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Saving and Growth under Borrowing Constraints: Explaining the "High Saving Rate" Puzzle*

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Abstract

This paper shows that uninsured risk and borrowing constraints can make an individual's marginal propensity to consume negatively dependent on his/her permanent income. Therefore, higher income growth can lead to higher saving rates without requiring (or causing) high interest rates — in sharp contrast to implications of the permanent income hypothesis. For example, the model predicts that household saving ratio can rise from 5% to 25% when the annual rate of income growth increases from 1% to 10%, despite a fixed 1% real deposit rate. The predictions are consistent with the experience of emerging economies, such as Japan (in the 1950-70s) and China (over the past 30 years).

Keywords: Permanent Income Hypothesis, Borrowing Constraints, High Saving Rate Puzzle, Chinese Economy, Growth and Development, Global Imbalance, Global Savings Glut.

JEL Codes: D91, E21, O16.

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1 Introduction

The canonical permanent income hypothesis (PIH)[1] predicts that forward-looking consumers should save less when income growth is high because they expect to be richer in the future than they are today. Accordingly, saving rates should be lower in fast-growing countries than in slow-growing countries.

This prediction has led to the well-known "high saving rate" puzzle for fast-developing economies (such as Japan in the 1950-70s and China in the past 30 years). Much effort has been devoted to resolving this puzzle without reaching a consensus. For example, Horioka (1985, 1990) provides a comprehensive list of various life-cycle factors that may contribute to Japan's high saving rate, including precautionary saving, housing finance, and income growth. However, careful analysis of the implications of the life-cycle hypothesis leads Hayashi (1986) to reject it as a plausible theory of Japan's saving behavior. In particular, Hayashi concludes that high income growth cannot explain Japan's high saving rate because the life-cycle hypothesis depends on heterogeneous cohort effects to generate the positive link between growth and aggregate saving, but such cohort effects are inconsistent with Japanese data.

Yet, empirical evidence suggests that saving and growth are strongly positively correlated, and the positive correlation holds largely because high growth leads to high saving, not vice versa.[2] Nonetheless, most theoretical effort has been devoted to understanding the growth-to-saving causality in a life-cycle framework, because it is thought that this fact is inconsistent with optimal consumption behavior in an infinite-horizon permanent-income framework.

A leading alternative view is that the PIH fails because it is based on, among other things, the assumption of exogenous rates of returns to financial assets (i.e., the real interest rate). In a production economy with productive assets (such as capital), the real rates of return are determined by the marginal products of such assets. When asset returns are so determined, they will respond to changes in productivity growth, which is the fundamental source of changes in permanent income. A permanent increase in total factor productivity (TFP) raises the rate of return to capital, so investment demand will increase, resulting in a higher equilibrium saving rate through a higher real interest rate. Consequently, in contrast to the prediction of the PIH, standard general-equilibrium growth theory suggests that household saving may increase rather than decrease in response to a higher permanent income (as implied by the analysis of Chen, Imrohoroglu, and Imrohoroglu, 2006).

[1] Friedman (1957).

[2] See, e.g., Carroll, Overland, and Weil (2000) and the references therein.

However, fast-growing economies tend to have not only high saving rates, but also low and essentially fixed interest rates. For example, in Japan in the 1950-70s, the household saving rate was as high as 23%, while the real 3-month time deposit rate was negative for the entire period.[3] Similarly, in China over the past 30 years, the average personal saving rate has been about 25%, yet the real 1-year deposit rate has been below zero.[4] Hence, the puzzle is not just why high growth can lead to high saving (or vice versa), but also why a high saving rate is possible when the interest rate is so low.[5]

This paper argues that sufficiently large uninsured uncertainty and borrowing constraints may hold the key for understanding the triple phenomena of "high saving, high growth, and low interest rate" for fast-growing countries. That households are subject to borrowing constraints and uninsured idiosyncratic risk has been extensively documented and studied in the literature for developed countries (see, e.g., Aiyagari, 1994, and Gross and Souleles, 2002, and the references therein). Agents in developing countries are typically far more borrowing constrained and far less insured for idiosyncratic shocks than those in developed countries, because of the lack of social safety nets and under-developed financial and insurance markets. Skinner (1986) presents empirical evidence that income uncertainty is a key factor determining households' precautionary savings in developing countries.

Precautionary saving under borrowing constraints can completely alter the relationship between permanent income and consumption by making the marginal propensity to save a positive function of permanent labor income, so that faster income growth generates an increased propensity to save regardless of interest rates. As a result, high growth can lead to high saving without requiring or causing high interest rates. In other words, even if the deposit rate is fixed at zero, household saving may still be excessively high and respond *positively* to income growth.[6] For the same reason, borrowing constraints can support a large spread between the deposit rate and the rate of return to capital in general equilibrium. Consequently, fast-growing economies may appear to have undiminished high rates of return to capital despite a high investment-to-output ratio.[7]

[3] An extensive literature argues that the Japanese government repressed interest rates in that period to promote investment and stimulate growth (see, e.g., Horiuchi, 1984; Patrick and Rosovsky, 1976). Dekle (1993) and Homer and Sylla (2005) also note that real interest rates were never particularly high in Japan, but were higher in the low-saving, low-growth 1920s than in the high-saving, high-growth 1960s. Although the rates of return on corporate equities were very high in Japan during that period, households' share of equities was quite low (see, e.g., Horioka, 1990).

[4] The fact that the real interest rates in China have been kept very low since the economic reform is well known (see, e.g., Zou and Sun, 1996). Because households' access to financial markets and investment opportunities are typically very limited and stockmarket is extremely risky, deposits have been the major means of saving and the most important contributor to China's aggregate saving ratio despite the low interest rates (see, e.g., Kraay, 2000).

[5] As a reference point, in the standard neoclassical growth model presented in the next section, when the rate of income growth reaches 10% per year, the aggregate saving rate can reach about 20%. However, to support this high saving rate, the real interest rate must be about 15%.

[6] This would not be the case without uninsured risk and borrowing constraints. In a standard growth model, the saving rate would decrease with income growth if the interest rate were constant, as predicted by the PIH.

[7] For example, the real rate of return to capital has been over 20% a year for the past 30 years in China, and it has shown little sign of diminishing despite the 40% investment-to-output ratio (Bai, Hsieh, and Qian, 2006).

The main intuition is that uninsured risk and borrowing constraints induce precautionary savings, even if the interest rate is low (imagine why rational agents hold inventories despite negative rates of return — because inventories yield a liquidity premium). If the degree of uncertainty remains constant relative to the income trend, agents would want to maintain a stable stock of savings relative to trend because of the need for self-insurance; namely, they prefer to maintain a constant buffer stock to income ratio along a balanced growth path. Since income is a flow, when it grows, a larger portion of the flow must be devoted to the accumulation of the stock ("investment"); otherwise, the stock-to-flow ratio would decline sharply, which would hinder the buffer-stock function of savings and reduce the extent of self-insurance. Thus, the saving rate should rise with income growth. In other words, the effective rate of return to saving is the real interest rate compounded by a liquidity premium. When the growth rate of income rises, the liquidity premium increases if the saving stock falls relative to the income trend. A higher liquidity premium thus induces a higher saving rate.[8] Therefore, the predictions of the canonical PIH are completely altered: High growth leads to high saving, even with low (and fixed) interest rates.

These arguments are formalized in this paper in an infinite-horizon neoclassical growth model (with or without capital), where long-run income growth is driven by exogenous TFP changes and households face uninsured idiosyncratic shocks and are borrowing constrained. To facilitate the analysis, some simplifying assumptions are necessary to make the model analytically tractable. When markets are incomplete and agents are heterogeneous, analytical tractability not only reduces the computational costs of comparative analysis, but also makes the mechanisms transparent.[9] The key simplifying assumptions include (i) The utility function is quasi-linear.[10] (ii) Labor supply decisions of all households must be made before observing their idiosyncratic shocks in each period. The second assumption implies that a spot labor market does not always exist and it ensures the buffer-stock function of savings — otherwise, current wage income can be adjusted immediately to fully buffer any shocks to income or preferences within the same period.[11] In addition, we need to place the idiosyncratic shocks on preferences or gross wealth instead of labor earnings.[12] Under these assumptions, the optimal growth model with borrowing constraints has closed-form solutions

[8] However, to keep the stock-to-flow ratio exactly constant as income grows faster is very costly — the opportunity cost of not consuming the rising income increases with income growth. Hence, although the saving rate may increase, it may not increase enough to achieve the original stock-to-flow ratio. So in a new steady state with a higher growth rate, the optimal stock-to-flow ratio is lower than before despite a higher saving rate. This means that the liquidity premium increases by just enough to balance the marginal costs of saving and the marginal benefits of saving.

[9] Given that our results are in sharp contrast to conventional wisdom and may appear to be counterintuitive, closed-form solutions are preferred to numerical results.

[10] Although this assumption implies an infinitely elastic labor supply, it is nonetheless realistic for developing economies such as China — the large rural population in provides an abundant supply of labor.

[11] This assumption is not needed if labor supply is inelastic or the leisure function is nonlinear, but then the model becomes analytically intractable.

[12] Even with quasi-linear preferences, the model is not analytically tractable if the idiosyncratic shocks come directly from labor income (see, e.g., Imrohoroglu, 1989; Zeldes, 1989; Deaton, 1991; Carroll, 2001; Aiyagari, 1994; Krusell and Smith, 1998).

for individuals' optimal consumption and saving plans. Aggregating by the law of large numbers, the general equilibrium of the model can be solved by standard methods in the representative-agent real business cycle (RBC) literature, and dynamic impulse responses of aggregate savings to both transitory and permanent changes in the growth rate of TFP (the source of permanent income) can be easily analyzed.

An important property of the model is that it reduces to a representative-agent, frictionless neoclassical growth model with aggregate uncertainty when the distribution of idiosyncratic shocks becomes degenerate. This property makes the model easily comparable to standard growth models by changing the parameter values that control the strength of precautionary saving and borrowing constraints.[13]

This paper is closely related to those by Jappelli and Pagano (1994), Carroll, Overland, and Weil (2000), and Chen, Imrohoroglu, and Imrohoroglu (2006). Jappelli and Pagano (1994) study the relationships between saving and growth under borrowing constraints in a simple overlapping-generations model. They show that borrowing constraints can enhance the positive effect of growth on saving. However, as pointed out by Modigliani (1970), Hayashi (1986), and many others, in life-cycle models the positive effect of growth on saving is largely the result of aggregation. To the extent that the economy is growing, workers' savings will increase relative to retirees' dissavings, thus, measured aggregate savings will increase. In contrast, this paper studies the issue in an infinite-horizon growth model in which the positive growth-to-saving effect originates from a different mechanism. That borrowing constraints can increase precautionary saving at a given level of income growth is well known, but whether borrowing constraints can also generate a positive causal effect of growth on saving in an infinite-horizon permanent-income framework is unclear. In this regards, this paper complements the analysis of Jappelli and Pagano (1994).[14]

Alternatively, Carroll, Overland, and Weil (2000) use an infinite-horizon endogenous-growth model with habit formation to explain the positive effect of growth on saving. Habit formation can generate a positive growth-to-saving effect because it makes consumption "sticky"; consequently, an increase in permanent income will raise saving first before it has a full impact on consumption. However, in this endogenous-growth model, high growth necessarily implies a high real interest rate, which is inconsistent with the experience of Japan and China. Also, Chamon and Prasad's (2009) empirical analysis based on Chinese household data does not support habit formation as a plausible explanation for China's high household saving rates.

[13] The solution techniques applied in this paper are similar to those in Wen (2009).

[14] The idea that borrowing constraints can lead to excessive saving and can be a possible source of failures of the PIH is hardly new. For example, see Zeldes (1989), Imrohoroglu (1989), Deaton (1991), Carroll (1992, 1997), Aiyagari (1994), and Huggett (1997), among many others. However, the implications of borrowing constraints for the aggregate relationship between saving and growth have not previously been examined in a rigorous infinite-horizon growth model, to the best of my knowledge. In addition, none of the above-cited works provides closed-form saving functions under borrowing constraints in an infinite-horizon model as is done in this paper.

More recently, Chen, Imrohoroglu, and Imrohoroglu (2006) use a standard neoclassical growth model to offer a quantitative account of the time path of Japan's saving rate in the postwar period. Their simulations, based on actual time-series data and the assumption of perfect foresight, reveal that stochastic TFP growth is the main force driving Japan's saving rate. But their model also requires high interest rates to induce high saving rates and they do not address the low interest rate issue.

This paper also has implications for global imbalances and global savings glut. If fast-growing economies opt to save excessively more because of higher income growth and less developed financial markets, it is then natural to observe outflows of savings from these economies toward developed countries where the interest rates are high, and inflows of foreign direct investment (FDI) from developed countries toward these emerging economies with faster TFP growth and high marginal products of fixed capital. In this regard, this paper is also closely related to the work of Caballero, Farhi, and Gourinchas (2008), Ju and Wei (2006), Mendoza, Quadrini, and Rios-Rull (2009), and Song, Storesletten, and Zilibotti (2009). These papers all emphasize inefficient financial system in emerging economies as the key contributing factor to their financial capital outflows. However, none of these papers aims at explaining why high growth can lead to high saving despite low interest rates.[15]

The rest of the paper is organized as follows. Section 2 presents the model and derives closed-form decision rules for household consumption and saving. Section 3 studies the general-equilibrium effects of growth and borrowing constraints on saving behavior along balanced growth paths. Section 4 studies the transitional and short-run dynamics of the aggregate saving rate. Section 5 reconsiders the growth-to-saving effects in a counterfactual experiment with a fixed deposit rate. Section 6 shows that the predictions of the model are consistent with the experience of China and Japan during their high-growth periods. Section 7 concludes the paper with remarks for future research. The robustness of the results for more general utility functions is analyzed in the Appendix.

