AEX

Aerospace Education Excellence

for

ADVANCED MATH

CIVIL AIR PATROL

AEX

The Aerospace Education EXcellence Award Program

ACTIVITY BOOKLET
for Advanced Math

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Lesson 1: A Bag of Trigs

OBJECTIVE

Students will apply both right triangle and non-right triangle trigonometry within the context of aviation.

NATIONAL STANDARDS

Mathematics

Number and Operations
- compute fluently and make reasonable estimates

Algebra
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and apply mathematics in contexts outside of mathematics

Representation
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science

Unifying concepts and processes in science
- Change, constancy, and measurement

Physical science
- Motions and forces

Science and technology
- Abilities of technological design
- Understanding about science and technology

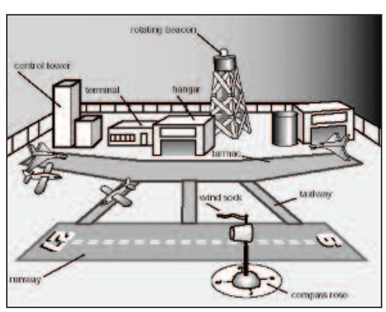
History and nature of science
- Science as a human endeavor
- Historical perspectives

Technology

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS

- Handout of FAA Airport Diagram—one per student group. Download diagrams from the National Aeronautical Charting Office - **http://www.faa.gov/airports/runway-safety/diagrams** or use one from a set of "approach plates". (Approach Plates is a common term used to describe the printed procedures or charts, more formally Instrument Approach Procedures, that pilots use to fly approaches during Instrument Flight Rules (IFR) operations.) See the example in Appendix II
- Chalk, masking tape, or other suitable method of reproducing an airport diagram
- Die-cast airplane models (hopefully) scaled to the airport diagram
- Meter sticks—at least two
- A copy (or actual) of your area's aviation sectional chart. Consider also downloading the pdf of the chart to project during whole-class discussions
- Twine (or other non-stretchy string)



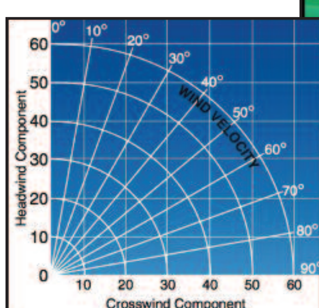
BACKGROUND INFORMATION

Applications of trigonometry abound in aviation. Two examples of computations involving trignonometry and aviation are the crosswind component at an airport given runway heading and wind information, and the heading and estimated time enroute (ETE) for a basic flight plan. The former employs right-triangle trigonometry while the latter usually involves non-right triangles.

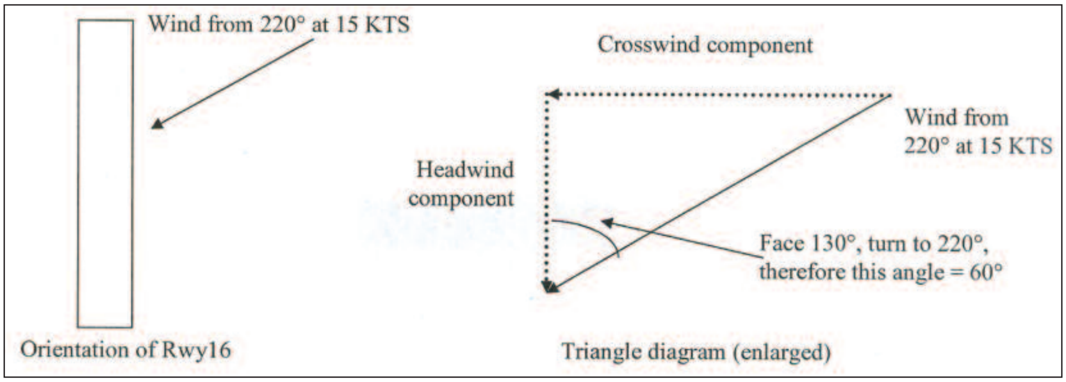
Computing a crosswind component *before* takeoff or landing helps prepare the pilot for the action, suggests how much "correction" might be needed, and helps determine whether the action should even be attempted. Specific techniques must be used for crosswind takeoffs and landings, which have a practical maximum given aircraft design and pilot skill. Consider teaching students a "rule of thumb" that adequately estimates the crosswind calculation after demonstrating the trigonometry upon which they are derived–which immediately follows.

One must first know the orientation of a runway. For example, if Mr. Smith stood on the end of a runway as it stretched ahead and the heading on his compass read 160°, then it would be called "Runway 16." Note the trailing zero is dropped from the compass heading. The opposite direction runway (standing from the opposite end) would be "Runway 34" because 340° is an "about face" (a rotation of 180°). Note that this can be confusing when an airport has reversed digits for the opposing runways (Rwy 13 and Rwy 31). Runway heading is magnetic (rather than "true") because the primary navigation instrument in airplanes is a magnetic compass.

When wind direction is spoken, such as from surface observations broadcast over aviation band radios, it is given as a magnetic direction "from" its source. Imagine holding a compass and facing directly into the wind–the compass reads the wind direction. Note that the wind is actually traveling in the opposite direction, as with a 220° wind comes from the Southwest it is actually traveling in the Northeast direction. Reporting wind this way simplifies calculations.

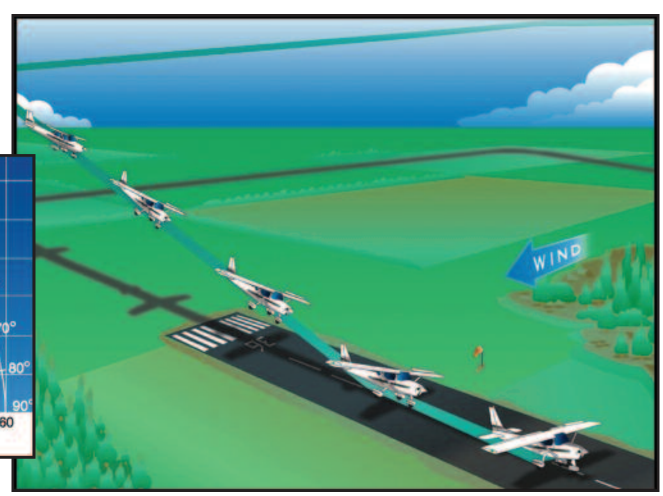


Use the facts above: Runway 16 and Winds 220° at 15 knots. Subtract the runway heading from the wind direction. Then, take the sine of the resulting angle (60°) and multiply it by 15 to get the crosswind component. 15•sin(60°) ≈ 13. The following diagram shows why the angles were subtracted and why sine is used. Note that the cosine results in the "headwind component" of the wind, which could necessitate a change in the airspeed of an airplane on approach.

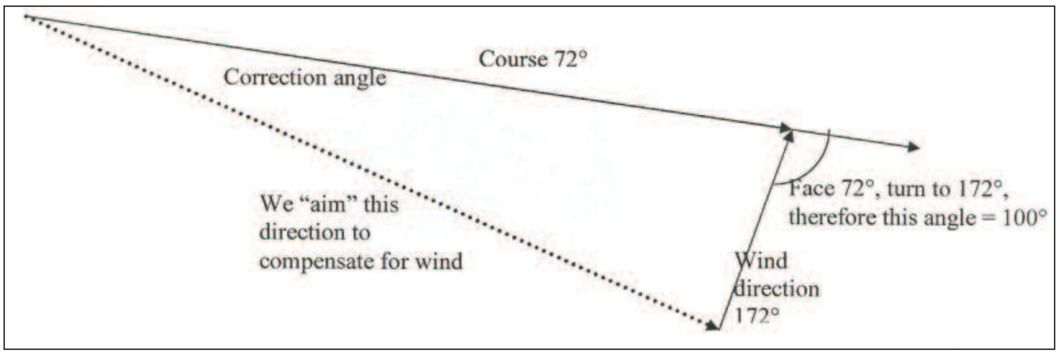


Student pilots should memorize three sine ratios (sines of 30°, 45°, and 60° which are 0.5, 0.7, and 0.9 respectively). The cosines, of course, have the same ratios, only reversed in order because they are from the complementary angle of the triangle. It makes little sense to fumble for a calculator during critical phases of flight when a mentally computed estimate is just as good. All wind angles will be close enough to 30°, 45°, or 60° suggesting one uses the actual wind speed for wind directions of 60° or more. Note that wind gusts are not addressed in the classroom application.

Knowing where to aim the aircraft (heading), its speed over the ground (groundspeed), and the amount of time a flight should take (estimated time enroute) are the primary components of flight planning. The process is very simple when there is no wind: heading and distance are read directly from the aviation chart, groundspeed = airspeed, and ETE = (distance) ÷ (airspeed). This, of course, is rare because the atmosphere is constantly in motion.



To compute airspeed, use actual forecast winds aloft (see ADDS link at the end of this lesson), convert the direction into magnetic (wind directions are true when in textual form), and complete a triangle diagram. Assume the course (magnetic heading between departure and destination airports) is 72° magnetic and at a distance of 86 NM (nautical miles). The winds at 3000', 6000', and 9000' respectively read "9900 1906+16 1914+10" and are translated to "calm, from 190° at 6 KTS, and from 190° at 14 KTS." The +16 and +10 are the forecast temperatures (in Celsius). The magnetic variation is about 18° East, so the wind direction to use in the calculation is 190° – 18° = 172°. At this point, students make a scale drawing on graph paper (North up) so they can "get the feel" of the relative distances and angles. When the drawings are reasonably well understood, transition to sketches like the following:



The course line extends past where the wind vector intersects so one can visualize this external angle. The angle from the course to the wind origin is 100° in this example, making its supplement 80°. Here's what is known so far: one angle of the triangle measures 80° and the side opposite it will be the airspeed of the aircraft (use 102 KTS). Solve for the "correction angle" and know the opposite side has length 14 KTS (the wind speed). Solve using the *Law Of Sines as follows:*

$$\frac{\sin(100)}{102} = \frac{\sin(\theta)}{14}$$

which equals a correction angle of approximately 7.8°, rounded up to 8°. This makes the heading 72 + 8 = 80°– aim the aircraft at this heading and the aircraft should fly along the course line. Solve for the groundspeed by computing the "length" of the side of the triangle along the course (note: all these computations involve speeds). The unknown angle of the triangle is 180 – (8 + 80) = 92° so one can use the *Law Of Sines* again, as follows:

$$\frac{\sin(100)}{102} = \frac{\sin(92)}{x}$$

although the *Law Of Cosines* would also work. Although the answer is approximately 103.5, round this down to 103 KTS because it's better to underestimate speed, and travel slightly slower, to make sure the plane has enough fuel! The ETE would be

$$\frac{86}{103} =$$

0.835 hours, which is reported as 50 minutes. Allow students to obtain practice with this by taking "fantasy trips" to various airports. Check answers with the *Low Approach* online calculator (Heading, Groundspeed, and Wind Correction Angle section).

PROCEDURE

Headwind and Crosswind

1. Create a model of your local airport that is a reasonably large scale and oriented the same as the airport. Reproduce the FAA airport diagram for students to replicate using chalk or using masking tape on the floor of the classroom. Scaling the diagram to a die cast model makes for a good additional exercise.
2. Place a meter stick adjacent to the "runway" on a heading the wind might have (such as use 190° if the runway were at 160°). Let every ten centimeters (a decimeter) represent one knot of wind speed. Have students measure the angle between the runway and the "wind" and repeat several times until they discover all they have to do is subtract the wind direction from the runway heading. Measure with meter sticks the legs of the right triangle formed relative to the runway (see the first diagram).
3. Leave one of the set-ups and have students create scale drawings using the *Crosswind and Headwind Practice* handout.
4. Introduce the trigonometry (using sine and cosine) that will facilitate computation (without the tedium of scale drawings).

Flight Planning

1. Create a model of your area's aviation sectional chart in the classroom or another location that will remain undisturbed for a few days. This has the potential to be a long term project where terrain, highways, and other geographic features are included along with airport symbols and navigation aids. Take care to orient the scale chart to true North.

2. Select a departure airport and a destination airport on your chart. Students will then measure the distance between them (in nautical miles–the number of minutes of latitude) and compass heading (taken off the chart, corrected for magnetic variation, and confirmed with a compass). Measure the distance with meter sticks or trundle wheel (not string–the reason to avoid string will be revealed in Step 4).
3. Obtain (or invent) wind speed and direction. Place a box fan (as in the variations above) to signify the wind. (You can use winds aloft from Aviationweather.gov.)
4. Stretch twine (or other non-stretchy string) between the departure and destination airports. Announce that the length of the twine represents the SPEED of the aircraft through the air, and, therefore, its ground speed in a no-wind condition. Make marks on the twine at each end of the course. Compute the length of the twine proportional to the wind speed. (For example, if the twine is 4.5 m long, the airspeed is 100 KTS, and the wind is 10 KTS then the wind twine is 1/10 as long, therefore 45 cm.) Place another length of twine on the destination airport, mark on the twine the proper length, and stretch the twine in the direction of the wind. While holding one end of the course twine at the departure airport, move the twine from the destination end until it intersects the wind twine at the mark representing the wind speed. The angle between the course line and the airspeed twine is the "correction angle" and the length of twine from the destination airport and the intersection point is the ground speed of the aircraft. Refer to the diagram in the Teacher Notes area.
5. Solve a few more situations using the process in Step 4.
6. Debrief this part of the activity by requesting suggestions for advantages and disadvantages of this method. *Hopefully* someone will say it is fairly cumbersome (even though it can be implemented on an actual chart rather than a larger replica). Lead the students towards "there MUST be a more efficient way!" Conclude this session with the reasons trigonometry was invented and how it easily translates to this application.

7. Using the example in the Teacher Notes and the *Law of Sines*, the correction angle is solved using the equation: $\frac{\sin(100°)}{102} = \frac{\sin(\theta)}{14}$ which equals a correction angle of approximately 7.8° after the algebra is completed. Continue as in the Teacher Notes and work another example.

SUMMARY

The calculations are performed by pilots every day, although the traditional method (using an "E6B Flight Computer") avoids the actual mathematics and, instead, depends upon using a step-by-step method. With the actual mathematics revealed, students (including student pilots) better understand what they are computing and can better judge the reasonableness of the outcome.

EVALUATION

Choose both departure and destination airports not already practiced. Provide winds aloft and winds at the destination. Then, allow students to perform computations. They should measure both course and enroute distance before beginning computations. Consider performing the same calculations over several days using current conditions to reinforce that the atmosphere is constantly changing and that flight plans must be adjusted accordingly.

LESSON ENRICHMENT/EXTENSION

Headwind and Crosswind Variations

- Place a box fan at the departure end of a classroom "runway" to signify the wind (turn it on, of course). Its angle to the runway should replicate that of the wind in the exercises.
- Have students approximate using the sines of 30°, 45°, and 60° rather than worrying about exact answers (as explained previously). The rapidity of mental math and having values to the nearest knot are sufficient for our purposes (and those of actually flying aircraft).

Flight Planning Variations

- Consider beginning the activity without the scale chart if students had a reasonable understanding of charts and had access to them (i.e., every student or work group had a chart). See the links below for (inexpensive) practice aeronautical sectional charts and the Seattle Sectional in PDF form.
- Bring in a ground school instructor to teach students how to solve for heading and ground speed using a traditional "flight computer" (known as an E6B). They will quickly realize that using trig is actually less complicated.

- Obtain performance data from an aircraft flight manual (see Appendix I) to determine how much time and distance will be devoted to the climb to cruise altitude. Note that the Cessna 172 P is expected to take 20 minutes over a distance of 26 nautical miles (NM) in a climb to 9000 feet.
- Account for descent to the destination airport—typically the winds are different and the descent speed is greater than cruise speed, so further calculation would be needed. A Cessna 172 can usually (and safely) descend at 120 KTS, however the descent rate is kept to around 500 feet per minute for the comfort of pilot and passengers. This means if the plane must lose 7000 feet from cruise altitude to enter the traffic pattern above an airport, then the descent would take 7000 ÷ 500 = 14 minutes. This means the descent must begin 28 NM from the destination.

ASSOCIATED WEBSITES AND/OR LITERATURE

- Airport diagrams and terminal procedures
http://www.faa.gov/airports/runway_safety/diagrams

- Geodetic Calculations software
http://avn.faa.gov/index.asp?xml=aeronav/applications/programs/compsys

- Airport/Facility Directory - digital
http://aeronav.faa.gov/afd.asp?cycle=afd_05APR2012&eff=04-05-2012&end=05-31-2012

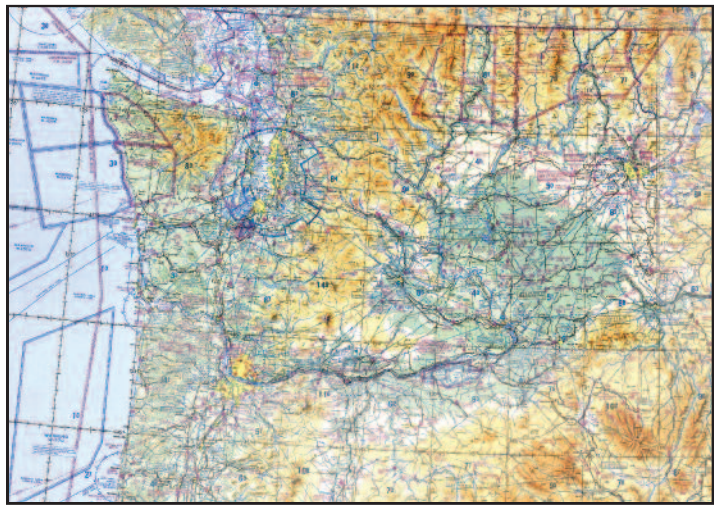
- E6B online calculator
http://www.theultralightplace.com/E6B%20Calculator.htm

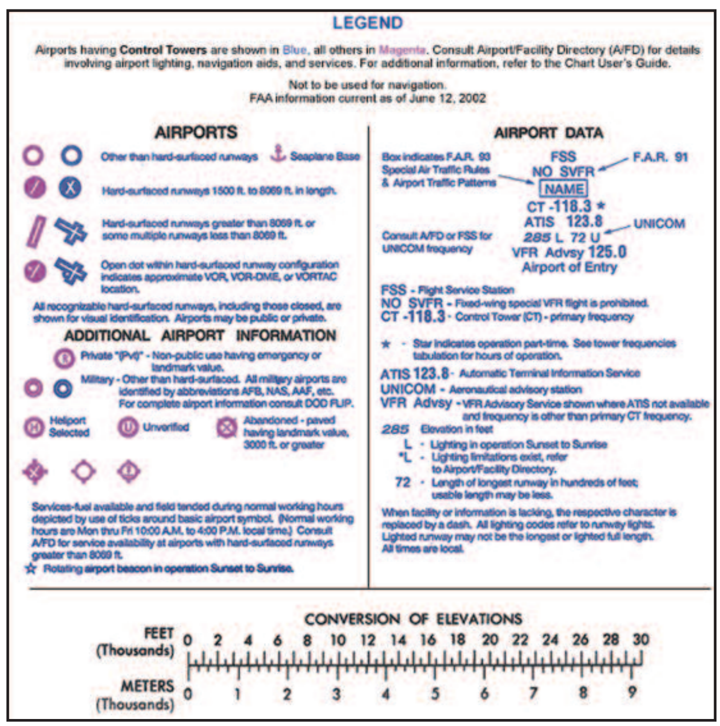
- Live ATC feed
http://www.liveatc.net/

- Aviation Digital Data Services (ADDS) Forecast Winds Aloft
http://aviationweather.gov/products/nws/winds/

- Paper navigation charts available through
FAA, AeroNav Services Team
REDIS, AJW-379
10201 Good Luck Road
Glenn Dale, MD 20769-9700
(800) 638-8972 toll free, U.S. only
9-AMC-Chartsales@faa.gov
aeronav.faa.gov

- Washington State Sectional chart - digital version, for classroom training only!
http://www.wsdot.wa.gov/aviation/Charts/default.htm





- Reasons trigonometry was invented - Reasons Trigonometry was invented
http://www.math.rutgers.edu/~cherlin/History/Papers2000/hunt.html

Crosswind and Headwind Practice

Assume the wind is blowing FROM 190° at 16 KTS. Calculate the crosswind and headwind components for each of the given airports relative to their named runway.

Boeing Field, RWY 13	Bremerton, RWY 19
Paine Field, RWY 16	Tacoma Narrows, RWY 17
Paine Field, RWY 11	Shelton, RWY 23
Port Townsend, RWY 09	Crest Airpark, RWY 15

Appendix I
Cessna 172 P Climb Performance

TIME, FUEL, AND DISTANCE TO CLIMB

MAXIMUM RATE OF CLIMB

CONDITIONS:
Flaps Up
Full Throttle
Standard Temperature

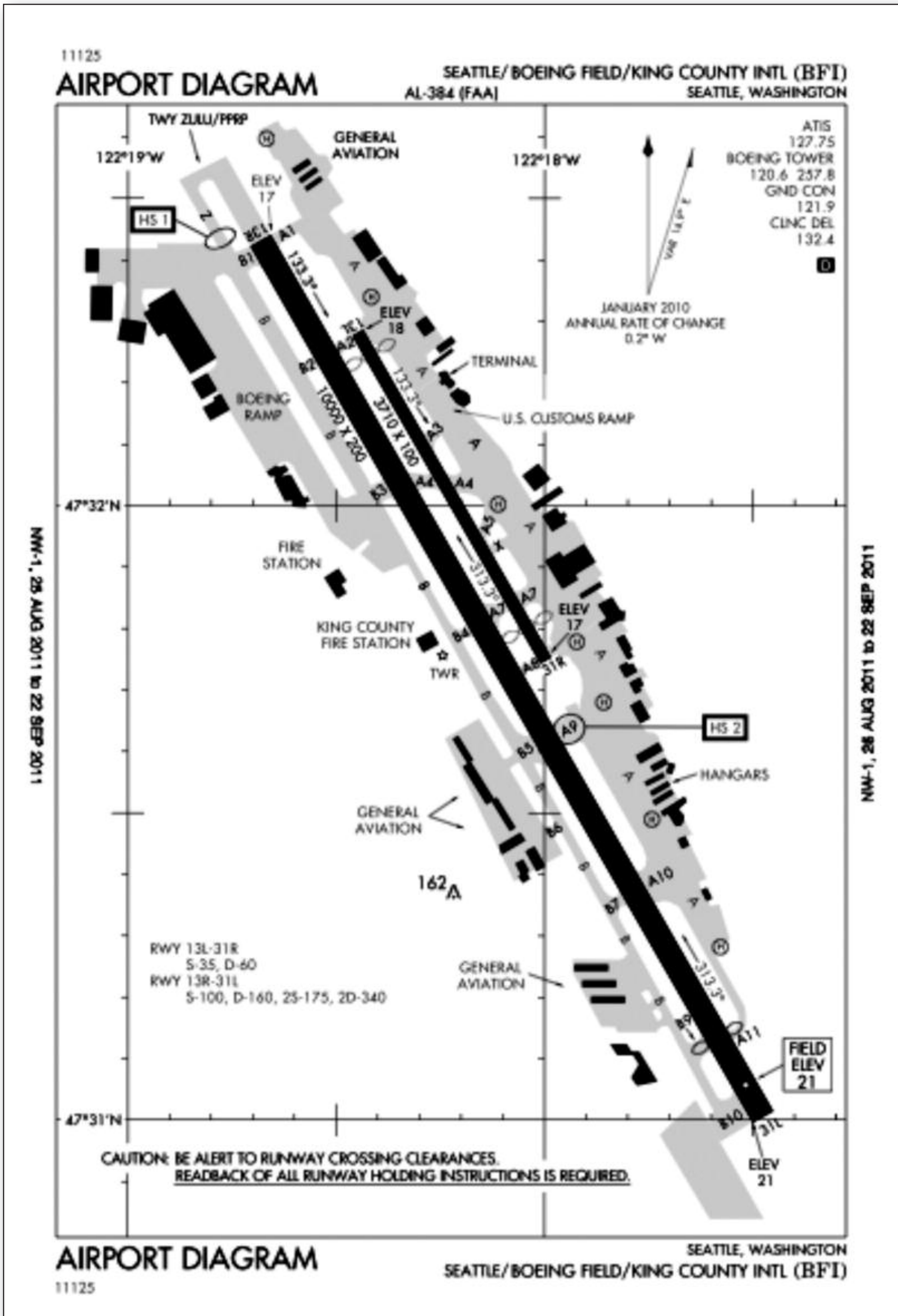
NOTES:
1. Add 1.1 gallons of fuel for engine start, taxi and takeoff allowance.
2. Mixture leaned above 3000 feet for maximum RPM.
3. Increase time, fuel and distance by 10% for each 10°C above standard temperature.
4. Distances shown are based on zero wind.

WEIGHT LBS	PRESSURE ALTITUDE FT	TEMP °C	CLIMB SPEED KIAS	RATE OF CLIMB FPM	FROM SEA LEVEL		
					TIME MIN	FUEL USED GALLONS	DISTANCE NM
2400	S.L.	15	76	700	0	0.0	0
	1000	13	76	655	1	0.3	2
	2000	11	75	610	3	0.6	4
	3000	9	75	560	5	1.0	6
	4000	7	74	515	7	1.4	9
	5000	5	74	470	9	1.7	11
	6000	3	73	425	11	2.2	14
	7000	1	72	375	14	2.6	18
	8000	-1	72	330	17	3.1	22
	9000	-3	71	285	20	3.6	26
	10,000	-5	71	240	24	4.2	32
	11,000	-7	70	190	29	4.9	38
	12,000	-9	70	145	35	5.8	47

Figure 5-6. Time, Fuel, and Distance to Climb

Page 5-15 from the *Information Manual* for a 1981 Cessna 172P.

Appendix II
Example of Airport Diagram



http://aeronav.faa.gov/d-tpp/1205/00384AD.PDF

Lesson 2: Day Length For Various Latitudes Along A Longitude

OBJECTIVE

Students will model real-world data using the trigonometric function *Tangent*.

NATIONAL STANDARDS

Mathematics

Algebra
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
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- monitor and reflect on the process of mathematical problem solving

Communication
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Connections
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Representation
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science

Unifying concepts and processes in science
- Change, constancy, and measurement

Science and technology
- Understanding about science and technology

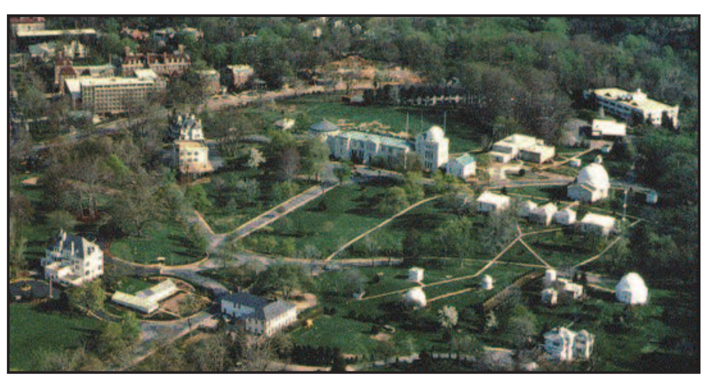
History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge

Technology

Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:
- Internet access to the US Navy Observatory Website
- Computer-based graphing program (such as Grapher on the Macintosh)
- One hand out (Day Length Along A Longitude) for each student



BACKGROUND INFORMATION:

During the debrief of an activity comparing day length over a year for Boston versus Seattle, a precalculus student asked, "What would a graph of the day lengths along a line of longitude look like?" The teacher became excited because he suspected the outcome to appear like a tangent function. The teacher's suspicions were later confirmed after collecting and plotting the "data" from the US Navy Observatory (USNO) Website (see Appendix I). Because modeling with trigonometric functions is typically limited to using sine (or a shifted cosine), the teacher, who is also the author of this book, created this project as a way to model with tangent.

The USNO's *Astronomical Applications* Website was accessed to make the initial plot (see Appendix II for both the "data" and the modeling function). The 122° W longitude was selected (the closest to Seattle) and 21 June 2010 (the Summer solstice). After entering the longitude and 90° N along with "8 hours West of Greenwich," the Website was unable to calculate for latitudes beyond 82° (both North and South). Both 80° and 70° returned no times for either sunrise or sunset, so the day length was inferred to be 24 hours. See the images in Appendix IV for an interesting pictorial confirmation. Times of sunrise and sunset were given for the remaining latitudes (entered in 10° increments) through 60° S, after which "0" was inferred for the day length. Because a tangent function was assumed, y = a•tan(x) + 12 was chosen as the general format where "x" represents the latitude and "y" represents the day length in hours. The "12" belongs in the equation because the day length at the Equator is approximately 12 hours regardless of the time of the year. The "formula" needed to fit Seattle's actual day length (as you will for your location), so another area of the USNO Website was used to get actual day length (see Appendix III) to fit the curve. The day length was found to be 16.00 hours and 47.0° was used for the latitude to create the function:

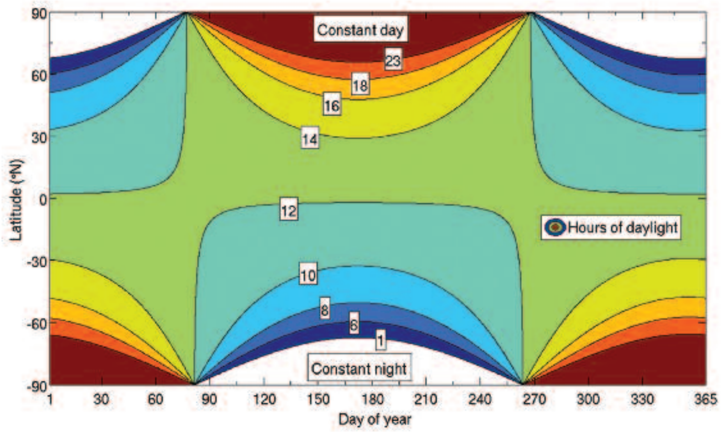
$16.00 = a \cdot \tan(47.0°) + 12.00$

$4.00 = a \cdot \tan(47.0°)$

$a = 4.00/\tan(47.0°) \approx 3.73$

Therefore the function for the 122°W longitude on 21 June 2010 is $y \approx 3.73 \cdot \tan(x) + 12$. This is represented by the curve in the plot in Appendix II. Note the function give a pretty close fit, deviating by only a few minutes at most.

