Deposit Relationships and Bank Portfolio Selection

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Deposit Relationships and Bank Portfolio Selection

R. Alton Gilbert

Based upon a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Graduate School of Texas A&M University

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ABSTRACT

Deposit Relationships and Bank Portfolio Selection
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In most models of bank portfolio selection, relations between the sources of bank funds and the way the funds are invested are ignored. A model is developed in which a bank competes for some of its deposits by offering loans to the depositors who demand loans at interest rates lower than the rates charged other borrowers. In a competitive banking industry, depositors will be offered better loan terms than non-depositors. The bank portfolio selection model is used to determine the effects of a tight money policy on the bank loan market. Under the assumptions used in constructing the model, bankers discriminate in favor of the borrowers who hold demand deposits at their banks as monetary policy becomes more restrictive.

Banks are prohibited from paying interest on demand deposits. Since banks can increase their interest earning assets by increasing deposits, bankers have an incentive to compete for demand deposits by indirect means. One way to compete for demand deposits is to offer loans at reduced interest rates to the depositors who demand loans. The bank's revenue from business with a customer includes
the interest payment on his loan and the interest income from investing his deposit. The opportunity cost of establishing such a relationship with a depositor is the market rate of interest times the loan to the depositor. Competitive banks reduce the loan interest rate for depositors until the profit from the total business with the depositor is zero.

Bank portfolio selection models are used in analyzing the optimal allocation of a bank's funds among a few groups of assets. In such studies there are generally random variables, such as demand deposit withdrawals, and constraints on the bank, such as the reserve requirement, and the banker maximizes some objective function. A model is developed in which depositors withdraw their deposits only if their loan demands are refused. Depositors are offered loans at the prime rate, and with a given deposit, the maximum prime rate loan offered is the one that makes total bank profits from business with the depositor equal to zero. The one random variable in the model is the demand for loans by depositors. The banker reacts to this source of uncertainty by allocating a part of the bank's funds to an inventory for loans to depositors. The banker's objective is to minimize the expected loss from holding the inventory for loans to depositors. If the loan demand by depositors is less than the inventory, the excess is invested in liquid securities. If the loan demand by depositors is greater than the inventory, the banker either borrows the funds needed to make the loans or refuses to make some loans and loses deposits.
The bank portfolio selection model is used to investigate the impact of a restrictive monetary policy on the interest rates banks charge depositors and non-depositors and the allocation of loans among these two groups of borrowers. The conclusions are based upon the condition that the share of deposits owned by the depositors who demand loans increases as monetary policy becomes more restrictive. Given that the demand for bank loans by non-depositors is not perfectly elastic, the difference between the interest rates charged non-depositors and depositors increases and the share of bank funds going to depositors increases as monetary policy becomes more restrictive. Hypotheses drawn from the model are not tested; the appropriate data on the demand deposit balances of borrowers are not available.
ACKNOWLEDGMENTS

Suggestions from members of my graduate committee have improved the organization and presentation of this dissertation. Dr. Thomas Saving has helped me a great deal in developing the ideas in this dissertation and presenting them in an acceptable form. Donald Hooks read early drafts of this paper and had useful comments. The Research Department of the Federal Reserve Bank of St. Louis has given me time to work on this dissertation and provided typing and copying facilities.
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CHAPTER I

BANK ASSET SELECTION

1.1 Introduction

Commercial banks have an important role in the transmission of monetary policy to economic activity. One approach to the study of this role is that of developing and testing hypotheses based upon portfolio selection models. The effects of changes in a bank's environment on the allocation of its funds can be analyzed with these models; the banker's environment includes rates of return on assets, risks, constraints, such as the required reserve ratio, and his anticipations of future events. Implications can be drawn from such models about the influence of monetary policy on the allocation of bank assets among various categories of assets.

1.2 The Nature of Bank Portfolio Selection Models

Bank portfolio selection models has been developed by specifying a few asset classes among which bank assets are allocated; assuming an objective function for the banker, such as expected bank wealth maximization or maximization of a risk averter utility function; and identifying constraints and sources of uncertainty. With a given

The citations on the following pages follow the style of the American Economic Review.
rate of return for each asset class, probability distributions for variables about which the banker is uncertain, and constraints, the optimal allocation of bank funds among the asset classes can be determined. Rates of return are assumed to be uniform within each asset class and to be net of repayment risk. With such models, the reaction of bankers to monetary policy changes can be predicted.

The bank portfolio selection models that have been published generally have either two or three asset classes. A common asset classification scheme for the models with two asset classes is reserves and interest earning assets; these models are constructed to predict the reserve ratio. A common asset classification scheme for models with three asset classes is reserves, securities, and loans. In these models, securities are sold to pay for demand deposit withdrawals and earn a lower rate of return than loans; loans are assumed to be illiquid.

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1 The role of repayment risk in the bank asset selection literature is discussed in section 1.3.2.

2 Morrison (1966); Orr and Mellon (1961).

3 Kane and Malkiel (1965); Pierce (1964), (1967), Porter (1961); Russell (1964).

4 Although there is a price at which loans can be sold, the percentage loss in value for loans is so much greater than for securities if they are sold on short notice that selling loans is disregarded as a source of cash to pay for deposit withdrawals.
1.3 The Differential Treatment of Borrowers

1.3.1 Assumptions in Bank Portfolio Selection Models

In order to keep the number of variables in a bank portfolio selection model manageably small, the number of asset classes must be small, and much of the detail of banking operation must be ignored. The following two assumptions are made either implicitly or explicitly in the development of most of the models discussed above:

(a) Loans are assumed to be homogeneous assets which have a uniform rate of return; no reason for differential treatment of borrowers are assumed in the bank's objective function or the bank's constraints.\(^5\)

(b) The amount of the bank's funds for investment, deposits plus bank capital, is independent of the way in which these funds are invested; the objective of the banker is to invest a given amount of funds in such a way as to maximize the bank's objective function.\(^6\)

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Pierce defends this approach to the study of bank portfolio selection by assuming that the decision as to the share of the bank's funds for each asset class, with loans as one of those asset classes, can be separated from the decision as to the types of assets purchased within each asset class.\(^7\)

The model developed by Kane and Malkiel is an exception to the statements made above about portfolio selection models.\(^8\) In their model, bankers attract depositors by offering them special terms on bank loans. The quality of relationships between a bank and some of its depositors depends upon whether or not the loan demands of the depositors are met, and the quality of these relationships affects the profitability and risk of the bank.

A portfolio selection model is developed in Chapter III in which the amount of assets a bank has to invest depends upon whether or not the bank meets the loan demands of some of its depositors who bargain for implicit interest on their deposits in the form of interest rates on loans below those charged non-depositors. Some conclusions are drawn from the model about the influence of monetary policy on the differential treatment of depositors and non-depositors as two groups of borrowers by an expected profit maximizing banker.

\(^7\)Pierce (1967, p. 1972).

\(^8\)Kane and Malkiel (1965).
Some sources in the banking literature that discuss differential treatment of borrowers are summarized in this chapter. The topics summarized below are (i) credit rationing due to risk, (ii) the role of credit rationing in the availability doctrine, and (iii) the differential treatment of large and small borrowers during periods of restrictive monetary policy. These summaries are followed by a discussion of the differential treatment of borrowers who are depositors of the lending bank and borrowers who are non-depositors. These models of bank asset selection are compared in Chapter V.

1.3.2. Bank Credit Rationing Due to Risk

Credit is rationed to a borrower if he has an excess demand for loans at the rate at which he can borrow and if he cannot increase the size of his loan by offering to pay a higher rate of interest. The effects of risk on the supply function of loans to a borrower by a bank can be analyzed by deriving a loan offer function based upon the bank's objective function and a probability distribution of the terminal value of the borrower's investment of the loan. Models that present the loan offer function of a banker in this way have been developed by Hodgman, Freimer and Gordon, and Jaffee and Modigliani.  

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9Hodgman (1960).

10Freimer and Gordon (1965).

The loan supply model of Freimer and Gordon is presented as representative of the loan supply functions in the credit rationing models. The borrower invests $Y$ of the bank's funds for a given period of time in a risky venture that has a terminal value of $X$. The banker forms a probability distribution $f(X)$ for the terminal value $X$, $\infty \geq X \geq 0$. Under the most favorable conditions, the borrower's total payment to the bank is the principle of the loan plus contracted interest payments, and the liability of the borrower is limited to the returns from the investment. Let $r$ be the interest rate the borrower promises to pay and $i$ the opportunity cost to the banker of using the bank's funds to make this loan. The expected profit to the bank from this loan is:

$$P = \sum_{X=0}^{\infty} Xf(X)dX + \frac{Y(1+r)}{1+i} - \frac{Y(1+i)}{1+i}$$

For a given rate $r$, there is a loan size $\hat{Y}$ that maximizes the expected profit from this loan. There are conditions under which the loan size offered by the profit maximizing banker stops increasing as the interest rate charged the borrower increases.

Hodgman and Freimer and Gordon concentrate on the construction of loan supply functions to individual borrowers. Jaffee and Modigliani stress the role of demand functions for loans as well as supply
functions in the determination of loan interest rates in their discussion of credit rationing; the analysis by Jaffee and Modigliani is summarized in the rest of this section. The discriminating monopolist would charge all borrowers the interest rate at which the size of the loan demanded equals the size of the loan supplied. Jaffee and Modigliani assume that banks collude by setting up a small number of classes of borrowers. Borrowers within each class are charged the same rate, and if the demand functions and risk characteristics differ among borrowers within the classes, some borrowers may have credit rationed to them. To enforce such a policy, banks would have to acquire information about the demand function for credit of each borrower, and banks would have to have complete cooperation in setting interest rates. Jaffee and Modigliani show that a banker will not ration credit to a risk free borrower.¹²

1.3.3 Bank Credit Rationing in the Availability Doctrine

According to the availability doctrine, monetary policy affects economic activity by changing the amount of funds lenders make available to private (non-government) borrowers. Monetary policy is assumed to be implemented through open market operations. A restrictive monetary policy has two effects on lenders: (a) the rise in the rate of return

on government securities relative to the rate of return on private securities makes buying government securities a more attractive investment than before, and (b) a restrictive monetary policy causes the risk attached to private securities to rise as expectations about future economic activity change.\textsuperscript{13}

In Scott's exposition of the availability doctrine, a restrictive monetary policy can be effective if open market operations increase the risk of return on government securities with the risk of return on private securities unchanged. Private securities are riskier investments than government securities. Banks are assumed to have a maximum risk tolerance for the bank's portfolio of assets and are assumed to have chosen a combination of assets that has such a level of risk. If the risk on government securities increases and the risk on private securities remains unchanged, banks must increase the share of government securities in their assets in order to remain within the risk tolerance.\textsuperscript{14}

The effectiveness of monetary policy has been challenged on the grounds that demand functions for loans are inelastic. The following statement by Scott gives the position of advocates of the availability doctrine in this debate.

\textsuperscript{13} Wallich (1946, p. 765); Roosa (1951); Scott (1957a).

\textsuperscript{14} Scott (1957a)
The availability doctrine has been cited in defense of monetary policy when the latter's efficacy has been questioned on the ground that borrowers may be insensitive to the cost of borrowing. According to the availability doctrine, a restrictive monetary policy may cause a reduction in the quantity of credit supplied private borrowers by private lending irrespective of the elasticity of demand for borrowed funds.\footnote{Scott (1957a, p. 41)}

A restrictive monetary policy causes the supply curves of funds to private borrowers to shift to the left according to advocates of the availability doctrine. A restrictive monetary policy must restrict the supply of funds to private borrowers if it is to be effective. If the demand curves for borrowed funds by private borrowers are extremely inelastic, and if supply curves of funds from lenders to private borrowers are not perfectly inelastic,\footnote{Scott (1957b, p. 536).} what must be explained is why the private borrowers cannot keep the level of funds supplied to them nearly the same by bidding up the interest rates they pay. Credit rationing by lenders to private borrowers is the explanation offered.\footnote{Ellis (1951, pp. 255-56); Kareken (1957, pp. 301-2).} The administration of monetary policy through credit rationing by lenders is discussed in the following two statements in the availability doctrine literature.

Such successes as monetary policy has scored in the past appear to have been due more to the rationing of credit than to changes in its cost.\footnote{Wallich (1946, p. 769).}

The 'equilibrium' or market solution is ... in general not the competitive solution. But the "equilibrium" point will of course lie on the supply curve,
at least in an inflationary situation when there is likely to be excess demand, since it is the lenders who administer the price. 19

Unfortunately, these statements are not supported with evidence. Imperfections in the bank loan market due to an oligopolistic or monopolistically competitive market structure keep the interest rate charged on loans from being the rate which just equates market supply and market demand. 20 There is no clear agreement among the contributors to this literature about whether the imperfections in the bank loan market exist at all times or only during the periods of restrictive monetary policy. The following statement by Paul Samuelson to the Patman Committee in 1952 expresses a widely held view of the functioning of the bank loan market.

According to this argument, if you change the terms of Government bonds ... what [the banker] will do is not post a sign outside his doors saying I am going to raise my interest charges. But, on the contrary, for a while at least, he might hold the same interest charges, but he is going to be more choosy in that margin of to whom he makes a loan. In other words, he rations out credit.

.................................................................

Now it is unthinkable that over a period of time, of a few months, let us say, or of over a year, or more than a year, that a banker should act so irrationally that when credit is scarce he will hold his rates perfectly inflexible. ...

.................................................................

19 Kareken (1957, p. 301).

What I mean is the following: the tightness of rationing that the central banker can induce by his ordinary operations will disappear after a few months and be replaced by a firming of interest charges and a return to normal stringency of rationing. ...

...rationing and discretionary decisions will always characterize the loan market in the short run and in the long run. 21

In his graphical presentation of the bank loan market, Kareken assumed that there were imperfections in the market that keep the loan rate below the equilibrium rate at all times; a more restrictive monetary policy causes an increase in the level of excess demand in the market for bank loans. 22

In graphical representations of the bank loan market by Kareken and Scott, 23 there is one loan demand curve and one interest rate on bank loans. With this approach the bank rations credit to borrowers if there is an excess demand for bank loans at the one bank loan rate. But if a bank has reasons for treating groups of borrowers differently, the bank rations credit if there is excess demand by each of these groups of borrowers at the loan rates quoted to them. 24 In the model developed in Chapter III there are two groups of borrowers: (1) the deposit customers, borrowers who have deposits at the lending bank, and (2) non-depositors. An excess demand for loans

21 Samuelson (1952, pp. 695-97).
22 Kareken (1957, p. 301).
23 Kareken (1957, p. 296, p. 301); Scott (1957 b, p. 536).
24 Jaffee and Modigliani (1969, p. 859) consider the implications for credit rationing of dividing borrowers into a small number of groups, they concluded that such actions must be based upon an oligopolistic organization of the bank loan market.
at the rate charged deposit customers does not necessarily mean that the banker is rationing credit. The higher interest rate charged non-depositors may clear the market for bank loans to non-depositors. See sections 2.5 and 2.6 for a discussion of why competitive bankers offer loans to deposit customers at lower interest rates than the interest rate charged non-depositors for loans.

