A Model of the Markets for Consumer Installment Credit and New Automobiles

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A Model of the Markets for Consumer Instalment Credit and New Automobiles

H. Albert Margolis

A Thesis

Submitted to the Faculty
of
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by
H. Albert Margolis

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of
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price of automobiles and credit terms, and a positive function of disposable income. Finally, the supply of automobiles is directly related to their retail price and disposable income, and negatively related to their wholesale price.

A simultaneous-equations model is constructed by assuming that credit terms and the price of automobiles vary each quarter to produce equilibrium in each market.

The structural form of the model is transformed into the reduced form and tested against a maintained hypothesis involving a system of equations regressing the four endogenous variables on the three exogenous variables. The logical implications of the structural model are derived and presented as propositions about reduced form parameters. The propositions present a set of identifiability conditions and a closed outcome space which is compatible with the model. The model tests whether estimates of the reduced form coefficients satisfy these propositions.

The set of observations used in testing include seasonally adjusted quarterly data for the period of 1960 through 1965, excluding the fourth quarter of 1964.

The results of the test were that each of the identifiability conditions were satisfied but that two of the 16 reduced form parameter estimates fell outside the weakly formulated hypothesized range. This suggests that, while the model is rejected as it stands, its essential structure is appropriate for the investigation of the problem. It is also indicated that this exploratory effort toward a supply of credit function is worthwhile but not entirely successful.
INTRODUCTION

This dissertation is concerned with the problem of determining the short-run effects of consumer instalment credit on purchases of new automobiles. Some observers have maintained that in the long run, durable expenditures in general seem to represent a rather stable percentage of total consumer outlays.\(^1\) Others have discerned a rising trend in investment by households in both owner-occupied homes and in major consumer durables relative to both GNP and total personal consumption expenditures over the last seven decades.\(^2\) It is possible that in the long run the increase in the proportion of income spent may be partially an effect of the utilization of credit. In the shorter run, however, expenditures on durables show a cyclical pattern. It is possible that consumer credit affects the timing of purchases and thus contributes to the cyclical behavior. If there were a high demand elasticity for durables with respect to credit terms, then an easing of the latter might result in a concentration of purchases of durables in a short period, followed by below normal


purchases as consumers wear out the goods while repaying the debt.

In addition, if the premises of the preceding sentence were demonstrated, it would be useful to policy makers to note the impact on credit and credit terms of influential factors from the money markets.

Important as this broad area of study is, there are rather severe limitations to quantitative analysis because of the unavailability of key data. Especially is this true for such variables as credit terms. Fortunately, the lack of availability and quality of the relevant data is not so disheartening for a strategically important component of durable expenditures, i.e., the purchases of new automobiles. For the most part, data are available on both volume and prices of new automobiles and volume and terms of the associated credit. Moreover, the automobile sector is sufficiently important to the economy to justify study. Its investigation in the context of the interaction with the associated credit market should add to the interest and significance of the study.

The principal goal of this research is to determine quantitatively the response of automobile sales to credit terms. Conceptually, however, automobile sales, through their effect on demand for credit, may in turn, feed back or influence credit terms. This view would argue for a simultaneous-equations system approach to the problem. The formulation of such a simultaneous-equations system undertaken here is based on the concept of a market for each of two goods, one good being used only in conjunction with the other.

Consumer instalment credit is thought of as a service auxiliary
to the purchase of new automobiles. There is a market for each good, with individual supply and demand functions. The interrelationship between the two markets is explicitly formulated by inserting credit terms as a variable influencing purchases of new automobiles and the latter as a factor in the demand for consumer instalment credit.

An additional by-product of this approach is the attempt to formulate a supply of credit function. From this formulation it may be possible to quantify the response of consumer instalment credit, and thus, indirectly, automobile purchases to changes in money market conditions.

The rationale for pursuing an aggregate time series analysis has been spelled out by Smith. In the first place, there are a large number of cross-sectional studies that have been based on a few sets of survey-acquired data. Smith provides a list and points out that these studies generally get insignificantly low multiple correlation coefficients. He suggests that important explanatory variables may have been left out. In the second place, the earlier time series studies such as that of Kisselgoff are limited by the paucity of data and the unusual circumstances included in the period with which they deal. To a large extent, the periods for which data

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are available include periods of unusual disturbances such as the Great Depression, World War II, the postwar boom, and the Korean conflict.

The Evans and Kisselgoff study is the only one so far which takes advantage of the availability of quarterly data in the postwar period.\(^5\) A detailed exposition of their work is presented in Chapter I.

The model is a linear system of four equations based on the structural equations for the supply and demand for each of the two goods and the equilibrium assumption that the market clears each quarter. The model is tested against quarterly data from IQ60 to IVQ65 with the exception of IVQ64 when there occurred a strike in the automobile industry.

The plan of the study is as follows: A brief survey of previous studies is made in Chapter I. A development of the basic hypothesis and a definition of the variables is described in Chapter II. Also, propositions concerning the structural relations and the numerical restrictions on their coefficients are presented. The complete simultaneous equations model is then formulated. Chapter III contains the maintained hypothesis, the reduced form equations, the identifying functions and the identifiability conditions. Chapter IV presents the empirical test and the conclusions. There is an appendix in which the definitions and the procedures of the collection of the data are described.

I. A SURVEY OF SOME RELEVANT STUDIES

An early often-cited empirical study by Kisselgoff used a simultaneous-equations approach to determine the effects of consumer credit terms on total consumer expenditures.\(^6\) Basically, however, his formulation included only a demand for credit and a demand for consumer goods along with an income identity and a price relationship. As in other studies to which we shall refer, this seems an incomplete account of the forces operating in the two markets. The usual justification for this simplification is an assertion that the supply of credit is perfectly elastic.

Kisselgoff's problems with the data were a result, on the one hand, of the time period with which he was concerned, namely 1929-41 on an annual basis. It has been pointed out that the data are even less complete in the prewar period and almost non-existent before 1929.\(^7\) On the other hand, he was dealing with aggregate consumer expenditures, a non-homogeneous category which would complicate even more the construction of a relevant credit terms index. His formulation of the credit term variable was a retail price index divided by an index of the average monthly duration of all instalment credit payment. The average

\(^6\)Kisselgoff, *Factors Affecting the Demand for Consumer Instalment Credit*.

duration is considered as "essentially determined by past experience and therefore may be regarded as a predetermined variable". 8

He felt he had shown a significant relationship between credit terms and the demand for credit, but he felt less confident about such a relationship between credit terms and the demand for consumer goods in general.

Another study which is similar in format was undertaken by Ball and Drake. 9 They set up simply a demand for credit function and a demand for consumer durables function. They dealt with quarterly data and used a dummy variable approach to seasonal variation problems. In their demand for credit function the credit term variable is an officially specified minimum proportion of the purchase price of a durable good that had to be deposited. In a sense then, they did not even have the "market determined" credit terms. They view the impact of credit on the demand for durables as operating through the volume of new credit which is introduced as a variable in the durables demand function. They conclude, "The introduction of new credits issued resulted in marked improvement in our ability to account for the movement of the consumption of durable goods over the period."

