# An Empirical Analysis of the Demand for International Liquidity

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An Empirical Analysis of the
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## I. Introduction

The international monetary system has experienced significant changes during the 1970s. The most dramatic of these changes was the movement from a system of pegged exchange rates to one where exchange rates were allowed to float. This paper attempts to analyze the effects of this change on the demand for international liquidity by both private individuals and central banks. The demand by a private individual to hold foreign currencies is typically a transactions demand. That is, the motive for holding reserves is to finance transactions in foreign markets. The analysis of this demand for international liquidity is developed following Baumol's [4] inventory-theoretic approach for cash balances. On the other hand, central banks typically hold stocks of foreign reserves so that they can intervene in foreign currency markets in order to "stabilize" the value of their currency. That is, their demand is a precautionary one. Conceptually, one would expect central banks to hold stocks of foreign reserves for precautionary reasons only before March 1973. Before this date a central bank that was a participant in the Bretton Woods Agreement had to intervene in foreign currency markets if the value of its currency moved outside of a predetermined range. After the Bretton Woods Agreement (and also the Smithsonian Agreement) collapsed, one would not expect central banks to continue to hold foreign reserves (or at least to hold foreign reserves for purposes other than exchange-market intervention) since exchange rates are supposedly market determined. Central banks, however, still maintain their stocks of foreign reserves and in some cases have actually added to them.

Several reasons have been offered to explain this maintenance of stocks of foreign reserves by central banks. The most obvious is that exchange rates are not completely market determined in the new system; instead, central banks use their reserve holdings to manage exchange rates a la Bretton Woods. Second, there may exist factors other than exchange market intervention that motivate a central bank to hold foreign reserves. In particular, a central bank may wish to hold foreign reserves as a way of diversifying its portfolio of assets or to serve as a basis for foreign borrowing. Third, some central banks may desire to return to a system of pegged exchange rates and consequently, hold a stock of foreign reserves appropriate for such a system.1/ Regardless of their reasons, even if a completely clean floating exchange rate system existed, central banks would certainly continue to hold assets, some of which may have been termed reserves in a pegged exchange rate world.

This paper investigates the above issue and several other issues that influence the demand for international liquidity. In particular, Section II contains a brief description of the determinants of reserve demand by both private individuals and central banks. Section III contains an empirical analysis of the demand for international reserves by private individuals. The distinguishing feature of this section is that a measure of opportunity cost is explicitly included in the estimation of the demand equation, correcting the positive bias due to specification error that has existed in antecedent work. Central banks' demand for foreign reserves is studied in Section IV. Again, a measure of the opportunity cost of holding reserves is explicitly included. Both short-run and long-run demand functions are estimated. The issue of structural change is also investigated. Specifically, the questions investigated are if and when the movement from pegged to floating exchange rates affected central banks' demand for foreign reserves. In Section V a simple asset choice model is developed based on the assumption that central banks act as if they are utility-maximizers in determining the asset composition of their portfolio. The question of structural changes caused by the movement from pegged to floating exchange rates is also investigated in this framework. Finally, some concluding remarks are presented in Section VI.

## II. Determinants of Reserve Demand

The holding of international reserves by private individuals in order to finance transactions in international markets is analogous to an individual's holding a stock of domestic currency in order to finance transactions in domestic markets. Consequently, the conceptual analysis of the transactions demand by private individuals for international reserves can be based on that of the transactions demand for money - the inventory-theoretic approach.[2/] In particular, stocks of international reserves are demanded to bridge the gaps in time between the receipts and the expenditures of individuals. The size of the stocks held depend fundamentally upon two variables - the value of international transactions and the opportunity cost of holding stocks of foreign reserves. That is:

(2.1) $E_{it} = f(T_{it}, r_{it})$

where $E_{it}$ = private individuals' stocks of foreign reserves in time period t in country i,

$T_{it}$ = the value of i's international transactions in t, and

$r_{it}$ = the opportunity cost of holding reserves during t in i.

An increase (decrease) in the value of international transactions, *ceteris paribus*, should increase (decrease) the transactions demand for international reserves. In fact, Baumol [4] has found that the elasticity of demand for cash balances with respect to the *number* of transactions (with the size of each transaction held constant) is 0.5. (This result is often termed the "square-root law.") Baltensperger [2] has shown that if the number of transactions is held constant, the elasticity of demand for cash balances with respect to the size of transactions is one. Since aggregate data over time include changes in both the size and the number of transactions, we hypothesize that the transactions elasticity of private demand for reserves should be between 0.5 and one. Obviously, an increase (decrease) in the opportunity cost of holding reserves, ceteris paribus, should decrease (increase) the transactions demand for reserves.

The demand by central banks to hold stocks of international reserves is typically analyzed within the framework of exchange market intervention. That is, central banks demand reserves so that they can intervene in foreign exchange markets in order to keep the value of their currency within a predetermined range. (In fact, this was a commitment by participants in the Bretton Woods Agreement.)[3/] Recent studies have enumerated three major determinants of reserve demand by central banks (motivated by the desire to intervene) - the variability of international receipts and payments, the propensity to import, and a scale variable measuring the size of international transactions (usually the value of imports).[4/] The variability of receipts and payments indicates the likelihood of external disequilibrium occurring which would require a central bank to intervene in foreign currency markets. The larger the variability of a country's receipts and payments, the more susceptible is that country to external disequilibrium; consequently, the larger is the optimal stock of reserves desired for precautionary purposes.

