Inflation Dynamics: A Cross-Country Investigation

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Inflation Dynamics: A Cross-Country Investigation

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July 3, 2006

Abstract

We document that "persistent and lagged" inflation (with respect to output) is a worldwide phenomenon in that these short-run inflation dynamics are highly synchronized across countries. In particular, the average cross-country correlation of inflation is significantly and systematically stronger than that of output, while the cross-country correlation of money growth is essentially zero. We investigate whether standard monetary models driven by monetary shocks are consistent with the empirical facts. We find that neither the new Keynesian sticky-price model nor the sticky-information model can fully explain the data. An independent contribution of the paper is to provide a simple solution technique for solving general equilibrium models with sticky information.

Keywords: Sticky Information, Sticky Prices, Inflation Dynamics, Inflation Comovement, Money.

JEL Codes: E31, E32, E40.

*We thank Barseghyan Levon, Bill Gavin, Ed Green, Ed Nelson, Ricardo Reis, Karl Shell, Dan Thornton, Tao Zhu, seminar participants at Cornell-Penn State Macroeconomic Conference (Fall 2005) and the Federal Reserve Bank of St. Louis, an anonymous referee and the Editor, Bob King, for helpful comments, and John McAdams for excellent research assistance. The views expressed in the paper and any errors that may remain are the authors' alone. Correspondence: Yi Wen, Research Department, Federal Reserve Bank of St. Louis, St. Louis, MO, 63144. Phone: 314-444-8559. Fax: 314-444-8731. Email: yi.wen@stls.frb.org.
1 Introduction

The nature of short-run inflation dynamics is one of the most eminent issues in macroeconomics. It can be traced back in the literature to at least as early as Phillips (1958). Although sophisticated, the modern incarnation of the sticky-price theory, based on the early work of Taylor (1980) and Calvo (1983), is incapable of explaining inflation dynamics in two important aspects (as noted by Fuhrer and Moore, 1995): First, it cannot explain the persistence in the inflation rate. Second, it cannot explain why inflation systematically lags output. The lack of inflation persistence in the models implies that monetary policy can drive a positive rate of inflation to zero with virtually no loss of output (see, e.g., Fuhrer and Moore, 1995). It also implies that announced disinflations can cause booms rather than recessions, which contradicts the historical experience of the U.S. economy (Ball, 1994). Further, the forward-looking property of the inflation dynamics embodied in the standard sticky-price model implies that a policy of permanently falling inflation can keep output permanently high. This implication is criticized by McCallum (1998) on the grounds that it violates a strict form of the natural rate hypothesis, according to which there is no inflation policy that will keep output permanently high.

In response to this challenge, Mankiw and Reis (2002) develop a "sticky-information" model in which information diffuses slowly throughout the population. This slow diffusion could arise because of costs of acquiring information or costs of reoptimization. As a consequence, firms' pricing decisions are not always based on current information. Mankiw and Reis show that a rational expectations monetary model featuring sticky information is capable of overcoming the aforementioned shortcomings of the sticky-price models.¹

The purpose of this paper is three-fold. First, we ask whether the typical output-inflation dynamics emphasized by the literature are a world-wide phenomenon. Our empirical investigation shows that the answer is "Yes". Just like in the U.S., inflation in other industrial countries is also highly persistent and systematically lags output by several quarters. More importantly, we show that such inflation dynamics are highly synchronized across countries. Namely, a high inflation in the U.S. after an output boom is often associated with a high inflation in Europe at the same time. This phenomenon of positive cross-country correlation in inflation is linked to the fact that inflation systematically lags output in each country and that output is positively correlated among countries. However, the data also indicate that the cross-country correlation of inflation is much stronger than the cross-country correlation of output. For example, in our sample of 18 OECD countries, the average correlation of inflation is about 0.6 while that of output is about 0.2.

What forces are responsible for the global comovements in inflation dynamics? Modern mon-

¹State-dependent pricing or hybrid sticky-price models can also overcome these shortcomings. See, e.g., Dotsey and King (2005) and Christiano, Eichenbaum, and Evans (2005).
etary theories attribute the persistent short-run inflation dynamics to exogenous monetary shocks under nominal rigidities (see, e.g., Fuhrer and Moore, 1995; Christiano et al., 2005; Dotsey and King, 2005; Mankiw and Reis, 2002; and Woodford, 2001, among many others). Yet we find that movements in the money stock are not significantly and systematically correlated across countries. This fact alone, however, may not necessarily imply a low correlation in inflations since business cycles can propagate across country borders even though they may be driven by country-specific monetary shocks. For this reason, the second purpose of our paper is to investigate whether standard monetary models with nominal rigidities can simultaneously account for the within-country output-inflation dynamics and the cross-country output-inflation correlations based on the calibrated international covariance of monetary shocks. We find the answer to be negative. Under country-specific money growth shocks, the models are not able generate strongly positively correlated inflation rates across countries while maintaining their ability to account for the domestic output-inflation relationship. This finding casts doubt on the view that exogenous monetary shocks are the main driving force of short-run inflation dynamics.

The third purpose of this paper, which can be viewed as an independent contribution to the literature, is to provide a simple solution technique to solve DSGE models with sticky-information. While conceptually simple, a dynamic general equilibrium model with sticky-information is difficult to solve. The difficulty arises because of the potentially large number of lagged expectation operators, which can create an extraordinarily large state space. In our method, variables with lagged expectations are replaced by their forecast errors with undetermined coefficients. Hence the problem of a large state space is avoided (see Appendix B for details of our solution method). Given that the sticky-information model is becoming increasingly popular, we think this solution technique is a timely contribution to the literature, especially for those who want to estimate sticky-information models with a large number of lagged expectation operators by traditional econometric methods.2

The remainder of the paper is organized as follows. We first present three stylized facts about short-run inflation dynamics: (a) inflation is highly persistent and it lags output within each country; (b) inflation is strongly correlated across countries and the correlation is stronger than that of output; and (c) money growth is not significantly correlated across countries. We then show that models with nominal rigidities and independent monetary shocks cannot simultaneously explain these stylized facts. Since the literature has already found that the sticky-information model is a good model to explain fact (a), we focus first on the sticky-information model’s ability to explain fact (b). After showing that the sticky-information model cannot explain fact (b) without deteriorating its ability to explain fact (a), we then show that the same is true for sticky-price models as well.

2For an example of solving DSGE models with sticky information by traditional methods, see Andrès, López-Salido, and Nelson (2005). The literature studying sticky-information models includes Andrès, López-Salido, and Nelson (2005), Ball, Mankiw, and Reis (2005), Coibion (2006), Jensen (2005), Keen (2005), Koenig (2004), Mankiw and Reis (2002), Mankiw, Reis, and Wolfers (2003), and Trabandt (2005), among others.
2 Stylized Facts

We begin by looking at postwar inflation dynamics for 18 developed countries. The data are from the IMF financial statistics and the OECD database (see Appendix A for detailed data descriptions). The inflation rates are computed based on the consumer price index (CPI).\footnote{Since CPI is not available in Germany until the 90s, we use the GDP deflator for Germany.} We use the Band-Pass filter to define the output gap (business cycle components of GDP). The results are similar if the HP filter is used instead. Some OECD countries have only seasonally unadjusted data available for prices and money stocks. In that case, seasonal adjustment by the X-11 filter is performed whenever needed.

Figure 1 graphs the domestic relationships (correlations) between output ($y_t$) and inflation ($\pi_t$) for each of the individual countries in our sample, for leads and lags up to six quarters. Several patterns emerge from the graphs. First, the contemporaneous correlation between output and inflation is positive for all individual countries except Norway and Portugal, where the correlation is zero. Second, inflation systematically lags output. The maximum correlations between output and inflation always take place at a lag $k \leq 0$ for all of the 18 countries, with an average of $k = -2.2$ quarters. This is why the curves tend to be "z" shaped with a negative slope around $k = 0$. Letting $j$ be the index for the 18 countries in alphabetic order (as in Figure 1), the number of periods that inflation lags output for each country is given by the sequence

$$\{k_j\} = \{-2, -2, -1, -3, -4, -4, -1, -2, 0, 0, -3, -1, -4, -4, -2, -2, -3\}.$$
Figure 1. Lead-Lag Relationship ($Cor(y_{t+k}, \pi_t)$) between Output and Inflation.

This lagged relationship of inflation to output is not a consequence of the filters used to extract the business cycle components (the Band-Pass filter has no phase effects on the time series). We also find a similar pattern if instead the HP filter is used or the inflation rates are not filtered.\textsuperscript{4}

Inflation in each country is also highly persistent. If we measure the persistence of inflation by its $AR(1)$ coefficient ($\rho_j$), where $j$ is the country index, then the following sequence shows the distribution of the persistence in inflation for the full sample:

$$\rho_j = \left\{ 0.56, 0.75, 0.74, 0.83, 0.74, 0.81, 0.88, 0.93, 0.94, 0.89, 0.69, 0.72, 0.53, 0.63, 0.84, 0.80, 0.76, 0.86 \right\}$$

which has a mean of 0.77 with a standard deviation of 0.12.\textsuperscript{5}

It is well known that standard sticky-price models cannot explain the persistence in inflation and the lead-lag relationships between output and inflation. This inability of sticky-price models

\textsuperscript{4}Den Haan and Sumner (2004) use a VAR forecast errors method and they also find that output leads inflation in the G7 countries.