Satisfied with the graphs, the teacher wrote an activity for his precalculus classes, assigned each student a different date (the class was able to cover the entire year with one-week increments), and posted student graphs in chronological order so the class could see the trend over the year. The process, detailed below, will (of course!) lead to further mathematical explorations!



One sample of a diagram comparing hours of daylight to latitude.

PROCEDURE:

1. Consider engaging in other trigonometric modeling activities before this one, especially one that uses the Earth's grid lines (such as *Modeling Satellite Orbits*).
2. Remind students of the basic gridlines on the Earth. They should understand the similarity between Cartesian graphing relative to using the intersection of the Equator and Prime Meridian as the "origin" with W longitudes "negative x-values" while the E longitudes represent "positive x-values." The y-axis, of course, extends from ⁻90 to 90 (from the South Pole to the North Pole respectively).
3. Ask students to sketch or graph the day length for your location along with the day length for a city they know that is a few degrees North or South of your location. Have them speculate how graphs would change as one moved closer to the poles (more extremes) or towards the equator (less extreme). The graphs would all have their maxima and minima on the same day. Invite speculation on the reasons behind these hypotheses.
4. Show the example in Appendix II or create a similar one for your area. Follow with the illumination diagrams (Appendix IV).
5. Derive the modeling function for the data using y = a•tan(x) + 12 as the general format. Show your solution for "a."
6. Assign specific days of the year to students with hopes of representing the entire year at reasonable intervals. Avoid 21 June because it has already been done.
7. Moderate as students locate and record the sunrise and sunset times. Suggest that a "number buddy" check the arithmetic used to compute the decimal equivalent for day length.

8. Check students' work as they solve for "a" to derive their modeling function.
9. Assist students as they graph the data and function, which should be on the same axes.
10. Post students' papers in chronological order. Ask students to record the date and "a" then write a short paragraph on how the graphs change over the year.
11. As a challenge question, consider asking students to create a function that results in the day length given the day of the year (e.g. 5 February is day #36) and latitude. Note: this procedure and model are merely a "rough and dirty" approximation of the day length and NOT what the USNO uses.
12. Find out what new questions were generated by students as they worked on this activity. Perhaps one of your students will "kick start" your next activity!

SUMMARY:
Interesting questions emerge when exploring day-length data. Students develop a greater understanding of how the sun illuminates the globe by comparing changes over a year at various latitudes. Note that the solstices are easy to determine from 5 day-length data but the equinoxes are somewhat variable by latitude.

EVALUATION:
Have students write a response to the following question: What pattern emerges in the graphs from 1 January through the year? Address each of the following in your response:
- how well the data is approximated by the Tangent function
- how the shape of the graph changes over the year
- the "end behavior" of the graphs over the year
- how the graphs would change if a different longitude were chosen for the activity (e.g. 90° to the East or West)

ASSOCIATED WEBSITES AND/OR LITERATURE:
- **http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-day-world**

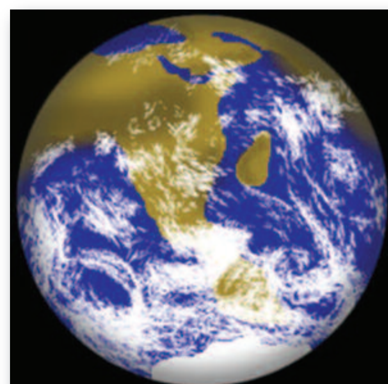
- **http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html**

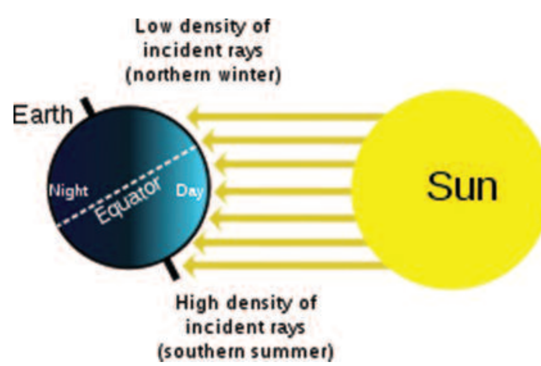
- **http://www.usno.navy.mil/USNO/astronomical-applications/data-services/earthview**

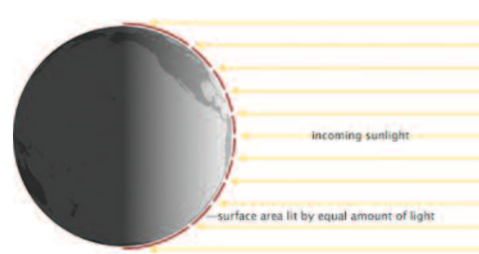
- **http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-year-us**

- **http://www.usno.navy.mil/USNO/astronomical-applications/data-services/duration-us**

- **http://www.jgiesen.de/SME/** - This website contains an applet that allows you to find the altitude and azimuth of the sun and moon at any point on earth at any time.





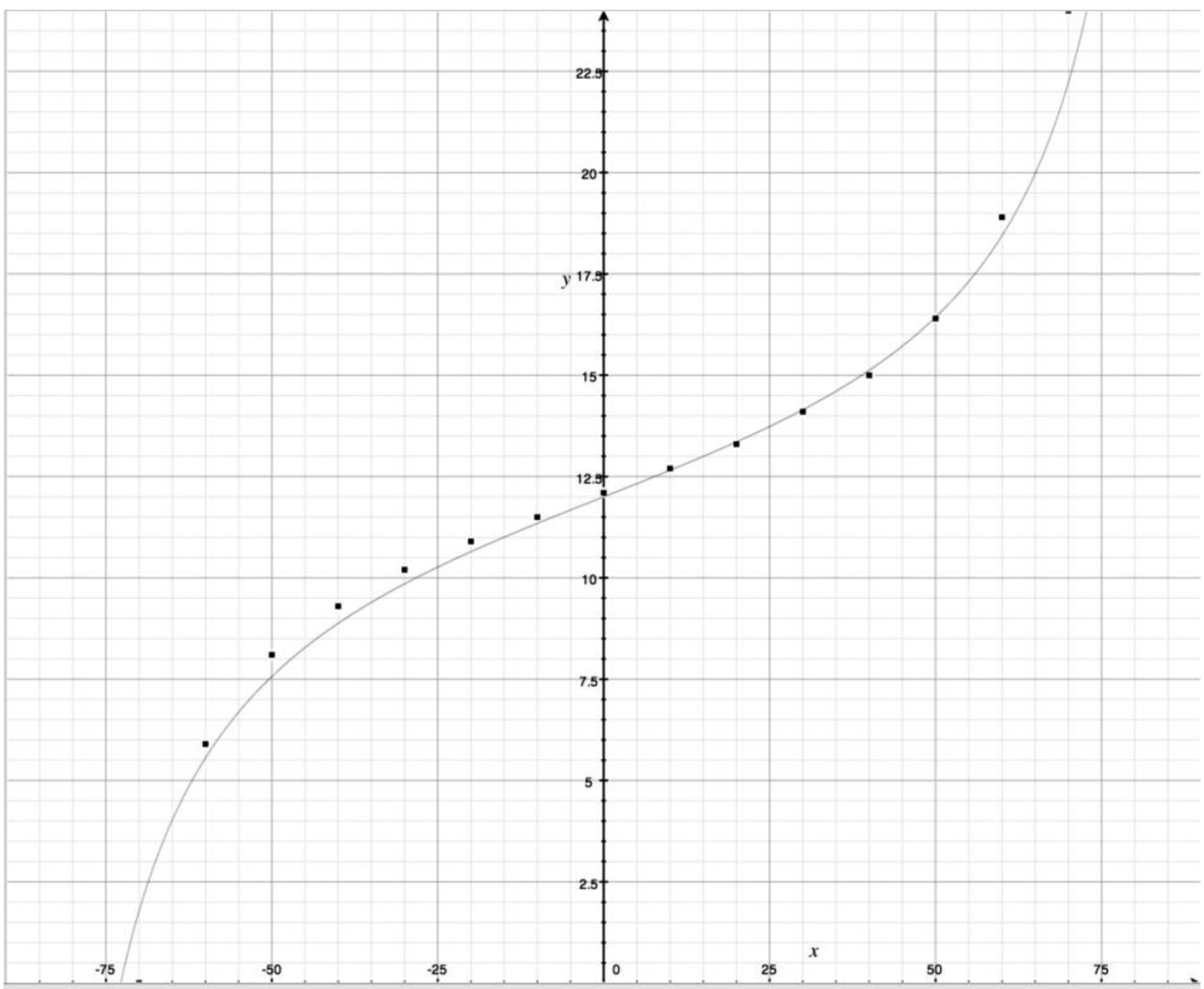


Appendix I
USNO Astronomical Applications
Day Length of Specified Day on Given Longitude & Latitude

Appendix II

Sample Plot: 21 June 2010 along the 122°W longitude

Latitude	Day Length Sunrise – Sunset	Difference # hours
90 N	☹	☹
80 N	24 (?)	24 (?)
70 N	24 (?)	24 (?)
60 N	21:36 – 02:44 =	18:52 = 18.9
50 N	20:21 – 03:59 =	16:22 = 16.4
40 N	19:40 – 04:39 =	15:01 = 15.0
30 N	19:12 – 05:07 =	14:05 = 14.1
20 N	18:50 – 05:30 =	13:20 = 13.3
10 N	18:31 – 05:49 =	12:42 = 12.7
0	18:14 – 06:06 =	12:08 = 12.1
10 S	17:56 – 06:24 =	11:31 = 11.5
20 S	17:37 – 06:42 =	10:55 = 10.9
30 S	17:16 – 07:03 =	10:13 = 10.2
40 S	16:50 – 07:30 =	9:20 = 9.3
50 S	16:12 – 08:08 =	8:04 = 8.1
60 S	15:06 – 09:14 =	5:52 = 5.9
70 S	0 (?)	0
80 S	0 (?)	0
90 S	☹	☹



Note: The plot in Appendix II was constructed in *Grapher* (a Macintosh computer application).

Appendix III

Sunrise and Sunset for Seattle, 2010

o , o ,
Location: W122 20, N47 38

SEATTLE, WASHINGTON
Rise and Set for the Sun for 2010

Astronomical Applications Dept.
U. S. Naval Observatory
Washington, DC 20392-5420

Pacific Standard Time

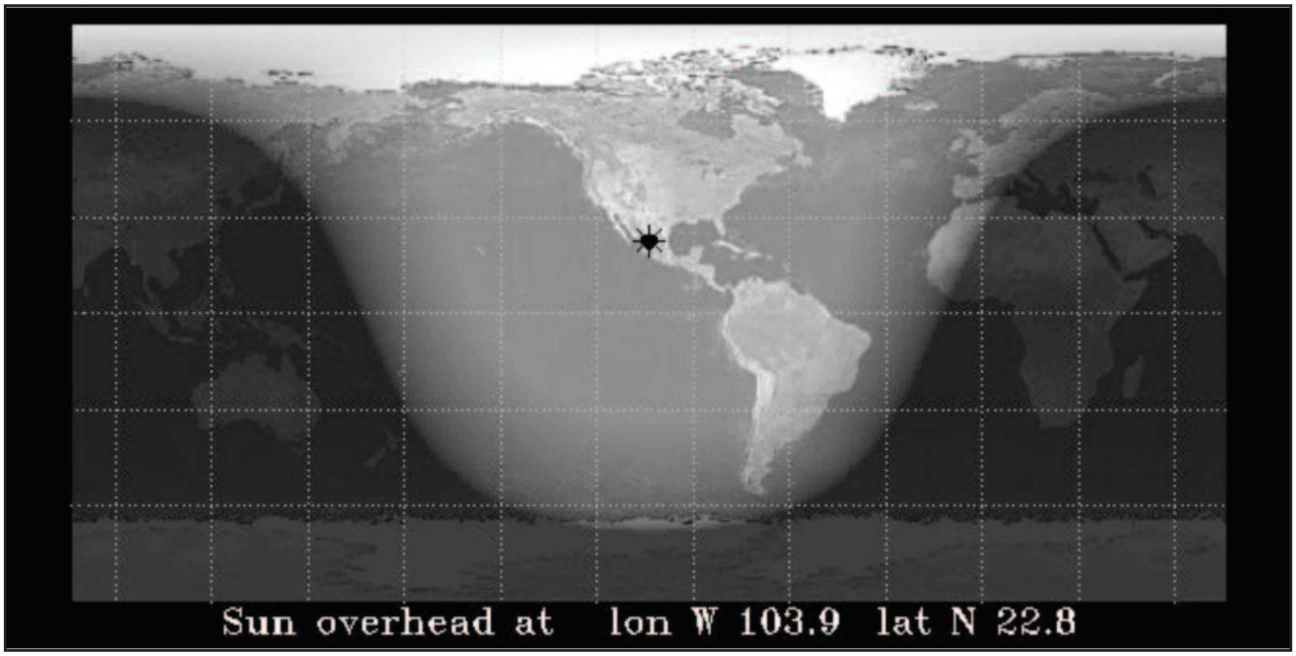
	Jan.		Feb.		Mar.		Apr.		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.	
Day	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set	Rise	Set
	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
01	0758	1629	0736	1711	0650	1754	0548	1839	0452	1922	0416	1959	0416	2011	0447	1944	0528	1850	0608	1749	0654	1652	0737	1620
02	0758	1630	0734	1712	0648	1756	0546	1841	0450	1923	0415	2000	0416	2010	0448	1942	0529	1848	0610	1746	0655	1650	0738	1619
03	0758	1631	0733	1714	0646	1757	0544	1842	0449	1924	0414	2001	0417	2010	0449	1941	0531	1846	0611	1744	0657	1649	0739	1619
04	0757	1632	0732	1715	0644	1759	0542	1844	0447	1926	0414	2002	0418	2010	0451	1939	0532	1844	0613	1742	0658	1647	0740	1619
05	0757	1633	0730	1717	0642	1800	0540	1845	0446	1927	0413	2003	0418	2009	0452	1938	0533	1842	0614	1740	0700	1646	0741	1618
06	0757	1634	0729	1719	0640	1802	0538	1846	0444	1928	0413	2004	0419	2009	0453	1936	0535	1840	0615	1738	0701	1644	0743	1618
07	0757	1635	0727	1720	0638	1803	0536	1848	0443	1930	0413	2004	0420	2008	0455	1935	0536	1838	0617	1737	0703	1643	0744	1618
08	0756	1636	0726	1722	0636	1805	0534	1849	0441	1931	0412	2005	0421	2008	0456	1933	0537	1836	0618	1735	0704	1641	0745	1618
09	0756	1637	0724	1723	0634	1806	0532	1851	0440	1933	0412	2006	0421	2007	0457	1931	0539	1834	0620	1733	0706	1640	0746	1618
10	0756	1639	0723	1725	0632	1808	0530	1852	0438	1934	0412	2006	0422	2007	0459	1930	0540	1832	0621	1731	0707	1639	0747	1618
11	0755	1640	0721	1726	0630	1809	0528	1854	0437	1935	0411	2007	0423	2006	0500	1928	0541	1829	0623	1729	0709	1637	0748	1618
12	0755	1641	0720	1728	0628	1811	0526	1855	0436	1937	0411	2007	0424	2005	0501	1927	0543	1827	0624	1727	0710	1636	0749	1618
13	0754	1643	0718	1730	0626	1812	0524	1856	0434	1938	0411	2008	0425	2005	0503	1925	0544	1825	0625	1725	0712	1635	0749	1618
14	0753	1644	0716	1731	0624	1814	0522	1858	0433	1939	0411	2009	0426	2004	0504	1923	0545	1823	0627	1723	0713	1634	0750	1618
15	0753	1645	0715	1733	0622	1815	0520	1859	0432	1940	0411	2009	0427	2003	0505	1921	0547	1821	0628	1721	0715	1633	0751	1618
16	0752	1647	0713	1734	0620	1816	0518	1901	0431	1942	0411	2009	0428	2002	0507	1920	0548	1819	0630	1719	0716	1632	0752	1618
17	0751	1648	0711	1736	0618	1818	0517	1902	0429	1943	0411	2010	0429	2001	0508	1918	0549	1817	0631	1717	0718	1631	0752	1619
18	0751	1649	0710	1737	0616	1819	0515	1903	0428	1944	0411	2010	0430	2000	0509	1916	0551	1815	0633	1715	0719	1629	0753	1619
19	0750	1651	0708	1739	0614	1821	0513	1905	0427	1945	0411	2010	0431	1959	0511	1914	0552	1813	0634	1714	0721	1628	0754	1619
20	0749	1652	0706	1741	0612	1822	0511	1906	0426	1947	0411	2011	0432	1958	0512	1912	0553	1811	0636	1712	0722	1628	0754	1620
21	0748	1654	0704	1742	0610	1824	0509	1908	0425	1948	0411	2011	0434	1957	0513	1911	0555	1809	0637	1710	0723	1627	0755	1620
22	0747	1655	0703	1744	0608	1825	0507	1909	0424	1949	0412	2011	0435	1956	0515	1909	0556	1807	0639	1708	0725	1626	0755	1621
23	0746	1657	0701	1745	0606	1827	0506	1910	0423	1950	0412	2011	0436	1955	0516	1907	0558	1805	0640	1706	0726	1625	0756	1621
24	0745	1658	0659	1747	0604	1828	0504	1912	0422	1951	0412	2011	0437	1954	0517	1905	0559	1803	0642	1705	0728	1624	0756	1622
25	0744	1700	0657	1748	0602	1829	0502	1913	0421	1952	0413	2011	0438	1953	0519	1903	0600	1801	0643	1703	0729	1623	0757	1623
26	0743	1701	0655	1750	0600	1831	0500	1915	0420	1953	0413	2011	0439	1952	0520	1901	0602	1759	0645	1701	0730	1623	0757	1623
27	0742	1703	0653	1751	0558	1832	0459	1916	0419	1954	0413	2011	0441	1950	0521	1859	0603	1757	0646	1700	0732	1622	0757	1624
28	0741	1704	0652	1753	0556	1834	0457	1917	0418	1955	0414	2011	0442	1949	0523	1857	0604	1755	0648	1658	0733	1621	0757	1625
29	0740	1706			0554	1835	0455	1919	0418	1956	0414	2011	0443	1948	0524	1855	0606	1753	0649	1656	0734	1621	0757	1626
30	0738	1707			0552	1837	0454	1920	0417	1957	0415	2011	0444	1946	0525	1854	0607	1751	0651	1655	0736	1620	0758	1626
31	0737	1709			0550	1838			0416	1958			0446	1945	0527	1852			0652	1653			0758	1627

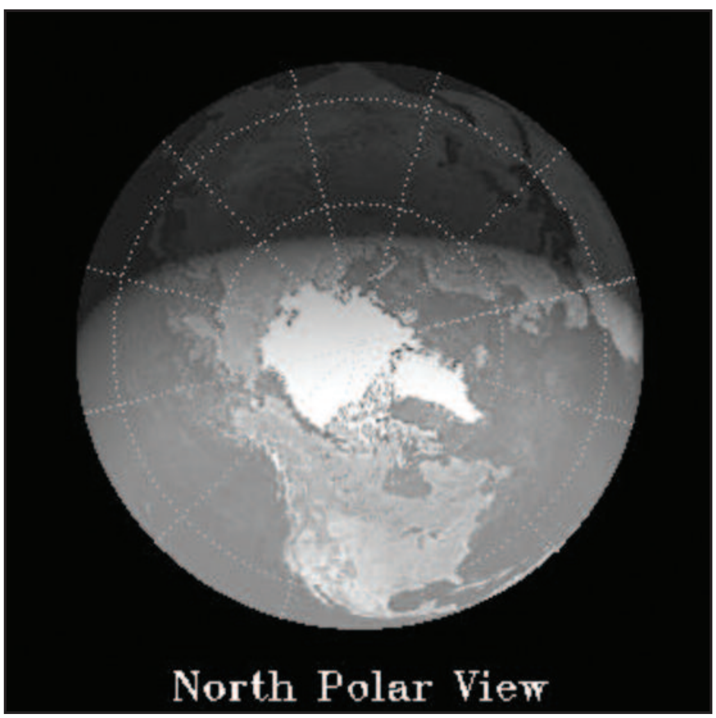
Add one hour for daylight time, if and when in use.

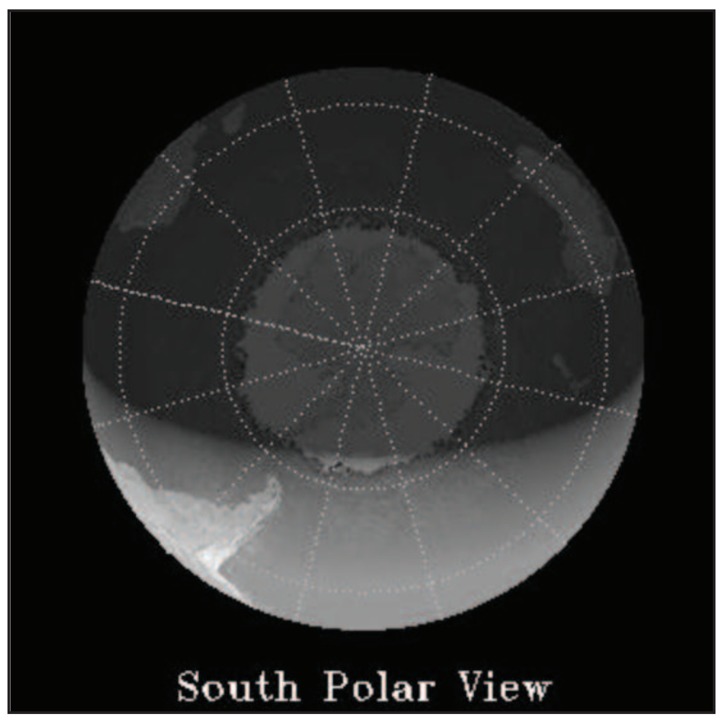
Source: http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-year-us

Appendix IV

Illumination of Earth, 04 July 2010







Source: http://www.usno.navy.mil/USNO/astronomical-applications/data-services/earthview

Day Length Along a Longitude

Your name: ____ .

The date you were assigned for data: ____ .

The latitude our class is using:____ .

"Reference city" our class is using:____ .

Number of "time zone" hours our class is using: ____________________________________ .

The purpose of this project is to provide you an opportunity to employ some unusual trigonometric modeling. Your product will be a scatter plot of day length along the given line of longitude for various latitudes from the North to South pole. Superimposed upon the scatter plot will be a graph of the trigonometric function you will derive to model the data (see below).

Obtain sunrise and sunset data from
http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-day-world

Obtain day length data for our reference city from
http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-year-us

1. Record the sunrise and sunset times along the given longitude for every 10° of latitude (input the appropriate "time zone" value). Write the times in the included table following those in the example table.
2. Subtract the times to give the number of hours (and decimal hours) of "daylight." Note: there are sixty (60) minutes in an hour so 21:36 – 02:44 = 18:52 = 18.87 hours.
3. Make a scatter plot of the data using a full sheet of graph paper. Use $-90 \leq x \leq 90$; $0 \leq y \leq 24$. Ignore repeated "24" and "0" readings (graph, at most, one of each).
4. Model the data as a tangent function and draw the function on the scatter plot. Use your reference city's day length for your date then solve $y = a \cdot \tan(x) + 12$ where x represents the latitude and y represents the day length in hours and decimal hours. Consult the USNO Website for the day length for our city. Show your work!
5. Graph the data and model using Excel, computer-based, or an online graphing resource. The goal is to have the scatter plot and function graphed together.
6. Participate in the "gallery walk" that displays a sequence of days of the year. Take some notes on the similarities and differences of the graphs over time. Record the day of the year and its associated "a" value.
7. Special challenge: create a function that results in the day length given the day of the year (e.g. 5 February is day #36) and latitude.
8. Write any new questions that came to mind while you worked on this activity.

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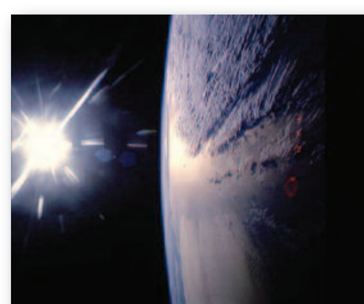


Day Length For Various Latitudes Along The _________________ Longitude

Your Date:

Latitude	Day Length Sunset – Sunrise (hr:min)	Difference # hours	Predicted Value	Difference Data – Predicted
90 N	☹	☹	☹	☹
80 N				
70 N				
60 N				
50 N				
40 N				
30 N				
20 N				
10 N				
0				
10 S				
20 S				
30 S				
40 S				
50 S				
60 S				
70 S				
80 S				
90 S	☹	☹	☹	☹

Note: "☹" represents data unavailable.

Lesson 3: Dreamliner Designer

OBJECTIVE:
Students “reverse engineer” the wing shape of the Boeing 787 Dreamliner by modeling with a polynomial function.

NATIONAL STANDARDS
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Form and function

Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
Standard 9 - Students will develop an understanding of engineering design.

MATERIALS:
- Handout (Dreamliner Designer)
- Graph paper—one sheet per student
- Scissors—enough for the class to all finish cutting out the diagram in a reasonable amount of time
- Glue sticks—enough so everyone can glue their diagram to the graph paper in a reasonable amount of time
- Graphics-based calculator capable of manipulating matrices (such as a TI-83+)
- Computer graphing program

BACKGROUND INFORMATION:

Polynomials are used for a wide variety of applications—from the design of fonts to the creation of figures in digital animation. Despite their power, teachers may rarely see them explored beyond roots and asymptotes. This lesson teaches precalculus students how to develop a polynomial equation that contains specified points and use that knowledge in a project in which they determine a pair of equations that follow the leading and trailing edges of a Boeing 787 Dreamliner wing.

Fitting a polynomial to a set of points is distinctively different from modeling data wherein one seeks a "best fit" function. As the class transitions to curve-fitting, the class also explores the limit of graphing calculators at quartic regression and overlays it with the intrigue regarding the development of a formula to solve for the roots of cubic polynomials (see the story of Ferrari, Cardano, Fior, and Tartaglia). The class should arrive at creating a system of equations from the general form from which they generate the coefficients of the selected polynomial.

Students infer from a short activity that one can "fit a curve" to a given set of points if the number of points is one greater than the order of the polynomial. Two points are needed for linear (1st order polynomial), three for cubic, four for quartic, etc. The terrifying part of the process may be the realization that one might be tasked with solving a huge system of equations. Technology comes to the rescue!

As long as there is a unique solution, *Cramer's Rule* (using matrices) will find it. Students can use their TI-83+ calculators to handle matrices up to 9 x 9, which allows one to obtain the coefficients of an eighth-order polynomial (which should be sufficient for our purposes). The following are the general steps of understanding to use Cramer's Rule for this purpose:

Agree that

$$\text{If } Ax = B$$
$$\text{Then } x = B/A$$

Which can also be written as $A^{-1}B$

(using our understanding of multiplicative inverses).

$$\text{Therefore}$$
$$[A]\,[x] = [B]$$
$$\text{and } [x] = [A]^{-1}[B]$$

Apply the above to polynomials by first understanding

The general formula for a polynomial is:

$$y = a_1x^n + a_2x^{(n-1)} + a_3x^{(n-2)} + \ldots + a_{n+1}$$

Then the specific points (x_1, y_1); (x_2, y_2), etc. are substituted into the general form to arrive at the system of equations. The numerical coefficients are then transcribed into matrices to solve for the unique polynomial that contains the given points.

Example: Create the polynomial that contains the given points $(-2, 15)$, $(-1, 2)$, and $(2, 11)$.

Solution: the polynomial is quadratic because it has three points, so use the general form to get each of the equations from the general form

$$a(-2)^2 + b(-2) + c = 15$$
$$4a - 2b + c = 15$$

$$a(-1)^2 + b(-1) + c = 2$$
$$1a - 1b + c = 2$$

$$a(2)^2 + b(2) + c = 11$$
$$4a + 2b + c = 11$$

So the system of equations becomes

$$4a - 2b + c = 15$$
$$1a - 1b + c = 2$$
$$4a + 2b + c = 11$$

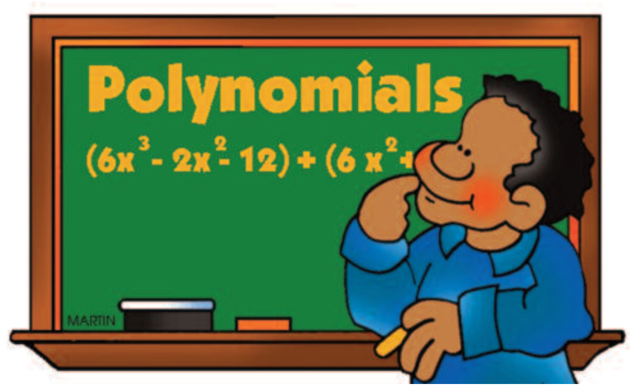
And the equivalent matrix equation is

$$\begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \\ 11 \end{bmatrix}$$

Which is solved by entering the first matrix as [A] and the last as [B] then performing $[A]^{-1}[B]$ so

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}$$

Yielding the polynomial $y = 4x^2 - x - 3$ (which you can now use to solve for the roots, if you want, using the quadratic formula or by graphing).