There are two possible rationales for including the propensity to import as a determinant of reserve demand. First, the average propensity to import can be considered a measurement of the degree of openness of a country, thus indicating the degree to which its economy is vulnerable to external disequilibrium.[5/] A second, alternative, rationale stems from the Keynesian model of an open economy. Specifically, an external disequilibrium can be corrected (without changing the exchange rate) by a change in output proportional to the foreign trade multiplier (which is inversely related to the marginal propensity to import). This cost of output adjustment can be avoided if central banks use their stocks of reserves to finance (or sterilize) the disequlibrium. Since the output cost of adjustment is directly related to the size of the foreign trade multiplier and the multiplier is inversely related to the marginal propensity to import, then the output cost of not having adequate reserves (and therefore, the demand for reserves) must be inversely related to the marginal propensity to import. Because data on the marginal propensity to import are usually difficult to obtain, most empirical studies have substituted the average propensity as a proxy. However, when the average propensity to import is used as both a proxy for the marginal propensity and a measure of openness, the sign of its effect on reserve demand is ambiguous.

Most authors have recognized that there exists an opportunity cost of holding reserves. Nevertheless, only a few have explicitly included some measure of the opportunity cost in their estimating equation and those that have included it have not been very successful. It has been hypothesized that the overall poor performance of this variable is caused by its strong positive relationship with the supply of reserves. That is, the higher the opportunity cost of holding reserves, the higher also is the domestic rate of return on financial capital which motivates capital inflows and consequently, increases the supply of reserves. Even though this argument lessens our expectations concerning the success of including a measure of opportunity cost, we do so anyway primarily because failure to include opportunity cost biases coefficient estimates.[6/]

## III. Private Demand for Foreign Reserves

The conceptual development above has depicted the private demand for foreign reserves as being analogous to the transactions demand for domestic currency. That is, following Baumol's model [4], the transactions demand for foreign reserves should be a function of the number (and/or size) of transactions and of the opportunity cost of holding reserves. Officer [32] attempted to test Baumol's result - the square-root law - for twenty-one countries. His findings were quite startling: of the twenty-one estimated transactions elasticities, the hypothesis that the estimate lay between 0.5 and one was rejected eighteen times. In fact, only one estimate of the transactions elasticity was less than one. However, Officer did not include a measure of opportunity cost which caused his estimates to be biased. Assuming that the opportunity cost of holding reserves is negatively correlated with the measure of transactions, it can be shown that the omission of opportunity cost imposes a positive bias on the estimated transactions elasticity. That is, Officer's estimates of the transactions elasticity should overstate the true value.

With this point in mind, we set out to estimate the transactions demand for foreign reserves by private individuals explicitly incorporating a measure of opportunity cost. In particular, the demand function estimated is as follows:

(3.1) $\ln E_{it} = a_0 + a_1 \ln M_{it} + a_2 \ln r_{it} + u_{it}$

where $E_{it}$ = the foreign reserve holdings of commerical banks in country i at the end of quarter t,

$M_{it}$ = the imports of country i during quarter t (proxy for transactions),

$r_{it}$ = one of three different specifications of i's opportunity cost of holding foreign reserves during quarter t,

$u_{it} = \rho_i u_{it-1} + \rho_2 u_{it-2} + \epsilon_{it}$
and
$\epsilon_{it} \sim N(0, \sigma^2)$. 7/

It is implicity assumed in the construction of equation (3.1) that the supply of foreign reserves is elastic enough to accommodate demand. (Or alternatively, this equation may be viewed as a long-run demand function.) The three specifications of the opportunity cost of holding reserves are (1) the yield on long-term securities (ILT), (2) the yield on short-term securities (IST), and (3) the ratio of ILT to IST. The rationale for the use of ILT is that the opportunity cost of holding foreign reserves is represented by foregone investment; the greater the marginal productivity of investment, the greater is the cost of tying up resources in the form of reserves. Assuming equilibrium in capital markets, the yield on long-term securities can be considered a proxy for the rate of return on capital and also, for the marginal productivity of investment.8/ The yield on short-term securities is employed simply to represent a relatively liquid, interest-earning alternative to holding foreign reserves. The use of the ratio of ILT to IST introduces the concept of yield differentials. Specifically, it is conceivable that individuals may invest some of their stocks of foreign reserves in relatively liquid, short-term paper. Consequently, the opportunity cost of holding these reserves is the difference between long-term and short-term yields. Of these three specifications only the use of IST yielded results that could be considered meaningful (i.e., a negative estimate of $a_2$). As a result, the remainder of the analysis concentrates on this measure of opportunity cost.

Table 3.1 contains the results for the entire sample IST as the opportunity cost of holding reserves; Table 3.2 contains the individual country results. In general, the sample period is from 1960/I to 1979/IV. (Due to the unavailability of data, sample periods for some countries begin as late as 1968/I). The two most striking features of these results are:

(1) the full sample transactions elasticity and eight of the twelve individual country transactions elasticities exceed one (however, only five are significantly greater than one) and (2) the full sample interest elasticity, although negative, is insignificant, and eight of the individual country interest elasticities are either insignificant or have the incorrect sign. These results can be interpreted in at least two ways. They may indicate that individuals'

transactions demand for foreign reserves does exhibit diseconomies of scale and also that the opportunity cost of holding reserves has a negligible influence on the amount of reserves held. Alternatively, these results may imply that the transactions demand for foreign reserves by individuals is too complex to be adequately explained simply by measures of transactions and opportunity cost. In particular, individuals may hold foreign currencies for reasons other than the purchase of foreign goods and services - e.g., foreign securities. One way to include this possibility is to use a measure of transactions that contains capital flows as well as flows of goods and services.[9/] An even more complex alternative is to develop a model of reserve demand which explains reserve holdings, imports, and capital flows simultaneously. Finally, the use of an interest rate as a measure of the opportunity cost of holding reserves implicitly assumes that a domestic, interest-earning asset is the only important alternative to holding reserves. However, another important alternative exists - domestically produced goods and serivces. That is, an individual may choose to hold fewer foreign reserves because he has decided to purchase relatively more domestically produced goods and services (vis-a-vis foreign produced). This possibility again requires the construction of a model within which imports and reserve demand are simultaneously determined. Whatever the interpretation, it is clear from these results that more conceptual work needs to be done in this area.