\textsuperscript{5}If higher order processes are estimated, the persistence is even stronger. For example, based on AR(2) model, the average persistence (measured by the sum of the two coefficients) is 0.84.
is the primary reason for Mankiw and Reis proposing to replace the New Keynesian sticky-price model with their sticky-information model.

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<th>Canada</th>
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<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
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<td>0.83 (0.60)</td>
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<td>0.74 (0.87)</td>
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<td>0.34 (0.34)</td>
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<td>0.70 (0.29)</td>
<td>0.78 (0.72)</td>
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<td>0.60 (0.26)</td>
<td>0.76 (0.65)</td>
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<tr>
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Table 1. Cross-Country Correlations in Inflation (Mean = 0.62)

What is more interesting about the output-inflation dynamics, however, is that the dynamic movements in inflation are highly synchronized across individual countries. Namely, a high rate of inflation in one country following an output boom is also associated at the same time with a high rate of inflation in another country. For example, Table 1 reports the cross-country correlations of inflation for the G7 countries based on $CPI$ (numbers in parentheses are based on the $GDP$ deflator). It shows that the inflation rates between any country pairs are positively correlated. The minimum value is 0.26 (between Germany and the USA), the maximum is 0.92 (between France and Italy). The sample average is 0.62. Table 1 also shows that the pattern is robust when the GDP deflator is used instead (see numbers in parentheses). For the entire sample, there are a total of 18 countries and 153 possible pairs. Consequently, we have 153 correlation coefficients. The information is summarized in the upper-left window in Figure 2, which shows that, with no exceptions, inflation ($\pi$) is positively correlated across developed countries. The average of the correlations for the entire sample is 0.57.

This strong comovement in inflation among countries is striking. It could be associated with the long-standing puzzle that output is positively correlated among countries. The upper right window in Figure 2 reveals that output ($y$) is also positively correlated across countries, but with the correlations significantly weaker than those of inflation. For example, the sample mean of the output correlations is only 0.18 (with a relatively large standard deviation of 0.29), while the sample mean of the inflation correlations is 0.57 (with a relatively small standard deviation of 0.17). In addition, the lower panel in Figure 2 shows the relationship between a country pair’s inflation correlation and the same country pair’s output correlation for all of the 153 country pairs (each point on the graph represents one country pair with its inflation correlation on the vertical axis and its output correlation on the horizontal axis). Two important patterns are worth noticing: First, country pairs with higher cross-country correlations in inflation also tend to have higher
correlations in output (the slope of the regression line is 0.17, which implies a correlation of 0.41 between inflation correlation and output correlation for the full sample of country pairs). Second and more importantly, the inflation correlations are stronger than output correlations for most of the 153 country pairs because most of the points in the graph lie above the 45° line.\textsuperscript{6}

![Figure 2. Synchronization in Inflation and Output.](image)

The stylized fact that cross-country correlations in inflation are significantly positive and systematically higher than those of output is puzzling from the viewpoint of real-business-cycle theory, according to which real shocks explain the international comovements in output, which in turn dictate the comovements in prices. Since in different countries inflation lags output for different periods (as seen in Figure 1), the cross-country correlations for output are thus expected to be higher than those for inflation if movements in output drive the movements in prices (see the arguments in Kydland and Prescott, 1990). But the data indicate the opposite – namely, that inflation correlations are much stronger than output correlations.

Oil shocks may be responsible for the cross-country inflation conomovements. To investigate this possibility, we have re-computed the cross-country correlations of inflation using core inflation, which is based on a measure of CPI excluding food and energy. The results are very similar. For

\textsuperscript{6}The international synchronization in output has been under intensive investigation by the real-business-cycle literature (see, e.g., Backus, Kehoe, and Kydland, 1992), but no consensus has been reached regarding the underlining forces of the output correlations (see Wen, 2005, for a more recent analysis).
example, based on core inflation, the average correlation of inflation among all country pairs is 0.59 with a standard deviation of 0.16. Hence the oil shocks, even if they are responsible, cannot be the full story behind the international synchronization in inflation.

What are then the causes of this international synchronization in inflation? A natural candidate is coordinated monetary policies among the developed countries. However, we document that movements in the money stock, measured either by currency in circulation, by total monetary reserves, by M1, or by the velocity of money are not significantly or systematically correlated across countries. Table 2, for example, shows that the correlations in money growth (M1) among the G7 countries do not have a systematic pattern: they are either negative or positive. The average of the correlations for the G7 countries is essentially zero (mean = 0.05). For the entire sample, the correlations of money growth range from −0.46 (between Finland and Denmark) to 0.36 (between Sweden and Belgium). The mean of the full sample is 0.06 with a standard deviation of 0.14. Hence the correlations are not significant. The results are very similar when M0 and total monetary reserves are used as the measure of money supply for each country. For example, among the G7 countries, the average correlation of money growth in terms of M0 is 0.14 with a standard deviation of 0.15. For the entire sample, the average correlation is 0.11 (with a standard deviation of 0.13) based on M0, and it is 0.03 (with a standard error of 0.13) based on total monetary reserves.

Table 2. Cross-Country Correlations in M1 Growth (Mean = 0.05)

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<tr>
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<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>USA</th>
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<td>1.00</td>
<td>0.05</td>
<td>-0.26</td>
<td>0.18</td>
<td>0.02</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>England</td>
<td>1</td>
<td>0.08</td>
<td>-0.09</td>
<td>-0.02</td>
<td>0.35</td>
<td>0.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>France</td>
<td>1</td>
<td>-0.01</td>
<td>0.31</td>
<td>0.00</td>
<td>-0.10</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>1</td>
<td>0.02</td>
<td>0.04</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>1</td>
<td>-0.04</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Japan</td>
<td>1</td>
<td>-0.06</td>
<td></td>
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<tr>
<td>USA</td>
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Notice that in our sample many of the countries had their currencies pegged or linked to each other for significant periods of the sample (i.e., during the European Monetary System), thus the

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7 We have also examined shorter samples that exclude the first and second major oil shock periods in the 70s and the 80s, and we find that the cross-country correlations in inflation are smaller but still significantly and systematically stronger than those in output.

8 We are not the first in the literature to document the international synchronization in inflation. Other people have also noticed that inflation is highly correlated across countries, such as Ciccarelli and Mojon (2005), and Guo et al. (2006). Our results, however, not only reinforce their findings from a different perspective, but also highlight a new stylized fact: inflation correlations are significantly and systematically stronger than output correlations across countries, regardless of which price index is used.

9 Countries like Finland, England, Norway, Denmark, and Belgium do not have M1 data over the full sample. We use the currency in circulation instead for these countries.

10 We also computed the cross-country correlations in the velocity of money and we found no evidence of significant correlations across countries. For more details, see Section 5.4.
inflation rates of these countries may be tied to each other by policy. To check whether this has an important effect on the stylized facts we presented, we have also divided our sample into two groups: Group 1 includes the EMU members and Group 2 includes the rest of the countries. The results are summarized in Table 3. They show that both groups have similar degrees of cross-county correlations in inflation, output, and money growth. In particular, money growth is not significantly correlated while inflation is highly correlated across countries within each group, regardless of which measure of money stock or price index is used. This suggests that the stylized facts we presented above are robust.11

Table 3. Correlation Statistics for EMU Members and Non-EMU Members*

<table>
<thead>
<tr>
<th></th>
<th>$M_0$</th>
<th>$M_1$</th>
<th>$\pi$</th>
<th>$\pi_{\text{core}}$</th>
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<td>Group 1</td>
<td>0.17 (0.11)</td>
<td>0.09 (0.13)</td>
<td>0.58 (0.20)</td>
<td>0.62 (0.19)</td>
</tr>
<tr>
<td>Group 2</td>
<td>0.06 (0.11)</td>
<td>0.05 (0.14)</td>
<td>0.57 (0.13)</td>
<td>0.58 (0.12)</td>
</tr>
<tr>
<td>All Countries</td>
<td>0.11 (0.13)</td>
<td>0.06 (0.14)</td>
<td>0.57 (0.17)</td>
<td>0.59 (0.16)</td>
</tr>
</tbody>
</table>

*Numbers in parentheses are standard errors. Group 1 includes Belgium, Denmark, England, France, Germany, Italy, Netherlands, Portugal, and Spain. Group 2 includes Australia, Austria, Canada, Finland, Japan, New Zealand, Norway, Sweden, and the USA.

Figure 3. Synchronization in Money Growth.

