



**ECONOMIC RESEARCH**  
FEDERAL RESERVE BANK OF ST. LOUIS  
WORKING PAPER SERIES

## An Analysis and Development of the Brunner-Meltzer Non-linear Money Supply Hypothesis

<b>Authors</b>	Albert E. Burger
<b>Working Paper Number</b>	1969-007A
<b>Creation Date</b>	May 1969
<b>Citable Link</b>	<a href="https://doi.org/10.20955/wp.1969.007">https://doi.org/10.20955/wp.1969.007</a>
<b>Suggested Citation</b>	Burger, A.E., 1969; An Analysis and Development of the Brunner-Meltzer Non-linear Money Supply Hypothesis, Federal Reserve Bank of St. Louis Working Paper 1969-007. URL <a href="https://doi.org/10.20955/wp.1969.007">https://doi.org/10.20955/wp.1969.007</a>

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Federal Reserve System, the Board of Governors, or the regional Federal Reserve Banks. Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment.

Working Paper No. 7

An Analysis and Development of the Brunner-Meltzer  
Non-linear Money Supply Hypothesis

Albert E. Burger

May 15, 1969

This working paper is circulated for discussion and comment.  
Quotations and references only by permission of the author.



## TABLE OF CONTENTS

INTRODUCTION . . . . .	1
SECTION A: Basic Relations Involved in the Brunner-Meltzer Hypothesis . . . . .	9
SECTION B: Derivation of Expressions for the Money Stock, Money Defined to Include Time Deposits, and Bank Credit . . . . .	20
SECTION C: Equilibrium in the Bank Credit Market . . . . .	29
SECTION D: The Credit Market Interest Rate $i$ . . . . .	33
SECTION E: Dependence of the Money and Bank Credit Multipliers on $i$ . . . . .	43
SECTION F: Effects of Changes in the Policy Parameters . . . . .	63
SECTION G: Historical Examples . . . . .	91
SECTION H: Implications of the Brunner- Meltzer Hypothesis for the Choice of Indicators of Monetary Policy . . . . .	100
APPENDIX I . . . . .	105
APPENDIX II . . . . .	107
BIBLIOGRAPHY . . . . .	109

A SUMMARY OF THE BRUNNER-MELTZER  
NON-LINEAR MONEY SUPPLY HYPOTHESIS

Introduction

What is the purpose of devoting time, energy and intellectual talent to developing a money supply hypothesis? First of all, let us consider just what we mean by the expression "an hypothesis." An hypothesis is a carefully formulated attempt to explain some phenomena of the real world. An hypothesis, or mathematical formulation of the hypothesis, is not the "real world". It is only a representation of the real world. Also, of necessity it must abstract from many facets of the real world situation it is attempting to represent, and only include those aspects that are considered crucial to the explanation of the variables it is designed to explain. From a carefully designed hypothesis we can derive logical consequences of the hypothesis which may be confronted with empirical observations. We call such a confrontation of logical consequences of an abstract representation of the real world and empirical observations a "test" of the hypothesis. An hypothesis that has been tested and found repeatedly to have its logically derivable consequences in good agreement with empirical observations, may serve as a valuable aid in our understanding of a segment of the real world.

For purposes of discussion, economic stabilization policies are frequently divided into two broad classes, fiscal policy and monetary policy. The goals, such as the level or rate of change of employment and prices, for both classes of policy are the same. However, there is considerably less agreement as to the meaning of the term "monetary policy" than the meaning of the term "fiscal policy." One view of monetary policy is that it should be central bank actions to control market interest rates to achieve employment and price objectives. An alternative view is that monetary policy should be policy employing the Federal Reserve's control of the stock of money as an instrument for achieving the objectives of general economic stabilization policy.

Practically, the Federal Reserve System by its direct actions does not directly control either market interest rates or the money stock. The policy actions which the central bank may perform are limited to such actions as purchase or sale of Government securities, changes in the legal reserve requirements on member bank demand and time deposits, changes in the discount rate and changes in the Regulation Q ceiling rate on time deposits. If policymaking groups such as the Federal Open Market Committee and Board of Governors are told they should control the level of market interest rates or the money stock, this advice is of little use to them unless they are also advised how the policy actions they may perform will result in the desired levels or rates of change of interest rates or the stock of money. <sup>1/</sup> Individuals, who have advised that

---

<sup>1/</sup> See Albert Burger and Leonall Andersen, "The Development of Explanatory Economic Hypotheses for Monetary Management," Southern Journal of Business (forthcoming).

by expanding the Federal Reserve System's holdings of Government securities the equilibrium level of market interest rates would decrease, have in recent years found to their dismay that the opposite effect occurred. Advisors who have assumed that the money stock is a simple expansion of bank reserves have also on occasion been dismayed by the results of their policy recommendations. In the classic phrase, "there has often been a slip twixt cup and lip," even where the target of policy actions was not a goal such as employment or prices but a more intermediate goal such as money or interest rates.

A few of the minimum requirements that an hypothesis about the determination of the money stock must satisfy, if it is to be useful to policymaking groups such as the Board of Governors and the Federal Open Market Committee, are:

- (1) It must explicitly include the policy variables under the direct control of the monetary authorities.
- (2) It must establish a definite link between these control variables and the monetary aggregates.
- (3) It must take into account the existing institutional framework within which the policymakers must operate, and it must be able to demonstrate the effects of a change in one of these institutional conditions on the impact of changes in policy variables.

In recent years an approach to monetary theory that might be labeled the Portfolio Theory of Monetary Policy has been gaining wide acceptance within the economics profession. In this theoretic

approach, monetary theory is viewed as being part of the broader theory of the portfolio management of economic units. It seeks to explain holdings by economic units of stocks of assets (among which is included money) and the value and yields of these claims.

In this approach, a monetary disturbance such as an injection of base money into the economy system, is viewed as changing the terms on which all existing assets will be held. A policy-induced change in the existing stock of assets leads to a behavioral reaction on the part of economic units as they attempt to adjust their holdings of stocks of all assets (both real and financial) to the amounts desired. The reallocation of assets in the balance sheet of individual economic units -- households, commercial banks, business firms, and government units -- spills over to the real sector via a change in the quantity demand of real assets such as capital goods and consumer goods and thus affects current output and the aggregate price level.

One of the consequences of this approach has been to view the magnitude of the economic quantity the money stock as being determined, both on the supply and demand side, by the behavioral choice and reaction of economic units. Money is viewed as one of many assets that economic units hold. The amount of money balances that economic units demand depends not only on the volume of money transactions, but also on the alternative assets available and the yields available on these assets, the cost of switching between these assets and money, and the information available to economic units. The stock of money balances is no longer viewed as an exog-

enously determined magnitude. It is viewed as a quantity whose magnitude is partly exogenously determined by the policy actions of the central bank and partly endogenously determined within the economic system by rational portfolio decisions of the commercial banks and the public.

Some persons (economists included) might at this point say why worry about explaining changes in such quantities as money and bank credit? What is really important are changes in money income, real income, prices, and employment. "Money doesn't matter." However, there is a substantial body of economic theory and empirical evidence that strongly indicates that changes in the money stock can and have had an important impact on changes in the aggregate price level, money income, and employment. One of the most comprehensive studies of the relationship between changes in money (defined to include time deposits) and money income and prices is the work of Milton Friedman and Anna Schwartz. From a study of the period 1867-1960, their major conclusions were:

1. Changes in the behavior of the money stock have been closely associated with changes in economic activity, money income, and prices.
2. The interrelationship between monetary and economic change has been highly stable.
3. Monetary changes have often had an independent origin; they have not been simply a reflection of changes in economic activity.<sup>2/</sup>

---

<sup>2/</sup> Milton Friedman and Anna Schwartz, A Monetary History of the United States: 1867-1960. Princeton University Press, Princeton, 1963, p. 676.



Although there is still considerable disagreement as to whether the relationship between money and income should be discussed in terms of money including only demand deposits and currency held by the non-banking public or money defined to include time deposits, and permanent income vs. measured income, other studies have substantiated the close relationship between changes in "money" and changes in "income." <sup>3/</sup>

The non-linear money supply hypothesis developed by Professors Karl Brunner and Allan Meltzer is an attempt to explain the determination of the magnitudes of the economic quantities, money defined to include demand deposits and currency held by the nonbanking public, money defined to include time deposits, and bank credit. The Brunner-Meltzer work is more than an attempt to forecast the magnitudes of these economic quantities. It is an attempt to explain the process by which the stocks of money and bank credit are determined. <sup>4/</sup>

Within the Brunner-Meltzer hypothesis the equilibrium values of money and bank credit are determined jointly by the policy actions of the Federal Reserve System and the portfolio decisions of the

---

<sup>3/</sup> Karl Brunner and Allan Meltzer, "Predicting Velocity: Implications for Theory and Policy", Journal of Finance, May, 1963. And Leonall Andersen and Jerry Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance In Economic Stabilization", Federal Reserve Bank of St. Louis Review, November, 1968.

<sup>4/</sup> For an excellent elementary discussion of the difference between forecasting and explanation see Stephen Toulmin, Foresight and Understanding, Indiana University Press, 1961.

commercial banks and the public. Federal Reserve open market actions are incorporated in changes in an item called the adjusted monetary source base. Federal Reserve reserve requirement actions and discount rate changes are incorporated via behavioral reactions by the banks and the public in what are called money and bank credit multipliers. Their hypothesis clearly brings out not only the impact of a given policy action by the central bank, but also how changes in the decisions of the banks and the public to acquire assets and emit liabilities affects the equilibrium stocks of money and bank credit. The partial-equilibrium responses of the endogenous variables, money and bank credit, to policy-induced changes in market interest rates are also analyzed.

This paper does not attempt to discuss fully all of the implications and applications of the Brunner-Meltzer non-linear hypothesis. For a detailed discussion of many of these points and mathematical derivations the reader is referred to the articles by Brunner and Meltzer cited in the bibliography at the end of the paper. What is attempted is to give the basic framework of the hypothesis in as rigorous a manner as possible while still retaining a degree of simplicity. In the first part of the paper we shall write down the basic relations underlying the hypothesis. Each of these basic relationships will then be discussed in some detail. We shall trace through the economic system the impact of changes in each of the policy parameters.

The author hopes the non-mathematically inclined reader will not be dismayed by the sudden abundance of mathematical equations

that appear in the first part of the paper. Mathematical notation and functional expressions are used simply as a convenient notational device that permits us to write in condensed form many ideas that would require much more space to write out in words. Also, the use of the analytical system of mathematics allows us to proceed carefully and check our deductions arrived at in different stages of the analysis. Each introduction of a mathematical form will be carefully explained and discussed.

The author wishes to express his thanks to Professor Alan Meltzer and Dr. Jerry Jordan who read and commented on drafts of the paper. The author is especially indebted to Professor Karl Brunner who gave to the author very generously of his time and comments. The author has tried to the best of his ability to reproduce faithfully the results of Professors Brunner and Meltzer's work. Errors could occur, and the author takes full responsibility for any errors or misinterpretations. Any comments by readers of this working paper would be appreciated by the author.



## SECTION A

### BASIC RELATIONS INVOLVED IN THE BRUNNER-MELTZER HYPOTHESIS

#### Definitional Relations

$$B = B^a + A \dots\dots\dots 1a$$

$$B^a = P + U + TC - (g + f + c + o) \dots\dots\dots 1b$$

$$B = V + R + C^P \dots\dots\dots 1c$$

$$M^1 = D^P + C^P \dots\dots\dots 1d$$

$$M^2 = M^1 + T \dots\dots\dots 1e$$

#### Bank Behavioral Relations

$$R^d = R^r + R^e + V^d \dots\dots\dots 2a$$

$$R^r = r^d \delta (D^P + D^t) + r^t \tau T \dots\dots\dots 2b$$

$$R^e = e (D^P + D^t + T) \dots\dots\dots 2c$$

$$e = e (i, \rho, \pi_1) \dots\dots\dots 2d$$

$$V^d = v (D^P + D^t + T) \dots\dots\dots 2e$$

$$A = b (D^P + D^t + T) \dots\dots\dots 2f$$

$$b = b (i, \rho, \pi_2) \dots\dots\dots 2g$$

#### Public Behavioral Relations

$$C^P = k D^P \dots\dots\dots 3a$$

$$k = k (c, Y/Y_p, l, s, W) \dots\dots\dots 3b$$

$$\Gamma = t D^P \dots\dots\dots 3c$$

$$t = t (i, i^t, p, W/P_a, Y/Y_p) \dots\dots\dots 3d$$

$$i^t = i^t (i^f, Q) \dots\dots\dots 3e$$

#### Treasury Behavior

$$D^t = d D^P \dots\dots\dots 4$$

In the Brunner-Meltzer system the policy variables are:

$r^d$  = weighted average legal reserve requirements on member  
bank demand deposits

$r^t$  = average legal reserve requirements on member bank time  
deposits

$\rho$  = discount rate

$B^a$  = adjusted monetary source base, which includes Federal  
Reserve holdings of Government securities and, hence,  
includes open market operations as a policy variable

$d$  = the ratio of Treasury deposits at commercial banks to  
demand deposits of the public. This ratio reflects the  
administration of the Treasury's balance. It is a ratio  
partially under control of the Treasury and hence is a  
policy variable partially under Treasury control.

$Q$  = Regulation Q ceiling rates on time deposits

Relations 1a-1e define the two concepts monetary source base  
and adjusted monetary source base; and define two concepts of the money  
supply,  $M^1$  and  $M^2$ . The Brunner-Meltzer approach begins with the defini-  
tion of an item called the monetary source base (B). The monetary  
source base is defined as:

$$B = A + B^a$$

where A represents the sum of discounts and advances to member banks by  
the Federal Reserve Banks and  $B^a$  represents an item called the adjusted  
monetary source base.  $B^a$  is defined in terms of the consolidated balance  
sheet of the Federal Reserve and the Treasury.

$$B^a = P + U + TC - (g + f + o + c)$$

P = earning assets of the Federal Reserve System net of discounts and advances (i.e., holdings of Government securities)

U = gold stock

TC = total Treasury currency outstanding

g = sum of Treasury deposits at Fed

f = foreign deposits at the Fed

c = Treasury cash

o = other liabilities plus net worth minus other assets in the consolidated balance sheet of the Reserve Banks

Relation 1b defines the adjusted monetary source base in terms of the sources of the base. The three main sources of base money are (1) Federal Reserve holdings of Government securities (2) the nation's gold stock and (3) Treasury currency outstanding less Treasury deposits at the Federal Reserve Banks, Treasury cash balances and other deposits and accounts at the Federal Reserve Banks (see Table I). Viewed from the sources side, Federal Reserve holdings of Government securities, the major component of  $B^a$ , is completely under the control of the Federal Reserve System. The magnitude of the gold stock depends mostly on past movements of the gold stock. Treasury currency outstanding and Treasury cash balances and deposits at the Federal Reserve are under direct control of the Treasury. The major part of the source of  $B^a$  is therefore directly under the control of the Federal Reserve System.

Relation 1c defines the monetary base B in terms of its uses.  $B^a$  defined in terms of uses is

$$B - A = V + R + C^P - A.$$

The uses of the monetary base reflect the demands of nonmember banks for vault cash (V), member bank demands for reserves (R) and the public's demand for currency ( $C^P$ ).

The adjusted source base is viewed as an asset supplied to economic units that make up the economy. The magnitude of the quantity supplied of this asset is under complete control of the monetary authorities. <sup>5/</sup> Since the supply of the asset  $B^a$  is under the control of the monetary authorities, banks and the public must adjust their holdings of other assets, both financial and real assets, so that the magnitude of  $B^a$  demanded is equal to the magnitude of  $B^a$  supplied. The process by which the quantity demanded of  $B^a$  is adjusted to the quantity supplied leads to a change in the prices of real assets and a change in interest rates. <sup>6/</sup>

---

<sup>5/</sup> This does not mean that the Federal Reserve determines Treasury cash policy or that the Fed determines the surplus or deficit in the balance of payments. It means that, through open market operations, the Federal Reserve can offset any movements in Treasury cash balances and inflows or outflows of gold. Also, this does not mean the Fed will choose to offset changes in either of these factors affecting the supply of base money. However, by open market purchase and sale of Government securities, the Federal Reserve has the power, if it wishes to exercise that power, to determine the magnitude of base money and the rate at which base money is supplied to the economy.

<sup>6/</sup> For a more complete discussion of the sources and uses of the monetary base and its role in economic theory, see L. C. Andersen and J. L. Jordan, "The Monetary Base: Explanation and Analytical Use," Federal Reserve Bank of St. Louis Review, August, 1968.

Table I

Adjusted Monetary Source Base, December, 1968

	millions \$
Federal Reserve holdings of Government securities	\$52,594*
float	3,209
gold stock	10,367
Treasury currency outstanding	<u>6,810</u>
Total	\$72,980
less:	
Treasury cash holdings	756
Treasury deposits at Fed	360
Foreign deposits at Fed	225
Other (net)	<u>-647</u>
equals: adjusted monetary source base (B <sup>a</sup> )	<u>\$72,286</u>
Federal Reserve holdings of Government securities as % of B <sup>a</sup>	72.8%

\* includes \$98 million of acceptances not shown separately.

Uses of the Adjusted Monetary Source Base

December 1968

Member Bank Deposits at Federal Reserve	21,677
Currency held by Banks	6,309
Currency held by the public (Currency Component of money supply)	<u>44,300</u>
	<u>72,286</u>

\* Data are unadjusted



Relations 2a-2g specify bank behavior. Relation 2a expresses commercial banks desired holdings of reserves as composed of three components, required reserves, desired excess reserves, and desired holdings of vault cash of nonmember banks. <sup>1/</sup>  $R^d$  expresses bank's desired portfolio of base money. Relations 2b-2d express the determination of required reserves and excess reserves. In 2b, required reserves are expressed as a function of the policy variables  $r^d$  and  $r^t$  which are weighted average legal reserve requirements on member bank demand and time deposits respectively, and two factors  $\delta$  and  $\tau$  which are defined:

$$\delta = \frac{\text{member bank demand deposits}}{\text{total demand deposits}}$$

$$\tau = \frac{\text{member bank time deposits}}{\text{total time deposits}}$$

$\delta$ ,  $\tau$  are factors influenced by the decision of holders of bank deposits as to the allocation of such deposits between member and non-member banks. For example, other factors constant, a shift of demand deposits from member to nonmember banks causes a decline in  $\delta$  and, hence, a decline in required reserves. The introduction of  $\delta$  and  $\tau$  into the analysis permits explicit recognition of the institutional condition that reserve requirements for member and nonmember banks differ. The

---

<sup>1/</sup> Effective September 12, 1968 Regulation D, "Reserves of Member Banks" was amended, making several changes in the computation of reserve requirements of member banks. The major change was to use deposits two weeks earlier in calculating weekly required reserves for the current week. At the start of each settlement week, the Federal Reserve Banks send to their member banks statements of the amount of legally required reserves for that week based on deposits of two weeks prior. This change may affect banks' behavior with regard to desired holdings of excess reserves. Work is presently being done on a possible reformulation of the banks' behavioral relations taking into account this change in Regulation D.