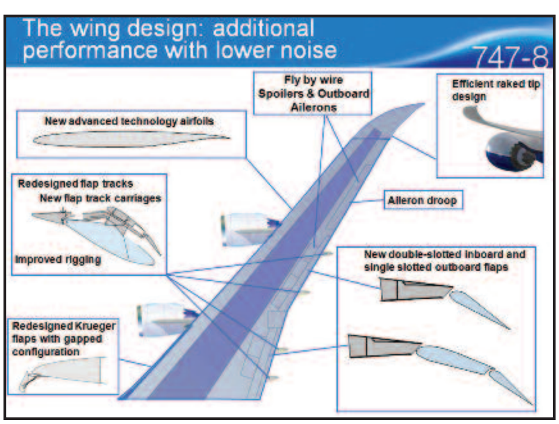


PROCEDURE:

1. Make sure students are grounded in curve-fitting with polynomials and solving a system of equations using Cramer's Rule.
2. Consider exploring the increase in level of complexity from quadratic, to cubic, then quartic formulae. This would also be a good place to include the intrigue about the development of a cubic formula.
3. Distribute the handout and allow students to read and then ask questions. Make sure they understand they must provide the points on which to create a polynomial of choice.
4. Allow work time in class to cut out the wing diagram, paste it onto the paper, and choose points. Hopefully each student will have chosen her/his points and began creating the system of equations before class ends.
5. Assign as homework to write and solve both systems of equations and write the associated polynomial.
6. Take students to the computer lab the next day to graph their equations on paper. Make certain students understand the scales of the printed graph must match the diagram (meaning if the x-axis of the wing diagram is 25 cm long the printed graph must be too).
7. Have students cut out their printed graphs (both should be on the same set of axes) and staple on top of the original wing diagram.
8. Culminate the project by posting the finished products "poster presentation" style and having students look at each to determine the similarities and differences between each of their classmates' and their own.
9. Celebrate!

SUMMARY:

The lowest order polynomial that approximates a shape is the best to use because it is the easiest to create and mathematically manipulate, should animation be desired. Polynomials of order higher than calculators can compute can be determined by writing a system of equations such that the order of the polynomial is one less than the number of points the polynomial must contain.



EVALUATION:

Use an old-fashioned overhead projector to overlay the graphs students produce from polynomials with the Dreamliner wing (see artwork of Dreamliner wing on next page). Display several approximations and the functions that produce them. Have students write a response to the question: What would you change about your solution process to obtain a better approximation of the Dreamliner wing?

LESSON ENRICHMENT/EXTENSION:

Provide other shapes to model with polynomials, including (but not limited to) fonts, logos, and outlines of animated characters (e.g. those from Toy Story).

ASSOCIATED WEBSITES AND/OR LITERATURE:

Cramer's Rule
- **http://mathworld.wolfram.com/CramersRule.html**
- **http://www.purplemath.com/modules/cramers.htm**

Lodovico Ferrari
- **http://www.gap-system.org/~history/Biographies/Ferrari.html**
- **http://fermatslasttheorem.blogspot.com/2006/11/lodovico-ferrari.html**
- **http://plus.maths.org/blog/2008/01/eventful-life-of-lodovico-ferrari.html**

Nicolo Tartaglia (AKA Niccolo Fontana)
- **http://www.gap-system.org/~history/Biographies/Tartaglia.html**
- **http://en.wikipedia.org/wiki/Niccolo_Fontana_Tartaglia**
- **http://www.answers.com/topic/niccolo-fontana-tartaglia**

Gerolamo Cardano
- **http://en.wikipedia.org/wiki/Cardano**
- **http://www.math.wichita.edu/history/men/cardano.html**
- **http://www.gap-system.org/~history/Biographies/Cardan.html**

Higher Order Polynomial Solutions
- **http://mathworld.wolfram.com/QuarticEquation.html**

Dreamliner Designer

Name:____

Your task is to create a pair of equations that follow the leading and trailing edges of a Boeing 787 Dreamliner wing (see the diagram on the next page). Show all your work on a separate sheet of paper and follow the following procedure:

1. Cut out the wing diagram as carefully as you can. Note that the “spike” on the leading edge near the wing root can be cut off and discarded.

2. Place the cut-out on a sheet of graph paper so the wingtip is at (0, 0) and the wing root is on the x-axis.

3. Label several points along the leading edge of the wing that will “force” a polynomial to follow the wing’s curve. Try to use the fewest number of points that will map the shape of the wing.

4. Select the polynomial that will fit those points exactly then solve for that polynomial by substituting the coordinates into the polynomial’s general form to create a system of equations. Use Cramer’s Rule to solve the system.

5. Check your polynomial on your calculator to ensure it contains the desired points.

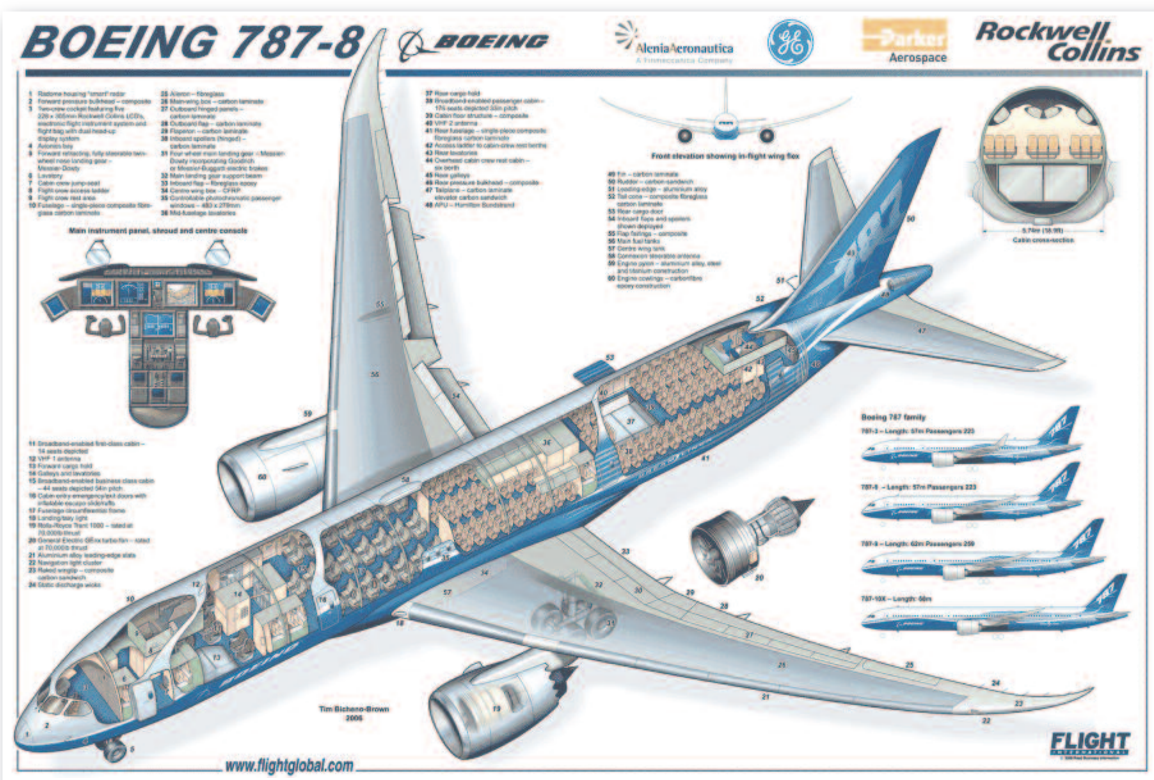
6. Repeat steps 3 & 4 for the trailing edge of the wing.

7. Confirm on your calculator the equation contains the chosen points.

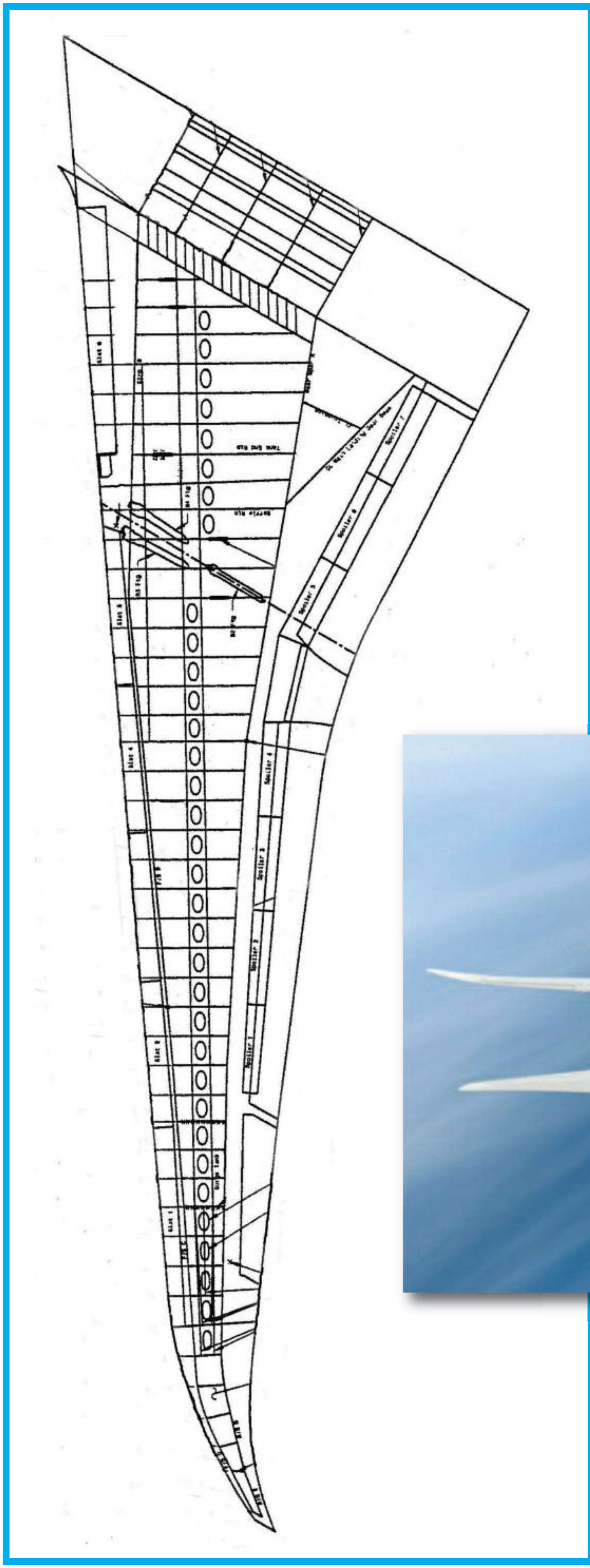
8. Graph your equations using Excel (or other appropriate graphing tool) so the scale and spacing matches the size and scale in step 2 above. Put both graphs on the same set of axes (so the output looks like the wing diagram).

9. Print your graph and turn it in with your work.

10. Write a few sentences that reflect upon your process and accomplishment.



Graphic of Dreamliner Wing provided by Russell Garlow of The Boeing Company and Aerospace Education Officer for Washington Wing, Civil Air Patrol



Lesson 4: Far And Away

OBJECTIVE:
Students compute global distances using latitude and longitude by employing the Pythagorean Theorem and an ancient (but new to most of them) global measurement system. Coordinates are found in a publication commonly used by pilots: the FAA Airport/Facility Directory (A/F D).

NATIONAL STANDARDS
Mathematics
Algebra
- use mathematical models to represent and understand quantitative relationships

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Data Analysis and Probability
- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Evidence, models, and explanation

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

MATERIALS:
- A copy (or actual) of your area's aviation sectional chart. Consider also downloading the pdf of the chart to project during whole-class discussions. (The pdf is available from National Aeronautical Charting Office – NACO. See the link at the end of this lesson.)
- Meter sticks – one per student group
- Protractors (as large as possible – one per student group
- Several copies of FAA Airport/Facility Directory (A/F D) or Internet access to use the online product

BACKGROUND INFORMATION:

Consider the following statement: "The Earth is covered by about 30% land and about 70% water BUT 100% of it is covered by air–that's why we fly!" In a less facetious sense, aircraft are typically faster than cars and have the added advantage of traveling on a (relatively) straight line from point-to-point rather than following the twists and turns of highways. One disadvantage of aircraft is running out of fuel can be catastrophic because one cannot merely "pull to the side of the road" as with a car. Knowing the distance between departure and destinations, therefore, is critically important!

Pilots learn in ground school to measure a route directly from an aeronautical chart using a "plotter" (a ruler-like tool featuring several scales). The distance is usually measured in *nautical miles* (NM) because airspeed is typically given in nautical miles per hour (known as "knots" and abbreviated as KTS). Recall that a *nautical mile* is one minute (one-sixtieth of a degree) of latitude whereas a *statute mile* (a statute is a "law" therefore it was dictated) is defined as 5280 feet. Of course, the length of a foot is currently defined in meters, but originated as the length of the king's foot (hence, the term ruler for the measuring device). Many believe the United States uses the length of King Henry I's foot.

Distances can be calculated without a chart using latitude and longitude along with the definition for nautical mile. This method is particularly useful when airports are on different charts or opposite sides of the same chart. The basic plan is to compute the differences in both latitude and longitude (in minutes) and then apply the Pythagorean Theorem to solve for the enroute distance. Lat and Long can be found in the FAA Airport/Facility Directory (both textual and online, referred to as A/F D). Note that correction is required for longitude because the lines converge at the poles, so measure a degree of longitude and see how many minutes of latitude it stretches (divide by 60 to get the number of nautical miles per minute of longitude). Example: a degree of longitude is about 40 NM at Seattle's latitude (near the 48th parallel North).

Here is a sample problem. Assume one will fly from SeaTac International Airport (SEA) to Yakima (YKM). Their respective latitude and longitude are N47° 26.99'; W122° 18.71' and N46° 34.09'; W120° 32.64'. Here, one must think like a Babylonian and convert 47° 26.99' into 46° 86.99' so one can subtract YKM's latitude. Note the difference is 52.90 minutes, which is also 52.90 NM. Similarly one converts 122° 18.71' into 121° 78.71' and arrives at 1° 46.07' which is equivalent to 106.07' (but NOT that many nautical miles). Because longitudes "squeeze together," the horizontal displacement is computed thusly: (40/60) (106.07') = 70.71 NM. The enroute distance is, therefore $\sqrt{(52.90)^2 + (70.71)^2}$ = 88.31 NM. Note that for flight planning purposes, only the whole number distance (88 NM) is used rather than all the significant digits even though they are significant. Note that if an individual is within a nautical mile of his/her destination by air, the airport appears to be very close, whereas by car, missing one's destination by a mile is, indeed, a miss!

The super-acute observer has already realized the method detailed above is correct only for "reasonably close" points on the globe. The chord of the great circle route between places is actually being calculated rather than the arc; one will always get an underestimate! More accurate calculations require solving for the angle at the Earth's center—an extension detailed in Lesson 6: *Going The Distance*.

1951 plotters and handbook

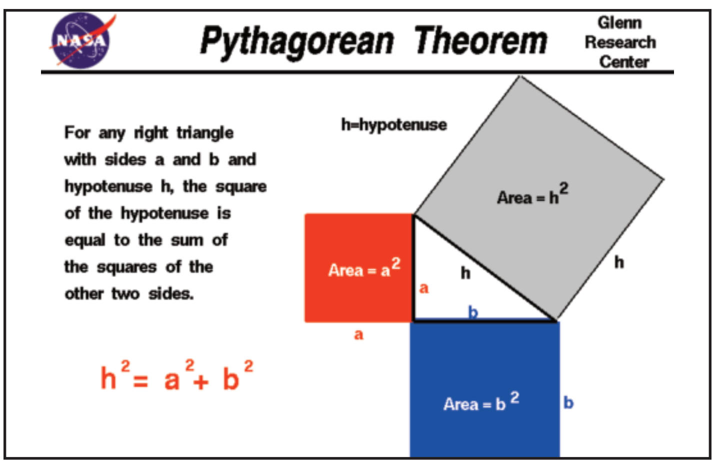
PROCEDURE:

1. Explore measures: mile, nautical mile, statute mile, cubit, foot, etc. An essential outcome is students understand a nautical mile is one minute of latitude.
2. Have students measure the distances, in nautical miles, between several points on an aeronautical chart. Consider letting them use only a meter stick so they will need to compare the measured length to the latitudes marked at the left to determine how many minutes.
3. Introduce the correction for longitude. Consider having students measure a degree of longitude and then counting the number of minutes of latitude.
4. Have students draw the vertical and horizontal displacements as the legs of a right triangle, label the lengths of the sides in nautical miles, and request a solution method for the hypotenuse. It is likely at least one student will suggest using the Pythagorean Theorem.
5. Obtain longitude & latitude for departure and destination airports from some official resource. Either textual or online Airport Facilities Directory (A/F D) are ideal (and authentic) sources.

6. Increase distances between airports, selecting a pair that are on opposite sides of the globe, such as Cairo, Egypt (N30° 2'; E31° 21') and Papeete, Tahiti (S17° 32'; W149° 34'). Note the students must accommodate for latitudes on opposite sides of the Equator and both East & West of the Prime Meridian. The result is $\sqrt{(47 \cdot 60 + 2)^2 + [(180 \cdot 60 - 55) \cdot 50/60]^2}$ 9340 NM if correction for longitude were "averaged" as 50 minutes per degree. Note that, by definition, the Earth would have 360° • 60 NM/degree = 21,600 NM for the circumference, and, therefore 10,800 NM for "half way around." From this analysis, one gets close but <u>not quire right</u>. Computing the distance by arc on the globe (as with *Going The Distance*) will provide a better estimate.

SUMMARY:

The Pythagorean Theorem provides reasonable estimates of distances between two points on the globe but only when near the Equator and "reasonably close" (within about 60 NM). Although a correction can be made away from the Equator by using a scaled-down number for the distance between degrees of longitude, computing distances across the surface (the arc length) must be performed as distances increase. Of course, these changes require deeper mathematical understanding and application.



EVALUATION:

Ask students to write the procedure to compute distances between two points on the globe given their latitude and longitude. State that they are writing for an audience much younger (e.g., ten years old) who can follow written instructions well in addition to having proficiency in addition, subtraction, multiplication, division, and square rooting on a calculator. No "complicated language" (i.e., "square") is allowed, and the instructions must allow for coordinates in both degrees, minutes, seconds and degrees, minutes, and decimal minutes.

LESSON ENRICHMENT/EXTENSION:

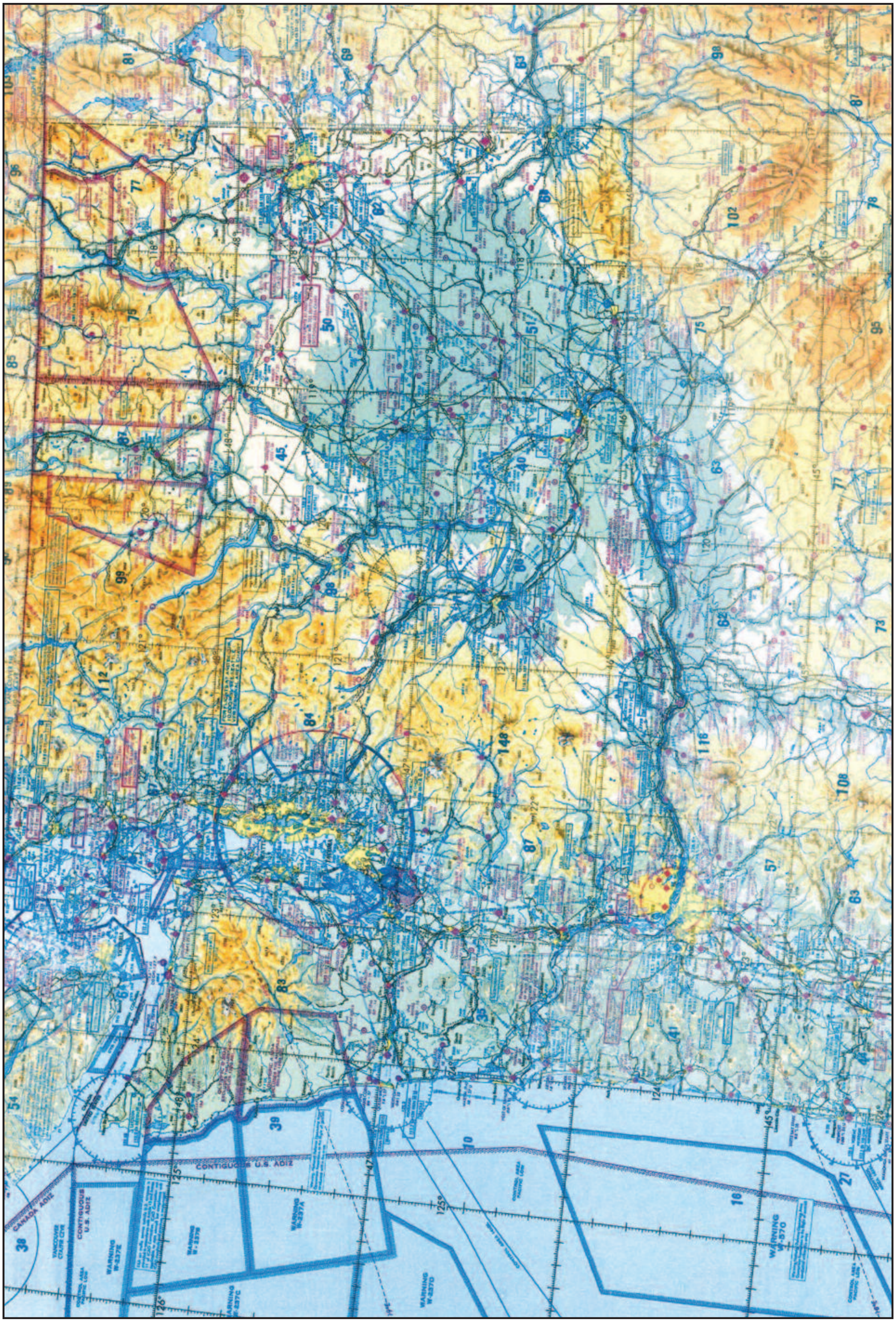
- Suggest students could "automate" the solution process by writing a program for their calculator. The inputs would be degrees and minutes for both longitude and latitude with the correction factor for longitude.
- Compare outcomes to those from measuring or online computation device to model "correction factors" for various conditions (distance between airports or extreme differences in latitude).

ASSOCIATED WEBSITES AND/OR LITERATURE:

- **http://aeronav.faa.gov/index.asp?xml=aeronav/applications/d_afd**
  - digital - Airport/Facility Directory.
- **http://www.wsdot.wa.gov/aviation/Charts/default.htm**
  - Washington State Sectional chart—digital version, for classroom training only: not for navigation!
- **http://avn.faa.gov/index.asp?xml=aeronav/applications/programs/compsys**
  - Geodetic Calculations software
- **http://www.infoplease.com/ipa/A0001769.html#axzz0yax0in4e**
  - Latitude & longitude of World cities
- **www.landings.com/_landings/pages/search/search_dist_apt.html**
  - Flight Route Planner
- **http://www.sgeier.net/tools/llp.php**
  - Great Circle calculator
- **http://faacharts.faa.gov/ProductDetails.aspx?ProductID=TRSSEA**
  - Seattle Sectional Training Chart - Product ID TRSSEA, cost $1.35 US

Paper charts available through
FAA, AeroNav Services Team
REDIS, AJW-379
10201 Good Luck Road
Glenn Dale, MD 20769-9700
(800) 638-8972 toll free, U.S. only
9-AMC-Chartsales@faa.gov
aeronav.faa.gov

Seattle Sectional Training Chart



Lesson 5:
Far Out!

OBJECTIVE:
Students use methods "of the ancients" to compute the distances from the Earth to the Moon and Venus.

NATIONAL STANDARDS:
Mathematics
Algebra
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Evidence, models, and explanation
- Change, constancy, and measurement

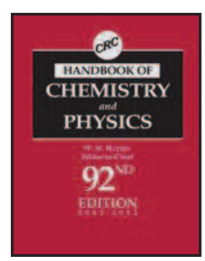
Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

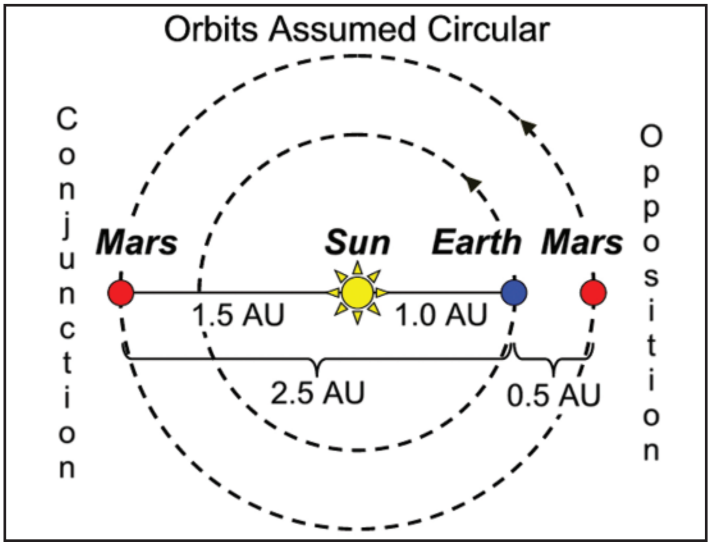
MATERIALS:
- Several copies of the *CRC Handbook of Chemistry and Physics* or Internet access to use the online product. They need not be particularly current because planetary data has not changed significantly in the past few decades.



- Twine (or other non-stretchy string)
- One large protractor with two straws (for sighting Venus's angle to the Sun). A surveyor's transit. Be careful about looking directly at the Sun!
- Internet access to **http://www.fourmilab.ch/cgi-bin/Solar**

BACKGROUND INFORMATION:

Although most people believe the Earth and other planets orbit the Sun, few appear to know the relative distances, periods of revolution, or how these values are determined. It is intriguing to imagine the observations and mathematics employed several centuries ago by which these were computed.



Given the radius of the Earth, Earth's distance from the Sun can be calculated using ge- ometry and trigonometry. Eratosthenes' method of noting shadow angle on two different locations and then measuring the distance between them yielded the Earth's circumference, by some accounts, to within 1% (there is some disagreement on the size of the unit used to measure the distance between Syene and Alexandria). Figure I shows how to use Earth's radius to compute the distance from the Earth to the Moon using right-triangle trigonometry (Cassini's method). This result allows one to compute the Sun-Earth distance.

Aristarchus noted that the Sun must be at a right angle relative to Earth when half illuminated (the half-Moon phases). Knowing the distance to the Moon and the angle from the Moon to the Sun allows one to, once again, employ right-triangle trigonometry to compute Earth's distance to the Sun (see Figure II).

One more application of right-triangle trigonometry will result in the computation of other planets' distance to the Sun. Awaiting Venus's apparent maximum distance from the Sun (as seen from Earth) will create a right triangle–its hypotenuse will be the line joining the Earth to the Sun, and the Earth-Venus-Sun angle is 90°. One then measures the Venus-Earth-Sun angle to provide the remaining needed fact to perform the calculations (see Figures III and IV).

Because people today "stand on the shoulders of giants" (Newton), they can accept the figures for the Earth-Sun and the Venus-Sun distances or compute them themselves using the previously employed methods. Either way, an individual can use non-right-triangle trigonometry to compute Earth's distance from Venus at any time it is visible.

Facts (from the *CRC Handbook of Chemistry and Physics*)

- Earth's orbit takes 1.0000 years.
- Venus' orbit takes 0.6152 (Earth) years.
- Mars' orbit takes 1.8808 (Earth) years.

