	82,12
1968-III	196,633	1 061.8	3.92	5,963	82,88
1968-IV	200,900	1 064.7	3.93	5,963	84,04
1969-I	204,267	1 074.8	3.93	6,657	84,95
1969-II	206,000	1 079.6	3.94	7,540	86,05
1969-III	207,200	1 083.4	3.94	8,487	87,40
1969-IV	208,733	1 077.5	3.95	8,620	88,48
1970-I	210,600	1 073.6	4.17	8,553	89,51
1970-II	213,000	1 074.1	4.42	8,167	90,91
1970-III	215,733	1 082.0	4.43	7,837	91,76
1970-IV	218,767	1 071.4	4.43	6,293	92,99
1971-I	222,700	1 095.3	4.44	4,590	94,40
1971-II	227,767	1 103.3	4.32	5,040	95,73
1971-III	231,633	1 111.0	4.32	5,743	96,53
1971-IV	233,233	1 120.5	4.39	5,067	97,38
1972-I	237,867	1 141.2	4.38	4,060	98,76
1972-II	242,100	1 163.0	4.30	4,577	99,45
1972-III	247,233	1 178.0	4.31	4,933	100,29
1972-IV	252,833	1 202.2	4.32	5,333	101,44
1973-I	258,166	1 229.8	4.33	6,283	102,89
1973-II	261,500	1 231.1	4.42	7,467	104,65
1973-III	264,933	1 236.3	4.70	9,873	106,57
1973-IV	268,467	1 242.6	4.77	8,980	109,05
1974-I	273,400	1 230.2	4.78	8,303	111,28
1974-II	276,233	1 224.5	4.80	10,457	114,34
1974-III	278,967	1 216.9	4.81	11,533	117,52
1974-IV	282,166	1 199.7	4.82	9,050	121,06
1975-I	283,600	1 171.6	4.83	6,563	124,16
1975-II	287,700	1 189.9	4.85	5,920	125,95
1975-III	292,933	1 220.0	4.90	6,667	128,19
1975-IV	295,100	1 227.9	4.90	6,120	130,14
1976-I	298,500	1 255.5	4.91	5,290	131,40
1976-II	303,267	1 268.0	4.92	5,370	132,92
1976-III	306,433	1 276.5	4.91	5,530	134,39
1976-IV	312,133	1 284.0	4.91	4,990	134,28
1977-I	317,900	1 306.7	4.90	4,810	138,27
1977-II	323,900	1 325.5	4.90	5,237	140,86
1977-III	330,500	1 343.9	4.90	5,807	142,53
1977-IV	336,966	1 354.3	4.90	6,593	144,56
1978-I	342,100	1 354.2	4.92	6,797	147,10
1978-II	350,366	1 387.6	4.93	7,200	150,98
1978-III	357,267	1 391.4	4.93	8,083	153,52
1978-IV	361,300	[illegible]	4.93	9,897	156,56

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