2 The Model

2.1 Households

There are a continuum of households indexed by $i \in [0,1]$. Taking as given the market real interest rate r_t, real wage W_t, and profit income V_t from firms, household i chooses sequences of consumption $C_t(i)$, savings $S_{t+1}(i)$, and labor supply $N_t(i)$ to maximize expected lifetime utility, $E\sum_{t=0}^{\infty}\beta^t\left\{\log C_t(i) - aN_t(i)\right\}$, subject to the budget constraint $C_t(i) + S_{t+1}(i) \leq \left[\varepsilon_t(i) + \theta\right] X_t(i)$,

[15] Song, Storesletten, and Zilibotti (2009) explain China's high growth rate while taking as given its high saving rate. This paper complements their analysis by offering an explanation of China's high saving rate.

where $X_t(i) \equiv [(1+r_t)S_t(i) + W_t N_t(i) + V_t]$ denotes household i's wealth income or cash in hand,[16] which includes initial wealth, $(1+r)S_t(i)$, labor income, $W_t N_t(i)$, and any lump-sum transfers such as firms' profit income V_t. Households are borrowing constrained, so we impose $S_{t+1}(i) \geq 0$ in each period following the standard literature. The log form of the utility function ensures the existence of a balanced growth path.[17]

Note that cash in hand is subject to a multiplicative idiosyncratic shock, $\theta + \varepsilon_t(i)$, where $\theta \in (0,1)$ is a constant multiplier, and $\varepsilon_t(i)$ is i.i.d. with mean $E\varepsilon(i) = 1-\theta$. This implies that θ fraction of the wealth income is never subject to idiosyncratic shocks. The expected value of the shock is normalized to $E\varepsilon(i) = 1-\theta$ so that the average value of $\theta + \varepsilon(i)$ equals 1; thus, idiosyncratic shocks do not cause distortions to the budget constraint on average or at the aggregate level.[18] The model reduces to a representative-agent model either if $\theta = 1$ or the variance of $\varepsilon_t(i)$ becomes zero.[19]

An interpretation of the wealth-income shock is disaster risk: Nature randomly destroys a portion of household wealth in each period. Since disaster risk is not fully self-insurable, the model can be easily calibrated to match the wealth inequality in the data if θ is small enough, even though $\varepsilon(i)$ is i.i.d.[20] This type of shocks helps to capture the severity of idiosyncratic risk in developing countries (such as traffic and various types of accidents, natural and social-political disasters, robbery, unemployment, illness, erratic changes in the value of wealth, and so on). However, with respect to the main results of the paper, this assumption is innocuous — it is mainly a technical device to obtain closed-form solutions because it is well known that with CRRA utility functions, an infinite-horizon consumption model with borrowing constraints is not analytically tractable if the idiosyncratic risk derives directly and solely from labor income. The model is also tractable if the idiosyncratic shocks are placed on preferences instead of cash in hand (see Wen, 2009) and the main results of the paper remain intact, which is reassuring since it suggests that the key mechanisms uncovered in this paper are robust to the assumptions of the sources of idiosyncratic risks.[21]

[16] In this paper, "wealth income" and "cash in hand" are used interchangeably.

[17] The Appendix shows that the qualitative results do not hinge on the log form of the utility function.

[18] The multiplicative assumption of individual wealth-income shocks implies that the degree of risk remains constant relative to the balanced growth path and does not diminish with growth — namely, wealth inequality remains stable as the economy grows. This implication is consistent with empirical evidence (see, e.g., Wolff, 1998). The same assumption is made in the literature of incomplete markets where idiosyncratic earning (or labor productivity) shocks are multiplicative to the real wage, suggesting that the degree of income uncertainty does not diminish as the real wage grows over time (see, e.g., Aiyagari, 1994). Still, without economic analysis, such assumptions do not indicate whether the saving rate should increase or decrease with growth. On the other hand, it is unrealistic to assume that the idiosyncratic shocks are additive to wealth or income, which would imply that as wealth or income grow over time the significance of such shocks diminishes to zero.

[19] When $\theta \to 1$, the distribution of $\varepsilon(i)$ becomes a degenerate delta function with a unit mass centered at 0.

[20] Aiyagari (1994) and Krusell and Smith (1998) show that idiosyncratic risk stemming from labor income alone cannot generate enough inequality across households to explain the wealth distribution in the United States because such shocks are almost fully self-insurable unless they are highly persistent.

[21] The point made in this paper is more general than it may appear under the specific assumptions of the stylized model, since (i) the source of the uninsured idiosyncratic risk is irrelevant, (ii) agents always need to accumulate a buffer stock of savings, and (iii) they want to maintain a stable stock-to-flow ratio on the balanced growth path.

The information structure and sequence of events are as follows. Within each time period t, there are two subperiods. All aggregate shocks are realized in the beginning of the first subperiod but the idiosyncratic shocks are realized only in the second subperiod. In the first subperiod, households choose labor supply $N_t(i)$ after observing period-t aggregate shocks but without observing the idiosyncratic wealth-income shocks. In the second subperiod, the idiosyncratic shocks $\varepsilon_t(i)$ are realized and households then choose consumption and savings to maximize expected lifetime utilities.[22] Without loss of generality, assume $a = 1$. The population is constant over time. Idiosyncratic shocks are assumed to be orthogonal to any aggregate shocks.

2.2 Firms

There is a unit mass of identical firms producing output (Y_t) according to the technology, $Y_t = K_t^{\alpha} (Z_t N_t)^{1-\alpha}$, where Z_t denotes a nonstationary process of labor-augmenting technology, which grows over time according to the process $Z_t = (1 + g_t) Z_{t-1}$. The capital stock (K) is accumulated according to $K_{t+1} = (1 - \delta) K_t + I_t$, where I is investment per firm. The stochastic growth rate g_t has mean $\bar{g} \geq 0$ and follows the law of motion:

$$g_t - \bar{g} = \rho_g (g_{t-1} - \bar{g}) + \mu_t, \tag{1}$$

where $\rho_g \geq 0$ measures the persistence of the aggregate growth shock and the innovation μ_t is i.i.d with zero mean. When $\bar{g} = 0$ and $\rho_g = 0$, the dynamic effects of Z_t are identical to a random-walk technology shock without drift. With a little abuse of language, we use the terms "technology" and "TFP" interchangeably, although Z_t reflects labor-augmenting technology. We assume the existence of capital rental markets, as in Aiyagari (1994). Firms behave competitively; hence, the real factor prices are determined by their respective marginal products: $W_t = (1 - \alpha) \frac{Y_t}{N_t}$ and $r_t + \delta = \alpha \frac{Y_t}{K_t}$, where $r + \delta$ is the user's cost of capital with capital depreciation $\delta \in [0, 1]$. Because of constant returns to scale, the profit income is zero, $V_t = 0$.

2.3 Transformation

The model is not stationary in the level but is stationary in the growth rate. In the absence of aggregate uncertainty (i.e., $g_t = \bar{g}$ for all t), the aggregate economy has a unique balanced growth path along which the real interest rate and aggregate hours worked, $N \equiv \int N(i) di$, are constant, and the other aggregate variables, such as $C_t \equiv \int C(i) di$, $S_{t+1} \equiv \int S(i) di$, K_{t+1}, Y_t, and W_t all

Therefore, the liquidity premium of the buffer stock must rise whenever the stock-to-flow ratio declines, which will trigger a higher saving rate.

[22] This timing structure implies that households may not be able to fully self-insure against idiosyncratic risk by adjusting labor income despite perfectly elastic labor supply.

grow at the same rate $\bar{g}$. Hence, to solve for the competitive equilibrium, we can transform the model into a stationary one by scaling it down by the growth factor $(1+\bar{g})^{-t}$. Using lowercase letters to denote the transformed variables (e.g., $y_t \equiv \frac{Y_t}{(1+\bar{g})^t}$),[23] the production function and the real factor prices become

$$y_t = k_t^{\alpha} \left(z_t N_t\right)^{1-\alpha} \tag{2}$$

$$w_t = (1-\alpha)\,\frac{y_t}{N_t} \tag{3}$$

$$r_t + \delta = \alpha \frac{y_t}{k_t}, \tag{4}$$

respectively. Other relationships such as the capital accumulation equation can be transformed analogously.

After the transformation, household i's maximization problem can be written more compactly as

$$\max_{\{c,s'\}} E_0 \left\{ \max_{\{N\}} \tilde{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log c_t(i) - N_t(i)\right] \right\} \right\} \tag{5}$$

subject to

$$c_t(i) + (1+\bar{g})\, s_{t+1}(i) \leq \left[\varepsilon_t(i) + \theta\right] x_t(i) \tag{6}$$

$$s_{t+1}(i) \geq 0, \tag{7}$$

where

$$x_t(i) \equiv (1+r_t) s_t(i) + w_t N_t(i) + v_t \tag{8}$$

defines the transformed cash in hand net of the idiosyncratic multiplier, and the expectation operator $\tilde{E}_t$ in the objective function denotes expectations conditional on the information set of time t excluding $\varepsilon_t(i)$, and the operator E_t denotes expectations based on the full information set in period t including $\varepsilon_t(i)$. These notations reflect the information and timing structure of the model. The idiosyncratic i.i.d. shock has support $\varepsilon \in [0, \varepsilon_{\max}]$, cumulative distribution function $F(\varepsilon)$, and the unconditional mean $\int \varepsilon dF = 1-\theta$. The mean requirement implies $\varepsilon_{\max} > 1-\theta$.

2.3.1 Decision Rules

Denoting $\{\lambda(i), \pi(i)\}$ as the Lagrangian multipliers for constraints (6) and (7), respectively, the first-order conditions for $\{c(i), N(i), s(i)\}$ are given, respectively, by

$$\frac{1}{c(i)} = \lambda(i) \tag{9}$$

[23] To obtain the equilibrium path of the untransformed variables, we can apply the inverse transformation, such as $Y_t = (1+\bar{g})^t\, y_t$ and $\frac{Y_t}{Y_{t-1}} = (1+\bar{g})\,\frac{y_t}{y_{t-1}}$.

$$1 = w_t \tilde{E}_t \left\{ \left[\varepsilon_t(i) + \theta \right] \lambda_t(i) \right\} = w_t \int \left[\varepsilon_t(i) + \theta \right] \lambda_t(i) dF(\varepsilon) \tag{10}$$

$$(1 + \bar{g}) \lambda_t(i) = \beta E_t \left\{ (1 + r_{t+1}) \left[\varepsilon_{t+1}(i) + \theta \right] \lambda_{t+1}(i) \right\} + \pi_t(i), \tag{11}$$

where equation (10) reflects the fact that labor supply $N_t(i)$ is determined before $\varepsilon_t(i)$ (and hence the value of $\lambda_t(i)$) is realized. By the law of iterated expectations and the orthogonality assumption of aggregate and idiosyncratic shocks, equation (11) can be written as

$$(1 + \bar{g}) \lambda_t(i) = \beta E_t \left\{ (1 + r_{t+1}) \int \left[\varepsilon_{t+1}(i) + \theta \right] \lambda_{t+1}(i) dF(\varepsilon) \right\} + \pi_t(i) = \beta E_t \frac{1 + r_{t+1}}{w_{t+1}} + \pi_t(i), \tag{12}$$

where the first equality applies the law of iterated expectations and the orthogonality assumption, and the second equality uses equation (10).

The optimal consumption and saving plans of each individual are characterized by a cutoff strategy, where the cutoff (ε_t^*) is related to the realization of the idiosyncratic shock. We assume interior solutions for labor supply and use a guess-and-verify strategy to derive the decision rules. A key step in the analysis is to show that the cutoff ε_t^*, as well as the optimal cash in hand x_t, are independent of i.

Proposition 1 *The decision rules for consumption, saving, and cash in hand for any household i are given by*

$$c_t(i) = \min \left\{ \frac{\varepsilon_t(i) + \theta}{\varepsilon_t^* + \theta}, 1 \right\} \times \left[\varepsilon_t^* + \theta \right] x_t \tag{13}$$

$$(1 + \bar{g}) s_{t+1}(i) = \max \left\{ \frac{\varepsilon_t(i) - \varepsilon_t^*}{\varepsilon_t^* + \theta}, 0 \right\} \times \left[\varepsilon_t^* + \theta \right] x_t \tag{14}$$

$$x_t = (1 + r_t) s_t(i) + w N_t(i) + v_t = w_t R(\varepsilon_t^*) \frac{1}{\varepsilon_t^* + \theta}, \tag{15}$$

where the cutoff ε_t^* is determined by the equation

$$\frac{1 + \bar{g}}{w_t} = \left[\beta E_t \frac{1 + r_{t+1}}{w_{t+1}} \right] R(\varepsilon_t^*), \tag{16}$$

in which the implicit function $R(\cdot)$ is given by

$$R(\varepsilon_t^*) \equiv \theta + \left[\int_{\varepsilon < \varepsilon_t^*} \varepsilon_t^* dF(\varepsilon) + \int_{\varepsilon \geq \varepsilon_t^*} \varepsilon dF(\varepsilon) \right] > 1. \tag{17}$$

Proof. In anticipation that the cutoff is independent of i, consider two possible cases:

Case A. $\varepsilon_t(i) \geq \varepsilon_t^*$. In this case, the effective cash in hand is high. To smooth consumption, it is optimal to save to prevent possible borrowing constraints in the future when cash in hand may be low. So $s_{t+1}(i) \geq 0$, $\pi_t(i) = 0$, and the shadow value of good $\lambda_t(i) = \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}$. Equation (9) implies that consumption is given by $c_t(i) = \left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}\right]^{-1}$. The budget constraint (6) then implies $(1+\bar{g})\, s_{t+1}(i) = [\varepsilon_t(i) + \theta]\, x_t - \left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}\right]^{-1}$. The requirement $s_{t+1}(i) \geq 0$ then implies

$$\varepsilon_t(i) + \theta \geq \frac{1}{x_t}\left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})\, w_{t+1}}\right]^{-1} \equiv \varepsilon_t^* + \theta, \tag{18}$$

which defines the cutoff ε_t^*.

Case B. $\varepsilon_t(i) < \varepsilon_t^*$. In this case, the effective cash in hand is low. To maintain a smooth consumption, it is then optimal *not* to save, so $s_{t+1}(i) = 0$ and $\pi_t(i) > 0$. By the resource constraint (6), we have $c_t(i) = [\varepsilon_t(i) + \theta]\, x_t$, which by equation (18) implies $c(i) = \frac{\varepsilon_t(i)+\theta}{\varepsilon_t^*+\theta}\left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}\right]^{-1}$. Equation (9) then implies that the marginal utility of consumption is given by $\lambda_t(i) = \frac{\varepsilon_t^*+\theta}{\varepsilon_t(i)+\theta}\left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}\right]$. Since $\varepsilon(i) < \varepsilon^*$, equation (12) confirms that $\pi_t(i) = \left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}}\right]\left[\frac{\varepsilon^*+\theta}{\varepsilon(i)+\theta} - 1\right] > 0$.