Brunner-Meltzer hypothesis is thus able to explicitly analyze the effect of shifts of deposits between member and nonmember banks and changes in the relative size of member banks vs. nonmember banks on the levels and rates of change of the monetary aggregates.

Relation 2c defines desired excess reserves as dependent upon the magnitude of private and Treasury demand deposits ( $D^P + D^T$ ) and time deposits (T) at commercial banks and a ratio  $\underline{e}$  which is the ratio of desired excess reserves to total deposits. 2d specifies the determination of the ratio  $\underline{e}$ ; or  $\underline{e}$  is shown to depend on the interest rate  $i$ , the rediscount rate  $\rho$ , and index of other factors  $\pi_1$  influencing the demand for excess reserves. Factors included in  $\pi_1$  are the variance of interest rates, variance of currency flows between the banks and the public, and banks anticipations about the average level and variance of the reserve ratio.

Brunner and Meltzer postulate that the ratio  $\underline{e}$  is negatively related to changes in  $i$ , and positively related to the rediscount rate  $\rho$ .

$$\frac{\partial \underline{e}}{\partial i} < 0 \quad \frac{\partial \underline{e}}{\partial \rho} > 0$$

Relation 2e expresses the desired holding of vault cash ( $V^d$ ) of nonmember banks as a proportion of total commercial bank deposits. This ratio is small and is approximated by a constant.

Relation 2f defines member bank borrowings as dependent upon the magnitude of private and Treasury demand deposits and time deposits and a factor which is the ratio of desired borrowings to total deposits. Equation 2g expresses  $\underline{b}$  as a function of the interest rate  $i$ , the rediscount rate  $\rho$ , and an index of other factors  $\pi_2$  affecting member banks

borrowing from the Federal Reserve Banks. The main factor included here is the restrictions placed on bank borrowing as part of the Federal Reserve Banks' administration of the discount window. <sup>8/</sup>

The ratio  $\underline{b}$  is postulated to be positively related to the interest rate  $i$ , and negatively related to the rediscount rate  $\rho$ , and other borrowing costs  $\pi_2$ .

$$\frac{\partial b}{\partial i} > 0 \quad \frac{\partial b}{\partial \pi_2}, \frac{\partial b}{\partial \rho} < 0$$

Relations 3a-3e describe the public's decisions as to the allocation their monetary wealth between currency and demand deposits, and the allocation of deposits between demand and time deposits.

Relation 3a expresses the public's holdings of currency as a fraction ( $k$ ) of their holdings of demand deposits. In 3b, the currency ratio  $\underline{k}$  is expressed as being dependent upon the factors:

$c$  = measure of net service charges on demand deposits

$l$  = public's tax liabilities

$\frac{Y}{Y_p}$  = ratio of net national product to permanent (Friedman)  
net national product

$s$  = mobility of the population and such factors as seasonal  
patterns introduced by vacation schedules

$W$  = nominal wealth of the nonbank public.

---

<sup>8/</sup> At times  $\pi_2$  may be a very important explanatory variable in explaining the  $\underline{b}$  ratio. For example, in the summer of 1966 as the cost of alternative short-term funds such as Federal funds rose well above the discount rate, member bank borrowings, after rising sharply over the first part of 1966, did not show a noticeable increase.

The public's allocation of money balances between demand deposits and currency has at times been an important factor influencing the magnitude of the money stock. However, there is little reliable evidence on what factors determine movements in the  $k$  ratio. In their hypothesis, Brunner-Meltzer take  $k$  as a datum that influences, but is not influenced by the monetary process. <sup>9/</sup>

Relation 3c expresses holdings of time deposits as a fraction (t) of the public's holdings of demand deposits. 3d then expresses the time deposit ratio  $t$  as a function of the factors  $i$ ,  $i^t$ ,  $W/P_a$ ,  $p$ ,  $Y/Y_p$ , where  $W/P_a$  is the deflated stock of wealth held by the public,  $p$  is a price index of current output (the income deflator),  $i^t$  is a composite index of interest rates on time deposits, and  $i$  is the composite interest rate on assets traded on the credit market.

Brunner-Meltzer postulate that the  $t$  ratio is positively related to  $i^t$  and real wealth ( $w$ ), and is negatively related to  $i$ .

$$\frac{\partial t}{\partial i^t}, \frac{\partial t}{\partial w} > 0, \quad \frac{\partial t}{\partial i} < 0$$

Relation 3e expresses  $i^t$  as a function of the interest rate ( $i^f$ ) and the Regulation Q ceiling rate ( $Q$ ) on time deposits. The interest rate  $i^f$  is a composite index of interest rates on financial assets traded on the credit market on which banks traditionally operate and interest rates on other market assets. The time deposit interest rate  $i^t$  is postulated to be positively related to  $i^f$ , subject to conditions involving the ceiling rate  $Q$ .

---

<sup>9/</sup> Specifically, Brunner-Meltzer assume that the  $k$  ratio does not depend on interest rates.

$$\frac{\partial i^t}{\partial i^f} > 0 \quad \text{subject to } Q$$

The relationship between  $i^t$  and  $i^f$  represents the response banks to changes in market interest rates. As yields on other market assets rise, banks will experience increasing difficulty in attracting and holding time deposits. Banks are postulated to react to this by raising the rate  $i^t$  they offer on time deposits. The ability of banks to respond to changes in  $i^f$  by raising  $i^t$  is constrained by ceiling rates set by the monetary authorities.

Relation 4 expresses Treasury demand deposits at commercial banks as a fraction of private demand deposits. The Treasury is viewed as making decisions concerning the allocation of its cash balances between the commercial banks and the Federal Reserve Banks. Strictly speaking it is  $D^t$  that is determined by the Treasury.  $D^p$  is primarily determined by the public, banks, and institutional factors such as different reserve requirements among classes of banks.  $D^p$  is little affected, except in short periods of time, by changes in  $D^t$ . Since the ratio  $\underline{d}$  is partially determined by the Treasury,  $\underline{d}$  is viewed as a policy variable partially under the control of the Treasury. As individuals and corporations pay taxes and buy Government securities, private demand deposits decrease and Treasury deposits at commercial banks rise. This is reflected in a rise in the  $\underline{d}$  ratio. When the Government spends it draws down its deposits at banks and private demand deposits rise, this is reflected in a fall in the  $\underline{d}$  ratio. In recent years the Government's balances at commercial banks have ranged from \$3 billion to \$9 billion in a few months time. Over a period of time, these changes in the  $\underline{d}$  ratio tend to average out. However, in short

periods of time, changes in the d ratio can have an important influence on the growth rates of the monetary aggregates.<sup>10/</sup>

---

<sup>10/</sup> See Jerry Jordan, "Relations Among Monetary Aggregates," Federal Reserve Bank of St. Louis Review, March, 1969.

## SECTION B

### DERIVATION OF EXPRESSIONS FOR THE MONEY STOCK, MONEY DEFINED TO INCLUDE TIME DEPOSITS, AND BANK CREDIT

Using basic relations 1a-4, expressions for  $M^1$ ,  $M^2$ , and bank credit may be derived. To derive the expressions for  $M^1$  and  $M^2$  we begin with the basic definition:

$$B^a = R - A + C^P$$

$R$  = total bank reserves = vault cash holdings of nonmember  
banks + reserves of member banks

$A$  = discounts and advances of the Federal Reserve

$C^P$  = currency held by the public

Divide by demand deposits of the public ( $D^P$ ) and total deposits  
( $D^P + D^t + T^d$ ):

$$\frac{B^a}{D^P (D^P + D^t + T^d)} = \frac{1}{D^P} \cdot \frac{R-A}{D^P + D^t + T^d} + \frac{1}{D^P + D^t + T^d} \cdot \frac{C^P}{D^P}$$

referring to our basic definitions and substituting we have:

$$\frac{B^a}{D^P (D^P + D^t + T^d)} = \frac{1}{D^P} (r-b) + \frac{k}{D^P + D^t + T^d}$$

multiplying through both sides by total deposits we have:

$$\frac{B^a}{D^P} = \frac{D^P + D^t + T^d}{D^P} (r-b) + k$$

referring again to our basic definitions we have:

$$t = \frac{T}{D^P}, \quad d = \frac{D^t}{D^P}$$

hence:

$$\frac{B^a}{D^P} = (1+t+d) (r-b) + k$$

multiply both sides by  $C^P$  and add  $B^a$  to both sides and let:

$$\Delta = (1+t+d) (r-b) + K$$

we have:

$$B^a + \frac{B^a C^P}{D^P} = \Delta C^P + B^a$$

since:  $B^a = \Delta D^P$

we have:

$$B^a (1+k) = \Delta (C^P + D^P)$$

since:  $C^P + D^P = M^1$

we conclude:

$$\frac{1+k}{(r-b) (1+t+d)+k} \cdot B^a = M^1$$

To derive the expression for  $M^2$  we begin with the derived relation:

$$\frac{B^a}{D^P} = \Delta$$

multiply both sides by  $(T + C^P)$  and add  $B^a$ :

$$B^a \left(1 + \frac{T}{D^P} + \frac{C^P}{D^P}\right) = \Delta(T + C^P) + \Delta D^P$$

therefore:

$$\frac{1+t+k}{(r-b) (1+t+d)+k} \cdot B^a = C^P + T + D^P = M^2$$

The dependence of the ratio  $r$  on reserve requirements, the distribution of deposits between member and non-member banks, vault cash holdings of non-member banks, and the excess reserve to deposit ratio is taken into account by writing  $r$  explicitly as:

$$r = u \delta r^d + (1-u) \tau r^t + v + e$$



where:  $u = \frac{1+d}{1+t+d}$

$v$  = ratio of vault cash holdings of non-member banks to total deposits

The relation of  $E$  = earning assets of banks net of capital accounts and Treasury deposits at commercial banks = bank credit, to the adjusted base  $B^a$  is expressed by: <sup>11/</sup>

$$E = (m^2 - 1) B^a$$

The relationship between the adjusted source base and total bank credit (total loans and investments = BC) may be expressed as:

$$BC = aB^a$$

where the total bank credit multiplier  $a$  is defined: <sup>12/</sup>

$$a = \frac{(1 + t + d) [1 + n - (r - b)]}{(r - b) (1 + t + d) + k}$$

$$n = \frac{\text{capital accounts}}{\text{total bank deposits}}$$

The introduction of  $n$  introduces another bank behavioral relation. The use of the total bank credit multiplier requires an extension of the hypothesis to include a specification of bank behavior with regard to their capital accounts.

---

<sup>11/</sup> See Appendix I at the end of the paper.

<sup>12/</sup> See Appendix II at the end of the paper.

The total bank credit multiplier  $\underline{a}$  is larger than  $m^2 - 1$ . The reader can check, by subtracting  $m^2 - 1$  from  $\underline{a}$ , that  $\underline{a}$  is larger than  $m^2 - 1$  by a factor:

$$\frac{d + n (1 + t + d)}{(r - b)(1 + t + d) + k}$$

To avoid further complications of our exposition of the Brunner-Meltzer hypothesis, we shall, in the remainder of the paper, work with bank credit adjusted (E). The results of the analysis are not essentially changed if one uses total bank credit and the bank credit multiplier  $\underline{a}$  instead of E and  $m^2 - 1$ . The major change is that the variability of the bank credit multiplier is increased, and hence divergent movements of  $M^1$  and bank credit discussed later may be amplified.

Given the values of the multipliers  $m^1$  and  $m^2$ , to each magnitude of  $B^a$  there will be associated a unique magnitude of the money stock ( $M^1$ ), the money stock including time deposits ( $M^2$ ) and bank credit ( $E$ ). The unique magnitudes of  $M^1$ ,  $M^2$ , and  $E$  associated with any given magnitude of the adjusted monetary source base  $B^a$  will depend upon the values of the multipliers  $m^1$  and  $m^2$ .<sup>13/</sup>

The values of the multipliers  $m^1$  and  $m^2$  are dependent upon the values of a vector of policy parameters, a vector of bank behavioral parameters, and a vector of behavioral parameters determined by actions of the public.

---

<sup>13/</sup> An alternative form in which  $M^1$  may be expressed is:

$$M^1 = m^{1*} B$$

where:  $B$  = monetary source base including discounts and advances, and  $B$  is adjusted by a factor  $RA$  which takes into account the effects of changes in reserve requirements on member bank deposits due to changes in legal requirements and deposit shifts between reserve classes of member banks.

$$m^{1*} = \frac{1 + k}{(r^*) (1 + t + d) + K}$$

$$r^* = r + \ell$$

$$\ell = \frac{RA}{\text{total deposits}} = \text{liberated reserves per dollar deposits}$$

In this formulation, member bank borrowings, since they are a form of credit extended by the Federal Reserve, are explicitly included in the monetary base (where the monetary base is the monetary source base adjusted by the factor  $RA$ ).

$$\text{Let: } X_1 = \begin{bmatrix} r^t \\ r^d \\ \rho \\ d \\ Q \end{bmatrix} = \text{vector of policy parameters.}$$

$$X_2 = \begin{bmatrix} t \\ k \\ \tau \\ \delta \end{bmatrix} = \text{vector of behavioral parameters determined by actions of the public.}$$

$$X_3 = \begin{bmatrix} v \\ e \\ b \\ i^t \end{bmatrix} = \text{vector of behavioral parameters determined by actions of the commercial banks.}$$

### Relationship of the Values of the Multipliers

Rewriting the expression for the  $m_2$  multiplier as:

$$m^2 = m^1 + \frac{t}{(r - b)(1 + t + d) + k}$$

we see that:

$$m^2 > m^1$$

The bank credit multiplier  $m^2 - 1$  may be rewritten as:

$$m^2 - 1 = m^1 + \frac{t}{(r - b)(1 + t + d) + k} - \frac{(r - b)(1 + t + d) + k}{(r - b)(1 + t + d) + k}$$

If:  $t > (r - b)(1 + t + d) + k$

then:  $m^2 - 1 > m^1$

The relationship  $m^2 > m^1$  is a derivable consequence of the hypothesis. The ordering relation between the  $m^1$  multiplier and the  $m^2 - 1$  (bank credit) multiplier is determined by empirical evidence. The question of whether  $m^1 = m^2 - 1$ ,  $m^1 > m^2 - 1$ , or  $m^1 < m^2 - 1$  is not derivable from the hypothesis, but depends on the empirical relationship between the magnitude of the  $t$  ratio and the magnitude of the denominator of the  $m^1$  multiplier.

Brunner-Meltzer, by directly computing the average monthly values for each of the ratios in the above inequality, show that for the period January, 1947 to November, 1948 the inequality did not hold and  $m^1 > m^2 - 1$ . In the period since then, their computations indicate that the inequality has held. We may therefore state the relationship between the multipliers as:

$$m^2 > m^2 - 1 > m^1.$$

Equilibrium Value of the Multiplier

vs

The Value of the Multiplier Calculated at a Point in Time

By definition the magnitude of the money supply at any point in time is equal to the product of the magnitude of the adjusted base and the value of the  $m^1$  multiplier. If at any point in time we calculate the value of the  $m^1$  multiplier (i.e. we calculate the existing value of  $r$  and the  $b$ ,  $k$ ,  $t$ , and  $d$  ratios), and we take the product of this value and the number which represents the current magnitude of  $B^a$ , we will obtain a number which represents the current magnitude of the money supply.

The equilibrium value of the multiplier is determined by functional relationships which determine equilibrium values for  $r$ , and the  $b$ ,  $k$ ,  $t$ , and  $d$  ratios. For example, given the interest rate and the value of the policy parameter the rediscount rate, there will be a definite ratio of excess reserves to deposits which banks desire to hold and a definite ratio of borrowings to deposits which banks desire to maintain.

Assume that initially the banks and the public are in an equilibrium position where their desired ratios equal their actual ratios. Assume the Fed conducts an open market purchase of securities. The instantaneous impact of this policy action by the Fed will be to increase the magnitude of  $B^a$ . If the entire amount of the purchase is from the commercial banking system, this purchase would instantly show up in the multiplier as an increase in the actual  $e$  ratio. If we calculate the value of the multiplier immediately after the purchase

we obtain a value that is lower than the equilibrium value of the multiplier. During the portfolio adjustment process, the calculated value of the multiplier will rise as it approaches its new equilibrium value. However, the value of the multiplier calculated at any point in time will be equal to the equilibrium value of the multiplier only after the adjustment process is completed.

To put the matter in another way, at any point in time we may calculate the presently existing values of the items which constitute the money and bank credit multipliers and obtain a definite numerical value for each of these multipliers. For example, at a point in time we may calculate the ratio of excess reserves to deposits, i.e. the  $\underline{e}$  ratio. However, the  $\underline{e}$  ratio calculated at that point in time may not be the ratio of excess reserves to deposits that banks desire to hold given the prevailing interest rate  $i$  and the rediscount rate  $\rho$ . If the prevailing  $\underline{e}$  ratio is not equal to the desired  $\underline{e}$  ratio, a portfolio adjustment process on the part of the commercial banks will occur and the magnitude of the stock of money and bank credit will change as a result of the adjustment process.

# SECTION C

## EQUILIBRIUM IN THE BANK CREDIT MARKET

In the bank credit market the commercial banks' stock demand for earning assets ( $E^d$ ) is confronted with the stock supply of earning assets supplied to banks ( $E^s$ ).

The stock supply of earning assets ( $E^s$ ) is composed of the dollar value of the public's desired loan liabilities to banks ( $L$ ), and the stock of government securities ( $S$ ) less the outstanding portion of government securities not absorbed into the public's asset portfolio ( $D^1$ ).

$$E^s = L + S - D^1 \dots\dots\dots 5$$

The stock of government securities  $S$  is composed of the stock of U.S. Government debt obligations ( $S^G$ ) and the stock of municipal securities. Therefore,  $S$  is a function of  $S^G$ :

$$S = f(S^G).$$

Correspondingly,  $E^s$  depends on the stock  $S^G$ . This becomes important, as we shall point out later, because an open market operation by the central bank results in a change in the outstanding stock  $S^G$ .

