



## Resource Overview

**Quantile® Measure:** 560Q

**Skill or Concept:** Use the commutative and associative properties to simplify numerical expressions. (QT-N-161)  
Use the distributive property to simplify numerical expressions. (QT-N-578)

**Excerpted from:**



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# Set A8 ★ Activity 1



## ACTIVITY

### The Commutative, Associative, & Distributive Properties

#### Overview

Although students often use the commutative, associative, and distributive properties to solve problems, they may not know the properties by name. This activity introduces the properties in the context of mental math.

#### Skills & Concepts

- ★ identify applications of commutative, associative, and distributive properties with whole numbers

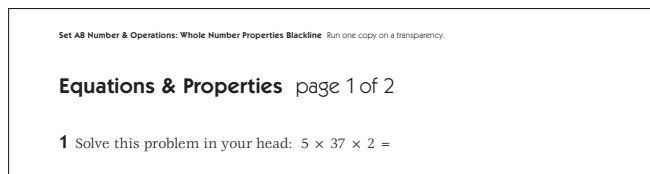
#### You'll need

- ★ Equations & Properties (pages A8.5 and A8.6, run a copy of each sheet on a transparency)
- ★ Defining & Using the Properties (page A8.7, run a class set)
- ★ paper to mask parts of the overhead
- ★ overhead pens

**Advance Preparation** Before you conduct this activity, write the terms Commutative Property, Associative Property, and Distributive Property on the board.

#### Instructions for The Commutative, Associative, & Distributive Properties

1. Draw students' attention to the terms you have written on the board. Read them together. Ask volunteers to share anything they know about these terms.
2. Explain that even if these terms don't sound very familiar, students may know more about them than they think. Each of the properties you have listed has to do with the way numbers behave, and today, you are going to investigate these behaviors together.
3. Place Equations and Properties on the overhead. Before you turn on the projector, cover the entire transparency with a piece of paper. Explain that when you turn on the projector, students will each think privately about the problem they see and raise their hand when they have the answer. Then turn on the projector and move the paper to reveal problem 1.



4. When most hands are up, ask students to pair-share their solutions and strategies. Then call on several volunteers to share their thinking with the class.

**Students** *It's 390. At first I thought it was hard. Then I realized I could just multiply  $5 \times 2$ . That's 10, and  $10 \times 39$  is 390. I got the same thing. I was going to round 37 up to 40, but then I saw  $5 \times 2$ .*

**Activity 1** The Commutative, Associative, & Distributive Properties (cont.)

5. Chances are, at least some students will report that they switched the order of the numbers, perhaps mentally reversing the position of 37 and 2 to make the problem read  $5 \times 2 \times 37$ . Is this okay? Is it true that  $37 \times 2$  is the same as  $2 \times 37$ ? Ask students to talk with one another about this issue, and then invite volunteers to defend the move.

**Students** *Sure it's okay. You can switch numbers when you multiply. Like  $3 \times 4$  is the same as  $4 \times 3$ , right?*

*You can do that when you add, too.  $7 + 3$  is 10, but so is  $3 + 7$ .*

6. Reveal the rest of the overhead one section at a time. Read and discuss the steps with your students to explain that when reformulating  $5 \times 37 \times 2$  to make it easier to solve mentally, you actually need to use both the associative and the commutative properties. Together, these two properties let you add or multiply a string of numbers in any order.

Set A8 Number & Operations: Whole Number Properties Blackline Run one copy on a transparency.

**Equations & Properties** page 1 of 2

**1** Solve this problem in your head:  $5 \times 37 \times 2 =$

Order of operations says we multiply the numbers in order from left to right:

$$(5 \times 37) \times 2$$

We can use two properties to make this problem easier to solve:

**Associative Property**  
*Changing the way you group three numbers or numerical expressions when you add or multiply does not change the answer.  $(2 + 3) + 4 = 2 + (3 + 4)$  and  $(2 \times 3) \times 4 = 2 \times (3 \times 4)$*

$$(5 \times 37) \times 2 = 5 \times (37 \times 2)$$

**Commutative Property**  
*Changing the order of two numbers or numerical expressions when you add or multiply does not change the answer.  $5 + 3 = 3 + 5$  and  $3 \times 4 = 4 \times 3$*

$$5 \times (37 \times 2) = 5 \times (2 \times 37)$$

If we use the associative property again, we can make the problem even easier to solve.

$$5 \times (2 \times 37) = (5 \times 2) \times 37$$

$$10 \times 37 =$$

What's the answer?

7. Use the second overhead to introduce the distributive property. When you have reviewed all three properties, ask students to identify which one(s) is/are most helpful in solving each of the four problems at the bottom of the second overhead.

**Activity 1** The Commutative, Associative, & Distributive Properties (cont.)

Set A8 Number & Operations: Whole Number Properties Blackline Run one copy on a transparency.

### Equations & Properties page 2 of 2

**2** Solve this problem in your head:  $6 \times 17 =$

Did you multiply  $6 \times 10$  and then  $6 \times 7$ , and then add the two products? If you did, you used the

**Distributive Property**  
*When you multiply a number you can break that number apart. Multiply each part separately, and then add the products. You will still get the same answer.*

$$6 \times 17 = 6 \times (10 + 7)$$

$$6 \times (10 + 7) = (6 \times 10) + (6 \times 7)$$

$$(6 \times 10) + (6 \times 7) = 60 + 42$$

$$60 + 42 =$$

What's the answer?