PROCEDURE:

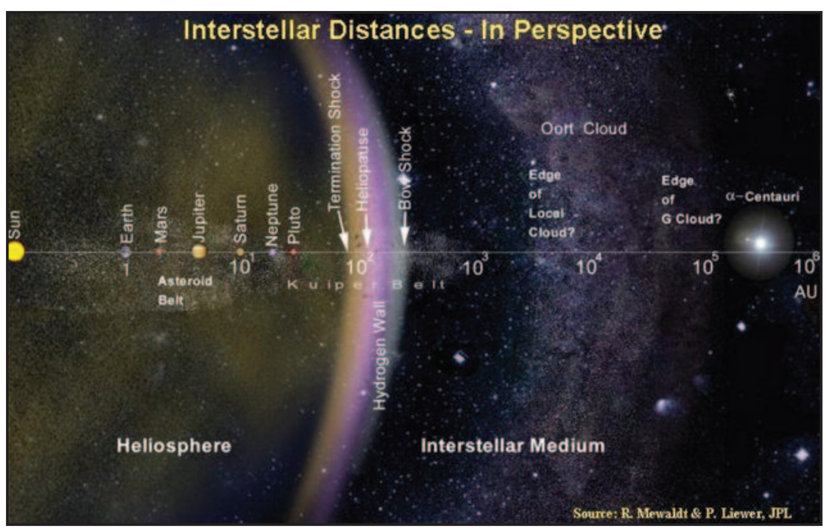
1. Complete *The Planet Dance* activity so students will understand relative position and distance. Set up the inner planets (the four closest to the Sun) in their current positions (follow the *Solar System Live* Website URL). **http://www.fourmilab.ch/cgi-bin/Solar - detailed set-up instructions:**
   - Set "Time" to the date on which the activity will be given. By default, the Website will use the current date.
   - For "Display" select the "Inner system" radio button and "600" for "Size" (you will get a larger and finer-grained graphic than the default value).
   - Select "Real" for "Orbits."
   - Select "Colour on white background" for "Colour Scheme" (this graphic will print better than the black background of the default view.)
   - Click the "Update" button.
   - Copy the graphic and paste into your worksheet document or save to disc.
   - Stretch string from your model's Sun to Earth then to Venus. Measure the angle. Have students sketch the situation and label the known values (the angle and the Earth-Sun and the Venus-Sun distances). Although the solar system depiction is for 4 July 2011 you can select any day you want (the default upon entering the URL is the current date).
2. Use the diagram and measures to solve for the Earth-Venus-Sun distance. Either Law of Cosines or Law of Sines will give solutions BUT the former will provide two solutions unless the Earth-Venus-Sun angle is 90°. Please see Figures III and IV. Note how this is a good opportunity to emphasize a weakness in the Law of Cosines and can lead to a discussion of why this happens (note the visual cue from the intersection of Venus' orbit with the Earth-Venus line).
3. Engage the class in developing a method to measure the Venus-Earth-Sun angle. The discussion should include when (at sunrise or sunset), where (the horizon must be unobstructed), and how (build a large protractor and sight through straws OR employ a surveyor's transit—be careful about looking directly at the Sun!).
4. Assign student teams (pairs work best) to make observations over one-week intervals so that each pair has measurements and estimated distance to Venus.
5. Celebrate the application of two millennia of astronomical and mathematical discoveries and developments!

SUMMARY:
Distances to the Moon, Venus, and several other planets can be computed using relatively simple observations and mathematics—simple enough to be performed by high school mathematics students. Therefore, knowledge of interplanetary distances was available to people much longer ago than most moderns realize.

EVALUATION:
Assign a variety of dates in the current year to student work groups to compute the distance to Venus using past, present, and future. Have students write a response to the question “About how frequently would conditions support computing the distance of the Earth to Venus? Show how you arrived at your estimate.”

LESSON ENRICHMENT/EXTENSION:
- Make all preliminary measurements (Earth’s circumference, Earth-Moon distance, etc.) and computations. This way you have the astronomical figures rather than looking them up.
- Use the orbital facts to calculate the greatest and smallest angle between Venus (or Mars), Earth and the Sun.
- Model (create a function) that outputs Earth’s distance from Venus (or Mars) given the number of days since the day the activity was given.
- Use orbital facts and current positions to solve for the number of years that must pass before Earth and Venus will be aligned with the Sun. A harder problem would be the amount of time before Earth, Venus, and Mars will be aligned. Still harder would be the amount of time before Mercury, Earth, Venus, and Mars will be aligned.

ASSOCIATED WEBSITES AND/OR LITERATURE:
- Distances within the Solar System
  http://spiff.rit.edu/classes/phys240/lectures/solar_sys/solar_sys.html; and, **http://galileoandeinstein.physics.virginia.edu/lectures/gkastr1.html**
- Calculating Earth’s radius
  http://sierra.nmsu.edu/morandi/CourseMaterials/RadiusOfEarth.html; http://eduwww.mikkeli.fi/opetus/myk/kv/comenius/erathostenes.htm; and **http://heasarc.gsfc.nasa.gov/docs/cosmic/earth_info.html**
- Calculating Earth-Moon Distance
  http://www.newton.dep.anl.gov/askasci/ast99/ast99155.htm
  See the second paragraph of Answer 2.
- Calculating Earth-Sun Distance
  http://www.newton.dep.anl.gov/askasci/ast99/ast99155.htm See Answer 2.
- Calculating Sun-Venus Distance
  http://curious.astro.cornell.edu/question.php?number=400;**http://eaae-astronomy.org/WG3-SS/WorkShops/VenusOrbit.html**; or, **http://athena.cornell.edu/kids/tommy_tt_issue3.html**
- Solar System Live
  http://www.fourmilab.ch/cgi-bin/Solar
- **http://www.astronomyforbeginners.com/astron omy/howknow.php**
- Handbook of Chemistry & Physics Online
  http://www.hbcpnetbase.com/

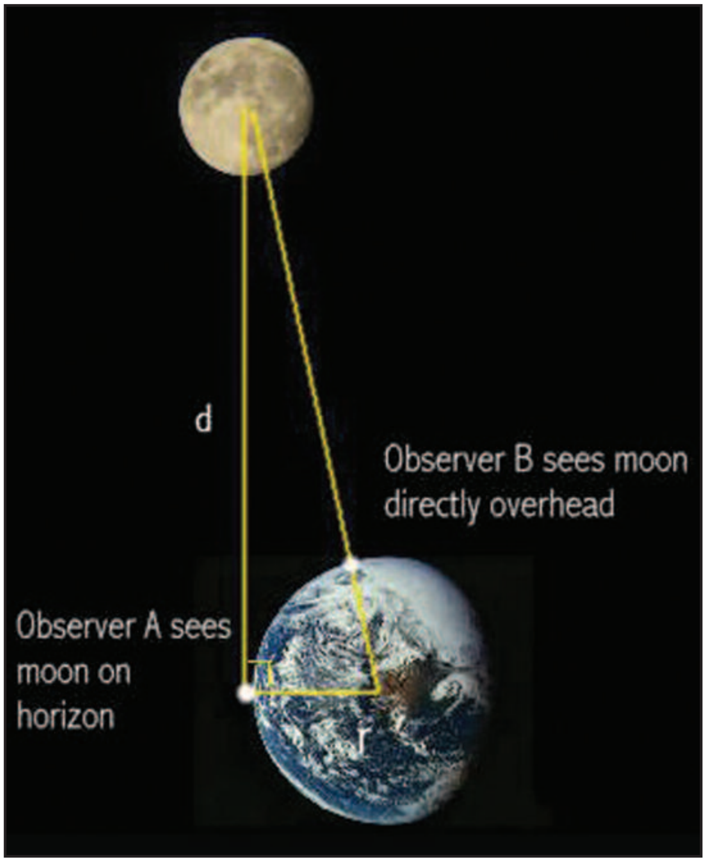


Surveyor’s transits - past (left) and present (above)

Websites with instructions for building a sextant:

- **http://wow.osu.edu/experiments/Measurement/Making%20A%20Sextant**
- **http://www2.jpl.nasa.gov/files/educator/sextant.txt**

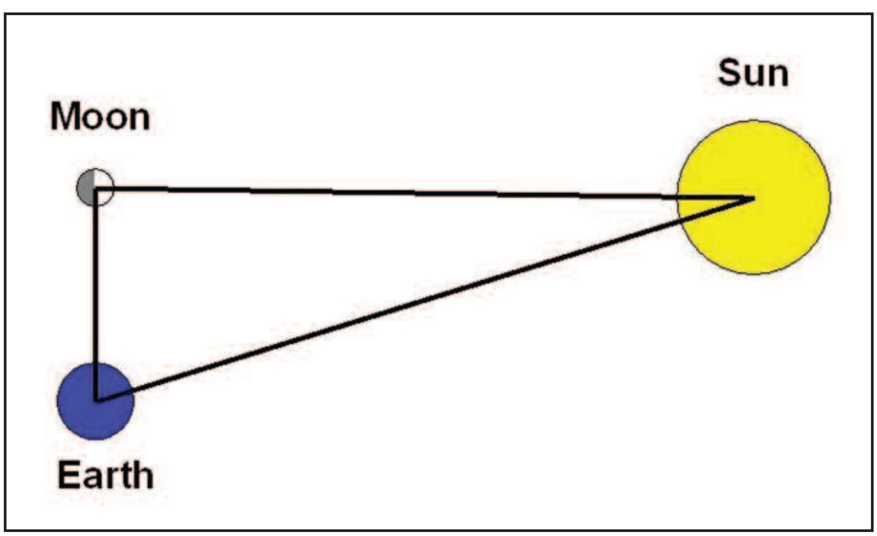
Figure 1
Computing distance from Earth to Moon



Assume we already know Earth's radius (let it be "*r*" in the diagram). Let the angle between the observers (located at the center of the Earth) be θ. The Earth – Moon distance ("*d*ˆ– *r*" noting that "*d*" is the tangent to the Earth) is $r \cdot \tan(\theta) - r$.

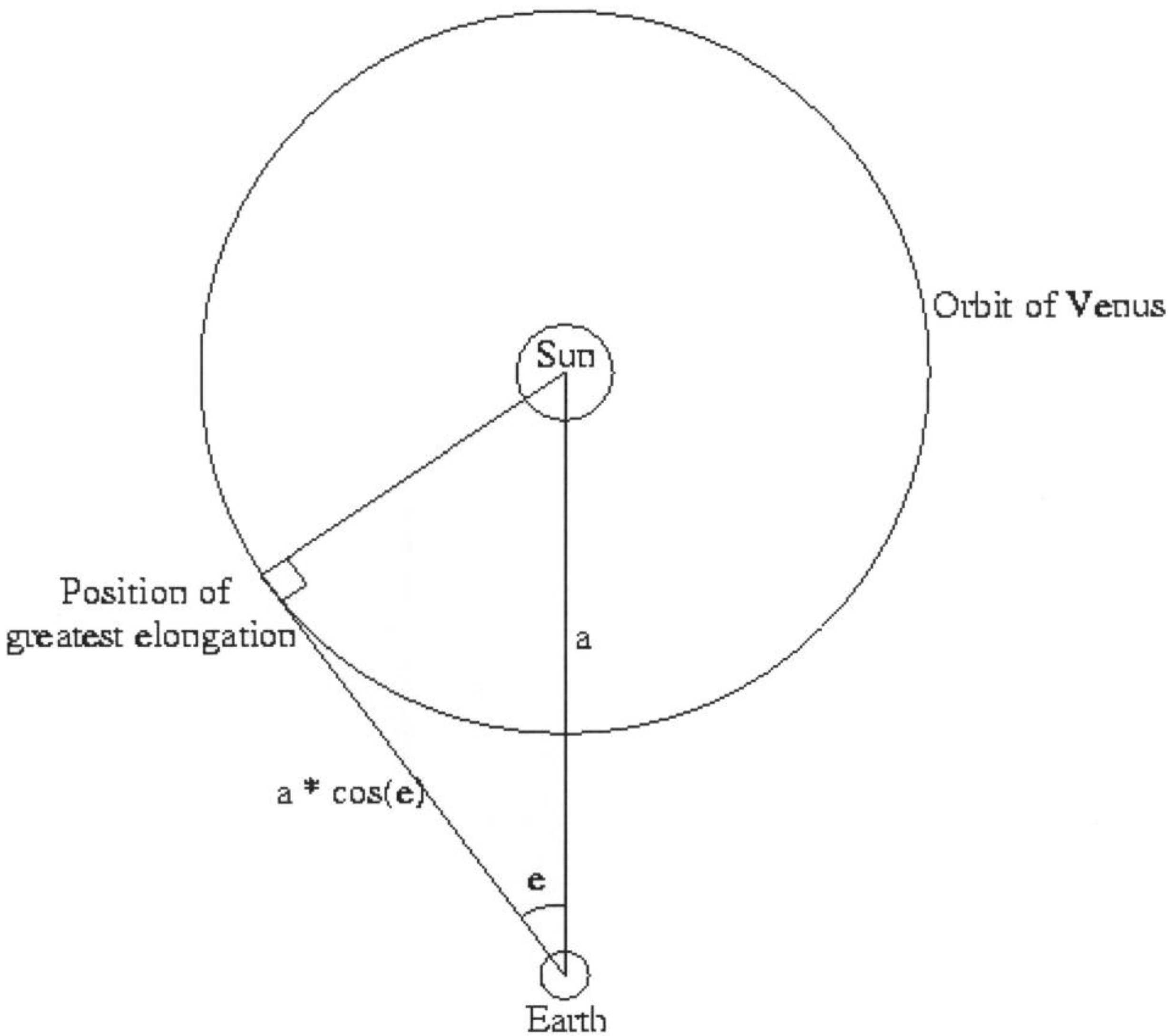
http://www.astronomyforbeginners.com/astronomy/howknow.php

Figure II
Computing distance from Earth to Sun using Aristarchus' method



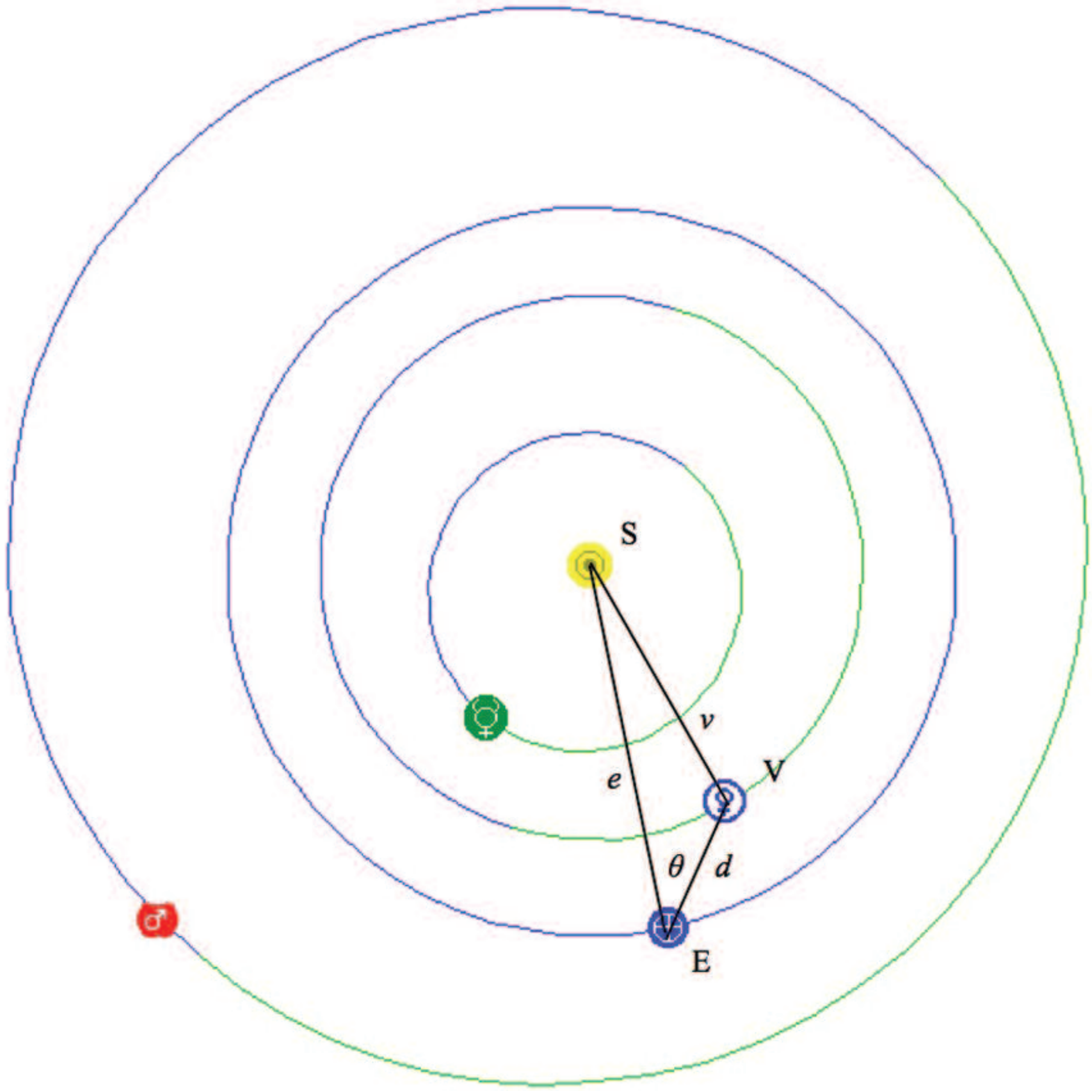
Assume we already know Earth – Moon distance (let it be "d" in the diagram). Let the angle between the Earth and Sun be θ. The Earth–Sun distance is $d \cdot \cos(\theta)$.

Figure III
Computing distance from Sun to Venus



http://curious.astro.cornell.edu/question.php?number=400

Assume we already know the Earth-Sun distance ("a" in the diagram) and measure the Sun-Venus angle ("e" in the diagram). Venus' distance from the Sun is $a \cdot \sin(e)$.



Let "e" be the Earth-Sun distance; "v" be the Venus-Sun distance; and "d" be the Earth-Venus distance. The viewed angle between Venus and the Sun is θ. The *Law of Cosines* gives

$$v^2 = d^2 + e^2 - 2de\bullet\cos(\theta) \text{ therefore } 0 = d^2 - 2de\bullet\cos(\theta) + e^2 - v^2$$

and we use the Quadratic Formula to get $d = \dfrac{2e\bullet\cos(\theta) \pm \sqrt{(-2e\bullet\cos(\theta))^2 - 4(1)(e^2 - v^2)}}{2}$ which will be two different answers (note the intersection point!). Use the *Law of Sines* twice and get a single answer

$$\frac{\sin(\theta)}{v} = \frac{\sin(\angle\text{EVS})}{e} \text{ therefore } \frac{\sin(\theta)}{v} = \frac{\sin(180° - \angle\text{EMS-}\theta)}{d} \text{ and } d = \frac{v\bullet\sin(180° - \angle\text{EMS-}\theta)}{\sin(\theta)}$$

Lesson 6: Going the Distance

OBJECTIVE:
Students compute global distances using latitude and longitude by employing a formula that combines angle measures in both degrees and radians, outputting an arc length that is a “great circle route.”

NATIONAL STANDARDS:
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate their mathematical thinking though communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Change, constancy, and measurement

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

Standard 8 - Students will develop an understanding of the attributes of design.

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:
- One large globe.
- Various maps of the World.
- Moldable material (clay or Play-Doh).

“Large” elastic (e.g. a rubber band, sliced to make a “string”)—long enough to stretch most of the way around the class’ globe.
- Non-stretchy string.
- Pencil stubs (or equivalent) to signify the poles of the Earth.

BACKGROUND INFORMATION:

Consider the question: if the distance between two points on a map is measured directly or computed from latitude and longitude, is the curve of the surface of the Earth adequately taken into account? One way to overcome the possibility of this oversight is to compute the "great circle route" (the arc formed from the intersection of a plane that passes through both points on the globe and the center of the Earth).

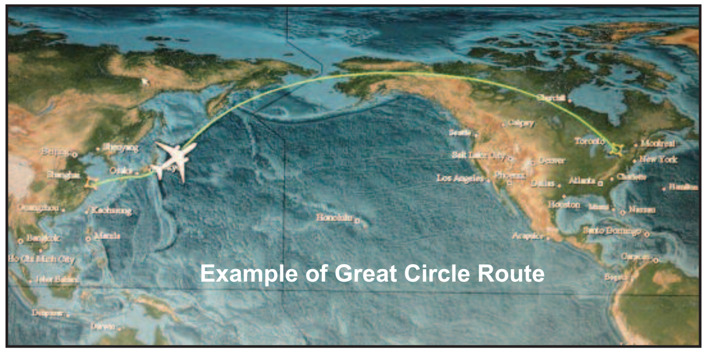
Two points on the globe form an angle with the center of the Earth whereby the length of the arc lying on the surface of the Earth formed by that angle is the distance between the points. Multiplying the angle's measure, in radians, by the radius of the Earth gives the arc length, and subsequently, the length of the great circle route. The difficulty in this approach is computing the angle knowing the latitude and longitude of the two points.

Latitude and longitude are in degrees and subdivided into minutes (1/60 degree). Further subdivisions are either decimal minutes or seconds (1/60 minute, which is 1/3600 degree). Airport coordinates, as listed in the FAA publication Airport/Facility Directory, are given in degrees, minutes, and decimal minutes. The Earth has a diameter of about 6875 NM ($360° \cdot 60$ NM/degree $\div \pi$), giving it a radius of about 3438 NM. Working backwards to obtain the desired information (an arc length between two points on the Earth), one would multiply the radius of the Earth by the measure (in radians) of the angle subtending the arc. The following process allows the distance from airport A to airport B to be computed using their latitude and longitude.

- Convert latitude and longitude of locations A and B to degrees and decimal degrees.
- Put your calculator in Degree mode.
- Let $a = \cos^{-1}[\cos(\text{lat A})*\cos(\text{lat B})*\cos(\text{lon B} - \text{lon A}) + \sin(\text{lat A})*\sin(\text{lat B})]$
- Convert a into radians ($a\pi/180$ = the angle measure in radians).
- Multiply the angle measure by the radius of the Earth.

Example:

- Let A = Sea Tac airport (N47° 26.99'; W122° 18.71') and B = Yakima (N46° 34.09'; W120° 32.64'). A = N47.45°; W122.31° and B = N46.57°; W120.54°. Note that both of these airports are in Washington State and appear on the Seattle Sectional Aeronautical Chart (available as a free pdf or as paper "practice charts" for $1.25).
- Therefore $a \approx \cos^{-1}(0.99966) \approx 1.4941° \approx 0.026068$ radians.
- The distance between the airports is $0.026068 \cdot 3438 \approx 89.6$ NM. Record 90 NM on the flight plan because the nearest whole number is good enough (it looks close when within one NM of an airport because of the view from the air).



Example of Great Circle Route

Procedure:

1. Prompt students for the possible disadvantages or inaccuracies resulting from determining distance between two points on the Earth by measuring on maps. The "answer we're looking for" involves the insight that the Earth curves and the maps do not, therefore, inaccuracy increases with increasing distance between the points. Most importantly, the estimates will be less than the actual value which could lead to fuel management errors.
2. Use a globe to demonstrate a "great circle route" between two points by stretching a rubber band from one point to the other. Allow students to speculate how the course could be constructed geometrically (e.g., a plane slicing a sphere). A 12" globe may be purchased from a local dollar store.

3. Make a large "Earth" from clay or Play-Doh and sketch the major land masses on its surface. Note the location of the poles by inserting pencil stubs (or equivalent). Mark two points, such as New York and Los Angeles. Use a wire to slice the globe through the two points and the center of the globe. Draw the central angle that contains the points on the surface then measure the central angle. Convert the angle to radians then multiply the angle measure by the radius of the globe (measured again). The result is the arc distance (AKA the great circle route between the two cities). "Confirmation" can be estimated by laying non-stretchy string along the arc and measuring its length—compare the outcome to the calculated value. Ask students to debrief this part of the activity by listing the steps of the procedure used to calculate the arc length.

4. Suggest there must be a mathematical way to get the central angle that leads to the arc length. Reveal the formula, and then allay students' panic by saying that, although the formula looks daunting, there are only a few values to input and two trig functions to use.
5. Lead a practice for students using New York (La Guardia: N40°46.64'; W73°52.36') and Los Angeles (N33°56.55' W118°24.43'). The result should be about 3140 NM.
6. Practice with several other routes. Consider allowing students the opportunity to indulge themselves on where they want to fly—many enjoy the fantasy of "taking themselves" to a new location. Students typically have to make several attempts (usually three) before inputting the values becomes reliable. This is an interesting and important subsidiary outcome.

SUMMARY:
The method revealed in this lesson compensates for the inaccuracies inherent in using the Pythagorean Theorem, as introduced in Lesson 4: *Far and Away*. Students typically need significantly more practice to produce an acceptable answer, which is only worth the extra effort when distances are becoming large (over 60 NM) or latitudes are noticeably away from the Equator.

EVALUATION:
Select two locations that students would not likely choose but would recognize (such as Anchorage, AK and Cape Canaveral, FL). Show the location of the locations on a globe and have students write an estimate of the distance between them (in nautical miles). Give the latitude and longitude for the locations and allow students to work independently on the solution.Have students write a an explanation of their method of estimation and how it compares to the computation.

LESSON ENRICHMENT/EXTENSION:
- Do Lesson 4: *Far and Away* first—it introduces global measurement using the Pythagorean Theorem.
- Provide clay or Play-Doh to student groups to slice, measure central angles, and compute arc length on the "surface" of their globes.
- Suggest students could "automate" the solution process by writing a program for their calculator. The inputs would be degrees & minutes for both longitude & latitude. Note that the program would have to change modes—starting in degrees and ending in radians.

ASSOCIATED WEBSITES AND/OR LITERATURE:
- **http://www.naco.faa.gov/**
(National Aeronautical Charting Office–NACO)

- **http://www.naco.faa.gov/index.asp?xml=aeronav/applications/d_afd**
(digital - Airport/Facility Directory)

- **http://www.wsdot.wa.gov/aviation/Charts/default.htm**
(Washington State Sectional chart—digital version, for classroom training only: not for navigation!

- **http://www.naco.faa.gov/index.asp?xml=aeronav/online/compsys**
(Geodetic Calculations software)

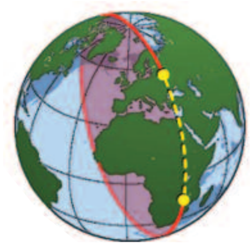
- **http://www.infoplease.com/ipa/A0001769.html#axzz0yax0in4e**
(Latitude and longitude of world cities)

- **www.landings.com/_landings/pages/search/search_dist_apt.html**
(Flight Route Planner)

- **http://www.sgeier.net/tools/llp.php**
(Great Circle calculator).

- **http://naco.faa.gov/ecomp/ProductDetails.aspx?ProductID=TRSSEA**
Seattle Sectional Training Chart
(Product ID TRSSEA, cost $1.25 US)

Paper charts available through
FAA, AeroNav Services Team
REDIS, AJW-379
10201 Good Luck Road
Glenn Dale, MD 20769-9700
(800) 638-8972 toll free, U.S. only
9-AMC-Chartsales@faa.gov
aeronav.faa.gov



Global measurement

Given the Earth has a diameter of 6875 NM (360°•60 NM/degree ÷ π). Calculate the distance over the Earth (the arc) between the airports. Note: The distance from one location to another using latitude and longitude is found from the following formula:

1) Let $a = \cos^{-1}[\cos(\text{lat } A)*\cos(\text{lat } B)*\cos(\text{lon } B - \text{lon } A) + \sin(\text{lat } A)*\sin(\text{lat } B)]$ where latitude and longitude of locations A and B are converted to degrees and decimal degrees. Put the calculator in Degree mode. The output is in degrees and decimal degrees.
2) Convert a to radians.
3) Multiply a by the radius of the Earth (~3438 NM). The answer will be in NM.

Lesson 7: Modeling Satellite Orbits

OBJECTIVE
Students use a current diagram showing the orbital path of the International Space Station (ISS) to create an equation that will trace its path. The equation will be a sine function requiring values for amplitude, frequency, and horizontal shift.

NATIONAL STANDARDS:
Mathematics
Algebra
- represent and analyze mathematical situations and structures using algebraic symbols
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Evidence, models, and explanation
- Change, constancy, and measurement

Science as inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
Standard 8 - Students will develop an understanding of the attributes of design.
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS: (one set for each group)
- Copy of the handout Space Station Tracker
- Copy of the ISS orbit track
- Graphing calculator
- Ruler or straightedge

BACKGROUND INFORMATION:

You have probably noticed NASA's tracking board features offset sine curves. This presentation began in early missions, continued with the Space Shuttle Orbiter, and continues today with the International Space Station (ISS). Most people have viewed a replica of it in movies, such as *The Right Stuff* and *Apollo 13,* while still others seek out Internet versions for various orbiters.

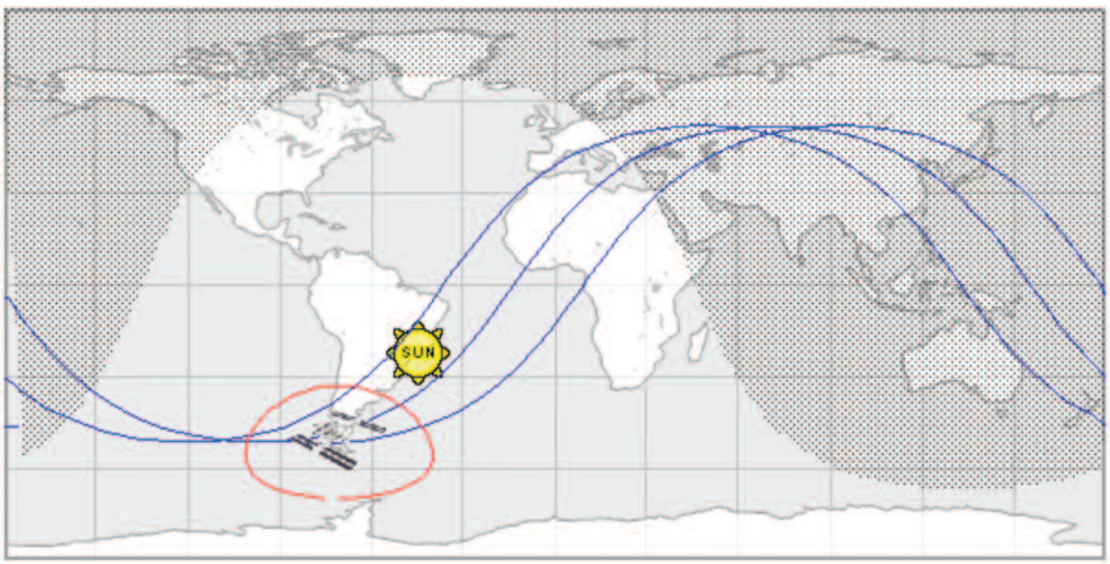
This activity will develop the formula required to track the ISS using information from the tracking board. The following diagram shows three orbits of the ISS on 5 January 2010.