The existence of a significant and systematic relationship among the inflation rates and the

11Since Sweden, Norway, and Austria did not join the EMU until 1996 and our data end in 1998, the three countries are included in the non-EMU member group. One possible explanation for the lack of significant cross-country correlations in money growth among the EMU members is that currencies among these countries are not exactly pegged, but linked with a fairly wide floating band.
simultaneous lack of a significant and systematic relationship among the money growth rates are recaptured by the graphs in Figure 3, where the top window shows the distribution of cross-country correlations of money growth, and the bottom window shows the relationship between the inflation correlation and money growth correlation for each country pair in our sample (each country pair is represented by one point in the graph). The slope of the regression line in the bottom window is 0.004, which implies a correlation of 0.006. It suggests that the international synchronization in inflation has little to do with coordinated monetary shocks across countries.

But this lack of a systematic relationship in money growth across countries does not necessarily imply that money is not responsible for the international comovements in inflation, since country-specific monetary shocks may still be able to generate synchronized price movements across countries via some international propagation mechanism. Thus, if monetary shocks, as argued by both the New Keynesian sticky-price literature and the recent work of Mankiw and Reis, are truly responsible for the output-inflation dynamics documented in Figure 1, then the data suggest that there must exist a strong international transmission mechanism that can propagate country-specific money shocks into synchronized price movements in neighboring countries. Are sticky prices or sticky information the culprit of the inflation synchronization? This question is addressed in the following sections. Since the literature has already shown that sticky information is the better model for explaining the domestic inflation dynamics, we examine its implications for cross-country comovements in inflation first before turning to sticky prices.

3 The Benchmark Model

3.1 Firms

The benchmark model consists of two identical countries, called Home and Foreign (H and F). Assume that each country produces a single final good using intermediate goods and that only the final goods are tradable. The production function is given by:

\[ Y_t = \left( \int_0^1 Y_t^{\frac{\sigma-1}{\sigma}} (i) di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1} \]

where \( \sigma > 1 \) measures the elasticity of substitution among the intermediate goods, \( Y(i) \). Letting \( P_t(i) \) denote the price of intermediate good \( i \), the demand for intermediate goods is given by \( Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\sigma} Y_t \), and the relationship between the final good price and intermediate goods prices is given by \( P_t = \left( \int_0^1 P_t(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \)

Each intermediate good \( i \) is produced by a single monopolistically competitive firm according to
the following technology, \( Y_t(i) = N_t(i) \). Intermediate good firms face perfectly competitive factor markets, and are hence price takers. Profits are distributed to households at the end of each period. The real profit in period \( t \) is given by \( \left( \frac{P_t(i)}{P_t} - MC_t \right) Y_t(i) \), where \( MC_t \) is the marginal cost, which is the same for all firms. Under constant returns to scale, it is also the average cost.

**Sticky Information.** In each period, a fraction \( 1 - \theta \) of firms update information about the state of the economy and set their optimal prices accordingly. The rest continue to set their prices based on old information. A firm who updated its information \( j \) periods ago maximizes its profit by solving \( \max E_t \left( \sum_{j=0}^{\infty} \theta^j P_t^{1-\sigma} Y_t(j) \right) \) subject to the demand constraint, \( Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} Y_t \). The optimal price is given by \( P_t(j) = \left( \frac{\sigma E_{t-j} \left( P_t Y_t MC_t \right)}{(\sigma-1) E_{t-j} \left( P_t^{1-\sigma} Y_t \right)} \right)^{\frac{1}{\sigma}} \). According to Mankiw and Reis, the aggregate price level is given by \( P_t = \left( 1 - \theta \right) \sum_{j=0}^{\infty} \theta^j P_t^{1-\sigma} \left( \pi_t + \Delta mc_t \right) \). Log-linearizing the above two equations, we can obtain what Mankiw and Reis call the "sticky information Phillips Curve":

\[
\pi_t = \frac{1 - \theta}{\theta} mc_t + (1 - \theta) \sum_{j=0}^{\infty} \theta^j E_{t-1-j} \left( \pi_t + \Delta mc_t \right),
\]

where \( \pi_t = \log(P_t/P_{t-1}) \) denotes the inflation rate. Clearly, inflation in the sticky information model depends not only on current economic activities, but also on past expectations of current economic conditions.

### 3.2 Households

There is one representative household in each country and only one type of final good in the world. In order to best capture the Mankiw-Reis idea in our general equilibrium model where money demand is endogenous, we assume that purchases of the final goods are subject to a cash-in-advance constraint. We also assume that financial markets are complete, hence households can trade for a complete set of state-contingent bonds to borrow and lend. The bonds are denominated in home currency. The history of events realized up to period \( t \) is denoted by \( s^t = (s_0, s_1, ..., s_t) \). The initial state, \( s_0 \), is given. Let \( B(s^{t+1}) \) denote the home household’s holdings of a bond purchased in period \( t \) that pays one unit of the home currency in period \( t+1 \) if state \( s^{t+1} \) occurs, and 0 otherwise. The price of this bond in units of the home currency is denoted by \( Q(s^{t+1}|s^t) \). To simplify notation, we denote \( x(s^t) \equiv x_t \) for any variable after the realization of the state \( s^t \).

Consider the home country. In the beginning of each period \( t \), based on the realization of \( s^t \), the household receives a money transfer in home currency in the amount \( X_t \), and a payoff of \( B_t \) additional units of home currency. Hence, the household’s total nominal assets at the beginning of period \( t \) is \( M_t + X_t + B_t \), where \( M_t \) is the household’s money holdings in home currency carried
over from the last period, which is contributed from wage and profit income. Because of the cash-in-advance constraint, the household needs home currency to purchase home goods and foreign currency to purchase foreign goods. Let \( M_{Ht} \) denote the household’s demand for home currency in period \( t \), and \( M_{Ft} \) the household’s demand for foreign currency in period \( t \). Hence the household decides its portfolio of home and foreign currencies as well as future contingent bonds holdings according to the budget constraint:

\[
M_{Ht} + e_t M_{Ft} + \sum_{s^{t+1}} Q(s^{t+1} | s^t) B(s^{t+1}) \leq M_t + X_t + B_t,
\]

(3)

where \( e = \frac{P}{P^*} \) denotes the exchange rate, where \( P^* \) denotes foreign price.\(^{12}\) After the portfolio choices, the currency market is closed. Namely, the household can no longer re-adjust its currency portfolio until the next period. The CIA constraints for the household’s demand of home and foreign goods are given by

\[
C_{Ht} \leq \frac{M_{Ht}}{P_t},
\]

(4)

\[
C_{Ft} \leq \frac{M_{Ft}}{P^*_t}.
\]

(5)

The household receives wage and profit income from intermediate good firms after consumption and portfolio decisions. The law of motion for the household’s money holdings is given by

\[
M_{t+1} = B_t + M_t + X_t - \sum_{s^{t+1}} Q(s^{t+1} | s^t) B(s^{t+1}) - P_t C_{Ht} - e_t P^*_t C_{Ft} + W_t P_t N_t + \Pi_t.
\]

(6)

The household maximizes lifetime utility,

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( C_t^{1-\gamma} - A_n N_t^{1+\gamma_n} \right)
\]

subject to (3)-(6). Since there is only one type of final good, we have \( C = C_H + C_F \), which implies perfect substitutability between home goods and foreign goods.

Denote \( \{\mu_t / P_t, \eta_{1t}, \eta_{2t}, \lambda_t / P_t\} \) as the set of Lagrange multipliers for constraints (3)-(6), respectively. The first-order conditions for the choice of \( \{C_{Ht}, C_{Ft}, M_{t+1}, M_{Ht}, M_{Ft}, N_t, B(s^{t+1} | s^t)\} \) are given, respectively, by

\[
C_t^{-\gamma} = \lambda_t + \eta_{1,t}
\]

(8)

\(^{12}\)We adopt the convention that an asterisk denotes variables in the foreign country.
\begin{equation}
C_t^{-\gamma} = e \frac{P_t^*}{P_t} \lambda_t + \eta_{2,t}
\end{equation}
(9)

\begin{equation}
\lambda_t = \beta E_t \left( \lambda_{t+1} + \mu_{t+1} \right) \frac{P_{t+1}}{P_t}
\end{equation}
(10)

\begin{equation}
\mu_t = \eta_{1,t}
\end{equation}
(11)

\begin{equation}
\frac{e_t \mu_t}{P_t} = \frac{1}{P_t} \eta_{2,t}
\end{equation}
(12)

\begin{equation}
\lambda_t W_t = A_t N_t^{\gamma}\alpha
\end{equation}
(13)

\begin{equation}
\left( \frac{\lambda_t + \mu_t}{p_t} \right) Q(s^{t+1}|s^t) = \frac{\beta \omega(s^{t+1}|s^t)}{p_{t+1}(s^{t+1})} \frac{\lambda_{t+1}(s^{t+1}) + \mu_{t+1}(s^{t+1})}{p_{t+1}(s^{t+1})}
\end{equation}
(14)

where \( \omega(s^{t+1}|s^t) \) is the conditional probability of \( s^{t+1} \) given \( s^t \).