The role of credit rationing due to risk has not been integrated into the availability doctrine view of credit rationing. Jack Guttentag developed a model for simultaneously determining the equilibrium interest rate and degree of risk on loans.\textsuperscript{25} He assumed a functional relation between the degree of risk on loans and a risk premium on the loans which is high enough to compensate for the risk. But if risk affects bank loan supply functions as analyzed in section 1.3.2., risk increases as the size of the loan increases, and beyond some level of risk, there is no increase in the loan interest rate that can compensate the bank for an increase in risk. The link that does exist between risk on loans and the credit rationing of the availability doctrine is that when bankers are rationing credit, loans are made to the less risky borrowers.

1.3.4. Empirical Investigations of Credit Rationing

Donald Hester was able to test hypotheses about the existence of credit rationing during the 1955-57 period of restrictive monetary policy. The results of his study are stated in the following quote.

\textsuperscript{25}Guttentag (1960).
Analysis of commercial and industrial loans revealed that borrowers of any particular size could obtain loans of at least the same maturity, the same amount, and with the same likelihood of security in 1957 as in 1955. If risk is associated with longer maturity, greater amount, or no security, we have the result that identically risky loans could be obtained by borrowers of given means, as measured by their total assets, regardless of the level of competing interest rates. There was no credit rationing.  

D. Jaffee and F. Modigliani established that a bank will not ration credit to a risk free borrower. Their measure of credit rationing is the fraction of loans made to risk free borrowers. Without considering the influence of customer relationships on the granting of prime rate loans, the risk free borrowers were assumed to be the prime rate borrowers, and the percentage of loans at the prime rate was used as a measure of credit rationing. Note that their approach to measuring credit rationing is based on the assumption that borrowers at rates above the prime rate have credit rationed to them. The percentage of loans at the prime rate is given a different interpretation in the model developed in Chapter III.

1.3.5. The Differential Effects of Tight Money

Several studies have analyzed the issue of whether restrictive monetary policy increases the difficulty of small business borrowers

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26 Hester (1967, p. 167).

27 Wojnilower and Speagle (1968, pp. 256-57).

relative to that of large business borrowers in obtaining bank loans. Three such studies are based upon data from the 1955-57 period of increasingly restrictive monetary policy; information from this period was used because borrowers were identified by the size of their loans in bank reports.

The Federal Reserve System made a study in 1958 of the allocation of bank loans during the 1955-57 period between large and small borrowers. The conclusion was that:

...monetary restraint may have contributed, to some extent, to the slower growth in loans to small than to large businesses within the various size classes of banks, particularly the larger banks, although the evidence from the loan surveys alone is not sufficient on this point. 29

In a study of G.L. Bach and C.J. Huizenga based upon the same 1955-57 loan survey data, banks were divided into groups of loose, medium, and tight banks. They found that the share of loans to large borrowers increased more at loose banks than at tight banks. With this as a test for banker discrimination, they concluded that there was no discrimination against small borrowers during this period. 30

William Silber and Murray Polakoff used the same data to show the existence of banker discrimination by the size of the borrower

29 Eckert (1958, p. 404).
30 Bach and Huizenga (1961).
under their indicator of discrimination. They assumed that banks have loan offer functions for loans to small borrowers as one group and large borrowers as another group. The dollar amounts of loans a bank has in each category of loans are assumed to be functions of (a) the interest rates charged the two groups of borrowers, (b) the average maturity and security of the two groups of loans, (c) total deposits of the bank, and (d) the ratio of demand to time deposits of the bank. Let $L^s$ be the dollar amount of loans to small borrowers, $L^b$ the dollar amount of loans to large borrowers, and $D$ total deposits at the bank. Then $\delta L_s/\delta D$ and $\delta L^b/\delta D$, indicate the allocation of an extra dollar of deposits between these two groups of borrowers. If there was no change in the treatment of large and small borrowers, there would be no reason for $(\delta L^b_{55}/\delta D)/(\delta L^b_{57}/\delta D)$ to be different from $(L^s_{55}/\delta D)/(\delta L^s_{57}/\delta D)$ (the subscripts 55 and 57 denote the years 1955 and 1957, respectively). Since the ratio was smaller for larger borrowers, they concluded that there was discrimination by banks against small borrowers as monetary policy became more restrictive from 1955 to 1957.

There is no discussion in these three studies of the reasons why banks would discriminate against small borrowers. If the hypothesis that discrimination of bankers against small borrowers increases as monetary policy becomes more restrictive is to be based upon economic theory,

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some criterion for differential treatment of borrowers by size of loan must be a part of the bank's objective function, cost function or constraints. In the article by Silber and Polakoff the bank offer functions for loans to large and small borrowers were not derived from a bank asset selection model but were simply assumed to exist in the forms given. In the Federal Reserve study, the differences in the interest rates charged large and small borrowers were related to differences in bank costs per dollar of loan. But the costs of lending to large borrowers relative to the cost of lending to small borrowers would have to change during a period of restrictive monetary policy to account for a change in the differential treatment of borrowers in response to monetary policy. The conclusions drawn from such studies about bank discrimination by size of borrower cannot be expected to hold in other periods unless

---

32 Eckert (1958, p. 406). Studies by Benston (1964), Landadio (1963), and Levenson (1962) investigate bank cost differences in making loans to large and small borrowers as a partial explanation for differences in the interest rates banks charge borrowers. The empirical work uses cross-section data, and the changes in the costs of making loans to large borrowers relative to the costs of making loans to small borrowers in response to monetary policy are not considered in these studies. H. Peter Gray (1963) considers the implications of bank cost changes over time for the effectiveness of monetary policy. Except for time deposit costs, the role of secular changes in bank costs is stressed, and he does not consider the implications of changes in bank costs for the interest rates charged large and small borrowers.
reasons are given for such discrimination.

One defense of an empirical study of bank discrimination among borrowers without theoretical foundation might be that changes in the treatment of borrowers in response to monetary policy can perhaps be predicted with enough study of the patterns in past periods. But such a predictable pattern might hold only under presently existing monetary institutions. The impact of changing these institutions for the differential treatment of borrowers can be predicted only if the differential treatment of borrowers is a conclusion drawn from a bank asset selection model which can reflect the institutional arrangements. One institutional arrangement with implications for the bank loan market is the prohibition of interest payments on demand deposits. The implications of this restriction on the form of competition among banks for the differential treatment of borrowers are developed in Chapter IV. See Chapter V for a contrast of the differential treatment of borrowers considered in this section and that developed in Chapter IV.

1.4 The Deposit Relationship and Bank Portfolio Selection

1.4.1 The Nature of Deposit Relationships

Banks are assumed to compete for those depositors who demand loans by offering them loans at rates below the loan rates offered other borrowers, the non-depositors. Other depositors demand their implicit interest on their demand deposits in the form of bank
services. A bank sacrifices the greater revenue that could be earned by making the loans to non-depositors because of the profitability of investing the deposits he can attract by his loan policies. 33 With this type of competition among banks for deposits, a differential treatment of borrowers exists; the two groups of borrowers are called deposit customers and non-depositors.

The prime rate is generally accepted as a floor on the loan rates offered to depositors. 34 With a given deposit size, there is a maximum loan the banker will make at the prime rate. A non-depositor is a borrower who does not have a demand deposit balance where he borrows with which to bargain for special terms. Since most non-depositor borrowers have deposits in some bank, why would they go to another bank for a loan? To make this distinction between borrowers appear more rational, a non-depositor is assumed to either have too small a demand deposit to make it worth the banker's trouble to establish a deposit relationship or to have a demand for bank loans which is larger than the maximum loan the banks offer the borrower as a deposit customer. The borrower enters the loan market with the rest of his loan demand under the same conditions as a borrower who owns no demand deposit balance. So the term non-depositor is used in the sense that the borrower

33 A model of the profitability of a deposit relationship to a bank is developed in section 2.5.

34 Wojnilower and Speagel (1968, p. 256).
has no deposit at the lending bank on which to bargain for interest rate reductions on a particular loan. One borrower may make loans both as a deposit customer and as a non-depositor.

1.4.2. The Effects of Deposit Relationships on Bank Asset Selection: Some Evidence

Donald Hester has estimated commercial bank offer functions for both term loans and commercial and industrial loans. His objective was to isolate those characteristics of borrowers which significantly influenced the size, maturity and interest rates of loans offered by banks:

An applicant's profits, the ratio of his current assets to current liabilities, his deposit balances, the number of years which he was a depositor at the bank, his profit rate, his location, and perhaps his total assets appear to influence significantly the terms at which banks lend.\(^{33}\)

With a sample of 48 borrowers at one large commercial bank, James Cooper found that, for prime rate borrowers, the size of the loan was significantly related to the number of banks at which the borrower had relationships and the profitability of the deposit relationship during the past year. Using the full sample, the rate of interest on loans was significantly related to the same two variables. The larger the number of banks at which a customer had relationships and the greater the past profitability of the deposit

\(^{35}\) Hester (1967, p. 165).
relationship to the bank, the larger the loan the customer gets and the smaller the interest rate he is charged. Murphy found that the closer the relationship between Massachusetts townships and banks buying their tax anticipation notes, the greater the difference between the rate the bank charged and the open market rate for tax anticipation notes of the same risk rating, size and maturity. Interviewers of bankers conclude that the nature of customer relationships is one of the most important factors in the allocation of bank funds during periods in which monetary policy is becoming more restrictive.

Benjamin Klein (1970) has tested the hypothesis that commercial banks pay interest on demand deposits as though the law did not restrain them from doing so against the alternative hypothesis that banks pay no interest on demand deposits. A money demand function which includes an interest rate on demand deposits was found to have a better fit than estimated money demand functions which exclude that variable. Klein concludes that the hypothesis that banks are not impeded from paying interest on demand deposits is more nearly correct than the hypothesis that they do not pay interest.

Donald Jacobs (1970) has estimated the influence of several variables involved in bank-customer relationships on the interest rates the

36 Cooper (1967, pp. 105-22).
37 Murphy (1969, pp. 60-67).
38 Hayes (1964, pp. 227-28); Hodgman (1963, pp. 21-33).
customers are charged on loans. Variables which are found to have statistically significant influences on loan interest rates include the average deposits of customers, fluctuations in their deposits, and the length of the relationship between the bank and the customer.

1.4.3 A Preview of Chapters II, III, and IV

In Chapter II the conditions under which it is to the advantage of both the banker and borrower to establish a deposit relationship are discussed, and a model for the profitability of a deposit relationship to the bank is developed in terms of interest rates and the customer's loan to deposit ratio.

In Chapter III the competition among banks for establishing deposit relationships is incorporated into a bank portfolio selection model. For convenience of exposition, all loans to deposit customers are assumed to be made at the same interest rate, called the prime rate. Since the bank attracts and retains some deposits by making prime rate loans to depositors, the bank loses deposits if the loan demands of depositors are refused and if other banks will meet the loan demands. As explained in the first part of this chapter, an important reason for developing bank portfolio selection models is to show the effects of uncertainty on bank asset selection. The random variable in the portfolio selection model of Chapter III is the demand for loans by deposit customers. Competition forces banks to offer depositors maximum prime rate loans which are some multiple of their deposits; a bank need not worry about the consequences of refusing a prime rate
loan demand which is larger than this maximum since no other bank will offer a larger prime rate loan to the borrower. The bank reacts to this source of uncertainty by allocating a part of its assets to an inventory for loans to deposit customers which is some fraction of the maximum prime rate loan commitments to its depositors, and the bank borrows reserves from other banks in order to make loans to deposit customers if the demand for loans by these customers is greater than the inventory of bank deposits held for making these loans.

An objective of the bank is to allocate that share of its assets to the inventory for loans to deposit customers which minimizes the expected loss of holding the inventory. If the inventory is too large, earnings on higher yielding loans to non-depositors are foregone. If the inventory is too small, the bank will have to refuse deposit customer loan demands which other banks will meet, and those depositors who have their loan demands turned down withdraw their deposits from the bank.

The four asset classes among which the bank allocates its assets are loans to non-depositors, loans to deposit customers, securities, and reserves; excess reserves are assumed to be zero. The inventory for loans to deposit customers equals loans to deposit customers plus securities, and since the demand for loans by deposit customers is assumed to be a random variable, the quantity of loans to deposit customers is partially determined by the deposit customers. An increase in the share of the bank's funds in the inventory for loans to deposit customers is a decrease in the share invested in loans to non-depositors. The
optimal share of the bank's funds in the inventory for loans to deposit customers is a function of the prime rate paid by deposit customers and the interest rate paid by non-depositors.

The differential treatment of deposit customers and non-depositors in this model is affected by monetary policy. Rational behavior by depositors is shown to lead to an increase in the share of deposits held by those who demand their implicit interest on deposits in the form of loans at reduced interest rates as monetary policy becomes more restrictive. The difference between the interest rates paid by deposit customers and non-depositors must increase if the optimal share of the bank's funds in the inventory for loans to deposit customers, and therefore, the share in loans to non-depositors, is to remain unchanged.

Differential treatment of groups of borrowers occurs if borrowers in one group must satisfy a different set of requirements to get loans than borrowers in another group. Since risk is ignored in the portfolio selection model of Chapters III and IV, such non-price bank requirements as security for loans are not relevant. The only distinction between borrowers in that model is the interest rate they pay, and the difference between the interest rates charged borrowers is a measure of their differential treatment. If the difference between the interest rates charged non-depositors and deposit customers increases, it has become relatively more difficult for non-depositors to get loans.

In his article entitled "The Deposit Relationship and Commercial Bank Investment Behavior," Hodgman discussed the effects of a restrictive monetary policy on the way banks allocate their loan funds between deposit customers and non-depositors. He concluded that
deposit customers get first claim on the bank's funds during a period of restrictive monetary policy; this position is expressed in the following quote:

It would be a foolish banker, indeed, who grasped at temporarily high yields at the expense of losing important long-term deposit accounts. ... there is no feasible increase in interest for such loans (to non-depositors) which would make them preferable to those which the banker expects to make to this preferred deposit customer in a period of increasing credit demand and increasing stringency. 39

Unfortunately, this conclusion is not a deduction from a bank portfolio selection model. Without assumptions about the bank's objective function, constraints and sources of uncertainty, there is no way to determine what increase in interest rate would induce the banker to switch funds from loans to deposit customers to loans to non-depositors.

