Identifying the Effects of U.S. Intervention on the Levels of Exchange Rates

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Identifying the Effects of U.S. Intervention on the Levels of Exchange Rates

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Abstract: Most intervention studies have been silent on the assumed structure of the economic system—implicitly imposing implausible assumptions—despite the fact that inference depends crucially on such issues. This paper identifies the cross-effects of intervention and the level of exchange rates using the likely timing of intervention, macroeconomic announcements as instruments and the nonlinear structure of the intervention reaction function. Proper identification of the effects of intervention indicates that it effectively changes the levels of exchange rates. Such inference depends on careful attention to nonlinearity and seemingly innocuous identification assumptions.

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Central bank intervention is the practice of monetary authorities buying and selling currency in the foreign exchange market to influence exchange rates. Although some major central banks have intervened less frequently recently, all central banks retain it as a tool in their arsenals and some still commonly employ it.

Intervention researchers seek to answer several questions: What effect does intervention have on the level and volatility of exchange rates? To what conditions do central banks respond? Secondarily, how do factors such as coordination, direction, secrecy and amount of intervention affect the answers to those questions? Understanding the dynamic impact of intervention on exchange rates is a first step toward understanding its potential impact on inflation and output through changes in exchange rates and expectations. And describing the impact of intervention could inform our understanding of exchange rate microstructure.

The literature on intervention has primarily addressed these questions with event studies—examinations of exchange rate behavior around intervention. Recently, these event studies have begun to use higher-frequency data, combined with some knowledge of the timing of intervention. High-frequency studies are handicapped, however, by the lack of accurately timed intervention data.¹

More importantly, while event studies find important correlations, they do not uncover structural relations without additional, often implausible, assumptions about the structure of the economy. That is, event studies describe how exchange rates behave around periods of intervention, rather than how intervention influences exchange rates.

¹ Only the Swiss National Bank publicly provides intervention times. The Bank of Canada has provided intervention times confidentially to certain researchers. Intervention times for other central banks must be inferred from press reports; Fischer (2007) shows that Reuters’s report times were fairly inaccurate for Swiss National Bank intervention.
In practice, because central banks often seek to counter recent exchange rate trends, a simple regression of exchange rate returns on intervention will typically imply a perverse (or zero) impact of intervention on exchange rates. This tells one nothing about the actual impact of intervention, however, because the regression coefficient does not consistently estimate any structural parameter.

To properly sort out the dynamic cross-impacts of intervention and exchange rates, one must explicitly confront the important issue of identification—assumptions necessary to obtain consistent estimates of structural parameters. Identification problems are ubiquitous in econometrics and especially important for those who advise policy makers. Economic policy reacts to economic conditions, which depend on economic policy.

This paper proposes and estimates an explicit, identified, dynamic structure for U.S. intervention and uses it to illustrate the dependence of inference on identification assumptions. Three elements underlie the construction of this system: 1) Modest knowledge of the likely timing of U.S. intervention; 2) macro announcement surprise data to be used as instruments for exchange rates; and 3) a nonlinear structure to identify and construct impulse responses.

The systems constructed with these elements imply that intervention has the desired effect on the level of exchange rates: USD purchases raise the value of the USD. And overnight exchange rate returns predict intervention in the expected way: U.S. authorities “leaned against the wind” with respect to overnight (and prior) returns. There is evidence, however, that U.S. authorities leaned with the wind during the business day of intervention.

To illustrate how important identification assumptions are to inference, this paper compares the nonlinear identified results with those of two other models: an identified linear
model and a nonlinear model with an unjustifiably restricted covariance matrix. While the data reject these models, they serve as useful benchmarks to judge the importance of identification assumptions.

The next section briefly discusses the literature on central bank intervention. The third section describes the data while the fourth confronts the issues of stability and identification to infer the cross-effects of U.S. intervention and exchange rates in a linear system. Then nonlinear systems are introduced and estimated. The final section concludes.

2. Studies of the Effects of Intervention

Event studies, which are the most common way to study the effects of intervention, can be informal examinations of price behavior or formal analyses that incorporate statistical tests.

2.1 Event Studies With Daily Data


"An event study framework is better suited to the study of sporadic and intense periods of official intervention, juxtaposed with continuously changing exchange rates, than standard time-series studies. Focusing on daily Bundesbank and US official intervention operations, we identify separate intervention ‘episodes’ and analyse the subsequent effect on the exchange rate." -- Fatum and Hutchinson (2003b)

Other papers can reasonably be described as event studies—even though they do not use that term—because they characterize the behavior of exchange rates around periods of

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2 Parameters that are derived from microeconomic models are often termed “deep” structural parameters. This paper

2.2 Intraday Event Studies

More recently, a third group of papers have used intraday data to evaluate the behavior of exchange rates at very high frequencies around the times of intervention. Fischer and Zurlinden (1999), Payne and Vitale (2003) and Pasquariello (2002) have exploited Swiss National Bank (SNB) data on the exact times of its intervention, not just the day and amount. Fischer and Zurlinden (1999) look at irregular observations at times of intervention to examine the effects of intervention. Payne and Vitale (2003) use 15-minute exchange rate data to quantify the effects of intervention operations on the USD/CHF rate. Pasquariello (2002) looks at a wider variety of exchange rate behavior—including spreads—in a similar exercise. Beattie and Fillion (1999) use confidential timed intervention data from the Bank of Canada to similarly investigate the effects of Canadian intervention. Fatum and King (2005) compare the effects of rule-based and discretionary Canadian intervention on high-frequency data. They find that intervention does systematically affect the CAD/USD and might be associated with reduced volatility. Finally, Dominguez (2003a, 2003b) regresses 5-minute exchange rate returns and volatility on leads and lags of news announcement and intervention news dummies, taken from newswire reports, during days of U.S. intervention from 1987 to 1993. Dominguez interprets the coefficients on intervention and news as the impact of those events on exchange rate behavior. Fischer (2007)

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uses the more traditional meaning of structural: The equations/parameters have economic interpretations.
implicitly criticizes the reliance on such newswire reports by showing that they were fairly inaccurate for SNB intervention, whose times are publicly known.

While intraday studies of intervention have been tremendously valuable in understanding the impact of intervention, at least two problems remain. First, while intraday studies only look at short event windows, intervention might have its full effects over days or weeks. Therefore such studies do not really answer the question of interest: What is the dynamic response of exchange rates to intervention? Second, the paucity of exact intervention timing prevents such studies’ conclusions from being cross-checked in other samples. Inference could be fragile.

2.3 Identified Studies of Intervention

The principal weakness of event studies is that while they characterize the behavior of exchange rates around intervention, they do not uncover the causal structure of the economy. To determine the effect of intervention on exchange rates, one must consider how the economic system simultaneously determines exchange rates and intervention. Even single-equation methods, e.g., two-stage least squares, must implicitly assume a structure for the system.

Kim (2003) and Kearns and Rigobon (2005) explicitly model structural economic relations to identify the effect of intervention on exchange rates. Kim (2003) used monthly data from 1974:1–1996:12 in a structural VAR to examine the effects of intervention and monetary policy on a trade-weighted exchange rate. Neely (2005a and 2005b), however, shows that the intervention effects in Kim’s (2003) structural VAR model are not identified. They might, however, be interpreted as one set of responses that is consistent with the data. Additionally, the intervention variable—purchases of USD—in Kim (2003) is misidentified as purchases of

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3 About 40 percent of central bankers surveyed by Neely (2000) believed that intervention takes at least a few days to have its full effect.
foreign exchange. Therefore Kim’s model actually implies an implausible perverse impact of intervention. In a novel study, Kearns and Rigobon (2005) exploit structural breaks in the Japanese and Australian authorities’ reaction functions to estimate a model of intervention.

