HMH Into Math®

Research Evidence Base

HMH research mission

HMH® is committed to developing evidence-based educational solutions, assessments, and professional services. To support this goal, the Efficacy Research Team collaborates with school districts and third-party research organizations to evaluate the impact of our programs and services on student outcomes, teacher practice, and school leadership.

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Introduction

In today's rapidly evolving educational landscape, deep mathematical understanding and strong reasoning skills are essential for student success in school, career, and life. As STEM fields continue to expand, mathematics education must equip all learners with the skills and confidence to thrive in a data-driven, technology-rich world. The equitable use of technology, access to high-quality instructional materials, and a focus on positive mathematical identity are now recognized as critical drivers of achievement and opportunity.

HMH *Into Math*, Version 2 builds on a proven foundation of research-based pedagogy and standards alignment, offering a comprehensive mathematics program for Grades K–8 that is intentionally designed to support growth for every student. Version 2 introduces streamlined structures, enhanced digital supports, and new resources to foster engagement, accessibility, and relevance in mathematics learning. The program's coherent learning progressions, differentiated supports, and integrated assessment tools empower educators to deliver rigorous, purposeful instruction while meeting the diverse needs of today's classrooms.

Grounded in the latest research and guided by the principles of equity, inclusion, and high expectations, *Into Math*, Version 2 provides:

- **Focused and purposeful content:** carefully crafted tasks and resources that prioritize conceptual understanding, procedural fluency, and real-world application
- **Ongoing support for teachers and students:** embedded differentiation, professional learning, and actionable data to inform instruction and promote continuous growth
- **Integrated assessment and reporting:** flexible, technology-enabled tools for monitoring progress and supporting data-driven decision-making
- **Commitment to equity and access:** resources and routines designed to empower all learners, including multilingual students and those with varied backgrounds and abilities

This evidence-base document details the research foundations, scholarly research, design principles, and evidence-based practices that inform *Into Math*, Version 2. It demonstrates how the program aligns with national standards, supports positive mathematical identity, and delivers measurable outcomes for students and educators. Through intentional design and ongoing innovation, HMH *Into Math*, Version 2 is dedicated to advancing mathematics achievement and preparing all students for success in a dynamic, interconnected world.

Purposeful, Connected Mathematics for All

"Math concepts and skills are important to all of school and life. Math provides a new way to see the world, the beauty of it, and the way you can solve problems that arise within it. However, math is much more: Math is critical thinking and problem solving, and high-quality math experiences also promote social, emotional, literacy, and general brain development" (Clements & Sarama, 2020, p. 2). Developing a solid in mathematics is essential for academic achievement and and real-world success. However, rather than serving as a gateway, mathematics often becomes a barrier to college and career advancement, with implications for economic growth and workforce readiness in STEM industries at both the national and individual level (Getz & Ortiz, 2016). Increasing accessibility and expanding student achievement within STEM education is crucial to expanding opportunities and meeting industry demands (Bland et al., 2024; IEEE Global Public Policy, 2023).

Mathematics programs that effectively support the development of essential 21st-century skills are structured by coherent learning progressions that build conceptual understandings as well as connections among areas of mathematical study and between mathematics and the real world (National Council of Teachers of Mathematics [NCTM], 2014). A process that includes making observations, posing problems, and representing and analyzing real-life situations via use of mathematical tools, numerical and spatial reasoning, and symbols and operations, mathematization activities "are designed to center students' experiences and ideas, to evoke curiosity, and to invite connections between mathematics and the world. Realistic mathematics education traditions emphasize mathematizing contexts that students can experience as real, which include authentic, real-world situations, imaginary or fantasy stories, or even situations in formal mathematics that students can imagine" (Aguirre et al., 2024, online).

HMH Into Math provides relevant, rigorous, real world mathematics learning that fosters students' positive mathematics identities. The program is structured around coherent learning progressions that utilize evidence-based pedagogy and practices to support engaging, interactive mathematical investigations with real-world relevance.

High-Quality, Standards-Aligned Instruction

Providing young children with extensive, high-quality early mathematics instruction establishes a strong foundation for later learning (Cross et al., 2009). For decades, U.S. education policy has emphasized rigorous standards, instruction, and accountability measures. Research demonstrates that standards-based environments positively affect mathematics achievement (Peterson & Ackerman, 2015; Stein et al., 2007; Tarr et al., 2008) and promote significant growth in problem-solving (Cai et al., 2011). High-performing schools share focused goals and align curriculum, instruction, and assessment to state standards (Darling-Hammond, 2010a; Shannon & Bylsma, 2007).

To ensure students master essential mathematics skills, teachers and students must clearly understand learning goals, expectations, and success criteria. Providing clarity about learning goals is among the most effective instructional practices (Almarode & Vandas, 2018; Hattie, 2009, 2023; Leahy et al., 2005). “Formulating clear, explicit learning goals sets the stage for everything else” (Hiebert et al., 2007, p. 57). Standards outline a clear roadmap for developing foundational knowledge and ensuring college and career readiness for all students (NCTM, 2014; Wiliam, 2011).

Within standards-aligned instruction, focus and coherence are essential (Schmidt et al., 2005), particularly as instruction adapts to individual students’ progress and needs (Pak et al., 2020). Prioritizing key standards enables educators to plan intentionally and achieve targeted learning goals. Through mathematics learning progressions, educators can monitor growth and adjust instruction accordingly (Sarama et al., 2004; Sztajn et al., 2012).

There is a growing recognition among policymakers and educators that instructional materials are central to both what students learn and how they learn. While curricula alone cannot guarantee learning, evidence shows that high-quality instructional materials (HQIM), when supported by professional learning, improve student achievement (Doan et al., 2022; EdReports, 2022). Research confirms that the core program used in a classroom has substantial effects on learning outcomes (Agondini et al., 2010; Bhatt & Koedel, 2012; Boser et al., 2015; Chingos & Whitehurst, 2012; Pak et al., 2020; Polikoff, 2018). A Johns Hopkins review found curriculum to be a “critical factor in student academic success,” with substantial cumulative benefits when schools shift from weak to strong materials (Steiner, 2017, p. 2).

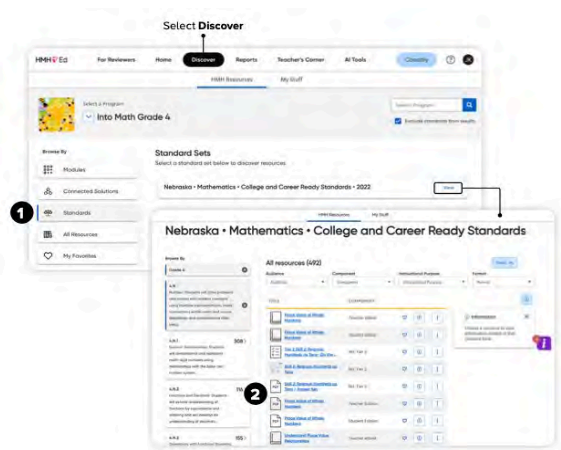
HQIM are standards-aligned programs and resources, such as textbooks, workbooks, and digital tools, that meet rigorous criteria for quality, including research evidence, state reviews, and EdReports ratings (Comprehensive Center Network, 2023; Doan et al., 2022). State policy increasingly drives districts to adopt HQIM and related professional supports (Boser et al., 2015). Such materials represent a cost-effective reform with a high return on investment (ROI): “Putting a high-quality curriculum in the hands of teachers can have significant positive impacts on student achievement” (Boser et al., 2015, p. 1). Because instructional materials directly affect student learning and shape teachers’ instructional choices, their quality is essential to effective mathematics instruction (Chingos & Whitehurst, 2012).

How *Into Math* Delivers

Today's education standards and workforce demand require high-quality, coherent, and rigorous learning solutions to ensure students develop an in-depth understanding of mathematical concepts and language. *Into Math* is a comprehensive learning system in which all resources have clear and intentional targets to optimize outcomes and support all students.

Standards-Aligned Mathematics Learning

Within *HMH Ed*®, teachers can browse through their state standards and identify all *Into Math*-aligned resources in one place. This tool allows teachers to quickly find, reference, and understand relevant standards for each resource during lesson planning, enabling highly strategic and focused mathematics instruction at the state, local, and individual learner level.



Additionally, each lesson is aligned with Mathematical Practices, clearly identified in both the Teacher Guide and Student Edition, to support goal-focused learning targeted toward essential proficiencies.

High-Quality Instructional Materials

Into Math, Version 2 brings fresh energy, smarter supports, and the trusted pedagogical foundation to HMH's proven pedological foundation that is already widely used nation-wide. *Into Math*, Version 2 is streamlined for ease and impact, enhancing teacher and student experiences. *Into Math* is designed to fit naturally into the school day to help educators teach with confidence and connect with every learner.

Into Math is crafted to captivate and inspire students while providing comprehensive support for teachers with high-quality instructional materials (HQIM), hands-on learning, tailored resources, and more.

Relevant, Real-World Mathematics

Research consistently supports the use of real-world contexts to boost student engagement and the relevance of mathematical concepts (e.g., Aguirre et al., 2024; Cross et al., 2009, Cai & Hwang, 2023; Miri et al., 2007). This should not be surprising. Mathematics is inherently connected to the natural and physical world; to be deprived of appreciation for its beauty and practicality is to be denied opportunity to flourish in one's life (Su, 2020). Decades ago, Piaget (1965) found that children, driven by innate curiosity, develop mathematical understanding through interactions with their environment. Humans are born with an intuitive and compelling sense of numeracy and pattern recognition (Devlin, 2006; Feigenson & Halberda, 2004). Effective curricula follow coherent learning progressions, linking mathematical concepts and real-world applications (NCTM, 2014).

Effective math instruction values students' lived experiences and contextualizes and demonstrates math's real-life applicability (Aguirre et al., 2012, 2013, 2024). It is problem-based, with important and useful math embedded within the learning activity (Cai & Hwang, 2023).

Embedding math in meaningful, real-world contexts enhances learning experiences, fosters creative and critical thinking, increases relevance and interest, and helps students draw richer connections across disciplines (Alberti, 2013; Czerniak et al., 1999; Drozda et al., 2022; Russo et al., 2011; Verschaffel et al., 2020). Recognizing mathematics as applicable to the world outside of school and relevant on a personal level is important for all students, but especially for underrepresented groups in STEM courses and careers (Anwar et al., 2019; Kang et al., 2019; Jackson et al., 2020; Lambert & Sugita, 2016; Young, 2014) and advanced learners (Gainsburg, 2008; VanTassel-Baska & Brown, 2007) for whom real-world relevance and applicability have been shown to increase motivation to pursue and persist in learning. Multilingual learners (MLs) also benefit from real-world and personal connections, as well as interdisciplinary contexts, which support both language development and content understanding (WIDA, 2020).

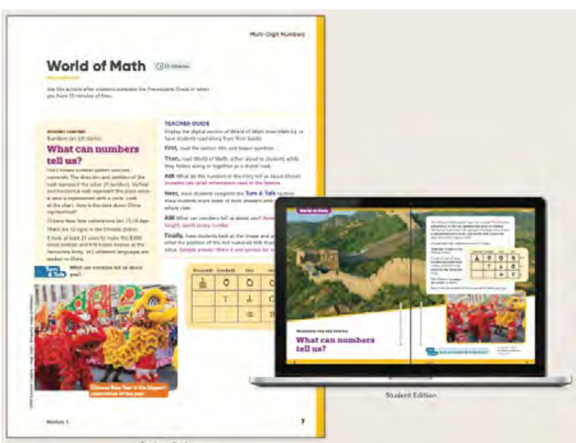
Students are more motivated when problem-solving tasks are connected to their experiences rather than presented as isolated facts (Alberti, 2013; Caine & Caine, 1991; Czerniak et al., 1999; David & Greene, 2007; Kovalik & Olsen, 1994; Verschaffel et al., 2020). Contextual problems promote mathematical modeling (Fosnot & Dolk, 2001) and interdisciplinary learning (Russo et al., 2011). Authentic problems—particularly those with real purpose, relevance, and mathematical value—promote discussion (Özgün-Koca et al., 2019). Similarly, authentic performance tasks help teachers link students' backgrounds and interests to mathematics (Lane, 2013).

How *Into Math* Delivers

Into Math is designed to make mathematics learning meaningful, relevant, and engaging for all students, encouraging learners to draw on their backgrounds and interests while deepening their understanding of how math permeates and offers possibility in the world.

Engagement With the World of Math

As students begin each program module, the World of Math offers an engaging way to connect math to history, culture, careers, and everyday life. Through teacher-led discussions, students use their prior knowledge to explore the cultural and historical significance of math while completing activities that showcase contributions from a wide range of figures. By understanding the real-world applications and historical roots of math, students experience joy and "wow" moments and come to see themselves as mathematicians who can recognize their own identities within the curriculum.



Engagement With a Math Learning Community

Each World of Math activity takes about 15 minutes. After a short introduction to the topic, a detailed teacher guide provides probing questions and answers. Turn & Talk questions engage students in examining the meaning of math in the world around them and in collaborating with classmates to devise collective understanding of and appreciation for mathematics and its relevance in their lives.

Fostering Positive Mathematical Identity

Developing a positive and valued mathematics identity at a young age is key to building a strong mathematical foundation. All students must be empowered, holistically, to participate meaningfully in learning mathematics. Effective, sustaining mathematics instruction nurtures within children a positive mathematical identity and a strong sense of agency; this type of instruction allows children to see themselves as capable, self-efficacious doers of math who construct their own mathematical understandings, as well as to experience firsthand the wonder, joy, and beauty of math (NCTM, 2020). Effective, accessible instruction nurtures students' confidence and agency, allowing them to see themselves as capable mathematicians and experience the joy of math (NCTM, 2020). Early math experiences predict later academic success, extending beyond classrooms to economic well-being and quality of life (Aguirre et al., 2024; Campbell et al., 2014; Clements & Sarama, 2016; Denton et al., 2002). A positive mathematics identity is crucial for building a strong mathematical foundation and fostering long-term achievement (Clements & Sarama, 2020) and must therefore be an underlying goal of mathematics education (Bishop, 2012; Larson, 2017).

Mathematical identity encompasses self-perceptions, emotions, and beliefs about math, influencing motivation, participation, and learning approaches. A strong mathematical identity is also shaped by each student's unique background (Bishop, 2012; Desai et al., 2021; Farrington et al., 2012; Turner et al., 2013). Developing persistence, agency, and self-efficacy is key to overcoming challenges (Bishop, 2012; Darragh, 2015). "Two pillars support a positive math identity: the belief that you can do math and the belief that you belong" (Froschl & Sprung, 2016, p. 320). However, many students lack this self-perception, resulting in disengagement from math or exclusion from advanced mathematical study (Desai et al., 2021; Darragh, 2015; Farrington et al., 2012).

Mathematics identity is dynamic, shaped by societal influences and classroom interactions. Supportive classroom environments for all students cultivate positive math identities (Desai et al., 2021; Farrington et al., 2012; Walton & Cohen, 2011). Aguirre and colleagues (2012, 2013) recommend deep mathematical learning, valuing comprehensive knowledge, fostering community, and affirming students' identities and backgrounds, including the funds of knowledge each student brings to school. For English learners (ELs), Turner et al. (2013) advocate for validating their reasoning and encouraging participation. A holistic approach is essential to empowering all students in mathematics.

How *Into Math* Delivers

Into Math encourages each student to connect to math on a personal level, seeing themselves as curious, capable, and confident learners. The program also inspires students to be problem solvers who feel like they belong as contributing members of the math community and as future STEM professionals. Welcoming mistakes and displaying persistence, students recognize the role math plays in the world and in their lives.

Cultivating Mathematics Learning Communities

The program's instructional framework provides a dynamic learning model that has students shift optimally and collaboratively among whole-class sessions, small-group activities, and independent practice. *HMH Classcraft* provides ongoing opportunities for students to engage in mathematical discourse via Turn & Talks and group discussions. The program's Games allow students to expand, apply, and challenge their learning while also having fun. Additionally, students go deeper and collaborate together on Module Projects. Mathematical Language Routines ensure that ELs participate fully in discursive activities and engage as contributing members of the classroom community.

Promoting Interest in STEM Learning and Careers

World of Math activities and Module Projects ignite students' interest in mathematics and help them recognize math as relevant to the real world and their own lived experiences. Students are encouraged to contribute their funds of knowledge to the classroom and use it to enrich their understanding in and curiosity about math learning. By building on students' interests, scaffolding and challenging resources support and foster a sense of belonging in STEM, preparing them for future opportunities.

Focus and Coherence in Mathematics Learning

"A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades" (NCTM, 2014, p.4).

A clear, structured learning path is essential for student success in mathematics. Effective mathematics instruction must be intentionally designed, focusing on the impact it can have on student learning (Hattie et al., 2017). Mathematics programs that build essential 21st-century skills are structured through coherent learning progressions that connect mathematical concepts and real-world applications (NCTM, 2014). These progressions reflect the logical and hierarchical nature of mathematics (Schmidt et al., 2005) and help teachers effectively guide students through the curriculum (NCTM, 2014); these are hallmarks of a coherent curriculum. Learning progressions are defined as carefully sequenced building blocks that students must master as they work toward higher mathematical understanding (Popham, 2008). Because mathematics learning is cumulative, instruction must emphasize coherence and connection rather than treating topics as discrete units (Fosnot & Jacob, 2010; Ma, 2010). Mathematics learning progressions support long-term student growth by articulating curricular frameworks, instructional sequences, assessments, and levels of learning sophistication (Fonger et al., 2018). Learning progressions clarify instructional goals, improve classroom assessment, and help teachers determine where students are in their learning trajectory (Moss, 2022; Sáez et al., 2013).