Orbiting patterns are sinusoidal because the ISS orbits at an angle to the Equator, and the speeds of the Earth and the ISS are different. The map is divided vertically into twelfths; hence, the lines are separated by 30° of longitude with the Prime Meridian at the center of the map. The Equator runs through the horizontal center, with the lines also separated by 30° of latitude. There are 360° from the left end to the right end of the map and 180° from the bottom to the top.

The challenge in this activity is for students to determine amplitude, frequency, horizontal shift, and vertical shift. Then, they will apply their findings to the general form **$y = A \cdot \sin[B(x - C)] + D$** to determine the equation by which the ISS position can be plotted. Note that if the equation is close enough to being correct, it will provide the position for several orbits into the future!

This activity is appropriate for students who have graphed a variety of trigonometric functions and had the opportunity to "tinker" a basic sine equation to fit a graph. They should be familiar with the terms in the first paragraph of this section, have a graphing tool available (computer or graphing calculator), and some practice modeling real-world data. This activity was originally developed for precalculus students for a unit on modeling periodic data. It may be used, however, in advanced algebra through advanced calculus.

Note: consider creating the equation yourself and compare to the sample solution.

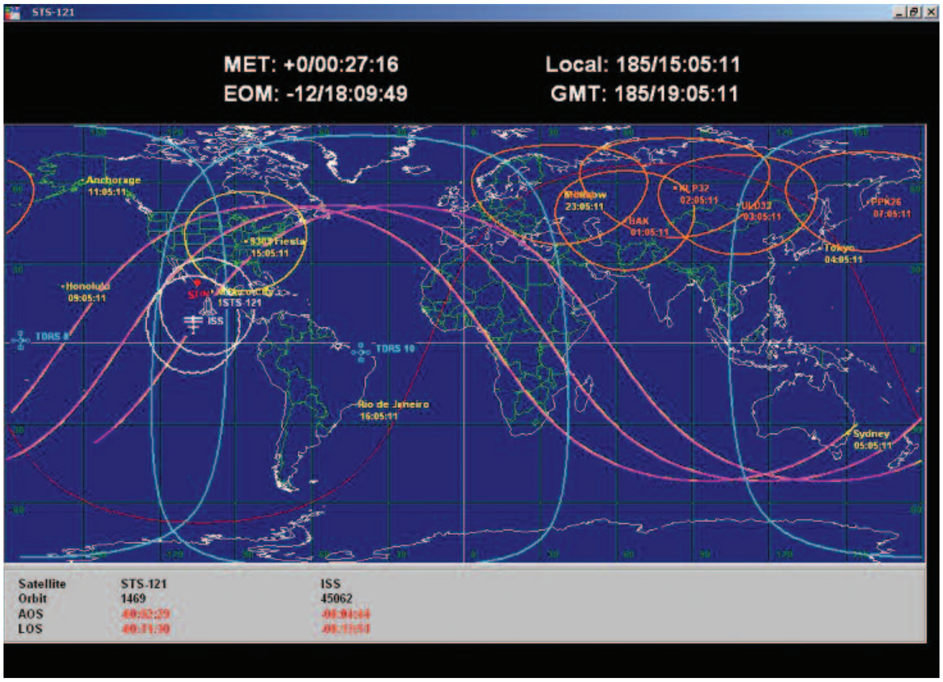


Space Station Tracker (http://spaceflight.nasa.gov/realdata/tracking/)

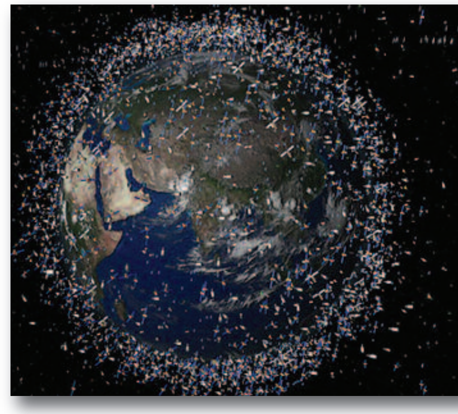
PROCEDURE:

Students should work in pairs on the following tasks:

1. Obtain a current tracking diagram (if possible) of the ISS from Space Station Tracker or show the one included in this module (if you cannot obtain a current one). Show it to the class and invite observations. Record the observations on chart paper, white board, etc., for future reference.
2. Clarify all dimensions on the map and their references: the Equator for the horizontal "zero" and the Prime Meridian. Make sure students understand that successive orbits trace successive curves (set to the left a measurable increment).
3. Consider showing Appendix I (Viewing Continuity of Orbits) so students will get a perspective of how the ISS transitions from the right hand side of the map to the left side. The diagram was created by capturing the graphic on the Website, reducing it to a width of 2.5 inches, and placing a copy of the diagram adjacent.
4. Confirm all students and calculators are operating in degrees rather than radians.
5. Inform students that they will have to measure amplitude, shift, and frequency to arrive at the appropriate equation for the ISS orbit.
6. Distribute handout Space Station Tracker and let students work.
7. Circulate and monitor.
8. Debrief by having student groups volunteer their equation. Write what they say so there is a record. Hopefully the equations are close.
9. Have students graph their equations for several orbits to see where they predict the ISS will be located.
10. Celebrate success! Consider awarding a sticker badge saying "Rocket Scientist In Training" to those groups who are closest to the actual position after 3, 6, and 9 orbits.



Tracking Map



Space Debris including Satellites

SUMMARY:
This real-world application of modeling with sine functions helps explain how periodic phenomena can be roughly forecast based upon observation.

EVALUATION:
Have students write responses to the following:

- How did you determine the amplitude, frequency, and horizontal shift that approximates the ISS orbits in the given diagram?
- Detail the measurements you made on the diagram and whatever assumptions you employed.

LESSON ENRICHMENT/EXTENSION:

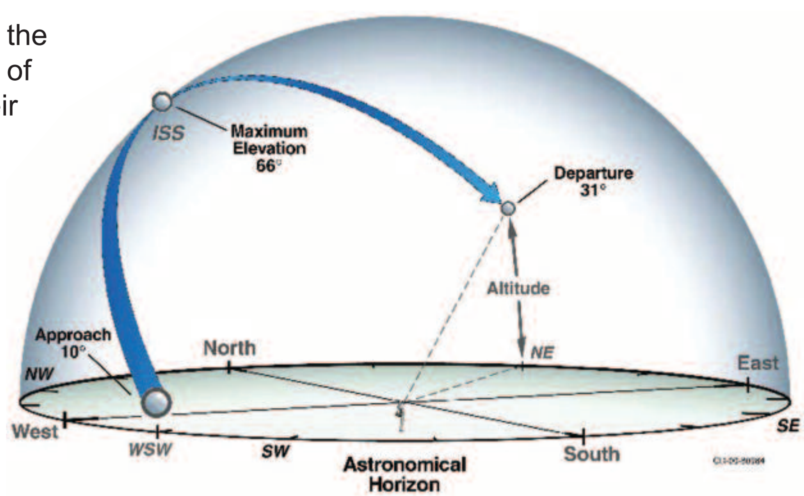
- Host a “star party” to view the major stars, planets, and constellations. Plan it when a reasonably long (at least 3 minutes) transit of the ISS can be seen in your area.
- Find relevant facts for orbiting bodies such as the typical angle of their orbit relative to the plane of Earth’s orbit around the Sun and period of their orbit—then develop a tracking diagram based upon the facts. This is how the tracking board is developed!
- Have students use their formula to predict when the ISS will be visible in their region (keeping in mind that it must be dark to see it). Check using the “Sighting Opportunities” Website. One fact students will need is that the ISS completes one orbit in about 92 minutes.
- Complete the Space Station Spotter activity in conjunction with this one. Solve the equation students write for the longitudes the ISS will be at your location’s latitude, and then use the other activity to determine if the ISS will be visible.

ASSOCIATED WEBSITES AND/OR LITERATURE:

- Space Station Tracker information
http://space flight.nasa.gov/realdata/tracking/

- ISS Sighting Opportunities website
http://spaceflight.nasa.gov/realdata/sightings/help.html

- ISS Operations info
http://www.nasa.gov/mission_pages/station/main/index.html



Modeling Satellite Orbits

Space Station Tracker

Group Member #1: ____

Group Member #2: ____

Determine amplitude, frequency, horizontal shift, and vertical shift, then apply your findings to the general form

$$y = A \bullet \sin[B(x - C)] + D$$

to determine the equation by which the ISS position can be plotted.

1. Draw the Equator on your map. Measure from the Equator to the "highest" point of the ISS orbit. Confirm that this is the same value as the "lowest" point. Note that this measure will be in "degrees latitude" and represents the amplitude of the function. This is "A" in the formula. Your findings:

2. Locate the crest of the wave that is furthest right. This is the ISS's first orbit in this tracking diagram. Draw a vertical line and estimate its longitude as accurately as you can. Trace the curve back to its trough and estimate its longitude as accurately as you can. The distance between these is one-half wave—double the value you just measured and you have the number of degrees for one wave (call this "T"). Note that a sine wave usually has 360° for a wave, so the frequency ("B") is 360° ÷ T.

3. Note where the "first" orbit intersects the Equator. This is the "horizontal shift" ("C") wherein the value is positive if the graph is "pulled left" and negative if "pulled right."

4. Explain why D = 0.

Modeling Satellite Orbits

Sample Solution

Herein is a solution for of the ISS on 5 January 2010 by determining amplitude, frequency, horizontal shift, and vertical shift, then apply your findings to the general form

$$y = A \bullet \sin[B\,(x - C)] + D$$

to determine the equation by which the ISS position can be plotted.

1. Draw the Equator on your map. Measure from the Equator to the "highest" point of the ISS orbit. Confirm that this is the same value as the "lowest" point. Note that this measure will be in "degrees latitude" and represents the amplitude of the function. This is "A" in the formula. Your findings:

Amplitude appears to be about 50° of latitude.

2. Locate the crest of the wave that is furthest right. This is the ISS's first orbit in this tracking diagram. Draw a vertical line and estimate its longitude as accurately as you can. Trace the curve back to its trough and estimate its longitude as accurately as you can. The distance between these is one-half wave—double the value you just measured and you have the number of degrees for one wave (call this "T"). Note that a sine wave usually has 360° for a wave, so the frequency ("B") is 360° ÷ T.

Crest to trough appears to be about 170°, therefore a whole wave would be 340°. B = 360/340.

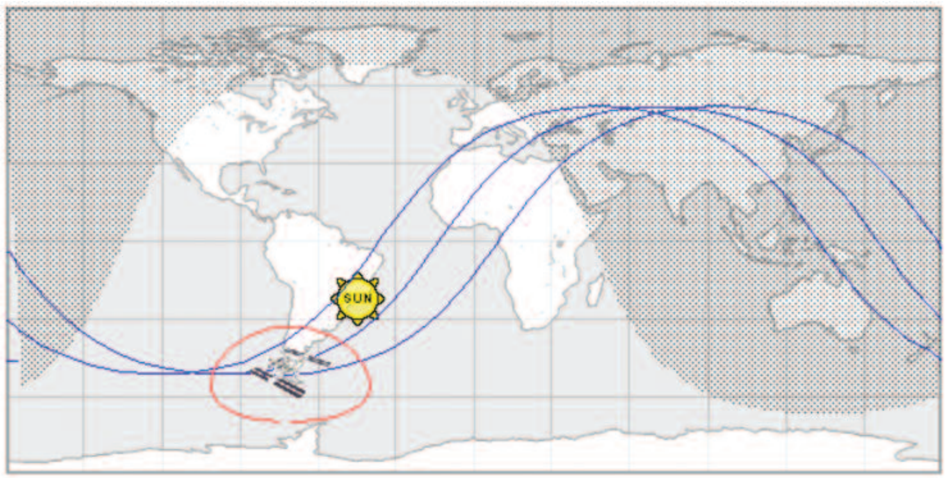
3. Note where the "first" orbit intersects the Equator. This is the "horizontal shift" ("C") wherein the value is positive if the graph is "pulled left" and negative if "pulled right."

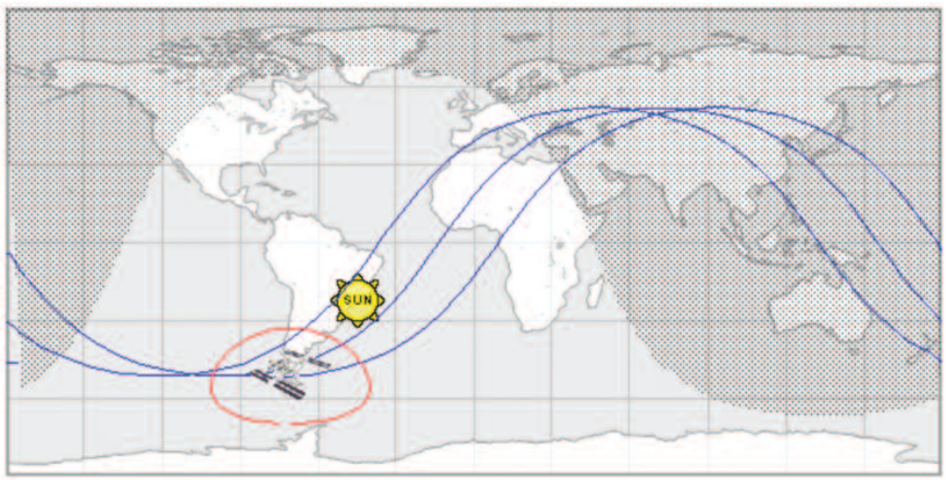
The first orbit appears to be about 10° "ahead of" the Prime Meridian, so C = –10.

4. Explain why D = 0.

The ISS orbits at an angle to the plane of the Earth's orbit around the Sun, so it will have an equal amount about and below the Equator resulting in NO vertical shift of the function.

Appendix I
Viewing Continuity of Orbits





Lesson 8: Out To Launch

OBJECTIVE:
Students calculate "area under the curve" to determine the distance a spacecraft has traveled in the first two minutes after liftoff.

NATIONAL STANDARDS
Mathematics
Algebra
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Evidence, models, and explanation

Science as inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:
- Computer with Internet connection connected to a projector
- Graphing calculators or computer-based modeling software (for students)
- One hand out, sheet of graph paper, and ruler for each student

Last Launch of Space Shuttle - Atlantis (STS-135) - July 8, 2011

BACKGROUND INFORMATION:

Consider allowing precalculus students an opportunity to experience the underpinnings of integral calculus. This activity was created with that goal in mind. Students will experience computing the “area under a curve” and attempt to bring meaning to the outcome.

Consider presenting the activity to students as a semester project wherein they may attempt modeling of actual data and try to make sense of it. The activity is appropriate for students at virtually all high school levels as long as they are comfortable with graphing & modeling data, familiar with power functions, and have some background (or facility) with dimensional analysis.

Area formulae

- Left endpoint approximation: $\sum_{i=1}^{n} f(x_{i-1})\Delta x$
- Right endpoint approximation: $\sum_{i=1}^{n} f(x_i)\Delta x$
- Midpoint approximation: $\frac{b-a}{n}\sum_{i=1}^{n} f(\frac{x_{i-1}+x_i}{2})$
  where *a* and *b* are the endpoints of the interval
- Trapezoidal approximation: $\frac{b-a}{2n}\sum_{i=1}^{n} f(x_i)$
  where *a* and *b* are the endpoints of the interval

PROCEDURE:

1. Discuss why the label “rocket scientist” has long been synonymous with “stellar” problem solving capabilities. If “it’s not rocket science” means “you can do this” then “it’s rocket science” probably means “only the best thinkers can do this.”
2. Remind students the basic form of a power model and how one differs from the others (the general form for power functions is $y = ax^b$). Consider reminding students how to derive the coefficient if the power is given (or known).
3. Lead a discussion on how students would “compute the areas under a curve” if the equation were y = 2*x* + 3; 0 = *x* = 4. Either demonstrate this or have students perform the area calculation independently. Hopefully, they will quickly determine that the shape is merely a trapezoid, and the area can be calculated easily.
4. Extend the discussion to how students would “compute the areas under a curve” if the equation were y = $2x^2$ + 3; 0 = *x* = 4. Place emphasis on how the “curve” makes the job a bit more complicated than for a line. Lead students to understanding they can approximate the area with small trapezoids and add them up.
5. Show the Space Shuttle launch (**http://www.youtube.com/watch?v=4FROxZ5i67k**) and explain how the Solid Rocket Boosters (SRBs) detach about two minutes after launch. From there, the main engines lift the Orbiter to its orbital altitude. Tell the class tol assume the Shuttle ascends pretty much vertically (which is, of course, not true).

Solid Rocket Boosters detach from Shuttle

6. Distribute the *Out To Launch* worksheet, graph paper, and rulers.
7. Monitor as students draw the interval lines and begin computing areas. The result should be around 173,000 feet.
8. Assure all students have completed all area computations, have a total, and have addressed the questions.
9. Discuss the answers on the paper and explore the variability of students’ area approximations. Ask if the approximation would be an underestimate or over estimate of the Shuttle altitude at SRB separation.
10. Conclude by announcing to students that they are now on their way to becoming rocket scientists!

Note: A reasonable squared power function is $y = (4151 \div 125^2)x^2$ where x is the number of seconds after launch and y is the speed of the Space Shuttle in feet/second. This is reasonable because a rocket’s acceleration should increase in an increasing rate because of lost mass during combustion, the curve should go through (0, 0) and (125, 4151), and the “approximation” the model provides will miss the values where the shuttle engines were throttled back; hence, the model provides the speed at 125 seconds if the engines delivered constant thrust. The area, therefore, is the altitude at SRB separation (assuming the Shuttle ascends vertically).

Area formulae

- Left endpoint approximation: $\sum_{i=1}^{n} f(x_{i-1})\Delta x$
- Right endpoint approximation: $\sum_{i=1}^{n} f(x_i)\Delta x$
- Midpoint approximation: $\frac{b-a}{n}\sum_{i=1}^{n} f(\frac{x_{i-1}+x_i}{2})$
  where *a* and *b* are the endpoints of the interval
- Trapezoidal approximation: $\frac{b-a}{2n}\sum_{i=1}^{n} f(x_i)$
  where *a* and *b* are the endpoints of the interval

SUMMARY:
Integral calculus need not be mysterious nor laden with cryptic formulae; it can be understood from the perspective of performing arithmetic (appropriately) on the graphical representation of an outcome. If the units on the axes support the computation (e.g., meters/second * number of seconds = number of meters) adding the areas of the representative columns (the "Riemann Sum") gives an approximation of the true area under the curve. Using narrower columns gives an even better estimate and making them infinitesimal in width is calculus!

EVALUATION:
See the Challenge Problem. (Page 49)

LESSON ENRICHMENT/EXTENSION:

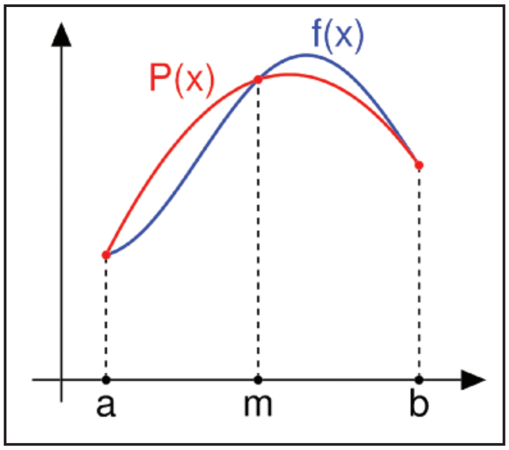
- Secretly produce four versions of the activity, each using a different method for calculating the areas of the intervals—rectangles using left endpoint, right endpoint, or midpoint for the height of the rectangle; or, trapezoids using both endpoints. Consider producing them on different colors of paper and having students with the same color of paper work together. Have students compare answers during the activity debrief, see if they can explain why the answers differ, and, for this function, which method *should* provide the closest approximation.
- Have different groups approximate the area using intervals of different widths. Then, compare the approximations. Draw students to the realization that the approximate approaches the correct value for the area the more intervals are used.
- Use a spreadsheet to calculate the areas and explore changing interval widths.
- Connect to Riemann Sum for beginning calculus students. (A Riemann Sum is a method for approximating the total area underneath a curve on a graph, otherwise know as an integral. It may also be used to define the integration operation. The method was named after German mathematician, Bernhard Riemann.)
- Connect to Simpson's Rule for advanced calculus students. (Simpson's rule is a staple of scientific data analysis and engineering. It is widely used, for example, by Naval architects to calculate the capacity of a ship or lifeboat.)

ASSOCIATED WEBSITES AND/OR LITERATURE:

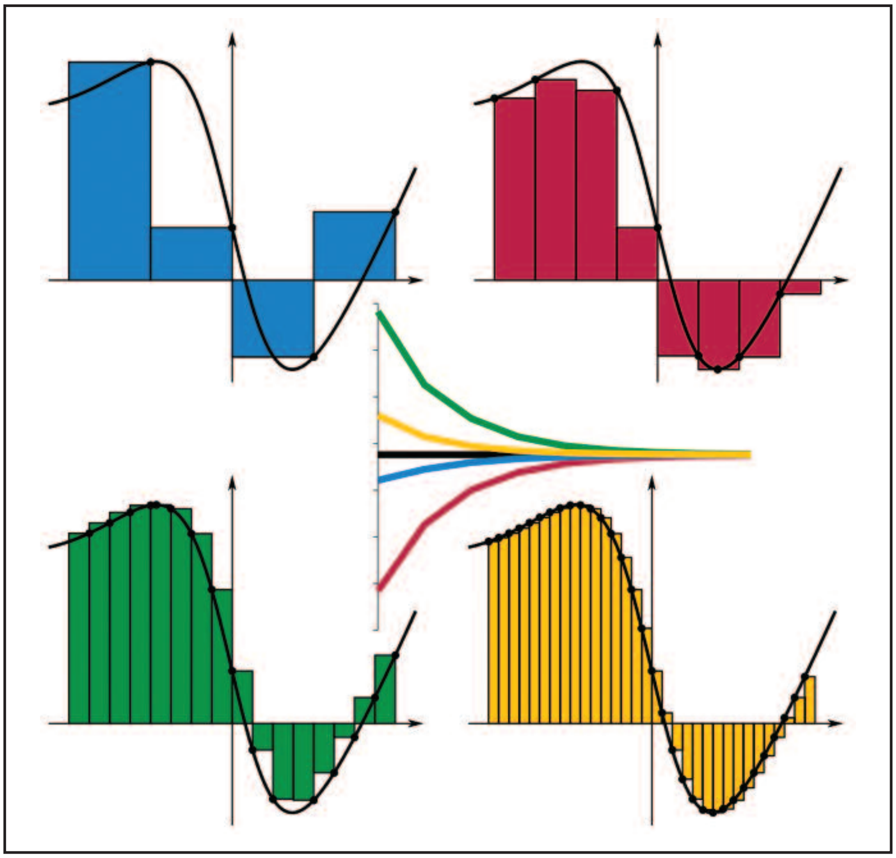
- You Tube video of shuttle launch: **http://www.youtube.com/watch?v=4FROxZ5i67k**

- Riemann Sum calculator: **http://mathworld.wolfram.com/RiemannSum.html**

Space Shuttle Launch



Simpson's Rule



Riemann Sum Convergence

Out To Launch

Name: ____

On 7 May 1992, the space shuttle Endeavour was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an *Intelsat* communications satellite. The following table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters (SRBs).

Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
SRB separation	125	4151

1. Make a scatter plot of the data using a full sheet of graph paper.

2. Model the data as a squared power function and draw the function on the same graph as the scatter plot.
   Note: this is NOT the same as a quadratic function!

3. Explain why this particular model is best/most appropriate for the context of a shuttle launch.

4. Draw vertical lines on the graph every 10.0 seconds and note where they intersect the graph of the model.
   Note that the last interval will be half the normal width, so the last vertical line will be at x = 125.

5. Determine the heights of the sides of each interval and transcribe the measurements into the "data table"

6. Substitute the values from #5 into an appropriate formula and calculate the area of each region obtained in #4.

Launch of STS-49, Endeavour, on 7 May 1992

7. Add up all the interval areas. Report the value with appropriate units.

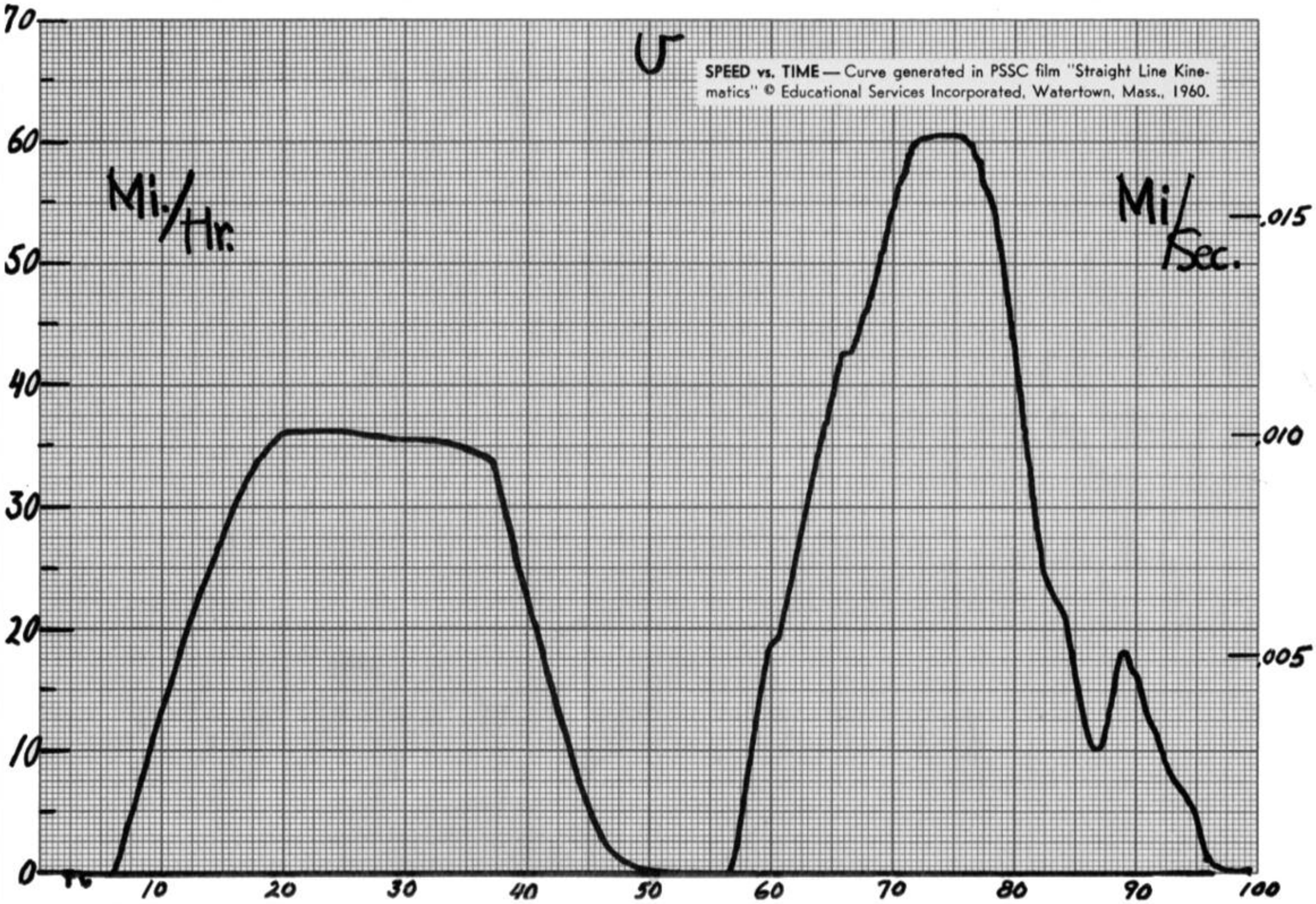
8. Answer the question, "What is the significance of the area under the curve of the shuttle launch data?"

9. What advantage would using intervals 5.0 seconds wide for the entire graph provide?

Interval	Left height	Right height	Width	Area Formula (with substitutions)	Area
10					
20					
30					
40					
50					
60					
70					
80					
90					
100					
110					
120					
125					

Challenge Problem

Velocity vs. time for a 1961 Thunderbird in a test area



Determine the distance the car has driven.
Watch out for the units (miles per hour are on the vertical axis and number of seconds on the horizontal axis).

Lesson 9: The Parachute Paradox

OBJECTIVE:
Students determine the relationship between the area of a parachute and the rate a given weight will fall.

NATIONAL STANDARDS:

Mathematics

Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science

Unifying concepts and processes in science
- Systems, order, and organization
- Evidence, models, and explanation
- Change, constancy, and measurement
- Form and function

Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology

Standard 1 - Students will develop an understanding of the characteristics and scope of technology.

Standard 7 - Students will develop an understanding of the influence of technology on history.

Standard 8 - Students will develop an understanding of the attributes of design.