Table 3.1

Transactions Demand for Foreign Reserves
Full Sample

| Parameter | Estimate Value | Standard Error |
|---|---|---|
| $a_0$ | -3.950* | .287 |
| $a_1$ | 1.489* | .030 |
| $a_2$ | -0.025 | .034 |
| $\rho_1$ | 0.707* | .037 |
| $\rho_2$ | 0.243* | .037 |

$\bar{R}^2 = .82$

$F(3,694) = 1583.48$

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

Table 3.2

Transactions Demand for Foreign Resources

Individual Country Analysis

| Country | $a_0$ | $a_1$ | $a_2$ | $\rho_1$ | $\rho_2$ | $\bar{R}^2$ |
|---|---|---|---|---|---|---|
| Belgium | - 4.320* | 1.536* | 0.027 | .459* | .396* | .80 |
| | (.732) | (.092) | (.062) | (.103) | (.103) | |
| Canada | 1.327* | 0.854* | 0.189** | .348* | .425* | .75 |
| | (.611) | (.080) | (.097) | (.111) | (.111) | |
| Demark | - 4.416* | 1.422* | 0.222* | .228 | .376* | .83 |
| | (1.090) | (.147) | (.091) | (.164) | (.164) | |
| France | - 3.761* | 1.500* | - 0.035 | .614* | .120 | .85 |
| | (.752) | (.097) | (.153) | (.127) | (.127) | |
| West Germany | - 2.394* | 1.306* | - 0.129* | .758* | .104 | .73 |
| | (.818) | (.091) | (.070) | (.112) | (.111) | |
| Italy | 0.422 | 1.214* | - 0.817* | .170 | .120 | .47 |
| | (1.695) | (.232) | (.287) | (.165) | (.165) | |
| Japan | 1.937* | 0.821* | - 0.155** | .512* | .133 | .87 |
| | (.462) | (.047) | (.088) | (.143) | (.143) | |
| Netherlands | - 3.849* | 1.510* | - 0.103 | .198 | .077 | .58 |
| | (.309) | (.038) | (.023) | (.112) | (.112) | |
| Norway | 1.277 | 0.647* | -0.093 | .761* | .127 | .74 |
| | (.783) | (.119) | (.126) | (.174) | (.174) | |
| Sweden | 3.644* | 0.582* | - 0.117 | .511* | .023 | .49 |
| | (1.116) | (.139) | (.127) | (.213) | (.213) | |
| United Kingdom | - 2.065* | 1.150* | 0.930* | .999* | .093 | .69 |
| | (1.039) | (.130) | (.201) | (.111) | (.111) | |
| United States | -0.066 | 1.067* | -0.093 | .761* | .127 | .74 |
| | (.694) | (.077) | (.068) | (.111) | (.111) | |

* Significantly different from zero (asymptotically) at the 5% level.
** Significantly different from zero (asymptotically) at the 10% level.
Asymptotic standard errors in parentheses.

## IV. Central Bank Reserve Demand for Exchange Market Intervention

The model of central banks' demand for foreign reserves presented in Section II is the result of theoretical work based on the assumption that central banks intervene in foreign exchange markets in order to keep the value of their currencies within certain ranges.[10/] Frenkel [14] and Heller and Khan [21] have each estimated a similar model except that neither included a measure of the opportunity cost of holding reserves. We correct this omission by estimating the following equation:

$$(4.1)\quad \ln R_{it} = b_0 + b_1 \ln M_{it} + b_2 \ln m_{it} + b_3 \ln \sigma_{it} + b_4 \ln r_{it} + u_{it}$$

where $R_{it}$ = the sum of counting i's holdings of gold, convertible foreign exchange, SDRs, and reserve position in the IMF at the end of time period t,

$M_{it}$ = imports of i during t,

$m_{it}$ = i's average propensity to import during t ( $\frac{M_{it}}{GNP_{it}}$ ),

$\sigma_{it}$ = the trend adjusted variance of i's stock of foreign reserves in t,

$r_{it}$ = i's opportunity cost of holding foreign reserves during t, and

$u_{it} = \rho\, u_{it-1} + \epsilon_{it}$

$\epsilon_{it} \sim N(0,\sigma^2)$.

The construction of equation (4.1) implicitly assumes that the supply of foreign reserves accommodates the demand. This assumption then implies that there exists sufficient time for this accommodation to occur. Consequently, we may interpret equation (4.1) as a long-run demand function. The measure of opportunity cost used in this analysis is the ratio of the discount rate in each country to the three-month U.S. Treasury bill rate. The discount rate represents a measure of the foregone earnings of central banks as a result of holding assets in the form of foreign reserves. The three-month U.S. Treasury bill rate is a measure of the income earned from invested foreign reserves. The rational for this is that most central banks hold their foreign reserves in the form of U.S. dollars on deposit in U.S. banks. When these reserves are invested, it is generally in some short-term asset in order to maintain a relatively high degree of liquidity. Therefore, an appropriate yield on invested foreign reserves is a short-term interest rate in the U.S. (e.g., the three-month Treasury bill rate).[11/] The variable $\sigma_{it}$ is the same proxy for the variability of receipts and expenditures that Frenkel [14] has constructed.