The methodology in this paper is different from that used by Hodgman. In Hodgman's analysis of bank profits from a deposit relationship, the loan demand by the deposit customer is assumed known, and the decision of the bank that he analyses is whether or not to make a loan to an individual deposit customer. The demand for loans by deposit customers is a random variable in Chapters III and IV, and the profitability of investing in loans to deposit customers as a type of asset is the profitability of investing in an inventory for loans to deposit customers. By assuming (1) expected profit maximization to be the banker's objective, and (2) a specific distribution of the demand for loans by deposit customers, the increase in the interest rate on loans to non-depositors that would keep the share of bank funds in the

inventory for loans to deposit customers from rising as monetary policy becomes more restrictive can be predicted.

Goldfeld discussed the role of the bank-customer relationship in his book *Commercial Bank Behavior and Economic Activity*. In his model of bank asset selection the banker is assumed to make all of the loans demanded by deposit customers. The choice variable relating to loans in his model is the share of the bank's funds in all loans. The approach taken in the next two chapters is to analyze the costs and returns from making loans to deposit customers and to make the part of deposit customer loan demand to be met an optimizing decision.

1.5 Summary

In most bank portfolio selection models, loans are assumed to be homogeneous assets with the same rate of return, and the amount of assets the bank has to invest is assumed to be unrelated to the way the bank invests the funds. Any differential treatment of borrowers violates such assumptions. The literature on credit rationing describes a type of differential treatment of borrowers. Bank credit rationing has been analyzed as the rational reaction of bankers to the riskiness of loans. Credit rationing is an important assumption in the analysis of the impact of monetary policy in the availability doctrine. Attempts have been made to measure the differential treatment of large and small borrowers. The objective of the studies was to determine if banks treat large borrowers more favorably than small borrowers during tight money periods.

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40 Goldfeld (1966, p. 26)
The type of bank behavior that is consistent with such objectives as expected profit maximization has been limited in most published studies by ignoring links between the sources and uses of bank funds. In the portfolio selection model developed in the following chapters, the bank competes for deposits by offering loans to depositors at lower interest rates than the rates charged non-depositors. The expected profit maximizing banks may discriminate in favor of its depositors who demand loans during tight money periods.
CHAPTER II

THE DEPOSIT RELATIONSHIP

The profitability of bank-customer relationships is analyzed in this chapter. The conditions under which banks and their customers find it to their advantage to establish these relationships are discussed by summarizing the literature on compensating balance requirements. A model for measuring deposit relationship profits for the banker is developed in the remainder of the chapter.

2.1 A Definition of a Deposit Relationship

A bank-customer relationship exists if a depositor is able to bargain with a bank for better prices on bank loans and services than the prices offered to non-depositors. A deposit relationship is a restricted form of a customer relationship in which special terms are given to a depositor only on bank loans. The deposit relationship is the only type of bank-customer relationship discussed in this chapter since bank asset selection is being considered and not the pricing of bank services.

2.2 The Rationale for Compensating Balance Requirements

The compensating balance requirement is the most uniform and widely known aspect of deposit relationships. Under the compensating balance arrangement for bank loans, the borrower must have
a deposit with the lending bank which is some fraction of the loan or credit line agreed upon. For some deposit customers the compensating balance is measured as the average balance and for others as the minimum balance; a reason for such a difference in treatment is discussed in section 2.4. Although compensating balances are required in some arrangements for the sale of bank services at reduced rates, Baxter and Shapiro found that over four-fifths of the banks in their survey requiring compensating balances used the requirement primarily as a condition for establishing lines of credit. 41

There is no regularly published series on compensating balance ratios; the available evidence indicates that the ratios vary from 10 percent to 20 percent of the credit line or loan, depending upon the stage of business fluctuations in which the negotiations take place. 42 Some banks do not require the same compensating balance ratio of all borrowers. When Hayes asked bankers what compensating balance ratios they required for various types of loans, some could not quote definite percentages because their policies were so flexible. 43

41 Baxter and Shapiro (1964, p. 487).


43 Hayes (1964, pp. 78-82)
Compensating balance requirements are considered in a discussion of deposit relationships because the profitability of deposit relationships for banks and the borrowers depends upon the rationality of compensating balance requirements. Under one interpretation requiring compensating balances is an inefficient means of raising loan interest rates, and under another interpretation this arrangement is advantageous to both the banker and the borrower. Establishing the case for the second interpretation is important for the later discussion of the profitability of deposit relationships for both bankers and deposit customers.

A compensating balance arrangement may be profitable for the banker if it increases bank lending capacity. The profitability of the arrangement for the borrower depends upon his reasons for holding his demand deposit balances. Hellweg has shown that if the customer borrows the compensating balance from the bank making the loan, and if the borrower would not have held as large a demand deposit balance if the compensating balance was not required to get the loan, the profits of the banker could be increased and the interest costs of the borrower reduced by eliminating the compensating balance requirement.44

The argument for the irrationality of a banker lending compensating balances to a borrower is developed symbolically. Let

44Hellweg (1961).
T be the total loan to the deposit customer, C the compensating balance, T-C the amount of the loan the borrower is free to use, i the contract rate of interest, i' the effective rate, and ρ the required reserve ratio. Then

\[(2.1) \quad i' = IT/(T-C)\]

The bank's reserves are not increased by this transaction, but required reserves are increased by ρ C above what they would have been if the bank had loaned T-C to the customer and required no compensating balance. If the borrower was not required to hold the compensating balance, the bank could increase bank profits by lending an additional ρ C, and the banker could share part of the increase in profits with the borrower of T-C by charging a rate between i and i'. So the compensating balance requirement may be interpreted as a rational means of increasing the interest rate on bank loans only if contract rates are rigid. With flexible contract rates lending compensating balances to the borrower is a less efficient way of increasing bank profits than increasing the contract rates on loans. The same argument is developed by Davis and Guttentag ⁴⁵ and Shapiro and Baxter ⁴⁶.

The compensating balance requirement is a rational arrangement for both the bank and the borrower if the borrower would have held a demand deposit balance at least as large as the required compens-

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⁴⁵ Davis and Guttentag (1962, pp. 121-26).
⁴⁶ Shapiro and Baxter (1964, p. 266).
sating balance even if compensating balances were not required to get a loan.\textsuperscript{47} If this is the case, the demand deposit balance is said to be held voluntarily. Of course the borrower might keep his deposit balance in a bank other than the one at which he borrows if the compensating balances were not required. But by holding the deposit at the lending bank, the borrower gets an interest rate reduction for holding a deposit balance that he would have held anyway. Through the compensating balance arrangement the bank can increase its lending capacity beyond what it would have been if it had made the loan to a non-depositor.

Writers on the rationality of compensating balance requirements have implicitly assumed that the reserve ratio is the only institutional constraint on the asset selection of bankers. If a capital adequacy constraint is effective, requiring forced (non-voluntary) compensating balances increases required bank capital more than lending with no compensating balances required. The guide to capital adequacy to be used in this discussion is that developed by the Federal Reserve Board.\textsuperscript{48} For every dollar of bank assets in a given category, there must be a specified number of cents in the capital accounts, and the larger deposits are relative to bank cash plus short-term assets, the greater the number of cents there must be in the capital accounts per dollar of demand deposits. If all demand deposits the

\textsuperscript{47} Shapiro and Baxter (1964, pp. 261-67).

\textsuperscript{48} Chamber and Charnes (1961); Crosse (1962, pp. 157-90).
bank creates by making loans are withdrawn from the bank except for the borrowed compensating balances, bank cash is reduced by T-C both if the loan T is made with the compensating balance C and if the loan of T-C is made with no compensating balance required. Required bank capital increases more with the loan of T and compensating balance C than with the loan of T-C because (a) loans are larger by C under the first transaction than the second, and (b) deposits are larger by C under the first transaction than the second with bank cash plus short-term securities the same in both cases.

2.3 Evidence on Demand Deposit to Bank Loan Ratios of Bank Customers

Emmer has estimated demand deposit to bank loan ratios for firms in several industries.\textsuperscript{49} Firms in non-financial industries had average demand deposit to bank loan ratios much above the existing required compensating balance ratio; these firms would hold such large balances only if they were held voluntarily. Mortgage companies, chosen as examples of firms in a non-bank financial industry, had demand deposit to bank loan ratios near the required compensating balance ratio; these observations imply that firms in non-bank financial industries may hold at least part of their demand deposits as idle (forced) balances. As is explained below, firms in financial industries are treated differently by banks as deposit customer than other large depositors.

\textsuperscript{49}Emmer (1957).
2.4 Compensating Balances as Average or Minimum Balances

The issue of whether compensating balances are held voluntarily or as forced balances is related to the issue of how banks measure compensating balances - as the minimum or average balance. If the borrower voluntarily holds a large average balance relative to the usual compensating balance associated with his borrowing demands, as is typical of firms in non-financial industries, the bank is more likely to measure the compensating balance for the loan or line of credit as the average balance during some period of time. Under this arrangement the demand deposit to bank loan ratio may be less than the required compensating balance ratio at any particular time; the bank anticipates that the depositor will hold a large enough balance at other times to make the deposit relationship profitable. If compensating balances are not voluntarily held, the bank is more likely to enforce the compensating balance requirement at all times. This means that compensating balances are measured as the minimum deposit balance held by the borrower during the period of the loan or credit line.

The reason presented in this section for measuring compensating balances as average or minimum balances is based upon a survey by Hayes of policies at nine large commercial banks. All of the banks in the survey measured compensating balances as the average balance.  

---

50 Hayes (1964, pp. 74-86).
Baxter and Shapiro found that about half of the banks that used compensating balance requirements allowed some sort of averaging of required balances, and of banks with more than $500 million in deposits, over 80 percent allowed balance averaging. The typical averaging period at the large banks was one year.\(^{51}\)

Firms in financial industries, such as mortgage and consumer finance companies, have their compensating balances for their credit lines measured as the minimum balance during the period of the relationship.\(^{52}\) The credit extended to firms in financial industries during periods of monetary restraint falls more than the decline in credit extended to other large business firms.\(^{53}\) As is shown in section 2.5, the higher the demand deposit to bank loan ratio, the greater the profitability of a deposit relationship. The relatively low ratio of demand deposits to bank loans of firms in financial industries make their deposit relationships only marginally profitable to banks. The contrast between the average demand deposit to bank loan ratios of firms in financial and non-financial industries is consistent with the differential treatment of firms in the calculation of compensating balances as average or minimum balances and with the loan priorities banks set during periods of monetary restraint.

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\(^{51}\) Baxter and Shapiro (1964, p. 487). For other banking literature sources in which bankers are said to measure compensating balances as average balances, see Robinson (1962, p. 143); David and Guttentag (1962, p. 122).

\(^{52}\) Baxter and Shapiro (1964, p. 490).

In the following analysis of deposit relationship profitability, bank lending capacity is assumed to increase when a deposit relationship is established, and the deposit relationship is assumed to be a profitable arrangement for the deposit customer. This assumption is important in the model of Chapter III.

2.5 Profits from a Deposit Relationship

Assume that the i'th deposit customer makes a loan $L_i$ and holds a deposit $D_i$ during a period of time. Let $r_p$ be the interest rate charged the deposit customer on his loan; $r$ is the rate of return on the investment of bank funds in interest earning assets other than loans to deposit customers; and $\rho$ is the reserve ratio. The revenue the bank earns from the loan to the i'th deposit customer and the investment of funds from the deposit which is attracted or retained by making the loan $L_i$ is:

\begin{equation}
(2.2) \quad r_p (L_i) + rD_i (1-\rho)
\end{equation}

The options open to the banker must be specified to know what opportunity cost to subtract from this revenue for a measure of profits from the deposit relationship. To be consistent with the opportunity cost used in the literature on this subject, the banker must have excess reserves at least as large as $L_i$. The banker can either invest $L_i$ at the interest rate $r$, or he can make the loan to the i'th deposit customer.

---

54 Cooper (1967); Hodgman (1961), (1963); Murphy (1969); Richards (1967).
customer and increase lending capacity by \( D_1(1-\rho) \). Therefore, the opportunity cost of lending \( L_1 \) to the \( i \)'th deposit customer is \( r(L_1) \). The service charges on the demand deposit \( D_1 \) are assumed to equal the bank's costs of servicing the demand deposit account to simplify the equations. The profit from the \( i \)'th deposit relationship is:

\[
P_i = (r_p - r) L_i + rD_1(1-\rho)^{55}
\]

The bank's profit from the \( i \)'th deposit relationship during one period is given in equation (2.3). If the present and future values of all variables in (2.3) are assumed to be the same, the present value of the stream of profits from the \( i \)'th deposit relationships is:

\[
(P_i)/r = L_i(r_p - r)/r + D_1(1-\rho)^{56}
\]

2.6 Indirect Interest Payments on Demand Deposits

Banks are assumed to pay interest on demand deposits indirectly by charging depositors lower interest rates on loans than they charge non-depositors. If the banking industry is assumed to be perfectly competitive, the interest rate deposit customers receive indirectly

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\( ^{55} \) This equation is similar to those in the works by Cooper (1967, p. 82), and Hodgman (1963, pp. 99-101).

\( ^{56} \) The present value of a stream of profits from a deposit relationship with a finite horizon and present and future values different is analyzed in section 2.8.
on their deposits through reduced loan interest rates is the same rate they would receive if banks were permitted to pay interest on demand deposits directly.\textsuperscript{57}

In a perfectly competitive banking industry equilibrium bank profits and net worth are zero. Let $r$ be the rate of interest at which banks can invest their assets. If $\text{DD}$ is the level of deposits in a particular bank, the bank's revenue during one period is $r\text{DD}(1-\rho)$. Assume that bankers may pay interest on demand deposits directly. Service charges on deposits are assumed to equal the costs of servicing deposit accounts. Then banks will bid up the rate $\hat{r}$ on demand deposits until:

$$
(2.5) \quad r\text{DD}(1-\rho) = \hat{r}\text{DD}; \quad \hat{r} = r(1-\rho)
$$

Now assume that direct interest payments on demand deposits are prohibited. The $i$'th customer at a particular bank holds a deposit $\text{D}_i$. During the current period, the bank earns $r\text{D}_i(1-\rho)$ on this deposit. Bankers cannot compete for this deposit by paying interest to the $i$'th customer, but they do compete by offering loans at the interest rate $r^*$ which is below $r$. The income the bank sacrifices in attracting the deposit of the $i$'th customer is $(r-r^*)\text{L}_i$. Because of competition among bankers,

$$
(2.6) \quad (r-r^*)\text{L}_i = r\text{D}_i(1-\rho)
$$

\textit{57} I am indebted to Dr. Thomas Smaung for the suggestion of viewing bank-customer relationships as indirect interest payments on demand deposits and for developing a part of the analysis in this section.
The i'th customer saves \((r-r^*)L_i\) in interest costs by getting the loan \(L_i\) at the bank where he has a deposit. This interest cost saving can be considered a return on holding the demand deposit \(D_i\). If \(\hat{r}\) is the rate of return the depositor receives indirectly from holding his deposit,

\[(2.7) \quad (r-r^*)L_i = \hat{r} D_i\]

By equations (2.6) and (2.7),

\[(2.8) \quad \hat{r} D_i = r D_i (1-\rho); \quad \hat{r} = r(1-\rho),\]

which is the same as equation (2.5).