A "big idea" in mathematics, as defined by Charles (2005), is a central, overarching concept that links multiple mathematical understandings into a coherent whole. Research strongly supports this approach, emphasizing its role in structuring knowledge, promoting transfer across topics and grade levels, and streamlining overcrowded curricula (Bruner, 1960; Cross et al., 2009; Siemon et al., 2012; Wiggins & McTighe, 2005). Nurnberger-Haag and colleagues (2024) argue that an approach focused on big ideas benefits not only students but also teachers, enhancing their content knowledge and instructional capacity. Effective programs build on children's intuitive mathematical thinking to help them solve problems and engage in reasoning (Carpenter et al., 2015). Foundational concepts such as number sense, geometry, and spatial measurement are particularly crucial for early learners (Cross et al., 2009).

A curriculum focusing on fewer but deeper mathematical concepts improves content mastery (Cobb & Jackson, 2011). The National Mathematics Advisory Panel (NMAP) (2008) recommends streamlining PreK–8 mathematics curricula to emphasize the most critical topics. Clearly identifying what students are expected to learn is a key component of effective instruction (Hattie et al., 2017; Wiggins & McTighe, 2005). Teachers must assess student progress and adjust instruction accordingly (Sarama et al., 2004; Sztajn et al., 2012). Additionally, instruction should be purposeful and connected to students' learning needs, ensuring they understand what they are learning and why (Kanold, 2018; Wiliam, 2011).

How *Into Math* Delivers

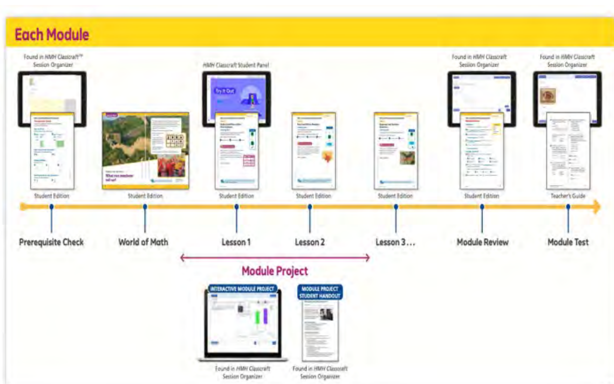
Purposeful, Connected, and Coherent Learning Progressions

With *Into Math*, students develop a deep, long-lasting mastery of mathematics. The sequence of lessons employs instructional design that builds conceptual understanding, connects concepts to skills, and then allows students to apply their thinking. Open Middle™ tasks invite all students to think deeply about mathematics.

Into Math follows a simple structure of 17 modules, providing consistent and coherent instruction throughout the school year. Each module has an average of six lessons that each last one day. *Into Math* enables both teachers and students to plan and predict the learning activities and routines for each day.

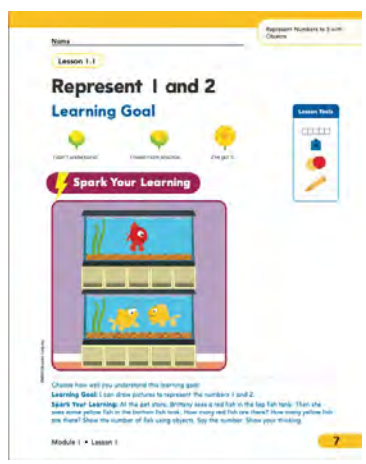
Every module provides formative and summative assessments to support flexible, adjustable teaching that meets students' needs and provides evidence of student progress in learning. Along with the lessons, each module has a project that students can complete during small-group time over multiple lessons.

Such coherence within and across *Into Math* lessons and modules helps teachers and students recognize the purpose and connectedness of content and tasks, as well as focus on learning goals.



Learning Guided by Explicit Goals

Each lesson within *Into Math* makes learning goals clear and explicit to students, both upfront and along the way. The Learning Goal is the instructional routine that starts every lesson and helps students set a goal and engage with the lesson's I Can statement. Lessons also launch with a metacognitive awareness activity, asking students to reflect on how well they understand the goal. This enables students to recognize expectations and engage in purposeful, focused learning. The program also provides varied support as needed to ensure students fulfill goals. *Into Math* provides rigor with appropriate levels of challenge. All students can meet rigorous mathematics standards when guided by clear and explicit learning goals in each math lesson.



Making Connections and Understanding the Whys of Math

Cognitive science affirms the benefits of multidisciplinary approaches to instruction. Cognitive neuroscientist Stanislas Dehaene (2020) emphasizes that the brain learns most effectively through integration, variation, and connection across domains. Dehaene advocates the use of multiple representations—symbolic, verbal, visual, and physical—and stresses the close relationship between language and math, particularly in early learning. In industry and research, there is interconnection between science, technology, engineering, and mathematics; in education, these subjects should be taught as they are practiced outside of school settings, in real-life contexts in which the world's issues and economies depend upon them (Russo et al., 2011; NRC, 2001).

Mathematical investigations play a crucial role in engaging students and fostering deeper understanding by emphasizing inquiry-based learning, real-world problem-solving, relevance and connections across disciplines, and collaborative discourse. "Worthwhile mathematical tasks" stimulate mathematical learning via opportunities for students to reinforce and extend what they know; they "should be intriguing, with a level of challenge that invites speculation and hard work. Most importantly, should direct students to investigate important mathematical ideas and ways of thinking toward learning goals" (Cai & Hwang, 2023, pp. 116–117).

Student-centered investigations mirror the inquiry-based methods of mathematicians and scientists. These tasks are rich, open-ended, and involve activities such as observing patterns, conducting experiments, controlling variables, and constructing models. Discourse is central, with students collaboratively discussing problems, planning strategies, interpreting data, and evaluating solutions (Dorier & Maass, 2020; Langer-Osuna & Avalos, 2015; von Renesse & Ecke, 2015). Furthermore, effective investigations are designed to be engaging for all learners—including ELs and students with disabilities—by encouraging multiple solution approaches, representations, and collaborative problem-solving. An investigative, discursive approach to instruction that values students' individually derived reasoning in open-ended problem-solving cultivates positive mathematics identities, encourages participation and agency (Engle, 2012; Langer-Osuna, 2016, 2017; Turner et al., 2013), and fosters mathematical habits of mind (Cuoco et al., 1996).

Research indicates that open-ended, student-driven learning enhances engagement, enjoyment, and participation in mathematical discussions, which likely has positive effects on both proficiency and interest in the discipline (Engle, 2012; Drozda et al., 2022; Sengupta-Irving & Enyedy, 2015). However, shifting to investigative instruction requires teachers to transition from traditional methods to a facilitator role, guiding students through active learning and ensuring broad participation (Sfard & Cobb, 2014; von Renesse & Ecke, 2015; Yow & Lotter, 2016).

Scaffolding strategies during investigative and inquiry-based activities, such as prompting, modeling, and task simplification, can focus attention and support sustained thinking and engagement (Fisher & Frey, 2021; NASEM, 2018; Shute, 2008) and are especially helpful for students with disabilities (Hughes et al., 2017).

How *Into Math* Delivers

Into Math's Module Projects provide integral opportunities for collaborative problem-solving, creativity, and advanced mathematical reasoning through engaging, hands-on, open-ended problems. Students explore mathematics working at a Depth of Knowledge (DOK) 4 level. With these activities, students apply math using data from real-world scenarios while considering the genuine impact math can have on the world. Each Module Project supports development of one or more Mathematical Practices.

Module 2 Project

Fish Race

Group Members: ______________________

Do all fish swim at the same speed? Can you calculate the distance a fish will swim in a certain amount of time?

Think about predicting the outcomes of races between individual fish or teams of fish. You get to predict the winners!

What is the Project About?

In this project, you will design a fish race and predict the winner of the race by:

- analyzing data about the top swimming speed of each fish participant and making a prediction of the winner,
- writing numerical expressions that tell the distance the fish swam,
- calculating the distance by which the race was won, and
- evaluating numerical expressions to confirm the predictions.

Module Project Correlations

Module	Project topic	Standards
1	**Recognizing Community Volunteers** Data Science Plan an event for community volunteers and describe the impact volunteers make.	MP: MP.2, MP.3
2	**Millions of Apples** Data Science Use data to determine which states produced the most and least apples; then compare the data.	MP: MP.2
3	**Who Has the Time?** Data Science Develop a plan for the activities you will do within four hours on a single evening.	MP: MP.1
4	**Classroom Cubbies** Data Science Write a plan to install cubbies, calculating how many cubbies are needed.	MP: MP.4
5	**Animals out of Place** Data Science Calculate the impact of two invasive species.	MP: MP.2
6	**A New Gym Floor** Data Science Decide which type of gym flooring to use based on different factors (cost, etc.).	MP: MP.1, MP.5
7	**Let's Watch a Parade!** Data Science Plan a parade viewing area according to the expected attendance.	MP: MP.4
8	**Planning a Science Experiment** Data Science Determine what experiment can be done using the available materials.	MP: MP.1, MP.2, MP.4
9	**Feeding Our Feathered Friends** Data Science Evaluate bird food consumption and plan for donations for a bird rescue.	MP: MP.2
10	**Fruit and Vegetables** Explain which fruit and vegetables students should receive as part of a balanced meal.	MP: MP.6, MP.7
11	**Our Community Garden** Use data to plan for a community garden.	

Module	Project topic	Standards
12	**Vegetable Farm** Decide where to plant vegetables on a farm.	MP: MP.2, MP.3, MP.6
13	**Farm Produce Fractions** Data Science Create a data set of produce in a produce box and evaluate that data.	MP: MP.7
14	**Planning a Playground** Decide where to place playground features.	MP: MP.1, MP.2, MP.3, MP.4
15	**Outdoor Shape Walk** Data Science Create a data set of shapes in the environment and evaluate that data.	MP: MP.7
16	**Drink More Water!** Data Science Determine which age group drinks the most water and which age group should drink more water.	MP: MP.1, MP.2
17	**Adopt a Pet!** Data Science Present data to support a persuasive argument for or against pet adoption.	MP: MP.3

Igniting Curiosity, Promoting Investigation, and Forging Connection: Real-World Math

Curious students are engaged students. The Module Projects draw from students' experiences and topics and present them in ways that leave students wanting to learn more. Each topic gives students an opportunity to connect the math from the module to their lives and the lives of others. The topics also encourage students to think about how math can be used to change their community and the world.

Many projects relate to issues that students may experience locally or more broadly. By making connections to their own environments, students come to understand how math can help them care for the world around them.

Optimizing Learning Experiences and Outcomes for All Students

Module Projects are accompanied by teacher supports that include suggestions for scaffolding each activity and grouping arrangements to benefit each learner and their collaborators. Module Projects can also be used as optional performance assessments, and each comes with a rubric to guide grading. Students can log in to *Ed*® to create data models using the data from their projects.

Purposeful Independent Practice

Learning is deeper and more durable when it is effortful (Brown et al., 2014). Deliberate, purposeful, and iterative practice builds knowledge and skills beyond rote repetition. Effective practice increases retention, automaticity, and motivation (Brabeck et al., 2015). Research identifies several high-utility strategies: spacing practice over time, practicing retrieval rather than restudy, and engaging with material in varied contexts (NASEM, 2018). Feedback focused on learning targets and metacognition is essential for progress (Brabeck et al., 2015; Dean et al., 2012; Hattie, 2009, 2023; Hattie & Clarke, 2018).

Within mathematics, students need consistent, distributed, and meaningful practice to strengthen conceptual understanding and procedural fluency (Baroody et al., 2009; Fuson & Beckmann, 2012; Rohrer, 2009). "[Math] practice does not mean rote experiences. Practice involves repeated experiences that give children time and opportunity to build their ideas, develop understanding, and increase fluency" (Fuson et al., 2015, p. 64). Cognitive science defines learning as a process of encoding, storage, and retrieval; however, classroom instruction often emphasizes encoding. Research shows that retrieval practice—actively recalling and applying learned information—most effectively consolidates learning and enhances transfer and higher-order thinking across subjects and populations (Agarwal & Bain, 2019).

Spacing, or distributed practice, has long been established as one of the most reliable findings in learning research (Carpenter et al., 2012). Spacing out learning sessions yields stronger long-term retention than massed practice (Cepeda et al., 2006; Underwood, 1961). Interleaving—mixing related problem types and targeted skills within a practice session—further enhances discrimination and flexibility, helping students choose appropriate strategies and improving learning outcomes (Agarwal & Bain, 2019; Dunlosky et al., 2013; Taylor & Rohrer, 2010).

Ongoing feedback is equally vital. Feedback during retrieval helps students assess what they know, address gaps, and refine understanding, improving metacognition (Agarwal & Bain, 2019; Brown et al., 2014; Molin et al., 2020). Current and emerging educational technology offers effective practice tools featuring adaptive task assignment and responsive feedback; see later sections of this paper on how strategic technology use to improve math learning optimizes practice at the individual student level.

Procedural fluency—the accurate, efficient, and flexible use of procedures—is a key component of mathematical proficiency, grounded in conceptual understanding (Rittle-Johnson et al., 2015; NCTM, 2014). When students connect procedures to concepts, retention and transfer improve; when they rely on memorization alone, learning is shallow and motivation declines (Fuson et al., 2005; Hiebert, 1999). To be proficient, students need flexible knowledge of multiple strategies and the ability to judge which to apply in varied situations (Baroody, 2006; Kling & Bay-Williams, 2014; NRC, 2001, 2005, 2012). Research shows that teachers foster fluency by building conceptual understanding, helping students notice patterns, offering choice among strategies, and implementing distributed, meaningful, and purposeful practice rather than rote repetition (Fuson & Beckmann, 2012; Rohrer, 2009). Research supports integrating worked examples with problem-solving, gradually reducing scaffolds as proficiency increases (Booth et al., 2013; Fuson et al., 2015; Pashler et al., 2007).

How *Into Math* Delivers

Within the program and in conjunction with *HMH Waggle*®, *Into Math* provides embedded adaptive, personalized learning and practice with clear, actionable insights rooted in effective, ongoing formative assessment.

Enhanced, Efficacious Independent Practice

Into Math's All New AI Tools generate quizzes and leveled practice sets* in multiple languages to measure students' knowledge and maintain district control over AI availability—safely, securely, and in the same ecosystem as teachers' HMH curriculum.

Purposeful Practice with Targeted Support

The Practice on Your Own feature allows students to practice what they have learned and includes Robert Kaplinsky's Open Middle® problems. These low-floor, high-ceiling activities are accessible to all students, encourage advanced problem-solving, and promote deep exploration and reasoning.

Students and parents can watch engaging, step-by-step videos demonstrating how to solve practice problems. These videos not only make learning enjoyable but also ensure students can grasp the concepts.

Mathematical Content and Practices: Exploration, Discovery, and Reasoning

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 7). Effective teaching and its development of students' mathematical knowledge are the driving forces behind powerful mathematics instruction and deep understanding. Research demonstrates the importance of instructional tasks that promote conceptual understanding, discourse, and procedural fluency, all of which contribute to deep mathematical comprehension (NCTM, 2000/2009 & 2014; NRC, 2012). High-achieving schools align curriculum, instruction, and assessment with standards, but standards alone are insufficient for real-world success (Peterson & Ackerman, 2015; Shannon & Bylsma, 2007; Tarr et al., 2008).

Mathematical literacy in the 21st century requires conceptual understanding (why), procedural fluency (how), application (when), and a strong math identity (Larson, 2017). Developing mathematical habits of mind is essential to mathematical proficiency, critical thought, college and career readiness, access to future opportunities, and productive participation in society (Goldenberg et al., 2015). Further, mathematical proficiency is developed via opportunities to engage in curiosity, investigation, argumentation, and collaboration around mathematical problem-solving (Aguirre et al., 2024; Cai & Hwang, 2023).

HMH Into Math empowers teachers with evidence-based tools, resources, and professional learning to improve outcomes and create an engaging classroom culture. The program is aligned to rigorous standards and promotes deep conceptual understanding and procedural fluency, number sense, problem-solving and reasoning, and data science competency by cultivating mathematical habits of mind, productive struggle, and discourse within a supportive learning community.

Mathematical Problem-Solving, Reasoning, Application, and Habits of Mind

“[S]olving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable” (Polya, 1965, p. ix). Problem-solving is an intrinsic part of human cognition, and within mathematics, it serves as a foundation for deeper conceptual understanding. However, as addressed later in this paper, K–12 mathematics content has long been presented and then perceived as a set of fixed facts or given knowledge that is difficult to master. This perception alienates many students and interferes with their learning. To improve conceptual understanding, procedural proficiency, and motivation, numerous researchers (e.g., Aguirre et al., 2024, Cai & Hwang, 2023; Jeannotte & Kieran, 2017; Li & Schoenfeld, 2019) call for the problematization of mathematics instruction as a transformative shift away from rote instruction and toward an approach that actively engages students in sense-making and experiencing math as a dynamic discipline of inquiry and reasoning. Such problematizing of mathematics instruction features intentionally designed learning activities in which students are challenged to grapple with meaningful, real-world problems, recognize patterns, confront uncertainty, construct viable arguments, and develop understanding through reasoning, justification, and critique—much like mathematicians do.

Proficient students of mathematics actively engage in problem-solving, experiment with strategies, and persist through challenges, embodying the habits of mind essential for mathematical success (Cuoco et al., 1996; Goldenberg et al., 2015).

To foster these skills, students must regularly engage in challenging problem-solving tasks with multiple solution paths (Cai & Hwang, 2023; Kaplinsky, 2019) and high cognitive demand (Li & Schoenfeld, 2019).

Engaging students in formulating problems and assessing solution approaches enhances their ability to think critically and apply mathematical knowledge (Aguirre et al., 2024; Cai & Hwang, 2023; Santos-Trigo, 2020). This engagement also allows students to actively construct deeper mathematical understandings with greater meaning, as opposed to being presented information and simply carrying out procedural exercises (Masingila et al., 2018). Research shows that student-centered problem-solving, rather than teacher-led procedural instruction, leads to longer-lasting comprehension and confidence in mathematics (Cai & Hwang, 2023; Marcus & Fey, 2003; Sengupta-Irving & Enyedy, 2015).