Equation (4.1) is estimated for a sample consisting of thirteen countries over the time period from 1964/I to 1979/IV. The countries are Austria, Belgium, Canada, Denmark, France, West Germany, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, and United Kingdom. The United States is not included because it is considered to be the primary supplier of foreign reserves. The data set consists of a cross section of time series. The assumption implicit in the construction of equation (4.1) is that parameter estimates across cross sections (i.e., countries) are the same. If this assumption is incorrect, equation (4.1) is misspecified. The conclusion of the pooling tests conducted is that the slope coefficients are not significantly different across cross sections, but the constant terms are. Consequently, an analysis of covariance model is the correct one for this pooled sample. Changing equation (4.1) to an analysis of covariance model simply entails substituting a dummy variable for each country in the cross section for the constant term as follows:

$$(4.2)\quad \ln R_{it} = \sum_{j=1}^{13} b_{oj} D_{ij} + b_1 \ln M_{it} + b_2 \ln m_{it} + b_3 \ln \sigma_{it} + b_4 \ln r_{it} + u_{it}$$

$$\text{where } D_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

and all other variables are defined above. The dummy variables capture the country-specific variation present in the data.

The results obtained from the estimation of equation (4.2) are reported in Table 4.1. To conserve space the estimates of the thirteen dummy variable coefficients are not reported; these estimates range from 1.849 to 3.200 and are all statistically significant at the 5% level. Of the remaining coefficient estimates, all are significant except for that of m (the average propensity to import) and have the expected sign. That $b_2$ is insignificant should not be surprising since, as discussed above, it is a proxy variable for two determinants of reserve demand that have opposite-signed effects. Two additional points are of particular interest. First, central banks' demand for foreign reserves appears to exhibit significant economies of scale: a one percent increase in imports causes foreign reserve demand to increase by .398 percent.12/ Second, the estimated coefficient of our measure of the opportunity cost of holding reserves has the expected sign (negative) and is statistically significant. In fact, in an analysis of individual countries, the estimated coefficient of the opportunity cost of holding reserves is negative for eleven of the thirteen countries. (Of these eleven, seven are statistically significant.) Even though far from conclusive, these results are certainly better than those in antecedent work that included a measure of opportunity cost as an explanatory variable and indicate that an interest rate differential is probably a better measure of the opportunity cost of holding reserves than is an interest rate.13/

The movement to a system of floating exchange rates occurred within the sample period. It is likely that this change has altered the structure of central banks' demand for foreign reserves. Quandt [37] has developed a method for detecting when (or if) a break in the structure has occurred and also has devised a test for the statistical significance of this break. The method for detecting the point of structural change is to find the value of t which maximizes

Table 4.1

Central Bank Foreign Reserve Demand

1964/I - 1979/IV

| Parameter | Estimate | Standard Error |
|---|---|---|
| $b_1$ | .398* | .027 |
| $b_2$ | -.002 | .010 |
| $b_3$ | .178* | .012 |
| $b_4$ | -.105* | .029 |
| $\rho$ | .858* | .018 |

NT = 832

SER = .121

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

the following likelihood function:

(4.3) $L(t) = -NT \ln \sqrt{2\pi} - NT \ln \hat{\sigma}_1 - N(T-t) \ln \hat{\sigma}_2 - \frac{NT}{2}$

where $N$ = the number of countries,

$T$ = the total number of quarters,

$\hat{\sigma}_1$ = the standard error of estimation the over first t quarters, and

$\hat{\sigma}_2$ = the standard error of estimation over second T-t quarters.

In our analysis, the highest value of L(t) occurs at 1973/II. Since the Smithsonian Agreement was terminated in March 1973, our analysis shows that central banks reacted very quickly in altering their demand for foreign reserves.14/ To test whether or not this structural change is statistically significant, we employ Quandt's likelihood ratio test:

(4.4) $$\lambda = \frac{\hat{\sigma}_1^{Nt^*} \hat{\sigma}_2^{N(T-t^*)}}{\hat{\sigma}^{NT}}$$

where $t^*$ = the quarter which maximizes L(t),

$\hat{\sigma}$ = the standard error of the estimation over the entire sample of T quarters, and

all other variables are defined above. According to the null hypothesis that no structural change occurred, $-2 \ln \lambda$ is distributed as a Chi square random variable with twenty degrees of freedom. In this case the calculated value of $-2 \ln \lambda$ is 118.606 which easily exceeds 37.566 - the critical value of $\chi^2$ (20) at the 99 percent confidence level.15/ The interpretation of this result is that it is inappropriate to pool data for the time period from 1964/I to 1973/II with that from 1973/III to 1979/IV.

Table 4.2

Central Bank Foreign Reserve Demand

| Time Period | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $\rho$ | SER |
|---|---|---|---|---|---|---|
| 1964/I - 1973/II | .484* (.035) | -.011 (.011) | .138* (.014) | -.106* (.039) | .864* (.023) | .110 |
| | Nt = 494 | | | | | |
| 1973/III - 1979/IV | .275* (.051) | .001 (.019) | .227* (.021) | -.118* (.046) | .780* (.034) | .139 |
| | N(T-t) = 338 | | | | | |

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

Standard errors are reported in parentheses.

Table 4.2 contains the results obtained by estimating equation (4.2) for the time periods 1964/I to 1973/II and 1973/III to 1979/IV respectively. The major changes that occur are that $b_1$ is smaller and $b_3$ is larger in the second sample period than they are in the first. This indicates that in the floating exchange rate system the variability of reserve flows increased in importance as a determinant of reserve demand while the level of imports decreased in importance.