A bank balance sheet can be used to show that competition among banks for deposits by both direct interest payments and loans at reduced rates for depositors have the same effect on bank net worth. Assume that (a) the bank has zero capital, (b) direct interest payments on demand deposits are prohibited, (c) all borrowers pay the market rate of interest \(r\) on bank loans, and (d) service charges equal the costs of servicing deposit accounts. Let \(DD\) be total demand deposits at a particular bank and \(\rho\) the reserve ratio. There are no time deposits at the bank. The bank's liabilities are \(\rho DD\), the present value of interest income foregone by holding assets in a reserve to pay for deposit withdrawals, and loans equal \((1-\rho)DD\). Balance sheet entries are

\[58\] Pesek and Saving (1967, pp. 138-62).
present values of future income and cost streams; the income and cost streams are discounted at the market interest rate $r$ which is assumed to remain the same in future periods.

<table>
<thead>
<tr>
<th>Bank Balance Sheet I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Loans $(1-\rho)DD$</td>
</tr>
<tr>
<td>Reserves $\rho DD$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Now suppose that the banker pays the interest rate $\hat{r}$ on demand deposits directly. During each period, the banker has a commitment to pay $\hat{r} DD$ in interest costs; the present value of the future stream of interest costs is $\hat{r}(DD)/r$. The effect of this change in bank competition is shown in Bank Balance Sheet II.

<table>
<thead>
<tr>
<th>Bank Balance Sheet II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Loans $(1-\rho)DD$</td>
</tr>
<tr>
<td>Reserves $\rho DD$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
If the banking industry is competitive, bankers will bid up the rate \( \hat{r} \) to the level at which net worth is zero:

\[
(2.9) \quad DD[(1-\rho) - \hat{r}/r] = 0; \quad \hat{r} = r(1-\rho)
\]

Assume that direct interest payments on demand deposits are prohibited. Competition among banks for deposits takes the form of loan rate reductions for those depositors who demand loans. Let \( CD \) be the value of deposits at the bank owned by those depositors getting prime rate loans, and let \( CL \) be the value of those loans. The present and future values of the prime rate are assumed to be the same. The bank receives \( r_CL \) each period from the loans to depositors. This stream of future interest receipts is discounted to the present with the discount rate \( r \), and the present value of this revenue stream value is \( r_p CD/r \).

Bank Balance Sheet III

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans ( (1-\rho)DD - CL + r_CL/r )</td>
<td>( pDD )</td>
</tr>
<tr>
<td>Reserves ( pDD )</td>
<td>Net Worth ( (1-\rho)DD - CL + r_CL/r )</td>
</tr>
</tbody>
</table>

The only way banks can compete for deposits is by offering interest cost savings to depositors who demand loans. If bankers complete for these deposits until the profitability of each deposit
relationship is zero, depositors are offered a ratio of prime rate loan to deposit equal to

\[(2.10) \quad \frac{r(1-\rho)}{(r-r_p)} \]

Therefore,

\[(2.11) \quad CL = CDr(1-\rho)/(r-r_p) \]

and the bank net worth in Bank Balance Sheet III will be written as:

\[(2.12) \quad (1-\rho)DD - \left[ \frac{(r-r_p)(1-\rho)}{r(r-r_p)} \right] CD = (1-\rho)DD - \left[ \frac{r(1-\rho)}{r} \right] CD \]

The deposit customers receive the interest rate \(r(1-\rho)\) indirectly on their demand deposits. During each period, the bank has a commitment to give interest cost reductions to depositors equal to \(r(1-\rho)CD\) in order to retain their deposits. The bank's balance sheet can be rewritten to show \(r(1-\rho)CD/r\) as a liability of the bank.

**Bank Balance Sheet IV**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans (\text{DD} \cdot (1-\rho))</td>
<td>(\rho\text{DD})</td>
</tr>
<tr>
<td>Reserves (\rho\text{DD})</td>
<td>((1-\rho)\text{CD})</td>
</tr>
</tbody>
</table>

Net Worth

\(\text{Net Worth} = (1-\rho)(\text{DD-CD})\)
Net worth of the bank would be zero if \( CD = DD \), i.e., if all depositors at each bank demand loans at that bank. In this situation each depositor would borrow \((1-\rho)\) times his deposit at a zero rate of interest.

\[
(2.13) \quad (1-\rho) = \frac{r(1-\rho)}{(r-r_p)}
\]

only if

\[
(2.14) \quad r_p = 0
\]

2.7 The Contribution of Future Deposit Relationship Profits to Bank Wealth

The profits which a bank anticipates making from deposit relationships in the future has been stressed as an important determinant of bank asset selection,\(^{59}\) and Hodgman has suggested that the present value of a future stream of profits from a deposit relationship be considered a part of bank net worth.\(^{60}\) Only Kane and Malkiel have specified how this source of bank wealth might be measured.\(^{61}\) Their equation for expected long-run profits from the \(i'\)th deposit relationship is:

\[
(2.15) \quad E(LR_{hi}) = \sum_{h=1}^{U} g_h E(D_{ih}/Rio)
\]

\(^{59}\) Kane and Malkiel (1965); Hodgman (1963).

\(^{60}\) Hodgman (1963, p. 119).

\(^{61}\) Kane and Malkiel (1965, p. 124).
\[ U = \text{the horizon of the banker;} \]

\[ g_h = \text{the rate of return on government securities in period } h; \]

\[ D_{ih} = \text{the deposit of the } i'th \text{ deposit customer in period } h; \text{ and} \]

\[ R_i = \text{the quality of the bank-customer relationship in period } 0. \]

This equation is consistent with the equations of deposit relationship profits presented in this chapter if:

(a) deposit customers are charged the same interest rate as the rate of return on alternative assets, i.e.,

\[ r_p = r, \]

(b) the reserve ratio is zero, and

(c) the deposit of the \( i'th \) deposit customer is invested only in government securities each period.

See section 3.2 for a discussion of the role of anticipated future profits from deposit relationships in the bank portfolio selection model.
CHAPTER III

AN INVENTORY MODEL OF BANK PORTFOLIO SELECTION

Analysis of the optimal level of reserves and short-term securities is often treated as an inventory problem with demand deposits a random variable. In the existing formal theory of banking, the basic assumptions are: (a) banks maximize expected profits (or minimize expected losses), (b) banks construct probability distributions of gains and losses from investment in assets, and (c) ... profit maximization takes place subject to a specified distribution of cash drains during the planning period.

The opportunity costs of having an excessive inventory of reserves and short-term securities is the higher rate of return on less liquid assets, and the cost of having an insufficient inventory is the cost of borrowing reserves.

Competition among banks for deposits through their loan policies is incorporated into this approach to banking theory in the following inventory model. The demand for loans by deposit customers is the one random variable in the inventory model, and the demand deposit withdrawals by deposit customers is made a function of their excess demand for loans. The bank is assumed to select the combination of assets that maximizes expected bank profits.

62 Edgeworth (1888); Morrison (1966); Orr and Mellon (1961); Porter (1967).

63 Morrison (1966, p. 8).
3.1 Assumptions about the Banking Industry

The banking industry is assumed to be competitive in the market for demand deposits, and there are assumed to be no time deposits. Banks are prohibited from paying interest on demand deposits, but banks do compete for those depositors who demand loans by offering them reduced rates on bank services and lower loan interest rates than are offered to non-depositors. Banks are assumed to operate without costs. Total bank funds for investment equal total deposits; banks are established with zero capital, and all earnings are paid to stockholders. The capital adequacy constraint cannot be applied to banks since there is no bank capital to serve as security for depositors.

Preferences for bank loans and services are assumed to vary among depositors. The only bank service some depositors value is demand deposits. Other depositors have a demand for bank services other than loans, and depositors in another group prefer to receive their implicit interest on deposits in the form of interest rate reductions on loans. Banks are assumed to be indifferent as to how they pay implicit interest on deposits. Bank services or loans are tied to demand deposits because of the prohibition of interest payments on demand deposits and competition among banks for deposits. All depositors are assumed to take their implicit interest in some form. The existence of these different groups of depositors has important implications for Chapter IV.
The loan terms offered deposit customers are determined by the supply function of loans by banks and the demand function for loans by deposit customers. Let \( r_n \) be the rate of return on loans to non-depositors, \( r_p \) the prime rate, and \( p \) the reserve ratio. If \( L_i \) is the prime rate loan offered to the \( i \)'th deposit customer and \( D_i \) is his deposit, the bank earns zero profits from the \( i \)'th deposit relationship if:

\[
\frac{L_i}{D_i} = \frac{r_n(1-p)}{(r_n-r_p)}
\]

The ratio of the prime rate loan to demand deposit banks offer to deposit customers and deposit customers demand is designated \( L \) in Figure 1. For given \( r_n \) and \( p \) a supply function is derived from (3.1) in Figure 1 in terms of the loan to deposit ratio and the interest rate deposit customers are charged for loans. Depositors have demand functions for loans in terms of the loan to deposit ratio and the interest rate paid. The equilibrium loan to deposit ratio and interest rate charged deposit customers for loans are determined by the intersection of the supply and demand functions. All bank customers who demand implicit interest on their demand deposits in the form of reduced interest rates on loans are assumed to have the same demand function for loans in terms of the interest rate and the ratio of loan to deposit. Banks are assumed to charge all depositors the same interest rate for loans, the prime rate.

3.2 A Summary of the Model

The implications of deposit relationships for the allocation of bank funds are analyzed by examining the allocation of assets by one bank. The objective of the bank is to allocate its assets
FIGURE 1  SUPPLY AND DEMAND OF LOANS TO DEPOSIT CUSTOMERS.
in such a way as to maximize expected bank wealth.

All interest earning assets held by the bank are assumed to have matured at the end of the previous period, and therefore, the bank is free to allocate all of its assets in the most profitable way. The assets held by the bank are reserves, loans to non-depositors, loans to deposit customers, and securities. Reserves held equal required reserves; the optimal reserve ratio is not analyzed in this model. Changes in the average demand deposit balance held by deposit customers are assumed to depend only on whether or not their loan demands are met, and the average balances held by other depositors are assumed to remain unchanged. These assumptions are made to separate the problem of the optimal fraction of bank funds to be reserved for loans to deposit customers from the problem of the optimal level of excess reserves and securities held to pay for the demand deposit withdrawals that are not related to the bank's portfolio selection.

The bank can invest any share of its assets in loans to non-depositors at the prevailing market interest rate on loans to non-depositors. The rates of return on all assets are assumed to be known with certainty. There is no risk of default on a loan. Loans are not sold by the bank during the current period since the capital loss and transactions costs of doing so would be large.

Loans to deposit customers plus securities equal the inventory of bank funds reserved for loans to deposit customers; this inventory is called the loans inventory. For a given loans inventory the value of loans to deposit customers depends upon their demand
for loans. The demand for loans by deposit customers is assumed to be a random variable. If that demand is less than the inventory for these loans, the excess funds in the loans inventory are invested in securities during the current period. To avoid calculating interest income for fractions of the current period, all loan demands by deposit customers are assumed to be made at the beginning of the period.

The problem of selecting the optimal loans inventory is different from the physical goods inventory problem or the excess reserves inventory problem in that the unused portion of assets in the inventory (the loans inventory less the demand for loans by deposit customers) earns a rate of return (the rate of return on securities) instead of being held as idle assets.

Since excess reserves are assumed to be zero, the optimal allocation of all the bank's funds can be studied by analyzing either the optimal share of the bank's funds invested in loans to non-depositors or the optimal share for the loans inventory. The share of the bank's funds in the loans inventory that minimizes the expected loss of holding the inventory also maximizes expected bank profits. The bank portfolio selection problem analyzed in this chapter is that of selecting the share of bank funds for the loans inventory that minimizes the expected loss of holding the loans inventory.

The returns from investing in the loans inventory depend upon the demand for prime rate loans by deposits. The maximum prime rate loan a bank will make to a depositor is $r_n(1-p)/(r_n-r_p)$.
multiplied by his deposit; this prime rate loan makes the profits from the deposit relationship zero according to equation (3.1). The demand for loans by deposit customers is assumed to be a random variable; if CD is total deposits by deposit customers at the bank, the upper bound on the probability distribution of prime rate loan demand is \( \frac{r_n (1-p)}{r_n - r_p} \) CD.

The two choices made by the banker in this model are (a) the share of the bank's deposits to put in the loans inventory, and (b) the amount of money the banker will borrow from other banks to make loans to deposit customers relative to the bank's demand deposits. The maximum amount the banker will borrow is called the borrowing limit in this chapter. If the demand for loans by depositors is less than the loans inventory, the excess is invested in securities, and if the prime rate loan demand is greater than the loans inventory, the banker may borrow to meet either a part or all of the loan demand. With a high enough rate of return on loans to non-depositors the bank may set the loans inventory and borrowing limit in such a way as to accept some probability of refusing loan demands that are within the limits of the maximum prime rate loans offered.

Since the demand for loans by deposit customers is assumed to be a random variable, there is some probability that the bank's deposit customers will not demand the full amount of
their indirect interest payments. The bank reacts to this source of uncertainty in the model by reserving a part of the bank's deposits for loans to depositors at the prime rate, borrowing additional funds if necessary, and perhaps refusing to make some of the indirect interest payments through prime rate loans if all deposit customers demand their maximum prime rate loans. A bank loses deposits if depositors demand their indirect interest payments and the bank does not pay them.