The stock supply of assets available to the banks is expressed as a function of the interest rate on these assets ( $i$ ), transitory income ( $Y/Y_p$ ); real stock of non-human wealth, including the stock of government debt outstanding ( $W/P_a$ ), a price index ( $p$ ) of current output (the income deflator), the rate of return on real capital ( $\beta$ ), the interest rate ( $i_o$ ) on financial assets other than bank earning assets, and  $S^G$ .

$$E^s = E^s(i, Y/Y_p, W/P_a, p, \beta, i_o, S^G) \dots\dots\dots 6$$



The postulated relations between these factors and  $E^S$  are:

$$\frac{\partial E^S}{\partial i}, \frac{\partial E^S}{\partial S^G} < 0$$

$$\frac{\partial E^S}{\partial Y/Y_p}, \frac{\partial E^S}{\partial W/P_a}, \frac{\partial E^S}{\partial p}, \frac{\partial E^S}{\partial \beta} \cdot \frac{\partial E^S}{\partial i_0}, > 0$$

Note: The introduction of  $\beta$  raises questions concerning its determination.

$\beta$  is determined in the real sector, and since the Brunner-Meltzer approach is at present devoted to an analysis of the monetary sector,  $\beta$  is taken as a magnitude exogenous to their system.

Equilibrium on the market for bank credit is specified by:

$$E^d = E^S \dots \dots \dots 7$$

Relation 7 states the equilibrium condition that the volume of earning assets demanded by banks is equal to the volume of earning assets supplied to the banks. In the process of adjustment by which equilibrium is reached in the credit market, the outstanding stock of government securities is absorbed into the asset portfolios of the banks and the public, additional loans are extended by the commercial banks; and interest rates are adjusted on bank loans, government securities and other financial assets traded on the credit market.

#### The Portfolio Adjustment Process of the Banks and the Public

In the Brunner-Meltzer hypothesis the adjustment mechanism by which the financial system moves to an equilibrium position is, as will be explained below, a portfolio adjustment process carried out by economic units. Hereafter when we speak of an adjustment process it should be understood as referring specifically to a portfolio adjustment process by the commercial banks and the public.

The commercial banks are viewed as holding a portfolio of assets which consists of borrowed reserves, unborrowed reserves, and earning assets. The proportion of their total assets which banks desire to hold in each of the three categories is functionally dependent upon such factors as the market interest rate  $i$ , the rediscount rate  $\rho$ , and the legal reserve requirement ratios  $r^d$  and  $r^t$ . These asset or portfolio decisions are summarized in the desired ratios  $e$ ,  $v$ , and  $b$ . A policy action by the central bank is viewed as altering the existing  $e$ ,  $v$ , and  $b$  ratios which leads to a restructuring of portfolios of assets by commercial banks. For example, an open market sale of securities to the banks by the Federal Reserve will initially lead to the commercial banks holding a larger proportion of their earning assets in the form of government securities. Since the commercial banks demand for excess reserves is considered a stable function of interest rates, the policy induced change in the banks asset portfolios will lead to a portfolio adjustment process on the part of the commercial banks.

Similarly the asset behavior of the public is summarized in the  $k$  and  $t$  ratios. The public is viewed as making a portfolio decision as to whether to hold currency or deposits at commercial banks (summarized in the  $k$  ratio) and a portfolio decision as to whether to hold their bank deposits in the form of demand deposits or time deposits (summarized in the  $t$  ratio). A change in the actual  $k$  or  $t$  ratio from the desired  $k$  or  $t$  ratio, or a change in one of the factors which functionally determine the desired or equilibrium values of the  $k$  or  $t$  ratio, will result in a portfolio adjustment process on the part of the public.

A policy action by the Federal Reserve System alters the values of parameters that enter into the decision functions determining the asset portfolios of the commercial banks and the public and/or alters the composition of the stock of assets held by the banks and the public. The equilibrium stocks of money and bank credit then emerge from the portfolio decisions of the banks and the public.

SECTION D

THE CREDIT MARKET INTEREST RATE  $i$

The interest rate  $i$  is a composite interest rate index which is a combination of the rates on loans and financial assets traded on the Government securities, municipal bond, mortgage, and loan markets on which commercial banks traditionally operate.  $i$  is expressed as being dependent upon the policy variables ( $B^a$ ,  $r^t$ ,  $r^d$ ,  $\rho$ ); the behavioral parameter ( $k$ ); and the factors transitory income ( $Y/Y_p$ ), the rate of return on real capital ( $\beta$ ), real non-human wealth ( $W/P_a$ ), the price level ( $p$ ), and the composite interest rate on other financial assets not usually traded in by banks ( $i_0$ ).  
 $i = i(B^a, r^d, r^t, \rho, k, Y/Y_p, W/P_a, p, i_0, \beta) \dots \dots \dots 8$

The interest rate  $i$  is determined on the credit market by the interaction of the banks and the public. The partial equilibrium condition on the bank credit market is that the dollar volume of banks' desired holdings of earning assets is equal to the dollar volume of assets (loan liabilities and government securities) the public desires to supply to the commercial banks. In the portfolio adjustment process by which the stock of assets traded on the credit market is absorbed into the portfolios of the banks and the public, interest rates on these assets adjust, and the composite index  $i$  is determined by the adjustment process.

We shall frequently also introduce some other interest rate notation.  $i^t$  will represent the interest rate on time deposits.  $i_0$  will represent an index of interest rates on assets not usually traded on the credit market on which banks traditionally operate. An example of such a financial asset would be corporate bonds.  $i^f$  will represent an index of interest rates on assets including those traded on the bank credit market and other

financial assets.  $i^f$  may be viewed as a weighted average of  $i$  and  $i_0$ .

As we discussed previously, the equilibrium condition on the bank credit market is that the stock of earning assets banks desire to hold equal the stock of assets the public desires to supply to the banks ( $E^d = E^s$ ). The banks' stock demand for earning assets may be expressed:

$$E^d = (m^2 - 1) B^a$$

and we can rewrite our equilibrium condition as:

$$(m^2 - 1) B^a = E^s$$

Referring to our previous discussion, we note that the asset multiplier  $(m^2 - 1)$  and the asset supply function  $E^s$  are both functions of the interest rate ( $i$ ).

The elasticity of  $i$  with respect to  $B^a$ . Using the above expression we may express the response of  $i$  to changes in the policy parameters and factors entering the public's asset supply function in terms of elasticities.<sup>14/</sup> For the elasticity of  $i$  with respect to  $B^a$  we have:<sup>15/</sup>

<sup>14/</sup>

When one economic quantity is dependent upon another quantity, economists frequently express the relationship between the dependent quantity and independent quantity in terms of elasticities. The concept of elasticity takes into account the fact that the two quantities may not be measured in the same units. Elasticity, instead of referring to absolute changes, refers to percentage changes. For example, the elasticity of the interest rate index  $i$  with respect to  $B^a$ , expresses the percentage change in  $i$  (the dependent variable) given a percentage change in  $B^a$  (the independent variable). For an excellent discussion of the concept of elasticity see R.H. Leftwich, The Price System and Resource Allocation, Revised Edition, Holt Rinehart Co., (1960), pp.34-47.

<sup>15/</sup>

The interested reader may check this by taking logs of both sides of the expression  $B^a = \frac{E^s}{m^2 - 1}$  and taking partial derivatives with respect to  $i$ .

$$\epsilon(i, B^a) = \frac{-1}{\epsilon(m^2 - 1, i) - \epsilon(E^S, i)}$$

We have already established that:

$$\epsilon(E^S, i) < 0.$$

Also, except in high interest rate periods under constraint of Q,

$$\epsilon(m^2 - 1, i) < 0. \quad (\text{see section E})$$

Therefore, the short-run elasticity of  $i$  with respect to  $B^a$  is negative.

$$\epsilon(i, B^a) < 0.$$

The numerical value of  $\epsilon(i, B^a)$  depends upon the numerical values of the two elasticities that appear in the denominator. The larger banks' and the public's response to interest rate changes, the greater the numerical value of  $\epsilon(m^2 - 1, i)$  and  $\epsilon(E^S, i)$ , and hence the smaller is the percentage response of  $i$  to a given percentage change in the supply of base money. If the public and banks are relatively insensitive to changes in the interest rate  $i$ , then  $\epsilon(m^2 - 1, i)$  and  $\epsilon(E^S, i)$  are small in value, and the resulting adjustment process to changes in the supply of base leads to relatively large percentage changes in the interest rates on the bank credit market.

For example, assume that  $\epsilon(m^2 - 1, i)$  and  $\epsilon(E^S, i)$  are small in numerical value, and the Federal Reserve increases the source base by an open market purchase of securities from the banking system. Given their larger holdings of base money, banks attempt to expand their holdings of earning assets. To induce the public to increase its supply of loan liabilities and to transfer a portion of the smaller stock of outstanding securities to the banks, the banks must, given the initial condition that the public's

asset supply function is relatively insensitive to changes in credit market yields, lower the interest rate they charge on bank loans and raise the price they offer to pay for securities more than under initial conditions where  $\epsilon(E^S, i)$  is greater.

Interest rate effects of an open market policy-induced change in the base compared to interest rate effects of a change in the base resulting from other sources.

A change in  $B^a$  brought about by an open market purchase or sale of Government securities would be expected to have a different impact on interest rates than a change in  $B^a$  resulting from changes in the gold stock, float, Treasury currency outstanding or some other component of the source base (See Table I). The essential difference being that a change in  $B^a$  resulting from an open market operation is matched by an identical change in the opposite direction of the stock of interest-bearing Government debt held by the public and banks. As an illustration, in the case of an open market purchase from the public,  $B^a$  increases, but this is matched by an equal decrease in the stock of interest-bearing Government debt held by the public. Precisely, the ratio of non-interest bearing debt of the Government to the interest-bearing debt of the Government held by the banks and the public rises. Where the change in  $B^a$  arises from other sources, there is not this offsetting change in the stock of interest-bearing Government debt held by the banks and the public.

In the portfolio adjustment process following an open market purchase (sale) there will be a smaller (larger) stock of interest-bearing assets available to satisfy the portfolio demands of the banks and the public. For an open market purchase (sale) the result will be a larger

short-term fall (rise) in the credit market interest rate  $i$  than in the case where the change in base money is not matched by an equal and opposite change in the stock of interest-bearing assets held by the banks and the public.

The elasticity of  $i$  with respect to reserve requirements ( $r^d, r^t$ ) and the discount rate ( $\rho$ ). The elasticity of interest rates with respect to other policy parameters may be expressed:

$$\epsilon(i, x) = \epsilon(m^2 - 1, x) \cdot \epsilon(i, B^a)$$

where  $x = r^d, r^t, \rho$

since:

$$\epsilon(i, B^a) < 0$$

$$\epsilon(m^2 - 1, x) < 0$$

then:  $\epsilon(i, x) > 0$

Also, since empirical estimates show that  $\epsilon(m^2 - 1, x) < 1$  we derive from the Brunner-Meltzer hypothesis the conclusion: <sup>16/</sup>

$$|\epsilon(i, B^a)| > |\epsilon(i, x)|$$

Changes in the magnitude or rate at which base money is supplied to the economy by the central bank, has a greater impact on the interest rate index  $i$  than do changes in the other policy parameters under the control of the central bank.

---

<sup>16/</sup> The symbols  $| \quad |$  represent that we are considering the absolute value, or the value of the term enclosed in the expression without regard to its sign. For example, in the real number system we would say +2 is greater than -3. However,  $|-3|$  is greater  $| 2 |$ .



Consideration of Feedback and Long-Run Effects of Changes  
in  $B^a$  on Interest Rates.

A policy induced change in the stock of base money can be accomplished in a very short period of time via open market operations. In the short-run the impact effect of such policy-induced changes in  $B^a$  will cause movements of interest rates in the opposite direction to the change in  $B^a$ . However, the increased or decreased stock of base money must be assimilated into the asset portfolios of the banks and the public. Over the long-run if the relative growth rates of base money and the factors determining the supply of earning assets to the banks are approximately equal, then changes in the public's supply of earning assets to banks principally determine interest rates on the credit market.

The expressions we used when discussing the short-run effects of a change in one of the policy parameters on  $i$ , for example:

$$\epsilon(i, B^a) = \frac{-1}{\epsilon(m^2-1, i) - \epsilon(E^s, i)}$$

are only partial equilibrium results. These expressions do not take into account the spill-over effects into the real sector of changes in the magnitudes of the money stock and changes in the levels of interest rates resulting from changes in the magnitude or rate at which base money is supplied to the economy. Consequently, feedback effects to the bank credit market from changes in the real sector induced by monetary policy actions are not taken into account. To analyze the feedback and long-run effects of changes in policy parameters on interest rates ( $i$ ), the hypothesis must be broadened to include propositions about the real sector. Taking these factors into consideration, important

consequences for the  $\epsilon (i, B^a)$  emerge. <sup>17/</sup>

We may express the current market interest rate ( $i_M$ ) as a combination of two components  $\alpha$  and  $\beta$  where:

$\alpha$  = expected real rate of return on capital

$\beta$  = expected rate of change of the aggregate price level

$$i_M = \alpha + \beta$$

The expected real rate of return on capital ( $\alpha$ ) is a function of the level of real output ( $y$ ). As  $y$  increases the expected rate of return on real capital increases:

$$\frac{\Delta \alpha}{\Delta y} > 0$$

For example, assume the supply of base money is increased via an open market purchase. A new partial equilibrium position is reached in the credit market by an increase in the banks' demand for earning assets. The attempt by banks to acquire a larger stock of earning assets results in a fall in the index of credit market interest rates ( $i$ ) and an expansion of bank credit and money. The short-run elasticity of  $i$  with respect to  $B^a$  is negative. However, the fall in  $i$  and the increase in  $M$  spills over the real sector in the form of an increase in aggregate demand. The increased level of aggregate demand has an impact on real output and prices. As the level of real output rises, the expected rate of return on real capital rises. This feedbacks to the credit market in

---

<sup>17/</sup> For a more complete discussion the reader is referred to, Karl Brunner and Allan Meltzer, "The Meaning of Monetary Indicators," Monetary Process and Policy: A Symposium, edited by G. Horwich, Purdue University Monograph Series #3, Irwin Co. (1967), pp. 187-217.

the form of a rise in public's demand for bank credit. To the extent that the monetary authorities do not attempt to peg  $i_M$  by continuing to increase the supply of base money, the base-induced increase in demand for credit results in a rise in the level of credit market interest rates.

The increase (decrease) in aggregate demand generated by an expansion (decrease) in the monetary aggregates following an increase (decrease) in the rate at which base money is supplied to the economy has an effect on the rate of change of real output ( $y$ ) and prices ( $p$ ). If we let  $Y$  denote aggregate demand, then  $\epsilon(p, Y)$  is positive. However, the numerical value of  $\epsilon(p, Y)$  depends upon the relationship between the current level of demand and the existing productive capacity of the economy. At low levels of demand relative to the productive capacity of the economy the numerical value of  $\epsilon(p, Y)$  is small. As the economy approaches full employment the numerical value of  $\epsilon(p, Y)$  increases.

As the economy approaches full employment, an expansionary monetary policy leading to a steady growth of base money and expansion of the money supply, through its impact on total spending, results in a sharp rise in the aggregate price level. Changes in the price level affect the demand for credit and hence interest rates. Although market interest rates fall in the short-run (the so-called liquidity effect), the price expectations generated as a result of the expansionary monetary policy may lead, over a longer period of time, to rising market interest rates.

Over a longer period of time as the rates of change of the aggregate price level changes, the public revises its expectations about the future rate

of change of the price level. <sup>18/</sup> In the form:

$$i_M = \alpha + \beta$$

this is reflected in a change in  $\beta$ . Over the longer-run, changes in the rate of change of the aggregate price level affect the demand for credit and hence affect the level of market interest rates ( $i_M$ ). Over a longer period of time, the price expectations generated as a result of an expansionary monetary policy may lead to an increase in  $\beta$  and hence a rise in  $i_M$ . <sup>19/</sup> Conversely, a restrictive monetary policy (a decrease in the growth rate of  $B^a$ ) may over the long-run cause a downward revision in  $\beta$  by the public and hence a fall in  $i_M$ .

Taking into account the feedback effects from the real sector and the longer-run effects of base-induced changes in prices and price expectations, in the new equilibrium position on the credit market, interest rates may be higher than before an increase in base money. Under these conditions

$$\epsilon(i, B^a)_{\text{Long-run}} > 0$$

Summary: To analyze the interest rate effects of a change in the magnitude or rate at which base money is supplied to the economy we must consider three effects.

- (1) The impact or short-run effect of changes in  $B^a$  on credit market interest rates. The impact effect is:

$$\epsilon(i, B^a) < 0$$

---

<sup>18/</sup> There is considerable disagreement among economists as to the length of time it takes the public to revise their expectations about the future rate of change of prices. However, almost all economists seem to agree that the effect of changes in  $\beta$  on  $i_M$  is a longer-run phenomenon than the impact effect of changes in  $B^a$  on  $i_M$  and than the feedback effects ( $\alpha$ ) of changes in real output in  $i_M$ . The lag may also be variable, decreasing as the rate of change of  $p$  increases.

<sup>19/</sup> Changes in the price level affect the real value of a stream of payments from a financial asset. A rise in the price level reduces the real purchasing power of the flow of payments to the holder of a financial asset, and reduces the real cost to the issuer of the financial asset. Consequently, when  $\beta$  rises, the amount of credit supplied at any given level of market interest rates decreases, and the amount of credit demanded at any given level of  $i_M$  rises.

- (2) Feedback effects from the real sector to the credit market.

The feedback effects of increases (decreases) in  $B^a$  act to raise (lower) interest rates.

- (3) Long-run or price expectation effects of a change in base money. This depends upon the change in the price level resulting from the effects in the real sector of base-induced changes in the monetary aggregates. These effects operate to raise (lower) interest rates as the base expands (contracts).

## SECTION E

### DEPENDENCE OF THE MONEY AND BANK CREDIT MULTIPLIERS ON $i$

In the Brunner-Meltzer hypothesis the composite interest rate  $i$  operates through the money and bank credit multipliers and through the public's supply of assets to the banks (basic relation 6). The dependence of the excess reserve ratio ( $e$ ), borrowing ratio ( $b$ ), and the time deposit ratio ( $t$ ) on the interest rate  $i$ , imposes a dependence of the money and bank credit multipliers on  $i$ .