2.5 Structural Stability

The use of structural breaks in Kearns and Rigobon (2005) ironically underscores another problem with intervention studies: structural instability. Structural stability is particularly problematic for intervention studies because the coefficients of reduced-form relations will generally change with the intervention reaction function. This paper will attempt to confront the issue of structural stability by looking for a period in which the reduced-form parameters are relatively stable.

3. The Data

This analysis uses two U.S. intervention series: U.S. in-market purchases of millions of USD in the Deutschemark/U.S. dollar (DEM/USD) market and the Japanese yen/U.S. dollar (JPY/USD) market. The intervention data begin in March 1973 and end in December 2001. Olsen and Associates provided 5-minute returns on the DEM, JPY, CHF, and GBP spot exchange rates against the USD from 1987 through 1998. The data were filtered to remove obvious errors. Haver Analytics provided various macroeconomic announcements, as well as survey expectations of those announcements. The surprises constructed from macro announcement data will be used as instruments for exchange rate returns.

Because—as will be shown—almost all U.S. intervention occurs during the business day, the intraday exchange rate data are valuable for identifying structural relations by separately constructing business-day and overnight exchange rate returns. The overnight return \( r_{pm,t} \) is
from 4 p.m. of day $t-1$ to 7 a.m. of day $t$, and the business-day return ($r_{\text{bus}}$) is from 7 a.m. to 4 p.m. of day $t$. Conversion from GMT to New York time accounted for daylight savings time.

Figure 1 shows the time series of the intervention and exchange rates over the post Bretton Woods era. Levels, frequency and direction of intervention appear to be unstable over the whole period. For example, there was a clear shift to more intervention in the JPY in the 1980s. Indeed, it appears that the most likely candidates for a “stable” intervention process—with both USD purchases and sales—are the eras of 1975 through 1980 and 1987 through 1991. (Only the latter period has transactions in both the DEM and JPY.)

More formal structural break statistics in a linear VAR framework—results omitted for brevity—confirmed what casual examination of Figure 1 suggested: The system of intervention with intraday returns (two per day) is generally unstable. That is, rolling structural break statistics are usually able to reject that the parameters of the VAR system are constant over successive two-year periods. The evidence against stability is relatively weak in the 1987-1990 period, however. Therefore the examination of the cross-effects of intervention and exchange rates will be undertaken in the 1987-1990 period.

The exchange rate panels of Figure 1 also display the purchasing power parity (PPP)-implied fundamental values—interpolated from monthly to daily frequency—for the exchange rate. Casual observation suggests that the U.S. authorities tend to buy the dollar when it is “overvalued” by the PPP measure and sell it under opposite circumstances. Statistical tests, such as those in Neely (2002), formally confirm this impression. That is, intervention tends to be positive (negative) when the exchange rate is below (above) its fundamental value. Deviations

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4 This tendency explains the results of Neely (1998) that the U.S. authorities tend to make excess returns on intervention by buying below long-run fundamentals and selling above those fundamentals. Frenkel, Pierdzioch and
from the daily PPP-fundamental help to explain intervention. Such deviations will be included as explanatory variables for intervention and exchange rates.

Table 1 presents summary statistics on these measures of intervention and exchange rates during their common sample, from 1987-1990. The U.S. authorities intervened on 15.7 percent of days in the DEM and 14.2 percent of days in the JPY. Intervention is autocorrelated. The conditional probability of DEM intervention on day \( t \), given that there was intervention on day \( t-1 \), is close to 55 percent, while the probability of intervention, given no intervention on day \( t-1 \), is about 8 percent. Statistics for intervention in the JPY were very similar. Mean exchange rate changes are almost zero and have little autocorrelation, as one might expect.

Table 2 shows the contemporaneous correlations among the variables. Several relations are worth noting. First, the correlation between intervention and exchange rate returns (red shading) are typically moderately negative in such data. Leaning-against-the-wind intervention doubtless causes this negative correlation, which probably accounts for frequent failure to find that intervention influences the exchange rate in the desired direction in studies that look at daily correlations. Second, adjacent a.m. and p.m. returns are correlated (green cells). Third, contemporaneous DEM and JPY returns are highly correlated (brown shading).

4. The Effects of Central Bank Intervention: A Linear Framework

The econometric problems associated with determining the effects of central bank intervention—stability, omitted variables, the unusual distribution of intervention, and identification—must be solved to infer the cross-effects of intervention and exchange rates. This paper will initially estimate dynamic impacts with a benchmark, linear near-VAR.

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representation—which ignores the unusual distribution of intervention—before comparing and contrasting these linear results with those of the preferred nonlinear frameworks.

4.1 Identifying Assumptions from Macro Announcements and the Timing of Intervention

Both exchange rate returns and volatility influence the size and probability of intervention within a day, but intervention simultaneously influences exchange rate behavior. To estimate the structural effect of intervention on exchange rates, or vice versa, one must assume a structure for the system governing exchange rates and intervention. An instrument for either variable, coupled with the assumption that the structural shocks are orthogonal, would be sufficient to identify the cross-effects of intervention and exchange rates in a linear system. A good instrument should be highly predictive of the regressor for which it serves as an instrument, but not structurally predictive of the dependent variable.

It is often difficult to find good instruments. The surprise components of macroeconomic announcements—the first report of the macro variable, less its median survey expectations—may be good instruments for exchange rates, however. Such announcements—typically made at 7:30 or 8:30 a.m., New York time—strongly predict exchange rates (Faust et al. (2003)), but presumably do not directly influence intervention. While macroeconomic surprises represent revisions to estimated fundamentals that could conceivably generate intervention, bias in the coefficients would require the intervention authorities to observe, (strongly) reassess fundamentals and react directly to the announcement within hours. This seems very unlikely.

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\[\text{5 Galati, Melick and Micu (2002) used macro announcement data as regressors to control for the effect of shocks in their study of intervention’s effect on option-implied moments. Announcements could indirectly influence intervention through exchange market effects and still be valid instruments.}\]

\[\text{6 Private conversations with central bankers involved in reserve management/intervention have confirmed that intervention is very unlikely to respond directly to announcements. While using macro announcements to identify the cross-effects of intervention and exchange rates seems very plausible, all identification assumptions—like models themselves—are necessarily approximations and subject to criticism. For example, Faust and Leeper (1997)}\]
Indeed, the extensive intervention reaction function literature ignores the idea that macro announcements themselves predict intervention, though announcements have been used in exchange rate equations.

One can estimate the system more precisely by exploiting the fact that both the policy action (intervention) and the potential instrument (macro announcements) occur during the business day. Breaking up the exchange rate return into overnight ($r_{pm,t}$) and business-day returns ($r_{am,t}$) isolates the business day return that is simultaneous with intervention and macro announcements. This increases the predictive power of the instrument and isolates the effect of intervention on exchange rates.

How does one know that U.S. intervention typically occurs during the business day? Although the U.S. authorities do not publicly release the times of intervention, one can estimate them from the timing of intervention news. Olsen and Associates conveniently provides Reuters’s headline news data on days of G-3 intervention, from August 18, 1989, through August 15, 1995. Figure 2 shows the hours of reported U.S. intervention for this sample in the DEM (top panel) and JPY (lower panel). The great majority of U.S. intervention reports for the DEM and JPY markets were between 8 a.m. and 4 p.m., New York time. This is consistent with Goodhart and Hesse (1993) and Humpage (1999), who report that U.S. intervention generally occurs before the London markets close at 11 a.m., New York time. 