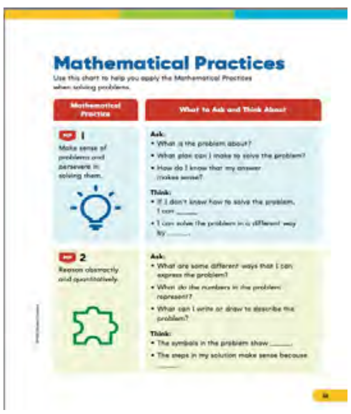
Studies indicate that learning from failed problem-solving attempts before direct instruction results in greater conceptual understanding and knowledge transfer (Kapur, 2010, 2014). Effective mathematics instruction shifts responsibility to students, allowing them to assess the validity of solutions and engage in meaningful discourse (Engle, 2012; Mueller et al., 2014; Sengupta-Irving & Enyedy, 2015). By fostering problem-solving mindsets, teachers help students develop intellectual resilience and motivation, ultimately empowering them as lifelong mathematical thinkers (Blackwell et al., 2007).

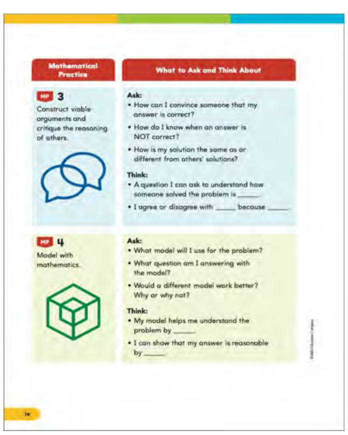
How *Into Math* Delivers

Into Math closely aligns to and provides comprehensive support to aid students in meeting rigorous mathematics standards and practices.

Promoting Mathematical Practices

The program supports students in developing Mathematical Practices (MPs). MPs are made clear and explicit for students via Anchor Charts and references throughout the SE.





Application of Mathematical Processes to Reasoning and Problem Solving and Cultivation of Habits of Mind

In addition to supporting all students, MPs serve to cultivate mathematical habits of mind and aid students in applying mathematical processes to reasoning and problem-solving.

Into Math Module Projects also provide consistent opportunities for students to collaboratively engage in open-ended problem solving, critical and creative thinking, and mathematical reasoning.

Number Sense

Having number sense entails deep understanding of numbers and their relationships. Number sense, which encompasses estimation, mental calculation, and number properties, is essential for mathematical fluency and flexible numerical thinking (Hiebert & Grouws, 2007; Baroody et al., 2012; McIntosh et al., 2005; Sood & Mackey, 2015). It underpins algebraic reasoning, including pattern recognition and equation manipulation (Chrysostomou et al., 2013), and serves as a predictor of future mathematics achievement (Sarama & Clements, 2009; Griffin, 2005; Jordan et al., 2010; Kirkland et al., 2024; Nelwan et al., 2022).

Number sense typically develops naturally in early childhood through environmental interactions with activities like sorting, comparing, counting, and sequencing. However, number sense continues to develop into the secondary level and even adulthood (Clements & Sarama, 2007; Dehaene, 2011; Jordan et al., 2010; Mix et al., 2002; Piaget, 1965). Home and classroom experiences, including reasoning and play, further nurture numerical reasoning (Baroody et al., 2012; Sarama & Clements, 2009). Cognitive styles also shape students' problem-solving approaches and flexibility with numbers (Sarama & Clements, 2009).

Number sense develops throughout K–12 education. In early grades, students focus on counting, place value, and arithmetic. In Grades 3–5, flexibility with numbers extends to multiplication, division, and fractions. Middle school students study rational numbers, proportional relationships, and number lines, while high schoolers explore polynomials, functions, and financial mathematics. Kirkland et al. (2024) found that mature number sense enables students to discern patterns and choose efficient problem-solving strategies, and it strongly predicts mathematics achievement.

Effective instruction requires hands-on experiences in early learning and explicit teaching starting in kindergarten (Baroody et al., 2012; Sood & Mackey, 2015). It is important that number sense is developed alongside strategic reasoning (Sarama & Clements, 2009) and that students be given frequent opportunities to build and apply number sense within real-world context, where knowledge of numbers is indeed ubiquitous (Griffin, 2005). Research shows that, across grade spans, games and math talks contribute to the development of number sense, as these have motivating effects and provide opportunities for feedback (Baroody et al., 2012; Fuson et al., 2015; Griffin, 2005; Pan et al., 2022). Games featuring spaced pathways promote number sense among younger students at-risk for math difficulties (Ramani & Siegler, 2008). Well-crafted, frequent, and brief math talks foster students' mental math and computation skills (Boonen et al., 2011; Fuson et al., 2014; Parrish & Dominick, 2016; von Sprecklesen et al., 2019). The language teachers use both within broad instruction and math talks specifically matters greatly, however. It must be accurate and avoid "tricks" or short-sighted rules; rather than constitute a set of memorized procedures, classroom discourse and instructional approaches must support students' authentic understanding of mathematical big ideas, including number sense (Fuson et al., 2014, 2015; Hiebert & Grouws, 2007).

How *Into Math* Delivers

In accordance with state standards as well as the research literature, *Into Math* helps students develop number sense in early math learning and at every level throughout the grade span. Number sense is embedded throughout program content.

Developing Number Sense via Play and Games

The program offers a variety of Games at every grade level to support the development of mathematical proficiencies, including specifically number sense.


Game Cards

10 Ahead

FOCUS
Recognize numerical patterns

MATERIALS
For partners: 2 playing pieces, 6-section spinner labeled 1–6, pencil, paper clip

USING THE GAME
Partners take turns. Each partner starts by placing his or her playing piece on START. Taking turns, one player at a time spins the pointer and moves that number of spaces on the hundred chart. If the player lands on green, he or she moves ahead ten spaces. If the player lands on orange, he or she moves back ten spaces. The first player who gets to END wins the game.

Grade 1 | 10 Ahead

HMH Into Math
1

Developing Number Sense via Math Talk

The program also fosters number sense through ongoing opportunities for students to engage in collaborative and supportive mathematical discourse with classmates. In such Math Talk activities, students are guided in recognizing and practicing numerical patterns.

Data Science: Mathematical Foundations and Reasoning with Data

Data literacy is an essential skill in today's data-driven world, influencing personal, professional, academic, and civic decision-making. For K–12 students, developing data literacy is crucial for engaging as informed global citizens. The study of data provides opportunities for authentic mathematical exploration, fostering conceptual understanding, reasoning, and problem-solving skills. Additionally, many careers require employees to interpret feedback data to refine practices, making data literacy a critical workforce skill. It also enables individuals to evaluate misleading information and make informed decisions regarding health, finances, and current events (Bargagliotti et al., 2020; Drozda et al., 2022; Lee et al., 2021; Makar et al., 2023; NASEM, 2018; Smith et al., 2023; Weiland & Engledowl, 2022; Wolff et al., 2016). "All students should develop the abilities not only to consume statistics and data with scrutiny, but also to think and work with data following a disciplined process of data inquiry" (Louie, 2022, p. 1).

Wolff and colleagues (2016) define data literacy as the ability to inquire about and analyze real-world data, with a focus on ethical considerations. Critical data literacy extends this definition to include ethical data collection, privacy concerns, and power structures in data use (Bargagliotti et al., 2020; Louie, 2022; NASEM, 2018).

Data literacy—also referred to as data science or data fluency—is a foundation for understanding within various fields of learning and industry, including healthcare, economics, and politics. Data science offers growing career opportunities, with job prospects expected to increase by 36% over the next decade (Bureau of Labor Statistics, 2022). Moreover, research shows that incorporating data science into K–12 education makes mathematics more engaging and relatable (Drozda et al., 2022), fostering a sense of belonging for all students, including groups underrepresented in STEM fields (Anwar et al., 2019; Kang et al., 2019; Weiland & Engledowl, 2022).

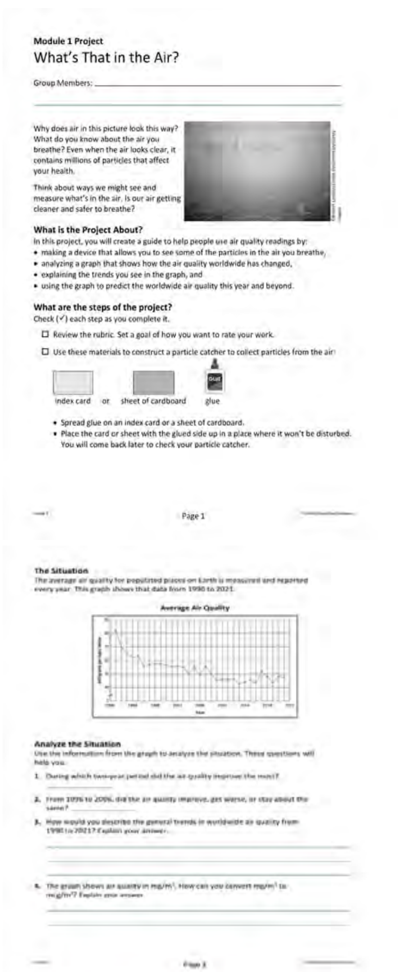
Embedding data science within mathematics instruction allows students to gain essential skills in data collection, interpretation, and communication. The *Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II)* (Bargagliotti et al., 2020) provides a framework for teaching data science; it is structured across three levels: elementary (basic data concepts), middle school (data variability and representation), and high school (sampling, probability, and inference). The guidelines advocate for expanding access to high-quality statistics education, hands-on learning, dynamic visualization tools, and assessment methods that emphasize conceptual understanding over rote memorization.

How *Into Math* Delivers

Into Math Module Projects provide ongoing opportunities for students to collect, display, and analyze data, as well as see the importance that such data plays in the world.

Data Literacy and Application in Real-World Math

Students working in print and digital Module Projects are guided step-by-step in carrying out the aims of the activity, which are centered around a series of probing questions that drive the investigation, making reporting more relevant and impactful.



Module 1 Project

What's That in the Air?

Group Members: ______

Why does air in this picture look this way? What do you know about the air you breathe? Even when the air looks clear, it contains millions of particles that affect your health.

Think about ways we might see and measure what's in the air. Is our air getting cleaner and safer to breathe?

What is the Project About?

In this project, you will create a guide to help people use air quality readings by:

- making a device that allows you to see some of the particles in the air you breathe,
- analyzing a graph that shows how the air quality worldwide has changed,
- explaining the trends you see in the graph, and
- using the graph to predict the worldwide air quality this year and beyond.

What are the steps of the project?

Check (✓) each step as you complete it.

☐ Review the rubric. Set a goal of how you want to rate your work.

☐ Use these materials to construct a particle catcher to collect particles from the air

index card or sheet of cardboard glue

- Spread glue on an index card or a sheet of cardboard.
- Place the card or sheet with the glued side up in a place where it won't be disturbed. You will come back later to check your particle catcher.

Page 1

The Situation

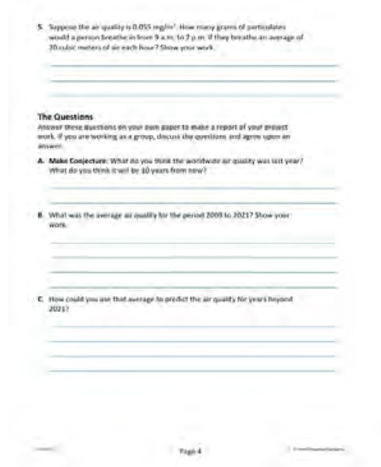
The average air quality for populated places on Earth is measured and reported every year. This graph shows that data from 1990 to 2021.

Average Air Quality

Analyze the Situation

Use the information from the graph to analyze the situation. These questions will help you.

1. During which two-year period did the air quality improve the most?
2. From 1996 to 2006, did the air quality improve, get worse, or stay about the same?
3. How would you describe the general trends in worldwide air quality from 1990 to 2021? Explain your answer.
4. The graph shows air quality in mg/m³. How can you convert mg/m³ to mg/m³? Explain your answer.



5. Suppose the air quality is 0.055 mg/m³. How many grams of particulates would a person breathe in from 9 a.m. to 2 p.m. if they breathe an average of 30 cubic meters of air each hour? Show your work.

The Questions

Answer these questions on your own paper to make a report of your project work. If you are working as a group, discuss the questions and agree upon an answer.

A. **Make Conjecture:** What do you think the worldwide air quality was last year? What do you think it will be 10 years from now?

B. What was the average air quality for the period 2009 to 2021? Show your work.

C. How could you use that average to predict the air quality for years beyond 2021?

Page 4

Developing Proficiency in Data Display and Communicating Ideas Visually

The digital portion of many Module Projects allows students to experiment with data displays to explore how different visualizations communicate ideas.

A partnership with Tuva Labs enables students to work with authentic data sets and tools throughout these activities.

Conceptual Understanding and Procedural Fluency: Teaching for Depth of Knowledge

U.S. education policy has long emphasized rigorous standards, aligned instruction, and accountability measures. While standards provide a foundation for effective teaching, they are insufficient on their own for improving learning outcomes. Focus and coherence are critical within standards-aligned instruction (Schmidt et al., 2005), particularly when adapting instruction to meet students' needs (Pak et al., 2020). Studies of American mathematics teaching reveal a lack of depth and rigor, with content coverage that is often too broad (NRC, 2001). In contrast, high-performing countries in international comparisons teach fewer topics but in greater depth and coherence (NMAP, 2008; Schmidt et al., 2005). Teaching for depth entails active learning methods that allow students to apply and test what they know within authentic, meaningful tasks that reflect how knowledge is used in its corresponding field (Noguera et al., 2015).

A longstanding debate in math education concerns the balance between conceptual understanding and procedural fluency. On one side is an emphasis on exploratory, hands-on learning, while the other prioritizes rote practice. Research underscores that both are necessary and should develop iteratively (Baroody et al., 2009; Fuson, 2009; Larson & Kanold, 2016; Li & Schoenfeld, 2019; NCTM, 2014; NRC, 2001; Rittle-Johnson, 2017). Procedural knowledge and conceptual understanding must be interwoven, as gains in one reinforce the other. Effective instruction fosters fluency with procedures while ensuring students understand the underlying concepts, allowing for flexible application of problem-solving skills and emphasizing meaning-making (Cross et al., 2009; Hattie et al., 2017; NRC, 2005; NCTM, 2014; Rittle-Johnson et al., 2015).

Conceptual understanding enables students to make connections between ideas, learn efficiently, and assess the reasonableness of their solutions (NRC, 2001; Yang & Sianturi, 2019). Procedural fluency goes beyond memorization, requiring accuracy, efficiency, flexibility, and the ability to modify strategies based on context (NRC, 2001, 2005). Developing fluency involves meaningful practice—engaging, purposeful, and spaced over time—rather than rote repetition (Baroody et al., 2009; Fuson & Murata, 2007; NCTM, 2015a; Rohrer, 2009).

To foster conceptual and procedural knowledge simultaneously, Rittle-Johnson (2017) suggests incorporating comparison (analyzing different problem-solving strategies), self-explanation (connecting new information to prior knowledge), and exploration before direct instruction. Students benefit from real-world applications of math throughout their learning, reinforcing understanding and motivation (Aguirre et al., 2024; Alberti, 2013; Cross et al., 2009; David & Greene, 2007; Hiebert et al., 1996; NCTM, 2014). Engaging students in complex, open-ended sense-making and problem-solving tasks promotes deeper mathematical understandings with greater meaning than when teachers merely present information to students and have them carry out procedural exercises (Cai & Hwang, 2023; Marcus & Fey, 2003; Masingila et al., 2018; Sengupta-Irving & Enyedy, 2015). Such approaches build a lasting and transferable foundation in math that also has motivating effects (Cross et al., 2009; Li & Schoenfeld, 2019).

How *Into Math* Delivers

Into Math is a research-based, high-quality comprehensive curriculum that centers on rich learning tasks.

Those tasks support student understanding of the Big Idea of each module and the Content Connections and Drivers of Investigation that weave these ideas together.

An Evidence-Based Instructional Framework That Builds Conceptual Understanding and Procedural Fluency

Students build conceptual understanding and procedural skills through collaborative discussions. Teachers facilitate these discussions with comprehensive supports in every lesson, including Math Language Routines and guidance for identifying common errors and misconceptions.

The sequence of lessons follows instructional design that builds conceptual understanding, connects concepts to skills, and then allows students to apply their thinking. Open Middle tasks invite all students to think deeply about mathematics.

Interactive Whole-Class Presentations with Real-Time Data Insights

Highly effective, interactive whole-class presentations provide teachers with curated resources to deliver engaging lessons supported by efficacious instructional routines.

Task-based Learning and Turn & Talk activities are facilitated via *HMH Classcraft*.

Deepening Mathematical Knowledge

Into Math's multifaceted program components build deep math learning. This includes the program's Module Projects, which offer rich, engaging Depth of Knowledge (DOK) level 4 projects in every module, allowing students to apply and extend learning through nterconnected opportunities for collaborative problem-solving, creativity, and advanced mathematical reasoning via open-ended problems. SMP Anchor Charts further foster mathematical habits of mind and encourage students to engage in mathematical processes to reason and problem solve.

Effective, Timely, and Adaptative Practice for Personalized Learning

Into Math provides practice of newly acquired and emerging skills in multiple modalities: print, interactive digital, teacher-led small groups, collaborative activities, and games.

AI-powered *HMH Waggle* allows students to practice math independently at the precise level that is most beneficial for them, regardless of where they are in their progression toward meeting learning goals.