The dependence of the money and bank credit multipliers on the interest rate  $i$  has two important implications:

- (1) Since  $i$  is postulated to be dependent upon the values of parameters under the control of the Federal Reserve, (i.e.,  $B^a$ ,  $r^d$ ,  $r^t$ , and  $\rho$ ) a change in one of these policy parameters will affect the value of  $i$ . Since  $m^1$  and  $m^2$  are dependent upon  $i$ , a change in one of the policy parameters results in an interest rate effect on the values of the multipliers.
- (2) Changes in factors affecting parameters determined in the real sector may alter  $i$  and hence, via the dependence of  $m^1$  and  $m^2$  on  $i$ , there will be a feedback effect from the real sector to the financial sector affecting the magnitudes of the stocks of money and bank credit.

Taking into account the dependence of the multipliers on  $e$ ,  $b$ , and  $t$ , the elasticities of the  $M^1$ ,  $M^2$ , and bank credit multipliers with respect to interest rates may be expressed:

$$\epsilon(m, i) = \epsilon(m, e) \cdot \epsilon(e, i) + \epsilon(m, b) \cdot \epsilon(b, i) + \epsilon(m, t) \cdot \epsilon(t, i)$$

The sign and numerical value of the elasticity of each of the multipliers with respect to interest rates depends upon the signs and numerical values of three terms. Rewriting the expressions for the multipliers:

$$m^1 = \frac{1 + k}{(r - b)(1 + t + d) + k}$$

$$m^2 = \frac{1 + k + t}{(r - b)(1 + t + d) + k}$$

$$m^2 - 1 = \frac{(1 + t) - (r - b)(1 + t + d)}{(r - b)(1 + t + d) + k}$$

and:  $r = u\delta r^d + (1 - u) \tau r^t + v + e$

we can see that an increase (decrease) in the  $e$  ratio decreases (increases) the numerical value of the multipliers. An increase (decrease) in the  $b$  ratio increases (decreases) the numerical value of the multipliers.

Hence we state:

$$\epsilon(m, e) < 0$$

$$\epsilon(m, b) > 0$$

For the  $t$  ratio, looking at the expression for  $m^1$  we see that:

$$\epsilon(m^1, t) < 0$$

By definition of  $t$ , a change in  $t$  involves a change in the proportion of demand and time deposits. Since the reserve requirements against these two classes of bank deposits are not the same, a change in  $t$  involves a change in the ratio of reserves to deposits ( $r$ ). This is reflected in the factor:

$$u = \frac{1 + d}{1 + t + d}$$

which appears in  $r$ . An increase (decrease) in  $t$  lowers (raises)  $u$  and lowers (raises)  $r$ , hence increases (decreases) the multipliers. The effect of the change in  $t$  on  $r$  is to decrease, but not to completely offset, the numerical value of the change in the denominators of the multipliers resulting from a change in  $t$ .

When estimating the elasticities of  $m^1$  and  $m^2$  with respect to  $t$ , this must be considered. For example, in the  $m^1$  multiplier, the dependence of  $r$  on  $t$  reduces the absolute value of the elasticity of  $m^1$  with respect to  $t$ . Having noted this point, for purposes of simplifying our exposition we ignore the effect of changes in the  $t$  ratio on  $r$ .

The  $t$  ratio appears in both the numerator and the denominator of the  $M^2$  and bank credit multipliers. For  $m^2$  if we disregard the effect of a change in  $t$  on the  $r$  ratio the change in  $m^2$  resulting from a change in  $t$ :

$$\frac{\Delta m^2}{\Delta t} = \frac{(r - b)(1 + t + d) + k - (r - b)(1 + k + t)}{[(r - b)(1 + t + d) + k]^2}$$

simplifying the numerator we have:

$$(r - b) \cdot d + k - (r - b) \cdot k$$

since:  $(r - b) \cdot d > 0$

$$k > (r - b) \cdot k$$

we conclude:  $\frac{\Delta m^2}{\Delta t} > 0$

For the  $m^2$  and  $m^2 - 1$  multipliers, although an increase (decrease) in  $t$  increases (decreases) the numerical value of the denominator, the corresponding change in  $t$  in the numerator of these expressions



offsets the effects of the changes in the denominator of the multipliers. Consequently, a rise (fall) in  $t$  results in a rise (fall) in the  $m^2$  and bank credit multipliers; and we conclude:

$$\epsilon(m^2, t) > 0$$

$$\epsilon(m^2 - 1, t) > 0$$

Also, since as the reader may check:

$$\frac{\Delta m^2}{\Delta t} = \frac{\Delta(m^2 - 1)}{\Delta t}$$

but:  $m^2 > (m^2 - 1)$

we conclude from the expression for elasticities:

$$\epsilon(m^2 - 1, t) > \epsilon(m^2, t)$$

From the conclusion that the elasticities of the  $m^2$  and bank credit multiplier with respect to  $t$  are opposite in sign to the elasticity of  $m^1$  with respect to  $t$ , it follows that, the elasticities of  $M^2$  and  $E$  with respect to  $t$  are opposite in sign to the elasticity of  $M^1$ . Empirical estimates by Brunner-Meltzer indicate that not only are the elasticities of opposite sign, but that in absolute value, the elasticities of bank credit and  $M^2$  with respect to  $t$  are larger than the elasticity of  $M^1$ .

As a consequence, a marked shift by the public from demand to time deposits leads to opposite movements in  $M^1$  and  $M^2$  and bank credit.

Calculations of the postwar monthly values of the  $t$  ratio reveal that changes in the  $t$  ratio have had a major effect on values of the  $m^2$  and  $m^2 - 1$  multipliers. Between January, 1947 and January,

1964 the  $m^2$  multiplier rose from 3.236 to 4.988, primarily reflecting the doubling of the t ratio. Over the same period, the  $m^1$  multiplier rose from 2.496 to 2.867, primarily as a result of changes in reserve requirement ratios and k.

Between January, 1964 and the start of 1969 the money defined to include time deposits multiplier has risen steadily. The rise in  $m^2$  primarily reflect a sharp rise in the t ratio. The yearly averages of monthly figures for this multiplier and the t ratio are:

	<u><math>m^2</math></u>	<u>t-ratio</u>
1964	4.029	.972
1965	5.213	1.080
1966	5.326	1.164
1967	5.427	1.264
1968	5.521	1.320

On the other hand,  $m^1$  (money defined to include only nonbank private demand deposits and currency multiplier) has declined slightly. The yearly averages for  $m^1$  are:

	<u><math>m^1</math></u>
1964	2.851
1965	2.884
1966	2.794
1967	2.739
1968	2.728

In other words, in January, 1964 a dollar increase in base money raised the stock of  $M^2$  by  $5.029 - 2.728 = \$2.30$  more than

the  $M^1$  money stock. By 1968 this difference had widened even more. In 1968 a dollar change in base money resulted on average in a change of \$2.79 more in  $M^2$  than in  $M^1$ .

Elasticities of the excess reserve and borrowing ratios  
with respect to the interest rate  $i$

As the yields on earning assets available to banks rise, the opportunity cost to banks of holding excess reserves rises. As  $i$  falls the opportunity cost of holding excess reserves decreases. When  $i$  rises (falls) banks are postulated to decrease (increase) their holdings of excess reserves relative to their deposit liabilities. Therefore:

$$\epsilon(e, i) < 0$$

As the yields on earning assets available to banks rise, borrowing from the Federal Reserve Banks becomes for the banks a more attractive source of base money to support deposits. Banks' desired ratio of borrowings to deposits rises. When  $i$  falls, banks' desired  $b$  ratio declines. Therefore:

$$\epsilon(b, i) > 0$$

The numerical value of the  $\epsilon(e, i)$  and  $\epsilon(b, i)$  depend upon the numerical value of  $e$ , and  $b$  and the level of interest rates. When interest rates, relative to their historical average, are at very low levels and the  $e$  ratio is large and  $b$  ratio small, then  $\epsilon(e, i)$  and  $\epsilon(b, i)$  tend to be large. As interest rates rise to medium levels,  $e$  falls and  $b$  rises. At medium levels of interest rates  $\epsilon(e, i)$  and  $\epsilon(b, i)$  are less than at very low levels of interest rates.

When interest rates are high relative to their past levels, the excess reserve ratio becomes small relative to its past values; and under these initial conditions the borrowing ratio becomes large. Under an initial condition of high interest rates,

banks attempt to minimize their holdings of excess reserves relative to their deposit liabilities. Also, if the discount rate rises less rapidly than yields on assets available to banks, member banks tend to maintain a high level of borrowings from the central bank relative to their deposit liabilities. As the b ratio rises, the central bank may become less willing to accommodate the borrowing demands of member banks. Tighter administration of the discount window is instituted. Consequently, under initial conditions of high levels of interest rates,  $\epsilon(e, i)$  and  $\epsilon(b, i)$  approach zero.

Elasticity of the t ratio with respect to i

The elasticity of the t ratio with respect to the index of interest rates (i) on assets traded by commercial banks depends upon three factors.

- (1) For a given percent change in interest rates ( $i^f$ ) on other assets, the response of the public with respect to their desired allocation of bank deposits between demand and time deposits. This is summarized as:

$$\epsilon(t, i^f)$$

- (2) For a given percent change in  $i^t$ , the response of the public with respect to their desired allocation of bank deposits between demand and time deposits. This is summarized as:

$$\epsilon(t, i^t)$$

- (3) The competitive response of commercial banks to a percent change in  $i^f$ . This is summarized as:

$$\epsilon(i^t, i^f)$$

We may combine these factors and express the elasticity of  $t$  with respect to  $i$  as:

$$\epsilon(t, i) = [\epsilon(t, i^f) + \epsilon(t, i^t) \cdot \epsilon(i^t, i^f)] \cdot \epsilon(i^f, i)$$

where  $\epsilon(i^f, i)$  is a weighting factor.

The signs and numerical values of the components of  $\epsilon(t, i)$ .

Referring to the basic definition of the  $t$  ratio:

$$t = \frac{\text{Time deposits}}{\text{Demand deposits held by the public}}$$

we can express  $\epsilon(t, i^f)$  and  $\epsilon(t, i^t)$  in the following manner:

$$\epsilon(t, i^f) = [\epsilon(T, i^f) - \epsilon(D^P, i^f)]$$

$$\epsilon(t, i^t) = [\epsilon(T, i^t) - \epsilon(D^P, i^t)]$$

As the spread between  $i^f$  (index of yields on other market assets) and  $i^t$  widens, these other assets become vis-à-vis time deposits, a more attractive means of holding wealth. The public switches from time deposits to other assets such as Treasury bills, commercial paper, savings and loan shares, and corporate bonds.  $T$  declines with a rise in  $i^f$  and increases with a rise in  $i^t$ .

$$\epsilon(T, i^f) < 0$$

$$\epsilon(T, i^t) > 0$$

With respect to  $D^P$ , as yields on time deposits and other market assets rise (fall) the opportunity cost of holding demand deposits increases (decreases). The public, in response to the change in available yields on other assets switches from demand deposits to other assets (time deposits and other market assets) when yields on these assets rise. We have:

$$\epsilon(D^P, i^f) < 0$$

$$\epsilon(D^P, i^t) < 0$$

Referring to the expressions above, we see that:

$$\text{Since: } \epsilon (T, i^t) > 0, \quad \epsilon (D^P, i^t) < 0$$

$$\text{then: } \epsilon (t, i^t) > 0$$

The numerical value, but not the sign of  $\epsilon (t, i^t)$  depends upon the numerical values of the elasticities of time deposits and demand deposits with respect to yields offered by banks on time deposits. The larger (smaller) the  $\epsilon (T, i^t)$  and  $\epsilon (D^P, i^t)$  correspondingly the larger (smaller) is the numerical value of the elasticity of the  $t$  ratio with respect to  $i^t$ .

The sign and the numerical value of  $\epsilon (t, i^f)$  depends upon the numerical values of  $\epsilon (T, i^f)$  and  $\epsilon (D^P, i^f)$ . In their hypothesis, Brunner-Meltzer postulate that the substitution between securities and time deposits dominates the substitution between demand deposits and securities resulting from a change in  $i^f$ .

$$|\epsilon (T, i^f)| > |\epsilon (D^P, i^f)|$$

and we derive the conclusion:

$$\epsilon (t, i^f) < 0$$

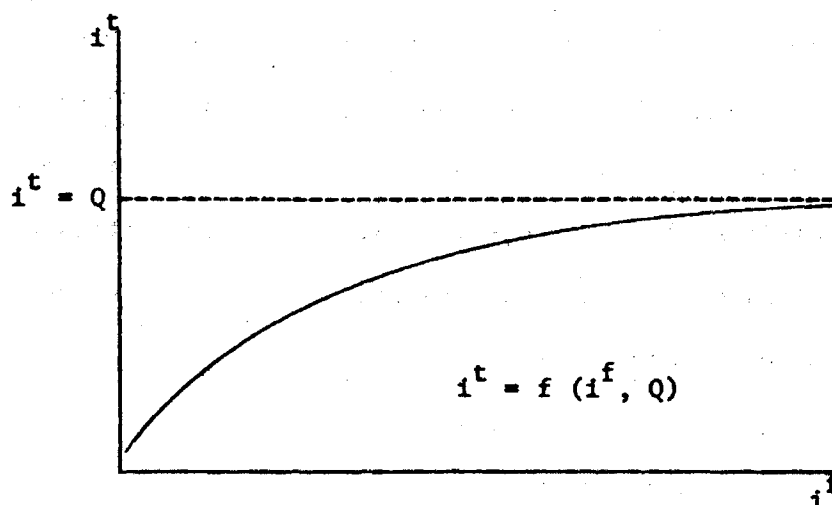
The banks' supply behavior with respect to time deposits is described as a price setting function of  $i^f$  and Regulation Q. The competitive response of banks to changes in the yields on other market assets is expressed by  $\epsilon (i^t, i^f)$ . As  $i^f$  rises, and the public's demand for time deposits decreases, banks respond by raising the yields they offer on time deposits. When  $i^f$  declines, banks respond by lowering the yields they offer on time deposits. This is expressed by:

$$\epsilon (i^t, i^f) > 0$$

Regulation Q. Given the institutional legal restraints under which banks operate, a constraint on banks' ability to raise  $i^t$  must be

taken into consideration. This constraint is the Regulation Q ceiling rates on yields banks can offer on time deposits. Once the regulatory imposed constraint of Regulation Q ceiling rates is introduced into the analysis, as yields offered by banks on time deposits approach their Regulation Q ceilings, the ability of banks to respond to a rise in  $i^f$  by raising  $i^t$  approaches zero.

Graphically, if we express  $i^t$  as a function of  $i^f$  and  $Q$  we have:



As  $i^t \rightarrow Q$  we can see that the change in  $i^t$  resulting from a change in  $Q$  becomes smaller:

$$\frac{\Delta i^t}{\Delta i^f} \rightarrow 0$$

And, correspondingly we have our result that as  $i^t \rightarrow Q$ .

$$\epsilon(i^t, i^f) \rightarrow 0$$

Total  $\epsilon(t, i)$ . Having discussed each of the components of  $\epsilon(t, i)$ , let us rewrite the total expression for  $\epsilon(t, i)$  and state each of the components with its expected sign.

$$\epsilon(t, i) = [\epsilon(t, i^f) + \epsilon(t, i^t) \cdot \epsilon(i^t, i^f)] \cdot \epsilon(i^f, i)$$

$$\epsilon(t, i^f) < 0$$

$$\epsilon(t, i^t) > 0$$

$$\epsilon(i^t, i^f) > 0 \quad i^t \text{ sufficiently below } Q$$

$$\epsilon(i^t, i^f) \rightarrow 0 \quad i^t \text{ approaches } Q$$

Brunner-Meltzer impose a further restriction on  $\epsilon(t, i^f)$  and  $\epsilon(t, i^t)$ . They impose the so-called Hotelling conditions that the direct elasticity is greater than the cross elasticity. This restriction can be stated:

$$|\epsilon(t, i^t)| > |\epsilon(t, i^f)|$$

Under these conditions, when  $i^t$  is sufficiently below  $Q$  so that  $\epsilon(i^t, i^f) > 0$ , the last two terms of the expression, which are positive, dominate the first term of the expression:

$$|\epsilon(t, i^t) \cdot \epsilon(i^t, i^f)| > |\epsilon(t, i^f)|$$

and the expected sign of  $\epsilon(t, i)$  becomes:

$$\epsilon(t, i) > 0$$

As market yields rise, and as  $i^t$  approaches  $Q$ , the numerical value of  $\epsilon(i^t, i^f)$  becomes smaller. Consequently, the absolute value of the product of the last two terms decreases:

$$|\epsilon(t, i^t) \cdot \epsilon(i^t, i^f)| \rightarrow |\epsilon(t, i^f)|$$

and:  $\epsilon(t, i)$  approaches zero.

When Regulation  $Q$  is effectively restraining banks from raising the yields they offer on time deposits, the  $\epsilon(i^t, i^f)$  becomes very small, and the first term of the expression dominates.

$$i^t \rightarrow Q \quad \text{and} \quad \epsilon(i^t, i^f) \rightarrow 0$$

$$|\epsilon(t, i^t) \cdot \epsilon(i^t, i^f)| < |\epsilon(t, i^f)|$$



Consequently, the numerical value of the first two products in  $\epsilon(m, i)$  decreases as interest levels rise from low levels to historically high levels. When interest rates are at very high levels relative to their past average levels, the numerical value of these first two terms approaches zero.

The  $\epsilon(m^1, e)$  and  $\epsilon(m^2, e)$  are equal. Also,  $\epsilon(m^1, b)$  and  $\epsilon(m^2, b)$  are equal.<sup>20/</sup> However,

$$|\epsilon(m^2 - 1, e)| > |\epsilon(m^1, e)|, |\epsilon(m^2, e)|$$

$$\epsilon(m^2 - 1, b) > \epsilon(m^1, b), \quad \epsilon(m^2, b)$$

When we examine the sign and numerical value of the last term in  $\epsilon(m, i)$  we must distinguish between the  $m^1$  and  $m^2$  and bank credit multipliers, and we must state out initial conditions about the relationship between  $i^t$  and  $Q$ . Reviewing, we have shown:

$$\epsilon(m^1, t) < 0$$

but that:

$$\epsilon(m^2, t) > 0$$

$$\epsilon(m^2 - 1, t) > 0$$

and:

$$\epsilon(m^2 - 1, t) > \epsilon(m^2, t)$$

Also, we have shown that as  $i^t$  approaches  $Q$  the numerical value of  $\epsilon(t, i)$  decreases and when  $i^t$  becomes sufficiently close to  $Q$ , the sign of  $\epsilon(t, i)$  reverses and becomes negative.