The above analyses imply that $\lambda_t(i)$ takes two possible functional forms, depending on the size of the realization of $\varepsilon_t(i)$. Hence, the expected value, $\tilde{E}\left\{[\varepsilon_t(i) + \theta]\, \lambda_t(i)\right\}$, can be expressed analytically. As a result, the optimal cutoff, ε_t^*, is determined by the Euler equation (16), which is based on the first-order condition for labor supply (equation 10). Equations (16) and (17) imply that the cutoff ε_t^* is independent of i because $\varepsilon(i)$ is i.i.d. Hence, equation (18), which defines the cutoff, implies that cash in hand (x_t) is also independent of i. ■

2.4 Discussion

Notice that these decision rules are consistent with the budget identity, $c_t(i) + (1+\bar{g})s_{t+1}(i) = [\varepsilon_t(i) + \theta]\, x_t$, and are very intuitive. Optimal consumption is a concave function of a target level of wealth or cash in hand, $[\varepsilon_t^* + \theta]\, x_t$, with the marginal propensity to consume given by the function, $\min\left\{\frac{\varepsilon_t(i)+\theta}{\varepsilon_t^*+\theta}, 1\right\}$. When the wealth-income shock is low ($\varepsilon(i) < \varepsilon^*$), the marginal propensity to consume is less than 1; when the wealth-income shock is high ($\varepsilon(i) \geq \varepsilon^*$), the marginal propensity to consume equals 1 and the individual does not save in this period. Therefore, saving is a buffer stock: The household saves ($s_{t+1}(i) > 0$) only if the wealth-income shock is high. These properties are consistent with the literature on buffer-stock saving (see, e.g., Deaton, 1991; Aiyagari, 1994,

and Carroll, 1992, 1997), except here they are expressed analytically instead of numerically.

The target-wealth policy is an important feature of optimal buffer-stock saving behaviors; this behavior has also been noted by Deaton (1991) in a simpler partial-equilibrium model where labor income is exogenous and subject to uninsured idiosyncratic shocks. Hence, this target behavior is not driven by the assumption of wealth shocks in our model. Nonetheless, depending on the particular models, the optimal target may or may not depend on an individual's history and initial wealth.[24]

Notice that $R(\varepsilon_t^*) > 1$ because it captures the extra rate of return to savings due to the liquidity value of the buffer stock under borrowing constraints. Hence, the effective rate of return to saving is determined by the real interest rate compounded by a liquidity premium $R(\theta^*)$: $(1+r)\,R(\theta^*) > 1+r$.

The intuition for optimal cash in hand x_t to be independent of i is that (i) it is predetermined before the realization of $\varepsilon_t(i)$ and (ii) labor supply $n_t(i)$ can adjust elastically to target any level of cash in hand under a constant marginal cost of leisure. That is, since all households face the same distribution of idiosyncratic wealth-income shocks, the quasi-linear utility function makes it feasible and optimal that households adjust their labor supply to target the same level of cash in hand regardless of history. Thus, x_t is the same across households regardless of the initial wealth $s_t(i)$. Consequently, all households start the second subperiod with the same cash in hand. Given this target level of cash in hand, equation (15) determines the optimal level of the labor supply.

2.5 General Equilibrium

Denoting aggregate variables as those without index i: $c \equiv \int c(i)di$, $s \equiv \int s(i)di$, and $N \equiv \int N(i)di$; and integrating the household decision rules over i by the law of large numbers, the policy functions for the aggregate variables are given by

$$c_t = D(\varepsilon_t^*)x_t \tag{19}$$

$$(1+\bar{g})s_{t+1} = H(\varepsilon_t^*)x_t \tag{20}$$

$$x_t = (1+r_t)s_t + wN_t + v_t = w_t R(\varepsilon_t^*)\frac{1}{\varepsilon_t^* + \theta}, \tag{21}$$

where the functions

$$D(\varepsilon^*) \equiv \theta + \int_{\varepsilon(i)\le\varepsilon^*} \varepsilon(i)dF(\varepsilon) + \int_{\varepsilon(i)>\varepsilon^*} \varepsilon_t^* dF(\varepsilon) \tag{22}$$

$$H(\varepsilon^*) \equiv \int_{\varepsilon(i)>\varepsilon^*} \varepsilon(i)dF(\varepsilon) - \int_{\varepsilon(i)>\varepsilon^*} \varepsilon_t^* dF(\varepsilon), \tag{23}$$

[24] But a model with a history-independent target wealth is much simpler to analyze.

which satisfy $0 < D(\varepsilon^*) < 1$ and $D(\varepsilon^*) + H(\varepsilon^*) = 1$.[25]

A general equilibrium is defined as the sequences of aggregate variables $\{\varepsilon_t^*, c_t, k_{t+1}, N_t, y_t, w_t, r_t\}$, such that (i) given prices $\{w_t, r_t\}$, households maximize utilities; (ii) given prices $\{w_t, r_t\}$, firms maximize profits; (iii) the law of large numbers hold; (iv) all markets clear: $s_t = k_t$ and $\int N_t(i)di = N_t$; and (v) the standard transversality condition hold:

$$\lim_{T\to\infty} E_0 \beta^T \frac{(1+\bar{g})\, k_{T+1}}{c_T} = 0. \tag{24}$$

Define the disposable income as $\wp_t \equiv r_t s_t + w_t N_t + v_t$, which includes labor income, capital gains, and lump-sum profit income. Hence, the aggregate wealth income is related to disposable income by the relation, $x_t = s_t + \wp_t$. Using this definition, the consumption function and the saving function together imply the household budget identity,

$$c_t + (1+\bar{g})\, s_{t+1} - s_t = \wp_t. \tag{25}$$

Namely, consumption plus net savings (i.e., net wealth accumulation) equals disposable income. The aggregate saving rate can thus be defined as the ratio of net savings to disposable income:

$$\tau_t \equiv \frac{S_{t+1} - S_t}{r_t S_t + W_t N_t + V_t} = \frac{(1+\bar{g})\, s_{t+1} - s_t}{\wp_t}. \tag{26}$$

Because of constant returns to scale, the profit income $v_t = 0$. Since $s_t = k_t$, $w_t = (1-\alpha)\frac{y_t}{N_t}$, and $r_t + \delta = \sigma \frac{y_t}{k_t}$, the household budget identity then becomes

$$c_t + (1+\bar{g})\, k_{t+1} - k_t = y_t - \delta k_t, \tag{27}$$

where $y_t - \delta k_t = \wp_t$ is an alternative expression of disposable income. This aggregate household budget identity is also the goods market clearing condition. The definition of saving rate in equation (26) in general equilibrium becomes

$$\tau_t \equiv \frac{(1+\bar{g})\, k_{t+1} - k_t}{y_t - \delta k_t}, \tag{28}$$

which is identical to the definition adopted by Chen, Imrohoroglu, and Imrohoroglu (2006).

The system of equations that determine the general equilibrium of the model consists of equations (2), (3), (4), (16), (19), (20), (27), and the transversality condition (24). It can be easily confirmed that this dynamic system has a unique saddle path near the steady state; that is, the

[25] Recall $E\varepsilon(i) = 1 - \theta$.

system has exactly the same number of stable roots as the number of state variables.[26] Hence, these seven equations plus the transversality condition uniquely solve for the equilibrium path of $\{\varepsilon_t^*, c_t, k_{t+1}, N_t, y_t, w_t, r_t\}$, given the distribution of ε_t, the path of $\{g_t\}$, and the initial condition k_0.

3 Steady-State Analysis

A "steady state" is defined as a situation without aggregate uncertainty wherein all variables and distributions in the transformed economy are time invariant. In a steady state the TFP growth rate $g_t = \bar{g}$ for all t. It can be shown that the transformed model has a unique steady state. In the steady state, equations (16), (19), (20), and (27) become

$$1 + \bar{g} = \beta(1 + r)R(\varepsilon^*) \tag{29}$$

$$c = D(\varepsilon^*)x \tag{30}$$

$$(1 + \bar{g})\, k = H(\varepsilon^*)x \tag{31}$$

$$c + \bar{g}k = y - \delta k, \tag{32}$$

respectively, where $x = k + \wp$.

In equation (29), $1 + \bar{g}$ represents the opportunity cost of saving (the opportunity cost of not consuming the growing income today increases with the rate of income growth), and $(1 + r)\, R(\varepsilon^*)$ is the effective rate of return to saving. Because saving provides liquidity to buffer unexpected shocks, its effective rate of return is compounded by the liquidity premium $R(\varepsilon^*) > 1$. In equilibrium, the marginal cost of saving equals its marginal benefits.

Using equation (31) and realizing that $H = 1 - D$ gives $x = \frac{1+\bar{g}}{\bar{g}+D}\wp$. Substituting this relationship into equations (30) and (32) gives the aggregate consumption and saving as functions of disposable income:

$$c = \frac{(1 + \bar{g})\, D}{\bar{g} + D}\wp \tag{33}$$

$$\bar{g}k = \left[1 - \frac{(1 + \bar{g})\, D}{\bar{g} + D}\right]\wp. \tag{34}$$

Therefore, the marginal propensity to consume from disposable income is given by $MPC \equiv \frac{(1+\bar{g})D}{\bar{g}+D}$, which is less than 1 because $D < 1$, provided that $\bar{g} > 0$. The aggregate saving rate is given by

$$\tau = 1 - \frac{(1 + \bar{g})\, D(\varepsilon^*)}{\bar{g} + D(\varepsilon^*)}. \tag{35}$$

[26] By equation (16), the cutoff ε_t^* is stationary along a balanced growth path as long as the growth rate g_t is stationary. Hence, the transversality condition is clearly satisfied by the consumption and saving functions.

The saving rate depends positively on growth $\bar{g}$. In particular, we have $\tau = 0$ if $\bar{g} = 0$, and $\tau > 0$ if $\bar{g} > 0$.[27] Note that the saving rate depends on the cutoff, which affects the distribution of wealth across households. The cutoff in turn depends on the growth rate.

Proposition 2 *If θ is small enough, an interior solution of the optimal cutoff ε^* exists and is uniquely determined by the following implicit equation:*

$$\frac{1+\bar{g}}{R(\varepsilon^*)} = \beta\left[1-\delta+\alpha\left(\bar{g}+\delta+(1+\bar{g})\frac{D(\varepsilon^*)}{H(\varepsilon^*)}\right)\right]. \tag{36}$$

Proof. Equation (29) implies a relationship for output-to-capital ratio: $(1+\bar{g}) = \beta\left[1-\delta+\alpha\frac{y}{k}\right]R(\varepsilon^*)$. Equations (30) and (31) imply the consumption-to-capital ratio, $\frac{c}{k} = (1+\bar{g})\frac{D}{H}$, which can be substituted into the resource constraint (32) to obtain another equation for the output-to-capital ratio: $\left(\bar{g}+\delta+(1+\bar{g})\frac{D}{H}\right) = \frac{y}{k}$. Combining these two restrictions yields equation (36).

Because $\frac{\partial R(\varepsilon^*)}{\partial \varepsilon^*} > 0$, the left-hand side (LHS) decreases monotonically with ε^*. In particular, since $\varepsilon^* \in [0, \varepsilon_{\max}]$ and $\varepsilon_{\max} > 1-\theta$, the LHS has a minimum equal to $LHS(\varepsilon_{\max}) = \frac{1+\bar{g}}{\theta+\varepsilon_{\max}} < 1+\bar{g}$ and a maximum equal to $LHS(0) = (1+\bar{g})$. On the other hand, because $\frac{D(\varepsilon^*)}{H(\varepsilon^*)}$ is monotonically increasing in ε^*, the right-hand side (RHS) has a maximum equal to infinity at $\varepsilon^* = \varepsilon_{\max}$ (because $H(\varepsilon_{\max}) = 0$) and a minimum given by $\beta\left[1-\delta+\alpha\left(\bar{g}+\delta+(1+\bar{g})\frac{\theta}{1-\theta}\right)\right]$. That is, the RHS is an upward-sloping curve.

Hence, as long as the maximum of the LHS exceeds the minimum of the RHS:

$$1+\bar{g} > \beta\left[1-\delta+\alpha\left(\bar{g}+\delta+(1+\bar{g})\frac{\theta}{1-\theta}\right)\right], \tag{37}$$

a unique interior solution for $\varepsilon^*(\bar{g})$ exists and this value is a function of the growth rate. Condition (37) is clearly satisfied if the multiplier θ is small enough. ■

3.1 Discussion

Proposition 2 shows that an interior solution for the cutoff exists only if the parameter θ is smaller than a critical value $\tilde{\theta}$, where $\tilde{\theta}$ is determined by setting the inequality (37) to an equality. It is clear that $0 < \tilde{\theta} < 1$. In contrast, if $\theta \geq \tilde{\theta}$, then the minimum of the RHS of equation (36) exceeds the maximum of the LHS, and we have a corner solution of $\varepsilon^* = 0$. In this case, only Case A in the proof of Proposition 1 is relevant, so we have $s_{t+1}(i) > 0$ and $\pi_t(i) = 0$ for all i. That is,

[27] Although net changes in the saving stock are zero ($s - s = 0$) in the steady state if $\bar{g} = 0$, the stock-to-income ratio ($\frac{s}{\wp}$) is always positive.

the borrowing constraint never binds. Thus, equation (12) implies that $\lambda_t(i)$ is independent of i. Equations (10) and (9) then imply $c_t(i) = w_t$, which is also independent of i. In other words, when θ is large enough, because the extent of idiosyncratic risk is small, agents are able to perfectly smooth their consumption through saving.[28] It can be shown that in this case the aggregate allocation of the model is similar to that of a representative-agent model. In the rest of the paper, we consider only the case where $\theta < \tilde{\theta}$.

With the cutoff ε^* determined, the capital-to-output ratio can then be derived from equation (29) through the link of interest rate and marginal product of capital,

$$\frac{k}{y} = \frac{\beta\alpha R(\varepsilon^*)}{1 + \bar{g} - \beta(1-\delta)R(\varepsilon^*)}. \tag{38}$$

Recall that in a standard, representative-agent neoclassical growth model with no borrowing constraints, the steady-state capital-to-output ratio is given by

$$\frac{k}{y} = \frac{\beta\alpha}{1 + \bar{g} - \beta(1-\delta)}. \tag{39}$$

Hence, at the aggregate level the current model differs from the standard growth model by the liquidity premium $R(\varepsilon^*) \geq 1$, which arises because of the possibility of binding borrowing constraints and precautionary saving motives. If the variance of the idiosyncratic shocks approaches zero (i.e., the idiosyncratic uncertainty vanishes), then the liquidity premium approaches zero and $R(\varepsilon^*) \to 1$ in the limit. Thus, borrowing constraints do not bind and the model reduces to a standard growth model in the limit. This reveals the design of the model: The setup makes it easy to compare this model with standard growth models regarding the influence of borrowing constraints on the growth-saving relationship.