7 Humpage (1999, 2000) used similar timing of morning and end-of-business exchange rates in some of his studies.

8 The figure is similar to one in Dominguez (2003a). Slight differences between the figures are probably due to different interpretations of news headlines. The inaccuracy of Reuters’s reports cited by Fischer (2007) is not nearly large enough to refute the conclusion from Figure 2, that the vast majority of U.S. intervention occurs during New York business hours.
The long literature on choosing instrument sets ultimately recommends a parsimonious instrument set that strongly predicts the regressor. For good distributional results, Stock, Wright and Yogo (2002) provide a function that specifies desired F-statistics as a function of the number of instruments. For one instrument, they recommend an F-statistic of 10. Therefore the optimal instrument set was chosen to maximize the F statistic for the linear regression of the business-day exchange rate return on every possible instrument set from the 6 announcement surprises with the highest univariate t statistics. The shock to the U.S. trade balance is the best instrument for both exchange rates; it very strongly predicts both DEM and JPY business day returns, having extremely high F statistics of 59.2 and 49.9, respectively.

4.2 The Structure of the Linear System

It seems desirable, in providing a benchmark against which to assess the importance of the nonlinear results to be presented later, to estimate a linear model with a similar identification scheme. The structural form of the linear model with one lag (for simplicity) is as follows:

\[
\begin{bmatrix}
1 & 0 & 0 \\
\delta_{pm} & 1 & \delta_{am} \\
\rho & \beta & 1
\end{bmatrix}
\begin{bmatrix}
I_{t} \\
I_{t-1} \\
r_{am,t-1}
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
r_{pm,t-1} \\
I_{t-1} \\
r_{am,t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
a_{1} \\
a_{2} \\
a_{3}
\end{bmatrix}
+ 
\begin{bmatrix}
0 & A_{1,ppp} & ma_{t} \\
0 & A_{2,ppp} & ppp_{t}
\end{bmatrix}
+ 
\begin{bmatrix}
\sqrt{D_{11}} & 0 & 0 \\
0 \sqrt{D_{22}} & 0 & 0 \\
0 & 0 & \sqrt{D_{33}}
\end{bmatrix}
\begin{bmatrix}
u_{t} \\
u_{2t} \\
u_{3t}
\end{bmatrix},
\]

where \( r_{pm,t} \) and \( r_{am,t} \) are the log exchange rate changes from 4 p.m. on \( t-1 \) to 7 a.m. on \( t \) and from 7 a.m. on \( t \) to 4 p.m. on \( t \); \( I_{t} \) is the amount of USD purchased by U.S. authorities on day \( t \); \( ma_{t} \) is the macroeconomic announcement shock (actual less expected) on day \( t \); \( ppp_{t} \) is the deviation from purchasing-power parity; and \( E(u_{t}u_{t}') = 1 \). The contemporaneous interaction matrix permits 1) \( r_{pm,t} \) to influence intervention \( (\delta_{pm}) \); 2) \( r_{am,t} \) to influence intervention \( (\delta_{am}) \); 3) correlation between \( r_{pm,t} \) and \( r_{am,t} \) \( (\rho) \); and 4) intervention can influence \( r_{am,t} \) \( (\beta) \).

\[ ^{9} \text{The regressor set is basically consistent with the practice in the literature. See the classic survey by Edison (1993).} \]
The following assumptions identify the linear model: Excluding the macroeconomic announcement from the intervention equation identifies the impact of returns on intervention, \( \delta_{am} \). This permits the assumption of uncorrelated structural shocks to identify the contemporaneous impact of returns on intervention, \( \beta \). The assumption that overnight returns are predetermined identifies the other contemporaneous coefficients, \( \delta_{pm} \) and \( \rho \).

The system can be estimated equation by equation with DEM/USD and JPY/USD returns and U.S. intervention data from 1987 through 1990. Because overnight returns \( (r_{pm,t}) \) are predetermined, the coefficients of the overnight return equation can be consistently estimated by ordinary least squares (OLS). The intervention equation \( (I_t) \) can then be estimated by instrumental variables, using the error from the first equation as an instrument for overnight returns \( (r_{pm,t}) \) and the macroeconomic announcement shock(s) \( (ma_t) \) as instruments for business-day returns \( (r_{am,t}) \). Finally the business-day returns \( (r_{am,t}) \) equation can also be estimated by instrumental variables, using the errors from the first two equations as instruments for evening returns \( (r_{pm,t}) \) and intervention \( (I_t) \), respectively.\(^{10}\)

The Bayesian information criterion (Schwarz (1978)) chooses a lag length of three lags for both exchange rates, from a maximum of 20 lags, for the reduced-form representation of \( (1) \).

4.3 What Effect Does Intervention Have on Returns in the Linear Model?

Figure 3 shows the effect of a one-standard-deviation U.S. purchase of USD in the linear model of DEM/USD and JPY/USD levels, along with a bootstrapped 80 percent confidence interval. The impacts at each horizon are measured as the cumulative sums of the impacts to the

\(^{10}\) An equivalent way to think about the identification is as follows: Because the macroeconomic announcement shock appears only in the equation for business-day returns \( (r_{am,t}) \), one can estimate/identify \( \delta_{am} \) as the ratio of the appropriate elements of the reduced forms for equations 2 and 3. The restriction that the structural shocks have a
a.m. and p.m. returns. Intervention has no significant impact on the DEM exchange rate; the point estimate is essentially zero. In contrast, the estimated effect on the JPY/USD is positive and significant, initially about 11 basis points, rising to 14 basis points over the impulse horizon. The identified linear model finds a positive effect of intervention on the JPY/USD exchange rate despite the negative contemporaneous correlations between the variables (Table 2).

There is no evidence that the dynamic effect of intervention is greater than the static impact. Many central bankers who actually conduct intervention, however, believe that the dynamic impact is greater than the static impact (Neely (2000)). All of the responding intervention practitioners believe that intervention is effective in influencing exchange rates and nearly 40 percent of them believe that its maximal effect takes at least a few days. The evidence in Figure 3 lacks the power to either refute or confirm this hypothesis.

4.4 What Effects Do Returns Have on Intervention in the Linear Model?

A ubiquitous finding in the intervention literature is that central banks “lean against the wind.” That is, central banks tend to buy (sell) the domestic currency if it has recently depreciated (appreciated). This stylized fact is consistent with the dynamic impact of a shock to returns on intervention, shown in Figure 4. Business day ($r_{am,t}$) and overnight ($r_{pm,t}$) return shocks have a significant, negative impact on intervention for both exchange rates. U.S. intervention in the JPY market seems to react more strongly to overnight returns than to business day returns. The impacts of overnight and business day returns are more similar to each other in the DEM market.

diagonal covariance matrix (D) identifies the rest of the contemporaneous interaction matrix ($\delta_{pm}, \rho$ and $\beta$) from the estimated reduced form covariance matrix.
5. Central Bank Intervention and Exchange Rate Returns: A Friction-Model Framework

5.1 The Friction Model System

The results from the linear representation are not entirely satisfactory because central bank intervention has an unusual distribution, being frequently equal to zero (see Figure 1). It is well known that linear models produce inconsistent estimates of the parameters of limited-dependent processes. To investigate whether the linear representation is adequate or misleading, one can construct a nonlinear model of the interaction between exchange rates and intervention that follows the spirit and basic assumptions of the benchmark linear representation.

The heart of the nonlinear system is a friction model that permits the dependent variable—intervention—to be insensitive to its determinants over a range of values (Rosett (1959)). This is appropriate for a variable like intervention that takes the value zero for a large proportion of observations. The following friction-model framework, for example, characterizes intervention:

\[
I_t = \delta_p r_{pm,t} + \delta_a r_{am,t} + A_1 X_t + a_1 + u_{I,t} \quad \text{if} \quad I_t < 0
\]

\[
I_t = 0 \quad \text{if} \quad I_t = 0
\]

\[
I_t = \delta_p r_{pm,t} + \delta_a r_{am,t} + A_1 X_t - a_1 + u_{I,t} \quad \text{if} \quad I_t > 0
\]

where \(X_t\) is a vector of all explanatory variables—lags of endogenous variables, the macro announcement instrument and deviations from purchasing power parity—excluding the constant. To maintain notational simplicity, \(X_t\) includes the macro announcement but the corresponding element of \(A_t\) equals zero.