---

<sup>20/</sup> The interested reader may check these assertions by taking logs of both sides of the explicit expressions for  $m^1$ ,  $m^2$ , and  $m^2 - 1$  and partially differentiating with respect to  $e$  and  $b$ .

and the sign of the elasticity of the  $t$  ratio with respect to interest rates is reversed from the case where  $i^t$  sufficiently below  $Q$  and:

$$\epsilon(t, i) < 0$$

Under these conditions, wealth holders switch from time deposits to other assets, a process that has been labeled disintermediation.

The signs and numerical values of the elasticities of the multipliers with respect to interest rates

The elasticities of the multipliers with respect to interest rates in a general form is:

$$\epsilon(m, i) = \epsilon(m, e) \cdot \epsilon(e, i) + \epsilon(m, b) \cdot \epsilon(b, i) + \epsilon(m, t) \cdot \epsilon(t, i)$$

From our previous discussion, since:

$$\epsilon(m, e) < 0$$

$$\epsilon(m, b) > 0$$

$$\epsilon(e, i) < 0$$

$$\epsilon(b, i) > 0$$

we may conclude:

$$\epsilon(m, e) \cdot \epsilon(e, i) > 0$$

$$\epsilon(m, b) \cdot \epsilon(b, i) > 0$$

The signs of the first two products in the expression for  $\epsilon(m, i)$  are positive. As we discussed above, the numerical value of these products depends upon the level of interest rates. As interest rates rise from low levels into the medium range of interest rates and then to high levels,  $\epsilon(e, i)$  and  $\epsilon(b, i)$  decrease in numerical value approaching zero as interest rates rise to high levels.

Relative responses of the  $M^2$  and bank credit  
multipliers to changes in interest rates

To examine the implications of the Brunner-Meltzer hypothesis for changes in interest rates on  $m^2$  and  $m^2-1$ , let us write out the elasticities of these multipliers with respect to interest rates as:

$$\epsilon(m^2, i) = \frac{\Delta m^2}{\Delta i} \cdot \frac{i}{m^2}$$

$$\epsilon(m^2-1, i) = \frac{\Delta(m^2-1)}{\Delta i} \cdot \frac{i}{m^2-1}$$

where the notation  $\Delta$  refers to a change in the variable it precedes, and

$\frac{\Delta m^2}{\Delta i}$  represents the change in  $m^2$  resulting from a change in  $i$

$\frac{\Delta(m^2-1)}{\Delta i}$  represents the change in the bank credit multiplier resulting from a change in  $i$ .

Let us now proceed to subtract:

$$\epsilon(m^2, i) - \epsilon(m^2-1, i)$$

by substituting and collecting terms we have:

$$\frac{i}{(m^2) \cdot (m^2-1)} \left[ \frac{\Delta m^2}{\Delta i} \cdot (m^2-1) - \frac{\Delta(m^2-1)}{\Delta i} \cdot (m^2) \right]$$

Since under the constraint of the hypothesis:

$$\frac{\Delta m^2}{\Delta i} = \frac{\Delta(m^2-1)}{\Delta i}$$

$$m^2 > m^2-1$$

we conclude that <sup>21/</sup>

---

<sup>21/</sup> The reader may note that, although the absolute changes in  $m^2$  and  $m^2-1$  for a given change in  $i$  are the same, since  $m^2-1$  is smaller than  $m^2$ , for a given absolute change, the percentage change in  $m^2-1$  is larger than the percentage change in  $m^2$ .

$$\epsilon(m^2, i) - \epsilon(m^{2-1}, i) < 0$$

$$\epsilon(m^{2-1}, i) > \epsilon(m^2, i)$$

Three cases for the elasticities of  
the multipliers with respect to interest rates

Case I

Initial conditions:

(1.1) Market interest rates at low levels

(1.2)  $i^t$  sufficiently below  $Q$  so that  $\epsilon(t, i) > 0$

Under these initial conditions, the large positive values of the first two terms in the expression for the  $\epsilon(m^1, i)$  dominate the negative value of the last term and:

$$\epsilon(m^1, i) > 0$$

For the elasticities of the  $m^2$  and  $m^{2-1}$  multipliers with respect to  $i$  all three terms in these expressions are positive and:

$$\epsilon(m^2, i) > 0$$

$$\epsilon(m^{2-1}, i) > 0$$

since the sign of the last term  $[\epsilon(m, t) \cdot \epsilon(t, i)]$  is negative for  $\epsilon(m^1, i)$  and positive for  $\epsilon(m^2, i)$  and  $\epsilon(m^{2-1}, i)$ :

$$\epsilon(m^1, i) < \epsilon(m^2, i), \epsilon(m^{2-1}, i).$$

Also, since as we have shown previously the elasticity of the bank credit multiplier with respect to  $e$ ,  $b$ , and  $t$  is greater than the elasticity of the  $m^1$  and  $m^2$  multipliers with respect to these ratios,

$$\epsilon(m^1, i) < \epsilon(m^2, i) < \epsilon(m^{2-1}, i).$$

Case II

Initial conditions:

(2.1) Market interest rates are at medium levels

(2.2)  $i^t$  approaches  $Q$  and  $\epsilon(t, i)$  approaches zero.

The major change in initial conditions for Case II involves the level of market interest rates. As market interest rates rise from the low levels of Case I, banks' desired holdings of excess reserves to deposits ( $e$ ) decreases, and banks' desired borrowings to deposit ratio ( $b$ ) rises. At the medium range of interest rates, the  $e$  ratio is smaller and the  $b$  ratio larger than at very low interest rates. The result is:

$$\begin{array}{ccc} \epsilon(e, i) & > & \epsilon(e, i) \\ \text{Case I} & & \text{Case II} \end{array}$$
$$\begin{array}{ccc} \epsilon(b, i) & > & \epsilon(b, i) \\ \text{Case I} & & \text{Case II} \end{array}$$

With respect to the  $t$  ratio, as market interest rates rise, the spread between  $i^f$  and the maximum yields banks can offer under constraint of  $Q$  on time deposits narrows. The ability of banks to offset the negative effect of a rise in  $i^f$  on  $t$  is constrained. Under these initial conditions:

$$\epsilon(i^t, i^f) \quad \text{decreases}$$

and:  $\epsilon(t, i)$  approaches zero.

In Case II, as interest rates rise from low levels into a medium range:

$$0 < \epsilon(m, i)_{\text{Case II}} < \epsilon(m, i)_{\text{Case I}}$$

However, as the level of market interest continues to rise and the spread between  $i^t$  and  $Q$  decreases, the numerical values of the elasticities of the multipliers with respect to the interest rate index ( $i$ ) become smaller. At the upper end of the medium range of interest rates, and as  $i^t$  comes closer to  $Q$  limits, the elasticities of all three multipliers with respect to interest rates approach zero.

$$\epsilon(m, i) \rightarrow 0$$

### Case III

Initial conditions:

(3.1) Interest rates are at high levels

(3.2)  $i^t$  is sufficiently close to  $Q$  so that  $\epsilon(i^t, i^f) = 0$

The consequence of (3.1) is that  $\epsilon(e, i)$  and  $\epsilon(b, i)$  become very small in numerical value and approach zero. Initial condition (3.2) leads to the result that  $\epsilon(t, i) < 0$ . Restating the expression:

$$\epsilon(m, i) = \epsilon(m, e) \cdot \epsilon(e, i) + \epsilon(m, b) \cdot \epsilon(b, i) + \epsilon(m, t) \cdot \epsilon(t, i)$$

we see that the first two terms of the expression are close to zero and the sign and numerical value of the term  $\epsilon(m, t) \cdot \epsilon(t, i)$  dominates the sign and numerical value of the elasticities of the multipliers with respect to the interest rate index ( $i$ ).

For the  $m^1$  multiplier, since:

$$\epsilon (m^1, t) < 0$$

$$\epsilon (t, i) < 0$$

then:  $\epsilon (m^1, i) > 0$

For the  $m^2$  and bank credit multipliers since:

$$\epsilon (m^2, t) > 0$$

$$\epsilon (t, i) < 0$$

then:  $\epsilon (m^2, i) < 0$

$$\epsilon (m^2 - 1, i) < 0$$

Also, since the elasticity of the bank credit multiplier with respect to the  $t$  ratio is greater than the elasticities of  $m^2$  and  $m^1$  with respect to  $t$  ratio:

$$|\epsilon (m^2 - 1, i)| > |\epsilon (m^2, i)| > |\epsilon (m^1, i)|$$

The initial condition (3.2):

$$i^t \text{ sufficiently close to } Q \text{ so that } \epsilon (i^t, i^f) = 0$$

has very important implications for the derivable consequences of the hypothesis. We notice that under initial condition (3.2) the sign of the elasticity of the  $t$  ratio with respect to the index of credit market interest rates ( $i$ ) is reversed from Case I and Case II. Under initial condition (3.2), the  $\epsilon (t, i)$  becomes negative.

The existence of initial condition (3.2) is a fairly recent phenomenon. Prior to 1966, every time  $i^t \rightarrow Q$  such that banks were effectively constrained from offering yields on time deposits that were competitive with yields ( $i^f$ ) on other market assets, the Federal Reserve authorities raised  $Q$ . During the period 1962 through 1965, the Regulation Q ceiling rates were raised four times. In December, 1965 when the discount rate was raised from 4 to 4-1/2 percent, the Regulation Q ceiling rate on time deposits other than passbook accounts was raised from 4 - 4-1/2 percent to 5-1/2 percent on all maturities of other time deposits. Indeed, in the period 1962-1965, the banking community came to expect that the Federal Reserve would not permit Regulation Q to effectively constrain banks from raising  $i^t$  to compete with yields on other market assets.

However, in the first half of 1966 market interest rates rose sharply. By the end of June, large commercial banks had increased the yields they were offering on other time deposits, even in the 30 day maturity range, to the Regulation Q ceiling of 5-1/2 percent. In the summer of 1966, as yields on commercial paper and Treasury bills continued to rise, the Federal Reserve held Regulation Q ceiling rates at 5-1/2% on large denomination CDs and in late July lowered the ceiling rates on multiple maturity time certificates. Not until April 19, 1968 were ceiling rates on time deposits raised, and this time only for large denomination single maturity CDs with maturities of over 60 days.

Therefore, beginning in 1966, we observe periods of time when initial condition (3.2) is in good agreement with empirical observations.



For short periods of time in the period 1966-1968 we find that the initial condition  $\varepsilon(t, i) > 0$  is not in good agreement with empirical evidence. To discuss the implications of the Brunner-Meltzer hypothesis for  $\varepsilon(m, i)$  and the responses of the monetary aggregates and bank credit to policy actions under the initial condition (3.2) we have introduced Case III.

## SECTION F

### EFFECTS OF CHANGES IN THE POLICY PARAMETERS

Changes in the policy parameters affect the magnitude of the money stock and the magnitude of the stock of bank credit in two ways:

- (1) Open market operations change the magnitude of the adjusted base  $B^a$ , and operate to alter the value of the money and bank credit multipliers.
  - (a) Changes in  $B^a$  cause changes in the composite market interest rate  $i$  and the change in  $i$  is transmitted via the dependence of the  $e$ ,  $b$ , and  $t$  ratios on  $i$  to the multipliers.
- (2) Other policy actions affect the values of the multipliers but do not affect the magnitude of the base.

We shall now trace through the Brunner-Meltzer system the impact of a change in each of the policy parameters under the control of the Federal Reserve System.

### OPEN MARKET OPERATIONS

An open market operation initially changes the magnitude of the adjusted monetary source base  $B^a$ . An open market purchase of Government securities by the Federal Reserve increases the magnitude of the base and an open market sale decreases Federal Reserve holdings

of Government securities and hence decreases the magnitude of the base. Corresponding to the open market policy-induced change in the source base is an equal change in the opposite direction of the stock of Government securities held by the banks and the public.

The Federal Reserve acquires a given volume of Government securities from banks and the public by bidding up the price of Government securities (alternatively forcing down yields on Governments) and hence induces the public and the banks to alter their portfolio of assets. At the higher prices of Government securities, the public is temporarily willing to exchange Government securities for demand deposits and the banks are willing to exchange Governments from their asset portfolios for excess reserves.

If the banks sell the entire volume of Government securities acquired by the Federal Reserve, the instantaneous impact of the open market policy action will be to increase the excess reserves to deposit ratio ( $e$ ) and hence the value of the multipliers will fall offsetting the rise in  $B^a$ . Initially there would be no change in the magnitude of the stocks of money and bank credit. If the public as well as the banks sell Government securities to the Federal Reserve, then the instantaneous effect will be to increase the  $e$  ratio and also to decrease the  $k$  ratio ( $k = \frac{C^P}{D^P}$ ). The instantaneous effect will be to increase the money supply by the amount of securities purchased by the public (i.e. the public exchanges Government securities for demand deposits).

Given  $i_G$ , banks and the public are willing to hold the smaller stock of Government securities. The interest rate on Government securities ( $i_G$ ) is however, only one interest rate entering into the determination of the composite interest rate  $i$  which determines the equilibrium values of the  $e$ ,  $b$ , and  $t$  ratios and hence the equilibrium values of the money and bank credit multipliers.

Assuming that initially commercial banks were holding the ratio of excess reserves to deposits they desired to hold given  $i$  and  $\rho$ , and assuming that individuals were holding the ratio of currency to demand deposits they desired to hold given  $c$ ,  $Y/Y_p$ ,  $1$ ,  $s$ , and  $W$ , then the policy-induced change in the asset portfolios of the commercial banks and the public will leave them holding a greater amount of excess reserves and demand deposits than they desire to hold relative to other assets, given the prevailing market interest rates on these assets. A portfolio adjustment process will follow.

In the adjustment process, the banks and the public attempt to acquire a larger portion of the stock of other assets. The commercial banks will attempt to acquire a larger stock of earning assets by inducing the public to increase their demand for commercial bank loans. As a consequence, the composite interest rate  $i$  will decrease. After the adjustment process, the yields on assets other than Government securities will have fallen and the rate on banks loans will have decreased. Banks and the public will be willing to hold the smaller total stock of securities at lower

interest rates. For the commercial banks, the likely result of the adjustment process will be that they are holding a larger volume of loans and other securities, a smaller volume of Government securities, and a larger amount of excess reserves than prior to the policy induced change in their asset portfolios. <sup>22/</sup>

The  $e$  ratio prevailing after the open market purchase falls as commercial banks switch from excess reserves to earning assets and consequently the stocks of money and bank credit rise.

If an open market purchase or sale of securities by the Federal Reserve does not affect the equilibrium values of the  $e$ ,  $b$ ,  $t$ ,  $k$ , ratios and  $r$ , (i.e. the multipliers  $m^1$  and  $m^2$  may be considered constants), and if the system was in equilibrium, the multipliers will return to their predisturbance values (their values prior to the open market operation). Following an open market purchase (sale)  $B^a$  remains at its higher (lower) magnitude, and the stocks of money and bank credit increase (decrease). The elasticity of  $M$  and  $E$  with respect to a change in  $B^a$  are equal to one. A percentage change in the base leads to an equal percentage change in the stocks of money and bank credit.

Proof:

$$\text{elasticity of } M^1 \text{ with respect to } B^a = \epsilon (M^1, B^a)$$

---

<sup>22/</sup> The final outcome of the adjustment process depends on the bidding of the banks and the public for the existing stock of securities and the willingness of the public to incur additional liabilities to the commercial banking system. Under certain conditions, the banks or the public may be unable to increase their portion of the total stock of securities and the end result will be that each sector holds the same portion of the existing stock of assets as before the open market purchase but at higher prices.

$$\epsilon (M^1, B^a) = \frac{\Delta M^1}{\Delta B^a} \cdot \frac{B^a}{M^1}$$

$$\frac{\Delta M^1}{\Delta B^a} = m^1$$

$$m^1 = \frac{M^1}{B^a}$$

$$\therefore (M^1, B^a) = \frac{M^1}{B^a} \cdot \frac{B^a}{M^1} = 1$$

likewise for  $\epsilon (M^2, B^a)$  and  $\epsilon (E, B^a)$ .

However, since the magnitudes of  $M^1$ ,  $M^2$ , and  $E$  are not the same, a given change in  $B^a$  leads to different absolute changes in  $M^1$ ,  $M^2$ , and  $E$ . If the magnitude of  $M^1$  is less than the magnitude of bank credit, a change in  $B^a$  leads to a greater magnitude of change in bank credit ( $E$ ) than  $M^1$ , even though the percentage changes are the same.

#### Interest rate effects of an open market operation.

Once we introduce the condition that changes in the source base affect interest rates, and the condition that the elasticities of the  $e$ ,  $b$ , and  $t$  ratio with respect to the interest rate are not equal to zero, then the elasticities of  $M^1$ ,  $M^2$ , and  $E$  with respect to changes in  $B^a$  are no longer equal to one. Changes in  $i$  caused by changes in  $B^a$  alter the equilibrium values of the multipliers.

Let:  $\epsilon (i, B^a)$  = elasticity of interest rates with respect to  $B^a$

$\epsilon (m, i)$  = elasticity of the multipliers with respect to  $B^a$

The effect of interest rate changes induced by changes in the source base on  $M^1$ ,  $M^2$ , and bank credit can be expressed as:

$$\epsilon (m, i) \cdot \epsilon (i, B^a)$$

The elasticities of  $M^1$ , and  $M^2$ , and  $E$  with respect to  $B^a$ , taking into account interest rate effects are as follows:

$$\epsilon (M^1, B^a) = 1 + \epsilon (m^1, i) \cdot \epsilon (i, B^a)$$

$$\epsilon (M^2, B^a) = 1 + \epsilon (m^2, i) \cdot \epsilon (i, B^a)$$

$$\epsilon (E, B^a) = 1 + \epsilon (m^{2-1}, i) \cdot \epsilon (i, B^a)$$

Writing the elasticities in this form we make explicit that the percentage changes in the monetary aggregates and bank credit depend not only upon the percentage change in  $B^a$  but also on the response of interest rates to changes in  $B^a$  and the response of the banks and public to base-induced changes in interest rates.

As we discussed in section E, the signs and numerical values of the elasticities of the multipliers with respect to the composite interest rate ( $i$ ) depend upon the relevant initial conditions about the level of interest rates and the relationship between  $i^t$  and  $Q$ . In section D we discussed that the sign and numerical value of  $\epsilon (i, B^a)$  depends upon the elasticities of the banks' asset demand and the public's asset supply functions with respect to  $i$ ; and upon whether we are considering a partial equilibrium (short-run situation) or whether we are taking into account the full feedback effects from the real sector and hence considering the long-run  $\epsilon (i, B^a)$ .