Next we turn to analyzing central banks' short-run demand for foreign reserves. This analysis is based on the concept that central banks have a desired stock of foreign reserves. (Explanation of this desired stock was the focus of the above analysis.) Deviations of the actual stock of reserves from the desired stock induces an adjustment process on the part of central banks. Theoretical models developed have defined an optimal trade-off between the stock of reserves held and the speed of adjusting to deviations between desired and actual reserve stocks. Empirical investigations of this topic have typically yielded estimated speeds of adjustment near zero and at times statistically insignificant.[16/] We believe that the following two characteristics of antecedent work are the basis for the lack of success. First, previous work that employed monthly or quarterly data has failed to correct for the presence of autocorrelation - an omission that yields biased and inconsistent parameter estimates. On the other hand, earlier studies that have corrected for autocorrelation have used annual data.[17/] The use of annual data alone presumes a relatively slow speed of adjustment and also, obfuscates the adjustment occurring within the year.

To correct the shortcomings present in each of the above characteristics of previous work, we estimate a partial adjustment model of foreign reserve demand corrected for autocorrelation using quarterly data. We also employ the covariance adaptation of the long-run demand analysis to capture country-specific variation.[18/] Our estimating equation is based on the following structure:

$$(4.5)\quad \ln R_{it} - \ln R_{it-1} = \delta(\ln R^{*}_{it} - \ln R_{it-1})$$

$$(4.6)\quad \ln R^{*}_{it} = \sum_{j=1}^{13} b_{oj} D_{ij} + b_1 \ln M_{it} + b_2 \ln m_{it} + b_3 \ln \sigma_{it} + b_4 \ln r_{it} + u_{it}$$

where $R^{*}_{it}$ = country i's desired stock of foreign reserves in time period t

and all other variables are defined above. Substituting equation (4.6) into (4.5) and solving for $\ln R_{it}$ yields the following estimating equation:

$$(4.7)\quad \ln R_{it} = \sum_{j=1}^{13} c_{oj} D_{ij} + c_1 \ln M_{it} + c_2 \ln m_{it} + c_3 \ln \sigma_{it} + c_4 \ln r_{it} + c_5 \ln R_{it-1} + u_{it}$$

Since the $u_{it}$'s are correlated across time, estimation of equation (4.7) using ordinary least squares (OLS) will yield estimates that are biased in both small and large samples.

In order to obtain consistent and asymptotically efficient estimates of the coefficients in (4.7), we employ an estimation procedure devised by Hatanaka [18]. The fundamental steps of this procedure are: (1) find an instrumental variable estimate of $R_{it-1}$; (2) using this predicted value of $R_{it-1}$, estimate (4.7) using OLS; (3) calculate the first-order autoregressive coefficient ($\rho$) from the residuals of step 2; (4) use the estimated $\rho$ to adjust the variables in (4.7); (5) finally, use OLS to estimate the $\rho$-adjusted version of (4.7) including $\hat{u}_{it-1}$ (the lagged estimated residual from step 2) as an explanatory variable. As proved by Hatanaka, the resulting estimates are consistent and asymptotically efficient.

Table 4.3 contains the results of applying Hatanaka's procedure to equation (4.7) for the time period from 1964/I to 1973/II. This end point was chosen because, as shown above, the movement to a system of floating exchange rates caused a significiant change in the structure of central banks' demand for foreign reserves. As before, the estimated coefficient of the average propensity to import is not statistically significant. Also, even though it has the expected sign, the estimated coefficient of the opportunity cost of holding reserves is not statistically significant. All other estimated coefficients are statistically significant and have the expected sign. Of particular interest is the estimated coefficient of $R_{it-1}$ ($c_5$); using this coefficient, we can calculate the estimated speed of adjustment ($\delta$). In the construction of equation (4.7)

Table 4.3

Central Bank Adjustment to Desired Foreign Reserves (1964/I - 1973/II)

| Parameter | Estimate | Standard Error |
|---|---|---|
| $c_1$ | .184* | .047 |
| $c_2$ | -.009 | .011 |
| $c_3$ | .086* | .015 |
| $c_4$ | -.049 | .037 |
| $c_5$ | .266* | .048 |
| $\rho$ | .858 | |

NT = 494

SER = .099

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

$c_5$ equals $(1-\delta)$. Consequently, to find the estimate of $\delta$, we simply calculate $1-c_5$. In this case $1-c_5$ equals .734 which means that a central bank accumulates 73.4% of the deviation between the desired stock and the actual stock of foreign reserves each quarter. At this rate, 99.5% of the adjustment to a change in the desired stock of reserves will be completed within one year. (This is substantially larger than the 54% annual rate of adjustment found by Bilson and Frenkel using a different method and annual data and the 29% annual rate implied by the dynamics of Heller and Khan's model.) Even though it is impossible to explain precisely why our speed of adjustment estimate is so much larger than those found in previous studies, at least two possible explanations do exist. First, our study is the only study to estimate explicitly a partial adjustment model corrected for first-order autocorrelation using quarterly data. (Bilson and Frenkel and Iyoha [25] use annual data; although Heller and Khan use quarterly data, it is difficult to ascertain whether their model is a partial adjustment model on a long-run demand function corrected for first-order autocorrelation.) Second, we explicitly capture country-specific factors affecting reserve demand through our analysis of covariance model.