Using equation (38), we can also express the saving rate alternatively as

$$\tau = \frac{\bar{g}\frac{k}{y}}{1 - \delta\frac{k}{y}} = \alpha\beta\frac{\bar{g}R(\varepsilon^*)}{1 + \bar{g} - \beta(1 - \delta + \alpha\delta)R(\varepsilon^*)}. \tag{40}$$

Without precautionary saving motives, as in the case of a representative-agent growth model, the saving rate becomes

$$\tau^o = \alpha\beta\frac{\bar{g}}{1 + \bar{g} - \beta(1 - \delta + \alpha\delta)} < \tau. \tag{41}$$

Notice the following implications:

[28] If θ is too large, the dispersion of $\varepsilon(i)$ is then too small.

(1) Since $R(\varepsilon^*) > 1$, equations (38) and (39) suggest that the steady-state capital-to-output ratio with uninsured risk and borrowing constraints is larger than that in standard growth models. This point is noted by Aiyagari (1994).[29]

(2) $\frac{\partial \tau^o}{\partial \bar{g}} > 0$. That is, in a standard neoclassical growth model, the saving rate is an increasing function of TFP growth. The intuition is that higher TFP growth raises the rate of returns to capital and induces higher investment demand. Thus, the saving rate of households increases in equilibrium due to a higher real interest rate. That is, high growth leads to high saving, as in the data. This prediction is consistent with the numerical simulations of Chen, Imrohoroglu, and Imrohoroglu (2006) for the Japanese economy under the assumption of perfect foresights.[30]

(3) Precautionary saving not only generates a higher saving rate for any given level of growth rate (i.e., $\tau > \tau^o$), it also magnifies the positive relationship between growth and saving in the neoclassical model. That is, the cross-partial derivative $\frac{\partial \tau^2}{\partial \bar{g} \partial R}$ is positive, because $\frac{\partial \tau}{\partial \bar{g}} > 0$ (taking R as given) and $\frac{\partial \tau}{\partial R} > 0$ (taking $\bar{g}$ as given); hence, if in addition $\frac{dR}{d\bar{g}} > 0$, then borrowing constraints enhance the positive effect of growth on saving. That is, if the liquidity premium is an increasing function of growth, then borrowing constraints magnify the positive relationship between growth and saving. This is ultimately the case because income growth induces consumption growth, which tightens borrowing constraints and raises the liquidity premium. This point is easier to grasp if the real interest rate is constant. In such a case, equation (29) indicates that $R(\varepsilon^*)$ rises with $\bar{g}$.

(4) Because of higher saving rates, the equilibrium interest rate in our model will be lower than that in the standard growth model.

Figure 1 presents a graphic illustration of the effects of growth on saving. Consider the standard growth model first. In the graph, a higher TFP growth from g_1 to g_2 rotates the investment curve to the left from $(g_1 + \delta)\, k$ to $(g_2 + \delta)\, k$ and decreases the steady-state capital stock per effective worker from k_1 to k'. As a result, consumption per effective worker would fall below the modified golden-rule level if the saving rate remain unchanged at τ_1. Hence, a higher saving (investment) rate is called for to raise the steady-state capital stock to k_2 so that consumption per effective worker can be higher than it would be without the adjustment in the saving rate. However, in the new steady state the capital stock is still lower than before ($k_2 < k_1$), because increasing the saving rate further would be so costly that the rate of return to saving is less than the marginal cost: $\beta(1 + r) < 1 + \bar{g}$. Therefore, the saving rate will increase only to the point where the discounted equilibrium real interest rate equals the growth rate.

[29] Aiyagari (1995) argues that taxing capital is optimal in this type of incomplete-market models because of too much savings.

[30] However, in the standard neoclassical growth model the real interest rate increases with TFP growth and is about 15% when the rate of growth reaches 10% per year.

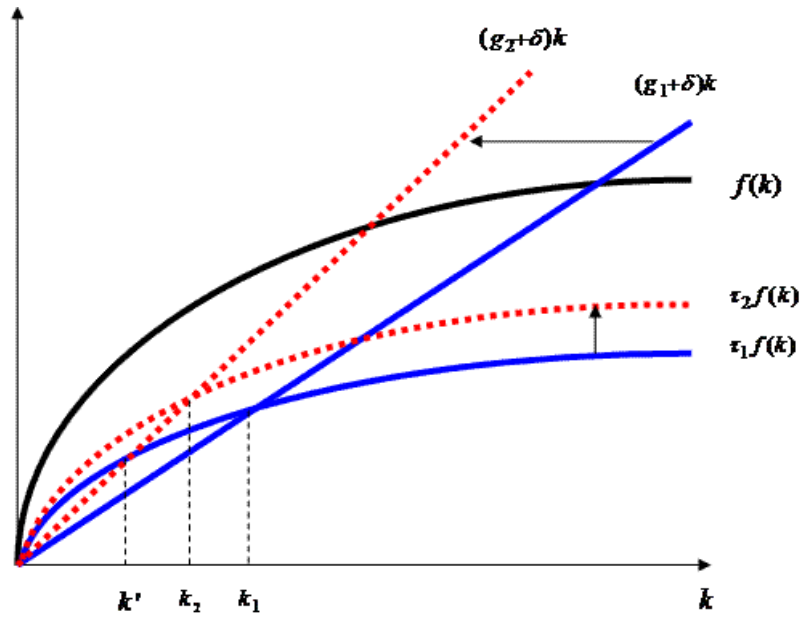

Figure 1. Effects of Growth on Saving.

With borrowing constraints, the upward shift in the saving curve ($\tau_1 f(k)$) is larger because a lower capital stock raises the liquidity premium (as savings are a buffer stock to self-insure against idiosyncratic uncertainty), which induces the saving rate to rise further. This results in a higher steady-state capital stock than k_2 (i.e., a lower real interest rate), which is dynamically inefficient because it yields lower consumption per effective worker. Hence, as argued by Aiyagari (1994, 1995), taxing capital (or the rate of return to savings) would be optimal when there exist precautionary saving motives. However, our analysis here suggests that the optimal capital tax rate should be an increasing function of the growth rate.

3.2 Calibration

To facilitate quantitative evaluation of the model, we calibrate the model by assuming that $\varepsilon_t(i)$ follows the power distribution, $F(\varepsilon) = \left(\frac{\varepsilon(i)}{\varepsilon_{\max}}\right)^{\sigma}$, with support $\varepsilon(i) \in [0, \varepsilon_{\max}]$ and the shape parameter $\sigma \in (0, \infty)$. We set the upper-bound parameter $\varepsilon_{\max} = \frac{1+\sigma}{\sigma}(1-\theta)$ to ensure $E\varepsilon = 1-\theta$. With this specification, we have

$$R(\varepsilon^*) = 1 + \frac{1}{1+\sigma}\varepsilon_{\max}^{-\sigma}\varepsilon^{*1+\sigma} \tag{42}$$

$$D(\varepsilon^*) = \theta + \varepsilon^*\left[1 - \frac{1}{1+\sigma}\varepsilon_{\max}^{-\sigma}\varepsilon^{*\sigma}\right] \tag{43}$$

$$H(\varepsilon^*) = 1 - \theta - \varepsilon^* \left[1 - \frac{1}{1+\sigma} \varepsilon_{\max}^{-\sigma} \varepsilon^{*\sigma} \right]. \tag{44}$$

The model's structural parameters are calibrated as follows: The time period is a year, the time discounting factor $\beta = 0.96$, the output elasticity of capital $\alpha = 0.4$, and the rate of capital depreciation $\delta = 0.1$. As a benchmark, we pick $\theta = 0.1$ and $\sigma = 0.15$. These values imply that the Gini coefficient of financial wealth in the model, $(1+r)\, s(i)$, is 0.79. This value roughly matches that of the major emerging economies in terms of wealth inequality. For example, the Gini coefficient of wealth is 0.71 for Thailand, 0.76 for Indonesia, and 0.78 for Brazil (see, e.g., Davies et al., 2006). The reported Gini coefficient for China is 0.55, which is likely significantly underestimated; the true value might be somewhere close to that in Indonesia and Brazil.[31] The calibrated parameter values are summarized in Table 1.

Table 1. Calibrated Parameter Values

Parameter	β	α	δ	θ	σ
Value	0.96	0.4	0.1	0.1	0.15

Table 2 presents the quantitative effects of growth and borrowing constraints on saving rates. In Table 2, Model A represents the counterpart representative-agent neoclassical growth model (called a "standard growth model" in this paper), and Model B represents the heterogenous-agent model with borrowing constraints. Consider the standard growth model first (the middle row). The table shows that high growth leads unambiguously to high saving. For example, when the growth rate is at 1% per year, the saving rate is less than 4%; when the growth rate increases to 10% per year, the saving rate rises to nearly 20%. Thus, theory predicts that high growth leads to high saving, which is consistent with the data, but in sharp contrast to PIH.

High growth leads to high saving in the standard growth model primarily because TFP growth enhances the productivity of capital, which raises the demand for investment, which in turn raises the interest rate and consequently leads to increased saving in equilibrium. In contrast, the conventional PIH is presented in a partial-equilibrium framework with a constant real interest rate, so high growth is not accompanied by high asset returns. Thus, consumers have no incentives to increase saving but opt to raise their marginal propensity to consume when permanent income rises.

Table 2. Saving Rate (τ) as a Function of Mean Growth ($\bar{g}$)

	$\bar{g}$	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Model A	τ	3.6%	6.5%	9.0%	11.2%	13.0%	14.6%	16.0%	17.3%	18.4%	19.4%
Model B	τ	5.4%	10%	13.9%	17.2%	20.1%	22.6%	24.9%	26.9%	28.7%	30.3%

* Model A, standard growth model; Model B, with borrowing constraints.

[31] The Gini coefficient is 0.8 for the United States.

Borrowing constraints can significantly amplify the neoclassical growth effect on saving. The bottom row in Table 2 shows that with borrowing constraints, the saving rate not only is much higher than in the standard model at each corresponding rate of growth, but the gap also increases with the growth rate. For example, when the growth rate is at 1% per year, the saving rate is 5.4% in Model B, which is 1.8 percentage points higher than that in Model A. However, when the growth rate is at 10% per year, the saving rate is 30.3% in Model B, which is about 11 percentage points higher than that in Model A. This implies that borrowing constraints magnify the positive effect of growth on saving.[32]

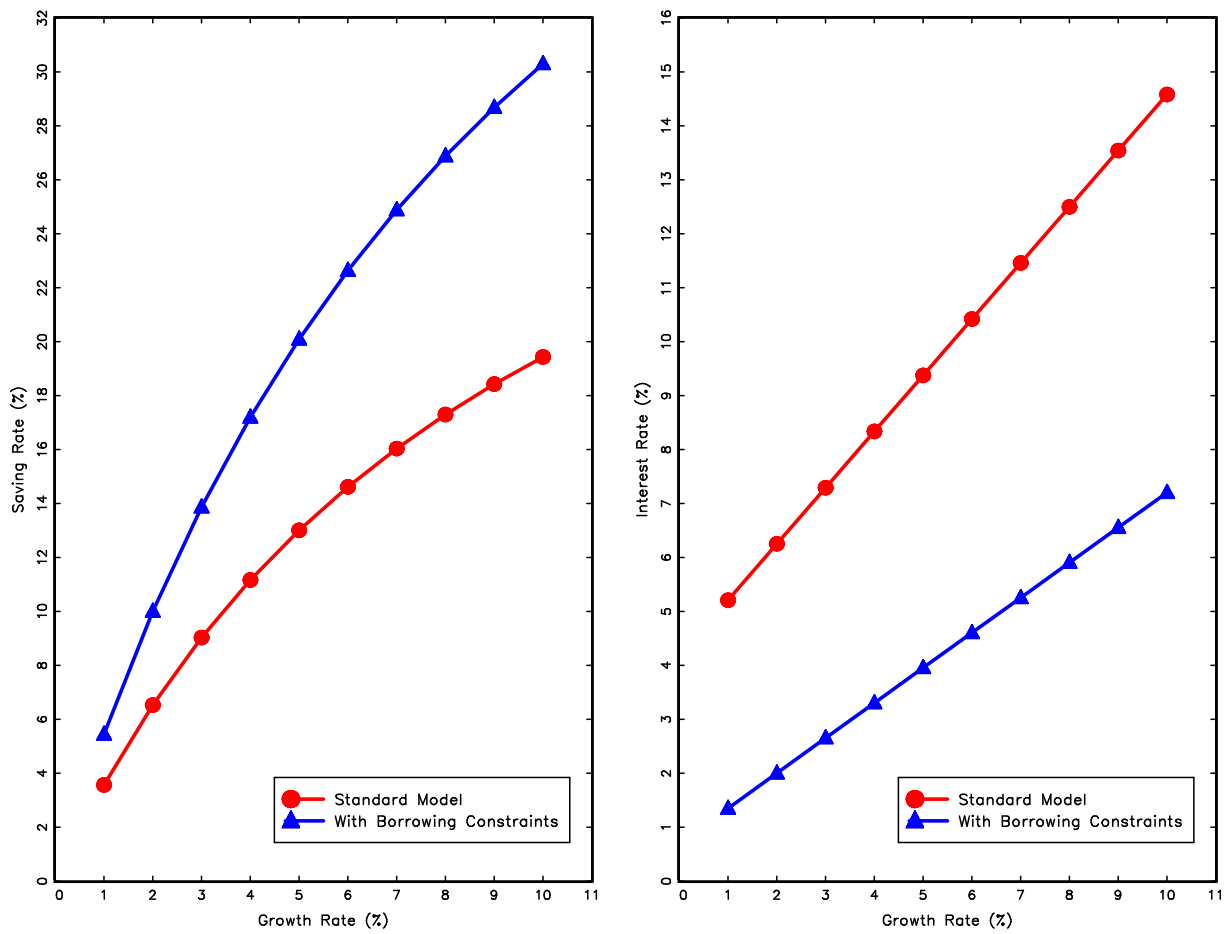

Figure 2. The Growth Effects on Saving and Interest.