The household in the foreign country has an analogous set of first-order conditions. In particular, with respect to bond choice, we have

\begin{equation}
\frac{(\lambda_t^* + \mu_t^*) Q(s^{t+1}|s^t)}{p_t^* e_t} = \frac{\beta \omega(s^{t+1}|s^t)}{p_{t+1}(s^{t+1})} \frac{\lambda_{t+1}^*(s^{t+1}) + \mu_{t+1}^*(s^{t+1})}{p_{t+1}(s^{t+1}) e_{t+1}(s^{t+1})}.
\end{equation}
(15)

These two conditions for contingent bonds imply perfect risk sharing:

\begin{equation}
C_t = C_t^*.
\end{equation}
(16)

### 3.3 Equilibrium Conditions

In equilibrium, all markets clear. Hence we have

\begin{equation}
M_{Ht} + M_{Ht}^* = M_t + X_t = M_{t+1},
\end{equation}
(17)

\begin{equation}
M_{Ft} + M_{Ft}^* = M_t^* + X_t^* = M_{t+1}^*,
\end{equation}
(18)

\begin{equation}
C_{Ht} + C_{Ht}^* = Y_t,
\end{equation}
(19)

\begin{equation}
C_{Ft} + C_{Ft}^* = Y_t^*,
\end{equation}
(20)

where the first two equations are money market clearing conditions, and the last two are goods market clearing conditions. The money market clearing conditions state that total demand of home (foreign) currency by both domestic and foreign residents must add up to total supply of
home (foreign) currency. The goods market clearing conditions state that total demand of home (foreign) goods by both domestic and foreign residents must add up to the total supply of home (foreign) goods. In addition, the CIA constraints must bind: \( C_{Ht} = \frac{M_{Ht}}{P_t}, \ C_{Ft} = \frac{M_{Ft}}{P_t}, \ C_{Ht}^* = \frac{M_{Ht}^*}{P_t}, \ C_{Ft}^* = \frac{M_{Ft}^*}{P_t} \). These CIA constraints and the market clearing conditions imply

\[
M_t + X_t = P_t Y_t, \tag{21}
\]

\[
M_t^* + X_t^* = P_t^* Y_t^*; \tag{22}
\]

which are identical to the money demand functions assumed in Mankiw and Reis (2002). Note that \( e = \frac{P}{P^*} \), hence the first-order conditions with respect to \( C_H \) and \( C_F \) imply \( \eta_1 = \eta_2 = \eta \).

Note that \( W_t P_t N_t + \Pi_t = P_t Y_t \), and \( M_t + X_t = M_{t+1} \), hence the budget constraint \( (6) \) implies

\[
\sum_{s=t+1} Q(s^t+1|s^t)B(s^t+1) - B_t \frac{B_t}{P_t} = Y_t - C_t \tag{23}
\]

where \( NX_t \equiv C_{Ht}^* - C_{Ft} \) is net exports. The above equation is the standard accounting equation that the current account (real net exports) plus the real capital account equals zero. This explains why the asset market portfolio constraint \( (\mu) \) is written the way it is, since the household’s total home currency holdings \( (M + X) \) may not equal its total money demand, \( M_{Ht} + e_t M_{Ft} \). The household’s excess demand of currency, \( M_H + eM_F - (M + X) \), is precisely the home country’s capital account, \( B - \sum_{s=t+1} Q(s^t+1|s^t)B(s^t+1) \).

Accordingly, we can also write the equilibrium CIA constraint as

\[
Y_t = C_t + NX_t = \frac{M_t + X_t}{P_t}. \tag{24}
\]

Note that this CIA constraint is different from the conventional one assumed in the international economics literature since the conventional CIA constraint, \( C = \frac{M+X}{P} \), does not take into account net exports. It is also this difference that makes the CIA constraint very different from a Money-in-Utility specification. We will discuss in a later section how this difference is crucial to preserve the domestic output-inflation dynamics of Mankiw and Reis (2002) in the two-country international model. Denote \( x_{i,t} = \log(M_{i,t+1}/M_{i,t}) \) as the growth rate of the nominal money stock for country \( i \). Assume that home and foreign money growth follow a jointly stationary process:

\[
\begin{bmatrix}
  x_{h,t} \\
  x_{f,t}
\end{bmatrix} =
\begin{bmatrix}
  \rho & 0 \\
  0 & \rho
\end{bmatrix}
\begin{bmatrix}
  x_{h,t-1} \\
  x_{f,t-1}
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_{h,t} \\
  \varepsilon_{f,t}
\end{bmatrix} \tag{25}
\]

where the innovations \( \{\varepsilon_{h,t}, \varepsilon_{f,t}\} \) are uncorrelated \( i.i.d \) shocks with the same variance \( \sigma^2 \).
3.4 Solution Method

The details of the method are described in Appendix B. Here we only briefly discuss how to apply this method to solve the two-country sticky-information model. The most crucial step in the method is to transform endogenous variables with lagged expectations, $E_{t-j}S_t$, into $j$-step ahead forecast errors, $S_t - E_{t-j}S_t$. To do so, we subtract $(\pi_t + \Delta w_t)$ from both sides of Equation (2) and rearrange terms to get

$$w_{i,t} = \theta w_{i,t-1} + (1 - \theta) \sum_{j=1}^{\infty} \theta^j [\pi_t + \Delta w_t - E_{t-j}(\pi_t + \Delta w_t)],$$

(26)

where the second term on the RHS is a weighted sum of $j$-step ahead forecast errors, which can always be expressed as functions of innovations in exogenous shocks. Thus, the lagged expected endogenous variables are replaced by exogenous forcing variables with undetermined coefficients (since the decision rules are still unknown). With this new representation, we can then apply any conventional method, such as that of Blanchard and Kahn (1980) or King and Watson (1998), to solve the model.

In order to apply the general solution method outlined in Appendix B to solve the sticky-information model, we also need to approximate the infinite sum of $j$-step ahead forecast errors on the right-hand side of (26) by a finite sum: $\sum_{j=1}^{N} \theta^j [S_t - E_{t-j}S_t] + o^{N+1}$, where $S_t \equiv [\pi_t, w_t, -w_{t-1}]'$, and $o^{N+1}$ is a higher-order residual term. Since $\theta < 1$, $\lim_{N \to \infty} \theta^N = 0$. Hence the error term $o^{N+1}$ approaches zero as $N$ increases. In practice we find that $N = 20$ gives very accurate results.

4 Predictions

Calibration. The time period is a quarter. In order to best match the output-inflation behavior of the closed-economy Mankiw-Reis model, we set the risk aversion coefficient $\gamma = 0.05$, and the inverse labor supply elasticity $\gamma_n = 0.05$. These parameter values are assumed because larger values tend to worsen the persistence of inflation and its lagged relationship to output in the Mankiw-Reis model.\(^{14}\) We set $\theta = 0.8$, which is slightly larger than the value assumed by Mankiw and Reis but is needed in our model in order to generate good results. This value implies that it takes information five quarters to spread across the entire population after an initial shock. We find that the larger $\theta$ is, the more inflation lags output. We set the time discounting factor $\beta = 0.99$, and the elasticity of substitution parameter $\sigma = 10$ (implying a markup of about 10%) as in Mankiw and Reis. We assume that money growth follows an $AR(1)$ process with persistence parameter $\rho = 0.6$, which is

\(^{13}\)Notice that the real wage equals the real marginal cost, $w = mc$.

\(^{14}\)More discussions on this issue can be found in the robustness analysis in the next section.
consistent with Mankiw and Reis and much of the sticky-price literature. The parameter values are summarized in Table 4.

### Table 4. Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Benchmark Model</strong></td>
<td></td>
</tr>
<tr>
<td>Preference</td>
<td>$\beta = 0.99, \gamma = \gamma_n = 0.05$</td>
</tr>
<tr>
<td>Production</td>
<td>$\sigma = 10$</td>
</tr>
<tr>
<td>Stickiness Parameter</td>
<td>$\theta = 0.8$</td>
</tr>
<tr>
<td>Money Growth</td>
<td>$\rho = 0.6$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>B. Model Variations</strong>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIU</td>
</tr>
<tr>
<td>Pricing Complementarity</td>
</tr>
<tr>
<td>Money Demand</td>
</tr>
<tr>
<td>Taylor Rule</td>
</tr>
<tr>
<td>Capital</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>C. Sticky Price Model</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 1$</td>
</tr>
</tbody>
</table>

*Note: The model variations use the same parameter values as the benchmark model except for those indicated in this table.