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11 Rosett (1959) describes the friction model as an extension of the Tobit model (Tobin (1958)). Maddala (1986) provides a very readable introduction to limited dependent variable models, like the friction and Tobit models. Almekinders and Eijffinger (1996) used a friction model to study central bank reaction functions.

12 This friction model imposes symmetry on the authority’s reaction function. One could permit the constants in (2) and (4)—\(\{a_1, -a_1\}\)—to differ in value as well as sign. One might also estimate an unobservable censoring threshold, but this introduces significant computational difficulties and seems unwise given that some values of intervention are very close to zero (e.g., $3 million) compared with the overall scale of intervention.
The structural model for the system is conditional on the value that intervention takes. If \( I_t \neq 0 \), the structural model is equations (2) and (4) along with the following:

\[
(5) \quad r_{pm,t} = a_0 + A_0 X_t + u_{pm,t}
\]

\[
(6) \quad r_{am,t} = \rho r_{pm,t} + \beta I_t + a_2 + A_2 X_t + u_{am,t}, \quad \text{for } I_t = 0.
\]

As with \( A_1 \), the element of \( A_0 \) corresponding to the macro announcement equals zero. The structural shocks—\( \{ u_{pm,t}, u_{am,t}, u_{am,t} \} \)—have covariance matrix \( \Omega \), with assumed structure:

\[
(7) \quad \Omega = \begin{bmatrix}
\omega_{00} & 0 & 0 \\
0 & \omega_{11} & \omega_{12} \\
0 & \omega_{12} & \omega_{22}
\end{bmatrix}
\]

Note that \( \omega_{12} \) need not be restricted to equal zero, as it must be in the linear model.

If \( I_t = 0 \), then the structural equation for \( r_{am,t} \) becomes:

\[
(8) \quad r_{am,t} = \rho r_{pm,t} + a_2 + A_2 X_t + u_{am,t}
\]

and (3) describes \( I_t \)'s behavior. Appendix A describes the identification of the structural parameters of the model from estimable moments. While much of the intuition for the identification of the model carries through from the linear case, the nonlinearity assists in identification. In particular, the Appendix shows that the system identifies the parameters of (6)—the \( r_{am,t} \) equation—without restricting \( \omega_{12} \) to be zero.\(^\text{14}\) Contrasting the results with and without that seemingly innocuous assumption will illustrate the sensitivity of the inference to identification assumptions.

\(^\text{13}\) Note that observed intervention appears on the right-hand side of the equation, not the value of shadow intervention, as in Nelson and Olson (1978). Sickles and Schmidt (1978) discuss the differences in interpretation and estimation between the two types of models.

\(^\text{14}\) This identification scheme is grossly similar to that of Kearns and Rigobon (2005) in the following way: That paper assumed a structural break in the intervention threshold, giving two sets of reduced-form moments—one set from the first subsample, one set from the second—and only one more parameter. Similarly, in the present case...
The likelihood function for the system is as follows:

\[
L = \prod_{\forall i} f_{\Omega_{ii}} \left( \mathbf{u}_{it} \right) \prod_{l=0}^{\mathbf{A}_{il}} (1 - \mathbf{b} \mathbf{d}_{l}) f_{\Omega_{il}} \left( \mathbf{u}_{l,t}, \mathbf{u}_{am,t} \right) \prod_{l>0} (1 - \mathbf{b} \mathbf{d}_{l}) f_{\Omega_{il}} \left( \mathbf{u}_{l,t}, \mathbf{u}_{am,t} \right)
\]

where \( f_{\Omega_{ii}} \) is the normal likelihood with variance term \( \omega_{ii} \) and \( f_{\Omega_{il}} \) is the bivariate normal likelihood with covariance matrix \( \Omega_{il} \), the intersection of the 2\(^{nd}\) and 3\(^{rd}\) rows and columns of \( \Omega \).

The system was estimated by maximum likelihood, subject to the constraints that \( \mathbf{b} \mathbf{d} < 1 \) and that the dynamic behavior be stable. Appendix A describes how an iterative process generated consistent starting values. The delta method produced confidence intervals for the impulse responses; bootstrapping and/or Monte Carlo simulation would be computationally burdensome.

5.2 Intervention’s Effect in the Friction Model

The left-hand side of Figure 5 shows the dynamic impact of a 3-standard-deviation shock to intervention on the foreign exchange levels and returns without restricting \( \omega_{12} \).\(^{15}\) The estimated impact is positive for both the DEM and the JPY but the confidence interval for the DEM is large enough to barely include zero. One should be careful about interpreting the delta-method confidence intervals, however. Correlation between the parameter estimates causes the Hessian to substantially overstate the true uncertainty about \( \delta, \beta \) and \( \omega_{12} \). Section 5.5 explores the reasons for this in greater detail. Contrary to the marginal significance implied by the confidence interventions in Figure 5, likelihood ratio tests clearly show that the initial impact of

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\(^{15}\) The figures use a 3-standard-deviation shock in the friction-normal models because smaller shocks would often fail to meet the intervention threshold and would therefore have no impact. Because the nonlinear model implies different effects of different-sized shocks, one cannot directly compare the linear and nonlinear results for some nonlinear study, the way the structural parameters map into reduced-form moments depends on whether intervention is zero or not. This provides different sets of reduced-form moments.
intervention ($\beta$) is statistically significant, with p-values of 0.012 for the DEM and 0.002 for the JPY. The estimated DEM point impact tends to fall slightly over time while that of the JPY tends to rise slightly.

The point estimates of the impact of intervention ($\beta$) imply that a $100$ million USD purchase causes a 5 to 6 basis point USD appreciation in either the DEM/USD or JPY/USD markets. Point estimates of $\beta$ from the two markets were very similar. It is difficult to cleanly compare the present results with those of the previous literature because the present model is dynamic and nonlinear, incorporating threshold effects. In addition, intervention’s impact surely varies over time and with the nature of the exchange rate market. Nevertheless, the present point estimates are broadly consistent with those from two well-known intervention studies. Dominguez (2003), who studied G-3 intervention at an intraday frequency, found that a $100$ million U.S. intervention in the DEM market had a maximal impact of almost 3 basis points. Using daily data, Kearns and Rigobon (2005) found that a $100$ million Bank of Japan intervention had a 20-basis-point impact in the JPY/USD market, while the same-sized Reserve Bank of Australia intervention had a 1.3 to 1.8 percent impact in the smaller AUD/USD market.\textsuperscript{16}

To illustrate the sensitivity of the present results to seemingly innocuous identification assumptions, the right-hand side of Figure 5 shows the same dynamic impacts under the restriction that $\omega_{12}$ equals zero. This seemingly minor change makes a very substantial impact on the impulse responses. The estimated initial impact of intervention is now significantly standard-sized shock. The responses are computed to single shocks, although the structural shocks may be correlated.\textsuperscript{16} The small size of estimated effects of intervention might cause one to wonder about whether intervention is important. Reitz and Taylor (2006) implicitly answer this question by laying out a nonlinear intervention model in which intervention serves to coordinate the expectations of rational speculators.
negative (perverse) and is never significantly positive for either exchange rate, using the standard delta-method confidence bounds.

To summarize, inference about the effectiveness of intervention depends on the treatment of $\omega_{12}$. When $\omega_{12}$ is freely estimated, $\beta$ is larger and positive, implying that intervention has the desired impact on exchange rates in the friction model. When $\omega_{12}$ is restricted to equal zero, the estimated impact of intervention is negative (perverse). The data, however, strongly reject the restriction that $\omega_{12}$ equals zero and therefore we conclude that the model that provides economically sensible results (with $\omega_{12}$ free) is also statistically appropriate.