If the loans inventory plus the borrowing limit are less than the total maximum prime rate loans offered all deposit customers, expected demand deposit withdrawals by the bank's existing deposit customers are positive. If deposits are withdrawn, the banker loses the interest that could have been earned from investing the deposits. Since excess reserves are assumed to be zero, the banker pays for the demand deposit withdrawals by borrowing reserves. Depositor loan demands and deposit withdrawals are two interdependent sources of demand for bank funds in this model.

The two reasons for which the banker might borrow are (a) to make loans to depositors, and (b) to pay for deposit withdrawals. He is assumed to borrow in the same market for both purposes and is able to borrow any amount he wants at the going rate that banks charge each other for loans. Under currently existing institutions the Federal Funds market is such a market in which bankers borrow from one another.
If a depositor withdraws his deposit from the bank because his prime rate loan demand has been refused by this bank but offered by another bank, the bank loses both the present and future profits which would have been derived from that deposit relationship. The contributions of future deposit relationship profits to bank wealth are discussed in greater detail in section 2.7. In the model developed by Kane and Malkiel the bank can anticipate making positive profits from long established deposit relationships in future periods because of imperfect knowledge among banks about deposit customers. The longer a customer and a bank have established a relationship, the better the banker understands the risks of loans to the customer and the better it can predict the customer's desired pattern of deposit balance and loan demand. Imperfect knowledge keeps banks from bidding away the profits of deposit relationships.\textsuperscript{64} Hodgman stressed the roles of imperfect knowledge among banks about deposit customers and the unequal bargaining power of deposit customers in his discussion of reasons for positive deposit relationship profits.\textsuperscript{65} The prime rate has the role of restricting price competition for deposits; banks compete for deposits by offering specialized services which are difficult for other bankers to match.\textsuperscript{66}

\textsuperscript{64} Kane and Malkiel (1965, pp. 122-23).

\textsuperscript{65} Hodgman (1963, pp. 118-9, p. 103).

\textsuperscript{66} Hodgman (1963, pp. 120-35).
The bank portfolio selection model developed in this chapter is simplified by assuming the reasons for long-run profits from deposit relationships discussed above do not hold. If a bank anticipates that the banking industry will remain competitive in future periods, he assumes that competing banks will offer depositors the maximum prime rate loans that will reduce profits from deposit relationships to zero. Therefore, the bank anticipates that the revenue he will receive from deposit relationships will just equal the opportunity cost of lending to the deposit customers in future periods. Under these assumptions the loss of anticipated future profits from deposit relationships is ignored as one of the costs of losing the business of a deposit customer. The same conclusions derived by Hodgman and Kane and Malkiel about the impact of monetary policy on the allocation of bank credit between depositors and non-depositors are derived in Chapter IV without assuming imperfect competition for deposits.

The opportunity costs of holding bank funds in the loans inventory are the earnings that could have been received from investing these funds in loans to non-depositors. The banker is assumed able to invest whatever fraction of the bank's funds he desires in loans to non-depositors at the prevailing market rate. Klein has criticized the approach to bank portfolio selection in which a banker forms a supply function for loans that is independent of the demand for loans because of the profitability of deposit relationships.
This objection is handled by having two loan supply functions — loans to deposit customers and loans to non-depositors. If the probability distribution of loan demand by deposit customers changes, the optimal share of bank deposits in the loans inventory might also change even if the prime rate remains fixed. This situation could occur if the probability that all deposit customers would demand all of their credit lines as loans rose from 50 percent to 100 percent. The loans inventory would probably be less than the total of all credit lines of deposit customers in the first case and would equal the total of all credit lines in the second case. The supply function for loans to non-depositors is independent of the demand for loans by non-depositors.

The directions of change in the optimal loans inventory and borrowing limit with respect to interest rates are derived from the function for the expected loss from holding the loans inventory. One result of special interest is that during periods of restrictive monetary policy, the difference between the rate of return on loans to non-depositors and the prime rate must increase if the share of bank deposits in the loans inventory is to remain constant. An increase in the difference between these two rates and an increase in the share of bank loans going to deposit customers is consistent with expected bank profit maximization.

67Klein (1968, pp. 29-44).
3.3 The Expected Loss Function

3.3.1 Returns from Investing the Loans Inventory

The probability distribution of the demand for loans by deposit customers is presented as the distribution of ratio $L$ of total loan demand by deposit customers to the total deposits by deposit customers at the beginning of the current period, $CD$. The probability distribution $f(L)$ of $L$ is assumed to be rectangular with lower bound $a$ and upper bound $r_n (1-\rho)/(r_n - r_p)$, the value of $L$ in case all deposit customers demand their maximum prime rate loan offered to them; therefore

$$f(L) = \left[ \frac{1}{r_n (1-\rho) - a} \right]$$

Changes in the interest rates $r_n$ and $r_p$ change both the rates of return on assets and the maximum loan to deposit ratio competitive banks offer depositors. An increase in $r_n$ increases the opportunity cost of holding a particular loans inventory, increases the return from investing the deposits of deposit customers, and reduces the maximum loan to deposit ratio deposit customers can demand:

$$\frac{\partial}{\partial r_n} \left[ \frac{r_n (1-\rho)}{r_n - r_p} \right] / \partial r_n = -\frac{r_n (1-\rho)}{(r_n - r_p)^2} < 0$$

An increase in $r_p$ increases the return from investing a given loans inventory and increases the maximum loan to deposit ratio deposit customers can demand:

$$\frac{\partial}{\partial r_p} \left[ \frac{r_n (1-\rho)}{r_n - r_p} \right] / \partial r_p = \frac{r_n (1-\rho)}{(r_n - r_p)^2} > 0$$
Let $L_p$ be the loans inventory divided by CD, or the value of $L$ at which the demand for loans by deposit customers equals the loans inventory. If the loan demand is less than the loan inventory, the excess funds in the loans inventory are invested in securities during the current period at the rate $r_s$. If

$$L > L_p,$$

(3.5)

the bank borrows up to some limit $B(CD)$ in order to make loans to deposit customers. The bank pays the market interest rate $r_n$ on the funds borrowed to make loans to deposit customers and to pay for deposit withdrawals. All of the deposit customers have their loan requests met if

$$\left( L + B \right) \geq L.$$  

(3.6)

If

$$\frac{r_n (1-p)}{r_n - r_p} > L > \left( L + B \right) \frac{1}{p},$$

(3.7)

deposit customers have their loan demands refused, and $(L_p + B)CD$ invested in prime rate loans.

The rate of return on loans to non-depositors and the rate at which banks borrow are assumed equal for two reasons: (a) the number of interest rates in the model is kept to a minimum, and (b) the interest rates on loans to non-depositors and the rates at which banks borrow from one another may be expected to move in the same direction, and making these rates the same is a simple
means of including this sort of reaction in the model. A bank can be both a supplier and demander of loans between banks; some of the bank's funds invested in loans to non-depositor at the beginning of the period may go to the other banks.

In the following equations the expected costs and returns from holding the loans inventory are divided by DD, the bank's total demand deposits at the beginning of the period. The objective of the bank is assumed to be minimization of the expected loss of

68 A symbol other than r could be used for the interest rate at which bankers borrow. The author wants the interdependence between the supply and demand functions in the markets for loans to non-depositors and the market for loans between banks to be explicitly built into the model. As an example of this interdependence, the reactions in the market for loans between bankers to a rise in the interest rate on loans to non-depositors are (1) a shift in the banks' demand curve for loans from other banks to the right because of the higher return on investing these funds, and (2) a shift in the banks' supply curve of loans to other banks to the left because of the increase in the bank's opportunity costs of making such loans. One way to catch the interdependence between these two market interest rates is to assume a functional relation between them. The most simple form of the function is assumed in the text; the two rates are assumed to be equal. The results of the model would not be changed by assuming a more complicated relation between the two rates.

Bankers are assumed able to borrow whatever amount of funds they want at an existing interest rate. Bank borrowing from the Federal Reserve Banks at the discount has been ignored since bankers cannot anticipate borrowing any desired amount at the discount rate. The results of the model would not be changed by assuming the banker can borrow up to some limit at the discount rate and borrow additional funds from other bankers.
holding the loans inventory divided by DD. The bank is assumed to
not be able to increase the level of demand deposits by his portfolio
selection but is able to reduce demand deposits by refusing the
loan demands of deposit customers.

For

\[ L_p \geq L \geq \alpha, \]

the returns from investing the loans inventory are

\[ \text{CD/DD} \left[ (L_p - L)r_s + Lr_p \right]. \]

For

\[ (L_p + B) \geq L \geq L_p, \]

the returns are

\[ \text{CD/DD} [L_p r_p + (r_p - r_n) (L_p - L_p) - r_n (L_p - L_p)]. \]

Bank borrowing equals

\[ (L_p - L_p) \text{CD} \]

\[ ^{69} \text{A banker may increase his bank's deposits by making loans to} \]
the deposit customers at other banks who have their loan demands re-
used by their bankers. The bank portfolio selection model could be
extended to include this effect by assuming that the banker forms a
probability distribution of deposit customer loan demands refused at
other banks. The banker might hold a part of the bank's deposits
as a reserve for such loans. None of the results of the portfolio
selection model as it is now constructed would be changed if such
an extension were added.
with \( L \) in the range of (3.10) and the explicit borrowing costs are

\[
(3.13) \quad r_n (L - L_p) CD
\]

The opportunity costs of investing borrowed funds in loans to deposit customers are the earnings that could have been received from investing in loans to non-depositors. Therefore, the net return from investing borrowed funds in loans to deposit customers is

\[
(3.14) \quad (r_p - r_n) (L - L_p) CD
\]

For

\[
(3.15) \quad \frac{r_n (1 - \rho)}{r_n - r_p} \geq L \geq (L_p + B)
\]

the returns from investing the loans inventory are

\[
(3.16) \quad CD/DD[L_p r_p + B(r_p - 2r_p)]
\]

The expected return from investing the loans inventory is:

\[
(3.17) \quad \left\{ \int_{L_p}^{L_p + B} \left( \frac{1}{(r_n (1 - \rho))} \right)^{-\alpha} \left[ (L - L_p)r_n + L r_p \right] dL \right\} + \int_{L_p}^{L_p + B} \left( \int_{L_p}^{L_p + B} [L_p r_p + B(r_p - 2r_n)] dL \right)
\]
3.3.2 The Cost of Demand Deposit Withdrawals

If the maximum amount of loans the bank will make to its deposit customers is less than the total of the credit lines of the deposit customers, i.e., if

\[(3.18) \quad r_n(1-\rho)/(r_n-r_p) > (L+B),\]

expected demand deposit withdrawals by deposit customers are positive. 70 A part of expected costs of holding the loan inventory is the earnings the banker would have received from investing expected demand deposit withdrawals by deposit customers. Deposit customers are assumed to withdraw their deposits from the bank only if their loan demands are refused by their present bank and met by another bank.

If

\[(3.19) \quad L > (L+B),\]

some deposit customers must have part or all of their loan demands refused. The profit maximizing bank will allocate the available

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70 Since deposit customers withdraw their deposits from a bank of this model only if another bank will give them better terms, the net loss of demand deposits of deposit customers within the banking system must be zero. The bank portfolio selection model would be consistent with this equilibrium property of the banking system if the extension to the model suggested in footnote 69 is made. If expected deposit withdrawals by the deposit customers at one bank are positive, other bankers may have an incentive to reserve assets for loans to those deposit customers who have their loan demands refused. The expected increase in deposits at other banks may be positive.
funds in such a way that demand deposit withdrawals are minimized, i.e., to those deposit customers with the lowest ratios of loan demand to demand deposit. To simplify the problem the loan demand to demand deposit ratios are assumed to be the same for all deposit customers. Therefore, the bank is indifferent as to which loan demands are refused. The fraction of loan demand refused is \((L-L_p-B)/L\); this is also the fraction of CD owned by deposit customers who have had their loan demands refused. Then the value of deposits by deposit customers who have had their loan demands refused is:

\[
(3.20) \quad \frac{[L-L_p-B]}{L}CD
\]

If some deposit customers withdraw their deposits, required reserves of the bank are reduced. Therefore, the opportunity costs to the bank of the deposit withdrawals equal the amount of revenue the bank could have earned from investing those deposits at the market interest rate multiplied by one minus the reserve ratio. The expected earnings foregone on demand deposit withdrawals is:

\[
(3.21) \quad r_n(1-\rho) \left[ \frac{1}{r_n (1-\rho)} + \frac{r_n - r}{r_n - r_p} - \alpha \right] \left( \frac{r_n (1-\rho)}{r_n - r_p} \right) \int_{L_p+B}^{(L-L_p-B)/L} dL
\]

When a deposit is withdrawn, the bank not only loses the earnings that would have been received from investing the deposit but also has
to borrow reserves in order to pay for the deposit withdrawal. The expected cost of paying for deposit withdrawals is $r_n (1-\rho)$ multiplied by the expected deposit withdrawals. As deposits are reduced, required reserves are reduced, and the funds required to pay for deposit withdrawals are $(1-\rho)$ multiplied by the deposits withdrawn. The expected cost of paying for demand deposit withdrawals due to an excess demand for loans by deposit customers is the same as (3.21) since banks borrow and invest at the same interest rate $r_n$.

3.3.3 The Opportunity Costs of the Loans Inventory

The banker has the option of investing the bank's funds in the loans inventory or in loans to non-depositors. Therefore, the opportunity cost of allocating the bank's funds to the loans inventory is:

\[ (3.22) \quad r_n \frac{L_{(CD/DD)}}{p} \]

3.3.4 The Expected Loss Function

Let $\epsilon$ be the expected loss from holding the loans inventory divided by DD.

\[ (3.23) \quad \epsilon = -(3.17) \quad \text{the expected returns from investing the loans inventory} \]

\[ +2(3.21) \quad \text{the expected cost of paying for demand deposit withdrawals plus the earnings foregone on the investment of these deposits} \]

\[ +(3.22) \quad \text{the opportunity cost of holding bank funds in the loans inventory}. \]
3.4 The Optimal Loans Inventory and Bank Borrowing Limit

The two decisions of the banker in this model are the sizes of

L_p, the loans inventory divided by CD, and B, the bank limit

on borrowing to make loans to depositors divided by CD. The optimal

values for L_p and B depend upon relations among interest rates.

Two relations among interest rates are specified before carrying

out any analysis:

\[ r_n > r_p; \]

(3.24)

\[ r_n > r_s \]

(3.25)

Inequality (3.24) holds if banks compete for deposits. Profits

from the i'th deposit relationship are

\[ p_i = (r_p - r_n)I_n + r_nD_i(1-\rho) \]

(3.26)

If bankers compete until \( p_i = 0 \),

\[ r_nD_i(1-\rho) = (r_n - r_p)L_i \]

(3.27)

The competitive rate \( r_p \) at which the banker lends to this

customer must be lower than \( r_n \) if (3.27) is to hold and the

\[ ^71 \text{See section 2.5 for a discussion of (3.26).} \]
deposit of the \( i \)'th customer is positive.