## V. Asset Choice of Central Banks

Thus far we have implicitly assumed that central banks hold reserves for the same reasons in a floating exchange rate system as they did in a pegged exchange rate system. This is essentially the conclusion that Frenkel [14] and Heller and Khan [20] draw. That is, the current floating exchange rate system is such a "managed" one that central banks' demand for reserves has not been significantly affected by the change from a pegged to a floating exchange rate system. This is a possible conclusion, but others could be drawn. (Some reasons for holding reserves other than exchange rate stabilization were enumerated above.) It is impossible to identify whether or not central banks are holding reserves for the same reasons before and after the movement to floating exchange rates from the model employed by Frenkel and by Heller and Khan. The reason for this is that the model used is developed on the assumption that central banks hold reserves to intervene in foreign currency markets. In order to really investigate this issue, a model has to be developed that is not based on the intervention assumption, but alternatively, is couched in an asset choice framework. It is possible that a central bank's optimal portfolio may contain foreign reserves even in an unmanaged system of floating exchange rates.

Assume that a typical central bank chooses its asset portfolio in order to maximize the following utility function:

(5.1) $U(x_{1t}, \ldots, x_{nt}) = \prod_{k=1}^{n} (x_{kt} - \gamma_k)^{\beta_k}$

where $x_{kt}$ = the quantity of asset k in time period t,

$\gamma_k$ = the committed or required level of asset k,

$x_{kt} - \gamma_k$ = the supernumerary quantity of asset k in t,

$\beta_k$ = the increase in the supernumerary quantity of asset k per dollar increase in supernumerary funds (i.e., k's share of the uncommitted portfolio).

This utility function possesses two properties that make it extremely applicable to this analysis. First, each asset's share appears as a parameter of the demand function derived from equation (5.1) and hence can be readily estimated. Second, this utility function explicitly represents the fact that, in the short run, some portion of each asset is committed, i.e., not under the short-run discretion of central bank management. These committed portions may arise for various reasons. A minimal value of each asset may be desired in order to insure a diversified portfolio. Central banks may specify a minimal value for their holdings of government securities so that they can adequately control this money supply. Likewise, they may require a minimal amount of foreign reserves in order to "stabilize" its foreign exchange rate or to prevent speculative capital flows. Whatever their reasons, it seems logical to consider part of a central bank's assets committed in the short run. Recognizing this, the utility function used here specifies utility as a function only of the supernumerary (or uncommitted) assets within the central bank's portfolio.

To derive the central bank's demand for each asset, equation (5.1) must be maximized subject to the following constraint:

$$(5.2) \qquad TA_t = \sum_{k=1}^{n} v_{kt}\, x_{kt}$$

where $v_{kt} = \frac{1}{1 + r_{kt}}$,

$r_{kt}$ = the return on asset k in time period t,

$TA_t$ = the total amount to be allocated among assets in the portfolio in t.

The resulting demand equation for asset k is as follows:

$$(5.3) \qquad x_{kt} = \gamma_k + \frac{\beta_k}{v_{kt}} \left[ TA_t - \sum_{j=1}^{n} \gamma_j\, v_{jt} \right] \quad k=1,\ldots, n$$

To estimate the demand equations, it was assumed that normally distributed random errors enter additively with zero mean and constant variance. One of the characteristics of introducing the error terms in this manner is that the error terms (summed across all equations in the system) must add to zero if the system is to be consistent. This restriction on the sum of the error terms has at least three important implications. First, since the errors are correlated across equations, single equation (or limited information) estimating techniques are not appropriate. Efficient estimation will require the use of a system (or full information) technique. Second, because the errors are linearly dependent, the covariance matrix of the entire system is singular. Consequently, a full information technique cannot be used on all n equations at once because the inversion of this covariance matrix is required during the estimation. It has been shown by Pollak and Wales [36] and others that if any equation is omitted, full information maximum likelihood can be used to estimate the parameters in the remaining n-1 equations and the estimates will not depend on which equation is omitted. Third, if any type of adjustment for autocorrelation is used, extreme care must be exhibited so that the sum of the errors remains equal to zero.

A dynamic specification is employed to give the model more flexibility. In particular, this specification allows the committed parameters ($\gamma_k$) to vary over time. Also, this procedure enables us to adjust for first-order autocorrelation in a manner that does not affect the restriction placed on the sum of the error terms.[19/] This dynamic feature is incorporated into the model by assuming that the committed level of each asset is a function of the total holding of that asset in the previous time period as follows:

(5.4) $$\gamma_{kt} = \theta_k x_{kt-1}$$

with $0 \leq \theta_k \leq 1$ for all k. The parameter $\theta_k$ reflects the proportional relationship between the committed level of an asset in the current period to the total holding of that asset in the previous period. The closer $\theta_k$ is to one, the larger the percentage of asset k which is committed.[20/]

The dynamic specification of the model is estimated using a full information maximum likelihood algorithm. The sample consists of the assets of the central banks of France, Japan, and West Germany aggregated into three categories - foreign reserves (FR), claims on government (GS), and claims on commerical banks (CBL).[21/] The interest rates used to specify the returns on these asset groups are the three-month Treasury bill U.S. rate (for foreign reserves), government bond yield (for claims on government), and the discount rate (for claims on commerical banks). The use of the U.S. rate as a measure of the return on a foreign central bank's reserve holdings is based on the assumption that most foreign reserves held by foreign central banks are in the form of U.S. dollars on deposit in U.S banks (primarily New York city banks). When these reserves are invested, it is usually in short-term paper in order to maintain a relatively high degree of liquidity. Consequently, the three-month Treasury bill U.S. rate appears to be an appropriate measure of the yield on foreign reserves.[22/]

The results obtained by estimating the system represented by equation (5.3) are reported in Table 5.1. The data consist of quarterly observations for the asset groups