**Impulse Responses.** Figure 4 graphs the impulse responses of output, consumption, the real wage, and the inflation rate for both the home and foreign countries. The intuition for the impulse response patterns of home and foreign countries can be understood as follows. After an unexpected monetary growth increase at home, real wealth increases, hence consumption in both countries increases due to risk sharing. Since leisure is a normal good, the labor supply curve shifts backward in both countries, which leads to a higher real wage. In order to finance the higher consumption demand both at home and abroad, production must increase either at home or abroad, or both. But a higher production in the foreign country is not an equilibrium since that implies an outward shift of the labor demand curve, which causes the real wage to further increase in the foreign country. A higher real wage implies a higher marginal cost of labor, hence a higher price level set by firms. This implies that the aggregate price level must also go up in the foreign country. The increased output combined with an increased price level implies a higher demand for money. But this cannot be an equilibrium since the foreign money supply has not changed. Thus output in the foreign country must decrease. Therefore, output in the home country must go up, which implies a much higher real wage and inflation rate at home. At the same time, since output in the foreign country decreases, the aggregate price level must increase to balance money demand with money supply

15 A larger value of $\rho$ also tends to generate more lagged inflation relative to output.
in the foreign country. A lower output in the foreign country also implies a downward shift of the labor demand curve, which explains why the real wage in the foreign country does not increase as much as it does at home.

Thus, in the initial impact period, the Mankiw-Reis sticky-information model is able to explain the comovements in inflation across countries under monetary shocks. However, as time goes by, while inflation at home gradually returns to zero because the shock to money growth is not permanent, inflation in the foreign country must become negative before returning to zero. This is because the foreign price level must eventually return to the steady state. If inflation is always non-negative, it would imply a permanently higher price level in the long run in the foreign country. Due to this dynamic inflation reversal in its transitional dynamics in the foreign country, the overall cross-country correlation of inflation is not significantly different from zero, since the initial positive correlation is offset by later negative correlations.

Notice that the inflation rate is highly persistent and it lags output both at home and abroad. Under our parameter calibration, the predicted persistence is \( \rho_h = 0.98 \) for the Home and \( \rho_f = 0.89 \) for the Foreign country. The predicted lag period is \( k_h = 2 \) for the Home and \( k_f = 3 \) for the Foreign country. These predictions are consistent with the data presented in Section 2. But, regarding the cross-country correlation in inflation, the model predicts \( \text{corr}(\pi_h, \pi_f) = 0.01 \), which is not significant at all. However, a higher correlation can be generated from the model if we reset the parameter \( \gamma \) to a much higher value. Higher values of \( \gamma \), unfortunately, completely destroy the

Figure 4. Impulse Responses to a Home Money Shock (home = asterisk; foreign = circle).
lead-lag relationship between output and inflation, deteriorating the model’s prediction on other dimensions.\textsuperscript{16} Furthermore, the predicted cross-country correlation for output is always negative. This negative correlation in output is a robust feature of the model. For this reason, the following robustness analyses focus on inflation only.

5 Robustness Analysis

5.1 Money-in-Utility

We consider several robustness analyses with respect to the low cross-country correlation of inflation implied by the sticky-information model.\textsuperscript{17} First, we show that a quantity relationship in terms of aggregate demand (not just consumption), $PY = M$, is crucial for the sticky-information model to explain the persistent and lagged inflation dynamics. Hence, adopting other specifications of money demand will not improve the model. In particular, if the money-in-utility (MIU) specification is used instead, the model not only is unable to explain the persistent and lagged inflation movements with respect to output, but also implies a strongly negative cross-country correlation for inflation. For example, based on the calibrated parameters reported in Table 4, the cross-country correlation predicted by MIU is $-0.74$. The intuition for the negative cross-country correlation in inflation is that under the MIU specification, consumption and real balances always move together within each country. Since a home-money injection leads to higher consumption both at home and abroad under risk sharing, demand for real balances also increases in the foreign country. This must imply a lower price level in the foreign country (since the money supply is fixed in that country) and hence a negative cross-country correlation in inflation.

Since the conventional CIA constraint in terms of consumption only, $PC \leq M$, is just a special case of MIU, a CIA specification in terms of consumption suffers from the same problems as the MIU specification. Thus, in order for the Mankiw-Reis model to generate persistent and lagged inflation dynamics consistent with the data in general equilibrium, and at the same time also generate non-negative cross-country correlations in inflation, a general CIA constraint in terms of aggregate demand ($C + NX$) must be imposed. For the same reason, if capital is included in the model, the CIA constraint must also include investment (see below).

5.2 Pricing Complementarity

Woodford (2003) shows that allowing for strong pricing complementarity can enhance inflation persistence in models with nominal rigidities. In particular, if the household’s utility function is

\textsuperscript{16}See the robustness analysis in the next section (5.2).

\textsuperscript{17}To preserve space, the impulse responses of each modified model are not reported. Interested readers are referred to our working paper (Wang and Wen, 2005c).
modified to
\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\gamma}}{1-\gamma} - A_n \int_0^1 \frac{N_t(i)^{1+\gamma_n}}{1+\gamma_n} \, di \right),
\]
then even large values of $\gamma$ and $\gamma_n$ can generate highly persistent and hump-shaped inflation. The reason for this is as follows. With the modified utility function on leisure, the log-linearized monopolist pricing rule is given by
\[
 p_t(i) = E_{t-j} p_t + E_{t-i} \frac{\gamma c_t + \gamma_n y_t + R_t}{1 + \gamma_n \sigma},
\]
where $R_t$ denotes the nominal interest rate. In the earlier model, the pricing rule is given by
\[
 p_t(i) = E_{t-j} p_t + E_{t-i} (\gamma c_t + \gamma_n y_t + R_t).
\]
Since persistence in inflation requires that monopoly prices be insensitive to aggregate demand, small values of $\gamma$ and $\gamma_n$ are required to generate persistent inflation in the earlier model. In the current model, since $(1 + \gamma_n \sigma)$ appears in the numerator and $\sigma$ is large, the requirement for small values of $\gamma$ and $\gamma_n$ is no longer needed. Thus, even under large values of $\gamma$ and $\gamma_n$, inflation can be persistent and hump-shaped. However, notice that the two models are the same when $\gamma_n = 0$. Hence for small values of $\gamma_n$, the two models’ implications are similar. On the other hand, large values of $\gamma_n$ in the current model are equivalent to small values of $\gamma$ and $\gamma_n$ in the earlier model, hence the implied cross-country correlation in inflation remains small in the current model for large values of $\gamma_n$. For example, when $\gamma = \gamma_n = 1$, the implied cross-country correlation of inflation is 0.05; and when $\gamma = 1$, $\gamma_n = 6.7$, the implied correlation is 0.009.

5.3 With Capital

The literature has shown that adding capital to sticky-price models can significantly deteriorate the dynamic properties of the models with respect to inflation persistence (see, e.g., Chari, Kehoe and McGrattan, 2000). We show here that adding capital to the benchmark sticky-information model does not change the model’s predictions significantly, as long as the CIA constraint is specified in the general form that applies to all components in aggregate demand, $C + I + NX \leq M/P$, where $I$ denotes investment. In particular, impulse response analysis shows that inflation remains highly persistent and lagging output both at home and abroad. Also, similar to the case without capital,

---

18 Namely, $R_t \equiv 1 + \mu_t / \lambda_t$, where $\mu$ and $\lambda$ denote Lagrange multipliers as in the benchmark model.
19 These values imply $\frac{\gamma + \gamma_n}{1 + \sigma n} \approx 0.1$, which is consistent with the calibration in Ball, Mankiw, and Reis (2005).
20 This result is first obtained by Wang and Wen (2005b) in closed-economy models with sticky-prices. To calibrate the model with capital, the production technology for intermediate goods is specified as $Y = K^{\alpha} N^{1-\alpha}$. The depreciation rate of capital is set to $\delta = 0.025$. Also see Table 3 for parameter calibrations in this model.
the cross-country correlations in inflation are not significant because the foreign inflation rate reverses itself to negative shortly after an initial increase, while the home inflation rate remains positive until the shocks die out. For example, based on the calibrated parameters reported in Table 4, the implied cross-country correlation of inflation is $-0.07$. The inflation reversal takes place because the price level in the foreign country has to return back to its steady state in the long run. In other words, with the money supply held constant in the foreign country, the accumulated changes in inflation must sum to zero. Thus, if any model is able to generate hump-shaped inflation persistence in one country under country-specific money growth shocks, then it must also imply a lack of strong cross-country correlations in inflation because of the necessary inflation reversal in the other country. This general statement holds true in all models with nominal rigidities regardless of capital.