The transactions costs of buying and selling securities are assumed to be less than those costs of buying and selling loans. If these transactions costs had been included in the model, inequality (3.25) would have to hold as an equilibrium condition in a model that includes risk. Inequality (3.25) is also shown to be a necessary condition for the minimization of the expected loss from holding the loans inventory.

The first order conditions for a minimization of (3.23) with respect to \( L_p \) and \( B \) are obtained by differentiating (3.23) with respect to \( L_p \) and \( B \) and setting the results equal to zero. Let \( \varepsilon_{L_p} \) and \( \varepsilon_B \) be the partial derivatives of \( \varepsilon \) with respect to \( L_p \) and \( B \), respectively.
(3.28) \[ \varepsilon_{L_p} = (\text{CD/DD}) \left[ \frac{1}{\left( \frac{r_n}{r_n - r_p} \right)^{-\alpha}} \right] \left\{ -r_s \left( L_p \right) \alpha - r \left[ \frac{r_n (1-p)}{r_n - r_p} \right] \right\} = 0 \]

(3.29) \[ \varepsilon_B = (\text{CD/DD}) \left[ \frac{1}{\left( \frac{r_n}{r_n - r_p} \right)^{-\alpha}} \right] \left\{ (25 - r_p) \left[ \frac{r_n (1-p)}{r_n - r_p} \right] \right\} = 0 \]
Equations (3.28) and (3.29) can be combined to give:

\[(3.30)\]
\[r_n \left( \frac{r_n (1-\rho)}{r_n - r_p} - \alpha \right) = r_s (L_p - \alpha) + 2r_n \left( \frac{r_n (1-\rho)}{r_n - r_p} - L_p \right)\]

One conclusion to be drawn from (3.30) is that \(L_p\) must be larger than \(\alpha\) if

\[(3.31)\]
\[\frac{r_n (1-\rho)}{r_n - r_p} > \alpha\]

Another conclusion is that

\[(3.32)\]
\[r_n > r_s\]

if

\[(3.33)\]
\[\frac{r_n (1-\rho)}{r_n - r_p} > L_p\]

The banker selects some level of \(L + B\) in the range

\[(3.34)\]
\[\frac{r_n (1-\rho)}{r_n - r_p} \geq L_p + B \geq \alpha,\]

and the optimal levels of \(L_p\) and \(B\) are functions of the variables in the model. To make the optimal \(L_p\) and \(B\) respond to changes in these variables, the relations among the interest rates are assumed to be such that the optimal value of \(L + B\) is at some interior point,

\[(3.35)\]
\[\frac{r_n (1-\rho)}{r_n - r_p} > L_p + B > \alpha\]

This condition is a conclusion to be drawn from the second order conditions for minimization of (3.23).
The second order conditions for minimization of (3.23) with respect to $L_p$ and $B$ require that the Hessian of that function be positive definite; therefore, the determinant of that Hessian,

\[(3.36) \quad (CD/DD)^2 \left[ \frac{1}{r} \frac{1}{1-\rho} \right]^2 \left[ 2r_n - r_s \right] \left[ \frac{r}{p} - 2r_n + \frac{2r_n (1-\rho)}{L+B} \right] \]

must be positive. Since $r_n > r_s$ by (3.25) and (3.31),

\[(3.37) \quad \frac{2r_n (1-\rho)}{L+B} > 2r_n - r_p \]

Inequality (3.37) can be rewritten as

\[(3.38) \quad \frac{2r_n (1-\rho)}{2r_n - r_p} > \frac{L+B}{2r_n - 2r_p} \]

And since

\[(3.39) \quad 2r_n - r_p > 2r_n - 2r_p, \]

\[(3.40) \quad \frac{r_n (1-\rho)}{r_n - r_p} \frac{2r_n (1-\rho)}{2r_n - 2r_p} \frac{2r_n (1-\rho)}{2r_n - r_p} \]

by (3.39). Therefore, by (3.38) and (3.40),
(3.41) \[ \frac{r_n (1-p)}{r_n - r_p} > (L_p + B); \]

the optimal value of \( L_p + B \) is less than the upper bound on \( L \).

Inequality (3.37) can be interpreted in terms of the costs and returns to the bank from borrowing to make loans to deposit customers. The bank's borrowing for loans to deposit customers is in the range

(3.42) \[ \frac{r_n (1-\phi)}{r_n - r_p} - L_p > B > 0 \]

At a particular volume of deposit customer loan demand \( L^*(CD) \) for

(3.43) \[ L^* > L_p, \]

the total cost of borrowing plus the opportunity cost of using the borrowed funds to make loans to deposit customers is

(3.44) \[ (L^*-L_p) \cdot 2r_n \cdot CD \]

One part of the total revenue to the bank from lending this amount to deposit customers is

(3.45) \[ r_p (L^*-L_p) \cdot CD \]
By making loans to deposit customers, the earnings from investing their deposits are retained, and the cost of paying for their withdrawal of deposits is saved. The dollar value of deposits retained per dollar of loans to deposit customers is \( (1/L^*) \), the customer's deposit to loan demand ratio. The return to the bank per dollar of deposits retained is

\[
(3.46) \quad 2r_n (1-\rho)
\]

The net return to the bank from borrowing \((L^*-L)\)CD for loans to deposit customers is

\[
(3.47) \quad (L^*-L_p) CD[r_p -2r_n +2r_n (1-\rho)/L^*]
\]

By (3.37), (3.47) is positive for

\[
(3.48) \quad L^* = L_p + B
\]

The total returns to the bank from borrowing reserves to make loans to deposit customers is positive if the ratio of loans the deposit customers demand to their deposits equals the maximum amount of loans the bank plans to make to deposit customers.

Equations (3.28) and (3.29) make up a system of two equations in the two unknowns \( L_p \) and \( B \) and parameters \( r_n, r_p, r_s, CD/DD, \rho \) and \( \alpha \). Therefore, \( L_p \) and \( B \) can be solved for in terms of the parameters. The solution for \( L_p \) and \( B \) are:

\[
(3.49) \quad L_p = \tilde{\delta}(r_n, r_p, r_s, CD/DD, \rho, \alpha)
\]

\[
B = \phi(r_n, r_p, r_s, CD/DD, \rho, \alpha)
\]
The parameters are numbered for expository convenience as:

\[ r_n = \delta_1; \quad r_p = \delta_2; \quad r_s = \delta_3; \quad \text{CD/DD} = \delta_4 \]

\[ \rho = \delta_5; \quad \alpha = \delta_6 \]

(3.50)

The derivatives of functions (3.49) can be derived by totally differentiating the system (3.28) and (3.29), which results in the following system of differential equations.

\[ \varepsilon_{L_p,L_p} dL_p + \varepsilon_{B,L_p} dB = - \sum_{i=1}^{6} \varepsilon_{i,L_p} d\delta_i \]

(3.51)

\[ \varepsilon_{L_p,B} dL_p + \varepsilon_{B,B} dB = - \sum_{i=1}^{6} \varepsilon_{i,B} d\delta_i \]

Denote the matrix of coefficients of system (3.51) as:

\[ S = \begin{bmatrix} \varepsilon_{L_p,L_p} & \varepsilon_{B,L_p} \\ \varepsilon_{L_p,B} & \varepsilon_{B,B} \end{bmatrix} \]

(3.52)

Then the solution for \(dL_p\) and \(dB\) are as follows:

\[ dL_p = - \sum_{i=1}^{6} \left[ \frac{S_{11}}{|S|} \varepsilon_{i,L_p} + \frac{S_{21}}{|S|} \varepsilon_{i,B} \right] d\delta_i \]

(3.53)

\[ dB = - \sum_{i=1}^{6} \left[ \frac{S_{12}}{|S|} \varepsilon_{i,L_p} + \frac{S_{22}}{|S|} \varepsilon_{i,B} \right] d\delta_i \]
where $|S|$ is the determinant of $S$ and $S_{ij}$ is the cofactor of $S$ associated with the element common to the $i$'th row and $j$'th column. The matrix $S$ is the Hessian of (3.23), and therefore, $|S| > 0$ is a necessary condition for a minimization of (3.23) with respect to $L_p$ and $B$.

3.5 The Optimal $L_p$ and $B$ as Functions of $r_n$

\[
\frac{\partial L}{\partial r_n} = -(CD/DD)^2 \left\{ \frac{1}{\left( \frac{n}{r_n (1-\rho)} \right)^{\alpha}} \right\}^2 \left\{ r_p - 2r_n + \frac{2n (1-\rho)}{L + B} \right\} \left\{ \frac{r_p - 2r_n + \frac{2n (1-\rho)}{L + B}}{L_p - \alpha + \frac{r_p r_n (1-\rho)}{(r_n - r_p)^2}} \right\} / |S| < 0
\]

by (3.37) and (3.30).

The fraction of bank deposits in the loans inventory is

\[(3.55)\] $L_p (CD/DD)$,

and the fraction invested in loans to non-depositors is

\[(3.56)\] $1-\rho - L_p (CD/DD)$,

which is inversely related to $L_p$. The optimal fraction of bank assets invested in loans to non-depositors is increased if the interest rate on loans to non-depositors is increased, assuming
all other things remain unchanged.

The sign of the partial derivative $\frac{\partial B}{\partial r_n}$ can most easily be determined by first determining the sign of $\frac{\partial (L_p + B)}{\partial r_n}$.

\[(3.57)\]

$$\frac{\partial (L_p + B)}{\partial r_n} = \frac{\partial L_p}{\partial r_n} + \frac{\partial B}{\partial r_n}\]

$$= (CD/DD)^2 \left[ \left( \frac{l}{r_n (1-p)} \right) - \frac{a}{r_n - r_p} \right]^2 \left( r_s - 2r_n \right) \left\{ \left( \frac{r_p}{r_n} \right)^2 - (L_p + B) \right\} / |S|$$

by (3.29), i.e., by introducing the constraint on the variables derived from assuming that B is chosen in such a way as to minimize the expected loss function. The sign of (3.57) depends upon the relations between $r_n$ and $r_p$; (3.57) is definitely positive if

\[(3.58)\]

$$2r_p - r_n > 0,$$

and if (3.58) is negative, the sign of (3.57) depends upon the value of $L_p + B$. By (3.57),
\[ (3.59) \quad \frac{\partial B}{\partial r_n} = \frac{\partial (L + B)}{\partial r_n} - \frac{\partial L}{\partial r_n} \]

If (3.58) holds,

\[ (3.60) \quad \frac{\partial (L + B)}{\partial r_n} > 0 \]

and by (3.54)

\[ (3.61) \quad - \frac{\partial L}{\partial r_n} > 0 \]

Therefore, by (3.59), (3.60), and (3.61),

\[ (3.62) \quad \frac{\partial B}{\partial r_n} > 0 \]

Let \( EL \) be expected loans to deposit customers divided by DD.

\[ (3.63) \quad EL = \frac{CE/DD}{(L + B) - \alpha} \int_{\alpha}^{L + B} LdL = \frac{(CD/DD)(L + B + \alpha)}{2} \]

By (3.60), expected loans to deposit customers is an increasing function of \( r_n \). And by (3.54), (3.56), and (3.60), the sum of expected loans to deposit customers plus loans to non-depositors is an increasing function of \( r_n \).

An increase in \( r_n \) has five effects on expected bank profits in this model. As \( r_n \) increases,

1. the opportunity cost of holding a given loans inventory increases;
(2) the opportunity cost of a given level of expected borrowing for loans to deposit customers increases;
(3) the cost of borrowing a given amount of funds increases;
(4) the costs associated with a given level of expected demand deposit withdrawals increases, and
(5) the competitive compensating balance ratio rises, which means that with a given level of \( L_p + B \), expected demand deposit withdrawals is reduced and that a given increase in the borrowing limit decreases expected demand deposit withdrawals more than before the increase in \( r_n \).

The effects of (1) - (3) tend to reduce the optimal size of \( (L_p + B) \), and the effects of (4) and (5) tend to increase the optimal size of \( (L_p + B) \) as \( r_n \) increases. Given the way the model is constructed, the optimal size of \( (L_p + B) \) is an increasing function of the interest rate \( r_n \) if \( 2r_p \) is greater than or equal to \( r_n \); under that condition effects (4) and (5) dominate effects (1) - (3).
CHAPTER IV

THE EFFECTS OF MONETARY POLICY ON THE BANK LOAN MARKET

Monetary policy is assumed to be carried out through open market operations. Open market operations change the rate of return on government securities. The share of demand deposits owned by deposit customers is shown to respond to changes in monetary policy. The model of bank portfolio selection developed in this chapter is used to demonstrate the role of bank–customer relationships in the response within the bank loan markets to monetary policy changes; the response within the bank loan markets are registered in terms of loan interest rates and the allocation of bank loans among borrowers. The impact of a change in monetary policy is illustrated for a monetary policy that is becoming more restrictive. The likely reactions in the bank loan market, under the assumptions of this model, are an increase in the difference between \( r_n \) and \( r_p \) and a rise in the share of loans going to deposit customers.

4.1 The Effects of a Change in \( r_s \)

The model has not been developed in such a way as to incorporate the effects of anticipated capital gains on assets. \(^{72}\)

\(^{72}\) For an approach to bank asset selection that incorporates expectations about capital gains and losses, see Porter (1967).
Therefore, a rise in the rate of return on government securities affects bank decision making only by changing the yield on government securities during the current period. A rise in \( r_s \) increases the profitability of investing a given loans inventory, all other things unchanged since it increases the earnings of the loans inventory if the demand for loans by deposit customers is less than the loans inventory.

\[
(4.1) \quad \frac{\partial L_p}{\partial r_s} = \frac{(L_p - \alpha)/(2r_n - r_s)}{0}
\]

A rise in \( r_s \) increases the optimal share of bank funds in the loans inventory.

\[
(4.2) \quad \frac{\partial B}{\partial r_s} = -\frac{(L_p - \alpha)/(2r_n - r_s)}{0} < 0
\]

\[
(4.3) \quad \frac{\partial (L_p + B)}{\partial r_s} = \frac{\partial L_p}{\partial r_s} + \frac{\partial B}{\partial r_s} = 0
\]

A change in \( r_s \), all other things unchanged, does not change the ratio of the maximum loans offered to deposit customers divided by DD, and therefore, does not affect expected loans to deposit customers. But a rise in \( r_s \) does increase the optimal level of \( L_p \), which means that the share of bank funds invested in loans to non-depositors is reduced at the existing rate \( r_n \). Aggregating across banks, the market supply function of loans to non-depositors shifts to the left with a rise in \( r_s \).
4.2 The Effects of a Change in CD/DD

By equations (3.28) and (3.29), the optimal value of \( L_p \) and \( B \), are not affected by changes in the ratio CD/DD, the fraction of demand deposits owned by deposit customers. Therefore, the fraction that the optimal loans inventory and borrowing limit are of total bank deposits, \( L_p (\text{DC/DD}) \) and \( B(\text{CD/DD}) \), change by the same percentage as the ratio CD/DD, all other things unchanged. According to the following analysis, the ratio of deposits held by deposit customers to total deposits is affected by monetary policy.