More recently Evans and Kisselgoff have estimated demand functions for automobile and parts expenditures and for instalment credit using

8 Kisselgoff, The Demand for Consumer Instalment Credit, 24.

ordinary least squares methods. They use seasonally adjusted quarterly data for the period 1948-1962. They compute an average monthly payment variable by using the following formula:

$$\text{AMP} = \frac{\text{PNA} \times (\text{DCR}) \times (1 + \text{AFR})}{\text{PC} \times (\text{IMM})}$$

Where

AMP = index of average monthly payment on consumer instalment loans for new cars

PNA = Consumer price index for new cars

DCR = Dealer cost ratio for new cars (ratio of new loans for new cars by major sales finance companies and commercial banks to dealer cost)

AFR = average finance rate for new automobile purchases

PC = implicit price deflator for personal consumption expenditures

IMM = implied maturity of consumer instalment automobile paper outstanding

The numerator is presumably supposed to represent the outstanding debt and is divided into even monthly payments.

They conclude that the average monthly payments term is "important" in the automobile demand equation, that repayment may be "marginally important", that credit terms depend on conditions in the whole economy rather than specific conditions in the credit markets, and finally that the various aspects of credit terms, such as down-payments and average length of contract, are interrelated.

In the course of their theoretical discussion, the authors say that the evidence suggests that the supply of funds to both instalment

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10 Evans and Kisselgoff, Demand for Consumer Instalment Credit and its Effects on Consumption.
credit institutions and to final users has been "fairly elastic."\textsuperscript{11} They attribute this largely to the relatively high rates which the consumers are charged. They see the volume of credit granted as responding principally to demand factors.

None of the studies cited so far have involved a supply function for either instalment credit or the consumer durable. There have been studies in which consideration has been given to the supply of the durable, namely, for automobiles. Suits has set out equations for both the supply and demand for new automobiles; however, he estimates only the demand expression, using least-square linear regressions with the variables expressed in first differences.\textsuperscript{12} He uses as his expression for the impact of credit terms the average real price of new cars divided by the number of months the average automobile instalment contract runs. He finds that "The explanation of new car demand is improved by even so crude a measure of credit conditions."

Dyckman sets out demand and supply equations for new automobiles in a fashion similar to Suits.\textsuperscript{13} He uses annual data for the period from 1929 to 1962. His measure of credit terms is simply a dummy

\textsuperscript{11} Evans and Kisselgoff, The Demand for Consumer Instalment Credit, 6.


variable which "was given a value of +1 in years in which a substantial easing of credit terms took place, and a value of zero in all other years." 14 As in Ball and Drake, there is no attempt to get at "market determined" credit terms. At least the credit variable appears as a separate variable. As in Suits, only the demand equation is estimated.

This survey shows that, while there have been various attempts to take into account the relationship of consumer instalment credit and consumer durable expenditures, some basic issues have been neglected. Principally, the interrelated nature of the markets for the two commodities has not been explicitly presented. Moreover, insofar as analysts have chosen to include credit terms in their models, they have not used market-determined variables such as those actually faced by the consumer. Partially this is because the effort has not been made to set up a supply of credit function.

The intent of this study is not to derive new theories of the dynamics of consumer instalment credit. Its more modest goal is to utilize some of the existing basic views and concepts in formulating a testable simultaneous equations system.

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14 Specifically he assigns the value +1 to the years 1950 and 1953, because of the relaxing of regulation W in the preceding years and to 1935 and 1955 as "periods singled out in the literature as periods when a substantial easing of credit took place." Dyckman, 256-257.
II. THE MODEL

2.1 Formulation of the Model

There is no established "theory of consumer credit." On an aggregate basis there have been Domar-like mechanical derivations of equilibrium growth conditions, but this seems open to the criticism that it assumes away too much of both borrower and lender behavior. As of yet there does not seem to have been published a derivation of supply and demand functions for consumer instalment credit from the basic economic concepts, i.e. utility function and production functions.

This situation is somewhat more satisfactory for the demand for durables, and more specifically, for automobiles. Essentially similar demand functions for durables or automobiles have been derived from two separate approaches. Dyckman and Suits follow Chow and

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utilize the stock-adjustment principle. Ball and Drake prefer a utility maximization formulation which they feel can lead to a "unified treatment of durable goods, non-durable goods, and personal savings." In any event, there is ample precedent for a basic model of the market for new automobiles.

The problem is to articulate a model of the market for consumer instalment credit and to formulate explicitly the interaction between the two markets.

Consumer instalment credit can be viewed as a service complementary to the transportation service flows derived from owning an automobile. Automobiles, and durables in general, involve relatively large expenditures, and the immediate lump sum lay-out of the cash value of the entire car would represent a sizable demand on the liquidity of the typical individual's portfolio. The automobile is not consumed all at once, of course, but depreciates over a period of time, yielding a flow of services all the while. The payment procedure optimal for the consumer would permit payment directly as the car depreciates. This would entail the least demand on the individual's liquidity and patience. The accumulation and maintenance of sufficient liquidity to purchase a car bears a sizable opportunity cost in addition to a possibly high psychic cost as a result of deferred purchase.

Consumer instalment credit more closely approximates this optimal

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17 Ball and Drake, "Impact of Credit Control," 182.
procedure in that it spreads the payments over the life of the real asset. From the point of view of the consumer, the longer the period over which payments are made the less the difficulty involved in adjusting his budget or his portfolio each month. At the same time, in the aggregate, a longer maturity on the payment contract is usually associated with a lower initial downpayment.

The producer and retailer of the car, as producer and retailer, would want to recover the full price of the car immediately. There are both risk and the tying-up of capital involved in the credit arrangement. The providers of credit may be viewed as playing a specialized role in the extension of credit. They are providing a financial service complementary to the purchase of the car. The price for the service is the interest or finance charges which they receive.

A complication in the analysis of consumer elasticities of demand for consumer instalment credit arises because there seems to be a greater concern for the size of the monthly payments than for the finance rates charged. Generally it is thought that consumers react less to the actual "price", i.e., the finance charges of the service, than to a dimension which might be termed the "quality" of the service, i.e., the credit terms. The concern for the "quality", that is, for the provision of an arrangement which more closely approximates a pay-as-you-go basis, may involve a rational calculation based on a higher weight placed on the psychic costs of portfolio adjustment than on the actual interest costs.

The principal empirical study on consumer sensitivity to finance rates concludes that "there will be little response observable in the
population as a whole under existing conditions."\textsuperscript{18} This is based in part on the consumer's lack of rate knowledge. In addition, the study observes that a majority of consumers may be rationed in the sense that, individually, they are unable to obtain longer maturity contracts by offering higher finance rates.\textsuperscript{19} It is observed that the pattern of greater response to the credit terms than to finance charges may be changing, and that, in the future, more borrowers may achieve the more secure status of being "unrationed" and then become more aware of finance charges.

Considerations such as these argue for the view that credit terms rather than finance charges play the role of the "market clearing" variable. In other words, credit terms will be considered endogenous, or determined within the consumer instalment credit market. The suppliers respond inversely to credit terms because of the greater risk to the lender involved in a longer contract. The longer the contract the greater the possibility that the remaining outstanding liability may be greater than the undepreciated value of the car pledged as security.

There are two dimensions to credit terms in practice, the maturity and the downpayment, or its inverse, the dealer-cost ratio. For the most part it is thought that they move together in the sense that


\textsuperscript{19} Juster and Shay, \textit{Consumer Sensitivity}, 3.
"Lower downpayments . . . tend to go hand in hand with lengthening maturities." For the purposes of verifying this intuitive and accepted judgment, a series representing the median dealer-cost ratio over the same period was constructed based on the same data source as the median maturity series. The two series, dealer-cost ratio and median maturity were found to be correlated at a significance level of 5 per cent. The decision was made to use median maturity on basis of a higher incidence of references to it in the literature.