Therefore, the derivable consequences of the Brunner-Meltzer with respect to the impact of a given open market operation for the monetary aggregates and bank credit depends upon what assumptions one makes about the initial conditions under which the open market operations is carried out. When applying or testing an hypothesis, the application or test involves the

statement of the initial conditions as well as the theory embodied in the hypothesis. If, when testing an hypothesis, one of the logically derivable consequences of the hypothesis is found to not be in good agreement with empirical evidence, this is evidence for the falsification or rejection of the theory embodied in the hypothesis if and only if the assumed initial conditions are shown to be in good agreement with empirical evidence. An hypothesis consists of initial conditions conjoined with a theory. The initial conditions are an integral part of the hypothesis. If we conjoin different initial conditions to a given theory we will derive different logical consequences of the hypothesis.

For purpose of demonstrating the workings of the Brunner-Meltzer Non-Linear Money Supply Hypothesis we shall now go through three different cases and analyze the predicted short-run and long-run effects of open market operations on the monetary aggregates and bank credit under differing initial conditions. The following cases will be labeled Case I-III to correspond to Case I-III in section E where we discussed the elasticities of the monetary multipliers with respect to 1.



CASE Ia: SHORT-RUN

Initial conditions:

(1.1a) Interest rates, relative to their past levels, are low.

(1.2a)  $\epsilon(i, B^a) < 0$

(1.3a)  $i^t$  sufficiently below  $Q$  so that

$$|\epsilon(t, i^t) \cdot \epsilon(i^t, i^f)| > |\epsilon(t, i^f)|$$

Initial condition (1.2a) states that changes in the magnitude of the stock of base money result in changes in the opposite direction in short-term interest rates.

Condition (1.3a) states that  $i^t$  is sufficiently below  $Q$  so that the response of banks to rising market yields on other assets offsets the negative influence on the  $t$  ratio of the rise in market yields relative to  $i^t$ . From (1.3a) we conclude:

$$\epsilon(t, i) > 0$$

Under initial condition (1.1a) where the level of interest rates is low relative to past levels, the large numerical values of  $\epsilon(m^1, e) \cdot \epsilon(e, i)$  and  $\epsilon(m^1, b) \cdot \epsilon(b, i)$  dominate  $\epsilon(m^1, t) \cdot \epsilon(t, i)$  and consequently:

$$\epsilon(m^1, i) > 0 \text{ [see Case I, section E]}$$

Rewriting the expression for the elasticity of  $M^1$  with respect to the base:

$$\epsilon(M^1, B^a) = 1 + \epsilon(m^1, i) \cdot \epsilon(i, B^a)$$

we conclude that under the initial conditions of Case Ia:

$$\epsilon(M^1, B^a) < 1$$

For the elasticities of the  $M^2$  and bank credit multipliers, under the initial conditions of Case Ia, the Brunner-Meltzer hypothesis implies that:

$$\epsilon (m^2, i) > 0$$

[see Case I, section E]

$$\epsilon (m^{2-1}, i) > 0$$

Writing the expressions:

$$\epsilon (M^2, B^a) = 1 + \epsilon (m^2, i) \cdot \epsilon (i, B^a)$$

$$\epsilon (E, B^a) = 1 + \epsilon (m^{2-1}, i) \cdot \epsilon (i, B^a)$$

we conclude that, since the sign of the product of the last two terms is negative,

$$\epsilon (M^2, B^a) < 1$$

$$\epsilon (E, B^a) < 1$$

Relative responses of  $M^1$ ,  $M^2$  and bank credit to changes in  $B^a$ . As we have shown in Case I, section E, under the initial conditions of Case I:

$$\epsilon (m^1, i) < \epsilon (m^2, i) < \epsilon (m^{2-1}, i).$$

Comparing our expressions for the elasticities of  $M^1$ ,  $M^2$ , and E with respect to  $B^a$  we have:

$$\epsilon (M^1, B^a) > \epsilon (M^2, B^a) > \epsilon (E, B^a)$$

Under the initial conditions of Case Ia, low interest rate regime where banks are not constrained by Regulation Q, a one per cent change in the monetary source base results in a less than one per cent change in the same direction of  $M^1$ ,  $M^2$ , and bank credit. Interest rate effects of a change in the magnitude of base money supplied by the central bank operate to dampen the increase (decrease) in the monetary aggregates resulting from an increase (decrease) in base money.

The relative responses of  $M^1$ ,  $M^2$ , and E to changes in  $B^a$  are not the same. Bank credit and  $M^2$  are less sensitive to changes in  $B^a$  than is  $M^1$ . A one per cent increase (decrease) in  $B^a$  leads to a larger percentage increase (decrease) in  $M^1$  than  $M^2$  and a larger percentage increase (decrease)

in  $M^2$  than E.<sup>23/</sup>

The difference in the relative responses of  $M^1$ ,  $M^2$ , and E to changes in the magnitude of  $B^a$  hinges on the difference in the responses of the multipliers to changes in interest rates. The elasticities of  $m^2$  and  $m^2-1$  with respect to the  $t$  ratio are opposite in sign to  $m^1$ . Also, the numerical value of the elasticity of  $m^2-1$  with respect to  $e$  is greater than  $m^2$  or  $m^1$ . Under a regime of extremely low levels of interest rates banks tend to be quite sensitive to a small percentage increase in interest rate. A policy induced decrease in  $B^a$ , resulting in an increase in interest rates may result in marked decline in banks' desired excess reserve to deposit ratio ( $e$ ) and a rise in their desired  $t$  ratio. The interest rate effects of the decrease in  $B^a$  may operate to offset much of the base-induced decrease in bank credit. However, to the extent that  $\epsilon(m^1, e)$  is less than  $\epsilon(m^2-1, e)$  and to the extent that the  $t$  ratio rises with increases in market yields, the base-induced percentage decrease in  $M^1$  is greater than the percentage decrease in bank credit and  $M^2$ .

---

<sup>23/</sup> The difference in relative responses of  $M^1$ ,  $M^2$ , and E also depends upon  $\epsilon(i, B^a)$ . As the  $\epsilon(i, B^a)$  approaches zero, the elasticities of  $M^1$ ,  $M^2$ , and E all approach one.

CASE Ib: LONG-RUN

In Case Ib we retain initial conditions (1.1a) and (1.3a). The only change we make is that we now take into account the longer-run interest rate effects resulting from a change in  $B^a$ . We state our new condition:

$$(1.2b) \quad \epsilon(i, B^a) > 0$$

The change in initial condition (1.2) reflects our discussion in section D of short-run and long-run interest rate effects of a change in  $B^a$ .

Using condition (1.2b) we can now trace through the impacts on  $M^1$ ,  $M^2$ , and  $E$  of a change in  $B^a$ . For  $M^1$ , we now have the situation that:

$$\epsilon(m^1, i) > 0$$

$$\epsilon(i, B^a) > 0$$

therefore  $\epsilon(m^1, i) \cdot \epsilon(i, B^a) > 0$

and:  $\epsilon(M^1, B^a) > 1$

For  $M^2$  and bank credit we have a similar result:

$$\epsilon(m^2, i) > 0$$

$$\epsilon(i, B^a) > 0$$

and:  $\epsilon(M^2, B^a) > 1$

$$\epsilon(E, B^a) > 1$$

In Case Ia we had the result that the short-run base induced changes in interest rates acted to dampen the response of the monetary aggregates and bank credit to changes in the source base. In Case Ib, as a result of altering initial condition (1.2), the long-run interest rate effects of a change in  $B^a$  act to increase the elasticities of  $M^1$ ,  $M^2$ , and  $E$  with respect to  $B^a$ . In the short-run the base induced decrease in  $i$  acts to dampen

the percentage changes in these aggregates resulting from an increase in  $B^a$ . Over the longer-run, taking the effects of the increase in monetary aggregates on total spending into account and the feedback from the real sector to the bank credit market into account, interest rates on the bank credit market rise, resulting in a rise in the multipliers and hence a further increase in the monetary aggregates and bank credit.

CASE IIa: SHORT-RUN

Initial Conditions:

(2.1a) Interest rates relative to their past levels are at about their average level. Interest rate levels are in a medium range.

(2.2a)  $\epsilon(i, B^a) < 0$

(2.3a)  $i^t$  approaches  $Q$  and the numerical value of  $\epsilon(i^t, i^f)$  decreases.

Case IIa, like Case Ia is a short-run analysis. This is reflected in initial condition (2.2a) which is identical to initial condition (1.2a).

In Case II we alter the initial conditions concerning the level of interest rates and the relationship between the yields offered by banks on time deposits ( $i^t$ ) and the ceiling rates ( $Q$ ) imposed by the Federal Reserve System.

The result of imposing initial conditions (2.1a) and (2.3a) is that the numerical value of the interest elasticities of the multipliers is decreased as compared to the situation under initial condition (1.1a) and (1.3a). As we discussed in section E, as interest rates rise from low levels into the medium range of interest rates, and as  $i^t$  approaches  $Q$  ceilings,  $\epsilon(e, i)$ ,  $\epsilon(b, i)$  and  $\epsilon(t, i)$  decrease in numerical value. At the upper end of the medium range of interest rates, and as  $i^t$  comes close to  $Q$ :

$\epsilon(e, i)$ ,  $\epsilon(b, i)$  become small in numerical value

$\epsilon(t, i)$  approaches zero.

Consequently, under these initial conditions:

$\epsilon(m, i)$  approaches zero.

Restating, our expressions:

$$\epsilon (M^1, B^a) = 1 + \epsilon (m^1, i) \cdot \epsilon (i, B^a)$$

$$\epsilon (M^2, B^a) = 1 + \epsilon (m^2, i) \cdot \epsilon (i, B^a)$$

$$\epsilon (E, B^a) = 1 + \epsilon (m^{2-1}, i) \cdot \epsilon (i, B^a)$$

we conclude that as interest rates rise from low levels to medium levels and  $i^t \rightarrow Q$ , the elasticities of the monetary aggregates and bank credit with respect to  $B^a$  increase. In the case where

$$\epsilon (m, i) = 0$$

then:  $\epsilon (M^1, B^a), \epsilon (M^2, B^a), \epsilon (E, B^a) = 1$

Under the initial conditions of Case IIa, as the level of market rates rises, short-run interest rate effects induced by changes in the supply of base money have progressively less of a dampening effect on the percentage change in  $M^1, M^2$ , and  $E$  for a given percentage change in  $B^a$ . Also, we note that as  $\epsilon (m, i) \rightarrow 0$  the percentage responses of the monetary aggregates and bank credit approach equality. Precisely, the difference in the percentage changes in  $M^1, M^2$ , and  $E$  generated by an open market operation are due to different responses of the multipliers to given interest rate changes. As the elasticities of the multipliers with respect to  $i$  approach zero, the divergent percentage changes in  $M^1, M^2$ , and  $E$  disappear.

CASE IIb: LONG-RUN

For Case IIb we shall retain initial conditions (2.1a) and (2.3a). In Case IIb, as in Case Ib we now take into consideration the long-run interest rate effects of changes in the magnitude or rate at which base money is supplied to the economy. This is reflected by changing initial condition 2.2 to:

$$(2.2b) \quad \epsilon(i, B^a) > 0$$

Using initial condition (2.2b) conjoined with initial conditions (2.1a) and (2.3a) we can now trace the implications of the Brunner-Meltzer hypothesis for the impact of open market operations on  $M^1$ ,  $M^2$ , and bank credit.

For  $M^1$ ,  $M^2$ , and bank credit we now have the situation that:

$$\epsilon(i, B^a) > 0$$

$$\epsilon(m, i) \rightarrow 0$$

Checking the expressions for the elasticities of the monetary aggregates and bank credit with respect to  $i$ , we see that as:

$$\epsilon(m, i) \rightarrow 0$$

$$\epsilon(m, i) \cdot \epsilon(i, B^a) \rightarrow 0$$

and hence:

$$\epsilon(M, B^a) \rightarrow 1$$

$$\epsilon(E, B^a) \rightarrow 1$$

These results lead to the interesting implication, that, although in Case IIb we reversed initial condition (2.2) about the  $\epsilon(i, B^a)$  from Case IIa, if we retain our initial conditions about the level of interest rates and the relationship between  $i^t$  and  $Q$ , we get the same results under



Case IIb as under Case IIa. In both cases, although the short and long-run effects of changes in  $B^a$  on  $i$  may be opposite in direction, to the extent that initial conditions (2.1) and (2.3) hold, as interest rates rise to the upper end of their medium range, the total interest elasticity of the multipliers decrease approaching zero, and the feedback effect of base-induced changes in interest rates on the multipliers become very small.

CASE IIIa: SHORT-RUN

Initial conditions:

(3.1a) Interest rates relative to their past levels are high.

(3.2a)  $\epsilon(i, B^a) < 0$

(3.3a)  $i^t$  sufficiently close to  $Q$  so that  $\epsilon(i^t, i^f)$  approaches zero.

Initial condition (3.3a) states that the yields offered by banks on time deposits are sufficiently close to the Regulation Q ceiling rates such that the ability of banks to raise  $i^t$  in response to an increase in yields ( $i^f$ ) on other market assets is constrained. The result of (3.3a) is to lead to the conclusion that:

$$|\epsilon(t, i^f)| > |\epsilon(t, i^t) \cdot \epsilon(i^t, i^f)|$$

and hence:  $\epsilon(t, i) < 0$

Under initial condition (3.3a) the elasticity of  $t$  with respect to  $i$  has an opposite sign (negative) compared to our previous two cases. For Case I initial conditions, interest rates are at low levels and  $i^t$  is sufficiently below  $Q$  so that  $\epsilon(t, i)$  is positive. In Case II initial conditions, as market rates rise and  $i^t$  approaches  $Q$ , the  $\epsilon(t, i)$  remains positive but decreases in numerical value. As  $i^t$  approaches closer to  $Q$ ,  $\epsilon(i^t, i^f)$  decreases and  $\epsilon(t, i) \rightarrow 0$ . Under the initial conditions of Case III, because of Regulation Q constraints,  $\epsilon(i^t, i^f)$  becomes close to zero and the sign of the elasticity of  $t$  with respect to  $i$  is reversed, becoming negative.

Under a regime of high levels of interest rates (3.1), the elasticities of the  $e$  and  $b$  ratio with respect to interest rates become small.

The first two terms in the expression

$$\begin{aligned}\epsilon(m, i) &= \epsilon(m, e) \cdot \epsilon(e, i) + \epsilon(m, b) \cdot \epsilon(b, i) \\ &\quad + \epsilon(m, t) \cdot \epsilon(t, i)\end{aligned}$$

approach zero. The last term dominates the expression  $\epsilon(m^1, i)$ . Since, under Case III initial conditions, the elasticity of  $t$  with respect to  $i$  is negative, and as previously  $\epsilon(m^1, t) < 0$  then:

$$\epsilon(m^1, i) > 0$$

and we conclude:

$$\epsilon(M^1, B^a) < 1$$

For the  $M^2$  and bank credit multipliers we conclude that since:

$$\epsilon(m^2, t) > 0, \epsilon(t, i) < 0$$

then: 
$$\epsilon(m^2, i) < 0$$

From the expressions:

$$\epsilon(M^2, B^a) = 1 + \epsilon(m^2, i) \cdot \epsilon(i, B^a)$$

$$\epsilon(E, B^a) = 1 + \epsilon(m^2-1, i) \cdot \epsilon(i, B^a)$$

we conclude:

$$\epsilon(M^2, B^a) > 1$$

$$\epsilon(E, B^a) > 1$$

Relationship between elasticities of  $M^2$  and bank credit with respect to  $B^a$ .

Under the initial conditions of Case IIIa:

$$\text{since: } \epsilon(m^2, i), \epsilon(m^2-1, i) < 0$$

and as we have shown previously:

$$\epsilon (m^2-1, i) > \epsilon (m^2, i)$$

by comparing the expressions for  $\epsilon (M^2, B^a)$  and  $\epsilon (E, B^a)$  we conclude:

$$\epsilon (E, B^a) > \epsilon (M^2, B^a)$$

Under the initial conditions of Case IIIa, the impact effect of an increase in the supply of base money is to lower credit market interest rates. The decrease in credit market interest rates makes time deposits more attractive vis-a-vis other assets. The public's demand for time deposits increase and the  $t$  ratio rises. The increase in  $t$  raises the equilibrium values of the bank credit and  $M^2$  multipliers and reinforces the increase in  $M^2$  and  $E$  given the increase in  $B^a$ . The rise in  $t$  lowers the  $M^1$  multiplier and this process dampens the rate of increase of  $M^1$  given an increase in  $B^a$ .

When the supply or rate at which base money is supplied by the central bank is decreased, the impact effect on  $i$  is to raise credit market yields. Because banks' offering rates on time deposits are at  $Q$  ceilings, no change in  $i^t$  occurs, the  $t$  ratio decreases. The decrease in  $t$  results in a decline in  $m^2$  and  $m^2 - 1$ , and this decline reinforces the rate of decrease of  $M^2$  and bank credit given the decrease in  $B^a$ . The fall in  $t$  raises the money multiplier ( $m^1$ ) and this change in  $m^1$  acts to dampen the rate of decrease of  $M^1$  given the decrease in  $B^a$ .

CASE IIIb; LONG-RUN

In Case IIIb we retain the initial conditions (3.1a) and (3.3a) but replace (3.2a) with:

$$(3.2b) \quad \epsilon(i, B^a) > 0$$

Under Case IIIb initial conditions, since:

$$\epsilon(m^1, i) > 0$$

$$\epsilon(i, B^a) > 0$$

then:

$$\epsilon(M^1, B^a) > 1$$

For  $M^2$  and bank credit the result of replacing initial condition (3.2a) with (3.2b) is that, since due to the constraint of Regulation Q:

$$\epsilon(m^2, i), \epsilon(m^2-1, i) < 0$$

then using condition (3.2b)

$$\epsilon(i, B^a) > 0$$

we conclude:

$$\epsilon(M^2, B^a) < 1$$

$$\epsilon(E, B^a) < 1$$

Under the initial conditions of Case IIIb the long-run interest rate effects of changes in the source base operate, as in Case Ib, to increase the elasticity of  $M^1$  with respect to  $B^a$ . However, in Case IIIb, due to the constraint of Regulation Q, the long-run interest rate effects of a change in  $B^a$  operate to decrease the elasticity of  $M^2$  and  $E$  with respect to  $B^a$ . This reverses our conclusions from Case Ib. The reason for the difference in results lies in initial conditions (1.3) and (3.3). In Case I banks were able to vary  $i^t$  to offset the negative influence of a rise in  $i^f$  on the  $t$  ratio. In Case IIIb under (3.3) banks were constrained from raising  $i^t$  and  $\epsilon(i^t, i^f) \rightarrow 0$  hence  $\epsilon(t, i)$  reverses sign and becomes negative. In Case

IIIb the long-run interest rate effects of an open market purchase operate to dampen the base-induced increase in  $M^2$  and bank credit, but operate to reinforce the base-induced increase in  $M^1$ .