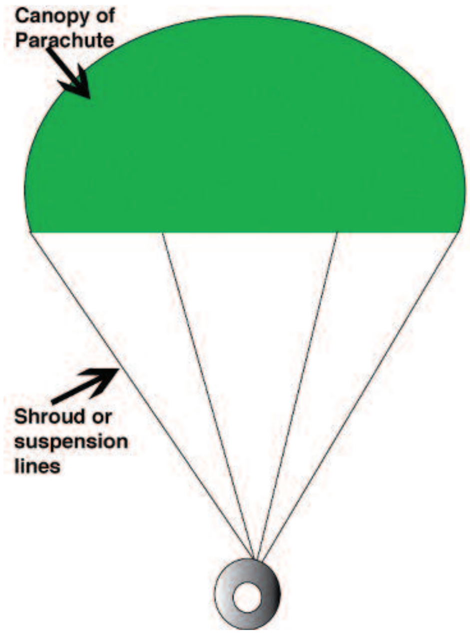
Standard 9 - Students will develop an understanding of engineering design.

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

Standard 20 - Students will develop an understanding of and be able to select and use construction technologies.

MATERIALS

- Plastic, large dry-cleaning sheath-type bags (Local dry cleaners will typically provide several of these for free if you tell them you are a teacher.)
- String (a large spool of medium weight
- Scissors (one per group)
- Several rolls of clear cellophane tape (not frosted)
- Hand-held (single) hole punch
- Washers, bolts, clothes pins, or other constant weight objects
- Stopwatch (consider the APP on TI-Education that uses TI-83+ and TI-84+ calculators)
- A stairway that allows a drop of at least 15 feet, catwalk above a gymnasium, or equivalent (Ladders are a bit too risky.)



BACKGROUND INFORMATION:

The job of a parachute is to arrest acceleration and create a constant descent rate, which various online parachute organizations suggest should be about 15 feet per second (about 4.6 meters per second). Given practical constraints of the packing size of a folded parachute and its weight, the smallest parachute that gives the desired performance is most desired. Therefore, to address the question, “What is the relationship between the size of a parachute and the rate it allows an object to fall?” students will relate area of circular parachutes to the parachutes’ descent rate, tested with various weights. To simplify the investigation, students drop the test parachutes from the same height (as closely as possible) and time their drop, performing multiple trials of each parachute/weight combination.

Consider how the three variables—parachute area, amount of weight being dropped, and time of the drop—will be graphed and subsequently mathematically modeled. The two independent variables (parachute area and weight being dropped) will each affect the time of the drop (assuming height of drop is kept constant throughout the investigation). Decide how you want students to address this difficulty *before* beginning; that way you determine the level of their inquiry a *priori*. You could constrain the investigation by providing explicit direction (e.g., graph parachute area vs. weight being dropped for only those systems that do not exceed 15 fps and use specified colors to indicate better descent rates) or leave totally “open” by merely announcing the challenge of three variables and leave the analysis to the students.

PROCEDURE:

1. Ask students to explain the purpose of a parachute. Guide their discussion towards cessation of acceleration using air resistance rather than “practical applications” (e.g., recreation or safety during planned extreme aircraft maneuvers). Consider referring to the story of The Candy Bombers who delivered supplies to West Berliners during the 1948 blockade and also dropped candy to kids using handkerchiefs (the effort is often deemed “the most successful humanitarian action of all time”) for background on handmade parachutes.

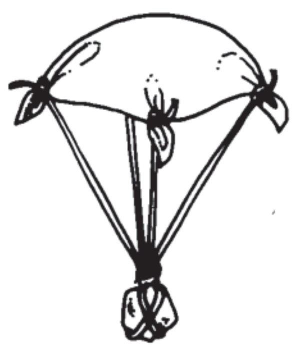
Candy Bomber, Col Gail Halvorsen, during the Berlin Airlift

2. Have students hypothesize the size of a parachute each individual student would need to make a safe descent. Reveal that “safe descent” could be defined as 15 feet per minute.

3. Announce the intent to perform an investigation that will use a mathematical model to compute the area of parachutes required for them to fall safely. Note that statements like "more area makes descent slower" is a naïve statement. There must be a practical limit to size because a VERY large parachute would likely be too bulky to be practical and would provide unpredictable descents rather than safe (and somewhat vertical) descents. Parachutes have weight and volume!
4. Suggest students construct parachutes of various sizes and drop them from the same height with varying weights. Make strong statements about increasing reliability of measurements (i.e., performing multiple trials with each set-up and not including trials suspected of mis-measurement).
5. Have students make parachutes from circles with six equally spaced hole punches around the perimeter. Tape the holes before punching to reinforce the hole. Then, tie strings on each punch-out. Tie a single weight onto the strings such that each string is the same length (see Appendix II).
6. Instruct students to have timers call out "ready, set, go!" in a predictable cadence. Start the stopwatch at "go" and simultaneously release the parachute.
7. Allow data collection and analysis to proceed through consensus of the student groups.
8. Conduct a debrief on the findings.

SUMMARY:

A parachute is not as simple as one may think because its size must be determined before one takes a leap, and larger does not necessarily mean it will be better. Once a reasonable model is determined through experimentation, the required size of a full-scale parachute can be computed.



EVALUATION:

- How do "conventional parachutes" differ from what DaVinci invented?
- What is the LIMIT of area to weight (where acceleration is adequately arrested)?
- How must this investigation be modified if a person were to jump from very high altitudes? Note that the stories of Joe Kittinger and Felix Baumgartner would be appropriate here.

LESSON ENRICHMENT/EXTENSION:

- Use two timers for each group in order to provide a cross-check for reliability. If the times do not agree acceptably, or if the timers were not confident in their start or stop, then the data is not considered in the analysis.
- Use photo gates and electronic data gathering devices (e.g. CBL and calculator) rather than stopwatches operated by students.

ASSOCIATED WEBSITES AND/OR LITERATURE:

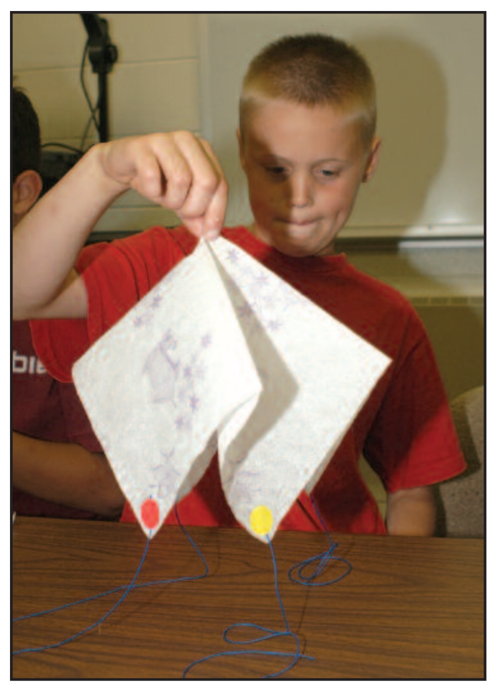
• Descent rate of a round parachute
http://www.pcprg.com/rounddes.htm.

• Descent rate calculator
http://www.aeroconsystems.com/tips/descent_rate.htm.

• The Candy Bombers
http://thecandy bombers.com.

• Example of displaying multiple variables: Napoleon's March by Charles Joseph Minard
http://www.edwardtufte.com/tufte/posters.

• Timer APP (application) for use on the TI-83+ and TI-84+ calculators
http://education.ti.com/educationportal/sites/US/homePage/index.html.



Another method for making parachutes is using a napkin, stick-on dots, and string. Another variable in parachute making can be type of material used to make the canopy.

Appendix I

Group Members:

____________________________ ____________________________

____________________________ ____________________________

Drop height: ____________________ **Weight:** ____________________

Trials

Diameter	Area	Time	Comments

Note: The person timing calls out "ready, set, go!" If there is any doubt about the release, the trial is not counted in the analysis BUT is still recorded. Use a new table for each change in area, weight, etc.

Lesson 10: Polar Expressions

OBJECTIVE:
Students construct polar graphs of Sun transit data. Comparisons can be made of transits of different days of the year and at different longitudes, latitudes, and altitudes.

NATIONAL STANDARDS:
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate their mathematical thinking though communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Systems, order, and organization
- Evidence, models, and explanation
- Change, constancy, and measurement

Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

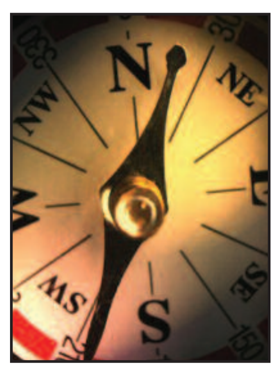
Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

Standard 7 - Students will develop an understanding of the influence of technology on history.

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.



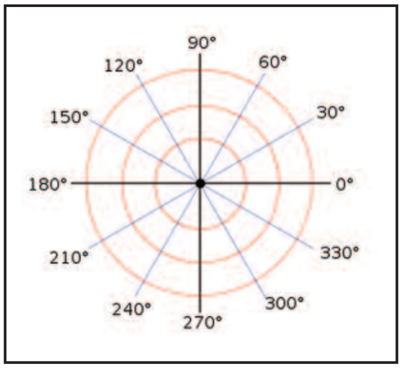
MATERIALS:

Materials—one set for each group

- Magnetic compass
- Device for measure angle of elevation of the Sun (see Appendix III for a crude diagram)
- Altitude & Azimuth data for your location

Materials—for each student

- Polar graphing paper
- Masking tape or tacks to affix graph to the wall



Polar Graphing paper

BACKGROUND INFORMATION:

Everyday examples of complex math problems give students some reason to remember and understand the lesson. For this lesson, the arc of the Sun as it travels accross the sky is likely overlooked but easily observed by all and lends itself to polar graphing. As an aid, published periodic weather data provides details to enhance the experience. Polar graphing is the plot of two angle measures. Mapping the Sun's path includes altitude (angle above the horizon) and azimuth (path along the horizon).

The students will graph the altitude and azimuth of the Sun for a particular day (data available from the US Navy Observatory). Polar graph paper can be printed from the online source listed below. Inform students that *azimuth* observations should be plotted on the ouside circle (0° through 360°) and *altitude* should be plotted on the concentric circles. One altitude plus one azimuth reading result in a point similar to an x,y coordinate. Provide no further details and see what happens. Note that *altitude* could be plotted with the zero at the origin or the outer-most circle and *azimuth* could be referenced from 0° at the top ("North"—as with a magnetic compass or lines of longitude) or at the right (as with standard protractor measurements). Four different graphs could result.

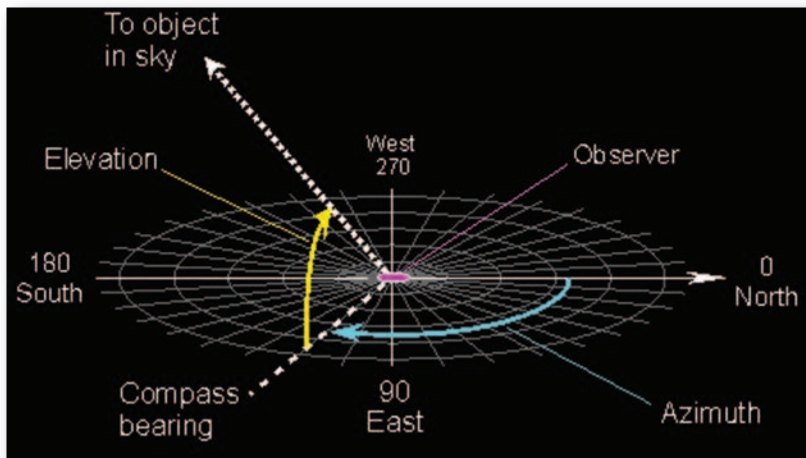
One graphical variation is given in Appendix II. Have students compare their graphing approaches. Ask the following questions:

1. Which mapping version most closely emulates the arc of the Sun across the sky?
2. What do the negative values (for "altitude") mean?
3. What is happening to the Sun when the points get further apart for successive half-hour intervals?
4. Was the Sun directly overhead at noon?
5. Explain how to identify "local noon" from the plot.
6. How will the plot differ for another date?
7. How will the plot differ for a different location?

To create a similar dataset, begin at the USNO Website, click "Astronomical Applications," then "more…" Under the heading *Positions of Selected Celestial Objects* choose "Altitude and Azimuth of the Sun or Moon During One Day." The data in Appendix I was created by selecting 4 July 2011 at Seattle, WA and 30 minute intervals, so choose an appropriate date and interval (30 minutes works well for most students).

PROCEDURE:

As shown in the figure below, azimuth is the angle of an object around the horizon, while altitude (or "elevation") is the angle of an object above or below the horizon.



1. Ask students to explain the path the Sun will make across the sky that day. Probe for what differences location or time of year would make.
2. Task students to measure the angle of elevation of the Sun and record the time at which the measurement was made. See Appendix III for a crude diagram of a device that will assist in this measurement. Caution: do NOT look directly at the Sun! Use a magnetic compass to approximate the Sun's direction.
3. Debrief step #2 by requesting speculation about how to graph the angle of elevation of the Sun over a day.
4. Show Azimuth vs. Altitude data, like that shown in Appendix I. Demonstrate how the azimuth (number of degrees clockwise from North) and altitude (number of degrees of elevation from the horizontal) are measured kinesthetically.
5. Hand out Altitude & Azimuth data (ideally, each student or student pair would get a different date for the same location).

6. Hand out polar graphing paper that has at least nine concentric circles and rays at least every 10° (every 5° is preferred). Explain that azimuth will be plotted around the circles with North at the top.That's all (leave whether students graph from inside out or outside in).
7. Students now graph their data.
8. Collect the papers that are plotted the same way (North at top, clockwise increase to angles, and the same treatment for "altitude" then post them around the room in chronological order. Students now engage in a "gallery walk" to understand the story the graphs tell.
9. Debrief by soliciting student ideas and insights. Share yours too.

SUMMARY:

Plotting Sun transit data in polar form gives practice in polar graphing and provides a way to both understand the phenomena and hypothesize what one would see at other global locations. This activity has the potential of inciting a great deal of incidental learning.

EVALUATION:

Have students write a response to the following questions and explain their reasoning:

- How would the Sun transit plot differ for a location 5° north of ours?
- How would the Sun transit plot differ for a location 5° south of ours?
- How would the Sun transit plot differ for a location 5° east of ours?

LESSON ENRICHMENT/EXTENSION:

- Plot a second location (but the same day) on the graph. Examine the plot for differences caused by longitude (there should be none) and latitude (the "altitude" difference should be connected to the difference in latitude.)
- Construct a gnomon (the part of a sundial that casts a shadow) where it will cast a shadow from the Sun from sunrise through sunset. Plot a point onto a sheet of paper every half hour where the tip of the gnomon casts its shadow. Record the time and the angle (clockwise from True North) for each point. Compute the angle of elevation of the Sun for each point using the height of the gnomon and arctan. Compare the computed angles with the table of *Azimuth and Altitude Of The Sun* from the USNO.

- Make multiple plots as in the previous bullet with observations spread apart by one week. Discuss how to use the USNO data to replicate such plots.
- Consider other graphs, such as day length for a location over a year (use the azimuth as day of the year because 360° ≈ 365 days).

ASSOCIATED WEBSITES AND/OR LITERATURE:

- Create your own graph paper
http://incompetech.com/graphpaper/

- US Navy Observatory altitude and azimuth of the sun on a given day and location
http://aa.usno.navy.mil/data/docs/AltAz.php

- NOAA's Solar Position Calculator
http://www.srrb.noaa.gov/highlights/sunrise/azel.html

- Derivation of The Elevation Angle of the Sun
http://www.sjsu.edu/faculty/watkins/elevsun.htm

Homemade Gnomon

Appendix I
Altitude and Azimuth of the Sun for Seattle, WA

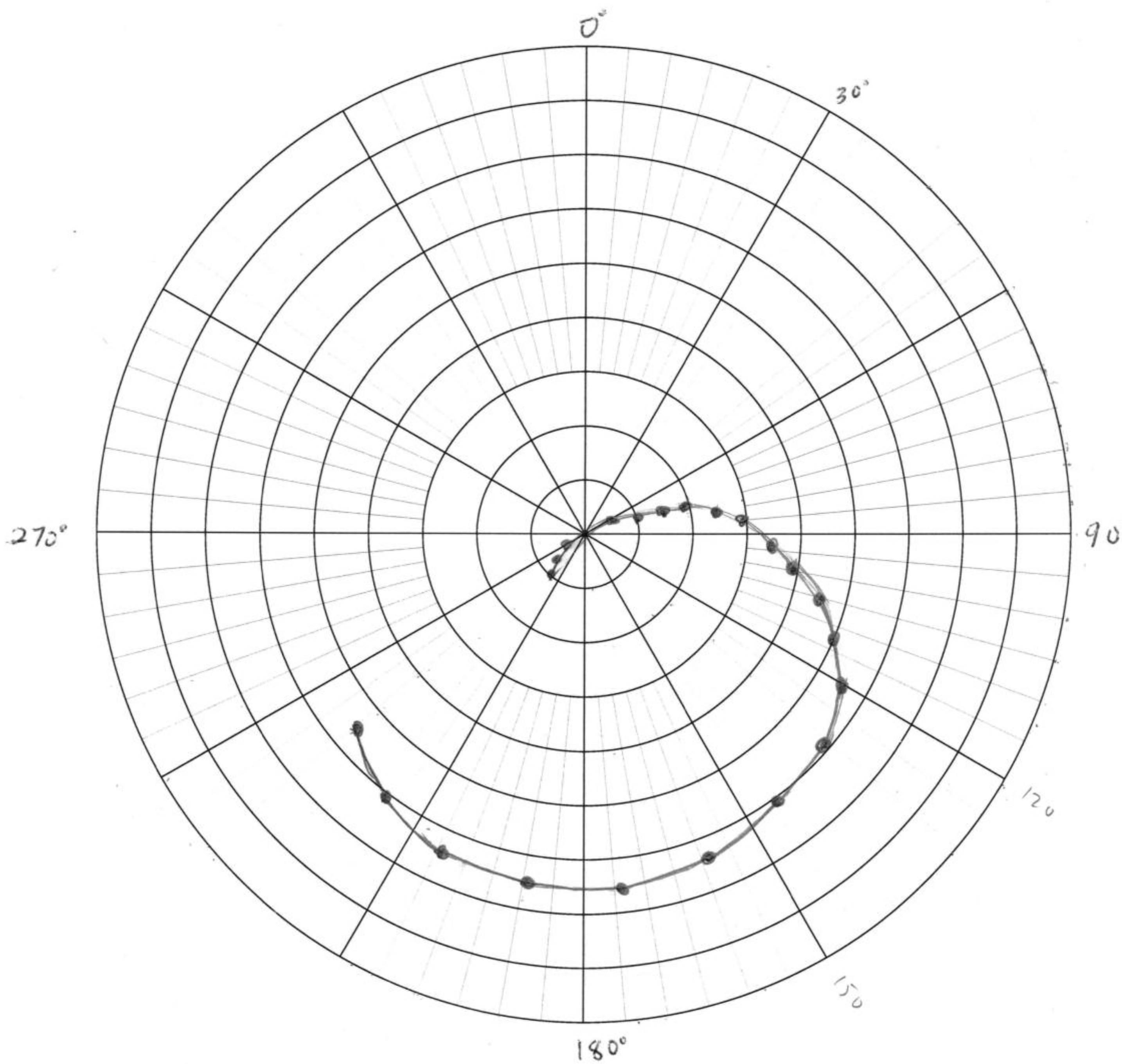
Astronomical Applications Dept.
U.S. Naval Observatory
Washington, DC 20392-5420

SEATTLE, WASHINGTON
o , o ,
W122 20, N47 38

Altitude and Azimuth of the Sun
Jul 4, 2011
Pacific Standard Time

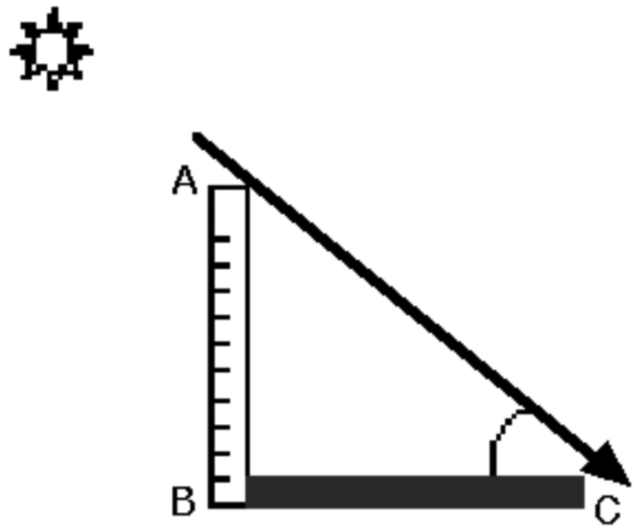
h m	Altitude o	Azimuth (E of N) o
03:00	-10.2	38.4
03:30	-6.9	44.5
04:00	-3.1	50.4
04:30	1.3	56.0
05:00	5.4	61.4
05:30	9.8	66.6
06:00	14.5	71.8
06:30	19.4	76.9
07:00	24.3	82.1
07:30	29.4	87.4
08:00	34.4	93.0
08:30	39.4	98.9
09:00	44.4	105.4
09:30	49.1	112.7
10:00	53.6	121.1
10:30	57.7	131.0
11:00	61.2	142.8
11:30	63.7	156.7
12:00	65.1	172.5
12:30	65.0	188.9
13:00	63.6	204.5
13:30	60.9	218.2
14:00	57.4	229.9
14:30	53.3	239.6
15:00	48.7	247.9
15:30	43.9	255.1
16:00	39.0	261.6
16:30	34.0	267.5
17:00	28.9	273.0
17:30	23.9	278.3
18:00	19.0	283.5
18:30	14.1	288.6
19:00	9.4	293.8
19:30	5.0	299.0
20:00	0.9	304.5
20:30	-3.5	310.1
21:00	-7.2	315.9
21:30	-10.5	322.1

Appendix II
Graph of Altitude and Azimuth of the Sun for Seattle, WA



This plot treats "azimuth" as a compass or true heading and "altitude" with) at the center.
The plotted times are from 0300 to 1400 PST.

Appendix III



Device for measuring the angle of elevation of the sun (the “altitude”).

Lesson 11: Prop Me Up!

OBJECTIVE:
Students explore the relationship between the diameter of a propeller and the thrust it produces.

NATIONAL STANDARDS:
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Systems, order, and organization
- Change, constancy, and measurement
- Form and function

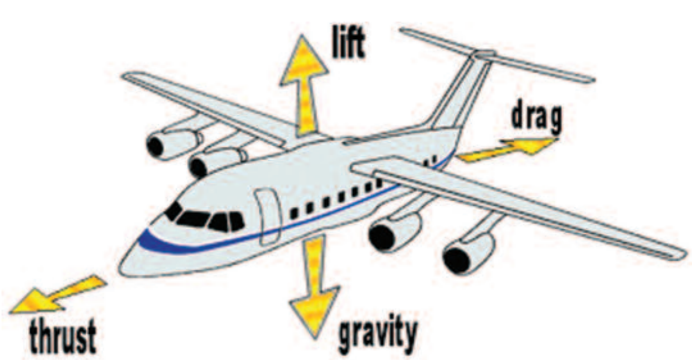
Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Physical science
- Motions and forces
- Interactions of energy and matter

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
Standard 7 - Students will develop an understanding of the influence of technology on history.
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.



MATERIALS: (USING AN R/C AIRCRAFT)

- Electric or fuel-powered airplane with an engine large enough to accommodate a wide variety of propeller diameters
- Propellers, in all diameters the engine can reasonably throw, making sure the pitch and propeller brand remain unchanged. Use propellers of the same pitch for the investigation (such as 12x6, 13x6, 14x6, etc.)
- Mini tachometer (about $18 from hobby shops)
- Spring scale, fish scale, Vernier Force Sensor, or other reasonably reliable method to measure static thrust
- Starter box, fuel, safety goggles, hearing protection, fire extinguisher, etc.
- Data tables (each student is given or makes one that has columns labeled Prop Diameter and Thrust along with rows with each prop diameter listed). A sample is located on page 63
- Graphing calculators or computer-based modeling software (for students)

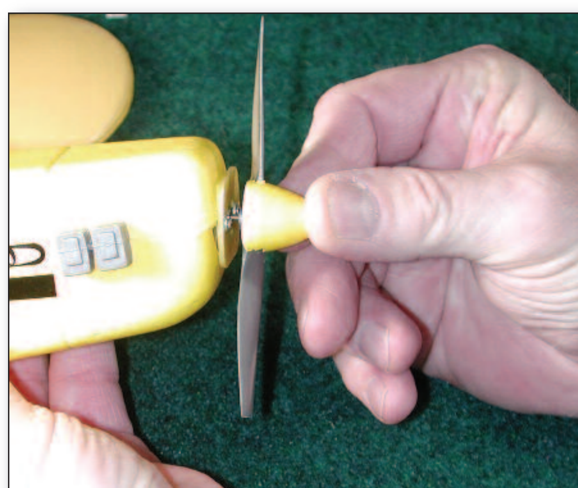
MATERIALS: (USING INTERNET RESOURCES)

- Computer with Internet connection connected to a projector
- Data tables (each student is given or makes one that has columns labeled Prop Diameter and Thrust along with rows with each prop diameter listed). A sample is located on page 63
- Graphing calculators or computer-based modeling software (for students)

BACKGROUND INFORMATION:

Radio controlled (R/C) airplanes are fully functional airplanes subject to all the forces of flight. For the purpose of scientific investigation, R/C airplanes are also a very affordable and accessible means of exploring the forces of flight. They provide a simple platform for testing ideas with very dramatic, tangible, and graphable results. This lesson will explore ways to optimize aircraft thrust, one of the basic forces of flight.

A quick fix for more power (thrust) in a road vehicle is to use a bigger engine. The balance of an airplane, however, does not allow for much change in engine size or weight. In most cases, the aircraft performance worsens if it changes at all. Another means to change aircraft thrust is to try various propellers on the same engine and measure the "pull" of the airplane using a fish scale. Surprisingly, the diameter of the propeller has a big impact on thrust. A larger diameter propeller turning slower can produce significantly more thrust than a slightly smaller one that is turning faster.



PROCEDURE: (USING AN R/C AIRCRAFT)

1. Remind students of the "four forces of flight." One of them, of course, is thrust, which differentiates an airplane from a glider.
2. Show a wide variety of propellers, including those made from wood, fiberglass, and plastic.
3. Demonstrate the meaning of "pitch" by moving a propeller through the air the correct number of inches as you revolve it once. A 12 x 6 propeller is 12 inches in diameter and will "move through" 6 inches of air in one revolution.
4. Agree, as a class, that RPM, pitch, air temperature, etc., will all be held constant throughout the data gathering trials while propeller diameter is varied.
5. Hook scale or force sensor to tail wheel or loop a cord around the airplane's empennage, making absolutely certain the airplane cannot get away.
6. Conduct a safety briefing: listen at all times for instructions, stand to the side of the airplane (never in front or behind), etc. Wear proper eye protection.
7. Run the engine to the agreed-upon RPM and measure the force. Students record it.
8. Repeat with other propellers at the agreed-upon RPM. Use at least six propellers.
9. Return to the classroom for students to use their calculators to compute the Power Model for the data. Hold a discussion as to why Linear and Exponential would be inappropriate.

PROCEDURE: (USING INTERNET RESOURCES)

1. Select a Website such as **http://personal.osi.hu/fuze sisz/strc_eng/index.htm** or **http://adamone.rchomepage.com/calc_thrust.htm** to generate the data. Spend some time with it on your own before you present to the class. See an example on page 63.

SUMMARY

Using an actual R/C airplane or online simulation software, we see that a small increase in propeller diameter makes a big difference in thrust when RPM and propeller pitch are kept constant.

EVALUATION:

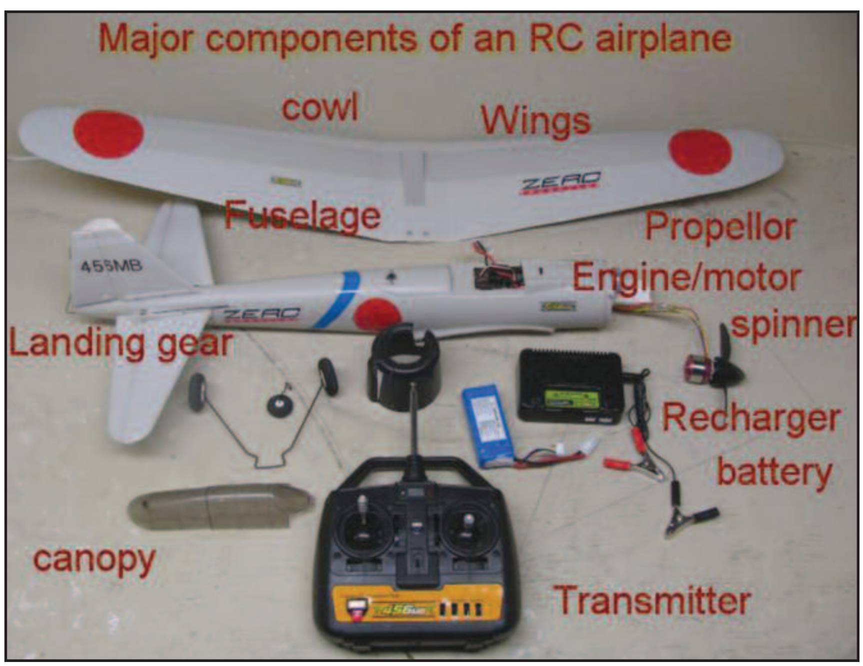
- Have students write a response to the question, "Why is a power function or polynomial a better model of propeller diameter vs. thrust rather than linear or exponential?"
- Have students write a response to the question, "What power model appears to be most appropriate for the relationship between propeller diameter vs. thrust?