The view that consumer instalment credit is a service complementary to the purchase of new automobiles suggests the nature of the interaction between the two markets. The demand for a good desired for its intrinsic flow of services should be influenced by the price of an ancillary complement. Thus the demand for automobiles should be influenced by credit terms which play the role of the usual price variable in the credit market. This is the crux of the study. In addition, because the flow of services from consumer instalment credit is tied inextricably to the purchases of automobiles, it should be the latter variable which influences the demand for credit.

As noted, none of the cited studies have formulated a supply of credit function. An initial attempt was made here to express the supply of credit as a function of credit terms, finance charges, and a

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money market interest rate faced by the automobile credit market. Conceptually it seemed reasonable to expect that suppliers of credit would respond positively to their "retail" price (finance charges) and negatively to their "wholesale" price (interest rate in the money market which they pay for funds.)

The series used to represent finance charges was constructed by Robert Shay. There was no alternative series on finance charges available. The series used to represent the cost of funds to the credit industry was the 4-to-6 month commercial paper rate for sales finance companies.

In preliminary statistical experiments, it developed that the results were unsatisfactory. It was also noted that there was a persistent downward trend in the series of finance charges over the period 1960-1965, with a pronounced drop in the years 1962 and 1963. It was also noted that over the 1960-1965 period there was a persistent upward trend in the percentage of the total credit that was extended by commercial banks. In particular, there was a pronounced jump in the fourth quarter of 1961. This suggests that the behavior of this particular series on finance charges was strongly influenced by the supply shift in the automobile credit market from sales finance companies to commercial banks during the period under consideration.

The present model seeks to relate automobile credit during this particular period to this basic institutional change, namely, the

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22 The data were communicated in a private letter, not to be quoted, as of yet not published. Shay based his data on a NBER sample of four large sales finance companies during the period 1956 to 1959. For 1960-1965 he extrapolated using data from one of the four companies.
increasing participation of commercial banks. A variable expressing the commercial banks' percentage share of the total is inserted in the supply of credit function.

From the fourth quarter of 1956 to the fourth quarter of 1965 this percentage grew steadily and consistently. Since the end of that period it has fluctuated around this new high level. With the exception of the fourth quarter of 1961 the period exhibits a striking and somewhat unique pattern of stable growth.

Reference has been made to the precedent for a basic model of the market for new automobiles.\textsuperscript{23} The general form would have the demand for automobiles depending upon credit terms, income, and the stock of automobiles. The supply of automobiles is determined by the price of automobiles and the wholesale price of automobiles plus an index of operation costs.

There are several deviations to this model from the precedents cited that were also motivated by the results of preliminary statistical experiments.\textsuperscript{24} One is the omission of a variable in the demand-for-automobile equation attempting to measure the existing stock of automobiles. Such a variable was constructed on a quarterly basis but was found to be very highly correlated to the income term. For this reason it was discarded in the present analysis. Dyckman gives two reasons

\textsuperscript{23} Above, 11.

\textsuperscript{24} Suits, "The Demand for New Automobiles"; Dyckman, "An Aggregate - Demand Model."
for the inclusion of a stock of automobile variable. On the one hand, demand might negatively be related to a larger stock which implies a smaller stock gap in a stock-adjustment model. In addition, the stock of used cars might conceptually exercise an indirect influence on the demand for new automobiles through its effect on the price of used cars.

Initial results also prompted the insertion of an income term in the supply of automobile function. A theoretical justification of this is not obvious. It is possible that the income variable exhibits the role that expectations play in the production plans of the automobile manufacturers and retailers. In addition, the income term might reflect the size of the capital stock in the automobile industry. By way of comparison, however, neither of the two studies referred to above cite their findings with regard to a supply function of automobiles.

The sources and values of observations used in testing the model are given in Appendix A.

This is a general sketch of the theoretical considerations underlying the model. It does not derive directly from the basic economic concept of utility curves, but it does draw on other basic ideas to spell out more explicitly the interaction between credit and expenditures.

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2.2 The Underlying Economic Premises

For each market, credit and automobiles, we offer a structural representation of the supply and demand for the respective commodity. Each representation is composed of a mathematical function, or structural equation, and a set of economic premises that restrict the parameters of the function in some definite way.

The variables used in the model, all of which are seasonally adjusted, are listed in Table 2.2.1 together with their generic and statistical symbols.

2.3 A Listing of Economic Premises

Proposition 2.3.1 (Demand for Instalment Credit to Buy New Automobiles, DICSA)

(2.3.1a) \[ DICSA \ (MM, RNAS) = \beta_1 MM + \beta_2 RNAS + \gamma_1 + U \]

where \( u \) denotes a random disturbance with mean and finite variance

(2.3.1b) \[ E(u) = 0 \quad \text{and} \]

(2.3.1c) \[ E(u^2) = \omega^2_u \]

with

(2.3.1d) \[ .5 < \beta_1 < .6 \]

(2.3.1e) \[ 1.6 < \beta_2 < 2.0 \]

(2.3.1f) \[ -15 < \gamma_1 < -5 \]

(2.3.1g) \[ .005 < \omega^2_u < .5 \]

The demand for credit given in proposition 2.3.1 is assumed to be a positive function of credit terms and automobile registrations with a negative intercept \( \beta_1 \). The coefficient of credit terms, is assumed positive because an increase in the credit terms variable, median monthly maturity, is considered to be an easing of credit,
<table>
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<th>Generic Symbol</th>
<th>Name of Variable</th>
<th>Statistical Symbol</th>
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<td><strong>Endogenous</strong></td>
<td></td>
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<tr>
<td>ICSA</td>
<td>Quarterly extension of consumer instalment credit to buy new automobiles</td>
<td>Y t,1</td>
</tr>
<tr>
<td>MM</td>
<td>Median maturity of new car contracts</td>
<td>Y t,2</td>
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<tr>
<td>RNAS</td>
<td>Quarterly registrations of new automobiles</td>
<td>Y t,3</td>
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<tr>
<td>PASA</td>
<td>Quarterly retail price index of new automobiles</td>
<td>Y t,4</td>
</tr>
<tr>
<td><strong>Exogenous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YSA</td>
<td>Quarterly personal disposable income at annual rates</td>
<td>Z t,1</td>
</tr>
<tr>
<td>WASA</td>
<td>Quarterly wholesale price index of new automobiles</td>
<td>Z t,2</td>
</tr>
<tr>
<td>BCSA</td>
<td>Commercial Banks' percentage of new automobile credit extended</td>
<td>Z t,3</td>
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i.e., equivalent to a lowering of "price".

**Proposition 2.3.2** (Supply of Instalment Credit to Buy New Automobiles, SICSA)

\[(2.3.2a) \quad \text{SICSA (MM,BCSA)} = \beta_3 \text{MM} + \gamma_2 \text{BCSA} + \gamma_3 + u\]

where \(u\) denotes a random disturbance with mean and finite variance

\[(2.3.2b) \quad \mathbb{E}(v) = 0 \quad \text{and} \]
\[(2.3.2c) \quad \mathbb{E}(v^2) = \omega_v^2 \]

with

\[(2.3.2d) \quad -.3 < \beta_3 < 0\]
\[(2.3.2e) \quad 0 < \gamma_2 < .2\]
\[(2.3.2f) \quad 0 < \gamma_3 < 90\]
\[(2.3.2g) \quad .005 < \omega_v^2 < .5\]

The supply of credit given in proposition 2.3.2 is assumed to be a negative function of credit terms and a positive function of the commercial banking system's participation in the market.