The information in Table 2 is graphed in the left panel in Figure 2, where the line with circles represents the standard growth model (Model A), and the line with triangles the model with borrowing constraints (Model B). It shows that (i) high growth leads to high saving in both models,

[32] Borrowing constraints per se will induce a higher saving rate because of the buffer-stock role of savings, other things equal. However, if there were no amplification effects, borrowing constraints would generate only a constantly higher saving rate than the standard growth model when the growth rate rises, instead of an increasingly higher rate, as shown in Table 2.

but (ii) the effects are much stronger in Model B than in Model A and the multiplier effect rises with growth.

Borrowing constraints not only significantly magnify the growth effect on saving, but also mitigate the growth effect on interest rates. Table 3 shows that the real interest rate increases with growth in the standard model (Model A). Since a higher TFP growth implies a higher opportunity cost of saving, the rate of return to saving (the real interest rate) must also increase accordingly to induce a higher saving rate. With borrowing constraints (Model B), however, the real interest rate not only is significantly lower than in the standard growth model at every level of growth, but also increases less rapidly with growth. For example, when the growth rate is 1% to 3%, the implied real interest rate is about 5% to 7% without borrowing constraints (Model A), but only about 1% to 3% with borrowing constraints (Model B). Also, when the growth rate rises to 8% to 10%, the implied interest rate jumps up to 13% to 15% without borrowing constraints (Model A), but increases only to about 6% to 7% with borrowing constraints (Model B). The intuition is that precautionary saving results in a higher steady-state capital-to-output ratio, so the real interest rate is lower than in the standard model for any given growth rate. In addition, since the liquidity premium (R) rises with growth, the strength of precautionary saving also increases with growth, hence leading to much higher capital-to-output ratios and more subdued real interest rates.

Table 3. Equilibrium Interest Rate (r) as a Function of Mean Growth ($\bar{g}$)

	$\bar{g}$	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Model A	τ	5.2%	6.3%	7.3%	8.3%	9.4%	10.4%	11.5%	12.5%	13.5%	14.6%
Model B	τ	1.3%	2.0%	2.7%	3.3%	4.0%	4.6%	5.3%	5.9%	6.6%	7.2%

* Model A, standard growth model; Model B, with borrowing constraints.

The information in Table 3 is graphed in the right panel in Figure 2, where triangles represent the model with borrowing constraints (Model B) and circles the model without borrowing constraints (Model A). Clearly, not only does the line with borrowing constraints lie significantly below that without borrowing constraints at all levels of growth rates, its slope is also less steep.

4 Dynamic Analysis

This section examines the relationship between growth and saving under transitional dynamics. We consider two scenarios. In the first scenario, there is no aggregate uncertainty ($g_t = \bar{g}$), and we study behaviors of the saving rate and its relationship to growth when an economy starts out "poor", in the sense of having a capital stock below the steady state. We show that the model with borrowing constraints will have not only a higher growth rate but also a significantly higher saving rate along the path converging to the steady state than the model without borrowing constraints,

although both models share the same steady state and rate of TFP growth. Since different degrees of borrowing constraints lead to different steady states, to ensure that the two model economies converge to the same steady state, we assume that in the borrowing-constrained economy the constraints are gradually reduced (by decreasing the variance of the idiosyncratic shocks, σ) along the transitional path so that they no longer bind in the long run.[33] The other parameters are calibrated to the same values as shown in Table 1, and the steady-state rate of TFP growth is set to $\bar{g} = 0.04$ for both economies.

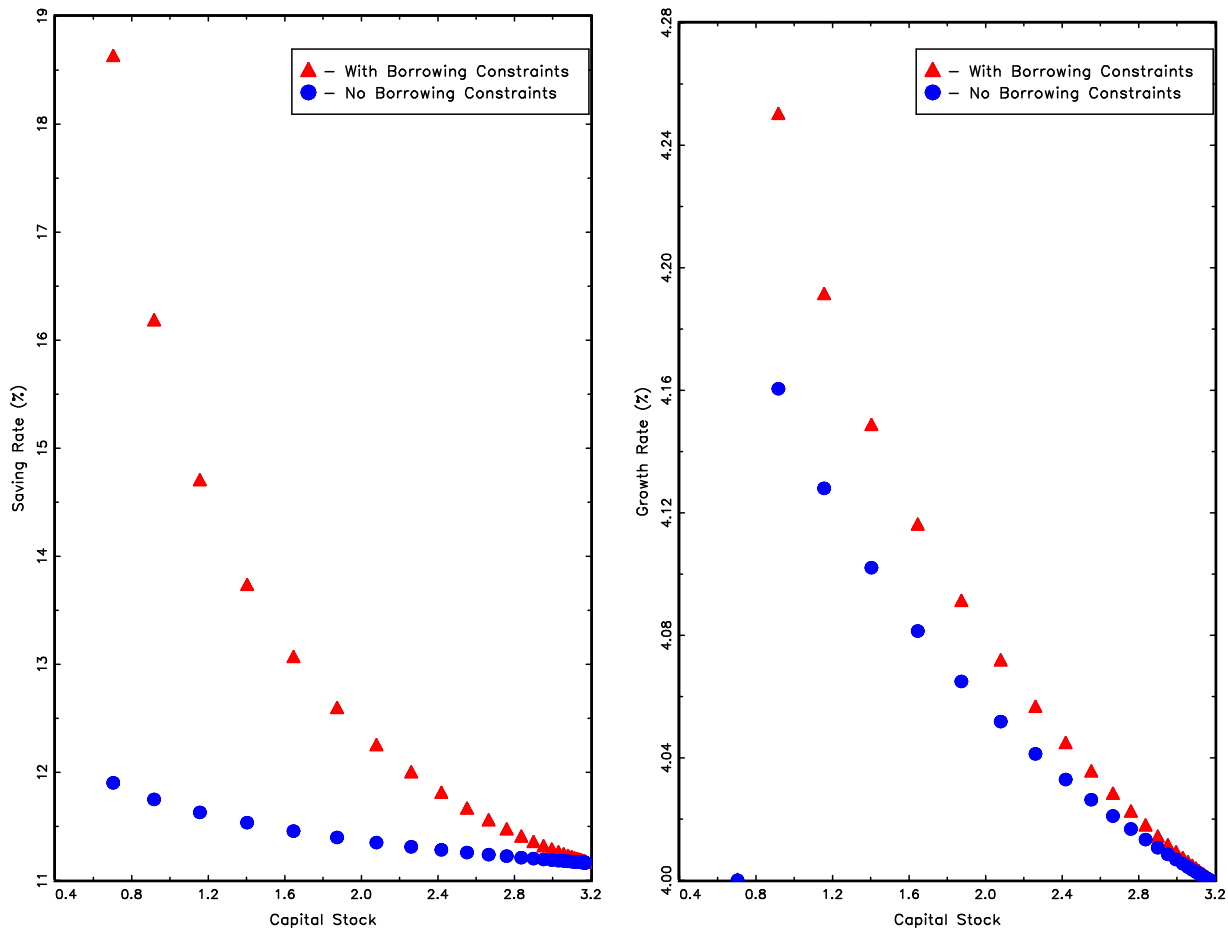

Figure 3. Transitional Dynamics.

Figure 3 depicts the saving rate (left panel) and the growth rate of investment (right panel) as a function of the capital stock for the two economies. The dots represent equally spaced points in time as the system evolves toward the shared steady state. The triangle symbols represent the model with borrowing constraints, and circles the counterpart model without borrowing constraints. Both models start with the same level of capital stock, but the model with borrowing constraints starts

[33] Namely, we have $\sigma \to \infty$ in the model without borrowing constraints.

with $\sigma = 0.15$ in the first period and this value increases over time (in every subsequent period) until it becomes large enough so that the two models converge to the same steady state in the long run.

Figure 3 shows that an initially poor economy will have both a higher-than-steady-state saving rate and a higher-than-steady-state growth rate, regardless of borrowing constraints. However, borrowing constraints reinforce this positive relationship between saving and growth so that the borrowing-constrained economy will exhibit a much higher saving rate and a moderately higher growth rate than the counterpart economy at any point in time.[34] For example, the saving rate is about 7 percentage points higher and the growth rate is about 0.1 percentage points higher with borrowing constraints than without in the initial period. The intuition for this pattern is that borrowing constraints induce excessive saving, and the lower the capital-to-output ratio, the greater is the need for a buffer stock and the larger is the implied liquidity premium, which results in stronger precautionary saving, more rapid capital accumulation, and higher growth along the transitional path, even though the TFP growth rate is the same (4% a year) across the two economies.

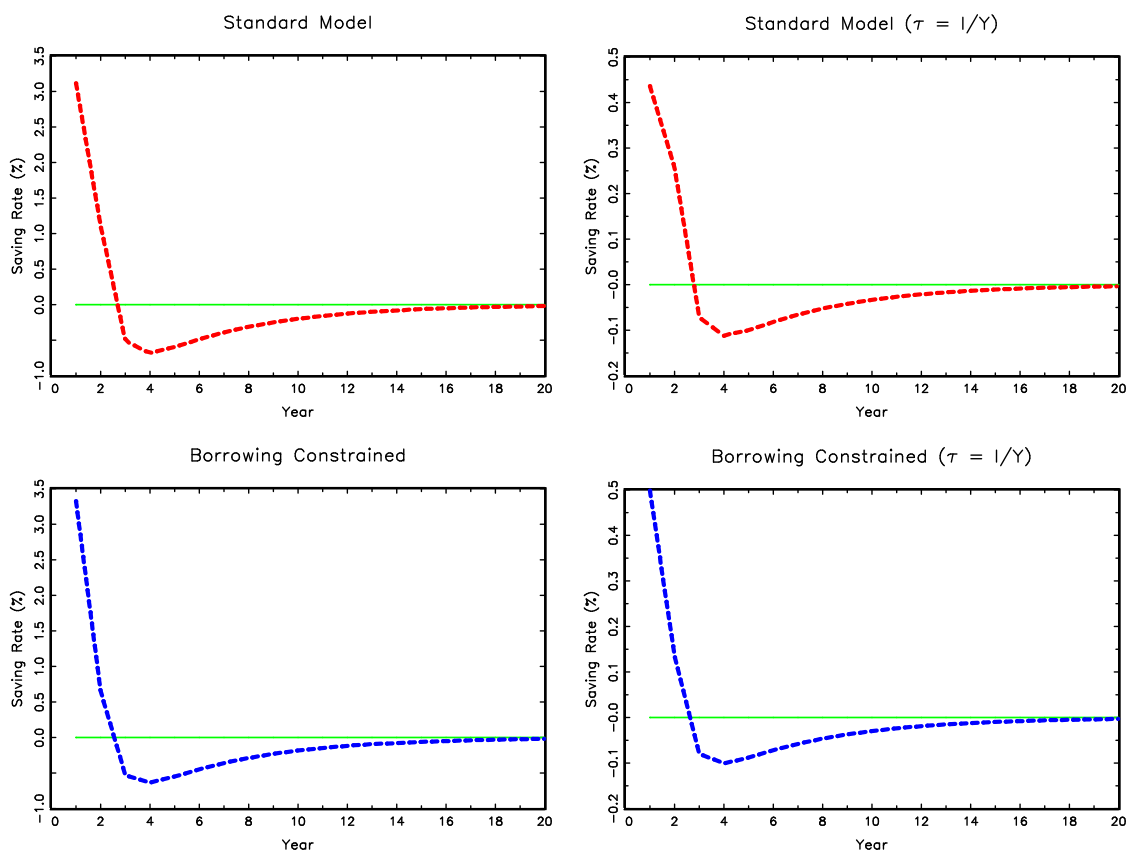

Figure 4. Impulse Responses of Saving to 1% Growth Shock.

[34] Since time is discrete, we assume that in the initial period the growth rate equals the steady-state rate and there is a sudden destruction of the capital stock.

In the second scenario, we introduce aggregate uncertainty and study the impulse responses of the saving rate to transitory TFP growth shocks. A transitory increase in the growth rate implies that income rises permanently from one level to another but not its growth rate. We use the growth process specified in equation (1) to drive the model by setting the mean annual growth rate to $\bar{g} = 0.04$ with persistence $\rho_g = 0.2$, consistent with postwar U.S. data. Figure 4 shows the responses (percentage deviations) of the saving rate (τ_t) to a 1 standard-deviation transitory increase in the growth rate, g_t. Since the transformed model is solved by the method of log-linearization around the steady state, all changes in Figure 4 represent percentage deviations relative to the steady state.

The top-left panel in Figure 4 depicts the standard growth model, and the bottom-left panel the borrowing-constrained model. In either case, the saving rate rises after a positive shock to TFP growth. This happens because the higher rate of returns to capital induces investment demand and stimulates saving.[35] Therefore, in sharp contrast to the prediction of the PIH, households increase rather than decrease their saving when permanent income rises, even though the higher growth rate and the consequently higher saving rate are purely transitory.[36]

The above results are not sensitive to the definition of the saving rate adopted in this paper (equation 28). For example, if the saving rate is defined as the ratio of gross investment to output ($\tau_t = \frac{I_t}{Y_t} = \frac{(1+\bar{g})k_{t+1}-(1-\delta)k_t}{y_t}$), it still responds positively to a growth shock, regardless of borrowing constraints (see the top-right and bottom-right panels in Figure 4). These analyses suggest that consumption growth responds less than one for one to income growth, whereas investment growth (saving) responds more than one for one to income growth. Hence, once the interest rate becomes endogenous, there does not exist the so called "excess smoothness puzzle" of consumption relative to income discussed in the consumption literature.[37]

A Deeper Puzzle. However, here arises another puzzle: In the general-equilibrium models presented previously, the real interest rate is also the rate of return to capital; hence, high TFP growth leads to high saving through a high real interest rate. But the empirical evidence suggests that for fast-growing emerging economies the rate of return to capital may be extremely high but the interest rates facing households may be extremely low. For example, the average 3-month nominal deposit rate in China was about 3.3% per year (the average 1-year rate was about 5.6% per year) from 1990 to 2006,[38] suggesting negative real deposit rates, yet the average real rate of return to capital was more than 20% per year in that period (Bai, Hsieh, and Qian, 2006). In the meantime,

[35] The absolute change in the saving rate is larger in the borrowing-constrained model because it has a higher steady-state saving rate.

[36] However, the income level has jumped up permanently.