LESSON ENRICHMENT/EXTENSION:

- Run the trials several times for each propeller to cross-check the force values.
- Mount the propellers on different airplanes to see if the thrust values are the same at the same RPM.
- Check the values you compiled experimentally with those from one of the Websites.

ASSOCIATED WEBSITES AND/OR LITERATURE:

- **http://personal.osi.hu/fuzesisz/strc_eng/index.htm**
- **http://adamone.rchomepage.com/calc_thrust.htm**



Thrust vs. Propeller Diameter

Diameter (in)	Thrust (pounds)
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

http://adamone.rchomepage.com/calc_thrust.htm

Lesson 12: Rules of Thumb

OBJECTIVE:
Students will evaluate a variety of "rules of thumb" for their "domain of acceptability" (values over which the "rule" is a reasonable approximation of the phenomena).

NATIONAL STANDARDS:
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Data Analysis and Probability
- select and use appropriate statistical methods to analyze data
- develop and evaluate inferences and predictions that are based on data

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Reasoning and Proof
- make and investigate mathematical conjectures

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- analyze and evaluate the mathematical thinking and strategies of others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Systems, order, and organization
- Evidence, models, and explanation
- Change, constancy, and measurement

Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
Standard 7 - Students will develop an understanding of the influence of technology on history.
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:

- Graph paper and/or geometric construction tools (Some students may need "labware," e.g., CBL and temperature probes).
- Poster board if presentations will be made

BACKGROUND INFORMATION:

Nearly every scientific principle can be resolved into a reasonably straight-forward function—typically linear, polynomial, exponential, or trigonometric. A few examples are orbits of planets, tides, and height of a column of mercury versus air pressure. Some relationships may be approximated by simple functions even though they require calculus. An example is predicting the temperature of a liquid as it cools because the relationship is a differential equation *(Newton's Law of Cooling)* but can be reasonably modeled as an exponential. The preciseness of the actual functions allows for predictability and extension to other areas; however, it may not be necessary for many "practical applications."

Often "close enough is good enough" in our daily lives. The same is true in aviation, where a precise answer is not only unnecessary, it might also require too much time to compute and, therefore, create a potentially dangerous distraction. "Rules of thumb" (ROT) are, therefore, commonly taught because they allow for mental math and provide an acceptable answer without unduly adding to the pilot's workload. This activity reveals many of the common ROT and examines them for their level of applicability.

PROCEDURE:

1. Ask students to plot the Altitude vs. Pressure data in Appendix I. When they are finished, announce that student pilots are taught in ground school that pressure decreases at the rate of one inch of mercury per thousand feet of altitude gain. Facilitate a discussion as to why this "rule" is taught when the data is clearly exponential. As the discussion subsides, ask students to draw the line with the slope (1"/1000') and a y-intercept of 29.92" (standard pressure). Group students in pairs to discuss, again, why this "rule" is taught. Hold a classroom discussion after the groups concur and steer the discussion towards the ROT existing over an acceptable domain (in this case, the altitudes applicable for general aviation) and being an easy relationship to remember.
2. Assign other ROT to student pairs or allow them to choose one that appeals to them. See Appendix II for a list of suggested ROT. Consider distributing the ROT on slips of paper or with the ROT written on the handout in Appendix III.
3. Monitor students as they work. Provide suggestions regarding geometric constructions or data gathering for the ROT that might need these methods.
4. Provide a venue in which to share student findings. For each one, have students explicate the domain of acceptability (values over which the "rule" is a reasonable approximation of the phenomena) for the ROT—after all, that's the reason they exist!

SUMMARY:

"Rules of Thumb" exist to both provide an easier way to understand a phenomena, relationship, or principle and a quick way to approximate computations. Fluency in "mental math" is prized and supported by the use of rules of thumb.

EVALUATION:

Explain in simple language (e.g., using terms understandable by someone much less sophisticated and less mathematically knowledgeable than you) how to determine the "domain of acceptability" of a rule of thumb.

LESSON ENRICHMENT/EXTENSION:

- Allow students time to produce a poster of their findings. The product should include the ROT, the actual relationship, any sources of information students used, and the domain under which the ROT is acceptable. Host a "gallery walk" where students can view all other posters.
- Facilitate a brainstorming session where students either "invent" or recall ROT they can test.

ASSOCIATED WEBSITES AND/OR LITERATURE:

- **http://www.phrases.org.uk/meanings/rule-of-thumb.html**
- **http://rulesofthumb.org/**

Appendix I

Table of pressure vs. altitude

Altitude (feet)	Pressure (inches of Mercury)	Altitude (feet)	Pressure (inches of Mercury)
0	29.92	18,000	14.94
1,000	28.86	19,000	14.33
2,000	27.82	20,000	13.74
3,000	26.82	25,000	11.10
4,000	25.84	30,000	8.89
5,000	24.89	35,000	7.04
6,000	23.98	40,000	5.54
7,000	23.09	45,000	4.35
8,000	22.22	50,000	3.43
9,000	21.38	55,000	2.69
10,000	20.57	60,000	2.12
11,000	19.79	65,000	1.67
12,000	19.02	70,000	1.31
13,000	18.29	75,000	1.03
14,000	17.57	80,000	0.81
15,000	16.88	85,000	0.64
16,000	16.21	90,000	0.50
17,000	15.56	95,000	0.32

Appendix II
Examples of *Rules of Thumb*

- SOMETHING IN THE SKY. You can describe the location of objects that are low in the sky by holding your hand in front of you at arm's length. With your palm facing in and your pinkie on the horizon, the width of your hand covers 15 degrees of arc above the horizon.

- ESTIMATING DISTANCES. Hold your thumb at arm's length against a distant background. Estimate how far your thumb jumps on the background when you look at it with one eye and then the other. The background is ten times that distance from you.

- HOW FAR OFF COURSE. You will be one nautical mile off course when traveling 60 NM for each 1° deviation.

- TEMPERATURE LAPSE RATE. Air temperature decreases at the rate of 2° C for each 1000’ of altitude.

- ESTIMATING ALTITUDE AND AZIMUTH. Your hands can be used to make fairly accurate measurements of altitude and azimuth. Close one eye and hold your fist at arms length, so that the thumb is toward the zenith. The amount of sky covered by the fist from little finger to thumb is approximately 10 degrees. Half a fist, or the width of two extended fingers is 5 degrees. Measure a star's altitude by estimating how many fists and fingers it is above the horizon; for measurements in azimuth, simply turn your fist 90 degrees.

- DISTANCE TO THE HORIZON. The distance to the horizon, in miles, is the square root of half again your height, in feet. If you're 6 feet tall, you can see 3 miles. From 600 feet, you can see 30 miles (sq. rt. of 900). Conversely, you can see a 150-foot building from 15 miles away.

- DISTANCE TO THE HORIZON. The distance to the horizon is equal to the square root of your altitude multiplied by 1.22.

- CROSSWIND COMPONENT. The crosswind component (the “wind vector” perpendicular to the runway) at an airport will be about 70% of the wind speed.

- DME IS THE SAME AS GROUND DISTANCE. DME is measured from an aircraft to a station on the ground. The distance over the ground from the point directly under the airplane is about the same as the DME.

- ESTIMATING WIND SPEED. Expect a 15 KT wind when small tree branches are moving and 25 KT when small trees are swaying.

- HYDROPLANING SPEED. The speed at which aviation tires will hydroplane is nine times the square root of the tire pressure.

- TAKEOFF DISTANCE. The distance to clear a 50’ obstacle is 80% more than the ground roll which is 80% more than the landing distance.

- MEASURING ANGLES. For a small angle x measured in radians, sin x = x. This approximation is frequently used in astronomy.

- CONVERT DEGREES CELSIUS TO DEGREES FAHRENHEIT. Double the Celsius temperature and add thirty.

- FUEL WEIGHT. Multiply the number of gallons of gasoline by six to get the number of pounds of fuel. Multiply by seven for Jet A fuel and eight for water.

- POUNDS OF JET FUEL TO GALLONS. To convert pounds of jet fuel to gallons, drop the zero and then add 50 percent to that.

Appendix II *(continued)*

• WHEN TO BEGIN A DESCENT. Divide cruise altitude by two and ignore the last two digits (usually zeros) to get the number of nautical miles away from a destination airport to begin a descent.

• ALTITUDE TO EXPECT CLOUD BOTTOMS. Subtract the dew point from the temperature to get the number of thousands of feet to expect cloud formations.

• ESTIMATING SPEED. Speed in feet per second is about 1.5 times speed in miles per hour.

• HEIGHT OF A HILL. The rate at which an object falls is independent of how fast it is traveling laterally. To determine the height in feet of a slope, throw a rock out level and time its fall. Square the number of seconds it takes the object to land, then multiply by 16. This will be your height in feet above the landing site.

• MULTIPLYING BY FIVE. To figure what any number times five is, take half the number and multiply by 10.

• THE SUM OF THE DIGITS OF ANY MULTIPLE OF NINE WILL EQUAL TO NINE OR A MULTIPLE OF NINE.

• BUILDING STAIRS. A set of steps will be comfortable to use if two times the height of one riser, plus the width of one tread is equal to 26 inches.

• FINDING YOUR DOMINANT EYE. To find your dominant eye, make a circle of your thumb and forefinger about 6 inches in front of your face. Look through the circle with both eyes at an object across the room. Now close one eye; if the object stays in the circle, the open eye is the dominant one.

• HANDWRITING ANALYSIS. If handwriting consists of all capitals, the chances are greater than 50 percent that it is a man's. If the capitals are slanted or joined, the odds are 75 percent that it is a man's. If the capitals are both slanted and joined, 85 percent of the time it was written by a man.

• DOLLAR BILL RULER. A US dollar bill is about six inches long (denomination is unimportant).

• QUARTER IS AN INCH. A US quarter is a trifle under an inch in width.

• HEIGHT BY ARMSPAN. Your "Wingspan"(Arm span) is about the same distance as your Height. If your 6 feet tall your arms from hand to hand is about 6 feet wide.

• CENTIMETER. A centimeter is about as long as the width of an average adult's little finger nail.

• WALKING SPEED. An average walking speed is about 5 km/h (3 mph).

• HEIGHT AS AN ADULT. Your adult height will be twice your height at the age of 22 months.

• LEANING A LADDER. When you use an extension ladder, you should put the bottom of the ladder one foot away from the wall for every four feet of vertical height.

• VOLUME OF A TANK. A cylindrical tank 15 inches in diameter holds about 1 gallon for each inch of height. A 30-inch diameter tank holds 4 gallons per inch.

• SPOTTING A CARNIVORE. Mammals with eyes looking straight ahead are carnivorous. Mammals with eyes looking to the side are vegetarian.

• THE RULE OF TWICE. Twice around the thumb is once around the wrist; twice around the wrist is once around the neck; twice around the neck is once around the waist.

Appendix III

Partner 1: ____

Partner 2: ____

Our Rule of Thumb: ____

____

____

____

____

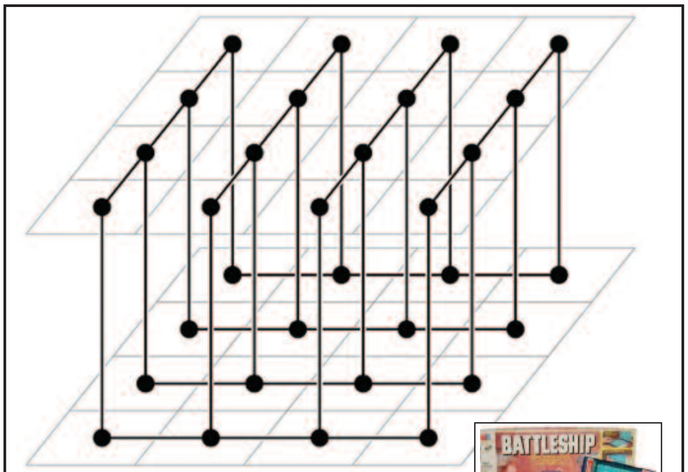
____

____

Instructions: Locate the actual function that governs the relationship approximated by your assigned *Rule of Thumb.* Sources would include your own geometric constructions, data you gather through a reasonably precise method, encyclopedias, and textbooks. Provide a graphical representation of both relationships (e.g., graphs or diagrams) and suggest a domain over which the *Rule of Thumb* is a reasonable approximation.

Lesson 13: Sky Wars!

OBJECTIVE:
Students practice 3-D graphing in the context of a game that is similar to Battleship, only in three dimensions rather than two.



NATIONAL STANDARDS:
Mathematics
Algebra
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Change, constancy, and measurement

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Historical perspectives

Technology
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS - (one set for each pair of students)
- Sky Wars rules

MATERIALS - (for each student)
- Sky Wars game sheets
  (three pages: Squadron Coordinates, Defense Grids, and Attack Grids)

BACKGROUND INFORMATION:

Plotting points in three dimensions is difficult to depict, especially when restricted to the two dimensions afforded by paper. One method, practiced with this game, is to "layer" several sets of x-y planes to depict graphing along the z-axis.

This activity is a derivation of the popular game Battleship™. The commercial version of the game has letters along one axis and integers along the other. Sky Wars was developed so 3-D graphing could be practiced in a similar way. Three identical grids represent the x, y, and z axes, so a point like (1, 2, 1) is located on the first grid, across one from the origin and "up" two along the y-axis. The z-axis begins at 1 to represent planes that are suspended above the ground at various heights. This lesson should help students extend their understanding of 3-D graphing.

Note: a typical game can take up to two hours from set-up to conclusion. Produce some extra packets for students to take home to play "just for fun."

PROCEDURE:

1. Remind students how to graph in two dimensions. Everyone must be clear on how the points (2, 3), (–2, 3), and (–2, –3) differ. If some doubt exists on students' understanding, consider having students play the 2-D version (http://edgerton.us/Battleship.pdf) first.
2. Demonstrate graphing in 3-D using the layered grids from one of the game pages.
3. Pass out copies of the first two pages of the activity (duplex copying works fine for this). Give students time to read the instructions and ask questions.
4. Clarify who is playing whom. Review the Special Provisions if you want or need to have three students play each other.
5. Decide whether to play on the whole x-y plane or to restrict the x-y planes to make finding aircraft easier to "find." Typical "shrinking" of the x-y planes would be to prohibit greater than or equal to x = 5, y = 5, or both.
6. Pass out copies of the last two pages of the activity and ask students to "hide" their aircraft in the Defense Grid. Remind students they may place aircraft "horizontally" or "vertically" in the space but not diagonally. Make sure students understand the fighter/attack aircraft is the only one that can be represented vertically and stay in the game space.
7. Check student papers to make sure their aircraft are depicted correctly in their Defense Grids.
8. Let play begin and monitor to see if points are recorded and graphed correctly.
9. Stop play and debrief the activity when the games are nearing conclusion (one participant is nearly out of aircraft) or when time is running out. Ask students what they learned and what should be changed about the game if they (or anyone else) were to play it again.

SUMMARY:

Graphing in three dimensions is difficult because one is restricted to two dimensions (paper, computer screens) unless one employs "tricks" such as stacking the grids (as with this game) or using color (as common in the computer application world). Sky Wars provides practice in "visualizing" one representation of 3-D graphing.

EVALUATION:

Explain in simple language (e.g., using terms understandable by someone much less sophisticated and mathematically wise than you) how to graph the points (1, 2, 3) and (3, 1, 2).

LESSON ENRICHMENT/EXTENSION:

- Allow two "flank bursts" which hit any adjacent coordinate in the x-y plane.
- Add additional x-y planes so the z-axis has more levels.
- Host a Sky Wars tournament.
- Show from 0:45 to 5:15 of the movie Pushing Tin to introduce a practical example of 3-D reasoning. Explain the purpose of the TRACON, the "blocks" used to keep track of aircraft, and language used in vectoring aircraft—all while the clip is playing.

ASSOCIATED WEBSITES AND/OR LITERATURE:

- **http://www.livephysics.com/ptools/online-3d-function-grapher.php**
- **http://www.math.uri.edu/~bkaskosz/flashmo/graph3d/**
- **http://www.houseof3d.com/pete/applets/graph/index.html**
- **http://www.ies.co.jp/math/java/misc/SimpleGraph3D/SimpleGraph3D.html**
- **http://www.archimy.com/**
- **http://reference.wolfram.com/mathematica/ref/Plot3D.html**
- **http://www.originlab.com/index.aspx?go=Prod ucts/Origin/Graphing/3D**
- **http://www.gnuplot.info/**

Sky Wars Student Sheet

Name ____

Opponent ____

INTRODUCTION

The purpose of this game is to practice visualizing and plotting points in 3-D. As you can tell, this game is merely a variant of the popular board game Battleship™. Play one opponent by shooting at a "hidden" aircraft while keeping track of all shots, hits, and misses. Hopefully, this exercise will be fun and educational for you and your opponent. Use the multiple four-quadrant grids which accompany this page that stack to make the z-levels.

PREPARATION FOR PLAY

"Hide" your aircraft by plotting their symbol (see below) on the graph using whole numbered values in your Defense Grids. Aircraft may be placed anywhere on the graph as long as the proper number of dots appears where the "grid lines" intersect. Place aircraft either in a single x-y grid or vertically–no diagonal aircraft! Before you begin firing, be sure to check with your teacher to make sure that your aircraft have been properly placed on the grid. Make sure no aircraft are placed between grid lines and all five aircraft are used. Write the coordinates of the "hidden" aircraft on the sheet entitled "Squadron Coordinates."

RULES

Take turns calling out the coordinates of a point that you think will “hit” an aircraft on your opponent’s grid. The attacker will call a point, such as (2, 3, 1), and the defender will respond either “hit” or “miss.” BOTH persons will record (on the paper) each point called in attack and mark with “H” for hit or “M” for miss. When all dots for an aircraft have been hit, it is identified to the attacker and crossed off both lists. The winner, provided you are able to play long enough, is the person who first destroys the other person’s squadron.

SUGGESTIONS

Record all shots on the appropriate grids, using a “dot” for each shot (•) and “x” when there is a hit. Check with your teacher a few times at the beginning of the game to make sure that your are proceeding properly.

SPECIAL PROVISION FOR GROUPS OF MORE THAN TWO

Three or four persons can also play together. However, the competition and complexity increases dramatically. Each person’s shots land in each of the opponent’s grids. It is, therefore, possible to have multiple hits with one shot. Each person records the shots of every competitor, and play continues until only one person remains.

Representing Symbols	Aircraft Type
* * *	Fighter / attack aircraft (three dots)
* * * *	Aerial tanker (four dots)
* * * * *	Transport aircraft (five dots) - Make two of these.
* * * * * *	Bomber (six dots)

Squadron Coordinates

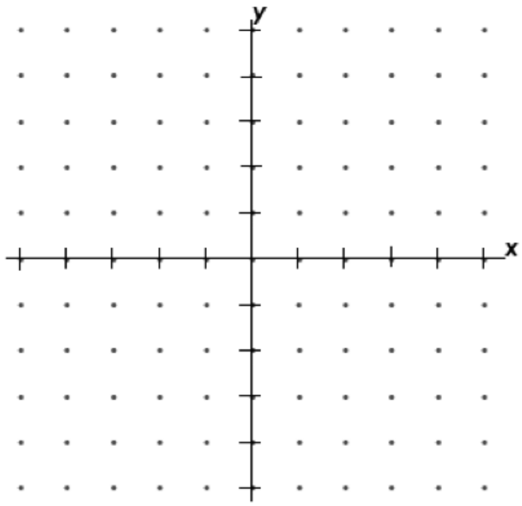
Write the coordinates of each point of your aircraft here.

Aircraft	Coordinates
Fighter/Attack	
Aerial tanker	
Transport #1	
Transport #2	
Bomber	

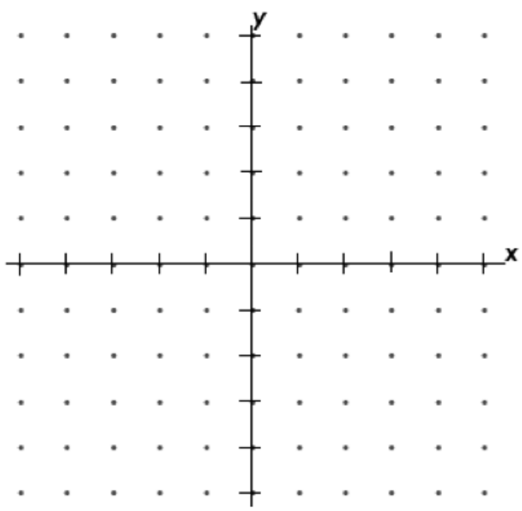
ATTACK GRIDS

Shoot at opponent's aircraft here

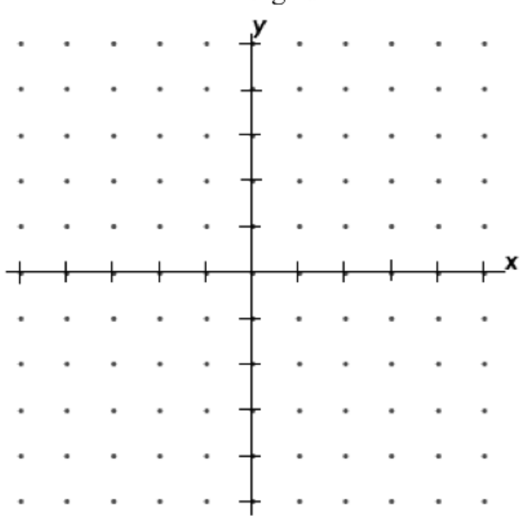
z = 1 grid



z = 2 grid



z = 3 grid



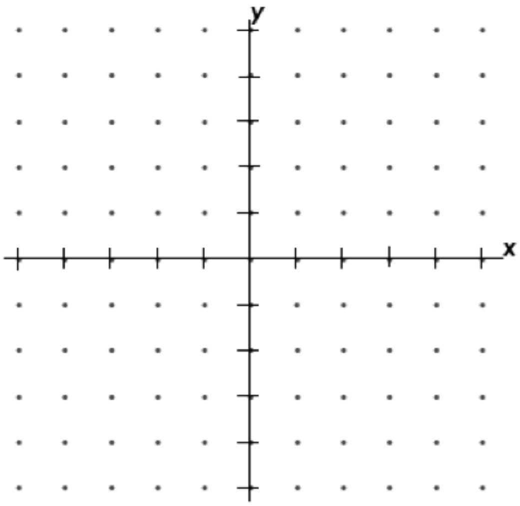
Your shots

Coordinates	Hit/Miss

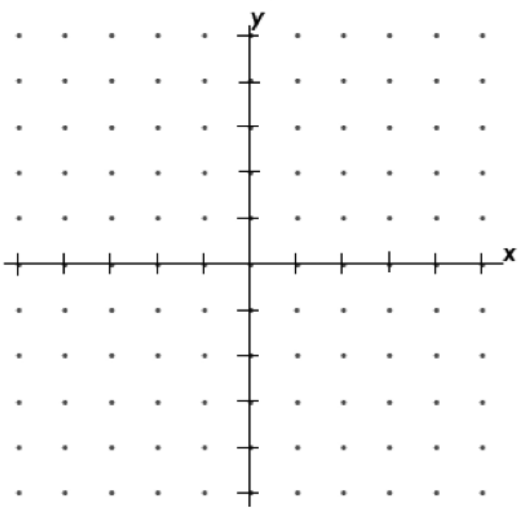
DEFENSE GRIDS

Hide your aircraft here

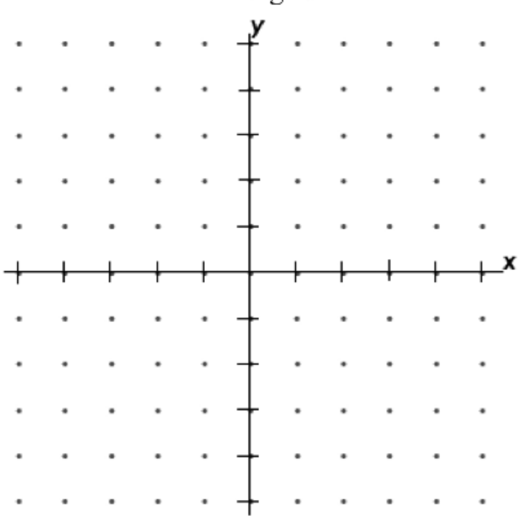
z = 1 grid



z = 2 grid



z = 3 grid



Opponent's shots

Coordinates	Hit/Miss

Lesson 14: Space Station Spotter

OBJECTIVE:
Students apply geometry to determine when the International Space Station (ISS) will be visible.

NATIONAL STANDARDS:
Mathematics
Algebra
- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry
- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

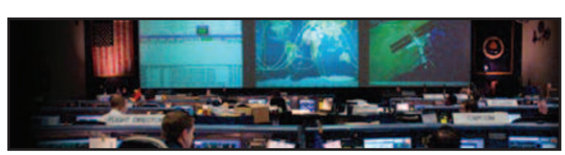
Science
Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

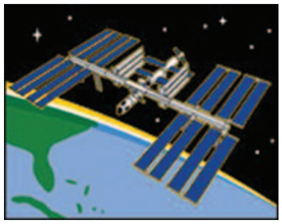
History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 1 - Students will develop an understanding of the characteristics and scope of technology.
Standard 3 - Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:
- Ruler, protractor, drawing compass, inclinometer, and a magnetic compass
- Table of sunrise and sunset times for your area
- Estimated sighting opportunities for the ISS (see Appendix I)


NASA Control tracking ISS

BACKGROUND INFORMATION:



A great thrill is to watch a celestial object made by people pass overhead, especially if there are people aboard. Putting humans into space and returning them safely is a difficult task and one now shared with several nations in the construction and servicing of the *International Space Station* (ISS). Personally seeing any part of this endeavor is an opportunity worth taking!

The activity in Lesson 7: *Modeling Satellite Orbits* leads students through creating an equation that provides the location of the ISS but stops short of whether it might be visible at a particular location. Viewing the ISS as it passes overhead requires, in addition to a relatively clear sky, sunlight reaching the satellite and a dark enough sky to provide contrast. This activity allows students to determine if the ISS will be visible when it passes nearby.

Assume the Earth is a sphere with a radius of 3960 statute miles (SM), and the ISS will orbit at an altitude of 250 SM. Using right triangles and the Pythagorean Theorem, the ISS must pass within about 1400 SM of the ground-based observer to be viewed. See Appendix II—the tangent line from a location on Earth's surface must contact part of the ISS orbit. Because the tangent forms a right angle, the remaining leg of the triangle is easily solved. Note that the central angle must be $\cos^{-1}(3960/4210) \approx 19.8°$. This analysis, of course, ignores refraction of the atmosphere, which the USNO indicates "the average amount of atmospheric refraction at the horizon is 34 arcminutes" (about 0.57°). The angle just derived will become more important when the illumination of the ISS by the Sun is determined.

One's knowledge of the stars "vanishing" after sunrise suggests the ISS will be invisible during daylight hours. One may also reason that the Earth would eclipse the sunlight most of the night and deprive the ISS illumination. One can estimate the time an ISS transit would be visible by drawing a line representing the Sun's rays from below the horizon so it will intersect the ISS transit at its apex (selecting this arbitrarily so at least half the transit is visible). Note in Appendix III that the central angle is identical to the one previously computed and, therefore, about 20°. Since the Earth rotates 15° per hour (360° ÷ 24 hours), the ISS should be visible from about 80 minutes before sunrise until sunrise itself and from sunset until about 80 minutes after sunset. Consult the sunrise/sunset table from the USNO for your area and overlay the times with those generated for the orbit of the ISS. You then have knowledge of whether the ISS will be visible.

PROCEDURE:

1. Brief students on the ISS orbits, missions, and crews. Consider completing the Modeling Satellite Orbits activity first. Plan this at a time when there will be several ISS sighting opportunities and during the winter months when students' availability will more likely coincide with opportunities.
2. Show the computer-generated sighting opportunities to students and ask them to examine the listings for an "interesting pattern." See Appendix I for an example. Hopefully, someone will notice the opportunities exist only just before sunrise or just after sunset (otherwise prompt for when people can see the ISS).
3. Host sightings of the ISS. Attempt to confirm the entry and exit latitudes of the transits along with the maximum elevation above the horizon the ISS is forecast to make. Use a magnetic compass and inclinometer for measurements. Note: find out the magnetic variation (difference between true and magnetic North) for your area.
4. Invite your students (in a whole-class discussion) to hypothesize why the ISS can be seen at some times and not others. Encourage sketches shared with all students that include the Earth, the ISS orbit, Sun position, etc.
5. Assign student pairs to develop the geometry by which one can "see" the ISS if it orbits at 250 SM and the Earth's radius is 3960 SM.
6. Once concordance is reached on ISS viewing, challenge students to develop Sun angles that allow ISS visibility. These should include disappearance of the ISS at sunrise but visibility at identifiable angles before sunrise (or after sunset).
7. Press for the amount of time the geometry suggests for ISS visibility. Students may not realize at first the Earth rotates at 15° per hour.
8. Overlay the conclusion of the above with the outcome of the Modeling Satellite Orbits activity from lesson 7.
9. Compare findings with both observations (time, rise and set latitudes, and visibility).
10. Debrief by, once again, showing the Sighting Opportunities printout and suggesting someone must have already worked out the formulae!

