Probabilistic theories, dilemma inferences, and decision making

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Summary

A new paradigm in the psychology of reasoning aims to replace the old traditional attitude. This new paradigm is Bayesian. In it, $P(\text{if } A \text{ then } C) = P(C|A)$.

Also newly stated, an inferentialist analysis holds that for "normal" conditionals $P(C|A)$ must be $> P(C)$.

There are problems with the traditional approach and inferentialism. The Bayesian alternative gives the best account of dilemma reasoning and decision making.

A glimpse ahead: Un petit d'un petit

Did Humpty Dumpty have a great fall?

(1) If he sat on a wall, he had a great fall.

(2) If he did not sit on a wall, he had a great fall.

(3) If he sat on a wall, cabbages are green.

Three conditional hypotheses

Material conditional hypothesis:

P(if A then C) = P(not-A or C)

Suppositional conditional hypothesis (Ramsey, 1929; De Finetti, 1937): ***P(if A then C) = P(C|A)*.**

Inferentialist hypothesis:

***if A then C* is a "normal" conditional only if *P(C|A) > P(C)*, or equivalently *P(C|A) > P(C|not-A)*.**

References for the hypotheses

Material conditional hypothesis

Johnson-Larid & Byrne (1991)

Suppositional conditional hypothesis

Cruz & Over (2026); Over & Evans (2024)

Inferentialist hypothesis

Douven (2016, 108-109); Douven et al., (2023, 2025);

Crupi & Iacona (2025)

Material conditional: *not-A or C*

Suppose we are considering whether to invest in a cryptocurrency scheme and consider:

If we invest in it (*I*), our money is in safe hands (*S*).

Suppose *P(if I then S*) = *P(not-I or S)* as implied by Johnson-Laird & Byrne (1991). Then, as we become more convinced the scheme is a scam, *P(if I then S*) will increase, an absurd result (Over & Evan, 2024).

Material conditional: Probability

In Johnson-Laird & Byrne (1991), *If A then C* and *not-A or C* have the same mental models, and thus *P(if A then C) = P(not-A or C) ≠ P(C|A)*.

But Johnson-Laird has revised his theory (Lopez -Astorga, Ragni, & Johnson-Laird, 2022), and he now holds that *P(if A then C) = P(C|A)*. Hence, he and suppositional theorists are on the same side at least in rejecting inferentialism.

Suppositional conditional

$P(if\ I\ then\ S) = P(S|I)$

Now, as we become more convinced that the scheme is a scam, *P(if I then S*) will decrease, a satisfying result intuitively (Over & Evans, 2024).

Support for this hypothesis comes from a wide range of experiments: See Over & Evans (2024) on these.

The new paradigm

The new Bayesian (or probabilistic) paradigm in the psychology of reasoning proposes that people usually make inferences from their degrees of belief and not from arbitrary assumptions (see Oaksford & Chater, 2020, and Over & Evans, 2024).

It has the truly psychological goal of accounting for belief-based reasoning. Its foundations in subjective probability theory go back to de Finetti (1937) and Ramsey (1926).

The conditional probability hypothesis

The foundation of the new paradigm is support for the conditional probability hypothesis:

$$P(\text{if } A \text{ then } C) = P(C|A)$$

The suppositional conditional satisfies this relation, and its normative system is what de Finetti called the logic of probability, probability logic for short. This logic is sometimes called System P (Pfeifer & Kleiter, 2009).

Inferentialism

Douven (2016, 13, 109-110) and Douven et al. (2023) argued a "normal" *if A then C* is such that *P(C|A)* > *P(A)*, or equivalently *P(C|A)* > *P(C|not-A)*.

Note that it would be incoherent to claim that *P(C|A)* and *P(C|not-A)* are both greater than *P(A)*.

Conditionals like "If the company is in debt, then it is not in its books" (Lopez –Astorga et al., 2022) are dismissed as "abnormal" and of no importance.

Conditionals: A distinction

Dependence conditional: $P(C|A) > P(C)$

$P(C|A) > P(C|not\text{-}A)$

Independence conditional: $P(C|A) = P(C)$.

$P(C|A) = P(C|not\text{-}A)$

For suppositional theorists, there is nothing wrong with independence conditionals, and indeed they are of great importance (Cruz & Over, 2023).

An independence conditional in French

“Si le Premier ministre est satisfait de la loi, le Président ne l'est pas.”