Mathematical Discourse: Communicating and Collaborating as a Community of Learners

Effective mathematics learning has sociological foundations that are developed via discursive, dialogic, and collaborative instructional approaches across all grade levels (Lambert & Sugita, 2016; Langer-Osuna & Avalos, 2015; NASEM, 2018; NCTM, 2014; Smith & Stein, 2018; Turner et al., 2013). Indeed, mathematical proficiency extends beyond individual cognitive ability, situating learning within social interactions. It underscores the integration of abstracting, generalizing, and argumentation to facilitate high-quality classroom discussions.

Research has long demonstrated that verbalizing mathematical thinking enhances understanding and problem-solving abilities. Encouraging students to articulate problems before writing responses increases accuracy (Lovitt & Curtiss, 1968), while verbal expression combined with feedback fosters deeper learning (Gersten & Chard, 2001). "Communication is an essential part of mathematics and mathematics education…[that] can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols…the communication process also helps build meaning and permanence for ideas and makes them public" (NCTM, 2000, pp. 59–60). Furthermore, mathematical language development is key to removing barriers to learning (Dacey et al., 2013; Thanheiser & Melhuish, 2023).

Discourse-based mathematics instruction has been shown to improve students' reasoning and problem-solving across grade levels (Hufferd-Ackles et al., 2015; Humphreys & Parker, 2015; Klibanoff et al., 2006; von Sprecklsen et al., 2019). Effective math instruction involves facilitating discussions in which students analyze and compare approaches, contributing to higher engagement and confidence (Clements et al., 2023; Kinsella, 2016; Michaels et al., 2008; Morgan et al., 2014; Parrish & Dominick, 2016; Saylor & Walton, 2018). Encouraging students to justify and refine their mathematical thinking fosters deeper conceptual understanding (Carpenter et al., 2003; Fuson et al., 2014). Additionally, discourse serves as a formative assessment tool, providing teachers insight into students' thought processes (Franke et al., 2007; Hattie, 2009, 2012, 2023; Wiliam, 2018).

Research has well established Math Talk as a beneficial instructional practice, including specifically for ELs (Langer-Osuna & Avalos, 2015; Turner et al., 2013). Quality discourse requires structured, predictable classroom protocols and routines to promote respectful, collaborative exchanges that engage all students and provide useful evidence of student understanding (Fuson et al., 2014; Hufferd-Ackles et al., 2015; Kinsella, 2016; Lambert & Sugita, 2016; Thanheiser & Melhuish, 2023). Math Talk conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Kinsella, 2016; Mercer & Howe, 2012; Zwiers et al., 2017).

How *Into Math* Delivers

Into Math fuels rich, teacher-facilitated student discourse and hands-on learning. The result is deep understanding of mathematics and a joy of learning, all within supportive, accessible learning communities where language development is fostered and every member contributes and holds a secure sense of belonging.

Structured and Supportive Classroom Learning Communities That Foster Language Development and Discourse

Students build conceptual understanding and procedural skills through collaborative discussions. *Into Math* features Math Language Routines and Activities, developed by Dr. Kate Kinsella and rooted in HMH's evidence-based *English 3D*®. These activities have been proven effective for all learners but show specific benefits for ELs. The provided Peer Coach videos model how teachers can effectively carry out these routines. Also included is detailed guidance on eliciting student responses. Mathematical Language Routines ensure that ELs participate fully in discursive activities and engage as contributing members of the classroom community.

Ongoing opportunities to share and co-construct understanding are provided by *HMH Classcraft*'s Turn & Talk activities. *Into Math*, in tandem with *HMH Classcraft*, provides ongoing guidance to teachers in structuring learning activities and supports in every lesson. Guidance includes identifying common errors and misconceptions. SMP Anchor Charts support language development and classroom discourse with sentence/thought starters for students. Module Projects offer additional collaborative and investigative work with classmates.

Dynamic Modalities Optimize Individual and Collaborative Differentiated Instruction

Into Math enables data-driven, fluid instruction modalities that allow students to rotate seamlessly among whole-class sessions, small-group collaborations, and independent practice, supporting efficacious differentiated and dynamic instruction. Students have ongoing opportunities to work either independently or with teachers and peers to develop conceptual understandings and procedural fluency, as well as a strong mathematical identity.

Mathematical Routines

Mathematical proficiency requires deep thinking, which is best developed through structured, interactive instructional routines that engage students in reasoning, communication, and sensemaking (Lampert, 2015; Berry, 2018; Thanheiser & Melhuish, 2023). Predictable and consistent classroom routines help students focus on mathematical tasks and one another, fostering engagement and conceptual understanding for all students, especially those with disabilities (Lambert & Sugita, 2016). Well-designed routines for reasoning allow students to articulate, revise, and refine their ideas both verbally and in writing (Aguirre et al., 2024; Cai & Hwang, 2023; Feldman & Kinsella, 2008; Kinsella, 2016; Zwiers, 2014; Thanheiser & Melhuish, 2023).

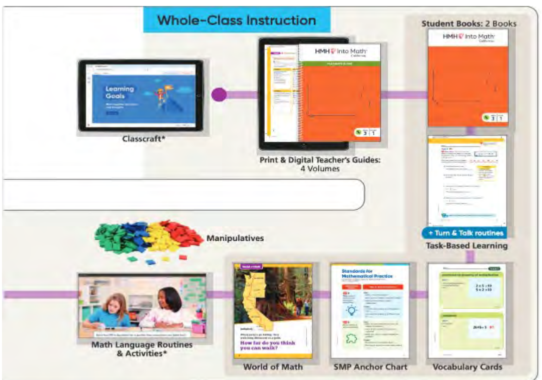
Effective instructional routines include “low floor/high ceiling” tasks, which provide accessible entry points for all students while scaling in complexity to challenge them at their individual levels (Sircar & Titus, 2015). In a 2016 research review, Lambert and Sugita found that consistent routines developed around problems serve as a potential positive factor, boosting engagement of students with learning disabilities (LDs) in problem-solving, as well as one that improves whole-class discussion.

Structured, predictable routines become second nature through repeated practice, much like classroom management routines (Kelemanik et al., 2016). Lampert (2015) suggests that structured routines should incorporate individual think time, peer discussions, public sharing of reasoning, and explicit connections to core mathematical concepts. Through repeated engagement in such routines, both teachers and students develop new intellectual and social skills that transform their perceptions of teaching and learning. Lucenta and Kelemanik (2020) emphasize a reasoning approach centered on mathematical modeling, guiding students through processes of sensemaking, analysis, model interpretation, adaptation, and reflection.

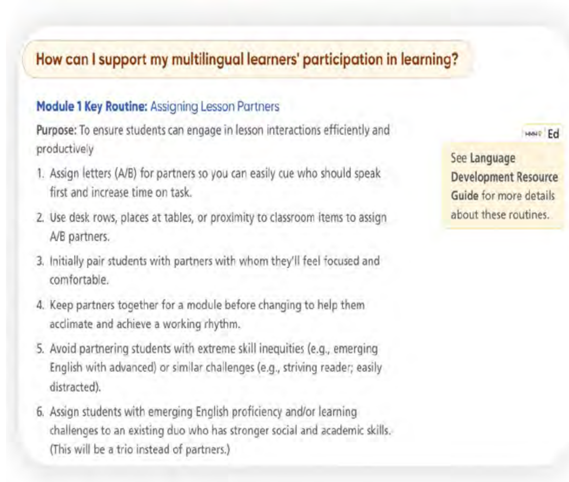
Establishing clear classroom norms ensures that students consider and build upon their peers' mathematical thinking, promoting collaborative learning (Fuson et al., 2014). The Claim, Evidence, and Reasoning (CER) (McNeil & Martin, 2011) routine helps students justify conclusions using scaffolded argumentation. Well-chosen routines can support "math talks" that build mathematical proficiencies. Another example of classroom norms is peer revoicing, a guided process in which students restate each other's thinking and co-construct understandings (Enyedy et al., 2008). Structured math talks improve student-to-student interaction via effective listening and questioning and allow students to engage in collaborative sense making and explanations, promoting deeper mathematical understanding (Thanheiser & Melhuish, 2023). These mathematical language routines are supportive of all students within both monolingual and multilingual classrooms (Enyedy et al., 2008; Kinsella, 2016; Zwiers, 2014). "Developing routines for complex cognitive behavior such as problem solving is beneficial for learners as it creates an external scaffold for internal processes" (Lambert & Sugita, 2016, p. 14).

How *Into Math* Delivers

Into Math provides structured, task-based whole-group learning via *HMH Classcraft*. This program architecture aids teachers in instructional planning and classroom management and creates predictable, organized routines that support student learning.



Additionally, for ELs specifically, as well as the benefit of all learners, *Into Math* provides Math Language Routines and Activities, developed by Dr. Kate Kinsella and rooted in HMH's evidence-based *English 3D*®. Peer Coach videos are also available for modeling of how to effectively carry out these routines.



How can I support my multilingual learners' participation in learning?

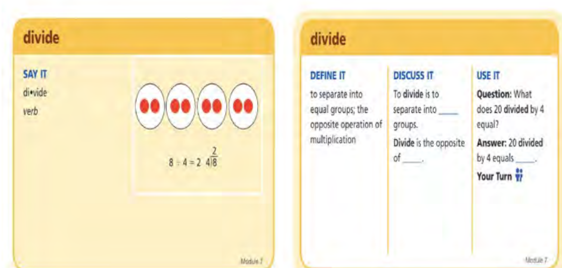
Module 1 Key Routine: Assigning Lesson Partners

Purpose: To ensure students can engage in lesson interactions efficiently and productively

1. Assign letters (A/B) for partners so you can easily cue who should speak first and increase time on task.
2. Use desk rows, places at tables, or proximity to classroom items to assign A/B partners.
3. Initially pair students with partners with whom they'll feel focused and comfortable.
4. Keep partners together for a module before changing to help them acclimate and achieve a working rhythm.
5. Avoid partnering students with extreme skill inequities (e.g., emerging English with advanced) or similar challenges (e.g., striving reader; easily distracted).
6. Assign students with emerging English proficiency and/or learning challenges to an existing duo who has stronger social and academic skills. (This will be a trio instead of partners.)

See **Language Development Resource Guide** for more details about these routines.

Dr. Kinsella helped create *Into Math*'s Words to Learn routine, integrated into vocabulary cards in every student book. These cards help students master essential math terms, guiding students to SAY, DEFINE, DISCUSS, and USE new words. Teachers can use copies of the cards for games and word walls, and the module planner provides a list of key terms and blank templates for further customization.



Productive Struggle

Success in mathematics depends on practice and perseverance (Larson, 2017). "An effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (NCTM, 2014, p. 11). When students engage in productive perseverance, they grapple with problems, persist through difficulties, and build resilience, ultimately gaining confidence in their mathematical abilities (Jackson & Lambert, 2010). Productive struggle also affords opportunities for students to engage in mathematical sense-making (Cai & Hwang, 2023; Li & Schoenfeld, 2019).

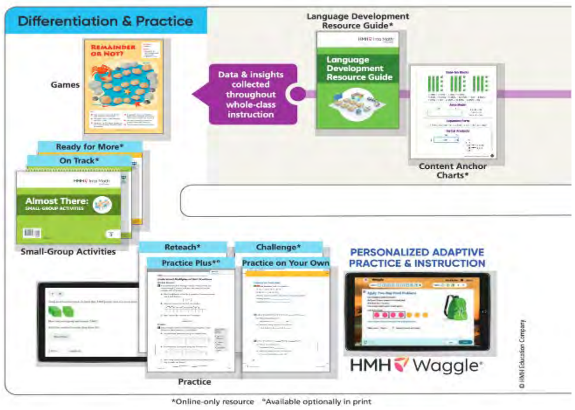
Perseverance is the "voluntary continuation of a goal-directed action in spite of obstacles" (Peterson & Seligman, 2004, p. 229). What makes perseverance *productive* is the combination of sustained effort with strategic thinking, reflection, and adaptability; positive mathematical struggle entails grappling with challenges, persisting through difficulties, and finding solutions through sense-making. Students who engage in effortful problem-solving—even when initially unsuccessful—achieve deeper learning and conceptual understanding (Kapur, 2010, 2014; Hiebert & Grouws, 2007; Warshauer, 2015). Productive perseverance results when students are equipped with the mindset and motivation to persist intelligently, not just persistently, and when instilled with the confidence that, through sustained effort, they will succeed in mathematics (Blackwell et al., 2007; Jackson & Lampert, 2010; NASEM, 2018; NCTM, 2014).

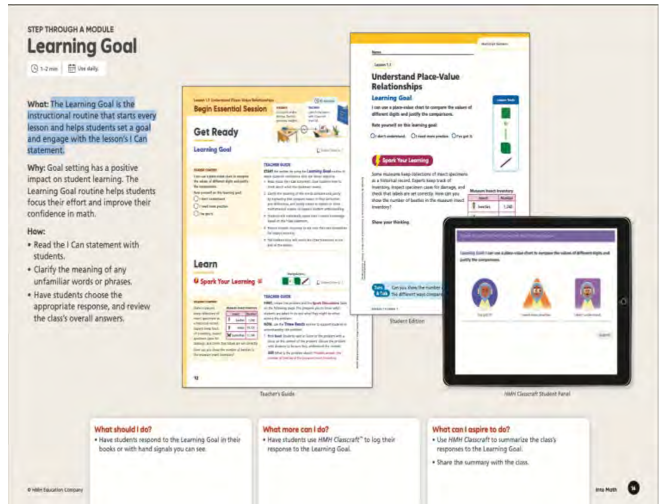
To develop mathematical habits of mind, educators must establish a classroom culture that normalizes challenge and emphasizes the benefits of perseverance (Star, 2015; Hiebert & Grouws, 2007; Goldenberg et al., 2015). Beliefs that effort and persistence lead to success are essential for fostering productive struggle and improving academic achievement, including particularly in math (NCTM, 2000, 2014; NRC, 2001). Learning contexts should reinforce positive messaging around the role of effort in learning, to make learning enjoyable and place the focus on that learning rather than on students' performance (Yeager et al., 2013, 2022). Research indicates that when students are allowed to make mistakes and persist through struggles, they demonstrate greater understanding (Hiebert & Grouws, 2007; Kapur, 2010, 2014; Warshauer, 2015). Perseverance in problem-solving also encourages metacognition, reinforcing the idea that meaningful learning comes from reasoning rather than simply arriving at correct answers (Hiebert & Grouws, 2007; Li & Schoenfeld, 2019).

Teachers play a crucial role in fostering perseverance by carefully selecting tasks, providing guidance without lowering cognitive demand, and allowing sufficient opportunity for students to struggle productively (Cai & Hwang, 2023; Dixon, 2024; Goldenberg et al., 2015). Effective questioning and scaffolding at the right moments are key to helping students develop persistence rather than frustration (Warshauer, 2015). However, timing is key. Excessive upfront scaffolding can reduce cognitive demand and limit students' opportunities to engage in challenging mathematical thinking (Boston & Wilhelm, 2015; Dixon, 2024). Additional strategies for promoting perseverance include heterogeneous grouping, contextualizing problems in familiar settings, goal setting, reflection, and using "low floor/high ceiling" tasks (Kaplinsky, 2019; Sircar & Titus, 2015).

How *Into Math* Delivers

Into Math promotes productive struggle by providing developmentally appropriate scaffolds to aid each student in meeting the rigorous learning goals—without eliminating challenge.





Program scaffolding resources include Content Anchor Charts, the Language Development Resource Guide, Manipulatives, Math Language Routines, Games, and leveled Small-Group Activities to incorporate within each lesson's differentiated instructional framework and *HMH Waggle*-supported adaptive practice—all aimed at explicit, rigorous, and attainable goals for learning for all students.

Accessible and Comprehensive Math Learning for All

Early experiences with mathematics yield effects well beyond classrooms, with consequences affecting economic prosperity, well-being, and quality of life (Campbell et al., 2014; Clements & Sarama, 2016, 2020). In our modern technological society where mathematics literacy is essential, lack of access to effective mathematics instruction and its myriad applications has adverse impacts on one's economic and civic stake within it (Aguirre et al., 2024).

While mathematics achievement on every scale requires that all students be expected to meet rigorous standards, each student comes to school with a unique background, skill set, and perspective, as well as personal strengths and needs. Therefore, each student must receive effective, individualized support to realize and enjoy success in math learning, as all children are capable of learning and performing in math at high levels (Aguirre et al., 2013; Clements & Sarama, 2020; Fuson et al., 2015; NCTM, 2014; Shapka et al., 2006). "Providing young children with extensive, high-quality early mathematics instruction can serve as a sound foundation for later learning in mathematics" (Cross et al., 2009, p. 2).

HMH Into Math supports students broadly and effectively by providing access to highest quality mathematics instruction with embedded differentiation to meet high expectations and wide-ranging needs. The program also supports teachers by providing tools to help create nurturing, comprehensive, and adaptive classroom environments that facilitate deep learning of mathematics for all.

Rigorous and Differentiated Mathematics Instruction

For over two decades, the National Council of Teachers of Mathematics (NCTM) has advocated for rigorous, accessible instruction that supports all learners (NCTM, 2000, 2014). “An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential” (NCTM, 2014, p. 59). Rigor in mathematics education involves intentionally balancing conceptual understanding, procedural fluency, and application, while establishing clear success criteria (Hattie et al., 2017). All students must be expected to meet rigorous standards, but since each learner brings unique backgrounds, skills, and needs, instruction must be differentiated to provide improved access and achievement (Clements & Sarama, 2020; Jones, 2010; NCTM, 2014, 2015, 2020; NMAP, 2008).