Why do the results change when $\omega_{12}$ is restricted? The restriction changes the estimated responses because the estimate of $\omega_{12}$ is correlated with the estimated initial impact of intervention on returns ($\beta$) and the estimated initial impact of returns on intervention ($\delta_{um}$). But these parameters play conceptually different roles in the assumed nonlinear data-generating process. Restricting $\omega_{12}$ forces the covariance between structural shocks to influence estimates of $\beta$ and $\delta_{um}$. But likelihood ratio tests clearly reject the null that $\omega_{12}$ equals 0. In other words, the restricted model is misspecified; one should expect incorrect answers. These results, in which $\omega_{12}$ equals zero, illustrate the sensitivity of inference to false identification restrictions.

5.3 The Intervention Reaction Function in the Friction Model

Figure 6 illustrates the friction-model-implied dynamic impact of a 3-standard-deviation shock to overnight and business-day returns on intervention in the DEM/USD (top two panels) and JPY/USD markets (bottom two panels). Again, the left-hand panels do not restrict $\omega_{12}$ while the right-hand panels restrict it to be zero. The restriction on $\omega_{12}$ makes little difference for the estimated impact of overnight returns on intervention. Intervention in both exchange rates show a (negative) leaning-against-the-wind reaction to overnight returns. The restriction on $\omega_{12}$ does
change the estimated impact of business-day returns, however. When $\omega_{12}$ is restricted to equal zero in the right-hand panels, the model implies a leaning-against-the wind reaction to DEM returns (second panel on the right) and the shock to JPY returns is insufficient to overcome the intervention threshold (fourth panel on the right).

The leaning-with-the-wind reaction implied by leaving $\omega_{12}$ unrestricted is very plausible, especially at very high frequencies. Although central banks usually lean against recent trends in exchange rates, private communications with central bankers suggest that they often lean with the wind at very high (intraday) frequencies. In addition, most central bankers who responded to Neely’s (2006) survey agree that skillful choices of trading opportunities within a day are important for an authority’s success in intervention.

5.4 The Linear model vs. the friction model

Comparing the linear cases, Figure 3 and Figure 4, to the friction-model system, Figure 5 and Figure 6, the unrestricted nonlinear case ($\omega_{12}$ free) implies a more consistently positive impact of intervention on exchange rates than does the linear case, while the restricted case ($\omega_{12} = 0$) implies an implausible, perverse effect. The linear model clearly implies a leaning-against-the-wind intervention reaction, which the nonlinear model confirms for overnight returns. The treatment of $\omega_{12}$ governs intervention’s reaction to business-day returns in the nonlinear models: Permitting $\omega_{12}$ to be free identifies a tendency to lean with the wind at intraday frequencies.

5.5 Identification in the friction model

Figure 5 shows that inference on the impact of intervention depends crucially on whether the covariance of the structural shocks $\omega_{12}$ is restricted. As discussed previously, this sensitivity reflects correlation between the estimates of $\omega_{12}$, $\beta$ and $\delta_{lm}$. The correlations between $\beta$ and $\delta_{lm}$ for the DEM and JPY cases are -0.75 and -0.68, for example. The correlations for $\beta$ and $\omega_{12}$ are
0.86 for the DEM but only 0.01 for the JPY.\(^\text{17}\)

The geometric counterpart of correlation in parameter estimates is a ridge in the likelihood function. An examination of the likelihood surface—in Figure 7—as a function of \(\{\beta, \delta_{am}, \omega_{12}\}\) pairs sheds light on the relation between the estimates. First, because the figure shows a maximum range of 0.8 standard errors for each parameter estimate, it is clear that these asymptotic robust standard errors substantially overstate the amount of uncertainty in the parameter estimates. That is, the likelihood contour plots (rows 2 and 4 in Figure 7) fall away from the maximum faster than the standard errors would suggest and therefore imply much tighter bounds on the parameter estimates than the asymptotic standard errors. The latter might be untrustworthy because the likelihood is maximized for values of the covariance terms \(\omega_{12}\) that are near the boundaries of their spaces.\(^\text{18}\) \(\omega_{12}\)’s value is implicitly bounded by the restriction that the covariance matrix must be positive definite.

Why are the estimates of these parameters correlated? In an unrestricted linear model, \(\beta, \delta_{am}\) and \(\omega_{12}\) are not separately identified.\(^\text{19}\) The intervention/return covariance is a function of all three parameters. The linear models estimated here—results in Figure 3 and Figure 4—used two restrictions, an instrument to identify \(\beta\) and \(\delta_{am}\), and \(\omega_{12}\) set equal to zero, as is usual in structural VARs. The friction model uses both an instrument and the nonlinearity of the model to separately identify \(\beta, \delta_{am}\) and \(\omega_{12}\).

The correlation between the parameter estimates leaves the Hessian nearly singular; its inverse imprecisely estimates and exaggerates the true sampling uncertainty. Therefore the

\(^{17}\) The restriction of \(\omega_{12}\) probably strongly influences the estimate of \(\beta\) indirectly, through other parameters.

\(^{18}\) An additional problem is the methods used to calculate numerical Hessians might not be very trustworthy in large nonlinear models with parameters of varying magnitudes. There is not much to be done about that, although rescaling the parameters might be of some help.
delta-method confidence intervals are oversized; they show that intervention’s effects are marginally significant, while the likelihood ratio tests strongly reject the null that intervention has no effect.

7. Conclusion

Recently, two research methods have furthered our understanding of the effects of intervention. The first strand uses high-frequency data and information on the timing of intervention to conduct event studies of the behavior of exchange rates around intervention, e.g., Beattie and Fillion (1999), Fischer and Zurlinden (1999), Payne and Vitale (2003), Dominguez (2003a, 2003b), Pasquariello (2002), Fatum and King (2005). Such research has very usefully characterized the correlations in the data, but has been mostly silent on the structure of the economic system, making it very difficult to identify causality. The second strand takes the identification issue seriously and uses lower-frequency data to identify the effect of intervention on exchange rates, as in Kim (2003) and Kearns and Rigobon (2005).

This paper combines the advantages of these strands of the literature in proposing and estimating a new identification scheme, based on the usual timing of intervention, using macro announcement surprises as instruments for exchange rate changes in a nonlinear system. The scheme identifies the dynamic cross-effects of intervention and foreign exchange rates.

Using data from the period 1987 through 1990—where the data show relatively little evidence against stability—the linear model finds that U.S. intervention in the JPY market has a significant effect in the desired direction but the estimated effect of intervention in the DEM market is essentially zero. The linear model picks up clear leaning-against-the-wind effects—U.S. authorities buy (sell) USD when it has been depreciating (appreciating). The linear model

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19 Instruments are equivalent to exclusion restrictions on regressors.
is used only as a benchmark, however, to illustrate the importance of nonlinearity and
identification assumptions.

Inference in the preferred nonlinear model generally differs from that of the linear system
and depends on the treatment of the covariance of structural shocks. When the covariance \( \omega_{12} \)
is restricted to equal zero—which statistical tests strongly reject—the nonlinear system implies
that intervention has a perverse effect on exchange rates but the business-day leaning-against-
the-wind reaction returns. In other words, imposing a seemingly innocuous identification
assumption, that the structural shocks may be correlated, implausibly implies that intervention
has a significant perverse impact.

When the covariance between contemporaneous exchange rate and intervention shocks \( \omega_{12} \)
is freely estimated, the nonlinear system implies that intervention moves exchange rates
significantly in the desired direction and intervention leans with-the-wind at intra-business-day
frequency. That is, all the nonlinear results with \( \omega_{12} \) free model are reasonable, consistent with
our knowledge of intervention policy and definitely statistically significant. The conclusions of
this \( \omega_{12} \) free model are economically sensible and statistically preferred to any other model
studied here. This sensitivity illustrates the importance of attention to identification issues.