“While the Prime Minister is satisfied with the law, the President is not.”

“If the Prime Minister is satisfied with the law, the President is not.”

Concessive conditionals

"Even if he did not sit on a wall, he had a fall."

"If he sat on a Wall, he had a fall."

Even if not-A then C **is usually used to convey that *A* and *C* are independent. But it can only do this when *if A then C* also holds in the background. When both hold, they are independence conditionals.**

Edgington's prediction

Edgington (1995) considered conditionals like:

"If Napoleon is dead, then Oxford is in England."

She predicted that such conditionals would be judged "not acceptable, or even, false".

For the suppositionalist, there is <u>pragmatic</u> problem with this example. For the inferentialist, there is a <u>semantic</u> problem with it.

Skovgaard-Olsen et al. (2016)

Skovgaard-Olsen et al. were the first to confirm the Edgington prediction about "missing-link" examples.

If Mark presses the power button, his TV turns on.
If Mark is wearing socks (*W*), his TV turns on (*T*).

***P(if W then T)* was found to be lower than *P(T|W)*, but there is clearly a <u>confound</u> above. A pragmatically OK dependence example is followed by an independence example that is <u>not</u> pragmatically OK.**

Walrus conditionals

Edgington's, and Skovgaard-Olsen et al.'s, strange examples are Walrus conditionals and pragmatically unacceptable. Should compare (Over & Evans, 2024):

If Mark presses the power button, his TV turns on.
If Mark presses the power button, his TV screen is blank.

The dependence conditional and the independence conditional above are both pragmatically OK.

Pragmatic coherence

Lassiter (2022) uses the notion of <u>pragmatic discourse coherence</u> to explain what is wrong with Walrus conditionals (see also Bourlier et al., 2023, Cruz et al., 2016, and Lassiter & Li, 2024).

Experiments on Walrus conditionals give no support to inferentialism (Cruz & Over, 2023, 2025; Over & Cruz, 2023). These experiments are limited as well by not having full <u>polarity</u> designs.

Different kinds of "nonsense"?

A <u>Jabberwocky</u> conditional:

(5) If it is brillig, then the toves are slithy.

A <u>*Walrus*</u> conditional:

(6) If cabbages are green, then kings use sealing-wax.

Suppositionalists claim: (5) and (6) are different. (5) is semantic nonsense. (6) is pragmatic "nonsense".

Bayesian reasoning: A contrast

Consider a pregnancy test that has an expiry date.

If Jane is pregnant (*H*), the test will be positive (*E*).

If she is not pregnant (*not-H*), it will be positive (*E*).

Suppositionalism gives a straightforward account of reasoning with these conditionals, but this not so for inferentialism.

What happens at the expiry date?

Consider a <u>pregnancy test</u> that has an <u>expiry date</u>.

"If Jane is pregnant, the test will be positive."

Nothing happens for the suppositionalist. But for inferentialists, the above conditional at the expiry date becomes no better than:

"If Jane is pregnant, cabbages are green."

Inferentialism and logic

For Douven et al. (2023), *I f A then C* is "true" if and only if there is a sufficiently strong relation between *A* and *C* - deductive, inductive, or abductive.

There is not yet a logic and formal semantics for this conditional, but MP, inferring *C* from *if A then C* and *C* is <u>in</u>valid for it.

Yet no inference is endorsed more highly than MP: it is at ceiling in psychological experiments.

Skovgaard-Olsen, Kellen, et al. (2017)

These authors find that *if A then C* is termed "true" when *A* and *C* are true whether *P(C|A) – P(C|not-A)* is positive, 0, or negative.

Douven et al. (2025) reply that *P(C|A) – P(C|not-A)* " ... is a poor proxy for inference strength."

But what then is "inference strength"?

Suppositionalism and logic

Probability logic, with its concepts of p-validity and logical coherence, is the formal logic for suppositional Conditionals (Adams, 1998).

A set of beliefs is coherent if and only if it is consistent with subjective probability theory. An inference is p-valid if and only if it preserves probability from its premises to its conclusion. MP and centering are both p-valid (Cruz & Over, 2026; Over & Evans, 2024).

De Finetti's analysis

if A then C **is true when *A* is true and *C* is true.**

if A then C **is false when *A* is true and *C* is false.**

if A then C **is uncertain, "void", when *A* is false.**

In a thoroughgoing de Finetti analysis, the third – "void" value is replaced by *P(C|A)*, the conditional probability itself.