Summary. The discussion of these three cases points out that, under differing initial conditions, the effects of open market operations on the growth rates of the monetary aggregates and bank credit will be different. In the short-run analysis, a given percentage change in  $B^a$  leads to a greater percentage change in  $M^1$  under conditions of Case IIa -- interest rates in a medium range and  $\epsilon (m^1, i)$  close to zero -- than for the initial conditions of Case Ia, or Case IIIa. For bank credit and  $M^2$ , a percentage change in  $B^a$ , in the short-run, leads to the greatest percentage change in  $M^2$  and bank credit under Case IIIa conditions, the next greatest under Case IIa conditions, and the smallest percentage change occurs under Case Ia conditions. Taking into account long-run effects of changes in  $B^a$  on  $i$ , these results for  $M^2$  and bank credit are reversed. In the long-run, changes in  $B^a$  lead to the greatest percentage changes in  $M^2$  and  $E$  under the initial conditions of low interest rates and  $i^t$  sufficiently below  $Q$  (Case Ib) and the smallest percentage change under initial conditions of high interest rates and the constraint  $Q$  (Case IIIb).

The analysis shows that the percentage changes in  $M^1$ ,  $M^2$ , and bank credit resulting from open market operations are not identical under all three cases. Only under Case II, as  $\epsilon (m, i)$  becomes very near to zero, do we reach the conclusion that the percentage changes in  $M^1$ ,  $M^2$ , and  $E$  resulting from a percentage change in  $B^a$  would be approximately equal. Observed divergent rates of growth of  $M^1$  and  $M^2$  and bank credit resulting from open market operations by the central bank are completely consistent with the derivable implications of the Brunner-Meltzer hypothesis. The hypothesis also provides the initial conditions under which such divergent patterns would occur.

RESERVE REQUIREMENTS AND THE REDISCOUNT RATE

In the Brunner-Meltzer system a change in member bank weighted average legal reserve requirements ( $r^d$ ,  $r^t$ ) and changes in the rediscount rate ( $\rho$ ), have their impact on the stocks of money and bank credit by altering the desired ratios of the commercial banks and the public and hence altering the equilibrium values of the money and bank credit multipliers.

A change in reserve requirements alters the magnitude of reserves that commercial banks desire to hold given  $D^p + D^t + T$ . The reaction of commercial banks to the policy induced change in their desired holdings of reserves and the resulting adjustment process leads to changes in the magnitudes of the stocks of money and bank credit. A change in the rediscount rate also alters the ratio of borrowing from the Fed to deposits (b) that banks desire to maintain. A change in  $r^d$ ,  $r^t$ , or  $\rho$  does not alter the magnitude of the adjusted base  $B^a$  (note:  $B^a$  has been adjusted by removing member bank borrowings from the broader concept of the base B--see basic relations 1a - 1b).

The initial effect of a change in  $r^d$  or  $r^t$  is to cause a change in the magnitude of required reserves (see basic relation 2b). A change in the magnitude of required reserves will alter the magnitude of reserves banks desire to hold (see basic relation 2a). In terms of the multipliers, a change in reserve requirements will be reflected in a change in  $r$ , where  $r$  is defined:



$$r = u\delta r^d + (1 - u) \tau r^l + v + e$$

For example, an increase in reserve requirements leads to portfolio adjustment process on the part of the commercial banking system. Given their existing liabilities, banks are no longer satisfied with the structure of their asset portfolios. They attempt to restructure their asset portfolios to contain more base money (reserves) and a smaller amount of earning assets relative to their deposit liabilities. This adjustment process leads to a decrease in the stock of bank credit and money. The increase in reserve requirements appears in the multiplier as an increase in  $r$  and hence a decrease in the equilibrium values of the multipliers. With  $B^a$  (or the growth rate of  $B^a$ ) unchanged, this leads via the adjustment process to smaller magnitudes (or slower rate of growth) of money and bank credit.

In the Brunner-Meltzer system, the rediscount rate appears in the bank behavioral relations specifying the banks' desired ratios of borrowings to deposits ( $b$ ) and excess reserves to deposits ( $e$ ). A change in  $\rho$  causes a change in these desired ratios and hence causes a behavioral reaction by commercial banks which alters the equilibrium values of the money and bank credit multipliers.

One way in which member commercial banks may obtain reserves is by borrowing from the Federal Reserve Bank. The cost associated with this channel of obtaining reserves is represented by the rediscount rate  $\rho$ . Alternatively commercial banks may hold excess reserves (non-earning assets) to meet expected or unexpected reserve losses. The commercial banks are viewed as making the decision on their

desired  $\underline{e}$  and  $\underline{b}$  ratios based on the relative costs of holding reserves in the form of borrowed reserves or excess reserves. As  $\rho$  rises, the cost of holding borrowed reserves rises and hence they become less attractive relative to excess reserves. Alternatively, a rise in  $\rho$  may be considered as a factor decreasing the opportunity cost of holding excess reserves. Also, as  $\rho$  rises the cost to member banks of obtaining base money to support deposits by borrowing from the Federal Reserve Banks rises and the desired ratio of borrowings to deposits declines.

An increase in  $\rho$  increases the commercial banks desired ratio of excess reserves to deposits and decreases their desired borrowings to deposit ratio. In the portfolio adjustment process following the policy induced change in  $\rho$ , commercial banks attempt to increase their holdings of excess reserves relative to deposits and attempt to reduce their borrowings from the Federal Reserve. In the credit market, the commercial banks attempt to shift part of their holdings of securities to the public and/or reduce their flow of credit to the public sector. The credit market interest rate  $i$  rises in response to this adjustment process.

In terms of the multipliers, the increase in  $\underline{e}$  leads to an increase in  $\underline{r}$ , and combined with the decrease in  $\underline{b}$ ,  $(r - b)$  increases. Consequently, the equilibrium values of the multipliers decrease and the impact of an increase in  $\rho$  is to lead via the adjustment process to a decline in the magnitude of the stock of money and

bank credit. In the case of a decrease in  $\rho$ ; the equilibrium value of  $\underline{e}$  decreases and  $\underline{b}$  increases and the equilibrium values of the multipliers rise, leading via the adjustment process to an increase in the stock of money and bank credit.

The responses of the stocks of money and bank credit to changes in member bank reserve requirements ( $r^d, r^t$ ) and changes in the rediscount rate ( $\rho$ ) may be expressed in terms of elasticities:

$$\begin{aligned} \epsilon (M^j, x) &= \epsilon (m^j, x) & j &= 1, 2 \\ \epsilon (E, x) &= \epsilon (m^2 - 1, x) & x &= r^d, r^t, \rho \end{aligned}$$

Since a change in  $r^d, r^t, \rho$  does not affect  $B^a$ , the percentage change in money and bank credit resulting from a percentage change in one of these policy parameters will depend on the percentage change in the multipliers resulting from the percentage change in the policy parameter.

In the case where the interest rate  $i$  is unaffected by changes in these policy parameters and/or where the values of the multipliers are not affected by interest rate changes, Brunner-Meltzer show that in their system:

$$\epsilon (E, x) = \epsilon (m^j, x) \alpha$$

The elasticity of bank credit with respect to one of the policy parameters is different from the elasticity of the stocks of money by a factor  $\alpha$ , where:

$$\alpha = \frac{M^2}{E}$$

Since

$$\frac{M^2}{E} = \frac{m^2 B^a}{(m^2 - 1) B^a}$$

then  $\alpha$  must be greater than one unless  $B^a < 0$ . Since these conditions

are not possible they conclude that  $\alpha > 1$ . Then, holding interest rates constant, a percentage change in one of the policy parameters should cause a larger percentage change in bank credit than in the stocks of money.

Since an increase in one of the policy parameters  $r^d$ ,  $r^t$ ,  $\rho$  leads to a decrease in the values of the multipliers via the adjustment process, we specify:

$$\begin{aligned}\epsilon(M, x) &< 0 & x &= r^d, r^t, \rho \\ \epsilon(E, x) &< 0\end{aligned}$$

#### Interest rate effects.

The policy parameters  $r^d$ ,  $r^t$ , and  $\rho$  also enter into the determination of the interest rate  $i$ . Considering interest rate effects of changes in  $r^d$ ,  $r^t$ , and  $\rho$  we may write the elasticities of  $M^1$ ,  $M^2$ , and  $E$  as:

$$\begin{aligned}\epsilon(M, x) &= \epsilon(m, x) + \epsilon(m, i) \cdot \epsilon(i, x) \\ \epsilon(E, x) &= \epsilon(m^2-1, x) + \epsilon(m^2-1, i) \cdot \epsilon(i, x) \quad x = r^d, r^t, \rho\end{aligned}$$

From our discussion of the adjustment process we specify:

$$\begin{aligned}\epsilon(i, x) &> 0 \\ \epsilon(m^1, x), \epsilon(m^2, x), \epsilon(m^2-1, x) &< 0\end{aligned}$$

Referring to our discussion of the elasticity of the multipliers with respect to  $i$  given in section E, we see that under conditions of low levels of interest rates and  $i^t$  sufficiently below  $Q$  (Case I):

$$\epsilon(m, i) > 0.$$

Therefore, under these conditions, interest rate effects of changes in  $r^d$ ,  $r^t$ , and  $\rho$  operate to decrease the numerical value of  $\epsilon(M, x)$  and  $\epsilon(E, x)$ .

Under conditions of Case II,

$$\epsilon (m, i) \rightarrow 0$$

then:

$$\epsilon (M, x) \rightarrow \epsilon (m, x)$$

$$\epsilon (E, x) \rightarrow \epsilon (m^2-1, x)$$

Under conditions of high levels of interest rates and the constraint of Q (Case III)

$$\epsilon (m^1, i) > 0$$

$$\epsilon (m^2, i), \epsilon (m^2-1, i) < 0.$$

In this case, interest rate effects of changes in  $r^d$ ,  $r^t$ , and  $\rho$  operate to increase the numerical value of  $\epsilon (M^2, x)$  and  $\epsilon (E, x)$ ; but decrease  $\epsilon (M^1, x)$ .

In summary, we have the result that, considering the interest rate effects of changes in  $r^d$ ,  $r^t$ , and  $\rho$

$$\epsilon (M, x), \epsilon (E, x) < 0.$$

At low levels of interest rates, changes in these policy parameters have relatively more effect on  $M^1$  than  $M^2$  or bank credit. At medium levels of interest rates, as  $i^t \rightarrow Q$ , interest rate induced effects of changes in these parameters on the multipliers become small and the percentage change in  $M^1$ ,  $M^2$ , and  $E$  depend upon the elasticities of the multipliers with respect to  $r^d$ ,  $r^t$ , and  $\rho$ .

At high levels of interest rates; as  $i^t \rightarrow Q$  and  $\epsilon (t, i) < 0$ , a change in  $r^d$ ,  $r^t$ , or  $\rho$  leads to a larger percentage change in  $M^2$  and bank credit than in  $M^1$ . The largest percentage change occurs in bank credit. This result follows from the increase in  $i$  resulting from the portfolio adjustment by the banks to an increase in  $r^d$ ,  $r^t$ , and  $\rho$ , leads to decline in the  $t$  ratio. This is reflected in a decrease in  $m^2$  and  $m^2-1$ , but a rise in  $m^1$ . The fall in the  $t$  ratio reinforces the decline in the  $m^2$  and  $m^2-1$  multipliers resulting from an increase in the policy parameters; but the decline in  $t$  dampens the decrease in  $m^1$ .

## SECTION G

### HISTORICAL EXAMPLES

In this section we shall present two examples covering recent periods to illustrate the above cases.

Example I: An Example of Case IIIa

Initial conditions :

- (1) Interest rates at high levels
- (2)  $\epsilon (i, B^a) < 0$
- (3)  $i^t$  sufficiently close to  $Q$  so that  
$$\epsilon (i^t, i^f) \rightarrow 0$$

In the seven month period from November, 1967 through June, 1968 the adjusted monetary source base grew at a 4.7 per cent annual rate, compared to a 6.5 per cent annual rate over the previous five months. Credit market interest rates rose sharply. As an illustration, Treasury bill rates rose from 4.64 per cent in early November to 5.35 per cent in June. Over this period, the growth rate of  $M^1$  moderated to 6.1 per cent annual rate, compared to a 7 per cent rate over the previous five months. The growth rates of money defined to include time deposits and bank credit fell much more sharply. The growth rate of  $M^2$  declined to 6 per cent, compared to an annual growth rate of 10.2 per cent over the previous five months. Bank credit grew at a 8.4 per cent rate compared to a 11.8 per cent rate in the previous period.

As yields on other credit market assets continued to rise, banks found that Regulation Q ceiling rates effectively limited their ability to acquire and hold time deposits. In this period the growth of time deposits fell to a 5.9 per cent annual rate compared to a 13.6 per cent rate over the

previous five month period. This was reflected in the  $t$  ratio. In November, 1967 the  $t$  ratio was 1.291, in June, 1968 the  $t$  ratio was 1.294. By comparison, the  $t$  ratio rose from 1.263 in June, 1967 to 1.291 in November. As the  $\epsilon (i^t, i^f)$  approached zero, and the rate of increase of  $B^a$  slowed, this was reflected in a much slower growth of  $M^2$  and bank credit, and a smaller reduction in the growth rate of  $M^1$ .

Over the period June, 1968-October, 1968 the growth rate of the base accelerated sharply to 7.6 per cent. The rise of other credit market interest rates slowed and by October they were below their June levels. The growth rate of  $M^1$  slowed to an annual rate of 4.6 per cent. The growth of  $M^2$  and bank credit, however, accelerated to rates of 11.7 per cent and 15.8 per cent respectively. During this period time deposits grew at a 19 per cent annual rate, about three times as rapid as over the previous seven months. The  $t$  ratio rose from 1.294 June, 1968 to 1.353 October, 1968. The difference in the growth rates of  $M^1$ , and  $M^2$  and bank credit during this period partly reflects the rapid rise in time deposits. As time deposits increased rapidly this was reflected in different relative changes in the  $m^1$  and  $m^2$  multipliers and hence in the relative growth rates of the monetary aggregates and bank credit.

From October, 1968 through January, 1969 credit market interest rates again rose sharply. Treasury bill rates which averaged 5.26 in early October rose to 6.16 in early January. During this period the adjusted base increased at a slower rate of 6.1 per cent. In this period the relationship between the growth rates of  $M^1$  and  $M^2$  and  $E$  were reversed from the previous four month period.  $M^1$  grew at an annual rate of 7.6 per cent

compared to 4.6 per cent over the previous four months.  $M^2$ , however, grew at a 6.8 per cent rate relative to an 11.7 per cent rate in the previous period. The growth rate of bank credit also decelerated, growing at an 11.9 rate compared to a previous rate of 15.8 per cent. As the spread between interest rates on other credit market financial assets such as Treasury bills and commercial paper rose well above the rate ( $i^t$ ) banks were able to offer under Regulation Q ceilings on time deposits, the growth rate of time deposits decreased markedly. In the three month period from October through January, time deposits rose at only a 6.2 per cent annual rate compared to a 19 per cent rate over the previous four months. The elasticity of  $i^t$  with respect to  $i^f$  was equal to zero. The  $t$  ratio declined slightly, going from 1.353 in October, 1968 to 1.349 in January, 1969. With the adjusted source base expanding at a rapid pace, the slower growth rate of time deposits led to a greater percentage change in the  $m^1$  multiplier than the  $m^2$  multiplier and hence a marked slowing resulted in the growth rates of  $M^2$  and bank credit while money ( $M^1$ ) accelerated its rate of expansion.



EXAMPLE II: LONG-RUN ANALYSIS

Initial conditions:

(1) Interest rates at medium to high levels

(2)  $\epsilon(i, B^a) > 0$

(3)  $\epsilon(t, i) > 0$

From initial conditions (1) and (3) we derive the consequences that:

$$\epsilon(m^1, i) < 0$$

$$\epsilon(m^2, i) > 0$$

$$\epsilon(m^2-1, i) > 0$$

Conjoining these results with condition (2) we derive:

$$\epsilon(M^1, B^a) < 1$$

$$\epsilon(E, B^a) > \epsilon(M^2, B^a) > 1$$

As we have shown in the previous sections, an analysis of the effect of an open market policy action on the relative changes in the major monetary aggregates and bank credit is not a simple procedure. Consideration must be given to the short- and long-run interest rate effects of a change in the supply of base money, and the responses of the banks and the public to interest rate changes. Frequently, what we may observe may be a combination of the separate cases we have outlined above.

During any period of time we may observe both the immediate (short-run) effects of a change in the source base on credit market interest rates and the feedback and long-run effects of previous periods' changes in  $B^a$  on interest rates. The net effect of the short- and long-run effects of changes in  $B^a$  in the present period depends on the direction of change and rates of growth of  $B^a$  in previous periods and the size of  $\epsilon(i, B^a)_S$  and  $\epsilon(i, B^a)_L$ .

In some cases, empirical evidence indicates that the feedback and long-run effect of changes in  $B^a$  (i.e.  $\epsilon (i, B^a)_L > 0$ ) appear to have overridden the short-run effects of changes in  $B^a$  on interest rates (i.e.  $\epsilon (i, B^a)_S < 0$ ).

This situation appears to have existed over the last five years. Except for a period including the last few months of 1966 and the first six months of 1967, credit market interest rates have for the period as a whole risen sharply. During the period 1964-1968 the monetary source base has also expanded markedly.

Table II

Credit Market Interest Rates and  
The Adjusted Monetary Source Base ( $B^a$ ).  
(Yearly averages of monthly figures)

	Prime Commercial	Market yields on		Adjusted
	Paper 4-6 month maturity	3 month maturity	Treasury bills 6 month maturity	Monetary Source Base (millions \$)
1964	3.97	3.54	3.68	\$54,818
1965	4.38	3.95	4.05	57,556
1966	5.55	4.85	5.06	60,798
1967	5.10	4.30	4.61	64,398
1968	5.90	5.33	5.48	68,756

The decline in the yearly average figures for credit market interest rates in 1967 reflects the decline in these yields in the first half of 1967. Following the "credit crunch" of August, 1966 and concurrent with the so-called mini-recession of late 1966 and the first quarter of 1967 credit market yields declined sharply. By the end of the second quarter of 1967 the economy was beginning a new stage of expansion. After June the rate of increase of the monetary source base accelerated. Beginning in July, 1967 credit market interest rates reversed their downward trend and began to rise sharply. By the end of 1967 yields on commercial paper and Treasury bills were at or above

their average yields for 1966.