It would be desirable to extend the model in the present paper to investigate secondary
questions in the intervention literature, such as the following: Through what channel does
intervention work? Are coordinated interventions more effective? Is the first intervention in a
series more effective? What are the cross-effects of intervention on volatility? Unfortunately,
it is not easy to answer such questions; the dynamic nonlinear models are difficult to estimate.
But such research is underway.
Appendix A: Identification and estimation of a friction-model system

This appendix explains how the parameters of the friction-model system are identified and initially estimated from the reduced-form moments. These iterative estimates are used as starting values for maximum likelihood estimates. The following friction-model framework characterizes intervention:

\[(A.1) \quad I_t = \delta_p r_{pm,t} + \delta_r r_{am,t} + A_1 X_t + a_1 + u_{t,t}, \quad \text{if} \quad I_t < 0\]

\[(A.2) \quad I_t = 0 \quad \text{if} \quad I_t = 0\]

\[(A.3) \quad I_t = \delta_p r_{pm,t} + \delta_r r_{am,t} + A_1 X_t - a_1 + u_{t,t}, \quad \text{if} \quad I_t > 0\]

where \(X_t\) is a vector of all explanatory variables, excluding the constant. The element of \(A_1\) corresponding to the macro announcement is restricted to equal zero. Note that the constant \((a_1 > 0)\) enters (A.1) and (A.3) with different signs and the three conditions—\(I_t > 0, I_t < 0, I_t = 0\)—can be rewritten as \(u_{t,t} > a_1 - \delta_p r_{pm,t} - \delta_r r_{am,t} - A_1 X_t\), \(u_{t,t} < -a_1 - \delta_p r_{pm,t} - \delta_r r_{am,t} - A_1 X_t\) and \(-a_1 > u_{t,t} + \delta_p r_{pm,t} + \delta_r r_{am,t} + A_1 X_t > a_1\). The figure illustrates the relation of intervention to its latent value in a friction model.

\[\begin{align*}
I(t) &> a_1 \quad \text{if} \quad I_t > 0 \\
&\quad \text{if} \quad I_t = 0 \\
&< -a_1 \quad \text{if} \quad I_t < 0
\end{align*}\]

\[A_1 X_t + e_t\]

---

20 Much work on the intervention/volatility relation has already been done, but some of the dynamic relations remain
The likelihood function for the friction model is given by

\[
L(\theta| I, X) = \prod_{i \in T_1} \frac{1}{\sqrt{\omega_{11}}} \phi \left( \frac{I - \delta_p r_{pm,t} - \delta_a r_{am,t} - A_i X_t - a_1}{\sqrt{\omega_{11}}} \right) \times \prod_{i \in T_2} \frac{1}{\sqrt{\omega_{11}}} \phi \left( \frac{-\delta_p r_{pm,t} - \delta_a r_{am,t} - A_i X_t - a_1}{\sqrt{\omega_{11}}} \right) \times \prod_{i \in T_3} \frac{1}{\sqrt{\omega_{11}}} \phi \left( \frac{I - \delta_p r_{pm,t} - \delta_a r_{am,t} - A_i X_t + a_1}{\sqrt{\omega_{11}}} \right),
\]

where \( \theta \) is the parameter vector, \( \{\phi, \Phi\} \) denote the normal density and cumulative normal density, respectively, and \( T_1, T_2 \) and \( T_3 \) denote the sets of observations for which \( I \) is negative, zero and positive, respectively.

The structural model for the system is conditional on the value that intervention takes. If \( I_i \neq 0 \) — the sets \( T_1 \) and \( T_3 \) — it is equations (A.1) and (A.3) along with the following:

(A.5) \[ r_{pm,t} = a_0 + A_0 X_t + u_{pm,t} \]

(A.6) \[ r_{am,t} = \rho r_{pm,t} + \beta I_t + a_2 + A_2 X_t + u_{am,t} . \]

The structural shocks — \( \{u_{pm,t}, u_{am,t}\} \) — have covariance matrix \( \Omega \), with assumed structure

(A.7) \[ \Omega = \begin{bmatrix} \omega_{00} & 0 & 0 \\ 0 & \omega_{11} & \omega_{12} \\ 0 & \omega_{12} & \omega_{22} \end{bmatrix} . \]

Note that the first dependent variable, \( r_{pm,t} \), is predetermined, so its structural form and reduced form coincide: \( \{a_0, A_0\} = \{\pi_0, \Pi_0\} \). These can be directly estimated by least squares, as well as the residual variance (\( \omega_{00} \)). Therefore, for identification and initial estimation purposes, imprecisely estimated, due to the elaborate nature of the model.
we can consider the 3-equation system to be a 2-equation system in \( I_t \) and \( r_{am,t} \) and treat \( r_{pm,t} \) as another predetermined variable in those equations, subsuming it into \( X_t \). The coefficients on \( r_{pm,t} \), \( \delta_p \), and \( \rho \), are subsumed into \( A_1 \) and \( A_2 \), respectively. For simplicity, the notation will remain unchanged.

If \( I_t = 0 \), then the structural equation for \( r_{am,t} \) becomes

\[
(A.8) \quad r_{am,t} = a_2 + A_2 X_t + u_{am,t}
\]

and (A.2) describes \( I_t \)'s behavior. (A.8) reflects the inclusion of the macro announcement in the matrix of predetermined variables, \( X_t \).

The reduced form in the case that \( I_t < 0 \)—the set of observations \( T_1 \)—is

\[
(A.9) \quad I_t = \frac{1}{(1 - \beta \delta_a)} \left( (a_1 + \delta_a a_2) + (A_1 + \delta_a A_2) X_t + (u_{1,t} + \delta_a u_{am,t}) \right), \quad \text{if } I_t < 0,
\]

\[
= \pi_1^- + \Pi_1 X_t + w_{1,t}
\]

\[
(A.10) \quad r_{am,t} = \frac{1}{(1 - \beta \delta_a)} \left( (\beta a_1 + a_2) + (\beta A_1 + A_2) X_t + (\beta u_{1,t} + u_{am,t}) \right),
\]

\[
= \pi_2^- + \Pi_2 X_t + w_{2,t}
\]

and if \( I_t > 0 \)—the set of observations \( T_3 \)—the reduced forms are as follows:

\[
(A.11) \quad I_t = \frac{1}{(1 - \beta \delta_a)} \left( (-a_1 + \delta_a a_2) + (A_1 + \delta_a A_2) X_t + (u_{1,t} + \delta_a u_{am,t}) \right), \quad \text{if } I_t > 0,
\]

\[
= \pi_1^+ + \Pi_1 X_t + w_{1,t}
\]

\[
(A.12) \quad r_{am,t} = \frac{1}{(1 - \beta \delta_a)} \left( (-\beta a_1 + a_2) + (\beta A_1 + A_2) X_t + (\beta u_{1,t} + u_{am,t}) \right),
\]

\[
= \pi_2^+ + \Pi_2 X_t + w_{2,t}
\]

where \( X_t \) includes \( r_{pm,t} \). Note that \( \pi_1^- = \pi_1^+ + \frac{2a_1}{(1 - \beta \delta_a)} \). The covariance matrix, \( W \), of the reduced form errors, \( \{w_{1t}, w_{2t}\} \), in \( \{T_1, T_3\} \) is as follows:
$$W = \begin{bmatrix} 1 & -\delta_a \\ -\beta & 1 \end{bmatrix}^{-1} \Omega_{12} \begin{bmatrix} 1 & -\beta \\ -\delta_a & 1 \end{bmatrix}^{-1}$$

(A.13)

$$= \frac{1}{(1 - \beta \delta_a)^2} \begin{bmatrix} \omega_{11} + 2\delta_\omega \omega_{12} + \delta^2 \omega_{22} & \beta\omega_{11} + (1 + \beta \delta_a)\omega_{12} + \delta_a \omega_{22} \\ \beta\omega_{11} + (1 + \beta \delta_a)\omega_{12} + \delta_a \omega_{22} & \beta^2 \omega_{11} + 2\beta \omega_{12} + \omega_{22} \end{bmatrix},$$

where $\Omega_{12}$ denotes the \{2,3\} submatrix of $\Omega$ defined in (A.7) and we denote the elements of $W$ as \{\$W_{11}, W_{12}, W_{22}\$\).