Rigor is achieved through clear, non-negotiable learning goals and aligned assessments that provide students with meaningful and flexible pathways toward success (Ainsworth, 2010; Darling-Hammond, 2010a; Jones, 2010; Marzano & Toth, 2014). Yet rigor alone is insufficient without responsiveness (Thompson et al., 2016). Differentiated instruction—an evidence-based, flexible approach to teaching—maximizes learning for all students by balancing challenge and success within a collaborative and accessible classroom community. Instruction is tailored to students’ readiness, interests, backgrounds, and learning profiles, with expectations adjusted based on ongoing assessment (Chamberlin & Powers, 2010; Tomlinson, 1997, 2001, 2017). “When teachers differentiate how they support student learning, they are ensuring that all students have access to the content and skills identified in the standards” (Rogers & Garii, 2013, pp. 34–35).

Learning occurs when activities are tailored to the appropriate level for each learner (Vygotsky, 1978), determined by continual, flexible assessment (Levy, 2008; Seo & Smith, 2013). Differentiation provides multiple avenues for engagement with content, process, and product: how students acquire information, make sense of ideas, and demonstrate understanding (Lawrence-Brown, 2004; Tomlinson, 2017). Research shows it significantly improves achievement across grade levels and demographics (Allan & Goddard, 2010; Beecher & Sweeney, 2008; Bogen et al., 2019; Tomlinson, 2004). Students with disabilities and specific needs benefit from environments that maintain high expectations while providing access to the general curriculum (Lawrence-Brown, 2004). Similarly, effective strategies for ELs emphasize developing academic and oral language proficiency (Francis et al., 2006; Kinsella, 2016). Advanced learners, too, require differentiation through opportunities for independent work and varied pacing (Rogers, 2007; Tomlinson, 1997, 2017).

A “funds of knowledge” approach (Aguirre et al., 2012; González et al., 2006) further supports students by connecting instruction to their lived experiences while maintaining high expectations and broad participation norms (Aguirre et al., 2013, 2024; Moll et al., 2006). Through rigor, responsiveness, and differentiation, teachers can ensure that all students engage meaningfully with mathematics and reach high levels of achievement.

How *Into Math* Delivers

Into Math promotes conceptual understanding through student driven discourse and hands-on learning. Students work with manipulatives throughout the lessons to ensure deep understanding of the math. The sequence of lessons follows an instructional design that builds conceptual understanding, connects concepts to skills, and then allows students to apply their thinking.

Ensuring Accessible and Effective Mathematics Learning

Universal Design for Learning (UDL) principles, resources for multilingual learners, and accessibility supports meet the needs of all learners. Furthermore, the program regards students' backgrounds, experiences, and prior math knowledge are funds of knowledge—valuable assets that are leveraged daily to ensure personal relevance for each student.

Into Math provides highly effective, interactive whole-class presentations with real-time data insights with *HMH Classcraft*, which offers curated resources for engaging lessons supported by efficacious instructional routines. Resources that promote academic language development are found throughout *Into Math*, including multilingual learner support, vocabulary cards, Dr. Kate Kinsella's language-building routines, and a language development resource guide. Dr. Kinsella also helped create the program's Words to Learn routine, which are integrated into vocabulary cards in every student book. These cards help students master essential math terms and guide students to SAY, DEFINE, DISCUSS, and USE new words. Teachers can use copies of the cards for games and word walls, and the module planner provides a list of key terms and blank templates for further customization. Additionally, every teacher and student component of *Into Math* is also available in Spanish.

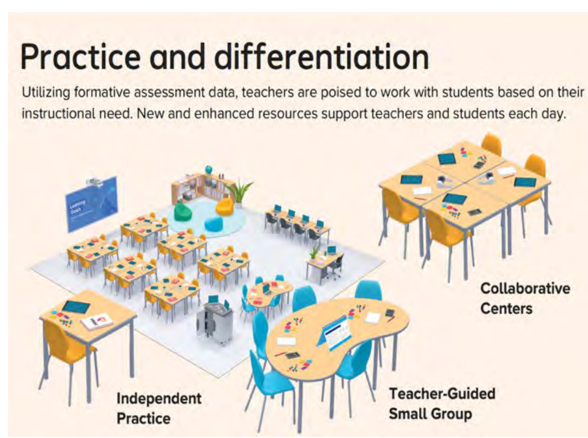
Ongoing, Seamless Differentiation and Practice Opportunities

Into Math pinpoints learning needs with detailed data and reporting through *HMH Ed*. Teachers access resources to support differentiation for students who need more support, are on track, or need greater challenge.

Instructional model that enables differentiation before practice

Formative assessment occurs before students practice so teachers can spend time with small groups to correct misconceptions before they stick. In addition to the practice built into each lesson, *Into Math* also includes:

- Practice videos that showcase the first problem in Practice on Your Own (Practice on Your Own is included in every lesson.)
- Practice Plus Workbook for every grade with Spiral Review
- Personalized Adaptive Practice via *Waggle*
- Almost There Small-Group Mini Lessons, Reteach, Challenge, and Fluency



Teaching for Engagement in Mathematics

Decades of extensive research has explored and continually confirmed that, across all education levels, content areas, and socioeconomic factors, active engagement in classroom activities is essential for meaningful and successful learning (Eccles et al., 1998; Frommelt et al., 2021; Jansen et al., 2022; Lee & Shute, 2010; Schunk & Mullen, 2012; Wang & Degol, 2014). Engagement consists of behavioral, emotional, and cognitive dimensions (Fredricks et al., 2004), and motivation plays a crucial role in academic success (Ainley, 2012; Bandura, 1986; Ryan & Deci, 2000; Schunk et al., 2008). Active participation helps students persist through challenges and improves retention of knowledge (Chi & Wylie, 2014; Finn & Zimmer, 2012). However, student motivation and engagement tend to decline in secondary education, impacting long-term academic success and STEM participation for many students (Duckworth et al., 2007; Kuo et al., 2021; Morgan et al., 2014).

Research continually underscores that active engagement that focuses on questioning, reasoning, explaining, and problem-solving optimizes learning (Freeman et al., 2014; Gresalfi et al., 2009; Hiebert et al., 1996; Kapur, 2014; NCTM, 2014; Smith & Stein, 2018). In mathematics education, research supports approaches that connect math to real-life applications, making it more relevant and engaging (Aguirre et al., 2024; Cai & Hwang, 2023; Li & Schoenfeld, 2019). Embedding math in meaningful, real-world contexts enhances learning experiences, fosters creative and critical thinking, increases relevance and interest, and helps students draw richer connections across disciplines (Alberti, 2013; Czerniak et al., 1999; Höfer & Beckmann, 2009; Martignon & Rechtsteiner, 2022; Russo et al., 2011; Verschaffel et al., 2020).

Inquiry-based learning encourages students to ask questions, explore multiple solutions, and develop curiosity (Deslauriers et al., 2019; Sengupta-Irving & Enyedy, 2015). Emerging research suggests that integrating data science within mathematics instruction holds much promise for increasing engagement in the domain as well as for generating interest in STEM careers. “Research referenced in this report suggests that the combination of content relatability, student autonomy, and inquiry-driven learning inherent to data science may partially explain the strong student excitement” (Drozda et al., 2022, p. 16).

Open-ended, problem-based tasks provide opportunities for students with divergent learning and linguistic skills and backgrounds to demonstrate their thinking in various ways and allow multiple modes of participation (CAST, 2018; Lambert & Sugita, 2016; Wong et al., 2003). Collaborative reasoning and group problem-solving also enhance engagement and mathematical understanding, as peer discussions help clarify concepts and increase interest (Langer-Osuna, 2017; Webb et al., 2014).

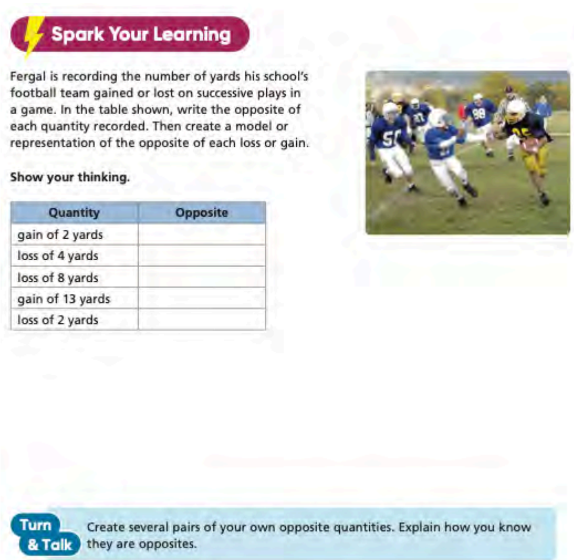
Using multidimensional teaching strategies, including visuals, manipulatives, and technology, aligns with UDL principles, making math more accessible to all students (CAST, 2018). Ultimately, engaging instructional practices foster positive math identities and help students see themselves as capable mathematicians, increasing their long-term success (Walton et al., 2012; Langer-Osuna, 2017).

How *Into Math* Delivers

Into Math infuses engagement with mathematics learning across all elements and learning arcs within the program.

Igniting Interest and Engagement in Learning

Lessons launch by identifying learning goals and engaging students in Get Ready activities, which generate self-assessment related to those goals. Next, Spark Your Learning activities engage interest and ignite curiosity. Turn & Talks engage students in interactive, collaborative peer learning to begin co-constructing understanding.



Spark Your Learning

Fergal is recording the number of yards his school's football team gained or lost on successive plays in a game. In the table shown, write the opposite of each quantity recorded. Then create a model or representation of the opposite of each loss or gain.

Show your thinking.

Quantity	Opposite
gain of 2 yards	
loss of 4 yards	
loss of 8 yards	
gain of 13 yards	
loss of 2 yards	

Turn & Talk Create several pairs of your own opposite quantities. Explain how you know they are opposites.

Making Learning Interactive and Dynamic

Other program features further engage students in math learning:

World of Math helps students see math learning as relevant, intriguing, and interconnected.

Dynamic classroom structure and lesson frameworks stimulate and re-engage interest and interactivity across the span of a lesson as students shift from whole-class to small-group learning, then to independent practice rotations.

Module projects provide hands-on, interactive engagement in interdisciplinary, real-world math learning.

Dynamic digital learning: *HMH Ed*, *Classcraft*, and *Waggle* offer multimedia appeal and interactivity. The *HMH Classcraft* Student Portal provides interactive learning experiences for students and keeps them on task during whole-group instruction, as well as during partner and individual work. All within the same classroom, students can follow along either in their book or in digital resources available online, and they can even use a mixture of the two. *HMH Waggle* provides engaging, adaptive practice that provides customized learning pathways to success.

Universal Design for Learning

Full, active, and contributory participation within the instructional environment is essential to academic success and developing positive math identities (Aguirre et al., 2013; Lambert & Sugita, 2016; Wolfe, 2022). Effective mathematics instruction for all learners, including students with disabilities, neurodivergent individuals, and multilingual learners (MLs), considers each student's unique learning style, strengths, and experiences, leveraging them to help every learner achieve high levels of mathematical proficiency and a positive math identity (Lambert, 2024).

Lambert and Sugita (2016) propose that limited engagement with mathematical problem-solving curriculum is a primary reason why students with LDs are underrepresented in STEM fields at the undergraduate level.

The research-based UDL guidelines developed and advocated for by CAST (2018) to support educators and other stakeholders in establishing optimal conditions for learning for all students emphasize three primary principles. First, multiple means of engagement (affective networks and the "why" of learning) provide students with varied ways to engage with mathematical content based on their interests, cultural backgrounds, and learning preferences. These variations include options for recruiting interest, sustaining effort and persistence, and self-regulation. Second, multiple means of representation (recognition networks and the "what" of learning) offer varying methods of processing and presenting information, such as visual aids, manipulatives, graphs, and real-world contexts. These methods include options for perception, language and symbols, and comprehension. The final principle is multiple means of action and expression (strategic networks and the "how" of learning), allowing students multiple ways to demonstrate their understanding, such as via written work, oral presentations, or hands-on projects, including options for physical action, expression and communication, and executive functions.

The UDL framework for mathematics instruction acknowledges that there are multiple ways to be a math learner. UDL for Math provides adaptive practices based on how each student perceives the world and approaches problem-solving (Lambert, 2024). Creating a nurturing classroom environment includes offering accessible resources, prompting students to explain their reasoning, and involving paraprofessionals to support student learning (Lambert & Sugita, 2016; Baxter et al., 2005).

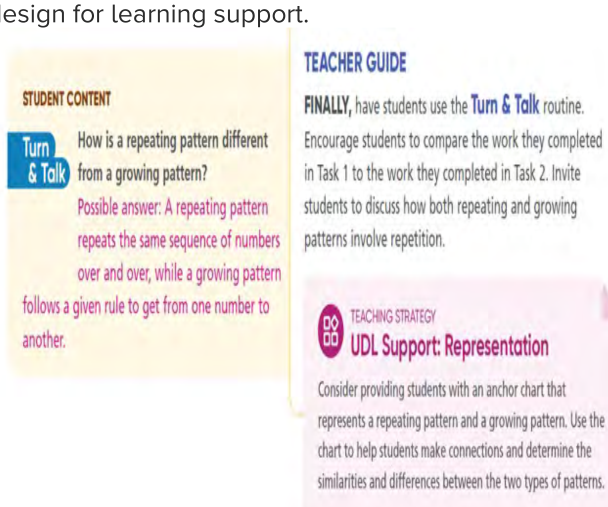
UDL frameworks promote multiple means of engagement, representation, and assessment, allowing students to demonstrate mathematical understanding in varied ways (Strangman et al., 2004). Accessible learning also extends to digital tools, ensuring all students—not just those in advanced courses—engage with high-cognitive-demand tasks using technology (White et al., 2021). Ultimately, improving accessibility in mathematics hinges on shifting traditional instructional models to foster success for all students, rather than reinforcing barriers (NCTM, 2014).

How *Into Math* Delivers

UDL principles, resources for multilingual learners, and accessibility supports are embedded within *Into Math* to meet the needs of all learners. Students' backgrounds, experiences, and prior math knowledge are leveraged to ensure personal relevance for each student. This accessible, supportive experience allows every student to thrive, regardless of their background.

Accessible and Supportive Learning Environments

Dr. Kate Kinsella's Key Routines in every module and grade level ensure multilingual learners feel comfortable in their classroom environment. The Turn & Talk activity included in every lesson fosters a supportive learning environment. Students participate in meaningful conversations with a partner, and their thinking becomes visible to the teacher and classmates. Point-of-Use scaffolds are embedded in every lesson, and targeted intervention supports students who need additional help in mastering key concepts. *HMH Classcraft* also includes embedded common error, multilingual learner, and universal design for learning support.



STUDENT CONTENT

Turn & Talk How is a repeating pattern different from a growing pattern?

Possible answer: A repeating pattern repeats the same sequence of numbers over and over, while a growing pattern follows a given rule to get from one number to another.

TEACHER GUIDE

FINALLY, have students use the Turn & Talk routine. Encourage students to compare the work they completed in Task 1 to the work they completed in Task 2. Invite students to discuss how both repeating and growing patterns involve repetition.

TEACHING STRATEGY

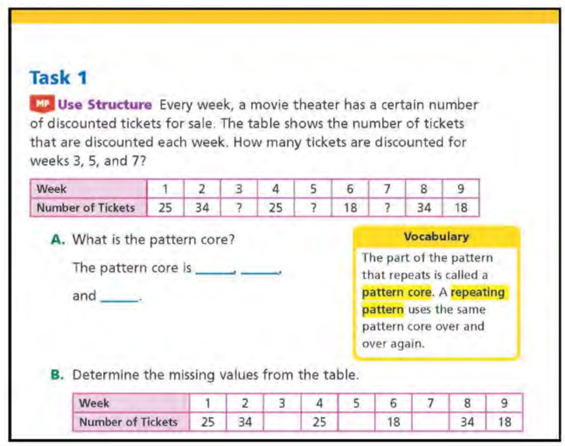
UDL Support: Representation

Consider providing students with an anchor chart that represents a repeating pattern and a growing pattern. Use the chart to help students make connections and determine the similarities and differences between the two types of patterns.

Almost There Small-Group Activities, tabletop flipchart mini-lessons, are perfect for co-teachers or resource math teachers. Detailed instructions, suggested questions, and additional materials support small-group instruction. The lessons enhance problem-solving and math talk.

Integrated Scaffolding

Language and math are integrated in *Into Math* Version 2. Sentence frames, sentence starters, and word backs are included in each lesson.



Task 1

MP Use Structure Every week, a movie theater has a certain number of discounted tickets for sale. The table shows the number of tickets that are discounted each week. How many tickets are discounted for weeks 3, 5, and 7?

Week	1	2	3	4	5	6	7	8	9
Number of Tickets	25	34	?	25	?	18	?	34	18

A. What is the pattern core?

The pattern core is ____, ____, and ____.

Vocabulary

The part of the pattern that repeats is called a pattern core. A repeating pattern uses the same pattern core over and over again.

B. Determine the missing values from the table.

Week	1	2	3	4	5	6	7	8	9
Number of Tickets	25	34		25		18		34	18

Additionally, the new Language Development Resource Guide is designed to enhance the existing program with practical, classroom-ready support for both teachers and MLs. It includes tools that foster academic language development, customizable vocabulary card templates to expand on existing materials, and practical guidance for implementing language-building routines.

Multilingual Support: Dual Content and Language Development

English language learners comprise approximately 5 million or 10% of the nation's public-school students (National Center for Education Statistics, 2024). These multilingual students across the United States come from wide-ranging backgrounds and have many cognitive, social, physical, and experiential differences. Each one brings unique assets, resources, and potential that educators must leverage to increase access, agency, and achievement for all (WIDA, 2020).