There is assumed to be a positive intercept.

**Proposition 2.3.3** (Demand for New Automobiles, DRNAS)

\[(2.3.3a) \quad \text{DRNAS (PASA, MM, YSA)} = \beta_4 \text{PASA} + \beta_5 \text{MM} + \gamma_4 \text{YSA} + \gamma_5 + w\]

where \(w\) denotes a random disturbance with mean and finite variance

\[(2.3.3b) \quad \mathbb{E}(w) = 0 \quad \text{and} \]
\[(2.3.3c) \quad \mathbb{E}(w^2) = \omega_w^2 \]

with

\[(2.3.3d) \quad -1 < \beta_4 < 0\]
\[(2.3.3e) \quad .4 < \beta_5 < 1\]
\[(2.3.3f) \quad 0 < \gamma_4 < 1\]
(2.3.3g) \[ 0 < \gamma_5 < 5 \]
(2.3.3h) \[ .005 < \omega_w^2 < .5 \]

The demand for automobiles given in proposition 2.3.3 is assumed to be a negative function of the retail price of automobiles and a positive function of credit terms and income. There is a positive intercept.

**Proposition 2.3.4** (Supply of New Automobiles, SRNAS)

(2.3.4a) \[ \text{SRNAS (PASA, WASA, YSA)} = \beta_6 \text{PASA} + \gamma_6 \text{WASA} + \gamma_7 \text{YSA} + \gamma_8 + x \]

where \( x \) denotes a random disturbance with mean and finite variance

(2.3.4b) \[ E(x) = 0 \text{ and} \]
(2.3.4c) \[ E(x^2) = \omega_x^2 \]

with

(2.3.4d) \[ .1 < \beta_6 < 1 \]
(2.3.4e) \[ -1 < \gamma_6 < 0 \]
(2.3.4f) \[ 0 < \gamma_7 < 1 \]
(2.3.4g) \[ 0 < \gamma_8 < 5 \]
(2.3.4h) \[ .005 < \omega_x^2 < .5 \]

The supply of automobiles given in proposition 2.3.4 is assumed to be a positive function of the retail price of automobiles and income and a negative function of the wholesale price of automobiles. There is a positive intercept term.

**Proposition 2.3.5** (Stochastic Premise)

The random vectors

\[ Q = (U_t, V_t, W_t, X_t) \]

are independently and identically normally distributed with non-singular covariance matrix of the form
(2.3.5a) \[
\Omega = \begin{bmatrix}
\omega_u^2 & \omega_v^2 & 0 & 0 \\
\omega_u^2 & \omega_v^2 & 0 & 0 \\
0 & 0 & \omega_w^2 & \omega_{wx} \\
0 & 0 & \omega_{wx} & \omega_x^2 \\
\end{bmatrix}
\]

and

(2.3.5b) \( .05 < (\omega_u^2 + \omega_v^2 - 2\omega_{uv}) < .5 \)

(2.3.5c) \( 0 < (\omega_w^2 + \omega_x^2 - 2\omega_{wx}) < .005 \)

\( -.001 < \omega_{vw} - \omega_u^2 < 0 \)

\( 0 < \omega_x^2 - \omega_{wx} < .006 \)

Proposition 2.3.6 (Domain Premise)

The preceding structural equation explains behavior only on the domain

(2.3.6a) \( t = 1, \ldots, 23 \) or the quarter from first quarter 1960 to fourth quarter 1965 excluding fourth quarter 1964.

(2.3.6b) \[
\begin{align*}
5 & > \text{ICSA} & > & 2 \\
32 & > \text{MM} & > & 33 \\
2.5 & > \text{RNAS} & > & 1.5 \\
104 & > \text{PASA} & > & 97 \\
490 & > \text{YSA} & > & 340 \\
102 & > \text{WASA} & > & 97 \\
5507 & > \text{BCSA} & > & 440 \\
\end{align*}
\]

A graphical presentation of the model is possible and is given in Figure 1.
Figure 1. -- Supply and Demand Curves for the New Automobile
Consumer Instalment Credit Market and the New Automobile Market
Along the axis in the credit market are the two endogenous variables, the amount of new automobile consumer instalment (ICSA) and the variable representing credit terms (MM). The usual direction of the supply and demand function is reversed. The demand for credit (DICS A) has as a parameter the automobile variable (RNAS). The supply of credit (SICS A) has as a parameter the commercial banks' share of the market (BCSA).

Along the axis in the automobile market are the two endogenous variables, RNAS and the price of new automobiles (PASA). The demand for automobiles (DRNAS) has as a parameter MM and personal income (YSA). The supply of automobiles (SRNAS) has as a parameter the wholesale price of automobiles (WASA) and YSA.

2.4 The Simultaneous Equations Model

The structural equations presented in Section 2.3 represent economic behavior in isolation.

In reality, of course, the markets and their components interact. We simulate this by formulating an additional proposition about the market mechanisms.

Proposition 2.4.1 (Simultaneous Equation Model)

For every quarterly period, given that the background conditions remain constant \( t,i \) \( (i=1, \ldots, 4) \) are jointly determined by \( t,j \) \( (j=1,2,3) \) according to the following system of equations

\[
(2.4.1a) \quad \{\text{DICS A}(\text{MM},\text{RNAS}) - \text{SICS A}(\text{MM},\text{BCSA})\} = 0
\]

\[
(2.4.1b) \quad \{\text{ICSA} - \text{DICS A}(\text{MM},\text{RNAS})\} = 0
\]

\[
(2.4.1c) \quad \{\text{DRNAS}(\text{PASA},\text{MM},\text{YSA}) - \text{SRNAS}(\text{PASA},\text{WASA},\text{YSA})\} = 0
\]

\[
(2.4.1d) \quad \{\text{RNAS} - \text{SRNAS}(\text{PASA},\text{WASA},\text{YSA})\} = 0
\]
Equation (2.4.1a) asserts that for every quarter under study the excess demand for credit is equal to zero. Similarly equation (2.4.1c) asserts that the excess demand for new automobiles is zero. Equations (2.4.1b) and (2.4.1c) assert that for each quarter the respective functions, evaluated at the observations of their arguments, are equal to their observed values.
III. MATHEMATICAL ANALYSIS OF THE MODEL

The test of the model set forth in Chapter II takes place in the reduced form in the context of a maintained hypothesis.