5.4 Money Demand Shocks

Suppose the source of monetary disturbances comes from money demand or from the level of the money supply, instead of money growth. Then the model is able to generate significantly positive cross-country correlations in inflation. For example, let the household in each country face modified CIA constraints in each period,

\[ C_{Ht} + D_{H,t} \leq \frac{M_{Ht}}{P_t}, \]
\[ C_{Ft} + D_{F,t} \leq \frac{M_{Ft}}{P_t}; \]

where $D_{H,t}$ and $D_{F,t}$ are random variables with zero mean, which represent shocks to money demand. In a log-linearized world, they are also equivalent to shocks to the velocity of money. The additive property of $D_{H,t}$ and $D_{F,t}$ simplifies the household’s first-order conditions. Thus, all first-order conditions of the household remain the same as in the benchmark model. Consequently, in equilibrium we have

\[ M_t + X_t = P_t(Y_t + D_{H,t} + D_{H,t}^*), \]
\[ M_t^* + X_t^* = P_t^*(Y_t^* + D_{F,t} + D_{F,t}^*). \]

Denote $D_t \equiv D_{H,t} + D_{H,t}^*$ and $D_t^* \equiv D_{F,t} + D_{F,t}^*$ as aggregate money demand shocks in the home country and the foreign country. Without loss of generality we assume that in the steady state $D = D^* = 0$. The log-linearized versions of the two equations are given by $m_t = y_t + d_t$ and $m_t^* = y_t^* + d_t^*$, respectively, where $d_t \equiv \frac{D_t}{Y}$ and $d_t^* \equiv \frac{D_t^*}{Y}$. We assume that money demand shocks follow an $AR(1)$ process for each country, $d_t = \rho_d d_{t-1} + \varepsilon_{dt}$. Since $d_t$ also measures the negative of
the velocity of money, we can use data from our sample to calibrate the process of money demand shocks. We find that the cross-country correlations of money demand shocks are essentially zero. The sample mean of the correlations for all country pairs is 0.18 with a standard deviation of 0.27 under the band-pass filter (the sample mean is 0.13 with a standard deviation of 0.17 under the HP filter). The sample mean of the \( AR(1) \) coefficients is 0.7 under the HP filter and 0.93 under the band-pass filter. The effects of money demand shocks on inflation can be summarized as follows. First, regardless of the value of \( \rho_d \), the predicted volatility of inflation relative to output is too small to match the data. For example, the volatility ratio \( \frac{\sigma_x}{\sigma_y} \) is 0.02 when \( \rho_d = 0.7 \) and 0.04 when \( \rho_d = 0.95 \). On this ground alone, money demand shocks can be ruled out as a good candidate for explaining the inflation dynamics. Second, if \( \rho_d \) is sufficiently close to one, then the prediction of the model starts to look similar to that under money growth shocks. Namely, although inflation is persistent and hump-shaped in the home country, the cross-country correlation of inflation is not significantly different from zero because of the inflation reversal in the foreign country. Third, only if \( \rho_d \) is sufficiently less than one can the cross-country correlation in inflation be made large enough to match the data, but then the persistence of inflation disappears. Given that the variance of money demand shocks and money growth shocks are similar in our sample (\( \sigma_d = 0.027 \) and \( \sigma_u = 0.022 \)), having a model with both types of shocks are unlikely to solve the problems since the movements of inflation under money supply shocks dominate those under money demand shocks.22

5.5 Endogenous Money Rule

Monetary policy shocks may take place in the form of interest rate shocks instead of money growth shocks, and this may imply a different cross-country propagation mechanism with respect to inflation. For this reason, we also consider endogenous monetary policy shocks to the Taylor rule:

\[
i_t = \alpha_y y_t + \alpha_n \pi_t + v_{it},
\]

where \( i_t \) represents the nominal interest rate and \( v_{it} = \rho_i v_{it-1} + \varepsilon_{it} \) represents shocks to monetary policy. Our analyses show that regardless of the values of the Taylor rule parameters, the cross-country correlation of inflation is always negative under independent shocks to the interest rate

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21 The fact that the cross-country correlations are high for inflation but not for output, money supply, and the velocity of money appears to be puzzling based on the identity, \( py = vm \). This can be explained by the fact that output is more volatile than prices, hence if nominal GDP is not highly correlated across countries (due to a weak correlation in output), then so is the effective supply of money. Indeed, the estimated cross-country correlations for nominal GDP are weak (the sample average is 0.17). This explains why the velocity of money is not highly correlated (given that money stock is not highly correlated) across countries.

22 The variance of inflation under a one-standard-deviation shock to money growth is 745 times that under a one-standard-deviation shock to money demand when \( \rho_d = 0.7 \). However, this may not necessarily rule out the possibility that there may be parameter combinations under which the model’s performance can be significantly improved. For example, one may find such particular parameter combinations by estimating a full model with multiple shocks. However, even though this is possible, the results are clearly not robust to perturbations in parameter values and model structures, as shown in this paper.
For example, based on the calibrated parameters reported in Table 4, the implied cross-country correlation of inflation is $-0.56$. Although correlated shocks may resolve this problem, the model does not perform well in explaining the domestic output-inflation relationship. For example, in order to generate highly persistent and hump-shaped inflation within a country, not only do the shocks need to be highly persistent (e.g., $\rho_i = 0.95$), but also the interest rate elasticities to output and inflation need to be relatively large (e.g., $\alpha_y = 0.5$, $\alpha_\pi = 1.6$). But in this case, domestic output is negatively correlated with domestic inflation, which is inconsistent with the data. If the interest rate elasticity parameters are chosen such that the domestic output-inflation correlation becomes positive, inflation then stops being persistent and hump-shaped.

The intuition of the negative cross-country inflation correlation is as follows. A negative home interest-rate shock is equivalent to a money injection at home. Hence it implies a higher price level at home (positive inflation). Consumption in both countries will increase due to risk sharing. In the foreign country, inflation must be negative since a positive inflation cannot be an equilibrium. Under the Taylor rule, a positive inflation implies an even higher nominal interest rate, which implies an increase in the real interest rate in the foreign country. This higher real rate induces agents to save more and consume less, contradicting the original consumption increase. This explains the negative cross-country correlation of inflation. Under Taylor rules the output response in the foreign country must be negative because a positive output response implies a higher labor demand, which increases the marginal cost and consequently the price level, hence contradicting the conclusion that the inflation response in the foreign country is negative.

### 5.6 Correlated Money Supply Shocks

All of the previous robustness analyses regarding different model variations indicate that independent money growth shocks generate negative cross-country correlations in output and zero (or negative) cross-country correlations in inflation, in sharp contrast to the stylized facts reported in Section 2. This subsection investigates whether correlated money shocks are able to improve the sticky-information model in accounting for the data. The results are summarized in Table 5. In the table, $\rho_m$ denotes the cross-country correlation in money growth shocks assumed in each of the model variations considered above, and $\{\rho_y, \rho_\pi\}$ denote the implied cross-country correlations in output and inflation, respectively. For the model variation with money demand shocks considered in Section 5.4, $\rho_m$ is replaced by $\tilde{\rho}_d$, which denotes the assumed value for the cross-country correlation in money demand shocks. The other parameters of the models are calibrated as in Table 4.
Table 5. Predicted Correlations

<table>
<thead>
<tr>
<th></th>
<th>$\rho_m$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
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<tbody>
<tr>
<td>Benchmark Model</td>
<td>$\rho_y$</td>
<td>-0.08</td>
<td>0.12</td>
<td>0.33</td>
<td>0.54</td>
<td>0.77</td>
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<tr>
<td></td>
<td>$\rho_\pi$</td>
<td>0.01</td>
<td>0.21</td>
<td>0.41</td>
<td>0.61</td>
<td>0.8</td>
</tr>
<tr>
<td>MIU</td>
<td>$\rho_m$</td>
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<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\rho_y$</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi$</td>
<td>-0.74</td>
<td>-0.63</td>
<td>-0.48</td>
<td>-0.25</td>
<td>0.15</td>
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<td>Complementarity</td>
<td>$\rho_m$</td>
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<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\rho_y$</td>
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<td>-0.02</td>
<td>0.23</td>
<td>0.47</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi$</td>
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<td>0.24</td>
<td>0.44</td>
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<td>0.82</td>
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<tr>
<td>Capital</td>
<td>$\rho_m$</td>
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<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
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<tr>
<td></td>
<td>$\rho_y$</td>
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<td>0.11</td>
<td>0.32</td>
<td>0.53</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi$</td>
<td>-0.07</td>
<td>0.13</td>
<td>0.34</td>
<td>0.56</td>
<td>0.77</td>
</tr>
<tr>
<td>Money Demand</td>
<td>$\tilde{\rho}_d$</td>
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<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\rho_y$</td>
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<td>0.18</td>
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<tr>
<td></td>
<td>$\rho_\pi$</td>
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<td>0.73</td>
<td>0.83</td>
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<tr>
<td>Taylor Rule</td>
<td>$\rho_m$</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\rho_y$</td>
<td>-0.77</td>
<td>-0.68</td>
<td>-0.54</td>
<td>-0.32</td>
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<tr>
<td></td>
<td>$\rho_\pi$</td>
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<td>-0.37</td>
<td>-0.17</td>
<td>0.10</td>
<td>0.46</td>
</tr>
<tr>
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<td>$\frac{\sigma^2_d}{\sigma^2_m}$</td>
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<td>25</td>
<td>100</td>
<td>225</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>$\rho_y$</td>
<td>0.03</td>
<td>0.13</td>
<td>0.17</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi$</td>
<td>0.10</td>
<td>0.13</td>
<td>0.17</td>
<td>0.24</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 5 indicates that in order for the sticky-information model to match the average cross-country correlation in inflation (0.6) observed in the data, highly correlated money growth shocks with a cross-country correlation around 0.6 are needed for the benchmark model and the model with pricing complementarity. In this case, the two models are also able to generate significant cross-country correlation in output. However, the MIU model and the model with the Taylor rule are still unable to explain the cross-country correlations in output and inflation even when the cross-country correlation of money growth shocks is as high as 0.8. Notice that under a realistic value of the cross-country correlation in money growth (0.1 ~ 0.2), none of the models listed in Table 5 are able to explain the strong correlations in inflation across countries. For example, when $\rho_m = 0.2$, the largest value for the inflation correlation is 0.24, which is obtained in the model with pricing complementarity.\(^{23}\)