**3** Solve the problems below in your head. Use one or more of the properties to help?

**Commutative Property** *Switch the order of two numbers.*  
**Associative Property** *Group it differently.*  
**Distributive Property** *Break the number apart and multiply one part at a time.*

<b>a</b> $(28 \times 50) \times 2 =$	<b>b</b> $(40 \times 267) \times 60 =$
<b>c</b> $5 \times 37 =$	<b>d</b> $4 \times (25 \times 298) =$

8. Next, give students each a copy of Defining and Using the Properties. Review the sheet with the class and provide clarification as needed. When students understand what to do, let them go to work. Encourage them to share their thinking with others nearby as they work.

Set A8 Number & Operations: Whole Number Properties Blackline Run a class set.

NAME \_\_\_\_\_ DATE \_\_\_\_\_

### Defining & Using the Properties

**1** Write your own definition of each property, and give an example so you can remember how it works.

Property	Definition	Example
<b>a</b> Commutative		
<b>b</b> Associative		
<b>c</b> Distributive		

**2** For each problem below:

- Use one or more of the above properties to rewrite the equation in a form that's easier to solve.
- Solve it and write the answer.
- Circle all the properties you used to rewrite the equation (C for commutative, A for Associative, and D for Distributive.)

Problem	Rewrite	Answer	Property
<b>ex.</b> $(70 + 469) + 30$	$(70 + 30) + 469$	569	(C) (A) D
<b>a</b> $(5 \times 39) \times 2$			C A D
<b>b</b> $(27 \times 25) \times 4$			C A D
<b>c</b> $4 \times 27$			C A D
<b>d</b> $(40 + 579) + 60$			C A D
<b>e</b> $6 \times 28$			C A D
<b>f</b> $(16 \times 50) \times 2$			C A D

**Activity 1** The Commutative, Associative, & Distributive Properties (cont.)**Extension**

- Consider doing the Factor Puzzles activity described below with some or all of your students. Write  $12 \times 25 = 75 \times 4$  on the overhead and ask students to help you prove this statement is true by using factoring and the commutative and associative properties. Can they start with  $12 \times 25$  and help you take whatever steps are necessary to transform  $12 \times 25$  into  $75 \times 4$ ? List the steps students advise you to take on the overhead. After they are finished you could also go back and label each step taken with F (factoring), C (using the commutative property) or A (using the associative property), again, as advised by students.

**Students** *I see what to do... Turn 12 into  $3 \times 4$  and write  $3 \times 4 \times 25$ .*

*Then have the 4 and the 25 switch places.*

*Then you can put the 3 and the 25 together because they'll multiply to make 75.*

$$\begin{aligned}
 &12 \times 25 \\
 &(3 \times 4) \times 25 \quad \text{F (factoring)} \\
 &3 \times (4 \times 25) \quad \text{A (associative factor)} \\
 &3 \times (25 \times 4) \quad \text{C (commutative property)} \\
 &(3 \times 25) \times 4 \quad \text{A (associative property)} \\
 &75 \times 4
 \end{aligned}$$

Next, have students advise you on the steps required to prove that  $5 \times 100 = 10 \times 50$ .

$$\begin{aligned}
 &5 \times 100 \\
 &5 \times (10 \times 10) \quad \text{F (factoring)} \\
 &(5 \times 10) \times 10 \quad \text{A (associative)} \\
 &(10 \times 5) \times 10 \quad \text{C (commutative)} \\
 &10 \times (5 \times 10) \quad \text{A (associative)} \\
 &10 \times 50
 \end{aligned}$$

Now write the following statements on the whiteboard or overhead:

$$4 \times 50 = 5 \times 40$$

$$12 \times 25 = 6 \times 50$$

$$25 \times 16 = 20 \times 20$$

$$30 \times 20 = 60 \times 10$$

Ask student pairs to choose any two of these statements, write them in their journals and prove they are true by using factoring and the commutative and associative properties of multiplication. (All the expressions require factoring but some might involve only one of the properties, depending on how students order the factors.) If your students enjoy these puzzles, you might make up some more or ask interested children to create additional puzzles for their classmates.

**INDEPENDENT WORKSHEET**

Use Set A8 Independent Worksheets 1 and 2 to provide students with more practice identifying applications of the commutative, associative, and distributive properties with whole numbers.

# Equations & Properties page 1 of 2

1 Solve this problem in your head:  $5 \times 37 \times 2 =$

Order of operations says we multiply the numbers in order from left to right:

$$(5 \times 37) \times 2$$

We can use two properties to make this problem easier to solve:

### ***Associative Property***

*Changing the way you group three numbers or numerical expressions when you add or multiply does not change the answer.  $(2 + 3) + 4 = 2 + (3 + 4)$  and  $(2 \times 3) \times 4 = 2 \times (3 \times 4)$*

$$(5 \times 37) \times 2 = 5 \times (37 \times 2)$$

### ***Commutative Property***

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If we use the associative property again, we can make the problem even easier to solve.

$$5 \times (2 \times 37) = (5 \times 2) \times 37$$

$$10 \times 37 =$$

What's the answer?

## Equations & Properties page 2 of 2

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### ***Distributive Property***

*When you multiply a number you can break that number apart. Multiply each part separately, and then add the products. You will still get the same answer.*

$$\begin{aligned} 6 \times 17 &= 6 \times (10 + 7) \\ 6 \times (10 + 7) &= (6 \times 10) + (6 \times 7) \\ (6 \times 10) + (6 \times 7) &= 60 + 42 \\ 60 + 42 &= \end{aligned}$$

What's the answer?

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