Researchers increasingly reject deficit-based models of English instruction that define MLs by their lack of English proficiency rather than their linguistic assets (August & Shanahan, 2007; Cummins, 2021; García, 2009; Hamman-Ortiz & Prasad, 2025). Evidence supports dual-language instruction, which strengthens foundational skills in students' heritage language (L1) and facilitates transfer to English (L2) (Baker et al., 2016; Escamilla et al., 2014; Lindholm-Leary & Block, 2010). This model builds on Cummins's (1991) Common Underlying Proficiency and Krashen's (1996) theory of comprehensible input, both emphasizing the cognitive and linguistic benefits of bilingualism.

Translanguaging, an extension of dual-language theory and assets-based approach, empowers MLs to use their full linguistic repertoires to construct meaning and engage with academic content (García, 2009; Li, 2022; Wei, 2018). By fluidly shifting between L1 and L2, students make cross-linguistic and home-to-school connections that deepen understanding and participation (Creese & Blackledge, 2015; Cummins, 2021; Hamman-Ortiz et al., 2025; Seltzer, 2019; Vallejo & Dooly, 2020).

Effective instruction for MLs combines grade-level content learning with dedicated English development. Within both contexts, flexible grouping and intentional collaboration are essential for providing targeted support and peer interaction while enabling translanguaging (Kinsella, 2016). Structured opportunities for communication among students of varied proficiency levels promote academic discourse, confidence, and comprehension (August & Shanahan, 2007; Foster & Ohta, 2005; Saunders & Goldenberg, 2010). Supporting MLs in mathematics is especially critical because math learning increasingly depends on linguistic comprehension as problem-solving becomes more contextualized and language-rich (Janzen, 2008). These demands often hinder MLs' understanding and assessment performance (Alt et al., 2014; Arizmendi et al., 2021). Dual-language and translanguaging approaches have been shown to improve MLs' math outcomes, literacy skills, and confidence while fostering mathematical identity and executive functioning (Bibler, 2021; Castellón et al., 2024; Morita-Mullaney et al., 2021).

Explicit vocabulary instruction focused on high-utility and domain-specific words that support conceptual understanding and academic discourse is vital (Kinsella, 2013). Consistent instructional routines further enhance learning by reducing cognitive load and supporting both language and content acquisition (Feldman & Kinsella, 2008; Saunders & Goldenberg, 2010; Zwiers et al., 2017). Structured reasoning routines allow MLs to articulate and refine ideas while receiving embedded language instruction in vocabulary, syntax, and grammar (Dutro & Kinsella, 2010; Lucenta & Kelemanik, 2020; Zwiers, 2014).

How *Into Math* Delivers

HMH knows that a student's ability to communicate a mathematical idea is closely related to the student's mastery of mathematical skills and concepts. *Into Math* is carefully designed to provide teachers with opportunities to help students develop their command of mathematical language and content simultaneously. All students benefit from the program's language routines and discourse activities. Peer Coach videos are available to model how routines are effectively carried out in classrooms.

Kate Kinsella's Key Routines

Dr. Kate Kinsella, author of HMH's *English 3D*, brings her powerful key routines to *Into Math*. These routines appear at the beginning of each module to support students in their development of academic language and set them up for success in their module work and in their classroom conversations and interactions.

Observable, Measurable Language Objectives

Each *Into Math* lesson includes Language Objectives that teachers can use to observe and measure the academic language growth of students.

Teaching Strategies That Support Mathematical Communication

The Teacher's Guide includes specific Teaching Strategies that help you support your students as they work through each lesson's Language Objective. Each Teaching Strategy provides tools that you can use to help your students demonstrate the skills described in the Language Objective. Many strategies offer different levels of scaffolding, allowing students to choose the level of scaffolding they feel is most appropriate for them.

Words to Learn Routine and Vocabulary Cards

The Words to Learn (WTL) instructional routine, which can be found in *HMH Classcraft*, is designed to engage students in guided, interactive word study, aid student comprehension of key academic vocabulary, and help students improve their understanding of high-utility words. The WTL routine works in tandem with the Vocabulary Cards, which are arranged by module and can be found toward the back of each Student Edition. The Vocabulary Cards complement the WTL routine, but they can also be incorporated into classroom activities, such as vocabulary games or word walls. Key vocabulary is also available at the start of each lesson and can be taught at any point during instruction.

Evidence-Based Mathematics Assessment

"Assessment really is the bridge between teaching and learning" (Wiliam, 2013, p. 15). High-quality and highly evolved assessment at the classroom level is essential to accurately determine how all students are progressing across a range of domains and progressions over time, as well as what students need currently and going forward (Fonger et al., 2018; Jimenez & Modaffari, 2021; NCTM, 2014, 2016; Thanheiser & Melhuish, 2023). Effective mathematics programs feature assessment as an integral part of instruction; these embedded assessments provide educators with evidence of proficiency in important mathematics content and practices, leverage a range of strategies and data sources to guide instructional decisions at the individual student level, and generate useful feedback to students (NCTM, 2014). Research has firmly established that well-designed assessment conducted regularly and used by teachers to alter and improve instruction has a tremendous positive impact on students' mathematics learning and achievement (Black & Wiliam, 1998a, 1998b; Hattie et al., 2017; NRC, 2005; Wiliam & Thompson, 2017).

Data-driven instructional decision-making is the systematic collection, analysis, and application of many forms of data from multiple sources to identify students' strengths and weaknesses regarding learning objectives. Subsequently, it addresses student learning needs and optimizes performance in future instruction. Rigorous, ongoing assessment that yields meaningful data is a fundamental component with an effective data-driven decision-making system. Research indicates that when it is implemented well, data-driven instruction has the potential to dramatically improve student achievement (Bambrick-Santoyo, 2014; Dunn et al., 2013; Marsh et al., 2006; Schifter et al., 2014). Effective assessment allows teachers to collect data about what is working and what is not so that they can take precise, swift, and effective action to better serve students (Hattie et al., 2017; Wiliam, 2013). Just as we want students to be agents in their own learning, they must be active participants in their own assessment (Brookhart, 2008; Frey et al., 2018; Hattie & Clarke, 2018; Heritage, 2013). As Fisher and Frey (2021) advise:

> Teachers should be adjusting their lessons in real time as they collect and analyze the data that they get from students, whether that be during focused, guided, collaborative, or independent learning. Assessment is the engine that drives instructional decisions; it's what allows teachers to know if we are having an impact. When we are not achieving the desired impact—learning—we have to change course and try something else (p. viii–ix).

HMH Into Math provides ongoing, balanced assessment and reporting that additionally utilizes digital technologies to empower teachers with data-driven decision-making and tools for effective instructional planning. The program also provides grouping and resource recommendations. This solution yields critical feedback loops that encourage students' self-assessment and reflection while freeing teachers from guesswork and time-consuming assessment reporting and subsequent material selections and planning. These approaches to evaluation of learning support optimal instructional practices and drive positive outcomes for every student.

Formative Assessment

Assessment for Learning (AfL) is assessment that is formative: an ongoing process in which assessment is used to inform and improve teaching and learning, rather than to measure student achievement (Wiliam, 2018). Both well-established and continually emerging research and evidence on formative assessment illustrates its integral role in improving student learning on a timeline that yields targeted outcomes. “Formative assessment is a process teachers and students use during instruction that provides feedback to adjust ongoing teaching moves and learning tactics. It is not a tool or an event, nor a bank of test items or performance tasks. Well-supported by research evidence, it improves students’ learning in time to achieve intended instructional outcomes” (Linquanti, 2014, p. 2).

Formative assessment is a dynamic process that involves gathering evidence of student thinking through formal (e.g., quizzes, assignments) and informal (e.g., discussion, observation) methods; it serves as the bridge between teaching and learning, ensuring that instruction continually adapts to student needs (Greenberg & Walsh, 2012; Hattie & Clarke, 2018; Heritage, 2013; Jimenez & Modaffari, 2021; Wiliam, 2018). Effective formative assessment provides immediate, meaningful, and actionable feedback and actively involves students in monitoring their progress (Brookhart, 2008; Frey et al., 2018; Hattie & Clarke, 2018; NASEM, 2018; NRC, 2012; Stecker et al., 2005; Wiggins, 2012).

Meaningful learning depends on the brain’s capacity to revise internal models through feedback, which enables students to refine understanding and become active learners (Dehaene, 2020). Via effective feedback, students recognize and correct errors, strengthen metacognitive skills, and stay motivated (Agarwal & Bain, 2019; Brookhart, 2008; Fisher & Frey, 2007, 2021; Hattie, 2009, 2023).

Diagnostic assessments identify students’ prior knowledge and skill levels, enabling them to tailor instruction accordingly. Screening tools help pinpoint students needing additional support (Ketterlin-Geller & Yovanoff, 2009; Schneider et al., 2013). Ongoing assessment ensures that teaching is responsive to students' individual progress and learning goals (Carpenter et al., 2015; Fonger et al., 2018; Thanheiser & Melhuish, 2023).

Effective formative assessment is crucial for closing achievement gaps through tailored instruction (Black & Wiliam, 1998a, 1998b; Heritage, 2013; Jimenez & Modaffari, 2021). Research supports data-driven mathematics instruction, showing that effective formative assessment prevents difficulties and improves learning outcomes for individual students and broadly (Baker et al., 2002; Clarke & Shinn, 2004; Hattie et al., 2017; Klute et al., 2017; Lee et al., 2020; Stecker et al., 2005; Wiliam, 2018).

Maximizing formative assessment benefits requires fostering student agency by making learning goals, success criteria, and feedback explicit, empowering students in self-assessment (Frey et al., 2018; Hattie & Clarke, 2018; Heritage, 2013). Encouraging metacognitive self-reflection enhances achievement (Bond & Ellis, 2013; Desoete & De Craene, 2019; Lee et al., 2020; Schneider & Artelt, 2010).

How *Into Math* Delivers

Into Math equips educators with a flexible, comprehensive, and accommodative formative assessment toolbox for monitoring student progress through fair and accurate measurement to continually guide and inform instruction.

Effective Formative Assessment That Helps Teachers Clarify, Elicit, Interpret, and Act

Into Math provides a variety of print and digital assessment options, allowing teachers to check student understanding and use both formative and summative data to make instructional decisions. These include module-level prerequisite checks, lesson-level quick checks, module review, module assessments, module projects, and beginning-, mid-, and end-of-year assessments as *MAP Growth*.

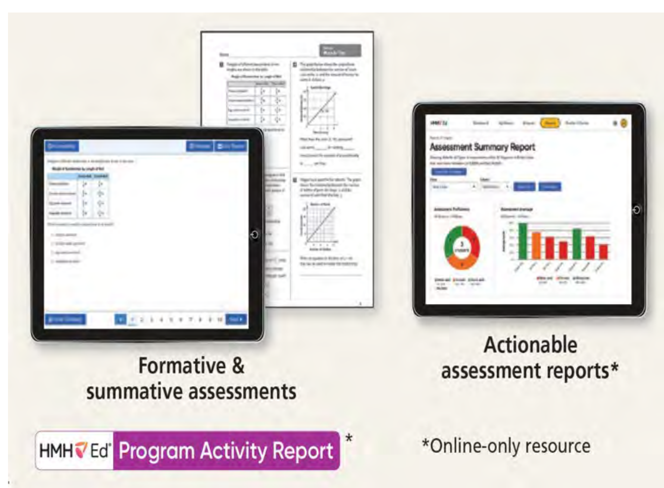
Addressing Misconceptions to Support Accurate, Long-Term Learning

Formative assessment occurs before students practice, so teachers can spend time with small groups correcting misconceptions before they "stick."

Students practice in ways that best support their learning, whether independently, in teacher-led small groups, or in collaborative groups. Teachers also have the choice of lesson practice, leveled practice, or ongoing adaptive practice with every lesson.

Formative Assessment That Yields Data Insights to Inform Instruction and Practice

HMH Classcraft offers embedded, ongoing Quick Checks that, along with Program Assessments, provide data insights to inform instruction.



Summative Assessment

“Assessments should drive excellent teaching and ensure that all students learn at high levels. To do so, education policy and practice must encompass a broader range of assessments so that schools have complete and effective assessment systems. This system would include predictive, informative, and evaluative assessments based on the state’s standards and curriculum” (Jimenez & Modaffari, 2021, p. 18).

Effective K–8 mathematics instruction relies on a comprehensive, data-driven assessment system designed to predict student performance, inform instructional decisions, and evaluate learning (Greenberg & Walsh, 2012; Jimenez & Modaffari, 2021; Mandinach et al., 2006). Assessments—whether predictive, formative, or evaluative—must consistently support high-quality instruction that leads to improved student outcomes (Darling-Hammond, 2010b; Hattie, 2009, 2023; Wiliam, 2013). Key aspects of effective mathematics assessment practices include a focus on evidence that identifies indicators of students' mathematical thinking along learning progressions that show how their mathematical thinking develops over time (Fonger et al., 2018; NCTM, 2014, 2016; Thanheiser & Melhuish, 2023).

Summative assessment measures overall student achievement at a specific point in time, distinguishing it from formative and diagnostic assessments. It serves as a tool for evaluating student understanding, shaping instruction, and personalizing learning. Summative assessments also function as accountability measures, aligning student performance with standards. It includes classroom-based assessments, interim or benchmark tests, and large-scale standardized exams.

Research suggests that well-designed classroom summative assessments can positively impact learning (Harlen, 2005; Moss, 2013; NCTM, 2016). While often linked to high-stakes testing, summative assessments in the classroom can provide valuable insights to inform future instruction (Black & Wiliam, 1998a, 1998b; NCTM, 2016). NCTM (2016) cautions against using large-scale assessments as the sole measure for high-stakes decisions regarding schools, teachers, or students. Instead, a balanced approach incorporating both formative and summative assessments is recommended, alongside systemic support for high-quality mathematics instruction.

Research supports the continued use of these assessments despite the pressures of high-stakes testing, as they offer meaningful measures of student competency and promote effective classroom instruction (Schneider et al., 2013). However, it is critical that assessment should not be merely imposed on students but should serve as a tool to enhance their learning (NCTM, 2014).

How *Into Math* Delivers

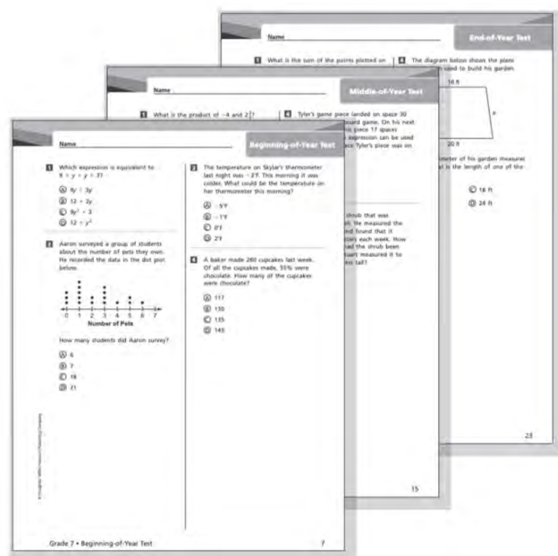
Effective Summative Assessment That Helps Educators Monitor Growth Over Time

In addition to a wide range of flexible formative assessments available in print and digital formats, *Into Math* provides summative measures and data reporting to allow teachers and administrators to track students' progress and growth over time. The program also includes professional development supports to aid teachers in optimal administration and usage of summative assessment.

Educators can also use *HMH Ed* reports to determine students' proficiency levels and learning needs based on program assessments.

Optimizing Interim Assessment

Into Math provides a range of interim assessments with reporting that can be administered throughout the school year. These include *MAP Growth* (an optional Connected Resource); the Beginning-of-Year Test, the Middle-of-Year Test, and the End-of-Year Test; and assessment(s) supplied by districts. Each functions distinctly and in conjunction to provide specific data, measure student progress, and utilize results in support of varied learners. These interim assessments should be used at strategic points over the school year to identify and track each student's academic growth and achievements.



Assessment Strategies for Accessibility and Language Development

Accessible assessment practices that best support every learner, including those with disabilities, neurodivergent individuals, and MLs, take an assets-based view that anticipates and accommodates differences in how each learner approaches content and demonstrates thinking and understanding. Accessible assessment is AfL that is characterized by ongoing feedback to inform teaching, learning, and further assessment; support for language variability; and flexible opportunities for students to show what they know through multiple modalities (Lambert, 2024). Accessible assessment practices, then, should offer multiple means of engagement, expression, and representation, consistent with UDL principles (CAST, 2018).

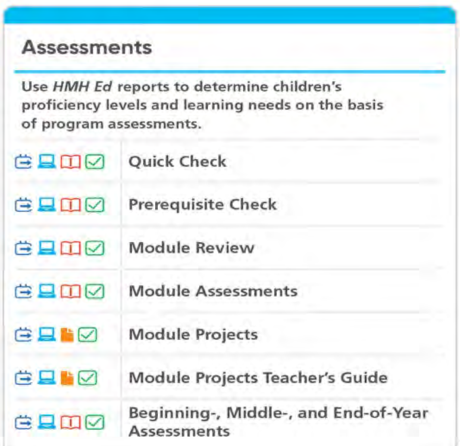
Friesen (2016) reports a series of compelling findings demonstrating the transformative potential of integrating AfL and UDL within mathematics instruction. The study revealed significant improvements in student achievement: every participant demonstrated measurable gains on the assessments used, underscoring the effectiveness of the design-based intervention in promoting mathematical learning. Gains extended beyond procedural fluency; students showed growth across all five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Friesen further found that when students are supported through such adaptable, flexible formative assessment practice guided by individual skills and needs, they engaged successfully with complex and challenging material. The study additionally concluded that technological tools, particularly those that supported visualization, exploration, and dynamic manipulation of mathematical ideas, were essential for enabling all students to access and represent mathematical concepts in multiple ways.