\(^{23}\) Notice that this model behaves very similar to the benchmark model.
However, the model with money demand shocks (the fifth panel in Table 5) is able to generate strong cross-country correlations in output and inflation. For example, when \( \tilde{\rho}_d = 0.2 \), the predicted correlation is 0.18 for output and 0.60 for inflation, matching the data closely. Unfortunately, as explained in Section 5.4, money demand shocks are not able to generate a sufficiently volatile inflation rate. Thus, in a model with both money supply shocks and money demand shocks, it is still unable to explain the data because the effect of money supply shocks dominates that of money demand shocks. For example, the last panel in Table 5 reports predicted correlations of the benchmark model under different mixtures of money demand and money supply shocks (assuming \( \rho_m = 0.1 \) and \( \tilde{\rho}_d = 0.2 \), consistent with the data). When the variances of the two shocks are equal (\( \frac{\sigma^2_d}{\sigma^2_m} = 1 \)), which is consistent with the data reported in Section 5.4, the model is not able to generate strong correlations in output and inflation (\( \rho_y = 0.03 \) and \( \rho_x = 0.1 \)). Even when the variance of money demand shocks is 400 times that of money supply shocks, the predicted cross-country correlation in inflation is 0.29, still substantially below the data (0.6). Thus, although a mixture of both money demand and money supply shocks offers some hope in reconciling the model with the data, there still remains a substantial distance between the two.

5.7 Fixed Exchange Rate

Although the data (Table 3) suggest that the cross-country correlations in inflation and money growth are similar regardless of the exchange rate system, it is nonetheless interesting to consider whether the sticky-information model is able to generate a sufficiently large correlation in inflation with only mild correlations in money supply when money demand shocks are present. Since purchasing power parity (PPP) holds in the model, a fixed exchange rate requires that inflation in the foreign country equals inflation in the home country. The Phillips curve then implies that the real wage is the same across countries. Due to perfect risk sharing, Equation (10) then implies that the Lagrangian multiplier (\( \lambda \)) for constraint (3) is the same across countries. Based on these, Equation (13) implies that the output level is the same across countries. The money market equilibrium condition, \( m_t = y_t + d_t \), then implies \( m_t - d_t = m_t^* - d_t^* \), or equivalently, \( u_t^* = u_t + \Delta d_t^* - \Delta d_t \). Suppose that the foreign country pegs its exchange rate. This equation suggests that the foreign country’s money growth is endogenously determined by the home country’s money growth shocks and the difference between the two countries’ money-demand growth rates.

Assuming that both countries have similar money demand shocks and all shocks are independent, the cross-country correlation of money supply is then determined by \( \rho_m = \frac{\sigma_u}{\sqrt{\sigma_u^2 + 2\sigma_d^2}} \), where \( \sigma_u \) is the standard deviation of monetary-supply growth and \( \sigma_{\Delta d} \) is the standard deviation of money-demand growth. Since \( \Delta d_t = d_t - d_{t-1} \) and \( d_t = \rho_d d_{t-1} + \varepsilon_d t \), we can substitute \( \sigma_{\Delta d}^2 = 2(1 - \rho_d)\sigma_d^2 \).
into the expression to get

\[ \rho_m = \frac{\sigma_u}{\sqrt{\sigma_u^2 + 4(1 - \rho_d)\sigma_d^2}}. \]  

(27)

In the data the variances of monetary demand and money supply are almost the same, hence Equation (27) implies \( \rho_m \approx 0.67 \) if \( \rho_d = 0.7 \) (\( \rho_m \approx 0.85 \) if \( \rho_d = 0.9 \)). Such a high correlation of money growth is not supported by the data. Alternatively, Equation (27) can also be written as

\[ \frac{\sigma_d^2}{\sigma_u^2} = \frac{1 - \rho_m^2}{\rho_m^2} \frac{1}{4(1 - \rho_d)}. \]  

(28)

For different values of \( \rho_m \), Equation (28) implies different values of the variance ratio between money demand and money supply shocks. In the data, \( \rho_m \approx 0.1 \). Based on this, the required variance ratio is 83 under a fixed exchange rate. Table 5 shows that under a flexible exchange rate, a variance ratio of 100 can generate a cross-country inflation correlation of about 0.17. Here, under a fixed exchange rate, it only needs a variance ratio of 83 to generate a perfect cross-country correlation in both inflation and output. This shows that a linked exchange rate can improve the model’s performance. However, the required variance ratio is still too high to be consistent with the data. In addition, as shown in Table 3, countries without linked exchange rates also tend to have a high correlation in inflation and low correlation in money growth. Therefore, even if linked exchange rates could explain the data associated with the EMU countries, there would still remain the challenge to explain the data associated with the non-EMU countries.

6 Sticky Prices

In a standard Calvo-type sticky-price model, the Phillips curve equation (2) is replaced by the following form:

\[ \pi_t = \beta E_t \pi_{t+1} + \left(1 - \beta\theta \right) \left(1 - \theta \right) w_t, \]  

(29)

which shows that inflation depends on the current expectation of future economic variables instead of on lagged expectations of economic variables. In this model, output and inflation lack persistence and inflation does not lag output. For this reason, it is rejected by Fuhrer and Moore (1995) and Mankiw and Reis (2002) as a candidate for understanding inflation dynamics. However, under the general form of the CIA constraint in terms of aggregate demand \( (C + NX \leq M/P) \), a sticky-price model is able to generate hump-shaped output persistence and explain the positive cross-country correlations in inflation, as the top panel in Figure 5 shows. As is also clear from the graph,
however, the model cannot simultaneously explain the positive cross-country correlation in output, the hump-shaped inflation dynamics, and its lagged relationship with output.\footnote{If the Money-in-Utility (MIU) specification is used instead, the sticky-price model then implies negative cross-country correlations in inflation. Kollmann (2001) uses a two-country model with sticky prices and sticky wages to study the comovement of output among G7 countries. He also found in his model a negative correlation between home and foreign price levels under monetary shocks. Gavin, Keen, and Pakko (2005) show that a shopping-time specification of money demand cannot generate inflation persistence under monetary shocks.}

In an enriched version of the Calvo sticky-price model, Christiano et al. (2005) adopt a hybrid Phillips curve,

$$\pi_t = \frac{1}{1+\beta} \pi_{t-1} + \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{(1-\beta \theta)(1-\theta)}{(1+\beta)\theta} w_t,$$

and show that allowing for lagged inflation in the Phillips curve can generate hump-shaped persistent and lagged inflation dynamics similar to those in the Mankiw-Reis model. Hence we also consider the Christiano et al. type of Phillips curve in our two-country model.

![Figure 5. Impulse Responses to a Home Money Shock (Top Panel: Calvo Model; Bottom Panel: Christiano et al. Model. Home = asterisk; Foreign = circle).](image)

The bottom panel in Figure 5 shows that the hybrid model fails to predict positive cross-country correlations for inflation. As in the sticky-information model, although it can successfully predict hump-shaped inflation and its lagged relationship with output within each country, the predicted cross-country correlation in inflation is negative ($-0.28$). In addition, the predicted output correlation remains negative. Similar to the sticky-information model, the results are very robust to parameter perturbations and model modifications. The reasons for the failure are the
same as in the sticky-information model. Thus we conclude that sticky-price models, just like sticky-information models, cannot simultaneously explain the within-country output-inflation dynamics and the cross-country output-inflation synchronization.

7 Conclusion

In this paper, we document stylized facts about output, inflation, and money using a cross-country analysis. We show that 1) persistent inflation and its lead-lag relationship with output are a common feature of developed economies; 2) such inflation dynamics are highly synchronized across countries with the cross-country correlations in inflation significantly and systematically stronger than those in output; and 3) changes in money stocks are not significantly correlated across countries. Since conventional wisdom attributes short-run inflation dynamics to monetary shocks, these stylized facts (especially 2 and 3) appear to be puzzling. We investigate whether monetary models with nominal rigidities can explain these stylized facts simultaneously. We find the answer to be negative.