Table 5.1

Estimated Parameters of Asset Choice Model
1968/I - 1979/IV

| Parameter | FR | GS | CBL |
|---|---|---|---|
| $\beta$ | .184* | .394* | .422* |
| | (.043) | (.057) | (.068) |
| $\theta$ | .902* | .857* | .747* |
| | (.029) | (.044) | (.088) |
| $\bar{R}^2$ | .976 | .964 | .938 |

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

Asymptotic standard errors are reported in parentheses.

and countries enumerated above for the time period 1968/I to 1979/IV. All parameter estimates are statistically significant (asymptotically) and lie within conceptually acceptable ranges. The $\bar{R}^2$ values are generally high which is not surprising since the data are time series. These values are reported primarily to indicate that the model did explain a relatively large percentage of each dependent variable. However, since the estimation procedure involves a system of equations and a least squares criterion is not used, $\bar{R}^2$ cannot be interpreted in the same way as it is in single equation, least squares estimation procedures.

The estimates of the asset share parameter ($\beta$) indicate that foreign reserves compose the smallest share of central banks supernumerary (discretionary) assets. This fact is not surprising when one considers the opportunity cost of holding supernumerary foreign reserves vis-a-vis the other two asset groups. Also, the estimates of $\theta$ indicate that a greater portion of foreign reserves than other assets is committed. That 90% of foreign reserve holdings is committed does lend credence to the conclusion that central banks are holding reserves today primarily for purposes of intervention in foreign currency markets. To investigate this possibility further we must analyze commitment parameters before and after the movement to floating exchange rates.

In Section IV we found that even though the movement to a system of floating exchange rates occurred in March 1973, the demand equation did not reflect this change until the second quarter of 1973. Consequently, we will choose this same point for the analysis of the asset choice model. Tables 5.2 and 5.3 contain the results of estimating the system represented by equation (5.3) for the time periods 1968/I to 1973/II and 1973/III to 1979/IV respectively. All parameter estimates are statistically significant and within a conceptually acceptable range of values. One readily notices the differences in the parameter estimates for the two time periods, especially for the foreign reserves parameters. In a system of pegged exchange rates, central banks committed 98% of their foreign reserve holdings (presumably to intervene in foreign exchange markets); in a floating exchange rate system only 66% of their foreign reserve holdings are committed. Also, in a pegged system central banks chose to hold 16% of their supernumerary (discretionary) portfolio in the form of foreign reserves while in a floating system they choose to hold 35% of their supernumerary (discretionary) portfolio in this form. Consequently, it appears that the movement to a floating exchange rate system has induced central banks to commit less of their foreign reserves to foreign exchange market intervention and also to hold a larger share of their discretionary portfolio in this form.

Table 5.2

Estimated Parameters of Asset Choice Model
1968/I - 1973/II

| Parameter | FR | GS | CBL |
|---|---|---|---|
| $\beta$ | .156*<br>(.053) | .503*<br>(.046) | .341*<br>(.052) |
| $\theta$ | .977*<br>(.032) | .502*<br>(.144) | .743*<br>(.115) |
| $\bar{R}^2$ | .970 | .930 | .930 |

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

Asymptotic standard errors reported in parentheses.

Table 5.3

Estimated Parameters of Asset Choice Model
1973/III - 1979/IV

| Parameter | FR | GS | CBL |
|---|---|---|---|
| $\beta$ | .353* | .245* | .402* |
| | (.027) | (.041) | (.042) |
| $\theta$ | .656* | .855* | .613* |
| | (.075) | (.050) | (.093) |
| $\bar{R}^2$ | .974 | .963 | .938 |

* Significantly different from zero (asymptotically) at the 5% level using a two-tailed test.

Asymptotic standard errors reported in parentheses.

To test whether or not the parameter estimates in the two time periods are significantly different in a statistical sense, a multivariate likelihood ratio test is developed based on the assumption that the error terms in the system of equations represented by equation (5.3) are normal random variates. Under the null hypothesis that the parameter estimates in both time periods are equal, the likelihood ratio statistic ($\lambda$) is

$$(5.5) \qquad \lambda = \frac{\left|\hat{\Sigma}_1\right|^{\frac{Nt}{2}} \left|\hat{\Sigma}_2\right|^{\frac{N(T-t)}{2}}}{\left|\hat{\Sigma}\right|^{\frac{NT}{2}}}$$

where $N$ = total number of countries,

$T$ = total number of quarters,

$t$ = the number of the quarter within which the hypothesized structural change occurred,

$\hat{\Sigma}_1$ = the covariance matrix of the estimation over the first t quarters,

$\hat{\Sigma}_2$ = the covariance matrix of the estimation over the second T-t quarters, and

$\hat{\Sigma}$ = the covariance matrix of the estimation over the complete sample of T quarters.

An attractive feature of the likelihood ratio test is that under certain general conditions, $-2 \ln \lambda$ will be asymptotically distributed as a Chi square ($\chi^2$) with p-q degrees of freedom, where p is the dimensionality of the denominator and q is the dimensionality of the numerator of $\lambda$. (In this case, p-q equals 9.) The calculated value of $-2 \ln \lambda$ is 172.46 which exceeds 27.877 - the critical value of $\chi^2(9)$ at the 99% level. This result indicates that the null hypothesis must be rejected or in other words, the parameter estimates in the two sample periods are significantly different.