For MLs, traditional assessments often fail to capture their full capabilities, as such measurement tools are designed for monolingual students. Instead, assessment should be embedded and formative, with accommodations made to allow MLs to leverage their full linguistic repertoires and to yield useful feedback (De Backer et al., 2016). Diagnostic, formative, and summative assessment for MLs should focus on student strengths (content understanding and linguistic assets) and areas for growth (skills needing improvement) (Lambert, 2024; Lopez, 2023). When possible, students should be allowed to receive assessment instruction and demonstrate understanding in their native language (Abedi & Hermann, 2010; Lopez, 2023). Mathematical understanding can be effectively communicated through speech, writing, drawings, and models (Lambert, 2024), consistent with UDL principles (CAST, 2018).

How *Into Math* Delivers

Into Math includes a wide range of print and digital assessment options in multiple flexible formats to effectively meet the needs of all learners and provide optimal personalized instruction.

Varied and Flexible Assessment Options


Assessments

Use *HMH Ed* reports to determine children's proficiency levels and learning needs on the basis of program assessments.

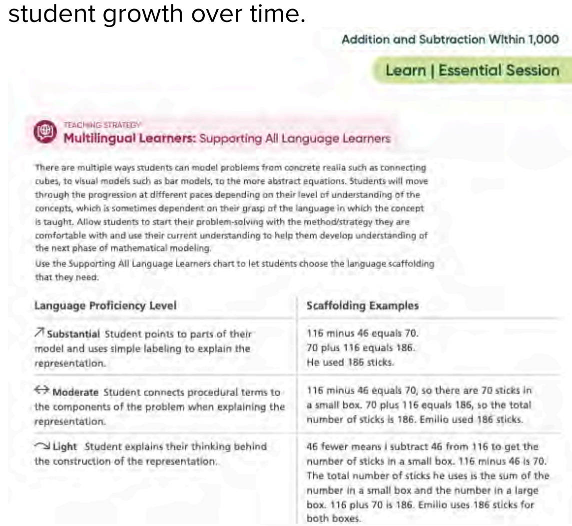
	Quick Check
	Prerequisite Check
	Module Review
	Module Assessments
	Module Projects
	Module Projects Teacher's Guide
	Beginning-, Middle-, and End-of-Year Assessments

Data-Driven Prerequisite Checks Foster Student Success

Each module's Prerequisite Check assesses the prerequisite concepts required to succeed with new learning. This provides educators with data that highlight students' strength and areas for improvement. This data guides just-in-time differentiated instruction before module learning begins.

Ongoing, Formative, Flexible Assessment for MLs That Aligns With Best Practices

Into Math offers multiple means for assessing and tracking students' language development and progress. In the Quick Check for each lesson, the Teacher's Guide describes a formative assessment that can be used to check students' mastery of the lesson's Language Objective. Teachers can also use the language routines and Turn & Talks of each lesson to observe and chart the growth of each student's academic language. Additionally, the relevant observational rubric found in the Language Development Resource Guide provides a ready means for documenting students' academic language growth. This documentation can help teachers, students, and students' families track student growth over time.


Addition and Subtraction Within 1,000

Learn | Essential Session

TEACHING STRATEGY

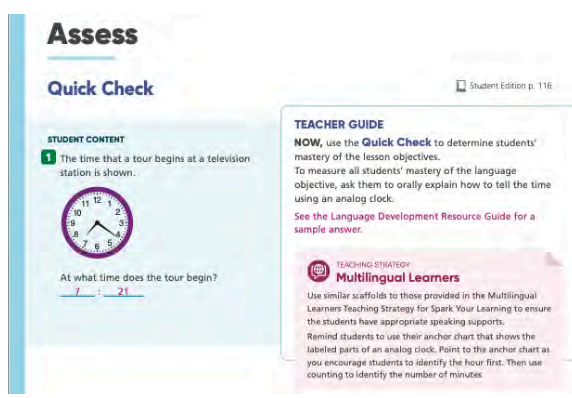
Multilingual Learners: Supporting All Language Learners

There are multiple ways students can model problems from concrete realia such as connecting cubes, to visual models such as bar models, to the more abstract equations. Students will move through the progression at different paces depending on their level of understanding of the concepts, which is sometimes dependent on their grasp of the language in which the concept is taught. Allow students to start their problem-solving with the method/strategy they are comfortable with and use their current understanding to help them develop understanding of the next phase of mathematical modeling.

Use the Supporting All Language Learners chart to let students choose the language scaffolding that they need.

Language Proficiency Level	Scaffolding Examples
Substantial Student points to parts of their model and uses simple labeling to explain the representation.	116 minus 46 equals 70. 70 plus 116 equals 186. He used 186 sticks.
Moderate Student connects procedural terms to the components of the problem when explaining the representation.	116 minus 46 equals 70, so there are 70 sticks in a small box. 70 plus 116 equals 186, so the total number of sticks is 186. Emilio used 186 sticks.
Light Student explains their thinking behind the construction of the representation.	46 fewer means I subtract 46 from 116 to get the number of sticks in a small box. 116 minus 46 is 70. The total number of sticks he uses is the sum of the number in a small box and the number in a large box. 116 plus 70 is 186. Emilio uses 186 sticks for both boxes.

All these learning activities provide ongoing opportunities for effective, evidence-based formative assessment that can be used with confidence to guide instructional decision-making and monitor language proficiency development levels as well as provide appropriate scaffolding accordingly (Substantial, Moderate, or Light) for each student. The program also includes ongoing support for educators in how to implement these assessment strategies optimally.



Using Evidence to Inform Learning, Grouping, and Differentiation

Data-driven instructional decision-making involves systematically collecting and analyzing qualitative and quantitative data from multiple sources to identify students' strengths and challenges, inform instruction, and improve achievement (Bambrick-Santoyo, 2014; Dunn et al., 2013; Mandinach & Schildkamp, 2021; Schifter et al., 2014). Continuous, data-driven assessment is essential for responsive and differentiated mathematics instruction. Drawing on assessment data gathered regularly from varied, standards-aligned sources helps teachers tailor instruction and provide individualized support, forming the foundation of effective teaching and learning (Frey et al., 2018; Hattie, 2009, 2012, 2023; Jimenez & Modaffari, 2021; Wiliam, 2013).

Globally high-performing education systems emphasize formative, flexible, and responsive assessments. Effective data-driven instruction relies on structured use of a wide range of assessment types—both qualitative and quantitative—to optimize learning and ensure broad progress toward shared goals (Darling-Hammond, 2010b; Greenberg & Walsh, 2012; Wiliam & Thompson, 2017). These assessments are most effective when used formatively, allowing teachers to adapt instruction in real time based on emerging patterns in student learning (Hoogland et al., 2016).

Digital learning environments expand the capacity for data-driven, formative assessment by enabling real-time progress monitoring, personalized feedback, and continuous adaptation of instruction (Anthony, 2019; Curtis & Werth, 2015; Hamilton, 2018; Hattie & Clarke, 2018; Johnson et al., 2023; Molin et al., 2020; O'Byrne & Pytash, 2015; Pulham & Graham, 2018). Technology-supported formative systems can collect data from multiple sources, generate immediate feedback, and provide teachers with actionable insights to target specific learning goals and differentiate instruction effectively.

Empirical studies demonstrate that digital tools designed to personalize mathematics instruction, practice, and formative assessment yield positive outcomes across varying achievement levels. These tools enhance students' engagement and motivation by offering individualized feedback, supporting self-regulation, and improving alignment between instructional tasks and learning needs (Alhadi et al., 2023; Faber et al., 2017; Haelermans & Ghysels, 2013). When integrated purposefully, technology enhances the quality and precision of formative feedback while supporting data-informed instructional decisions.

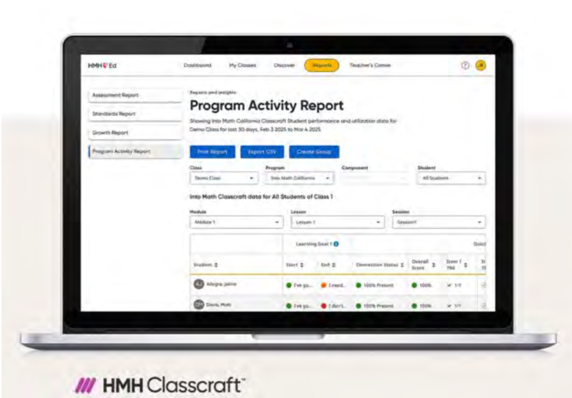
Overall, evidence consistently links well-implemented data-driven instruction with improved student outcomes (Bambrick-Santoyo, 2014; Dunn et al., 2013; Hamilton et al., 2009; Mandinach & Schildkamp, 2021; Marsh et al., 2006; Schifter et al., 2014). Successful systems integrate continuous assessment, professional learning, and technology to ensure that data are used not merely for accountability but as a catalyst for instructional improvement and student growth at-scale. Advanced digital tools, when employed thoughtfully, strengthen educators' capacity to interpret and act on data to enhance both teaching and learning (Mandinach & Schildkamp, 2021).

How *Into Math* Delivers

Into Math was designed to support differentiation that most effectively meets the evolving needs of every student. The program includes abundant practice as well as three Small-Group Activities for each lesson. Teachers can choose the best practice and/or activity according to their classroom observations and the formative data collected during whole-class instruction.

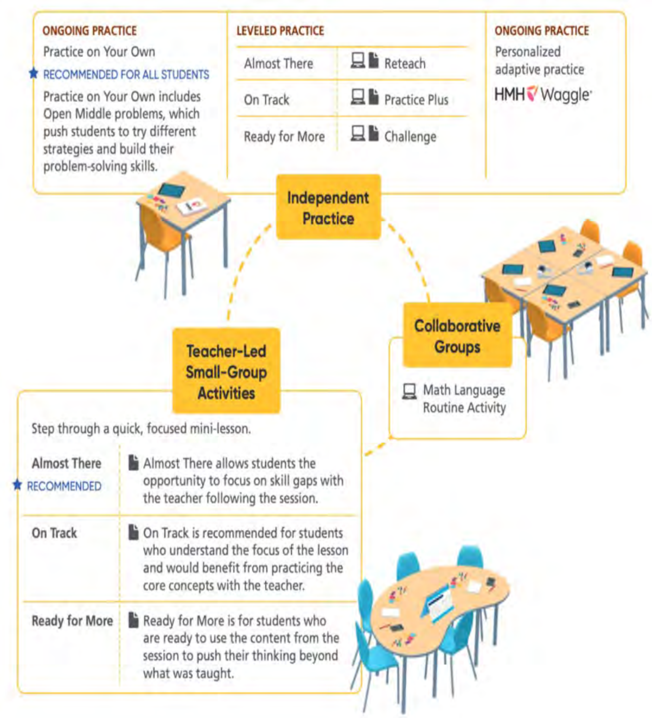
Rely on Real-Time Data

Teachers use real-time formative assessment data from the *HMH Classcraft* Quick Check on learning goals to make instructional decisions and group students on the spot.



Group and Assign Practice for Optimal Differentiated Learning

Educators are empowered to make data-driven decisions about which students will benefit most from the teacher-led Small-Group Activities and can work with additional groups as time allows. For students not currently working in groups, teachers choose the best type of practice for them provided in *HMH Waggle*.



Strategic Technology Use to Improve Mathematics Learning

"An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014, p. 78). Technology plays a critical role in STEM education, enhancing learning experiences through dynamic representations, simulations, interactive models, communicative tools, and real-time feedback. Digital platforms also facilitate formative and summative assessments, supporting interdisciplinary and project-based learning (U.S. Department of Education [ED], 2019).

Strategic technology integration, as endorsed by the National Council of Teachers of Mathematics (NCTM, 2015b), enhances mathematics instruction when used purposefully rather than continuously. Teachers should prioritize mathematical learning objectives while leveraging technology's capabilities (Dick & Hollebrands, 2011; NCTM, 2015b). Effective technology use should avoid an approach where the focus is on technology itself rather than its pedagogical value (Fisher et al., 2020; Hamilton, 2018; Zinger et al., 2017). Two primary types of educational technologies should be used strategically to develop students' mathematical reasoning: content-specific tools that support conceptual exploration and content-neutral tools that aid problem-solving and communication (NCTM, 2015b).

Advances in educational technology provide tremendous opportunities to expand the possibility of growth for all students in the form of enhanced, efficacious, and engaging content; agency regarding pace, environment, interest; more frequent and timely feedback; and increased access to high-quality educational programs (Curtis & Werth, 2015; ED, 2017; Johnson et al., 2023; O'Byrne & Pytash, 2015). This will better prepare students to organize and direct their learning in their lives even after formal schooling (ED, 2017). The increasingly utilized blended or hybrid learning environment in particular can offer a more personalized, student-centered and active pedagogy that allows for self-regulated, self-paced learning pathways with increased agency and improved feedback loops, helping each child learn more deeply as well as feel and be successful at school (Agosto et al., 2013; Dikkers, 2018; Hamilton, 2018; Hattie & Clarke, 2018; Johnson et al., 2023; Molin et al., 2020; Moore et al., 2017). Ultimately, digital tools help educators "take advantage of the things that leading brick-and-mortar schools do well, such as creating a strong, supportive culture that promotes rigor and high expectations for all students, as well as providing healthy, supportive relationships and mentorship" (Horn & Staker, 2011, p. 7).

HMH Into Math harnesses technology to provide dynamic, interactive, adaptive, and personalized instruction along with evidence-based practice and assessment solutions addressing individual students' ongoing needs and engaging their interest in math learning.

Developing Meaningful, Effective Digital Learning Environments

Research highlights the benefits of blended learning, which increases student engagement and motivation while providing personalized instruction (Anthony, 2019; Halverson & Graham, 2019; Kwon et al., 2019; Moore et al., 2017; O'Byrne & Paytash, 2015; Patrick & Sturgis, 2015). Blended learning fosters student success by offering flexibility, autonomy, and timely feedback that is beneficial to all students (Curtis & Werth, 2015; Johnson et al., 2023; SRI International, 2018; ED, 2017).

Digital learning tools promote engagement by integrating multimedia, interactivity, and student agency. Studies show that students learn better when verbal instruction is supplemented with visual elements (Johnson et al., 2023; Mayer, 2013, 2017). Multimedia and digital tools create engaging environments, increasing motivation and persistence in challenging tasks (Chen et al., 2018; Taylor & Parsons, 2011). Within STEM education, technology-enhanced simulations, models, and representations become dynamic, interactive, and more engaging and impactful (ED, 2019). Digital learning platforms can also reduce cognitive load, freeing up students' capacity to acquire new information and build frameworks for conceptual understanding (Mayer, 2017; Sweller et al., 2019). When instructional technologies adapt content and feedback based on an individual student's pace and performance, and when both are matched to each student's current level of understanding, learners are not exposed to material that is too simple or complex, making information processing resources more available for short- and long-term learning (Shute & Zapata-Rivera, 2012).

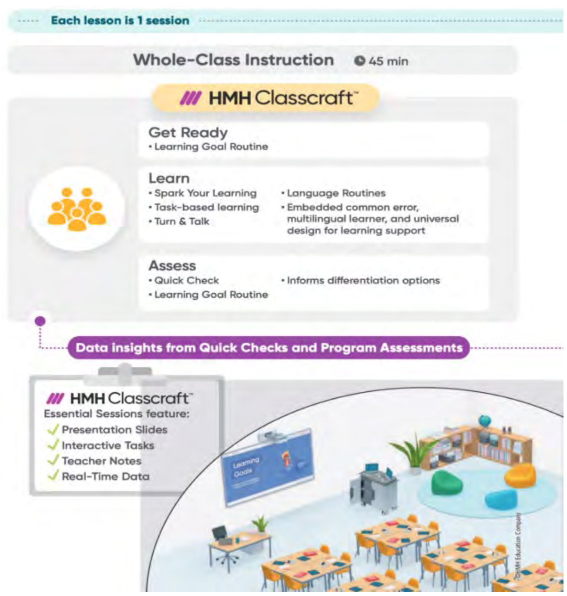
Student autonomy in digital learning, which includes making meaningful choices around content and pacing, has been linked to improved motivation and resilience (Ryan & Deci, 2000; NASEM, 2018). Online collaboration tools can strengthen classroom communities and communication skills (SRI International, 2018; Wolfe & Poon, 2015). Further, by providing a wide array of online resources, technology supports learning drawn from real-world challenges and students' personal interests and passions, aiding the organization of a project-based curriculum, all of which are especially salient and effective within STEM instruction (ED, 2017, 2019). Advanced technology that increases adaptive learning mechanisms and provides guidance to support instruction can offer the functionality needed for optimal responsiveness within a formative feedback system (Gerard et al., 2015; Hattie & Clarke, 2018). Increased student agency and adaptivity combined with more timely and effective formative feedback afforded within digital environments enhances self-regulation and metacognition (Johnson et al., 2023; Molin et al., 2020; Moore et al., 2017). Metacognitive prompts embedded within adaptive feedback systems can strengthen learning monitoring. Ultimately, technology helps students "become their own teachers," developing self-evaluative and self-directed qualities that are essential for deep learning (Hattie, 2009, p. 22).

For educators, technology simplifies tasks such as data management and assessment scoring, freeing time for personalized instruction and small-group learning (Moore et al., 2017; Wolfe & Poon, 2015). Automated scoring tools aid teachers in evaluating student reasoning and providing adaptive guidance (Gerard et al., 2015).

How *Into Math* Delivers

Technology-Enhanced, Optimized Math Learning Experiences

Into Math classroom environments are enriched and empowered with *HMH Classcraft*. This digital learning solution increases students' engagement and interactivity with math content while providing teachers with a broad range of effective instructional planning and classroom-management tools.



Additionally, *HMH Classcraft* yields real-time data from ongoing, varied, and embedded formative assessment measures to guide differentiated instruction at optimal challenge levels, ensuring all students meet rigorous mathematics standards.