3.1 The Maintained Hypothesis

Proposition 3.1.1

For every quarter \( t=1, \ldots, 23 \) if the background conditions remain constant, then the mechanism that determines \( Y_{t,1}, \ldots, Y_{t,4} \) can be represented by the following invariant system of regression equations.
(3.1.1a) \[ Y_{t,1} = \alpha_1 z_{t,1} + \alpha_2 z_{t,2} + \alpha_3 z_{t,3} + \alpha_4 + \eta_{t,1} \]

(3.1.1b) \[ Y_{t,2} = \alpha_5 z_{t,1} + \alpha_6 z_{t,2} + \alpha_7 z_{t,3} + \alpha_8 + \eta_{t,2} \]

(3.1.1c) \[ Y_{t,3} = \alpha_9 z_{t,1} + \alpha_{10} z_{t,2} + \alpha_{11} z_{t,3} + \alpha_{12} + \eta_{t,3} \]

(3.1.1d) \[ Y_{t,4} = \alpha_{13} z_{t,1} + \alpha_{14} z_{t,2} + \alpha_{15} z_{t,3} + \alpha_{16} + \eta_{t,4} \]
where the vectors
\[ \eta_t = (\eta_{t,1}, \eta_{t,2}, \eta_{t,3}, \eta_{t,4}) \]
are independently and identically normally distributed with
\[ E(\eta_t) = 0 \]
and non-singular covariance matrix
\[ \Sigma = \{\sigma_{hi}\} \quad h,i = 1,2,3,4 \]

The maintained regression model has 26 parameters: 16 regression coefficients in equations (3.1. a-d) and the 10 covariances in (3.1.3). The parameters \( \alpha_1, \ldots, \alpha_{16}, \sigma_{11}, \ldots, \sigma_{34} \) are regarded as coordinates of a definite but unknown point \( \bar{\alpha} \) in 26-dimensional Euclidian Space \( \mathbb{R}^{26} \). Actually \( \bar{\alpha} \) will be contained in a 20-dimensional subspace because of dependencies implied by the model.

The \( Y \)'s are regressed on the \( Z \)'s for the periods in the domain and an estimate of the point \( \bar{\alpha} \) can be found. This estimate will be called \( \hat{\alpha} \). An important aspect of the testing procedure is that the set of structural propositions presented in Chapter II implies a range of values with which \( \hat{\alpha} \) must be consistent in order that the model be accepted.

### 3.2 The Reduced Form

This range is derived by examining the parameters of the reduced form expressed in terms of the \( \beta \)'s, \( \gamma \)'s and \( \omega \)'s. The reduced form is the solution of the system of equations (2.3.1 a-d) for the \( Y \)'s. In matrix notation equations (2.3.1a-d) can be expressed as
\[ (3.2.1) \quad \beta Y_t + \Gamma Z_t = \eta_t \]
The reduced form is
\[ (3.2.2) \quad Y_t = -\beta^{-1} \Gamma Z_t - \beta^{-1} \eta_t \]
We shall see the $|\beta| > 0$ over the hypothesized range of value of the $\beta$'s and $\gamma$'s and that therefore $\beta^{-1}$ exists. $\beta^{-1}\Gamma$ represents the terms in the $\alpha$ space. The expectation $E\{-\beta^{-1}\eta_t, \eta_t, \eta_t\cdot(-\beta^{-1})\}$ represents the terms on the $\sigma$ coordinates.

In the case of this particular model it is simplest to solve for $Y_t$ by elementary simultaneous equations techniques. The following substitutions are useful:

(3.2.4a) \[ H = \beta_1 - \beta_3 \]

(3.2.4b) \[ K = \beta_4 - \beta_6 \]

(3.2.4c) \[ D = \gamma_3 - \gamma_1 \]

(3.2.4d) \[ E = \gamma_7 - \gamma_4 \]

(3.2.4e) \[ F = \gamma_8 - \gamma_5 \]

(3.2.4f) \[ \Delta = -HK + \beta_5 \beta_2 \beta_1 \]

(3.2.4g) \[ G = -(HE - \beta_5 \beta_2 \beta_1) \]

The coefficients are presented in columns, each column representing the coefficients of an exogenous variable ($Z_1, Z_2, Z_3, \text{and the constant terms}$). Also presented is the vector of disturbance terms.

---

$^{26}$This is $|B|$ and $0.114 < \Delta < 3.8$. 
(3.2.5a-d) \[ \alpha_{11} = \beta_1 \alpha_{21} + \beta_2 \alpha_{31} \]
\[ \alpha_{21} = \frac{-K \alpha_{41} + \epsilon}{\beta_5} \]
\[ \alpha_{31} = \beta_6 \alpha_{41} + \gamma_7 \]
\[ \alpha_{41} = G/\Delta \]

(3.2.6a-d) \[ \alpha_{12} = \beta_1 \alpha_{22} + \beta_2 \alpha_{32} \]
\[ \alpha_{22} = \frac{-K \alpha_{42} + \gamma_6}{\beta_5} \]
\[ \alpha_{32} = \beta_6 \alpha_{42} + \gamma_6 \]
\[ \alpha_{42} = \frac{-\gamma_6 (H + \beta_5 \beta_2)}{\Delta} \]

(3.2.7a-d) \[ \alpha_{13} = \beta_1 \alpha_{23} + \beta_2 \alpha_{33} \]
\[ \alpha_{23} = \frac{-K \alpha_{43}}{\beta_5} \]
\[ \alpha_{33} = \beta_6 \gamma_{43} \]
\[ \alpha_{43} = \frac{\beta_5 \gamma_2}{\Delta} \]
\( (3.2.8a-d) \)

\[
\begin{align*}
\alpha_{14} &= \beta_1 \alpha_{24} + \beta_2 \alpha_{34} + \gamma_1 \\
\alpha_{24} &= \frac{-\kappa \alpha_{44} + F}{\beta_5} \\
\alpha_{34} &= \beta_6 \alpha_{44} + \gamma_8 \\
\alpha_{44} &= \frac{\beta_5 (D - \beta_2 \gamma_8) - HF}{\Delta}
\end{align*}
\]

\( (3.2.9a-d) \)

\[
\begin{align*}
\eta_1 &= \beta_1 \eta_2 + \beta_2 \eta_3 + v \\
\eta_2 &= \frac{-\kappa \eta_4 + (X-W)}{\beta_5} \\
\eta_3 &= \beta_6 \eta_4 + x \\
\eta_4 &= \frac{\beta_5 (V-U) - H (X-W) - \beta_5 - \beta_2 x}{\Delta}
\end{align*}
\]
The variances of the reduced form are listed next with the aid of the following abbreviations:

(3.2.10a)

\[ M = \omega_v^2 - 2\omega_{uv} + \omega_v^2 \]

(3.2.10b)

\[ N = \omega_x^2 - 2\omega_{xw} + \omega_w^2 \]
\begin{align*}
\sigma_{11} &= \beta_1^2 \sigma_{22} + \beta_2^2 \sigma_{33} + u^2 + \frac{2}{\Delta} (\beta_2 \beta_6 \beta_5 - \beta_1) (\omega uv - \omega u^2) \\
&\quad + \frac{2\beta_1 \beta_2}{5} - K \beta_6 \sigma_{44}^2 + \left(1 + \frac{HK}{\Delta} - \frac{\beta_6 \beta_5 \beta_2}{\Delta}\right) (\omega x^2 - \omega xx) + \frac{K \beta_6 \beta_2}{\Delta} \omega x - \frac{HB_6}{\Delta} N \\
\sigma_{22} &= \frac{1}{\beta_5^2} \frac{K^2 \sigma_{44}}{\Delta} N - \frac{2K}{\Delta} (HN + \beta_5 \beta_2 (\omega x^2 - \omega xx)) \\
\sigma_{33} &= \frac{\beta_6^2}{6} \sigma_{44}^2 + \sigma x^2 - \frac{\beta_6^2}{\Delta} H (\omega x^2 - \omega xx) + \beta_5 \beta_2 \omega x^2 \\
\sigma_{44} &= \frac{1}{\Delta^2} \beta_5^2 M + H^2 N + \beta_5^2 \beta_5^2 \omega x^2 + 2HB_5 \beta_2 (\omega x^2 - \omega xx)
\end{align*}
The covariances of the reduced form are listed in Appendix B. Analysis of these will not be pursued, but ideally the testing procedure would not be complete in the absence of this.