## VI. Conclusions

The purpose of this paper has been to identify the determinants of the demand for foreign reserves by both private individuals and central banks. The primary focus of the analysis of private reserve demand was to investigate the effect of opportunity cost on reserve demand. The findings here were inconclusive implying that the opportunity cost of holding reserves had little impact on private demand for reserves; these results also reinforced Officer's conclusion that the private demand for reserves exhibits substantial diseconomies of scale. Since these results are extremely counter-intuitive, this interpretation may be too superficial, and a reassessment of the conceptual framework should be undertaken in future work.

The analysis of central bank demand for foreign reserves yielded at least two enlightening conclusions. First, central banks appear to react very rapidly to changes in their economic environment. Several of our results support this observation. The change in the structure of reserve demand appeared in the very next quarter after the termination of the Smithsonian Agreement. Central banks' reserve demand was also found to be sensitive to the costs of holding reserves - a result that signifies a recognition of alternatives. Most importantly, central banks almost completely adjust their actual stocks of reserves to a change in the desired stock within one year.

Second, Heller and Khan found that the central banks of industrial countries are holding larger stocks of foreign reserves during the floating period than is implied by their behavior during the pegged exchange rate period. They conjecture that greater uncertainty in the floating period has motivated this change. However, their projections of reserve demand for the floating period were generated from a model based on the assumption that central banks hold reserves solely for purposes of exchange-market intervention. We reject this assumption and develop an asset choice model of central bank reserve demand based on the desire (explicit or implicit) of central banks to hold a utility-maximizing portfolio of assets, one of which is a stock of foreign reserves.[23/] Using this model we found that the interventionist model used by Frenkel and by Heller and Khan predicted quite the pegged exchange rate period primarily because central banks held 98% of their reserves for purposes of intervention. Alternatively, these models underestimate the reserve holdings during the floating period because central banks are currently holding only 66% of their reserves for purposes of intervention in foreign exchange markets. The remainder is being held because it represents the foreign reserve component of a utility-maximizing portfolio. This result is an extremely important one from the point of view of the adequacy of international liquidity. It signifies that the adequacy of international liquidity is not solely dependent upon balance of payments deficits of reserve-currency countries nor upon the creation of an international money (e.g., SDRs). Instead, the availability of international liquidity is, in general, a portfolio decision made by central banks.

## FOOTNOTES

1/ See 1974 IMF Annual Report [24].

2/ See, among others, Baltensperger [2], Baumol [4], and Tobin [41].

3/ It's not clear that these same motives exist in a period of floating exchange rates. This is also an empirical issue and will be investigated in Section IV.

4/ See Clark [10], Frenkel [15], Hipple [22], Iyoha [25], and Kelly [26].

5/ This argument is clearly presented by McKinnon [30].

6/ In particular, if imports and interest rates are negatively correlated (a reasonable assumption), then the coefficient estimates have a positive bias. This bias is especially important when investigating the issue of economies of scale in reserve demand.

7/ The data are pooled cross section and time series. Pooling tests were performed to insure the validity of this error structure.

8/ Of course, this argument overlooks the inflationary expectation premium included in long-term yields.

9/ Officer [32] attempted to construct a more comprehensive measure of international transactions. His results differed very little from ours.

10/ See Claassen [8], Clark [10], and Kelly [26].

11/ See Hipple [23] for a reinforcement of this argument. With the development of the Eurocurrency market and the declining role of the U.S. dollar as a reserve currency, it can be argued that some Eurocurrency market rate may be a more appropriate measure of the yield on foreign reserve holdings. This possibility will be investigated in furture work.

12/ It has been suggested that the measure of variability also captures scale effects and consequently, the estimate of $b_1$ understates the "true" scale effect. A more appropriate measure may be $b_1 + b_3$. Even when the measure is used, the demand function exhibits economies of scale.

13/ A long-term interest rate was also employed as a measure of opportunity cost. The use of this measure was not as successful as was the use of the ratio of the discount rate to the three-month U.S. Treasury bill rate.

14/ Also using quarterly data, Heller and Khan [21] found that the structural change occurred at the end of 1973/IV. The only obvious explanation for our different finding is that our model includes a measure of opportunity cost and their's does not.

15/ The null hypothesis that the coefficient estimates are the same in both sample periods was also rejected when a Chow test was performed.

16/ Using a different method, Bilson and Frenkel [5] have calculated a speed of adjustment that is much larger than those in previous studies.

17/ Heller and Khan used quarterly data but confused the correction for first-order autocorrelation with a dynamic model. Consequently, we must interpret their work as either (1) a static model corrected for autocorrelation or (2) a dynamic model not corrected for autocorrelation.

18/ Bilson and Frenkel have argued that failing to account explicitly for country-specific factors may have led to positively biased estimates of the speed of adjustment in previous work employing a pooled data set.

19/ See Pollack and Wales [36] for a discussion of how this specification corrects first-order autocorrelation.

20/ If one questions the idea that central banks organize their portfolios (or at least, part of their portfolios) in a way that maximizes their utility, this feature of the model allows us to test the hypothesis that they do. Specifically, if $\theta_k$ is not significantly different from one in statistical terms (for all k) then there are no uncommitted assets to allocate within the portfolio. Consequently, the hypothesis that central banks behave as utility maximizers would be rejected.

21/ These countries were chosen primarily because they reported the data in a form consistent with our aggregation yet were heterogeneous enough to provide a preliminary overview of the behavior of central banks in developed nations. The sample will be enlarged and asset groups will be disaggregated in subsequent work.

22/ This argument is identical to the one made in explaining the measure of opportunity cost in equation (4.1).

23/ This model does not preclude the holding of foreign reserves for interventionist reasons; however, this is just one of several reasons for holding reserve stocks while it is the sole reason in the model used by Frenkel and by Heller and Khan.

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