[37] For more analysis on the "excess smoothness puzzle", see Wen (2009) and the references therein.

[38] The average inflation rate was above 6% in that period. Data for interest rates are not available before 1990.

the household saving rate in that period was about 25%, the national saving rate was about 40%, and the average growth rate of real GDP per capita was about 10% per year. The situation is similar in Japan. During the high-growth and high-saving period of Japan in the 1960-70s, the average nominal 3-month deposit rate was about 4%,[39] and the average after-tax real rate of return to capital was above 16%, whereas the average household saving rate was about 17%, the national saving rate was about 20%, and the average growth rate of real income was above 10%.

In addition, in both economies (Japan and China) during their respective high-growth periods, bank deposits have been the major means of saving for households, as well as the most important source of funds for firms' investment. For example, in China, bank deposits accounted for 72% of total household financial assets in 2004 and 2005. In contrast, the total share of bonds and stocks accounted for less than 10% of household financial assets in that period. On the other hand, the share of bank loans in total corporate debt was about 64% in 2004 and 2005, while the total share of corporate bonds and stocks was only around 15% in that period.

Therefore, regardless of how interest rates are kept low in fast-growing economies, the puzzle is not just why high saving is positively correlated with high growth, but why high saving is possible under low interest rates. Namely, why would households save excessively to finance firms' investment when the returns to their savings are so low and do not reflect the high returns to capital or TFP growth?[40] We address this puzzle in the next section.

5 Fixed Deposit Rates

In developing economies, because of incomplete markets and various forms of financial repression (including distorted government banking regulations and monetary policies), large spreads may exist between deposit rates that households receive for their savings and the true rates of returns to capital that firms receive for their investment. If such spreads exist, how do they affect the relationship between saving and growth? This section analyzes this issue by conducting a counterfactual experiment.

5.1 With Capital

Suppose that the real interest rate faced by households is not the same as the marginal product of capital. In particular, suppose households have no access to investment opportunities except earning a low, fixed real interest rate ($\bar{r}$) on their deposits at financial intermediaries. On the other hand, firms must pay the market real interest rate (r_t) to obtain loans, and banks earn

[39] The real deposit rate is negative because the average inflation rate was also above 6% in Japan in that period.

[40] This puzzle may be related to the well-known empirical failure in the literature to identify a significantly positive relationship between saving and interest rates.

"monopolistic" profits from the spread in rates of returns, $(r_t - \bar{r})\, s_t \geq 0$.[41] For simplicity, assume that the profits, $v_t = (r_t - \bar{r})\, s_t$, are redistributed as lump sum to households.

In a standard growth model, a fixed deposit rate is inconsistent with general equilibrium because the equilibrium condition,

$$1 + \bar{g} = \beta\,(1 + \bar{r})\,, \tag{45}$$

must hold to ensure positive household saving in the steady state. This condition will be violated when the growth rate rises — that is, $1 + \bar{g} > \beta\,(1 + \bar{r})$ — so household saving will become zero (or negative if borrowing is allowed) and the marginal product of capital will become infinity. Hence, investment demand will drive up the deposit rate in equilibrium to induce positive saving. So a fixed deposit rate below the market interest rate cannot constitute an equilibrium in a standard growth model.

However, in models with uninsured risk and borrowing constraints, a fixed deposit rate below the market interest rate can be supported by general equilibrium. Because the liquidity premium, $R(\varepsilon^*)$, can rise endogenously with growth, equation (29) can continue to hold in equilibrium even with a fixed interest rate:

$$1 + \bar{g} = \beta(1 + \bar{r})R(\varepsilon^*), \tag{46}$$

where the cutoff $\varepsilon^*(\bar{g})$ is a function of growth. This equation shows that if the deposit rate lies below the market real interest rate, the liquidity premium will rise so that the effective rate of return to saving increases to balance the marginal costs and benefits of saving.

This current model with the spread in the rates of returns can be solved exactly as discussed in Section 2. In equilibrium, we still have the capital market-clearing condition, $s_t = k_t$; namely, the supply of capital equals its demand.[42] In the steady state, we have the following analogous relationships and decision rules:

$$c = D(\varepsilon^*)\,[(1 - \delta)\,k + y] \tag{47}$$

$$(1 + \bar{g})\,k = H(\varepsilon^*)\,[(1 - \delta)\,k + y] \tag{48}$$

$$c + \bar{g}k = y - \delta k \tag{49}$$

$$r = \alpha\frac{y}{k} - \delta. \tag{50}$$

The above equations imply that the saving rate is still determined by the formula in equation (35): $\tau = 1 - \frac{(1+\bar{g})D}{\bar{g}+D}$. However, the implied value of the saving rate given by this equation now differs

[41] In China, all banks are state owned and the government has monopoly power on setting the deposit rates and loan rates. In Japan, although banks are privately owned, the government nonetheless had great influence on banks' interest rates in the 1950-70s.

[42] Because of the below-equilibrium deposit rate, the demand of capital is "rationed." This implies that the marginal product of capital exceeds the real deposit rate $\bar{r}$. The profits from the spread are earned by banks by assumption, so firms still pay a market interest rate equal to the marginal products of capital and earn zero profits.

from that given by equation (40) because the deposit rate ($\bar{r}$) in equation (46) is no longer equal to the marginal product of capital, $r = \alpha\frac{y}{k} - \delta$. Hence, the value of the cutoff (ε^*) determined by equation (46) differs from that in the previous model. In particular, with a fixed deposit rate below the market rate, the optimal cutoff ε^* is higher and increases faster as $\bar{g}$ rises. That is, for any given level of the growth rate, the portion of the population with positive saving is smaller in the current model with a low fixed deposit rate.

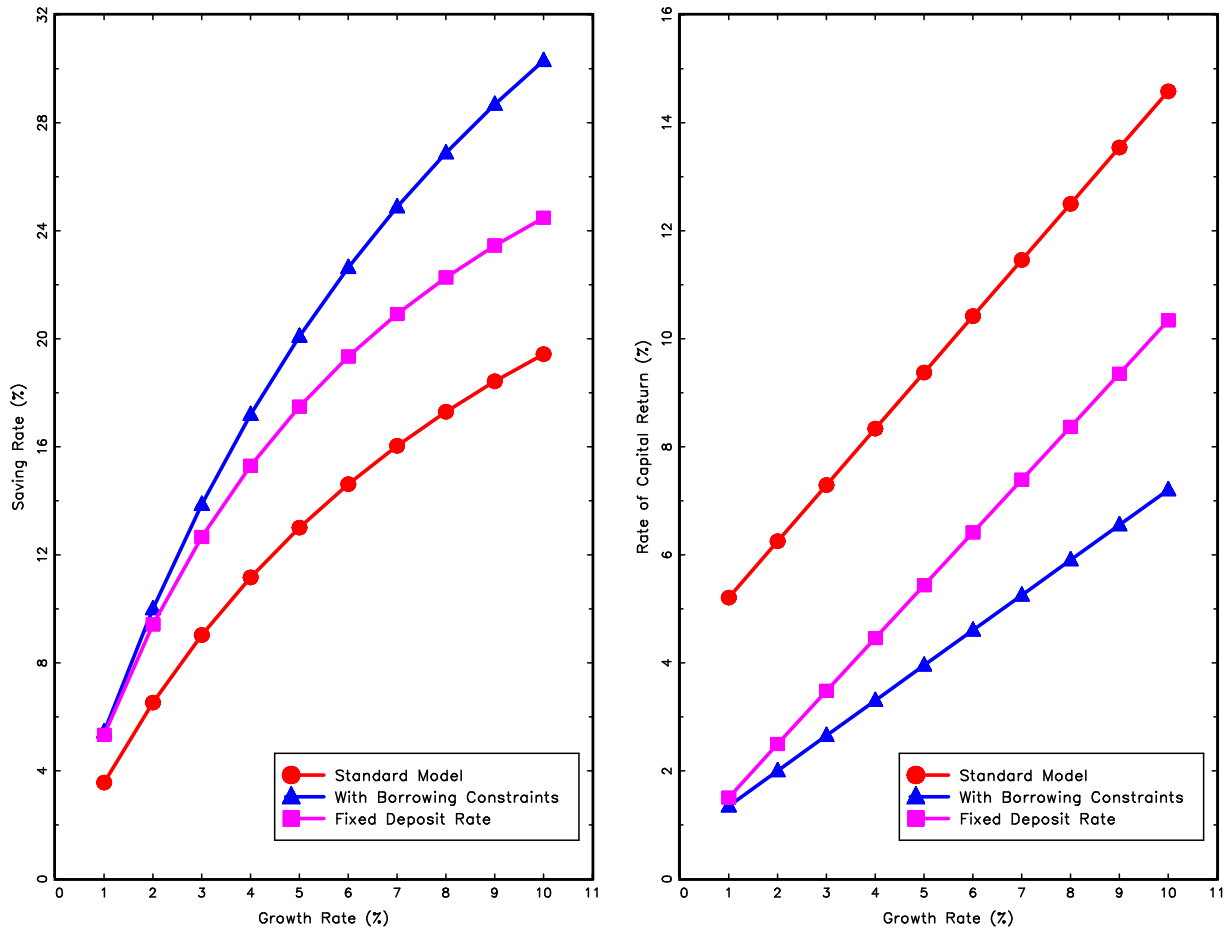

Figure 5. Effects of Fixed Deposit Rate.

In Figure 5, the line with squares in the left-panel shows the relationship between saving and growth when the real deposit rate is fixed at 1% per year. For comparison, in Figure 5 we also include the same curves shown in Figure 2. The left-panel in Figure 5 shows that, even with such a low and fixed deposit rate, households still save significantly more of their disposable income than they do in the standard growth model at various levels of the growth rate (compare the two lines with squares and circles), although the saving rates are lower than those in the counterpart model without the distorting interest spread (the line with triangles). For example, when the

growth rate is about 3% a year, the saving rate in the standard growth model without borrowing constraints is about 9%; however, this rate is about 13% in the current model despite an essentially zero deposit rate. Therefore, income uncertainty and borrowing constraints are able to generate excessive savings even under low interest rates.

More importantly, thanks again to borrowing constraints, as the growth rate increases, the saving rate in the current model also rises accordingly, despite the low and fixed deposit rate. That is, even though TFP growth does not transmit to the rate of returns to household savings, the saving rate still increases rapidly with economic growth. For example, when the rate of income growth increases from 3% to 10% a year, the saving rate increases from 13% to 25%. The reason behind this substantial rise in saving is that TFP growth affects real wages, and a faster wage growth leads to a lower buffer stock-to-income ratio. Hence, the liquidity premium rises, which induces a higher saving rate. That is, uninsured risk and borrowing constraints make the marginal propensity to save positively dependent on permanent income due to a rising liquidity premium with income growth. Hence, even if the real deposit rate is fixed at extremely low levels, households still opt to increase their propensity to save when permanent income increases. Similar results hold even if the real deposit rate is negative.[43]

On the other hand, the right-panel in Figure 5 shows that the real rate of return to capital (the marginal product) in the current model with a fixed deposit rate (the line with square symbols) is higher than that in the counterpart model with borrowing constraints and a flexible rate (the line with triangles), and it rises faster when the growth rate increases. This explains why fast-growing economies may simultaneously exhibit low deposit rates and high (and apparently undiminished) rates of returns to capital.

The property that the marginal propensity to save depends positively on the level of permanent income is the sole consequence of uninsured risk and borrowing constraints. That is, uninsured risk and borrowing constraints can completely alter the PIH and generate exactly the opposite prediction of the PIH. The intuition is that with uninsured risk and borrowing constraints, a higher income growth ($\bar{g}$) induces a larger liquidity premium (R) because of a lower buffer stock-to-flow ratio and thus a tighter borrowing constraint, which results in a higher effective rate of return to saving.

5.2 Without Capital

This powerful effect of precautionary saving on the growth-saving relationship can manifest itself even in a simpler equilibrium framework where there is no capital and both the real interest rate and the real wage are exogenous. For example, consider the special case where $\alpha = 0$ and household

[43] The liquidity premium has an upper bound given by $R(\cdot) \leq \theta + \varepsilon_{\max}$, which imposes a lower bound on the deposit rate, $\bar{r} \geq \frac{1}{\beta(\theta+\varepsilon_{\max})} - 1$. Clearly, this lower bound can be negative if $\varepsilon_{\max}$ is large enough.

savings are simply inventories that earn a constant real interest rate $\bar{r}$. The production function then becomes $Y_t = Z_t N_t$. The real wage then becomes completely exogenous, $W_t = Z_t$, which grows over time at the rate g_t. Since the household's maximization program (5) has not changed, this "partial-equilibrium" model[44] has closed-form solutions analogous to equations (46) through (49):

$$1 + \bar{g} = \beta(1 + \bar{r})R(\varepsilon^*) \tag{51}$$

$$c = D(\varepsilon^*)x \tag{52}$$

$$(1 + \bar{g})\, s = H(\varepsilon^*)x \tag{53}$$

$$c + \bar{g}s = \wp, \tag{54}$$

where $x = s + \wp$ and $\wp = \bar{r}s + wN + v$. Hence, the saving rate is still given by the same formula, $\tau = 1 - \frac{(1+\bar{g})D}{\bar{g}+D}$, and the relationship between saving and growth is identical to that implied by the graph in the left panel in Figure 5 (i.e., the line with squares).[45]

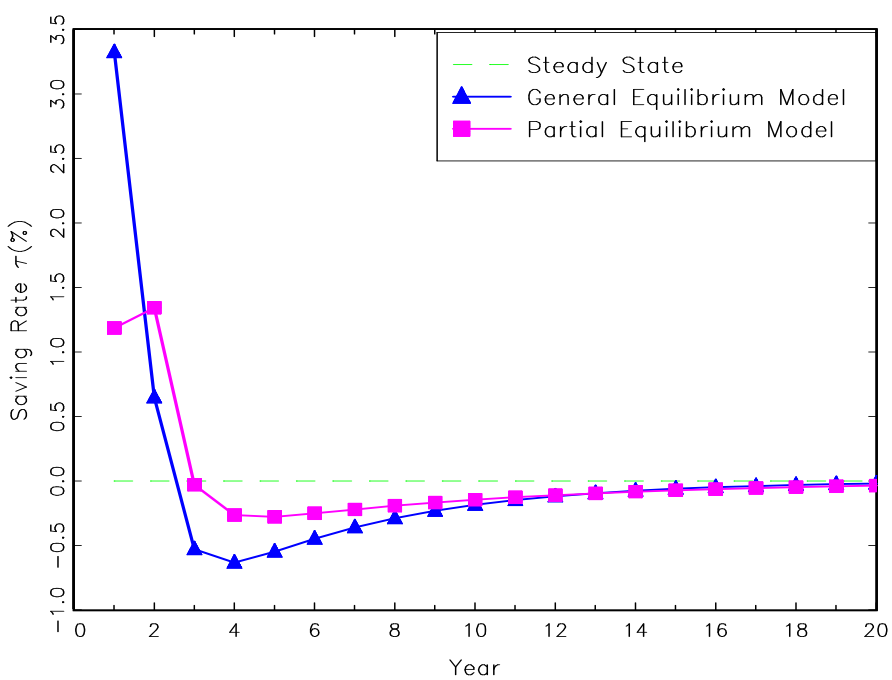

Figure 6. Impulse Responses of Saving to 1% Growth Shock.