The reduced form gives the coordinates of an unknown point in the A space. This point is defined by 26 parameters. Because there are 26 parameters in the reduced form space and only 20 parameters in the structural space B, when we map from B to A, there will be 6 functional dependencies among the A space variables. There are 16 coefficients in the reduced form and only 14 in the structural specification. Thus there are two dependencies which do not refer to the variances.

3.3 The Identifiability Relations

Altogether there are 26 identifying functions. In the present paper we have not given ten functions associated with the $\sigma$'s.
Table 3.3.1

The Identifying Functions

\( \phi_1 = -\alpha_1 + \beta_1 \alpha_5 + \beta_2 \alpha_9 \)
\( \phi_2 = -\alpha_2 + \beta_1 \alpha_6 + \beta_2 \alpha_{10} \)
\( \phi_3 = -\alpha_3 + \beta_1 \alpha_7 + \beta_2 \alpha_{11} \)
\( \phi_4 = -\alpha_4 + \beta_1 \alpha_8 + \beta_2 \alpha_{12} + \gamma_1 \)
\( \phi_5 = -\alpha_1 + \beta_3 \alpha_5 \)
\( \phi_6 = -\alpha_2 + \beta_3 \alpha_6 \)
\( \phi_7 = -\alpha_3 + \beta_3 \alpha_7 + \gamma_2 \)
\( \phi_8 = -\alpha_4 + \beta_3 \alpha_8 + \gamma_3 \)
\( \phi_9 = \beta_5 \alpha_5 - \alpha_9 + \beta_4 \alpha_{13} \)
\( \phi_{10} = \beta_5 \alpha_6 - \alpha_{10} + \beta_4 \alpha_{14} + \gamma_4 \)
\( \phi_{11} = \beta_5 \alpha_7 - \alpha_{11} + \beta_4 \alpha_{15} \)
\( \phi_{12} = \beta_5 \alpha_8 - \alpha_{12} + \beta_4 \alpha_{16} + \gamma_5 \)
\( \phi_{13} = -\alpha_9 + \beta_6 \alpha_{13} \)
\( \phi_{14} = -\alpha_{10} + \beta_6 \alpha_{14} + \gamma_7 \)
\( \phi_{15} = -\alpha_{11} + \beta_6 \alpha_{15} + \gamma_6 \)
\( \phi_{16} = -\alpha_{12} + \beta_6 \alpha_{16} + \gamma_8 \)
To insure that the mapping from A to B defined to (3.3.1 a–p) is 1-1 we make the hypothesis that Proposition 3.3.1 the functions (3.3.1 a–p) are zero at the point $\bar{a}$.

From this hypothesis and the relations (3.3.1 a–p) two dependencies are found, namely

Proposition 3.3.2 The regression coefficients $\alpha_i$, $i = 1, \ldots, 16$, in equations (3.3.1a–h) satisfy

$$(3.3.2a) \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_5 & \alpha_6 & \alpha_7 \\ \alpha_9 & \alpha_{10} & \alpha_{11} \end{vmatrix} = 0$$

$$\text{rank} \begin{vmatrix} \alpha_5 & \alpha_6 & \alpha_7 \\ \alpha & \alpha & \alpha \\ 9 & 10 & 11 \end{vmatrix} = 2$$

$$(3.3.3a) \begin{vmatrix} \alpha_1 & \alpha_2 \\ \alpha_5 & \alpha_6 \end{vmatrix} = 0$$

$$\text{rank} \begin{vmatrix} \alpha_5 & \alpha_6 \end{vmatrix} = 1$$
3.4 A Weaker Hypothesis as a Tentative Test

The restrictions assumed in the structural space imply that the point $\bar{\alpha}$ is in a 20-dimensional subset $A_B$, called the strict hypothesis of the 26-dimensional regression parameter space $A$. To derive the actual boundaries of this region is rather complicated. A feasible tentative test of the hypothesis is to derive a 20-dimensional rectangle set $A'_B$, called the weaker hypothesis, which contains $A_B$.

If the observed point falls outside the acceptance region associated with $A'_B$ then clearly it falls outside that acceptance region associated with $A_B$. The test is tentative because the observed point may fall within $A'_B$ but not $A_B$.

The sides of $A'_B$ are defined by the infima and suprema of the reduced form parameters (3.2.5 - 3.2.10).
Table 3.4.1

The Weaker Hypothesis $A_d^i$

(Infima and Suprema of Regression Parameters Admissible under Structural Premises)

<table>
<thead>
<tr>
<th>(3.4.1 a-d)</th>
<th>$0 &lt; \alpha_{11} &lt; 3.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.875$</td>
<td>$&lt; \alpha_{12} &lt; .0$</td>
</tr>
<tr>
<td>$.0$</td>
<td>$&lt; \alpha_{13} &lt; .2$</td>
</tr>
<tr>
<td>$.0011$</td>
<td>$&lt; \alpha_{14} &lt; 90.0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(3.4.2 a-d)</th>
<th>$-3.492 &lt; \alpha_{21} &lt; .0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-7.302$</td>
<td>$&lt; \alpha_{22} &lt; .0$</td>
</tr>
<tr>
<td>$.0$</td>
<td>$&lt; \alpha_{23} &lt; .3583$</td>
</tr>
<tr>
<td>$-8.7301$</td>
<td>$&lt; \alpha_{24} &lt; .11$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(3.4.3 a-d)</th>
<th>$.0 &lt; \alpha_{31} &lt; .9393$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.8538$</td>
<td>$&lt; \alpha_{32} &lt; .0$</td>
</tr>
<tr>
<td>$.0$</td>
<td>$&lt; \alpha_{33} &lt; .0952$</td>
</tr>
<tr>
<td>$.18691$</td>
<td>$&lt; \alpha_{34} &lt; 51.1905$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(3.4.4 a-d)</th>
<th>$-5.8442 &lt; \alpha_{41} &lt; 1.6883$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.0$</td>
<td>$&lt; \alpha_{42} &lt; 10.$</td>
</tr>
<tr>
<td>$.0$</td>
<td>$&lt; \alpha_{43} &lt; .9524$</td>
</tr>
<tr>
<td>$-38.2353$</td>
<td>$&lt; \alpha_{44} &lt; 511.9051$</td>
</tr>
<tr>
<td>(3.4.5 a-d)</td>
<td>( \sigma_{11} )</td>
</tr>
<tr>
<td>------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.0242</td>
<td>0.0026</td>
</tr>
<tr>
<td>( \sigma_{11} )</td>
<td>7.6221</td>
</tr>
</tbody>
</table>
IV. AN EMPIRICAL TEST AND RESULTS

In this Chapter the estimate $\hat{\theta}$ is compared with the identifiability conditions (3.3.6-7) and the weaker hypothesis defined in Table 3.4.1. The computation of the estimate was done on an IBM 7094 computer. Clee L. Childress wrote the program used, *Econometric Model Program--Version 2.* 27 This program also includes the identifiability test statistics and the GCL estimates of the structural coefficients.

A discussion and display of the data used is given in the Appendix A.