Let:

\[
U_i = U_i \text{ (DD}_i \text{)}
\]

be the total return the \( i \)'th depositor receives on his average demand deposit balance in the form of the utility demand deposits can offer, such as reducing transactions costs for a given volume of transactions. The \( i \)'th customer also receives implicit interest on his deposits in the form of bank services.

Let:

\[
U^*_i = U^*_i \text{ [S(DD}_i \text{)]}
\]

be the utility the \( i \)'th customer derives from bank services other than demand deposits. The amount of services offered depends upon the average deposit balance \( \text{DD}_i \). As specified above, service charges on

\[73\] This approach to presenting the returns from holding money balances is similar to the approach by Johnson (1969). He assumed a functional relations between the money supply and the rate of return society receives from holding money in his discussion of the returns to society from inventing money.
Demand deposits are assumed to equal the costs to the bank of servicing deposit accounts. Let the rate of return on government securities \( r_s \) be the opportunity cost of holding demand deposits, and let \( R^i \) be the net return to the \( i \)'th depositor from holding demand deposits. Then if the \( i \)'th depositor does not demand a loan from the bank,

\[
R^i = U_i(DD^i) + U_i[S(DD^i)] - r_s(DD^i)
\]

The depositor demands that level of demand deposits that maximizes the net return from holding demand deposits.

\[
\frac{\partial R^i}{\partial DD^i} = \frac{\partial U_i(DD)}{\partial DD^i} + \frac{\partial U_i^*}{\partial s} \cdot \frac{\partial s}{\partial DD^i} - r_s = 0
\]

The demand for demand deposits as a function of \( r_s \) is

\[
r_s = \frac{\partial U_i(DD^i)}{\partial DD^i} + \frac{\partial U_i^*}{\partial s} \cdot \frac{\partial s}{\partial DD^i}
\]

The \( j \)'th depositor has a demand for a loan of size \( L_j \). For given interest rates \( r_n \) and \( r_p \), the maximum prime rate loan he can demand is a function of his average demand deposit held,

\[
[r_n(1-\rho)/(r_n-r_p)]DD_j
\]

At the demand deposit level

\[
DD^*_j = L_j \begin{bmatrix} \frac{r_n-r_p}{r_n} \\ \frac{r_n}{r_n(1-\rho)} \end{bmatrix},
\]

the full amount of the loan demand \( L_j \) is made at the prime rate. At demand deposit levels above \( DD^*_j \), the \( j \)'th customer receives implicit interest on deposits in the form of bank services. His demand for loans affects his return from holding demand deposits.
\[ R_j^D = U_j(DD_j) - r_s(DD_j) + (r_n - r_p) \left( \frac{r_n}{r_n - r_p} \right) DD_j \]

\[ = U_j(DD_j^*) - r_s(DD_j^*) + r_n(1-\varphi)(DD_j) \quad \text{for } DD_j^* \geq DD_j \]

\[ = U_j(DD_j^*) - r_s(DD_j^*) + U_j[S(DD_j^* - DD_j)] \quad \text{for } DD_j^* \geq DD_j \]

\[ \frac{\partial R_j^D}{DD_j} = \frac{\partial U_j}{DD_j^*} - r_s + r_n(1-\varphi) \quad \text{for } DD_j^* \geq DD_j \]

\[ = \frac{\partial U_j}{\partial DD_j^*} - r_s + \frac{\partial U_j}{\partial S} \cdot \frac{\partial S}{\partial DD_j^*} \quad \text{for } DD_j^* \geq DD_j^* \]

The utility customers \( i \) and \( j \) derive from the use of demand deposits is assumed to be the same at each level of demand deposits. The marginal utility function is graphed in Figure 2. Up to the deposit level \( DD_j^* \), the \( j \)'th deposit customer receives an additional return from holding demand deposits at the interest rate \( r_n(1-\varphi) \). Beyond the deposit level \( DD_j^* \), customer \( j \) receives implicit interest in the form of bank services. Customer \( j \) receives diminishing marginal utility from bank services. Although the bank offers services to customer \( i \) at the cost of \( r_n(1-\varphi)DD_i \) to the bank, customer \( i \) receives diminishing marginal utility from the services, as indicated by the marginal utility function for customer \( i \).

\[ \text{Note that } r_n(1-\varphi) \text{ is the competitive interest rate on demand deposits established in section 2.7.} \]
Figure 2

Demand for Deposits

$$r_s = \frac{\partial U_i}{\partial DD_i} + r_n (1-\rho)$$

$$r_s = \frac{\partial U_i}{\partial DD_i} + \frac{\partial U_i^*}{\partial DD_i}$$

$$r_s = \frac{\partial U_i}{\partial DD_i}$$

$$r_s = \frac{\partial U_i}{\partial DD_i} = \frac{\partial U_i}{\partial DD_i}$$

0  DD_i^0  DD_i^1  DD_i^2  DD_i^3  DD_i^4  DD_i^5  DD_i^6

DD

rs

rs0

rs1
Assume the interest rate on securities is initially $r_s^0$ and rises to $r_s^1$. The demand for deposits by customer $i$ declines from $DD_i^0$ to $DD_i^1$, and the demand for demand deposits by customer $j$ declines from $DD_j^0$ to $DD_j^1$. In this example, the deposits on which the deposit customer $j$ demands an interest rate reduction on a loan as implicit interest is $DD_j^*$, which remains unchanged when $r_s$ rises. Aggregating this analysis over all depositors, the percentage of deposits on which deposit customers demand implicit interest in the form of loans rises with a rise in the interest rate on government securities, with other interest rates unchanged.

4.3 Reactions in the Bank Loan Markets

The way in which the bank alters its asset selection when the money supply is reduced by the monetary authorities depends upon the demand functions for bank loans by depositors and non-depositors. With a rise in the ratio of deposits owned by deposit customers, all other things unchanged, the bank increases the fraction of bank assets in the loans inventory and the ratio of the borrowing limit to bank assets. Therefore, a reduced share of bank deposits is allocated to loans to non-depositors at a given rate $r_n$. The market supply curve of loans to non-depositors shifts to the left because of the decline in bank deposits and the reduced share of these deposits invested in loans to non-depositors at the existing interest rate $r_n'$, as shown in Figure 3.
FIGURE 3  A SHIFT IN THE SUPPLY FUNCTION OF LOANS TO NON-DEPOSITORS
If the demand curve for loans by non-depositors is not perfectly elastic, the equilibrium interest rate \( r_n \) will rise. By (3.54), a rise in \( r_n \), all other things unchanged, will decrease the optimal ratio \( L_p \) and keep the share of bank deposits in the loans inventory \( (L_p/CD/DD) \) from rising as much as if \( r_n \) had remained unchanged.

The process of reaction in the bank loan market to monetary policy has been traced from rise in \( r_s \), as a response to a rise in CD/DD, to a change in \( r_n \). An increase in \( r_n \) causes the supply function of loans to deposit customers to shift to the left, and with \( r_p \) unchanged, the maximum ratio of prime rate loan to deposit that a deposit customer can demand falls. Unless the demand for loans by deposit customers is perfectly elastic, the equilibrium prime rate will be bid up when \( r_n \) rises, as shown in Figure 4. With a rise in \( r_n \) from \( r_n^1 \) to \( r_n^2 \) the supply curve in terms of \( r_p \) and \( L \) shifts to the left. If \( r_p \) remains unchanged, the equilibrium maximum loan to deposit ratio offered deposit customers is reduced from \( L_1 \) to \( L_2 \). With the given demand curve \( r_p \) rises from \( r_p^1 \) to \( r_p^2 \) and \( L \) is \( L_3 \).

A change in \( r_p \), all other things unchanged, affects the allocation of bank funds among borrowers.

\[
(4.13) \quad \frac{\partial L_p}{\partial r_p} = \left[ \alpha - \frac{(r_n)^2(1-p)}{(r_n-r_p)} \right] \left( \frac{1}{2r_n-r_s} \right) > 0
\]
FIGURE 4  A shift in the supply curve of loans to deposit customers.
By (4.14) a rise in $r_p$, other things unchanged, increases the optimal share of bank deposits in the loans inventory and decreases the optimal share in loans to non-depositors. If the demand curve for loans by non-depositors is not perfectly elastic, the equilibrium solutions in the bank loan markets include an increase in the difference between $r_n$ and $r_p$, and if the demand curve by non-depositors is not extremely inelastic, the optimal share of bank deposits in the loans inventory will rise and the share in loans to non-depositors fall.

The effect of a change in $r_p$ on the optimal level of $B$ can most easily be determined by first determining the sign of $\frac{\partial (L_p + B)}{\partial r_p}$.

\begin{equation}
(4.14) \quad \frac{(L_p + B)}{r_p} = (\text{CD/DD}) \left[ \frac{1}{r_n (1-\rho)} \left( \frac{r_n}{r_n - r_p} \right) - \alpha \right]^{2} \left( \frac{2r_n - r_s}{r_n} \right) \cdot \left[ \left( L_p + B \right) + \frac{r_n (2r_n - r_p) (1-\rho)}{(r_n - r_p)^2} \right] / |S|
\end{equation}

If

\begin{equation}
(4.15) \quad 2r_p - r_n > 0,
\end{equation}

\begin{equation}
(4.16) \quad \frac{\partial (L_p + B)}{\partial r_p} < 0
\end{equation}

since
(4.17) \[ 2r_n > r_s \]

by (3.25) and (3.31). If (4.15) is negative, the sign of (4.14) depends upon the initial value of \( L_p + B \). Since

\begin{align*}
(4.18) \quad \frac{\partial (L_p + B)}{\partial r_p} &= \frac{\partial L_p}{\partial r_p} + \frac{\partial B}{\partial r_p} \\
(4.19) \quad \frac{\partial B}{\partial r_p} &= \frac{\partial (L_p + B)}{\partial r_p} - \frac{\partial L_p}{\partial r_p}
\end{align*}

By (4.13),

\begin{equation}
(4.20) \quad \frac{\partial L_p}{\partial r_p} > 0
\end{equation}

and if inequalities (4.16) and (4.20) hold,

\begin{equation}
(4.21) \quad \frac{\partial B}{\partial r_p} < 0
\end{equation}

By (3.57) and (4.14) note that

\begin{equation}
(4.22) \quad \frac{\partial (L_p + B)}{\partial r_n} = \left[ -\frac{\partial (L_p + B)}{\partial r_p} \right] \frac{r_p}{r_n}
\end{equation}
The reaction within the bank loan markets to monetary policy can be traced through several rounds of reaction within individual markets. The next round of reaction could begin with the demand for demand deposits as a function of the interest rate \( r_s \). The increase in \( r_n \), \( r_p \), and \( r - r \) have effects upon the demand for demand deposits by deposit customers that may go in different directions. The increases in interest rates may reduce the deposit customer's loan demand. The maximum loan to deposit customers relative to their deposits may either increase or decrease, depending upon the relative increases in \( r_n \) and \( r_p \).

\[
\frac{\partial}{\partial r_n} \left[ \frac{r_n (1-\rho)}{r_n - r_p} \right] = -\frac{r_p (1-\rho)}{(r_n - r_p)^2} \\
\frac{\partial}{\partial r_p} \left[ \frac{r_n (1-\rho)}{r_n - r_p} \right] = \frac{r_n (1-\rho)}{(r_n - r_p)^2}
\]

With equal increases in \( r_n \) and \( r_p \), the ratio \( r_n (1-\rho)/r_n - r_p \) increases by (3.3) and (3.4). The ratio \( r_n (1-\rho)/r_n - r_p \) does not change if the percentage increases in \( r_n \) and \( r_p \) are the same.
Suppose \( r_p \) increases to \( r_p(L+X) \) and \( r_n \) increases from \( r_n \) to \( r_n^2 \). For no change in the maximum loan to deposit ratio:

\[
\frac{r_n^1(1-p)}{r_n^1 - r_p} = \frac{r_n^2(1-p)}{r_n^2 - r_p(1+X)}
\]

(4.23)

\[
r_n^2 = r_n^1(1+X)
\]

If the percentage increase in \( r_n \) is larger than the percentage increase in \( r_p \), the ratio \( r_n(1-p)/r_n - r_p \) decreases; deposit customers would be offered lower prime rate loans with the same level of demand deposits.

Conclusions can be drawn from this model of bank loan markets about the effects of monetary policy on the allocation among banks of loans to individuals. Since the interest rate at which banks borrow from one another is assumed to be the same as the rate of return on loans to non-depositors, only banks with deposit customers for whom other banks are competing will borrow from other banks. As \( r_n \) and \( r_n - r_p \) rise, banks with such deposit customers increase the ratio of expected loans to these customers to total bank deposits. These banks can make this change only by increasing expected borrowing from other banks. Therefore, the share of expected bank loans to
individuals by banks having deposit relationships increase during a period of restrictive monetary policy if the share of the loss in deposits at banks having deposit customers is not large enough to offset this effect.
CHAPTER V

SUMMARY AND CONCLUSIONS

The effect of changes in the environment of banks on the allocation of bank funds among groups of assets can be analyzed within a portfolio selection model of an individual bank. The issue analyzed in Chapter IV is the impact of monetary policy on the allocation of bank loans between borrowers who are depositors at the lending bank, the deposit customers, and borrowers who are not depositors at the lending bank, the non-depositors. Following a summary of this model, the methodology and conclusions of this model are compared with those of other banking studies.

5.1 A Review of Chapter II

The model of bank portfolio selection developed in Chapters II and III is designed to include some of the effects of bank-customer relationships on bank portfolio selection in the theory of banking. A bank-customer relationship exists if the terms at which a customer is able to purchase one bank output is linked to the nature of other business done with the bank. A deposit relationship is a restricted form of a bank-customer relationship in which a bank competes for the deposit of a customer having a demand for loans by offering him loans at interest rates below the rates charged other borrowers.
With a given level of bank capital bankers maintain or increase lending capacity by retaining or increasing deposits. With direct interest payments on demand deposits prohibited the competition between banks for deposits takes other forms, such as rate reductions on bank services and interest rate reductions on loans. Bank service cost reductions as a means of competing for deposits are not considered in the bank portfolio selection model.