[44] With a little abuse of language, we call this simpler model "partial-equilibrium" model even though it is not, which helps to distinguish this model from the model with fixed deposit rate and the model with flexible market-determined interest rate.

[45] Note that the optimal household saving stock in this simpler model would be zero without idiosyncratic risk and borrowing constraints. That is, if $\varepsilon(i) = 1 - \theta$ and $\bar{g} \geq \bar{r}$, we would have $s_{t+1}(i) = 0$ for all i and t. This is consistent with the prediction of the PIH: High growth leads to low savings.

This "partial-equilibrium" analysis further reveals that uninsured risk and borrowing constraints alone (without the neoclassical, general-equilibrium, endogenous interest mechanism) can completely alter the predictions of the PIH by making the marginal propensity to consume negatively dependent on the changes in permanent labor income. The reason is again precisely that the liquidity premium for savings rises with income growth because agents want to maintain a constant saving stock-to-wealth ratio to provide enough liquidity to buffer idiosyncratic shocks along the balanced growth path; since the buffer stock-to-income ratio decreases when income grows faster, the liquidity premium therefore rises to induce a higher saving rate. Hence, high growth leads to high saving despite zero interest rates.

This prediction holds true even if the higher wage growth is purely temporary in the partial-equilibrium model. This is illustrated in Figure 6, where the line with squares shows the impulse responses of saving in the partial-equilibrium model to a transitory 1% growth shock, and the line with triangles represents the counterpart general-equilibrium model with borrowing constraints (studied previously in Figure 4 with a market-determined deposit rate). It is clear from Figure 6 that the saving rate still responds positively to growth, albeit with a smaller magnitude because of a lower and fixed real interest rate, even though the change in growth is purely temporary and there is no capital.

6 The Experience of Japan and China

6.1 The Japanese Experience

Aoki (1986, p. 579) notes that Japanese families hold most of their savings in the form of safe assets such as bank deposits and postal savings accounts despite the much higher after-tax returns on stock holdings. According to the survey by the Central Council for Financial Services Information in Japan, even as recently as 2004, the share of deposits in total household financial assets was 41.5%, while the share of postal savings was 18.6%. Therefore, the share of bank deposits and postal savings together accounted for 60.1% of household financial assets (Kishi, 2005, p. 808). This ratio was much higher before the 1970s. Kishi (2005, p. 809) also notes that "In regard to the Japanese household portfolio, safety is the highest priority." Horioka (1990) notes that interest rates on bank and postal deposits have been regulated at relatively low levels, but the rates of return on corporate equities have been very high. One would therefore expect the much higher rates of return on equities to lead to a higher share of the household's portfolio being held in such assets. However, the share of equities is quite low (Horioka, 1990, p. 83). For example, the share of equities was 7.4% in 2001 (Korb, 2001). Such empirical evidence suggests that deposit rates, instead of the rates of returns to capital, were the most relevant interest rates for household saving

in Japan in the postwar period.

However, the household saving rate in postwar Japan is well known for being one of the highest in the world. The average household saving rate in the 1957-74 period was 18% and it reached 23% in 1972. The average income-growth rate was about 10% per year in that period. Yet the 3-month nominal deposit rate remained essentially fixed at 4% per year and the nominal 1-year deposit rate was also essentially fixed at 5.5% per year during the 1960-72 period (Patrick and Rosovsky, 1976, p. 261). The average inflation rate in the sample period was above 6% per year, higher than the deposit rates, making the average real deposit rates negative.[46] In addition, the spread between the real deposit rate and the real rate of return to capital was extremely large, close to 20 percentage points during that high-growth period (see, e.g., Chen, Imrohoroglu, and Imrohoroglu, 2006).

6.2 The Chinese Experience

According to Kraay (2000), between 1978 and 1995, the main source of the increase in household saving was the rapid growth of household deposits in the banking system, which accounts for the bulk of the increase in the saving rate. For example, by 1995 the net change in deposits was more than three times larger than individual investments. Wei and Zhang (2009) document that the household saving rate rose from 16% in 1980 to 30% in 2007, and they also argue that corporate saving is not yet quantitatively as important as household saving in modern China.

Xie (1992) presents data for the structure of household financial assets in China during the 1978-91 period. The data show that cash and deposits accounted for 100% of total household financial assets in 1978, and this number remained as high as 90% in 1991 despite rapid growth in income and financial wealth. Yi and Song (2008) present data for the 1991-2007 period. Their data show that bank deposits accounted for 72% of total household financial assets in 2004 and 2005. In contrast, the share of bonds and stocks accounted only for 3.5% and 6.3%, respectively, in 2004; and 3.1% and 5.5%, respectively, in 2005. On the other hand, bank loans have been the major source of external funds for nonfinancial firms. For example, the share of bank loans in total corporate debt was 63.3% in 2004 and 64% in 2005. In contrast, the share of corporate bonds and stocks was 0.6% and 15.3%, respectively, in 2004; and was 1.3% and 12.8%, respectively, in 2005.

Hence, bank deposits have been not only the major means of saving for Chinese households, but also the most important source of external funds for Chinese firms. Therefore, deposit rates are the most relevant interest rates for saving decisions in China, instead of the rates of returns to productive capital.

Figure 7 depicts the household saving ratio (left axis), the 3-month nominal deposit rate (left

[46] Horiuchi (1984) argues that the interest rates in Japan in the postwar period were not particularly low compared with those in the United States and developed European countries. However, with respect to Japan's extraordinary high growth and rates of return to capital, the interest rates in Japan were indeed very low.

axis), and long-term household-income growth rate (right axis) for the period of 1953-2006. The household saving rate is defined as the ratio between net wealth changes and disposable income, and the long-term income-growth rate is defied as the average growth rate of the past 14 years, following Modigliani and Cao (2004).[47] The figure shows that the household saving rate traces the long-term income growth rate very closely and has been increasing steadily since 1978. The household saving rate peaked in 2006 at 37%. The bulk of the household saving is contributed by bank deposits and government bonds (net changes), which, as a fraction of disposable income, remained at 27% in 2003, suggesting that safe and low yield assets have been the major means of saving for Chinese households. On the other hand, the interest rates remained very low in the post-reform period. For example, the average nominal 3-month deposit rate was 3.3% (and the average 1-year rate was 5.6%) while the average inflation rate was about 6% in the 1991-2007 period.[48] Yet the average real rate of return to capital was more than 20% per year in that period (Bai, Hsieh, and Qian, 2006).

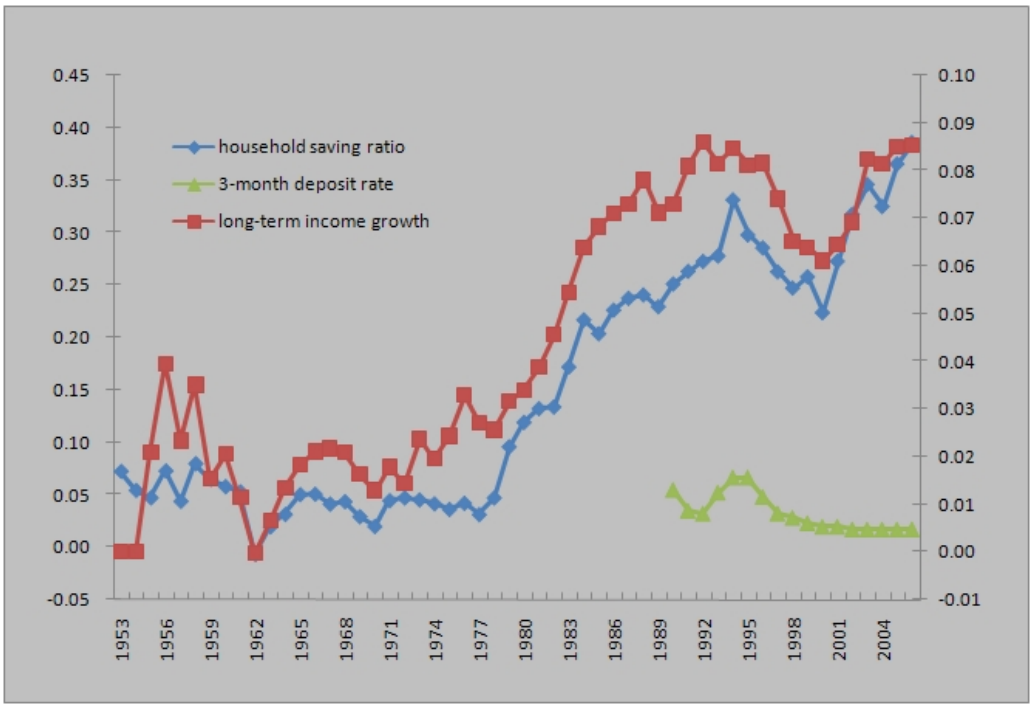

Figure 7. Household Saving Ratio (left scale) and Income Growth (right scale)

Such extremely high saving rates in conjunction with very low real interest rates are difficult to reconcile with standard growth theories. According to the conventional "saving-causes-growth" theory, high growth is the result of high saving. Based on the "growth-causes-saving" theory, high

[47] Household wealth includes deposits, bonds, and individual investments. Excluding individual investments lowers the saving rate slightly but does not change the dynamic pattern of the saving ratio.

[48] Data for the interest rates before 1990 are not available.

growth leads to high returns to capital, hence encouraging high saving through a high equilibrium interest rate. But both theories require high interest rates to induce high saving, so neither theory can explain why saving is so high while the interest rate is so low in fast-growing economies, such as Japan in the 1950-70s and China over the past 30 years.

7 Conclusion

This paper uses a simple growth model to show that uninsured risk and borrowing constraints can completely alter the relationship between the marginal propensity to consume and permanent income, so that higher permanent income can lead to increased saving, instead of higher consumption, counter to the prediction of the PIH. The results are independent of capital accumulation and the neoclassical linkage between the marginal product of capital and the real interest rate.

Uninsured idiosyncratic risk and borrowing constraints give rise to a liquidity premium to reward precautionary savings. This liquidity premium is shown to be an increasing function of income growth because faster income growth reduces the steady-state saving stock-to-flow ratio and raises the liquidity value of the buffer stock. Hence, high growth can lead to high saving through a higher rate of return to liquidity even if the interest rates on household deposits are low and repressed by banking regulations.

Therefore, precautionary saving under borrowing constraints can support a large spread between the deposit rates and the rates of return to capital. That China's rate of return to capital has been so high (about 23% per year) and shown little sign of diminishing despite an investment-to-output ratio in excess of 40% is consistent with this hypothesis. One immediate implication is that if the spread in China were eliminated by allowing interest rates to rise to market levels, the saving rate in China would rise even further. This suggests that, given the severity of idiosyncratic uncertainty and the lack of social safety nets and risk-sharing markets in China, the current Chinese saving rate may not be high enough, but lower than it should be, counter to popular views.[49]

Our analysis not only explains why high growth can lead to high saving despite low and fixed real interest rates, but also provides a rational for the phenomenon of global imbalances that financial capital flows from emerging economies toward developed countries but FDI flows into these fast-growing economies from industrial nations. We use a highly stylized model to make the arguments as straightforward as possible. While permitting closed-form solutions to inspect the mechanisms in a transparent way, the simplifying assumptions underlying the model also impose some limitations. Nonetheless, we believe that the main insight of this paper does not hinge on the specific assumptions of our model and should persist in more general and realistic settings. The main limitations of the current model include the following:

[49] On the other hand, financial development is expected to enhance risk sharing and reduce borrowing constraints, thus ultimately lowering China's high saving rates.

1. To obtain closed-form solutions, interior solutions for hours worked are assumed. However, if the variance of the idiosyncratic shocks is too large, the non-negativity constraint on labor supply ($N_t(i) \geq 0$) may start binding for some households with high realizations of $\varepsilon(i)$ in the previous period. Although this should not affect the main findings of the paper qualitatively, the quantitative results may be affected. For example, with a non-negative constraint on hours binding, households may opt to consume more when the realization of $\varepsilon(i)$ is large, anticipating that in the next period hours cannot go below zero. Although this may change the average saving rate for a given rate of growth, it should not affect the insight that a higher growth rate for income flow will lower the steady-state stock-to-income ratio, hence raising the liquidity premium and the saving rate. Also, analysis shows that (i) this non-negativity constraint on labor supply never binds if the idiosyncratic shocks are placed on preferences and (ii) with preference shocks the model generates very similar results.

2. The model is not designed to explain wealth distribution or income inequality. Wealth distribution in the model is exogenously driven by idiosyncratic wealth shocks and these shocks are i.i.d. Hence, it is not realistic to expect the model to explain the persistence and distribution of wealth in the data. We reverse-engineered the model by assuming a distribution of wealth to match the Gini coefficient in the data. Idiosyncratic wealth shocks in the model are purely a technical device to induce precautionary savings and make the model analytically tractable. The basic results are expected to hold under more realistic idiosyncratic shocks (such as persistent earning shocks).

3. Under quasi-linear preferences the implied elasticity of labor supply is infinity. Although developing countries such as China may have a highly elastic labor supply, this certainly does not last forever and does not apply to developed countries. If this assumption is relaxed (i.e., if the cost of labor supply is convex), agents will be less able to target a wealth level that is independent of initial wealth and, as a result, the optimal target level of wealth will be history dependent. In this case, numerical solution methods are required to solve the model. However, this should not affect the main findings of this paper because it remains optimal to form a target level of the buffer stock even though the target itself may be history dependent.