4.1 The Estimates

Table 4.1.1 contains the estimates of the coefficients of (3.1.1 s-d) along with their respective estimated finite sample standard deviations in parentheses below and the corresponding square roots of multiple correlation coefficients to the right. Table 4.1.2 shows the estimates of (3.1.3).

---

27This is based on the General Electric Document RM 61 TMP--12, Vol. 4.
Table 4.1.1
The Estimate of $\delta$

(4.1.1a) $Y_{t,1} = \frac{0.01619}{(0.0039)} Y_{t,1} - \frac{0.06335}{(0.13169)} Y_{t,2} - \frac{0.00282}{(0.00624)} Y_{t,3} + 4.33710 \quad R = 0.95260$

(4.1.1b) $Y_{t,2} = \frac{-0.00038}{(0.00104)} Y_{t,1} - \frac{-0.01964}{(0.03472)} Y_{t,2} + \frac{0.00614}{(0.00164)} Y_{t,3} + 31.6704 \quad R = 0.96603$

(4.1.1c) $Y_{t,3} = \frac{0.00867}{(0.00200)} Y_{t,1} - \frac{-0.03432}{(0.06649)} Y_{t,2} - \frac{-0.00344}{(0.00315)} Y_{t,3} + 3.5005 \quad R = 0.93990$

(4.1.1d) $Y_{t,4} = \frac{-0.05017}{(0.01222)} Y_{t,1} + \frac{0.44883}{(0.40712)} Y_{t,2} + \frac{0.04016}{(0.01928)} Y_{t,3} + 57.0349 \quad R = 0.89831$
Table 4.1.2
Matrix of Residuals

\[
\begin{bmatrix}
0.04109 & 0.00498 & 0.01931 & 0.02986 \\
0.00286 & 0.00227 & 0.02081 & \\
0.01048 & 0.01759 & \\
& & & 0.39273
\end{bmatrix}
\]

\[\sigma_{ij} = \]

\((i, j = 1, \ldots, 4)\)
4.2 A Test of Identifiability

The two identifiability conditions are tested following the procedures of R. L. Basmann.\(^\text{28}\)

The test statistic for (3.2.5-9) is called the identifiability test statistic or \(\bar{F}\) statistic. The distribution of the \(\bar{F}\) statistic is approximated by the Snedecor's \(F\). The acceptability of this procedure is dependent on analysis which is not carried out here because of its complexity.

The subsequent results, of course, are contingent on the requisite demonstration.

To test the identifiability conditions, a 5% critical region is employed i.e. \(\bar{F} > \text{Snedecor's } F_{.95} (m,n)\), where \(m\), the numerator's degrees of freedom, is equal to \((K_2 + G_\Delta + 1)\) and \(n\), the denominator's degree of freedom is equal to \((N-K)\) with \(N\) equal to the number of observations, \(K\) equal to the total number of exogenous variables, \(K_2\) equal to the number of exogenous variables excluded from the equation tested and \(G_\Delta\) equal to the number of endogenous variables included in the equation tested.

Table 4.2.1 gives the \(F\)'s calculated by the computer and the corresponding values of Snedecor's \(F\).

### Table 4.2.1

Identifiability Test Statistics

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>$(\bar{F})$</th>
<th>Condition</th>
<th>Snedecor's $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4.2.1a)</td>
<td>.07708</td>
<td>3.36</td>
<td>$F_{.95}(1, 19) = 4.38$</td>
</tr>
<tr>
<td>(4.2.1b)</td>
<td>1.76253</td>
<td>3.37</td>
<td>$F_{.95}(1, 19) = 4.38$</td>
</tr>
</tbody>
</table>

Inspection of Table 4.2.1 shows that the observations satisfy the presently used test for the restrictions (3.2.5-9).
4.3 Comparison of Regression Estimates with Predicted Ranges

Only two of the estimates shown in tables 4.1.1-2 fall outside the ranges predicted in the weaker hypothesis of table 3.4.1. These two estimates are those of $\alpha_{13}$, the coefficient of BC in the reduced form equation for CIC, and $\alpha_{33}$, the coefficient of BC in the reduced form equation for RNAS. In each case the estimate is negative, -.00282 and -.00344 respectively, whereas the predicted range has a lower bound of zero.

Inspection of the expressions for $\alpha_{13}$ and $\alpha_{33}$ in terms of the structural coefficients shows that no trivial alteration of the ranges of the latter would extend the lower bounds of the expressions for $\alpha_{13}$ and $\alpha_{33}$ so as to include the estimates. In order to arrive at negative lower bounds in these two cases, it would be necessary to permit the structural coefficients to take values opposite in sign to those they are here permitted. This represents a fundamental alteration of the theory being tested.

4.4 Conclusion

The estimates $\alpha_{13}$ and $\alpha_{33}$ reject the weaker hypothesis and therefore the model as it stands is rejected. A thorough investigation of the reasons as to why the model fails would include a check into the validity of the data (are they an accurate measure of an actual economic entity or phenomena?) and of the institutional assumptions (what are the effects of omitting references to the used car market?) in addition to a possible different specification of the model.

On the other hand, the fact that none of the identifiability conditions were rejected and that only two regression estimates were outside
a calculated range suggests that the essential structure of the model is appropriate for this investigation of the problem. This assumes of course, that had the strict hypothesis been derived, no further instances of inconsistencies would have occurred.

The fact that the estimates which fall outside the predicted range are of coefficients of the bank credit term suggests that the pioneer effort at a supply of credit function has not been entirely successful. The theory of the supply of credit needs further investigation. Conceptually it would have been more desirable to utilize some measure of the cost of funds to the industry, but as was discussed above, this did not work out.

Elimination of the term BC, possibly with an eye to substituting the term YSA in the supply of credit function, would leave the automobile market equations under-identifiable. The best hope for progress in this area is more extensive collection of data pertaining to finance charges and profit margins in the finance industry.

4.5 The Structural Estimates

The actual test has not taken place in the reduced form and the model has been rejected. The structural estimates then are of only academic interest and are present in table 4.5.1. Incidentally, the coefficient of MM in DRNAS is negative. The signs of PASA and WASA are the reverse of those expected.

For the purposes of facilitating comparisons of the hypothesized ranges on and estimates of the structural coefficient of the current model with the results of other studies, the elasticities of both the ranges and the estimates are given in Table C.1 in Appendix C.
Table 4.5.1

The Structural Estimates

| (est 2.2.1a) DCIC = 0.54220MM + 1.88621RNAS - 18.0512 |
|-------------|-------------|-------------|----------|
| (.21727)    | (.01535)    | (6.8147)    |
| (est 2.2.2a) SICSA = -8.95174MM + .07569BCSA + 257.563 |
| (17.9066)   | (.11307)    | (527.882)   |
| (est 2.2.3a) DRNAS = .07847PASA + -.04631MM + .0471YSA + 9.44087 |
| (.15611)    | (.75448)    | (.00799)    | (14.3575) |
| (sst 2.2.4a) SRNAS = -.08556PASA + .00407WASA + .00437YSA + 8.3853 |
| (.09804)    | (.06730)    | (.00364)    | (13.8315) |

\[
\begin{bmatrix}
0.00534 & -0.00560 & -0.00279 & -0.00285 \\
0.37295 & 0.05652 & 0.05666 \\
0.01602 & 0.01618 \\
0.01638
\end{bmatrix}
\]

(est 2.2.5b) \( \Omega = \)
BIBLIOGRAPHY


APPENDICES
APPENDIX A: THE DATA

The endogenous and exogenous variables used in the model are listed in Table A.1. Discussion of the sources and method of construction of each follows:

Sources used for obtaining the original data consist of the Statistical Releases published by the Division of Research and Statistics, Board of Governors of the Federal Reserve System, and the Survey of Current Business and its Supplement in 'National Income and Product Accounts.'