Most of Chapter II is a review of the existing literature on the role of deposit relationships in bank asset selection. The bank makes loans to deposit customers at rates below those rates charged non-depositors and imposes a compensating balance requirement on the deposit customers; the deposit customers must hold demand deposit balances at least as large as some fraction of their loans or the lines of credit. The closer a customer's ratio of demand deposit to bank loan is to the required compensating balance ratio, the more likely the bank is to measure the customer's required compensating balance as the minimum instead of the average balance during the period of the deposit relationship.

The profitability of deposit relationships is examined from the points of view of both the bank and the customer. A deposit relationship is profitable for the bank if the earnings from investing the deposits attracted by loan rate reductions are at least as large as the interest income sacrificed by making loans to deposit customers. A deposit customer who may count his
compensating balance as his average balance receives both the benefits from holding demand deposits that other depositors receive, such as reduced transactions costs, and reduced borrowing costs on his loans. A deposit relationship is profitable to a deposit customer if these two sources of income from a given level of demand deposits are greater than or equal to the opportunity cost of holding the demand deposits.

The revenue a bank receives from a deposit relationship is the interest income on the loan to the customer plus the interest from investing the deposit. The opportunity cost of establishing the deposit relationship for the bank is assumed to be the earnings the bank would have received if the loan to the deposit customer had been invested in loans to non-depositors. Banks are assumed to make loans to all deposit customers at the same interest rate, the prime rate. For a given prime rate, market interest rate charged non-depositors, and reserve ratio, there is a loan to deposit ratio at which revenue from the deposit relationship equals the opportunity cost. The bank receives profits on deposit relationships in which the customers hold higher average deposit to loan ratios. Loan interest rate reductions for deposit customers are analyzed as indirect interest payments on demand deposits.

5.2 A Review of Chapter III

In the analysis of costs and returns to the bank from investing in loans to deposit customers developed in Chapter II, the loan
demand by deposit customers is assumed known. Many of the models in banking theory are constructed to analyze the impacts of several sources of uncertainty on bank portfolio selection. To incorporate the effects of deposit relationships into this format for bank portfolio selection, the demand for loans by deposit customers is assumed to be a random variable, and one of the assets among which the bank distributes its funds in an inventory for loans to deposit customers.

Bank asset classes in the portfolio selection model are loans to non-depositors, loans to deposit customers, securities, and reserves; excess reserves are assumed to be zero. Loans to deposit customers plus securities equal the inventory for loans to deposit customers. All of the funds in the inventory are invested in loans to deposit customers if the demand for loans by deposit customers is greater than or equal to the inventory for these loans. On the other hand, the excess funds in the inventory are invested in securities if this loan demand is less than the inventory.

A bank's objective function and the nature of competition among banks must be specified in order to analyze the optimal portfolio selection of the bank. The bank is assumed to allocate its assets in such a way as to maximize expected bank profits, which in this model is the same as minimizing the expected loss from holding the inventory for loans to deposit customers. Banks are assumed to be competitive in the markets in which they are allowed to compete.
One such market in which bankers are assumed to compete is the market for depositors who demand loans. All such depositors are offered loans at the prime rate, and the competitive banks offer depositors the volume of loans at the prime rate which make profits from deposit relationship zero. The opportunity cost of investing bank funds in the inventory for loans to deposit customers is the income that could have been received from investing these funds in loans to non-depositors. The bank's penalty for having too small an inventory for loans to deposit customers is the withdrawal of deposits by those deposit customers who have their loan demands refused. Changes in the prime rate and the interest rate on loans to non-depositors alter (a) the profitability of investing in loans to deposit customers and (b) the maximum ratio of prime rate loan to deposit that deposit customers can demand. The optimal share of bank funds to be invested in the inventory for loans to deposit customers is an increasing function of the prime rate and a decreasing function of the interest rate on loans to non-depositors.

The bank has the option of borrowing funds from other banks if the demand for loans by deposit customers is greater than the inventory of bank deposits held for this purpose. If the bank sets some limit on the amount of funds to be borrowed for this purpose, then maximum loans to the bank's deposit customers equal the inventory of bank deposits held for loans to these borrowers plus the bank borrowing limit. The optimal loans inventory plus borrowing limit may be less than the bank's maximum loan commitments to the bank's deposit customers, as is assumed in the
development of the bank portfolio selection model in Chapter III.

Since the demand for loans by deposit customers is a random variable, the actual level of loans to deposit customers cannot be determined at the beginning of the period. But with a probability distribution of loan demand by deposit customers, expected loans to these borrowers is a function of the maximum volume of loans the bank plans to make to them. The responses of a bank to changes in its environment are discussed in terms of the share of bank deposits invested in the inventory for loans to deposit customers and in terms of the ratio of expected loans to deposit customers divided by bank deposits.

5.3 A Review of Chapter IV

The portfolio selection model is used to trace the effects of restrictive monetary policy on the allocation of bank funds among borrowers. The share of total bank deposits owned by deposit customers is assumed to increase during a period of restrictive monetary policy. In order to keep the optimal share of bank deposits in the inventory for loans to deposit customers from increasing, the interest rate on loans to non-depositors must increase more than the prime rate. The demand function for bank loans by non-depositors is assumed to be neither perfectly elastic nor extremely inelastic in order to illustrate the interaction of the market for loans to deposit customers and the market for loans to non-depositors. The difference between the interest rate on loans to non-depositors and the prime rate increases, and also the share of bank deposits invested
in the inventory for loans to deposit customers increases as monetary policy becomes more restrictive. If twice the prime rate is assumed greater than or equal to the interest rate on loans to non-depositors, expected loans to deposit customers is an increasing function of the interest rate on loans to non-depositors and a decreasing function of the prime rate. If an increase in the difference between these two interest rates is assumed to be a result of a restrictive monetary policy, expected loans to deposit customers divided by the bank's deposits are increased, and also the share of bank deposits invested in loans to non-depositors is decreased during the period of a restrictive monetary policy.

5.4 The Relation of This Study to Other Banking Literature

5.4.1 Other Bank Portfolio Selection Models

The significance of the approach and conclusions of this study are discussed in relation to those of several other banking studies. This theoretical study of bank portfolio selection differs from most of the portfolio selection models referred to in Chapter I in that:

(a) The bank forms distinct supply functions for loans to deposit customers and loans to non-depositors, and

(b) the source of bank funds for investment, deposits, is linked to the use of bank funds, the allocation
of bank loans between depositors and non-depositors. This methodology permits analysis of the impacts on bank portfolio section of a source of uncertainty not included in the other models, the demand for loans by deposit customers. By imposing constraints on the bank in the form of deposit relationships, a differential treatment of deposit customers and non-depositors may be analyzed as a rational policy for an expected profit maximizing bank.

5.4.2 The Deposit Relationship Literature

The analysis of the profitability of deposit relationships in the published sources on this topic is used to set the limits on the competition among banks for deposits. The maximum ratio of prime rate loan to deposits that deposit customers can demand is the ratio at which the profits from deposit relationships are zero. But the approach to studying bank asset selection in much of this literature, in particular by Hodgman and his students, is different from the approach taken in the bank portfolio selection models. The demand for loans by deposit customers is assumed known in this deposit relationship literature, and the profitability of individual

75 Morrison (1966); Orr and Mellon (1961); Porter (1961); Pierce (1964), (1967); Russell (1964).

76 Cooper (1967); Hodgman (1961), (1963); Murphy (1969); Richards (1967).
deposit relationships is examined instead of the profitability of investing in loans to deposit customers as a class of assets. The effects of deposit relationships on bank asset selection are incorporated into a bank portfolio selection model in this paper by assuming the demand for loans by deposit customers to be a random variable and the inventory of bank funds reserved for these loans an individual asset.

Kane and Malkiel incorporate some effects of customer relationships into a bank portfolio selection model. They assume the bank to be a risk averter and that maximization of a quadratic utility function is its objective. The loan requests of depositors affect the objective function of the bank differently than loan requests of other borrowers in that refusing the loan requests of depositors increases the risks of the bank. Their model differs from the model developed in Chapter III in the following ways:

1. The bank in Chapter III is assumed to be an expected profit maximizer; Kane and Malkiel assume the bank is a risk averter.

2. The demand for loans by deposit customers as a random variable is incorporated into the Chapter III model; Kane and Malkiel do not explicitly consider depositor loan demand as a random variable.

3. An inventory for loans to deposit customers is an individual asset in Chapter III; Kane and Malkiel have no such asset class in their model, but they

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77 Kane and Malkiel (1965).
consider the impacts of loan demands by individual deposit customers on the bank's objection function.

4. The excess demand for loans by deposit customers is linked directly to demand deposit withdrawals in Chapter III. Kane and Malkiel reflect the bank's actions on depositor loan requests in a variable called quality of customer relationship, and the qualities of customer relationships affect the expected return and risk of the bank's portfolio of assets.

5.4.3 Deposit Relationships or Credit Rationing

One objective of constructing the portfolio selection model is to examine how the ability of models with expected bank profit maximization as the objective function are restricted in explaining bank behavior by ignoring the influences of deposit relationships. Any differential treatment of borrowers violates the assumptions of a portfolio selection model in which loans are assumed to be homogeneous assets earning a unique rate of return. The pattern of differential treatment of borrowers who are depositors and borrowers who are non-depositors described in section 5.3 is consistent with the assumption of expected bank profit maximization. This pattern of differential treatment of borrowers may explain behavior which has been rationalized by assuming other types of bank competition and motivation.

The differential treatment of borrowers due to risk is not analyzed in the portfolio selection model in Chapter III. The implications of risk for the loan supply functions of a bank to
individual borrowers and for credit rationing have been analyzed for the expected profit maximizing bank.\textsuperscript{78} To incorporate the effects of risk into a bank portfolio selection model, loan demand functions and probability distributions of return from the investment of loans would have to be assumed for each potential borrower. A portfolio selection model would be complicated even more if the interbank competition analyzed by Jaffee and Modigliani is assumed to exist;\textsuperscript{79} they assumed that banks collude by putting borrowers into a few groups and offering all borrowers within a group the same loan terms. The portfolio selection model is simplified by assuming that banks are competitive.

The availability doctrine literature is discussed in section 1.3.3. In the sources in this literature monetary policy is assumed to be effective in restricting the availability of funds to private borrowers because lenders are assumed to allocate funds among private borrowers by non-price credit rationing.\textsuperscript{80} Statements about the prevalence of credit rationing in the availability doctrine literature are not defended with data.

\textsuperscript{78} Hodgman (1960); Freimer and Gordon (1965); Jaffee and Modigliani (1969).

\textsuperscript{79} Jaffee and Modigliani (1969).

\textsuperscript{80} Wallich (1946, p. 765); Roosa (1951); Scott (1957a, 1957b); Ellis (1951); Kareken (1957); Samuelson (1952, pp. 695-97).
It can be shown that actions which are consistent with expected bank profit maximization appear to be inconsistent with this objective if the effects of deposit relationships on bank asset selection are ignored. At each point in time the loan interest rates paid by deposit customers are less than the loan interest rates paid by non-depositors if banks compete for deposits. As monetary policy becomes more restrictive, the interest rates paid by non-depositors rise faster than the rates paid by deposit customers, and the share of loans going to non-depositors decreases under the assumptions about the bank loan market made in section 5.3. If the ownership of deposits is ignored in a study of the bank loan market, the above observations about the bank loan market would appear to be inconsistent with expected bank profit maximization. If the interest rate charged deposit customers is assumed to be the market loan rate, there will be an excess demand for loans at that interest rate, and it would appear that the loan rate has not risen enough to clear the market.

5.4.4 The Differential Effects of Tight Money

Another objective of this study is to establish a theoretical basis for a relation between monetary policy and the differential treatment of borrowers. Three studies of the differential effects of tight money on large and small borrowers are discussed in section 1.3.5. These studies are entirely empirical in content; no economic theory of banking is used to discuss the motivations of banks that
would cause the lending terms offered large borrowers relative to the terms offered small borrowers to respond to monetary policy. By making deposit customers and non-depositors the two groups of borrowers, a theoretical basis for differential treatment of borrowers in these two groups can be established for an expected profit maximizing bank. If monetary policy is assumed to operate through open market operations, changes in the loan terms offered these two groups of borrowers can be related to monetary policy. Under the assumptions made in sections 5.2 and 5.3, the interest rate charged non-depositors rises faster than the prime rate, and also the share of bank loans going to non-depositors declines as monetary policy becomes more restrictive.

The portfolio selection model developed in Chapter III reflects the prohibition of direct interest payments on demand deposits. The bank retains deposits if the loan demands of its depositors are met, and the banker does not have the option of retaining these deposits or attracting more deposits by direct interest payments on demand deposits. Banks might change their means of competing for deposits from loan policy to direct interest payments if these payments are permitted. Such a change in the form of competition would affect the

81 Eckert (1958); Bach and Huizenga (1961); Silber and Polakoff (1970).
treatment of deposit customers relative to the treatment of non-depositors. The rational bank will not sacrifice more loan interest income in attracting a deposit than it has to pay in interest directly to attract the same size of deposit. To the extent that the groupings of large and small borrowers correspond to the groupings deposit customers and non-depositors, the differential treatment of large and small borrowers is influenced by the prohibition of interest payments on demand deposits.

The prohibition of interest payments on demand deposits has been linked to the development of customer relationships in the market for loans to security dealers. These loans were used by banks during the 1920's for short-term investments of surplus funds and were called to meet reserve deficiencies. After the prohibition of direct interest payments on demand deposits, bank established customer relationships with these borrowers and ceased to consider calling these loans as a source of short-term liquidity. Garvey noted a link between compensating balances and the prohibition of interest on demand deposits. "Prior to 1933 commercial banks paid interest on demand deposits, no compensating balances in the current sense were held, and the practice of requiring commensurate balances was less prevalent than now." And the

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82 Board of Governors, Federal Reserve System (1959, p. 27, p. 31).
83 Garvey (1959, p. 32).
prime rate as it is now known dates back to 1934, one year after interest payments on demand deposits were prohibited. A hypothesis that can be drawn from such observations is that bank loan policy would cease to be influenced by borrowers' demand deposit balances if banks were permitted to pay interest on demand deposits. Unfortunately, this hypothesis cannot be tested empirically.

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84 Hodgman (1963, p. 120).
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