The "defective" truth table

Even traditional experiments on truth tables found that people do not make binary classifications.

They classify cases in which *A* is false as "irrelevant" to the truth or falsity of *if A then C*.

These results can be seen as supporting de Finetti's analysis of the conditional (Over & Baratgin, 2017).

Confirmation of de Finetti's analysis

There is confirmation of de Finetti's three-valued analysis of the conditional as descriptive of people's judgments about natural language conditionals. (Baratgin et al., 2018).

This analysis also implies the *conditional probability hypothesis* that the probability of the conditional, *P(if A then C)*, is the conditional probability, *P(C|A)*.

Conditional dilemmas

Conditional dilemmas come in two p-valid forms that are a problem for inferentialists:

One premise (1CD)

From *if A then C & if not-A then C* infer *C*.

Two premise (2CD)

From *if A then C* and *if not-A then* C infer *C*.

Inferentialism, (1CD), and (2CD)

Recall - it would be incoherent to hold *P(C|A) > P(C*), and *P(C|not-A) > P(C*).

Thus, inferentialism implies that the premise of (1CD)

if A then C & if not-A then C

is necessarily false, and one of the two (1CD) premises.

For inferentialists, dilemma inferences are not useful, as they always have a false premise.

The blocks problem (Levesque, 1986)

Hypothetically suppose that there is a stack of five blocks, some of which are green and some not green.

The second block from the top in the stack is green, and the fourth one down is not green.

Is there a green block in this stack directly on top of a non-green block? Yes, no, or cannot say?

Reasoning in the blocks example

The third block is either green or not green.

If the third block is green, the answer is “Yes”.
If the third block is not green, the answer is “Yes’.

Therefore, the answer is “Yes”. A green block is directly on top of a non-green block.

An actual example

Mill (1859) produced a famous argument of the same form in *On Liberty* (Ch. II) for free speech:

Given the background that view *A* is "silenced". If *A* is true, there is a loss in utility, and if *A* is false, there is a loss in utility.

Therefore, there is a loss in utility, given a context in which *A* is "silenced".

Sure-thing principle (Savage, 1954)

Following Savage's principle and Mill on free speech, it would be rational to reason in the following way.

If *A* then we prefer to tolerate *A*, and if *not-A* then we prefer to tolerate *A*. Therefore, we should tolerate *A* before *A* is found to be true or not-true.

This principle is like a deontic and temporal form of dilemma. It goes perfectly well with the suppositional conditional but not the inferentialist conditional.

Bayesian decision making

Jane's pregnancy test has reached its expiry date. She goes out to buy a new one. She is very thirsty and reasons:

"If I the test will be positive, I prefer to drink water now. If it will be negative, I prefer to drink now."

Should Jane drink the water now, before she learns the result of the test or not?

Reminder: Independence conditionals

Dependence conditionals: $P(C|A) > P(C|not\text{-}A)$

Independence conditionals: $P(C|A) = P(C|not\text{-}A)$

Independence conditionals can appear in dilemma inferences, establishing that a p-valid conclusion *C* holds whether or not *A*. Knowledge of independence, of whether *C* holds in any case, can be of great utility in human decision making and reasoning (Cruz & Over, 2023; Over & Evans, 2024).

Conclusions

I have covered three hypotheses about conditionals: the material, suppositional, and inferentialist.

I have argued that the suppositional conditional and the new Bayesian paradigm provide the best account of human reasoning.

In particular, the suppositional conditional allows us, e.g., in dilemma inferences, to acquire knowledge of independence, which is essential for rationality.

Appendix

A basic inferentialist claim (Douven, 2016):

if A then C **is** **<u>acceptable</u>** **if and only if** ***P(C|A)*** **is above some contextual threshold and** ***P(C|A) > P(C)*****.**

Other inferentialist analyses of "acceptability" can be found in Crupi & Iacona (2025).

The **<u>Finetti normal form</u>** **for** ***if A then C*** **is** ***if A then (A & C)*****.**

A specific de Finetti normal form

"If Mark is wearing socks (*W*) , his TV turns on (*T*)."

"If Mark is wearing socks, he is wearing socks and his TV turns on (*T*)."

By all the inferentialist definitions, *if W then T* has 0 "acceptability" in this example, but *if W then (W & T)* can be "acceptable". I leave this as an exercise (Over et al., 2026; Over & Evans, 2024)