If $I_t = 0$—the set of observations $T_2$—the reduced forms are

(A.14)

$$I_t^* = \left( -a_1 + \delta_a a_2 \right) + \left( A_1 + \delta_a A_2 \right) X_t + \left( u_{t,t} + \delta_a u_{am,t} \right)$$

$$= \psi_t^+ + \Psi_t^+ X_t + u_{t,t} + \delta_a u_{am,t} = (1 - \beta \delta_a) \left( \pi_t^+ + \Pi_t X_t + w_{t,t} \right)$$

$$I_t^* \leq 0$$

$$I_t^* = \left( a_1 + \delta_a a_2 \right) + \left( A_1 + \delta_a A_2 \right) X_t + \left( u_{t,t} + \delta_a u_{am,t} \right)$$

(A.15)

$$= \psi_t^- + \Psi_t^- X_t + u_{t,t} + \delta_a u_{am,t}$$

$$= (1 - \beta \delta_a) \left( \pi_t^- + \Pi_t X_t + w_{t,t} \right) = (1 - \beta \delta_a) \left( \pi_t^+ \frac{2a_1}{1 - \beta \delta_a} + \Pi_t X_t + w_{t,t} \right)$$

where $I_t^*$ denotes the unobserved “shadow” value of intervention. (A.14) and (A.15) imply that, in $T_2$, the reduced-form error obeys the following:

(A.16)

$$-\pi_t^+ - \Pi_t X_t \geq w_{t,t} \geq \pi_t^- - \frac{2a_1}{1 - \beta \delta_a} - \Pi_t X_t.$$  

And the reduced form for $r_{am,t}$—in $T_2$—is as follows:

(A.17)

$$r_{am,t} = a_2 + A_2 X_t + u_{am,t} = \psi_2 + \Psi_2 X_t + u_{am,t}.$$  

### A.1 Estimating the Model

We can estimate the reduced form for $I_t$, using all observations (sets $T_1$, $T_2$ and $T_3$). The likelihood function is given by the following:
\[ L(\theta | I_t, X_t) = \prod_{i \in T_i} \frac{1}{\sqrt{W_{i1}}} \phi \left( \frac{I_t - \pi_1^- - \Pi_1 X_t}{\sqrt{W_{i1}}} \right) \times \prod_{i \in T_i} \frac{1}{\sqrt{W_{i1}}} \phi \left( \frac{I_t - \pi_1^+ - \Pi_1 X_t}{\sqrt{W_{i1}}} \right) \]

(A.18)

where \( \pi_1^- > \pi_1^+ \). Maximizing this likelihood provides estimates of \( \{ \pi_1^+, \pi_1^-, \Pi_1, \} \) and \( W_{11} \).

For the \( r_{am,t} \) equations, the fact that the form of the equation depends on the behavior of \( I_t \) complicates estimation of the reduced forms. The equations hold only for a nonrandom subset of the observations and the error terms are not necessarily mean zero.

(A.19) \[ r_{am,t} = \pi_2^- + \Pi_2 X_t + w_{2,t} \quad \text{if} \quad I_t < 0 \]

(A.20) \[ r_{am,t} = \pi_2^+ + \Pi_2 X_t + w_{2,t} \quad \text{if} \quad I_t > 0 \]

(A.21) \[ r_{am,t} = a_2 + A_2 X_t + u_{am,t} = \psi_2 + \Psi_2 X_t + u_{am,t} \quad \text{if} \quad I_t = 0 \]

One can decompose the error in (A.19) through (A.21) into estimable functions and an uncorrelated mean-zero error:

(A.22) \[ E \left[ w_{2,t} \mid \frac{w_{1,t}}{\sqrt{W_{11}}} < -\pi_1^- - \Pi_1 X_t \right] = -\frac{W_{12}}{\sqrt{W_{11}}} \phi \left( \frac{-\pi_1^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) \left/ \Phi \left( \frac{-\pi_1^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) \right. \]

(A.23) \[ E \left[ w_{2,t} \mid \frac{w_{1,t}}{\sqrt{W_{11}}} > -\pi_1^- - \Pi_1 X_t \right] = \frac{W_{12}}{\sqrt{W_{11}}} \phi \left( \frac{-\pi_1^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) \left/ \Phi \left( \frac{-\pi_1^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) \right. \]

\[ E[u_{am,t} \mid -\Psi_1^- X_t \leq w_{1,t} \leq -\Psi_1^- X_t] = \]

(A.24) \[ \frac{\text{cov}(u_{am,t}, w_{1,t})}{\sqrt{W_{11}}} \left( \frac{\phi \left( \frac{-\Psi_1^- - \Psi_1^+ X_t}{\sqrt{W_{11}}} \right) - \phi \left( \frac{-\Psi_1^- - \Psi_1^- X_t}{\sqrt{W_{11}}} \right)}{\Phi \left( \frac{-\Psi_1^- - \Psi_1^+ X_t}{\sqrt{W_{11}}} \right) - \Phi \left( \frac{-\Psi_1^- - \Psi_1^- X_t}{\sqrt{W_{11}}} \right)} \right) \]
This implies that

\[ r_{am,t} = \pi_2^+ + \Pi_2 X_t - \frac{W_{12}}{\sqrt{W_{11}}} \phi \left( \frac{-\pi_i^+ - \Pi_1 X_t}{\sqrt{W_{11}}} \right) + \phi \left( \frac{-\pi_i^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) + \varepsilon_t \]

(I \leq 0 \quad \text{or} \quad w_{1,t} \leq -\pi_1^- - \Pi_1 X_t)

\[ r_{am,t} = \pi_2^+ + \Pi_2 X_t + \frac{W_{12}}{\sqrt{W_{11}}} \phi \left( \frac{-\pi_i^+ - \Pi_1 X_t}{\sqrt{W_{11}}} \right) + \phi \left( \frac{-\pi_i^- - \Pi_1 X_t}{\sqrt{W_{11}}} \right) + \varepsilon_t \]

(I > 0 \quad \text{or} \quad w_{1,t} > -\pi_1^+ - \Pi_1 X_t)

\[ r_{am,t} = \Psi_2 + \Psi_2 X_t + \frac{\text{cov}(u_{am,t}, w_{1,t})}{\sqrt{W_{11}}} \phi \left( \frac{-\psi_i^- - \Psi_1 X_t}{\sqrt{W_{11}}} \right) + \phi \left( \frac{-\psi_i^- - \Psi_1 X_t}{\sqrt{W_{11}}} \right) + \varepsilon_t \]

(I = 0 \quad \text{or} \quad -\pi_i^- - \Pi_1 X_t \leq w_{1,t} \leq -\pi_i^+ - \Pi_1 X_t)

where \( \text{cov}(u_{am,t}, w_{1,t}) = E(u_{am,t}(u_{1,t} + \delta_a u_{am,t})(1 - \beta \delta_a)) = (\omega_{12} + \delta_a^2 \omega_{22})/(1 - \beta \delta_a)^2 \). One can insert the consistent estimates of \( \pi_1^+ \), \( \pi_1^- \) and \( W_{11} \)—obtained by estimating the reduced form for the sets T1 and T2—in the above expressions and estimate them by least squares, using the T1, T3 and T2 observations, respectively, to get \( \hat{\psi}_2 \), \( \hat{\Psi}_2 \), \( \hat{\pi}_2^+ \), \( \hat{\pi}_2^- \), and \( \hat{\Pi}_2 \).