The inability of the monetary models to explain the stylized facts simultaneously is robust to model variations, such as endogenous monetary policy, different specifications of money demand, the presence or absence of capital, fixed exchange rates, and allowing for money demand shocks. Thus, we conclude that the short-run output-inflation dynamics and their strong synchronization across countries are unlikely a monetary phenomenon. The attempt to use monetary shocks to explain output-inflation dynamics could be misguided.\(^\text{25}\) Whether non-monetary shocks are responsible for these stylized facts, however, is a challenging topic for further research. According to our sub-sample analysis, the major oil shocks in the ’70s and the early ’80s may be partially responsible for the strong international comovements in inflation. But they cannot be the whole story since samples excluding the energy prices and the major oil shock periods also show significant international synchronization in inflation. In addition, it is not clear that oil shocks can simultaneously explain the within-country output-inflation dynamics. What is clear, however, is that understanding the sources of the international synchronization in inflation is important both for developing monetary models and for designing monetary policy.

Appendix A

This appendix describes the data source and data range. For within country statistics, we use the full sample available for each country. For cross-country statistics, we choose the common sample

\(^{25}\)The correlation between money growth and inflation within a country is weak in the data. Fitzgerald (1999), however, shows that the correlation is strong in the longer run (e.g., beyond a two-year horizon). This notwithstanding, our purpose in this paper is to use a cross-country analysis to scrutinize the view commonly held in the monetary literature that monetary shocks are the key driving force of short-run inflation dynamics in each country. If this view is correct, then monetary shocks should also be responsible for the cross-country comovements in inflation. Our analysis shows that this is not the case.
period shared by all countries. The data include Gross Domestic Product (nominal GDP), GDP deflator (2000 = 100), Consumer Price Index (CPI, 2000 = 100), Consumer Price Index excluding food and energy (to measure core inflation, 2000 = 100), and Money Supply (M0, M1 and total monetary reserve). The core inflation data are available for all countries except Australia, Portugal, and Sweden. For those countries where M1 data are not available, we use M0; this includes Belgium, Denmark, England, Finland and Sweden. All data are either from the International Monetary Fund (IFS data source) or from the OECD data bank. The following table gives details of the sample periods available. Since many countries do not have money supply data available until 1977, and since the money supply variables are only used for computing cross-country statistics, a common sample period of 77:1-98:4 is used in the paper for computing all cross-country statistics. The positive cross-country correlations in inflation, however, are not an artifact of our sample period. We have also used longer samples for countries where data are available, and we obtain similar results.

<table>
<thead>
<tr>
<th>Country Name</th>
<th>Sample Period</th>
<th>Country Name</th>
<th>Sample Period</th>
</tr>
</thead>
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<td>59:3 - 04:4</td>
<td>Italy</td>
<td>60:1 - 04:1</td>
</tr>
<tr>
<td>Austria</td>
<td>64:1 - 04:2</td>
<td>Japan</td>
<td>57:1 - 04:3</td>
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<td>80:1 - 04:2</td>
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<td>77:1 - 04:3</td>
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<td>87:2 - 04:3</td>
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<td>70:1 - 04:4</td>
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<td>Sweden</td>
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<td>Germany</td>
<td>60:1 - 04:2</td>
<td>USA</td>
<td>57:1 - 04:4</td>
</tr>
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</table>

Appendix B

This appendix describes briefly the solution method for linear difference models with lagged expectations (e.g., due to sticky information). More detailed discussions can be found in our working papers (Wang and Wen, 2005c, 2006). Let \( X_t = [x_{1t}, x_{2t}, \ldots, x_{pt}] \) be a vector of non-predicted variables in period \( t \), let \( Y_t = [y_{1t}, y_{2t}, \ldots, y_{qt}] \) be a vector of predetermined variables in period \( t \), and let \( Z_t \) be an exogenous forcing variable with the law of motion given by \( Z_{t+1} = \rho Z_t + \varepsilon_{t+1} \), where \( \rho \in [0, 1] \) and \( \varepsilon \) is i.i.d. Let \( S_t = [X_t', Y_t', Z_t] \) and let \( k = p + q + 1 \). A rational expectations DSGE model with lagged expectation/information structure can be reduced to the following canonical

---

26 The common sample period used is 77:1 - 98:4. We stop at 1998 because M1 data are available for most countries only up to 1998. Since Belgium, New Zealand, and Sweden do not have data available until the 80s, we use their growth rates to extrapolate the samples. Otherwise the common sample period becomes too short.
dynamic system of equations:

\[
AE_tS_{t+1} = BS_t + \sum_{i=1}^{N} \Gamma_i E_{t-i}S_t,
\]

where \( \{A, B, \Gamma_i, i = 1, 2, \ldots\} \) are \( k \times k \) coefficient matrices. System (1) can be solved in several steps:

**Step 1.** Express the forecast error \( S_t - E_{t-i}S_t \) for \( i \geq 1 \) as a finite moving average process with undetermined coefficients \( \{ \Phi_i \}_{i=0}^{N-1} \).

Since the solution for \( S_t \) should take the form

\[
S_t = \sum_{j=0}^{N} \phi_{t,j} \epsilon_{t-j},
\]

where \( \phi_{t,0} = 1 \), we have

\[
S_t - E_{t-i}S_t = \sum_{j=0}^{N-1} \phi_{t-j} \epsilon_{t-j},
\]

for \( i = 1, 2, \ldots, N \).

Using this property to substitute out \( E_{t-i}S_t \) by their forecast errors, Equation (1) can be rearranged into

\[
AE_tS_{t+1} = \tilde{B}S_t - \Omega \Psi \epsilon_t,
\]

where \( \Psi \) is a \( N \times N \) matrix with \( \Phi_i \) as its diagonal elements, and \( \epsilon_t \equiv [\epsilon'_t, \epsilon'_{t-1}, \ldots, \epsilon'_{t-N+1}]' \).

**Step 2.** Solve Equation (2).

Equation (2) is a standard system of linear difference equations without lagged expectations. Hence we can solve the system with standard methods such as those proposed by Blanchard and Kahn (1980), King and Watson (1998) and Sims (2002). The solution is a set of decision rules of the form

\[
X_t = H(\Psi) [Y'_t, Z'_t, \epsilon'_t]' \quad \text{and} \quad [Y'_{t+1}, Z'_{t+1}, \epsilon'_{t+1}]' = M(\Psi) [Y'_t, Z'_t, \epsilon'_t] + G\epsilon_{t+1},
\]

where \( H(\Psi) \) and \( M(\Psi) \) are coefficient matrices that depend on the undetermined coefficient matrix \( \Psi \).

**Step 3.** Solve for \( \{\Phi_0, \Phi_1, \ldots, \Phi_{N-1}\} \).

Denote the state vector \( Q_t \equiv [Y'_t, Z'_t, \epsilon'_t]' \). The equilibrium decision rules and law of motion are recaptured as\( X_t = H(\Psi)Q_t \) and \( Q_{t+1} = M(\Psi)Q_t + G\epsilon_{t+1} \). Hence, the forecast errors are given by

\[
Q_t - E_{t-i}Q_t = \sum_{j=0}^{i-1} B^j(\Psi)G\epsilon_{t-j},
\]

\[
X_t - E_{t-i}X_t = H \sum_{j=0}^{i-1} B^j(\Psi)G\epsilon_{t-j},
\]

for \( i = 0, 1, 2, \ldots, N-1 \). Equation systems (4) and (3) can be stacked into the following form after leaving out the exogenous variables \( \epsilon_t \) from the bottom rows in equation system (3):

\[
\begin{bmatrix}
X_t \\
Y_t \\
Z_t
\end{bmatrix} - E_{t-i} \begin{bmatrix}
X_t \\
Y_t \\
Z_t
\end{bmatrix} = \sum_{j=0}^{i-1} P_j(\Psi)\epsilon_{t-j}.
\]
By definition we also have

\[
\begin{bmatrix}
X_t \\
Y_t \\
Z_t
\end{bmatrix} - E_{t-i} \begin{bmatrix}
X_t \\
Y_t \\
Z_t
\end{bmatrix} = \sum_{j=0}^{i-1} \Phi_j \varepsilon_{t-j}.
\] (6)

Recall that \( \Psi \) is a diagonal super matrix with \( \Phi_j \) \((j = 0, 1, \ldots, N - 1)\) as its diagonal elements. Clearly, the equivalence of system (6) and system (5) constitutes \( kN \) equations with \( kN \) unknowns in \( \{ \Phi_j \}_{j=0}^{N-1} \). In particular, for \( i = 1, 2, \ldots, N \), a term-by-term comparison between (5) and (6) for all of the coefficients of \( \varepsilon_{t-i} \) suggests that \( P_0(\Psi) = \Phi_0 \), \( P_1(\Psi) = \Phi_1 \), \( P_{N-1}(\Psi) = \Phi_{N-1} \), which can be compactly expressed as \( P(\Psi) = \Psi \). The solution for the vector sequence \( \{ \Phi_i \}_{i=0}^{N-1} \) can thus be found as a fixed point. Although analytical solutions exist, it can also be solved numerically using standard packages in Gauss or Matlab.
References