4. As in the standard literature (e.g., Carroll, Overland, and Weil, 2000; Chen, Imrohoroglu, and Imrohoroglu, 2006), the model does not distinguish household saving from national saving. That is, firms' investment is assumed to be financed entirely by household savings. Although household saving is the single most important component of aggregate saving in China (Wei and Zhang, 2009), corporate saving nonetheless is also important and has risen rapidly in recent years. For this reason, we cannot fully account for the 40% average national saving rate observed in China, although our model does a reasonably good job in accounting for the Chinese household saving rate. In particular, without the spread in the rates of return, the model with borrowing

constraints implies that, under a 10% annual growth rate, the national saving rate is about 30%, whereas with low and fixed deposit rates, the implied saving rate is between 20% and 30%. Other factors besides TFP growth and borrowing constraints (e.g., corporate saving, international trade surplus, public finance) can affect the national saving rate. These factors are not analyzed in this paper.

5. As a first-order approximation and a first step in analyzing the effects of borrowing constraints on the growth-to-saving relationship in an infinite-horizon model, this paper has considered only situations around the balanced growth path. Since China is still in a transition period, it is not clear how this would affect the quantitative predictions of the model once we allow the model to deviate sufficiently far from the steady state. These issues and limitations can be addressed in future studies by extending the current simple framework to a richer environment.

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Appendix (not for publication)

This appendix shows that the results in this paper do not hinge on the log utility function. Namely, the results are not dictated by the special feature that the coefficient of relative risk aversion equals 1.

Suppose the utility function is given by $U(C,N) = \frac{C^{1-\gamma}}{1-\gamma} - aN$, where $\gamma \in (0,\infty)$ measures the degree of risk aversion. Since consumption grows over time, if the parameter a is constant, the relative weight of leisure will shrink to zero, unless a also grows over time accordingly. Hence, to ensure balanced growth, assume $a = (1+\bar{g})^{t(1-\gamma)}$. This implies that leisure time as a fraction of time endowment is constant over time, consistent with the empirical evidence (see, e.g., Ramey and Francis, 2009). By the same transformation as in the main text, we have $U(C,N) = (1+\bar{g})^{t(1-\gamma)}\left[\frac{c^{1-\gamma}}{1-\gamma} - N\right]$. Thus, household i's optimization problem becomes

$$\max_{\{c,s'\}} E_0 \left\{ \max_{\{N\}} \tilde{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1+\bar{g})^{t(1-\gamma)} \left[\frac{c_t(i)^{1-\gamma}}{1-\gamma} - N_t(i) \right] \right\} \right\}$$

subject to

$$c_t(i) + (1+\bar{g})s_{t+1}(i) \leq [\theta + \varepsilon_t(i)]\left[(1+r_t)s_t(i) + w_t N_t(i)\right] \tag{55}$$

$$s_{t+1}(i) \geq 0. \tag{56}$$

For households, except the utility function, the economic environment is exactly the same as before. Therefore, the decision rules of an individual's consumption and saving plans are also characterized by a cutoff strategy and are nearly identical to those in the benchmark model. Hence, the derivation steps are omitted.

The system of equations that determine the general equilibrium of the model is summarized as follows:

$$\frac{1}{w_t} = \left[\beta E_t \frac{1+r_{t+1}}{(1+\bar{g})^{\gamma} w_{t+1}}\right] R(\varepsilon_t^*), \tag{57}$$

$$(1+r_t)k_t + w_t N_t = [w_t R(\varepsilon_t^*)]^{\frac{1}{\gamma}} \frac{1}{\varepsilon_t^*} \tag{58}$$

$$c_t = D(\varepsilon_t^*)\left[(1+r_t)k_t + w_t N_t\right] \tag{59}$$

$$(1+\bar{g})k_{t+1} = H(\varepsilon_t^*)\left[(1+r_t)k_t + w_t N_t\right] \tag{60}$$

$$w_t = (1-\alpha)\frac{y_t}{N_t} \tag{61}$$

$$r_t + \delta = \alpha \frac{y_t}{k_t} \tag{62}$$

$$c_t + (1+\bar{g})\, k_{t+1} - (1-\delta)\, k_t = y_t \tag{63}$$

$$y_t = k_t^{\alpha} \left(z_t N_t\right)^{1-\alpha}, \tag{64}$$

where

$$R(\varepsilon_t^*) \equiv \left[\int_{\varepsilon<\varepsilon^*} \varepsilon^{*\gamma} \varepsilon(i)^{1-\gamma} dF(\varepsilon) + \int_{\varepsilon\geq\varepsilon^*} \varepsilon(i) dF(\varepsilon)\right], \tag{65}$$

and the functions $\{D(\varepsilon^*), H(\varepsilon^*)\}$ are the same as in equations (22) and (23). Note the function $R(\varepsilon^*)$ differs from the previous model unless $\gamma = 1$. In particular, the value of R exceeds that in the previous model if $\gamma > 1$, and it increases with γ. This suggests that highly risk-averse agents value the liquidity of the buffer stock more than do low-risk-averse agents.

In the steady state, we have

$$(1+\bar{g})^{\gamma} = \beta(1+r)R(\varepsilon^*) \tag{66}$$

$$c = D(\varepsilon^*)x \tag{67}$$

$$(1+\bar{g})\, k = H(\varepsilon^*)x \tag{68}$$

$$c + \bar{g}k = y - \delta k, \tag{69}$$

where $x = (1-\delta)\, k + y$ is the wealth income. Define the disposable income $\wp \equiv y - \delta k$. Thus, the consumption and saving functions are the same as before:

$$c = \frac{(1+\bar{g})\, D}{\bar{g} + D} \wp \tag{70}$$

$$\bar{g}k = \left[1 - \frac{(1+\bar{g})\, D}{\bar{g} + D}\right] \wp. \tag{71}$$

Therefore, the national saving rate is given by the same function as in the previous model:

$$\tau = 1 - \frac{(1+\bar{g})\, D(\varepsilon^*)}{\bar{g} + D(\varepsilon^*)}. \tag{72}$$

However, because the function R in equation (65) depends on the value of γ, the cutoff ε^* will also be a function of γ, as is the saving rate. The equation that determines the value of the cutoff is given by

$$\frac{(1+\bar{g})^{\gamma}}{R(\varepsilon^*)} = \beta\left[1 - \delta + \alpha\left(\bar{g} + \delta + (1+\bar{g})\, \frac{D(\varepsilon^*)}{H(\varepsilon^*)}\right)\right], \tag{73}$$

which is analogous to equation (36). Assuming that ε follows the same distribution as in the previous model, we have

$$R\left(\varepsilon^*\right) = 1 + \left[\frac{\sigma\gamma}{(1-\gamma+\sigma)\,(1+\sigma)}\right] \varepsilon_{\max}^{-\sigma} \varepsilon^{*1+\sigma} \tag{74}$$

$$D(\varepsilon^*) = \varepsilon^* \left[1 - \frac{1}{1+\sigma} \varepsilon_{\max}^{-\sigma} \varepsilon^{*\sigma} \right] \tag{75}$$

$$H(\varepsilon^*) = 1 - \varepsilon^* \left[1 - \frac{1}{1+\sigma} \varepsilon_{\max}^{-\sigma} \varepsilon^{*\sigma} \right]. \tag{76}$$

Notice that $\gamma < 1 + \sigma$ is necessary to ensure that the function $R(\varepsilon^*)$ is greater than 1. This is good news because $\sigma \in (0, \infty)$ and the inequality of the wealth distribution in the current model depends only on the relative magnitude $|\sigma - \gamma|$. Hence, to generate the same Gini coefficient in wealth distribution when $\gamma > 1$, we can simply raise the value of σ accordingly. In other words, with a large degree of risk aversion, even a small variance in the idiosyncratic wealth shocks can generate a large enough inequality across households.[50]

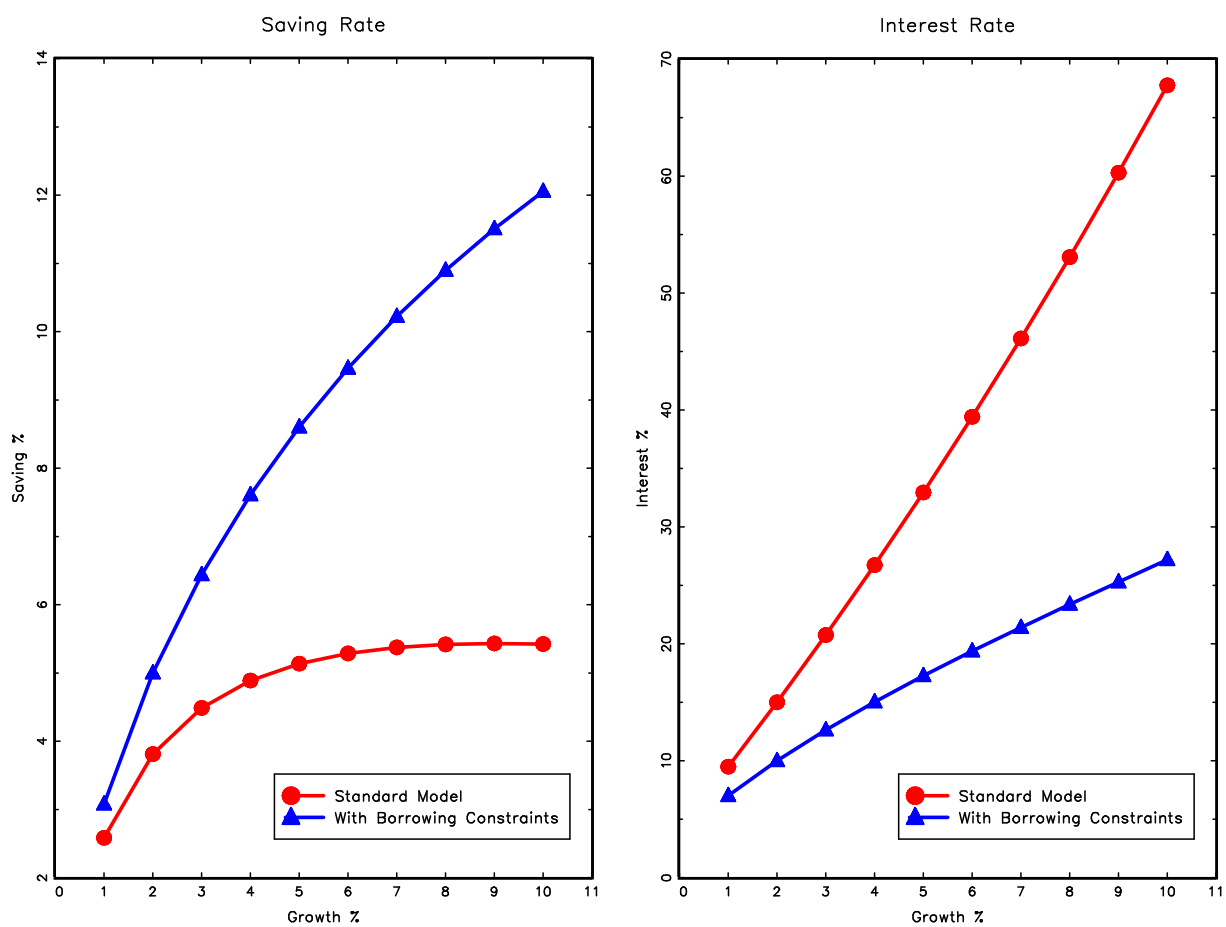

Figure 8. The Growth Effects on Saving and Interest ($\gamma = 5$).

Let the structural parameters take the following values: $\beta = 0.96$, $\delta = 0.1$, and $\alpha = 0.4$, as in the previous model. Also let the coefficient of risk aversion be $\gamma = 5$, which is a sufficiently large

[50] Recall that the variance ε in the power distribution is inversely related to σ. The variance approaches zero when $\sigma \to \infty$.

number. With this value of risk aversion, we set $\sigma = 4.05$, which satisfies the requirement $\gamma < 1+\sigma$ and implies a sufficiently large Gini coefficient. Figure 8 (left panel) shows the relationship between saving and growth, where the line with circles represents the standard growth without borrowing constraints and the line with triangles the counterpart model with borrowing constraints. In both models $\gamma = 5$. Figure 8 indicates that (i) high risk aversion reduces saving at all levels of growth, other things equal (compared with Figure 2); (ii) saving and growth are still positively related, although the positive relation is much weaker, regardless of borrowing constraints; and (iii) with borrowing constraints, this positive relation is significantly magnified. The stronger the growth, the larger the amplification. For example, when the growth rate increases from 1% to 10% per year, the saving rate rises from 2.5% to 5.4% in the standard model without borrowing constraints, but under borrowing constraints the saving rate rises from 3% to 12%.

The right panel in Figure 8 shows that the marginal product of capital rises too fast without borrowing constraints (the line with circles), while this is not the case with borrowing constraints (the line with triangles). Hence, borrowing constraints greatly mitigate the growth effect on the real interest rate.

To understand why the relationship between saving and growth remains positive even under high degrees of risk aversion, consider the saving rate in the standard growth model without borrowing constraints:

$$\tau = \frac{\bar{g}k}{\wp} = \frac{\bar{g}\beta\alpha}{(1+\bar{g})^{\gamma} - \beta(1-\delta) - \delta\beta\alpha}.$$

Differentiating this expression with respect to $\bar{g}$ yields $\frac{d\tau}{d\bar{g}} = \beta\alpha\frac{[(1+\bar{g})^{\gamma}-\beta(1-\delta)-\delta\beta\alpha]-\bar{g}\gamma(1+\bar{g})^{\gamma-1}}{[(1+\bar{g})^{\gamma}-\beta(1-\delta)-\delta\beta\alpha]^2}$. The sign of $\frac{d\tau}{d\bar{g}}$ depends only on the numerator of this expression, which is positive if $(1+\bar{g})^{\gamma-1}\left[1+\bar{g}-\bar{g}\gamma\right] > \beta(1-\delta) + \delta\beta\alpha$. The left-hand side of this inequality decreases with $\bar{g}$ and γ. Suppose $\gamma = 5$, $\bar{g} = \delta = 0.1$, $\alpha = 0.4$, and $\beta = 0.96$; then the left-hand side of the above expression takes the value 0.9317, whereas the right-hand side takes the value 0.9024. Hence, for values of γ within the empirical range (i.e., $\gamma \in (0,5)$), the relationship between saving and growth is positive even in a standard growth model — albeit a weak one — if γ is 5. This positive relationship can be greatly amplified by borrowing constraints.