### A.2 Identifying the Structural Parameters

The following summarizes the relation between the structural parameters and the reduced-form parameters:

\[
\begin{bmatrix}
A_1 \\ A_2
\end{bmatrix} =
\begin{bmatrix}
1 \\ -\beta
\end{bmatrix}
\begin{bmatrix}
\Pi_1 \\ \Pi_2
\end{bmatrix}
\]
Recalling that we are treating $r_{pmt}$ as a predetermined variable and that we obtain $\delta_p$ and $\rho$ as part of $A_1$ and $A_2$, the structural parameters to be estimated are $\{\beta, \delta_a, a_1, a_2, A_1, A_2, \text{ and the } 3\}$ elements of $\Omega$. Maximizing (A.18) provides estimates of $\pi_1^+, \pi_1^-, \Pi_1$ and $W_{11}$. Estimation of (A.25) to (A.27) provides $\hat{\psi}_2, \hat{\Psi}_2, \hat{\pi}_2^+, \hat{\pi}_2^-$, and $\hat{\Pi}_2$, as well as $W_{12}$. Implicitly, these can be used to construct $W_{22}$. The estimates of $\{\psi_2, \Psi_2\}$—which equals $\{a_2, A_2\}$—can be used to solve for $\beta$ using the relation implied by solving for $A_1$ and $A_2$ in the structural $\{T1,T3\}$ equations:

$$\pi_2^+ - \beta \pi_1^+ = a_2, \quad \Pi_2 - \beta \Pi_1 = A_2. \quad \delta_a$$

is the ratio of the appropriate elements of $\Pi_1$ to $\Pi_2$, because of the restriction on $A_1$, that the macro announcement has no direct effect on intervention. Equation (A.29) can then be solved directly to provide $\hat{a}_i$ and $\hat{\beta}$. An estimate of $A_1$ can be derived from (A.28) with $\hat{\delta}_a$.

Now, with consistent estimates of $A_1$, $\delta_a, A_2$ and $\beta$, we can use the reduced form in set $\{T1,T3\}$ to solve for the elements of the lower $\{2,3\}$ submatrix of the structural covariance matrix, $\Omega$, as follows:

21 The restriction on $A_1$ that identifies $\delta_a$, also permits us to take a more direct route to identifying the rest of the structural model than did Sickles and Schmidt (1978). They studied a similar 2-equation framework with a Tobit model, rather than a friction model. Usefully, they showed that the nonlinearity of the model always identifies the parameters of the second equation—the analogue of the $r_{am,t}$ equation in this paper—without restricting $\Omega$ or $A_2$. 

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Thus, we have obtained consistent estimates of all the structural parameters.

To summarize, the procedure for estimating and identifying the model described by (A.1) to (A.6) is given by the following:

1. Estimate (A.5) by least squares, directly estimating $A_0$ and $\Omega_{00}$. Because $r_{pm,t}$ can be treated as a predetermined variable in the $I_t$ and $r_{am,t}$ reduced forms, the contemporaneous coefficients $\delta_p$ and $\rho$ can be treated as elements of $A_1$ and $A_2$.

2. Estimate the reduced form of the $I_t$ equation in (A.18) by maximum likelihood, using all observations, to get $\hat{\pi}_1$, $\hat{\Pi}_1$ and $W_{11}$.

3. Estimate (A.25), (A.26) and (A.27) by least squares, using the consistent estimates—$\hat{\pi}_1$, $\hat{\Pi}_1$ and $W_{11}$—in the regressors on the right-hand side. (A.25) and (A.26) are estimated jointly.

These regressions provide $\hat{\pi}_2$, $\hat{\Pi}_2$, $\psi_2$, $\hat{\Psi}_2$ and $W_{12}$. Note that $\psi_2 = \hat{a}_2$ and $\hat{\Psi}_2 = \hat{A}_2$.

4. The restriction on $A_I$—that that the macro announcement has no direct effect on intervention—identifies $\delta_a$ from the ratio of the appropriate elements of $\hat{\Pi}_1$ and $\hat{\Pi}_2$.

5. Using $\hat{\delta}_a$, $\hat{\Pi}_1$ and $\hat{\Pi}_2$, one can solve for $\hat{\beta}$ and $\hat{A}_1$ with (A.28).

6. Solve for $\hat{\Omega}$ using previously estimated parameters—$\hat{\delta}_a$, $\hat{\beta}$ and $\hat{W}$—in (A.32).

These iterative estimates are then used as starting values in maximum likelihood estimation.


Neely, Christopher J. “Identification Failure in a Structural VAR with Intervention: Alternative Identification and Estimation.” unpublished manuscript, the Federal Reserve Bank of St. Louis, 2005b.


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### Notes:
The table provides summary statistics—mean, standard deviation, autocorrelations—on U.S. intervention (purchases of billions of USD), day and night log returns to the DEM/USD and JPY/USD rates. The sample period is 1987 through 1990.
Table 2: Correlation among endogenous variables

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<th>DEM statistics</th>
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Notes: The table shows correlation coefficients among the endogenous variables of the two data sets over the sample period: 1987 through 1990. The variables are intervention (I)—U.S. purchases of billions of USD—business day returns ($r_{am}$), and evening returns ($r_{pm}$).
Figure 1: Intervention, exchange rates and PPP fundamentals

US Purchases of USD in the DEM Market

US Purchases of USD in the JPY Market

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Figure 2: Hours of reported intervention

Notes: The figure shows times of Reuters’s reports of U.S. intervention between 1987 and 1995. Times are in New York time and are rounded down to the nearest half hour. The figure is similar to one reported in Dominguez (2003a).
Figure 3: Dynamic impact of intervention on exchange rates in the linear model

Notes: The figure shows the dynamic impact of a one-standard-deviation purchase of USD on the DEM/USD rate (top panel) and the JPY/USD rate (bottom panel), along with an 80 percent confidence interval. The x-axis denotes business days after the impact. Solid horizontal lines denote zero.
Figure 4: Impact of returns on intervention in the linear model

Notes: The figure shows the dynamic impact of a one-standard-deviation increase in exchange rates (foreign currency per USD) on U.S. intervention (USD purchases). The top panels show DEM results while the bottom panels show JPY results. Dashed lines denote 80 percent confidence intervals. The x-axis denotes business days after the impact. Solid horizontal lines denote zero.
Figure 5: Impact of shocks to intervention on exchange rate returns/levels in the friction-normal system

Notes: The figure shows the dynamic impact of a 3-standard-deviation shock to intervention on levels in the friction-model system, along with 80 percent confidence intervals, computed by the delta method. Results from DEM/USD are in the first two panels and those from the JPY/USD are in the bottom two panels. Panels on the right restrict $\omega_{12}$ to equal zero. The x-axis denotes business days after the impact. Solid horizontal lines denote zero.
Figure 6: Impact of returns on intervention in the friction-normal system

Notes: The figure shows the friction-model-implied dynamic impact of a 3-standard-deviation shock to business-day exchange rate returns on intervention, along with 80 percent confidence intervals, computed by the delta method. Results from DEM/USD are in the first two panels and those from the JPY/USD are in the bottom two panels. Panels on the right restrict $\omega_{12}$ to equal zero. The x-axis denotes business days after the impact. Solid horizontal lines denote zero.
Figure 7: The likelihood surface for the friction model as a function of $\beta$, $\delta_{am}$ and $\omega_{12}$

Notes: The figure shows the likelihood surface for the friction model as a function of $\beta$, $\delta$ and $\omega_{12}$. The first (third) row shows the likelihood surface for the DEM (JPY) as a function of $\{\delta, \beta\}$, $\{\delta_{am}, \omega_{12}\}$ and $\{\beta, \omega_{12}\}$, respectively. The second and fourth rows show the p-values for the likelihood ratio tests that the parameter pairs are significantly different than the value of the parameters that maximize the likelihood function.