Interactive Experiences and Tools to Enrich the Learning of Math Content and Practices

Effectively integrating digital tools into mathematics classrooms has the potency to transform learning into interactive, engaging, and empowering experiences and to improve outcomes for all students. Digital tools offer opportunities to personalize instruction, increase motivation, and expand access to high-quality education (Curtis & Werth, 2015; ED, 2017; Johnson et al., 2023; SRI International, 2018). Research consistently shows that blended learning—combining digital and traditional instruction—enhances engagement, motivation, and academic performance (Anthony, 2019; Halverson & Graham, 2019; Moore et al., 2017). When used purposefully, technology supports student-centered learning environments that are active, collaborative, and personalized (Attard & Holmes, 2022; Chen et al., 2018; Fisher et al., 2020; Horn & Staker, 2011; Johnson et al., 2023; Taylor & Parsons, 2011).

Interactivity lies at the core of effective digital learning. Technologies that provide real-time feedback, adaptive pathways, and opportunities for student choice enhance learners' sense of agency and autonomy, which in turn fuels motivation and persistence (Chen et al., 2018; O'Byrne & Pytash, 2015; Ryan & Deci, 2000). Effective self-paced learning in math blends student autonomy with structured guidance, using varied task difficulty and scaffolding to support self-regulation. This approach has been shown to enhance achievement, motivation, accuracy, effort, and independence (Billington et al., 2004; Kwon et al., 2019). Additional research affirms that such approaches benefit learners from varied backgrounds, including high-ability students (Balentyne & Varga, 2016; SRI International, 2018). These study results are illustrative. For example, Bang et al. (2023) found that an adaptive, game-based math app provided personalized instruction and significantly improved outcomes for young learners. Similarly, Barana et al. (2021) showed that interactive feedback systems promote conceptual understanding and encourage self-correction, helping students engage productively with errors.

Multimedia technologies further enrich mathematical understanding by presenting multiple representations of concepts through interactive visuals, animations, and simulations (Mayer, 2013, 2017; Johnson et al., 2023). Such dynamic elements, including game-like challenges and exploratory tasks, foster curiosity and sustained engagement (Moon & Ke, 2019; Taub et al., 2020). Dyrvold and Bergvall (2023) observed that interactive, dynamic content promotes deeper exploration than static materials, supporting persistence and resilience in problem-solving (NASEM, 2018; Rezat, 2021).

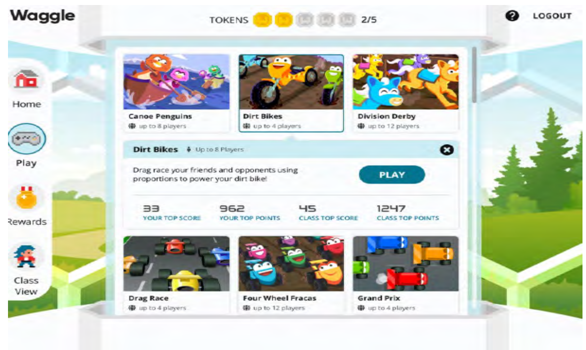
Technology also strengthens the social and cognitive dimensions of mathematics learning. When integrated thoughtfully, digital tools complement teachers and parents in nurturing students' intellectual and creative growth (Masnawati et al., 2022). Interactive technologies facilitate collaboration, communication, and reflection, which serve as key components of effective mathematical practice.

Additionally, technological tools improve formative assessment by enabling timely feedback, progress monitoring, and individualized instruction (Anthony, 2019; Hamilton, 2018; Johnson et al., 2023; Lee et al., 2020). Adaptive mathematics learning platforms offer personalized feedback that benefits all students, including those with special needs (Alhadi et al., 2023; Faber et al., 2017; Haelermans & Ghysels, 2013). When instructional goals are explicit, digital assessments help students demonstrate and deepen their understanding (Hattie, 2023; Hamilton, 2018). Such interactive and responsive technology-enhanced transformations allow students to be active participants in their mathematical growth rather than passive recipients of information (Kallio & Halverson, 2020; Wolfe & Poon, 2015).

How *Into Math* Delivers

Personalized adaptive learning that propels students toward success

Into Math provides personalized, adaptive instruction and practice supported by *HMH Waggle*.



Teachers can assign content directly aligned to daily instruction, while auto-assignment ensures students receive targeted practice. Powered by a skill-based AI engine, the adaptive practice adjusts in real time to meet each student's needs and provides powerful data insights to inform instruction.

Professional Learning

To support the delivery of effective instruction, *Into Math* features research-based approaches to professional learning that prepares teachers to facilitate high-impact learning experiences for their students. Comprehensive professional learning solutions are data and evidence driven. They are mapped to instructional goals, are centered on students, and build educators' collective capacity. HMH enables teachers to achieve agency in their professional growth through effective instructional strategies, embedded teacher support, and ongoing professional learning relevant to everyday teaching. In addition, school leaders can help support teachers' *Into Math* implementation through leader-facing reports, including reviewing teachers' participation and completion of *Teacher Success Pathways* and their teacher-coach engagement.

In this chapter, we describe *Into Math*'s professional learning offerings and how they align to three key areas of research on effective professional learning, including connected and personalized professional learning, program-aligned professional learning for actionable insights, and coaching to strengthen teaching and learning.

Connected and Personalized Professional Learning

Effective curriculum-based professional learning consists of ongoing, active experiences that focus on improving the rigor and impact of instructional practices and ideally replicate the learner-centered approaches that teachers are expected to provide for their students. Elements of effective curriculum-based professional learning include high-quality educative curriculum materials, transformative learning experiences that shift teachers' attitudes, beliefs, and practices, and a prioritization of accessibility to ensure all students meet high expectations. Functional design elements include learning designs that model inquiry-based instruction, experiences to shift teachers' beliefs, opportunities for reflection and feedback, and change-management strategies that address individual concerns and group challenges. Finally, structural design features include collective participation in which teachers practice and reflect on the curriculum, models of learning that evolve from initial use to ongoing support to building capacity, and effective use of teachers' time. These elements of effective curriculum-based professional learning must exist in a system with strong leadership, adequate resources, and coherence toward common goals (Short & Hirsh, 2020).

How professional learning is delivered has an impact on its effectiveness. Professional learning programs with teacher-to-teacher collaboration focused on instructional improvement—whether in professional learning communities (PLCs), teacher teams, or group work in professional learning sessions—have demonstrated improvement in teachers' instructional skills. Another effective practice is conducting follow-up meetings or coaching sessions after the initial implementation of a program so that teachers can share their experiences and receive feedback. The content of professional learning is equally important and should focus on subject-specific instructional practices (not merely content knowledge), prioritize specific supportive materials over general principles, and help teachers build stronger relationships with students (Hill & Papay, 2022).

Long-term connected professional learning includes cohesive features, including online coaching, observations, and collaboration. These features focus on ensuring student well-being and meaningful student learning in digital environments. A connection between workshops, coaching, and collaboration is essential for a professional learning program to make a difference in student achievement. Connecting workshops, follow-up coaching, and support among peers can help teachers retain new knowledge, apply and refine new skills, and share effective and scalable approaches (Aguilar, 2019).

Effective professional learning, whether in-person, online, or blended, offers teachers coherent experiences so that their learning is connected to their work in the classroom and builds proficiency. This approach includes alignment between the study of theory and practice, observation of theory and practice, individual coaching, and further practice and refinement through collaboration. Each of these components is essential to support and builds on the content and pedagogy that is learned, observed, and practiced in each of the other components (Rock, 2019).

For schools to support the implementation of high-quality instructional materials, effective professional learning during the launch of the curriculum, when teachers are learning and committing to an instructional approach, is critical (Gulamhussein, 2013). Teachers' initial exposure to a concept should engage them through varied approaches and active learning strategies so they can make sense of the new practice (Bill & Melinda Gates Foundation, 2014;

Garet et al., 2001; Gulamhussein, 2013). An effective professional learning program should be curriculum-based; focused on targeted content, strategies, and practices (Bill & Melinda Gates Foundation; 2014; Saxe et al., 2001); and be grounded in the teacher's grade level or discipline (Gulamhussein, 2013).

Online professional learning can help solve resource challenges in implementing a scalable and sustainable model. Online professional learning platforms can create a peer-to-peer support community, building the capacity of the teaching team to support each other. Perhaps most importantly, online professional learning allows teachers to experience the agency and personalized learning they are creating for students. Blended professional learning offers the unique opportunity to shift from viewing learning as a one-time or periodic event to seeing it as an ongoing, embedded practice (Tucker & Wycoff, 2019).

Many school districts and providers of teachers' professional learning are moving toward a more personalized model of professional learning, taking a cue from the movement toward personalized learning for students. This approach often focuses on short modules that teachers can choose and then complete on their own time. The modules can incorporate aspects of gamification, micro-credentialing, and online PLCs. By allowing teachers to choose their own professional learning courses and activities, and by giving them a place to complete and review content at their own pace, the learning will be better matched to their needs. Teachers will be able to set goals, find resources to help them meet those goals, refresh their learning, track their progress, and get feedback from supervisors and colleagues (Gamrat et al., 2014; Meeuwse & Mason, 2018).

When provided with time and frameworks to collaborate on improving instruction through PLCs or Teacher Study Groups (TSGs), teachers can improve their knowledge, enhance their instructional practices, and see increased student achievement (Gersten et al., 2010).

How *Into Math* Delivers

Into Math provides a continuum of connected professional learning designed to foster teacher agency, promote collaboration, and build collective efficacy and capacity to support teachers' roles as both facilitators and implementors of high-quality instruction. Through strategic planning, guided implementation support, and blended coaching. HMH helps schools and districts achieve measurable gains with professional learning centered on research-based practices and student outcomes.

HMH Into Math Implementation Support

Guided Implementation Support for *Into Math* assists educators in building confidence and success with their *Into Math* program. A Getting Started session provides teachers with learner-centered foundational program knowledge through exploration and hands-on activities, enhancing their instructional skills and building confidence for successful implementation. Subsequently, a Follow-Up session focuses on using data and *Into Math* resources to differentiate instruction, empowering teachers to effectively meet the widely varying needs of their students and deepen their classroom impact. In addition, a *Leader Success* session is included to provide school building and district leaders with essential program details and resources to support successful implementation.

Ongoing training and support resources are also provided through their *Into Math Teacher Success Pathway* on *HMH Ed*, HMH's learning platform. There, teachers access a guided learning pathway focused on the first 30 to 60 days of instruction. A recommended sequence of topics, which includes live events, videos, interactive media, and related resources, helps teachers plan, teach, and assess student learning using their new HMH program. The pathways allow teachers to personalize their learning by selecting the mode of learning (live event during back-to-school or on demand) to meet their needs. In addition, teachers have control over how much time they spend reviewing resources.

Teacher's Corner

HMH continues to provide a personalized model for professional learning, engaging teachers throughout the school year via *Teacher's Corner* on *HMH Ed*. Teachers have access to a searchable library that includes classroom videos, tips from other teachers, teacher-to-teacher collaboration through live events, and additional content and support from HMH's experienced instructional coaches. *Teacher's Corner* includes the following:

Live events: promotes learning from HMH coaches, thought leaders, and *Into Math* educators in live online sessions with active participation and feedback

Program support: provides on-demand teaching resources and professional learning tools, including model lesson videos, teacher tips, interactive support, and more

The Breakroom: inspires educators with ideas from other educators, new lesson resources, and reflection opportunities

Leader's Corner: offers leader access to program support and resources to assist teachers with program implementation (available to district and site-based leaders through their *HMH Ed* login)

Family Room: provides a dedicated space with personalized, easily accessible, and on-demand resources to support family engagement and learning at home (available to families through their student's *Into Math* login on *HMH Ed*)

Program-Aligned Professional Learning for Actionable Insights

Grounded in research principles of effective professional learning, HMH program-aligned courses provide flexible, continuous, and relevant learning for teacher groups. Through a blend of in-person and online study group sessions that may be scheduled anytime, HMH courses provide educators with deep learning that ties theory to practice and is proven to generate sustained impact (Tucker & Wycoff, 2019). HMH courses are designed to unpack the program's research, helping educators understand the *why* behind the program's evidence-based instructional approach and content.

Educators will benefit from opportunities to continuously review their learning, practice implementing strategies in the classroom, and exchange feedback through online study groups. Engaging in follow-up online study groups to practice and reflect after each session reinforces educators' growth through sustained, collaborative learning (Aguilar, 2019). HMH courses do more than share subject knowledge; they offer personalized content aligned with research-based practices, leveraging program resources, activities, and instructional strategies. This ensures professional learning is relevant and applicable to classroom practices (Hill & Papay, 2022).

HMH Program-Aligned Courses

The course, *From Reasoning to Action: Building Strong Math Thinkers with* Into Math, includes three in-person sessions and six online study groups. Participants will explore best practices in mathematical reasoning and discourse to improve student success. They will link students' proficiency to understanding and communication, learn to create a supportive math community, promote discourse and mathematical language, and use tools to enhance reasoning.

From Reasoning to Action: Building Strong Math Thinkers with Into Math California
This course provides teachers with immediately applicable learning that uses their program resources to connect pedagogy to practice.

Session 1: Conversations that Count: Fostering Reasoning and Discourse

Session 2: Talk Like a Mathematician: Using Language Routines for Deep Reasoning

Session 3: Make Math Thinking Explicit: Using Tools and Representations

Coaching to Strengthen Teaching and Learning

Research has demonstrated that sustained, job-embedded coaching is the most effective form of professional learning, whether it is delivered in person or online. Coaching delivered in person is most effective when coaches are highly expert and focus their work with teachers on a clearly specified instructional model or program. Other opportunities for teachers to develop their knowledge of the targeted instructional model (e.g., in courses, workshops or coach-led learning groups) are also an important component of successful coaching programs. Online coaching shows promise for being at least as effective as in-person coaching for improving outcomes, though the research base comparing delivery systems is thin. The balance of evidence to date, however, suggests that the medium through which coaching is delivered is less important than the quality and substance of the learning opportunities provided to teachers (Matsumara et al., 2019).

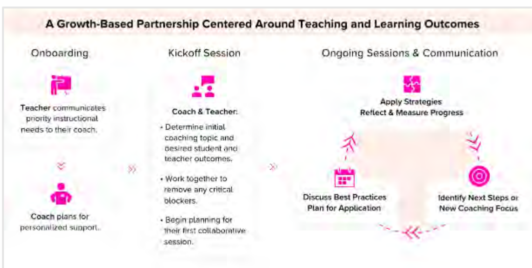
A recent meta-analysis of coaching programs found overall effect sizes of 0.49 SD on instructional practices and 0.18 SD on student achievement. Encouragingly, teachers who received online coaching performed similarly to teachers who received in-person coaching for improving both instructional practices and student achievement. The authors identified several aspects of online coaching as potential strengths: increasing the number of teachers with whom a high-quality coach can work, reducing educators' concern about being evaluated by their coach, and lowering costs while increasing scalability (Kraft et al., 2018).

The best evidence for coaching is found for one-to-one coaching, where coaches observe and offer feedback on teachers' practices. There is not a great deal of research on specific coaching practices, but effective coaches might engage in co-planning, modeling, or guiding teacher reflection. Effective coaching is time-intensive, and the most successful programs have invested in the selection, training, and ongoing support of their coaches (Hill & Papay, 2022).

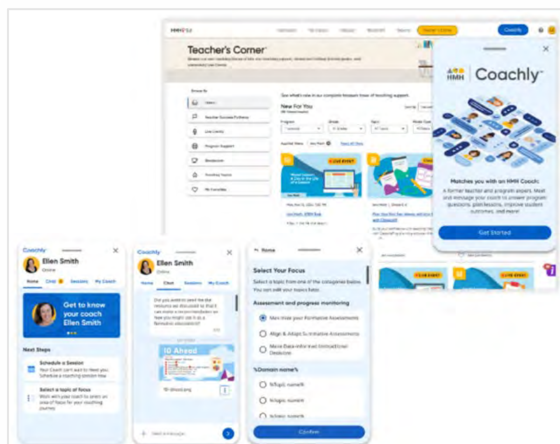
Online coaching can provide a framework for a shared leadership structure that focuses on facilitating teachers' autonomy, self-management, empowerment, and cooperation. Because the coach and the teacher jointly pursue the goal of increased student achievement, virtual coaching provides social support for both parties, leading to enhanced emotional and psychological strength. Any coaching relationship—traditional or virtual—builds on several underlying qualities of both teacher and coach: a willingness to change, a trusting relationship, a high level of initiative, and a personal and organizational commitment to the workplace (Blackman, 2010).

HMH Coaching for *Into Math*

HMH Coaching is grounded in a research-based coaching framework that leverages instructional best practices proven to impact student success. It offers sustained, data-driven, and personalized support aligned to each teacher's individual learning goals. Available through unlimited one-on-one virtual coaching sessions and in-person coaching days, *HMH Coaching* supports every teacher to elevate instructional practice, meet district goals, and raise student achievement. *HMH Coaching* provides teachers with the agency and insights to connect their own growth to student outcomes.



HMH Coachly is a yearlong digital coaching subscription that gives teachers unlimited access to a dedicated HMH coach. Teachers are matched with a highly experienced instructional coach, who will guide them through high-impact coaching topics such as implementing instructional best practices with *Into Math*, addressing classroom challenges, and goal setting and tracking. Once logged onto *HMH Ed*, teachers will be able to schedule unlimited one-on-one virtual coaching sessions, message their coach, and receive timely feedback in a single-platform experience. For additional support, *Coachly* licenses can be paired with in-person group Coaching days.



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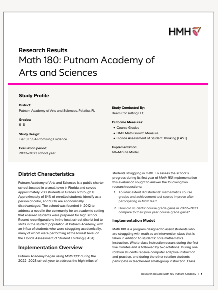

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