The method used here for the quarterly seasonal adjustment of all variables, excepting MM and YSA, is the X-11 variant of the Census Method II seasonal adjustment program. 29

Federal Reserve Statistical Release No. G-25 provides the data for the MM variable. This variable is taken directly from the release already seasonally adjusted.

The YSA variable, obtained from the Survey of Current Business, is also seasonally adjusted.

Due to a non-seasonal external shock effect, namely an automobile

---

<table>
<thead>
<tr>
<th>Year</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>I 2.715000000</td>
<td>32.29110</td>
<td>1.68240</td>
<td>103.60000</td>
<td>347.00000</td>
<td>101.40000</td>
<td>442.00000</td>
</tr>
<tr>
<td></td>
<td>II 2.630000000</td>
<td>32.29940</td>
<td>1.67669</td>
<td>102.80000</td>
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strike, the fourth quarter 1964 data of all variables have been deleted. Inspection of the data suggests that the effect was localized, and it is assumed here that this effect did not carry over into later periods.

For the purpose of seasonally adjusting the variables RNAS and ICSA, an estimate of the fourth quarter 1964 was substituted for the original data. This estimate was computed by averaging the original data for the fourth quarters of 1963 and 1965.
Endogenous Variables

\[ Y_{t,1} = \text{ICSA} = \text{the value of automobile instalment credit on new cars extended by all lenders in millions of dollars, seasonally adjusted.} \]
\[ Y_{t,1} \text{ is computed by adding monthly totals to obtain a quarterly total for each period. These data are then seasonally adjusted.} \]

Source: G.26 - Federal Reserve Statistical Release, (Automobile Instalment Credit Developments - Monthly)

\[ Y_{t,2} = \text{MM} = \text{the value of the median maturities of new car contracts made on loans extended by major sales finance companies, seasonally adjusted.} \]
\[ Y_{t,2} \text{ is computed by using quarterly averages of the monthly medians which are computed as follows:} \]

\[
\text{Median} = \text{Left End Limit of Class} + \\
\text{Total No. of Observations} = \frac{\text{No. of Observations to Left of Class}}{2} \times \text{No. of Units}^{30}
\]

Original data used are already seasonally adjusted.


\[ Y_{t,3} = \text{RNAS} = \text{the number (in thousands) of new passenger autos registered monthly, in seasonally adjusted terms.} \]
\[ Y_{t,3} \text{ is computed by determining quarterly totals from the monthly data, which are then seasonally adjusted.} \]

---

(under "Motor Vehicles")

\[ Y_{t,4} = \text{PASA} = \text{the value of the consumer price index for new cars for quarterly periods, seasonally adjusted.} \]

\[ Y_{t,4} \text{ is computed by obtaining monthly data which are determined relative to a base of 1957-59 = 100 to be averaged into quarters, then seasonally adjusted.} \]


(under "Commodity Prices - Consumer Prices")

**Exogenous Variables**

\[ Z_{t,1} = \text{YSA} = \text{the value of disposable personal income on a quarterly basis, in billions of dollars, seasonally adjusted.} \]

\[ Z_{t,1} \text{ is used as it appears in the source: quarterly averages which have already been seasonally adjusted.} \]


\[ Z_{t,2} = \text{WASA} = \text{the value of the wholesale price index of passenger cars on a quarterly basis, seasonally adjusted.} \]

\[ Z_{t,2} \text{ is computed by obtaining the data, which is related to a base of 1947-49 = 100 and is given in terms of monthly values, then averaging it into quarterly periods and seasonally adjusting it.} \]

(under "Commodity Prices")

\[ Z_{t,3} = \text{BCSA} = \text{the value of that share of automobile instalment credit held by commercial banks as a per cent of total consumer instalment credit in terms of quarterly periods, seasonally adjusted.} \]

\[ Z_{t,3} \] is computed by finding monthly percentages of bank-held auto instalment credit as it relates to total auto instalment credit using monthly averages (in millions of dollars) of each. The monthly percentages are then averaged into quarterly per cents and then are seasonally adjusted.

Source: G.19 - Federal Reserve Statistical Release monthly, p. 4 ("Consumer Credit")
APPENDIX B: THE COVARIANCES OF THE REDUCED FORM

The algebraic expressions for the covariances of the reduced form are listed in Table B.1.
Table B.1

The Covarainces of the Reduced Form

\( (B.1.1 \ a-f) \)

\[
\begin{align*}
\sigma_{43} &= \beta_6 \sigma_4 - \frac{1}{\Delta} \left[ \frac{\Delta}{H} (\omega x^2 - \omega x^2) + \beta_5 \beta_2 \omega x^2 \right] \\
\sigma_{42} &= -\frac{1}{\beta_5} \left[ \frac{\Delta}{K} \sigma_4 + (HN + \beta_5 \beta_2 (\omega x^2 - \omega x^2)) \right] \\
\sigma_{41} &= \beta_1 \sigma_{42} + \beta_2 \sigma_{43} + \frac{\beta_5}{\Delta} (\omega u - \omega u^2) \\
\sigma_{32} &= \frac{1}{\beta_5} \left[ -\frac{K}{\beta_6} \frac{\omega x^2}{\Delta} - \frac{\beta_6 H}{\Delta} N + \left( 1 + \frac{HK}{\Delta} - \frac{\beta_6 \beta_5 \beta_2}{\Delta} \right), \omega x^2 - \omega x^2 + \frac{K \beta_5 \beta_2}{\Delta} \omega x^2 \right] \\
\sigma_{31} &= \frac{\beta_1}{\beta_5} \frac{\omega x^2}{\Delta} \left[ \omega x^2 - \omega x^2 + \frac{K \beta_5 \beta_2}{\Delta} \omega x^2 \right] \\
&\quad + \beta_2 \left[ \frac{\Delta}{\omega x^2 - \omega x^2} \frac{HBC}{\Delta} \right] \\
\sigma_{21} &= -\frac{K}{\beta_5} \left[ \frac{\beta_1}{\beta_5} \sigma_{42} + \frac{\beta_2}{\Delta} \sigma_{43} + \frac{\beta_5}{\Delta} \omega u - \omega u^2 \right] + \frac{\beta_1 K}{\beta_5 \Delta} \left[ H - 1 \right] N + \beta_5 \beta_2 \left( \omega x^2 - \omega x^2 \right) \\
&\quad + \frac{\beta_2}{\beta_5} \left[ \frac{\Delta}{\omega x^2 - \omega x^2} \frac{H}{\Delta} N + \left( 1 - \frac{\beta_6 \beta_5 \beta_2}{\Delta} \right) \omega x^2 - \omega x^2 \right]
\end{align*}
\]
APPENDIX C: THE ELASTICITIES

The elasticities of the hypothesized ranges are given in Table C.1 and the elasticities of the estimates are given in Table C.2.
Table C.1

Elasticities of the Hypothesized Ranges

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Table C.2

Elasticities of the Estimates

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