If policy actions by the monetary authorities and changes in interest rates had an equal effect on both the money ( $m^1$ ) and ( $m^2$ ), and bank credit ( $m^2-1$ ) multipliers the ratio of these aggregates would remain constant. Over the last five years, however, these ratios have not remained constant. As shown by the table below, the ratio of  $E/M^1$  has risen sharply. Also, the ratio of  $M^2/M^1$  has risen. The ratio of  $M^2/E$  has fallen. The fall in the ratio  $M^2/E$ , implying a more rapid growth of bank credit than  $M^2$ , is consistent with our previous conclusions that the elasticity of bank credit with respect to the source base is greater than the elasticity of money defined to include time deposits with respect to  $B^a$ .

Table III

Yearly ratios of  $M^1$ ,  $M^2$  and bank credit<sup>24/</sup>

	$M^2/M^1$	$E/M^1$	$M^2/E$
1964	1.764	1.433	1.231
1965	1.847	1.514	1.219
1966	1.907	1.580	1.207
1967	1.982	1.651	1.201
1968	2.024	1.722	1.175

---

<sup>24/</sup> These ratios are calculated from averages of monthly figures. For  $M^1$  and  $M^2$  the data are taken from Annual Triangles of U.S. Economic Data, available from Research Department Federal Reserve Bank of St. Louis. The figures for E are calculated by taking total bank credit and subtracting Treasury deposits and capital accounts.

As this case indicates, we must not only consider the growth rate of the monetary source base and the  $\epsilon (i, B^a)$ , but we must also consider other key factors such as  $\epsilon (t, i)$ . As the table below illustrates, the initial condition that  $\epsilon (t, i) > 0$  appears, on average over the five year period considered, to be in good agreement with empirical evidence.

Table IV

Yearly Averages of Monthly Figures for  
the t ratio

1964	.972
1965	1.080
1966	1.164
1967	1.264
1968	1.320

For the two years 1964 and 1965, the condition that:

$i^t$  sufficiently below  $Q$  so that  $\epsilon (i^t, i^f) > 0$

is in good agreement with a study of the historical background conditions of this period. As discussed on page 61, during 1964 and 1965 as  $i^t \rightarrow Q$  the Federal Reserve raised  $Q$  so that  $\epsilon (i^t, i^f)$  remained positive.

For the period 1966-1968, as we pointed out under Case III section E, the Federal Reserve did not raise  $Q$  as  $i^f$  rose. In Historical Example I, we saw that for short periods of time during this period, as  $i^t \rightarrow Q$  and  $i^f$  continued to rise,  $\epsilon (i^t, i^f) \rightarrow 0$ . During this three year period, if we examine the relationship between changes in the yields on commercial paper and Treasury bills and the t ratio, we find clear evidence of the constraint of  $Q$ . We observe that during periods when the yields on Treasury bills and commercial paper rise sharply, the t ratio declines or remains approximately constant. During periods when the yields on these

other market assets decline, the  $t$  ratio increases.

What we observe in the three year period 1966-1968 is a switching between Case II and Case III initial conditions. In periods where Case II initial conditions hold:

$$\epsilon(i^t, i^f) > 0$$

$$\epsilon(t, i) > 0.$$

With an expanding supply of base money,  $M^1$ ,  $M^2$ , and bank credit expand. The  $t$  ratio rises and  $M^2$  and bank credit grow at more rapid rates than  $M^1$ . As market interest rates rise and  $i^t \rightarrow Q$ :

$$\epsilon(t, i) \rightarrow 0$$

$$\epsilon(m, i) \rightarrow 0$$

and the divergence between the relative growth rates of  $M^1$ ,  $M^2$ , and  $E$  is reduced.

As market interest rates continue to rise, and as  $i^t \rightarrow Q$  so that:

$$\epsilon(t, i) < 0$$

we move into Case III initial conditions. With an expanding supply of base money,  $M^1$ ,  $M^2$ , and bank credit continue to expand. However, as we move into Case IIIb conditions, the rise in  $i^f$  resulting from the feedback and long-run interest rate effects of an expanding supply of base money, coupled with the constraint of  $Q$ , operate to decrease the  $t$  ratio. The decrease in  $t$  increases the multiplier associated with  $M^1$ , but decreases the multipliers associated with  $M^2$  and bank credit. During periods of time when these initial conditions hold, the money stock ( $M^1$ ) expands at a more rapid rate than the money stock more broadly defined to include time deposits, and both of the monetary aggregates expand more rapidly than bank credit.

The empirical evidence indicates that, taking a long-run analysis, on average over the five year period 1964-1968, Case II initial conditions have dominated. Considering the period as a whole, credit market interest rates rose sharply over this period. However, on average, commercial banks were able to respond to the rise in  $i^f$  by raising the yields they offered on time deposits. For the period as a whole the  $t$  ratio showed a sharp rise. Consequently, the expanding supply of base money led to a more rapid rate of expansion for  $M^2$  than  $M^1$ , and a more rapid rate of expansion of bank credit than either of the monetary aggregates.

## SECTION H

### IMPLICATIONS OF THE BRUNNER-MELTZER HYPOTHESIS FOR THE CHOICE OF INDICATORS OF MONETARY POLICY ACTIONS

The Brunner-Meltzer work brings out that the aggregate under the direct control of the Federal Reserve System is the monetary source base. The central bank through its open market operations can determine the magnitude or rate at which base money is supplied to the economy. The distribution or uses (reserves of member banks, vault cash holdings of non-member banks, and currency held by the public) of a given stock of base money is determined by the actions of the banks and the public in response to certain economic variables and the given values of the policy parameters  $r^d$ ,  $r^t$ , and the discount rate.

The Brunner-Meltzer hypothesis also makes explicit that changes in the supply of base money affect the equilibrium stocks of the monetary aggregates, bank credit, and the level of market interest rates. Given that the Federal Reserve can control the base, a key question for policy becomes which of these three items, the monetary aggregates, bank credit or the level of interest rates can the Federal Reserve most closely control by controlling movements in the source base?

In our discussion in section E we saw that the  $m^2$  and bank credit multipliers were more sensitive to base-induced changes in interest rates than the  $m^1$  multiplier. Only under Case II conditions do interest rate changes on the multipliers become negligible. Under a regime of low interest rates and  $i^t$  sufficiently below  $Q$  (Case I), the  $M^2$  and bank credit multipliers are relatively sensitive to changes in market interest rates. Under these initial conditions interest rate changes operate to decrease the elasticities of the multipliers with respect to  $B^a$ . Of the three multipliers,

$\epsilon (m^1, i)$  is the smallest. Consequently, changes in the rate at which the Federal Reserve supplies base money are more closely related to changes in  $M^1$  than in money defined to include time deposits and bank credit.

For initial conditions such as Case III -- high levels of interest rates and the effective constraint of  $Q$  -- the  $t$  ratio is particularly sensitive to base-induced changes in interest rates. Changes in levels of market interest rates resulting from open market operations or changes resulting from the portfolio adjustment to changes in other policy parameters cause sharp outflows and inflows of time deposits to commercial banks. This process of disintermediation and reintermediation results in significant changes in the values of the  $m^2$  and bank credit multipliers. Although, the  $m^1$  multiplier is also affected by changes in the  $t$  ratio, the  $\epsilon (m^1, t)$  is less than the  $\epsilon (m^2, t)$  and  $\epsilon (m^2-1, t)$ .

If we had complete knowledge about the response of the components of the multipliers to changes in interest rates and about  $\epsilon (i, B^a)$ , the central bank, by controlling  $B^a$  could equally well control  $M^1$ ,  $M^2$ , or bank credit. However, given that our knowledge about these factors is far from complete, the conclusion follows that, given existing knowledge, the Federal Reserve by controlling  $B^a$  can best control movements in  $M^1$ .

Within some theoretical frameworks, specifically so-called Keynesian theory, the path by which monetary policy actions are transmitted to the real sector is through the impact of Federal Reserve actions on market interest rates. Therefore, a number of economists might opt for the level of market interest rates, rather than monetary aggregates, as an indicator of the impact of Federal Reserve policy on the real sector.



As we mentioned earlier, the Brunner-Meltzer hypothesis explicitly takes into account the condition that changes in the policy parameters under the direct control of the Monetary Authorities affect the level of market interest rates. The work of Brunner-Meltzer, however, makes a careful distinction between the short-run interest rate effects of changes in the policy parameters and long-run effects. In the short-run, open market operations are postulated to change credit market interest rates in the opposite direction and  $\epsilon(i, B^a) < 0$ . When considering the feedback and long-run interest rate effects of open market operations, Brunner-Meltzer take into account the impact of base operations on the real sector and the feedback from the real sector to credit market interest rates [see section D]. Taking into account the observable procyclical movements of the base, money stock and interest rates, the long-run interest rate effects of open market operations are postulated to be opposite in sign to the short-run effects, i.e.,  $\epsilon(i, B^a)_{\text{Long-run}} > 0$ .

The Brunner-Meltzer studies show that, using a money market indicator such as the level of interest rates as an indicator of the thrust of policy actions, results in accelerated movements in the monetary aggregates in a direction opposite to the direction desired by the policymakers.

Suppose the Federal Reserve authorities are attempting to follow a "tighter" policy and they use the level of interest rates as the indicator of the thrust of monetary policy. To implement the tighter policy, the Federal Open Market Committee directs the trading desk to, on balance, reduce the System's holdings of Government securities. By engaging in open market sales to increase the level of interest rates, Federal Reserve actions reduce the monetary base. The impact effect (short-run) of this policy is to raise the level of credit market interest rates. However, over a longer

period, interest rates may decline [ $\epsilon(1, B^a)_{\text{Long-run}} > 0$ ]. Observing the decline in interest rates, the central bank may reason it is not being "tight enough" and decrease the base further. Such a policy leads to quite sharp declines in  $M^1$ ,  $M^2$ , and bank credit.

Under Case I initial conditions -- low levels of interest rates -- if over the longer-run a decrease in  $B^a$  results in a decline in market interest rates the Federal Reserve may reason it is not "tight enough" and engage in further open market sales or increases in reserve requirements or the discount rate. These actions will accelerate the decline in the monetary aggregates and bank credit. Since  $M^1$  is most sensitive to decreases in  $B^a$ , this leads to the greatest relative decline in  $M^1$ .

For Case III conditions -- high levels of interest rates and banks constrained by  $Q$  -- if the central bank decreases  $B^a$ , over the long-run interest rates may fall. The central bank concludes it is not being "tight enough" and continues to pursue a contractionary policy. The result is an accelerated decline in the monetary aggregates and bank credit. The decline in the level of market rates may tend to increase the  $t$  ratio which dampens the decline in  $M^2$  and bank credit, but accelerates the decline in  $M^1$ .

In the opposite case, if the result of Federal Reserve policy actions is to increase the monetary source base, the feedback and long-run interest rate effects of this action may be to lead to a rise in interest rates. Using interest rates as their indicator, policymakers would conclude that their actions were having a restraining effect on the economy. However, as the Brunner-Meltzer work illustrates, the observed rise in credit market interest rates may reflect the feedback effects from the real sector to the credit market.

Under these conditions, using the level of interest rates as their indicator, policymakers may allow the base to grow at a rapid rate. The resulting expansion of  $M^1$ ,  $M^2$ , and bank credit have spill over effects into the real sector. The resulting rise in real output and prices generates increased demands for credit which feeds back to the credit market resulting in a rise in nominal interest rates.

For example, under Case III conditions, the result of an increasing stock of base money and  $\epsilon(i, B^a) > 0$  is that the growth rate of  $M^1$  accelerates. Due to the constraint of  $Q$ , the rate of increase of  $M^2$  and bank credit is dampened. The effect of the rapid growth of  $M^1$  on the real sector is to lead to an acceleration in the growth of aggregate demand. As the economy approaches full employment, prices begin to rise sharply. The impact of a rapidly growing money stock on the real sector feeds back to the credit market and the level of nominal interest rates rises sharply.

In the recent period as interest rates on the credit market have risen to historically high levels, with an expanding supply of base money, the ratio  $E/M_1$  has risen. The policymakers have been dismayed by the rapid rise in interest rates combined with a booming economy and a sharply rising price level. By viewing the rise in interest rates as an indication they are following a "tight" or "contractionary" policy, the results of the Brunner-Meltzer hypothesis suggest that the policymakers have discounted the procyclical movement of other policy indicators, for example the monetary source base. The rise in interest rates which has frequently been attributed to policy actions may instead have reflected a change in the public's demand for bank credit resulting from changes in other variables in the public's supply of financial assets to the banks (see section D on the long-run effects of changes in  $B^a$  on  $i$ ).

Appendix I

Derivation of the relationship  $(m^2 - 1) B^a = E$   
from the consolidated balance sheet of  
the banks and monetary authorities

Consolidated Balance Sheet of Commercial  
Banks, Treasury, and Federal Reserve

Assets	Liabilities
(1) Monetary gold stock	(6) Money stock
(2) United States securities	Nonmonetary liabilities
(2a) Held by Federal Reserve	(7) Time deposits
(2b) Held by commercial banks	(8) Treasury deposits
(3) Commercial bank loans	(8a) at Federal Reserve Banks
(4) Commercial bank holdings of other securities	(8b) at commercial banks
(5) Treasury currency	(9) Treasury cash holdings
	(10) Foreign deposits
	(11) Capital accounts - float *

Let: N = capital accounts  
BC = total bank credit  
 $C^p + D^p$  = money supply

$$(1) + (2a) + (5) - [8(a) + (9) + (10) + (11)] = B^a - N$$

$$2(b) + (3) + (4) = BC$$

$$\therefore B^a - N + BC = D^p + C^p + T^d + D^t$$

$$\text{since: } m^2 B^a = D^p + C^p + T^d$$

$$B^a - N + BC = m^2 B^a + D^t$$

$$\therefore BC - N - D^t = (m^2 - 1) B^a$$

$$\therefore E = (m^2 - 1) B^a$$

\* Federal Reserve float is entered as a negative item in capital accounts in the consolidated Balance Sheet, see Federal Reserve Bulletin, October, 1960, p. 1114.

## Appendix II

### Derivation of the total asset multiplier

In Appendix I we derived from the consolidated balance sheet of the banks and monetary authorities the relationship:

$$B^a + BC = D^p + C^p + T^d + D^t + N$$

$$BC = (m^2 - 1) B^a + D^t + N$$

From definition 4 on page 9:

$$D^t = dD^p$$

To simplify our notation let:

$$\Delta = (r - b) (1 + t + d) + k \quad (\text{i.e., the denominators of the multipliers})$$

By definition of the multiplier:

$$\frac{(1 + k) B^a}{\Delta} = M^1 = D^p + C^p$$

hence:

$$\frac{1}{\Delta} = \frac{D^p + C^p}{(1 + k) B^a}$$

Multiply both sides by  $dB^a$ :

$$\frac{d}{\Delta} B^a = \frac{d (D^p + C^p)}{1 + k}$$

Using the definition:  $k = \frac{C^p}{D^p}$

$$1 + k = \frac{D^p + C^p}{D^p}$$

hence:

$$\frac{d}{\Delta} B^a = \frac{d (D^p + C^p)}{1} \cdot \frac{D^p}{D^p + C^p}$$

$$\therefore \frac{d}{\Delta} B^a = dD^p$$

Substituting this result in our original equation, we have:

$$BC = (m^2 - 1) B^a + \frac{d}{\Delta} B^a + N$$

We now make the substitution:

$$N = n (D^p + D^t + T)$$

by substitution we have:

$$N = \frac{n (1 + t + d)}{\Delta} B^a$$

Substituting in our original equation, we now have:

$$BC = [ m^2 - 1 + \frac{d}{\Delta} + \frac{n (1 + t + d)}{\Delta} ] B^a$$

since:

$$m^2 - 1 = \frac{(1 + t) - (r - b) (1 + t + d)}{\Delta}$$

we have:

$$BC = \left[ \frac{1 + t + d - (r - b) (1 + t + d) + n (1 + t + d)}{\Delta} \right] B^a$$

rearranging terms:

$$BC = \frac{(1 + t + d) [1 + n - (r - b)]}{(r - b) (1 + t + d) + k} B^a$$

Hence the total asset multiplier:

$$a = \frac{(1 + t + d) [1 + n - (r - b)]}{(r - b) (1 + t + d) + k}$$

BIBLIOGRAPHY

- Andersen, Leonall C., and Jordan, Jerry L. "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization". Federal Reserve Bank of St. Louis Review, November, 1968.
- Andersen, Leonall C., and Jordan, Jerry L. "The Monetary Base: Explanation and Analytical Use". Federal Reserve Bank of St. Louis Review, August, 1968.
- Brunner, Karl. "The Role of Money and Monetary Policy". Federal Reserve Bank of St. Louis Review, July, 1968, pp. 9-24.
- Brunner, Karl, and Meltzer, Allan H. "Liquidity Traps for Money, Bank Credit, and Interest Rates". Journal of Political Economy, Vol. 76, No. 1.
- \_\_\_\_\_. "Some Further Investigations of Demand and Supply Functions for Money". Journal of Finance, Vol. XIX, May (1964), pp. 240-283.
- \_\_\_\_\_. "Predicting Velocity: Implications for Theory and Policy". Journal of Finance, Vol. XVIII, May (1963), pp. 319-354.
- \_\_\_\_\_. "The Meaning of Monetary Indicators". Monetary Process and Policy: A Symposium, edited by George Horwich, Purdue University Monograph Series #3, Irwin Co., 1967, pp. 187-217.
- Burger, Albert E. and Andersen, Leonall C. "The Development of Explanatory Economic Hypotheses for Monetary Management". Southern Journal of Business (forthcoming).
- Friedman, Milton, and Schwartz, Anna. A Monetary History of the United States: 1867-1960. Princeton University Press, 1963.
- Jordan, Jerry L. "Relations Among Monetary Aggregates," Federal Reserve Bank of St. Louis Review, March, 1969.
- Leftwich, R. H. The Price System and Resource Allocation, Revised edition. Holt, Rinehart and Winston, 1960.
- Meltzer, Allan. "Controlling Money", Federal Reserve Bank of St. Louis Review, May, 1969.
- Toulmin, Stephen. Foresight and Understanding. Indiana University Press, 1961.