SUMMARY:
Using geometry, construction, and/or trigonometry, it can be concluded that the ISS will be visible from about 80 minutes before sunrise until the Sun lights the sky enough that it (and the stars) are no longer visible. Likewise, the ISS can be viewed when the sky is sufficiently dark until about 80 minutes after sunset.

EVALUATION:
Explain in simple language (e.g., using terms understandable by someone much less sophisticated and less mathematically knowledgeable than you) why the ISS is not visible during each of the following situations: between sunrise and sunset, and between 80 minutes after sunset until 80 minutes before sunrise. Include a diagram with each situation.

LESSON ENRICHMENT/EXTENSION:
- Have students compute the closest the ISS will pass on a specified orbit by using its "maximum elevation" above the horizon. Note that if the elevation is 90°, it will pass directly overhead and will, therefore, be only 250 miles away!
- Build a scale model of the ISS. Use the CAP International Space Station curriculum module as a template. The finished product makes a great conversation piece hanging in a classroom! Show the ISS assembly video several times during the process.

ASSOCIATED WEBSITES AND/OR LITERATURE:
- NASA International Space Station page
http://www.nasa.gov/mission_pages/station/main/index.html

- ISS Orbital Tracking
http://spaceflight.nasa.gov/re aldata/tracking/

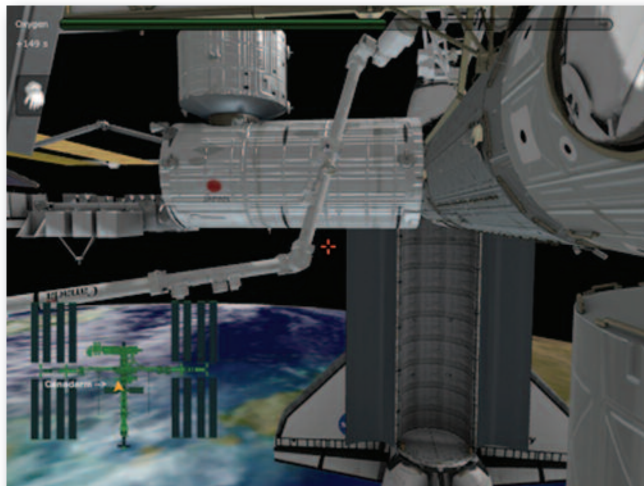
Tracking the ISS

- US Navy Observatory Astronomical Application for sunrise and sunset times
http://www.usno.navy.mil/USNO/astronomical-applications/data-services/rs-one-year-us

- ISS assembly
http://i.usatoday.net/tech/graphics/iss_timeline/flash.htm

- Life aboard the Space Station
http://www.nasa.gov/audience/foreducators/teachingfromspace/dayinthelife/index.html

- Station Spacewalk Game
http://www.nasa.gov/multimedia/3d_resources/station_spacewalk_game.html



Station Spacewalk Game

- NASA Human Space Flight
http://spaceflight.nasa.gov/

Appendix I

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

HUMANSPACEFLIGHT

+ SHUTTLE + STATION + REALTIME DATA + NEWS + GALLERY + QUESTIONS + HISTORY + INFO + SITEMAP + SEARCH

Host a Star Party!

SATELLITE SIGHTING INFORMATION FOR

NASA Johnson Space Center
Send Space Station Sighting Information

***** Seattle *****

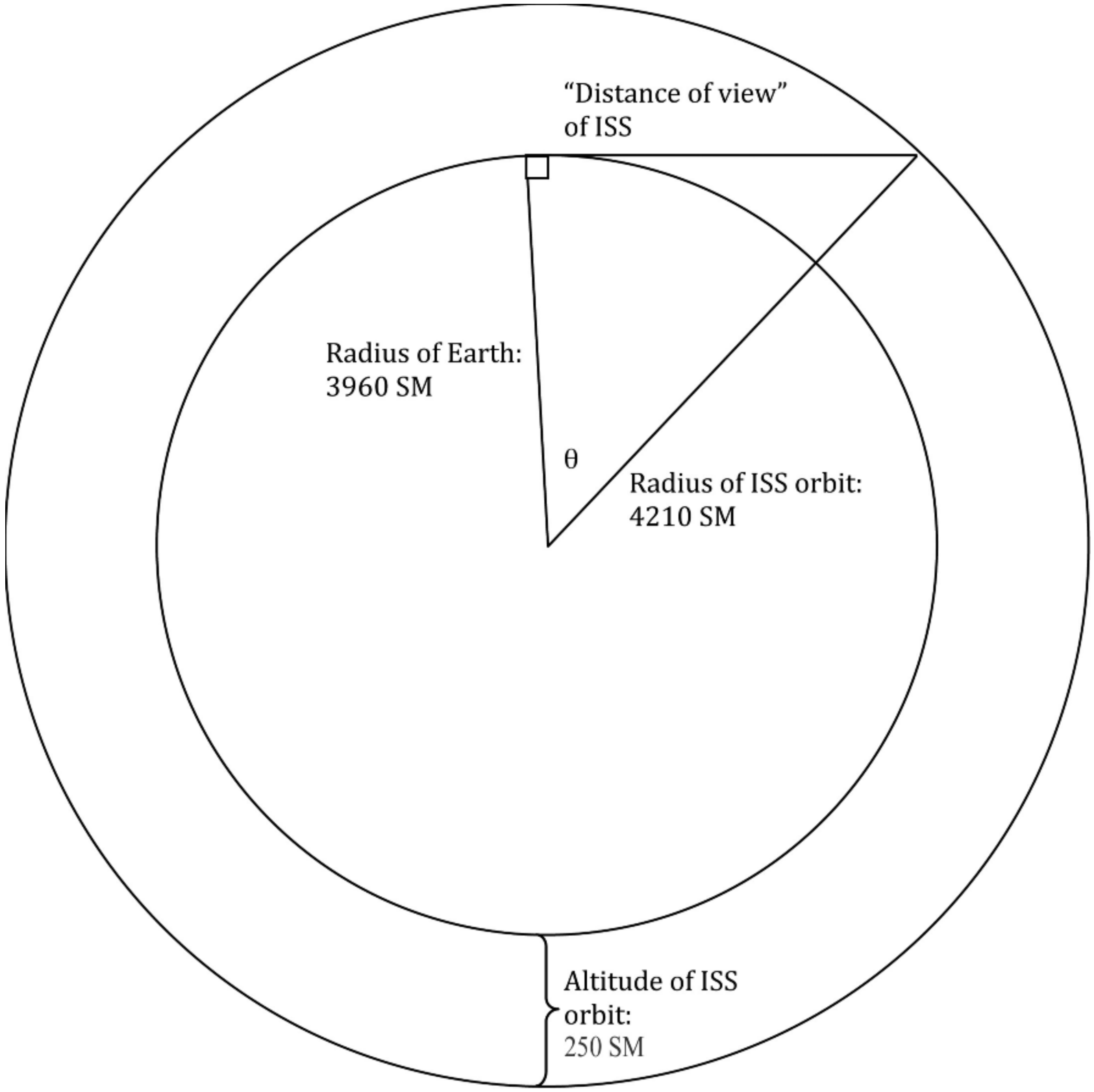
Subscribe via RSS

THE FOLLOWING ISS SIGHTINGS ARE POSSIBLE FROM WED AUG 17 TO THU SEP 01

SATELLITE	LOCAL DATE/TIME	DURATION (MIN)	MAX ELEV (DEG)	APPROACH (DEG-DIR)	DEPARTURE (DEG-DIR)
ISS	Wed Aug 17/10:07 PM	5	48	10 above W	18 above NE
ISS	Wed Aug 17/11:43 PM	1	17	10 above WNW	17 above NW
ISS	Thu Aug 18/09:09 PM	6	70	10 above WSW	11 above ENE
ISS	Thu Aug 18/10:46 PM	2	36	17 above WNW	34 above NNE
ISS	Fri Aug 19/09:49 PM	4	40	30 above NW	11 above ENE
ISS	Fri Aug 19/11:23 PM	1	22	10 above WNW	22 above NW
ISS	Sat Aug 20/08:51 PM	4	51	41 above WNW	10 above ENE
ISS	Sat Aug 20/10:27 PM	1	37	29 above NW	35 above NNE
ISS	Sun Aug 21/09:29 PM	3	36	33 above NNW	11 above ENE
ISS	Sun Aug 21/11:03 PM	1	28	12 above WNW	28 above NW
ISS	Mon Aug 22/08:34 PM	< 1	14	14 above ENE	12 above ENE
ISS	Mon Aug 22/10:07 PM	2	42	31 above NW	35 above NE
ISS	Mon Aug 22/11:41 PM	< 1	10	10 above WNW	10 above WNW
ISS	Tue Aug 23/09:09 PM	4	37	29 above NW	11 above ENE
ISS	Tue Aug 23/10:44 PM	1	37	15 above WNW	37 above WNW
ISS	Wed Aug 24/09:47 PM	2	52	34 above NW	34 above ENE
ISS	Wed Aug 24/11:21 PM	< 1	10	10 above WNW	10 above WNW
ISS	Thu Aug 25/08:49 PM	4	40	30 above NW	11 above E
ISS	Thu Aug 25/10:24 PM	1	46	19 above WNW	46 above W
ISS	Fri Aug 26/09:27 PM	2	72	37 above NW	31 above E
ISS	Fri Aug 26/11:01 PM	< 1	11	11 above W	11 above W

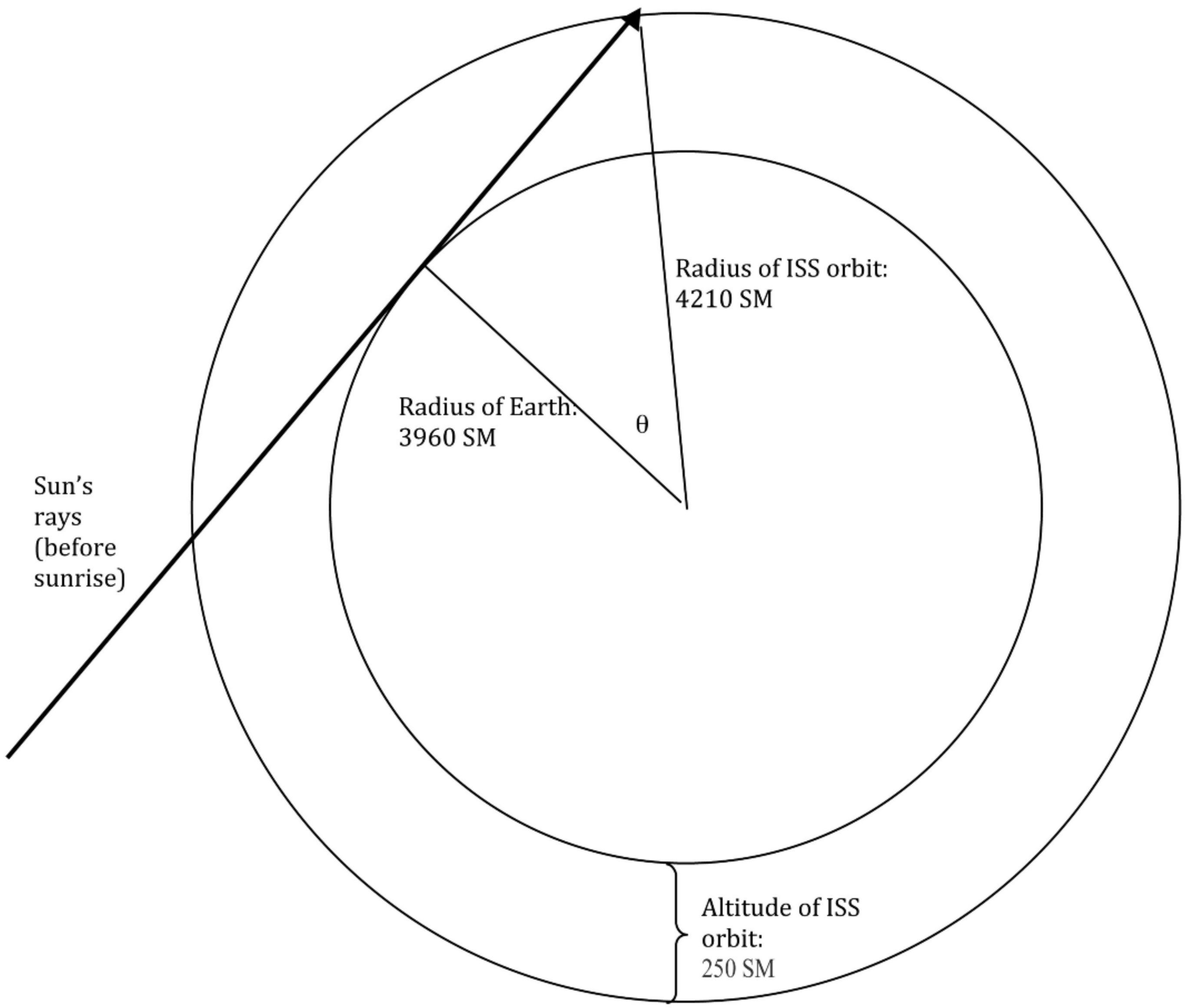
Only days with sighting opportunities are listed

Appendix II



Earth with ISS orbit. Not to scale.

Appendix III



Illumination for the ISS to be visible overhead. Not to scale.

Lesson 15: Planet Dance

OBJECTIVE:
Students create a scale model of our solar system and simulate planetary orbits by physically walking the "planets" around the "Sun."

NATIONAL STANDARDS
Mathematics
Algebra
- use mathematical models to represent and understand quantitative relationships

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science
Unifying concepts and processes in science
- Systems, order, and organization
- Evidence, models, and explanation
- Change, constancy, and measurement

Science as inquiry
- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

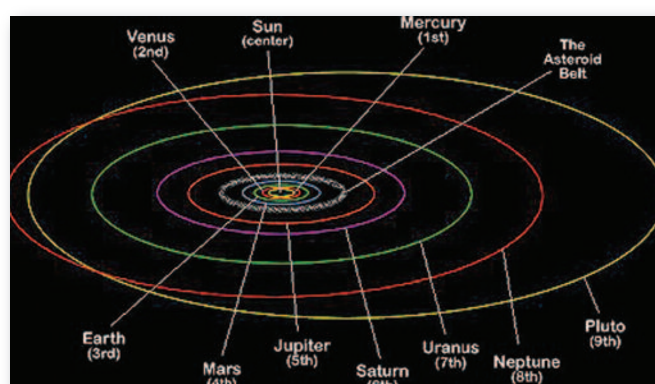
Physical science
- Motions and forces

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology
Standard 1 - Students will develop an understanding of the characteristics and scope of technology.
Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS - (one set for each group)
- A copy of the Scaling Our Planet handout
- Computer access (or copy of the data from Solar System Data at **http://hyperphysics.phyastr.gsu.edu/Hbase/Solar/soldata.html#c1**)
- Meter stick
- Magnetic compass



BACKGROUND INFORMATION:

The objective of this activity is to confront several common misconceptions about planetary orbits, particularly alignment and scale. Many people may believe the planets line up (at least most of the time), probably because posters, Websites, and books portray them that way. The second common misconception is the planets orbit closer to each other and move more slowly when compared to their size. For example, the Earth traverses about 200 of its diameters each day while orbiting and Mars orbits more than 6000 Earth diameters further from the Sun than Earth—difficult for most to believe!

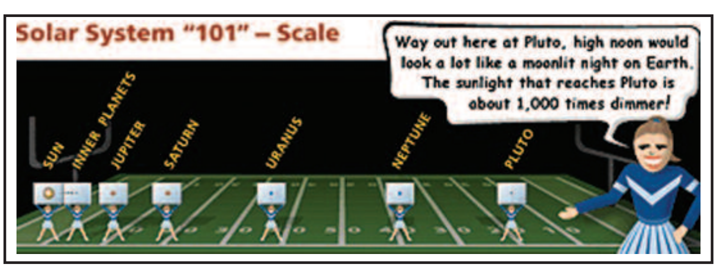
The age or grade-level appropriateness of this activity depends on the audience's maturity, resourcefulness, mathematical capability, prior understanding of the solar system, and ability to work independently. Generally speaking, the activity is in five parts:

1. Decide on a scale that will allow all the planets to orbit within the school yard (nearby field, etc.). Avoid arguments on whether Pluto should be included by saying the class is choosing to be egalitarian—including all good sized orbiting bodies.
2. Calculate the distances each of the "planets" must be from the center.
3. Construct scale models of the Sun and each planet. This scale must align with the one used for the planetary orbits.
4. Calculate the "scale pace" each planet revolves around the center.
5. Go out to your field and have student groups walk the orbits of the planets; hence, the planet dance!

Suggestion: Before proceeding, consider tinkering with the Build A Solar System Website hosted by the Exploratorium
http://www.exploratorium.edu/ronh/solar_sys tem/

PROCEDURE:

1. Announce your intent to have the class "construct" a model of the solar system that will fit in the school athletic field (or other large equivalent).
2. Ask the class what they know about the orbits of the planets. Write whatever they say (even incorrect facts—address them in the debrief) without editing (paraphrasing is acceptable). Record the responses on chart paper, whiteboard, overhead projector, etc., so they can be reviewed during the debrief of the activity. See if you can get the class to agree on the relationships between the sizes of the planets and their distances apart.
3. Invite the class to speculate how to create the scale factor that will fit the solar system in the field. A good response will require a couple of facts: the width of the field (say 150 meters) and the greatest distance a planet will orbit the sun (about 6 billion km). Note that the diameter of the solar system (about 12 billion km) would have to fit into the field. This means, of course, each meter on the field is about 80 million km!
4. Select groups (do not exceed three students per group—too many students per group means someone will be idle). Their responsibility will be to create the scale model of their planet and determine the scale radius and pace of their orbit. Assign dwarf planets such as Ceres, Pallas, Vesta, or Haumea if you have additional groups.
5. Distribute one Scaling Our Planet handout per group, and either provide them Internet access to get the necessary data to perform their calculations or provide the data for the class yourself (either by handout or projecting).
6. Provide time for students to find the appropriate data and perform calculations. Encourage groups to check with other groups to see if their answers make sense. Refer to the table in Appendix I to see if the groups' computations are reasonable (adjusting as needed for the size of your field).
7. Determine the speed (m/s) the scale "Earth" will revolve around the scale "Sun." Depending on the mathematical facilities of the students, you may give them the data and let them compute, lead them through the computation, or merely provide it. The key is how you want to define a "year" (one revolution about the Sun) and use the circumference formula to "walk" the planet at the correct rate. Check groups' calculations.
8. Have students practice walking the rate their planet is supposed to orbit. Have them all begin and stop on your command (both auditory and by hand signal).
9. Show a diagram of the current position of the planets (see Solar System Live). Adjust the parameters to your approximate latitude and longitude along with the day and time you expect the activity to be executed. See Appendix II for a model set in Seattle on 01 January 2010.
10. Show a diagram of your field, which way is North (or other recognizable point) with the position of the



planets overlaid. Measure approximate angles for each planet. Make sure all groups have their "azimuth" (compass heading) and distance to their initial position.

11. You stand at the center (as the solar system revolves around you). Upon your signal, the "planets" begin to move according to their proper pace. For younger students, hold your arms above your head and clap each second so the groups will advance at the same time. More mature students can take their experience from "walking" the distance while holding a watch to get the speed approximately right. Continue for about two Earth "years."
12. Reset all planets to their original positions and run the orbiting again.
13. Switch groups so the outer planet groups become inner planets and vice versa. Run two more times.
14. Return to the classroom and have students write, individually, a response to the question: How do the speeds of the outer planets compare to the speeds of the inner planets? Let students write silently for two minutes.
15. Invite comments, ideas, suggestions, and insights. During the discussion, prompt for how the size of the planets compared to the space between them.
16. Follow up at a later time with questions such as:
    - How do "we" send spacecraft to other planets (and about how much time that would take)?
    - If the solar system fits in the field, how much space is needed for our galaxy?
    - When will the planets be aligned?

SUMMARY:
The solar system is much larger than most people believe, particularly the size of the orbits relative to the size of the planets. The "dynamic" nature of the simulation punctuates the huge difference in orbital speed.

EVALUATION:
Explain in simple language (e.g., using terms understandable by someone much less sophisticated and less mathematically wise than you) why the amount of time it takes a planet to orbit the Sun is NOT related to its size. Include several diagrams with facts to support your argument.

LESSON ENRICHMENT/EXTENSION:
Recall that Kepler's Third Law Of Planetary Motion is: $\frac{T^2}{R^3} = k$ where T represents the orbital period (the amount of time for one revolution), R represents the radius of the orbit (or, in the case of elliptical orbits, the semi-major axis), and k is a constant for all bodies orbiting the same object.

- Use the Solar System Data found at **http://hyperphysics.phyastr.gsu.edu/Hbase/Solar/soldata.html#c1** to confirm the constant, k, is the same for Earth and Mars.
- Use the constant, k, derived above to compute the period of our solar system's largest asteroid, 4 Vesta, if it orbits at a radius of 375 * 10^6 km. Note that Ceres is much more massive than Vesta, but has been promoted to dwarf planet status.

ASSOCIATED WEBSITES AND/OR LITERATURE:

- Solar System Data
http://hyperphysics.phyastr.gsu.edu/Hbase/Solar/soldata.html#c1)

- The Nine Planets
http://www.nineplanets.org/

- Solar System Live
http://www.fourmilab.ch/cgi-bin/uncgi/Solar/action?sys=-Si

- **http://www.universetoday.com/15611/largest-asteroid-in-the-solar-system/**

- **http://pds.nasa.gov/**

Appendix I
Solar System Scale Models

Sun diameter 4 cm. (golf ball)

Planet	Size	"Distance" from Sun	Object	Bullet Font*
Mercury	0.16 mm	1.8 meters	Grain of sand	2
Venus	0.4 mm	3.4 meters	Grain of sand	4
Earth	0.4 mm	4.7 meters	Grain of sand	4
Moon	0.12 mm	12 mm from Earth	Dust speck	2
Mars	0.2 mm	7.2 meters	Grain of sand	2
Jupiter	4.5 mm	24.4 meters	Peppercorn	48
Saturn	3.8 mm	44.8 meters	Peppercorn	42
Uranus	1.6 mm	90.4 meters	Small peppercorn	16
Neptune	1.6 mm	142 meters	Small peppercorn	16
Pluto	0.08 mm	185 meters	Dust speck	1

*For the given scale (Sun diameter 4 cm) the bullet represents the relative size of the planet when using Times New Roman font.

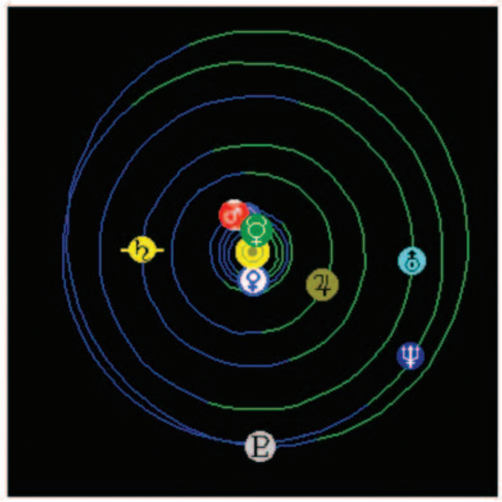
Quantity	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distance from sun (10^6 km)	57.9	108	150	228	778	1,430	2,870	4,500	5,900
Period of revolution, years	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Orbital speed, km/s	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Inclination of axis to orbit	<28	3	23.5	24	3.08	26.7	82.1	28.8	?
Inclination of orbit to earth's orbit	7	3.39	...	1.85	1.30	2.49	0.77	1.77	17.2
Eccentricity of orbit	0.206	0.0068	0.0167	0.0934	0.0485	0.0556	0.0472	0.0086	0.250
Equatorial diameter, km	4,880	12,100	12,800	6,790	143,000	120,000	51,800	49,500	3000?
Mass(earth=1)	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.01?
Density (water = 1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	?
Escape velocity, equator, km/s	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	0.9?
Satellites	0	0	1	2	16*	17*	15*	2*	1
Orbit speed, km/hr	172.7	126.3	107.4	87	47.2	34.8	24.5	19.6	17.1
High/Low temperatures, °C	350	480	58	27	...	...	...	...	-234
	-170	-33	-88	-123	...	...	...	...	-390
Atmosphere	none	CO2	N2,O2	CO2	H2,He	H2,He	H2,He	H2,He	none
Orbit in AU	0.387	0.723	1.000	1.524	5.203	9.539	19.18	36.06	39.44

Planetary Data from http://hyperphysics.phy-astr.gsu.edu/Hbase/Solar/soldata.html#c1

Appendix II

Screen Shot of Solar System Live

Solar System: Fri 2010 Jan 1 0:00



Time: ○ Now ⊙ UTC: 2010-01-01 0:00:01 ○ Julian: 2455197.50001

Update

Show: ⊙ Icons ○ Images

Display: ⊙ Full system ○ Inner system Size: 320 ☐ Stereo: ⊙ Cross ○ Wall

Orbits: ○ Real ⊙ Logarithmic ○ Equal

Observing site: Lat. 47° ⊙ N ○ S Long. 122° ○ E ⊙ W

Heliocentric: Lat. 90° ⊙ N ○ S Long. 0°

Colour scheme: Colour

Ephemeris:

	Right Ascension	Declination	Distance (AU)	From 47°N 122°W: Altitude	Azimuth	
Sun	18h 45m 26s	-23° 1.7'	0.983	3.101	50.765	Up
Mercury	19h 21m 17s	-20° 28.4'	0.703	9.782	45.145	Up
Venus	18h 34m 17s	-23° 38.5'	1.708	1.111	52.489	Up
Moon	6h 57m 47s	+23° 30.1'	57.3 ER	-4.309	-131.850	Set
Mars	9h 29m 54s	+18° 45.1'	0.739	-22.672	-163.502	Set
Jupiter	21h 55m 41s	-13° 36.6'	5.637	28.780	10.680	Up
Saturn	12h 20m 10s	+0° 18.6'	9.323	-37.328	145.864	Set
Uranus	23h 35m 48s	-3° 25.7'	20.368	37.776	-19.607	Up
Neptune	21h 48m 5s	-13° 43.3'	30.722	28.405	12.743	Up
Pluto	18h 13m 18s	-18° 18.1'	32.735	2.396	59.622	Up

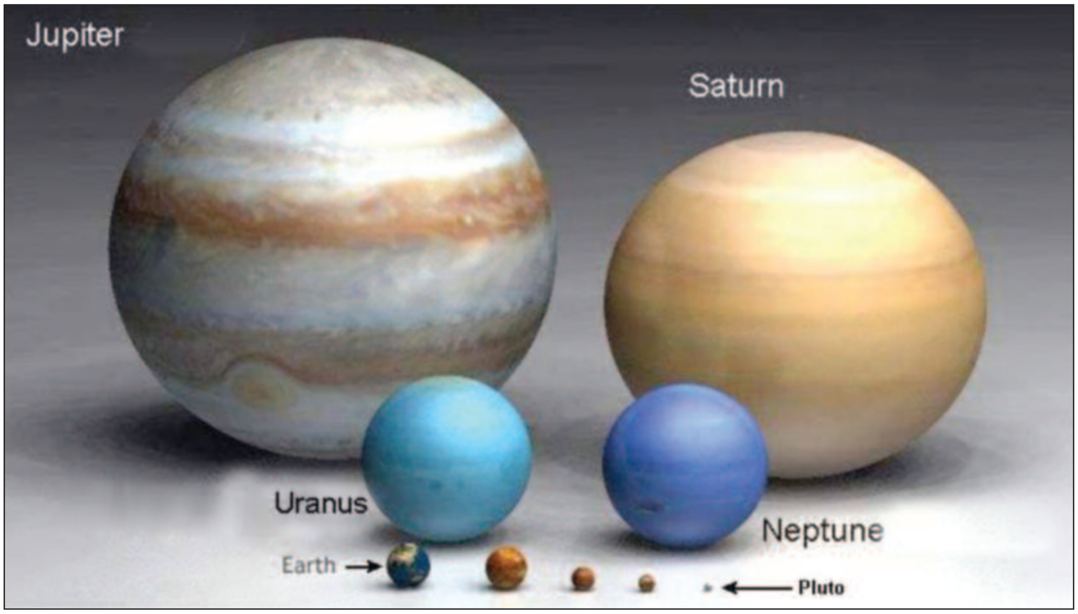
To track an asteroid or comet, paste orbital elements below: ☐ Echo elements

Return to Solar System Live   Details   Credits   Customise   Help

by John Walker

Appendix III

Relative sizes of “the nine planets”



THE PLANET DANCE
Scaling Our Planet

Group Member #1: ____

Group Member #2: ____

Group Member #3: ____

Our planet's name: ____

Our class' agreed upon scale factor: ____________________________________

Planetary facts

See Solar System Data (**http://hyperphysics.phy-astr.gsu.edu/Hbase/Solar/soldata.html#c1**)

Our planet's diameter:________________________________ (km)

Our planet's orbital radius:____________________________ (10^6 km)

Our planet's orbital period:____________________________ (Earth years)

Calculations
(Show your work!)

Our planet's diameter: ________________________________ (m)

Our planet's orbital radius:____________________________ (m)

Our planet's orbital period: ____________________________ (Earth years)

Orbiting speed

Amount of time our class agreed is one Earth year:_____________ (min.)

Amount of time for our planet to make one orbit: _____________ (min.)

Scale distance of our planet's orbit:_____________ (m)

Scale speed to walk our planet in its orbit:_____________ (m/sec)

When we get in the field, we will find our position by:

- A compass heading of _____________.
- A distance from the "Sun" of _______m.

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