RESEARCH EVIDENCE BASE

Go Math!®

THE HMH RESEARCH MISSION STATEMENT

Houghton Mifflin Harcourt® (HMH®) is committed to developing innovative educational solutions and professional services that are grounded in learning science evidence and efficacy. We collaborate with school districts and third-party research organizations to conduct research that provides information to help improve educational outcomes for students, teachers, and leaders at the classroom, school, and district levels. We believe strongly in a mixed-methods approach to our research, an approach that provides meaningful and contextualized information and results.

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INTRODUCTION

During most of the 20th century, the United States was a world leader in mathematics, achieving feats of engineering, scientific discovery, and economic growth on every scale. Much of how the nation viewed its success was based on the financial prosperity of its citizens and its international competitiveness. However, for the past several decades, American students' performance on measures of mathematics skills rank the United States at mediocre levels worldwide, while the math, science, and technical training of other countries' workforces develops and benefits their job growth, especially in a technology-driven economy. Without substantial and sustained changes to its educational system, specifically in mathematics, the United States will continue to slip when stacked against other nations, and the gap between the country's highest and lowest achievers will continue to grow (National Center for Education Statistics [NCES], 2021; National Mathematics Advisory Panel, 2008; Schmidt, 2012). Fundamentally, rigorous mathematics standards and high-quality mathematic instruction for all students are the solution. However, pervasive equity and access issues additionally beset efforts and outcomes in educating our nation's most vulnerable children. Any systemic improvement to mathematics success must also address systemic disparities (National Council of Teachers of Mathematics [NCTM], 2014).

"Math concepts and skills are important to all of school and life. Math provides a new way to see the world, the beauty of it, and the way you can solve problems that arise within it. However, math is much more: Math is critical thinking and problem solving, and high-quality math experiences also promote social, emotional, literacy, and general brain development" (Clements & Sarama, 2020, p. 2).

Houghton Mifflin Harcourt's *GO Math!*® was developed to provide high-quality instruction and assessment aligned with rigorous standards and high expectations for all students to thrive in their learning of mathematics. The *HMH GO Math!* pedagogy is built on a solid foundation of effective mathematics teaching and learning that educators have relied upon for years. The new edition has been completely rebuilt and infuses the program with additional embedded supports to nurture young mathematicians and meet the needs of all learners. *GO Math!* celebrates the unique assets each student brings to the classroom and provides ongoing opportunities for exploration, independence, and collaboration that foster successful, strategic mathematics learning. The program additionally utilizes effective teaching practices as advocated by the National Council of Teachers of Mathematics (2014).

CURRICULUM DESIGN AND STANDARDS

Mathematical thinking and reasoning and the skills to engage in procedural reliability, fluency, and automaticity are vital capacities for 21st-century learners. In the modern era, the United States has benefitted greatly from the economic, social, and health advances made possible by a workforce with expertise in science, technology, engineering, and mathematics (STEM)—and both the importance of and demand for jobs in STEM fields continues to increase (Granovskiy, 2018). Within mathematics programs that effectively support the development of essential 21st-century skills, instruction and assessment are closely aligned to rigorous standards (Peterson & Ackerman, 2015). The NCTM (2014, 2020) additionally recommends that mathematics programs are structured by coherent learning progressions that build conceptual understandings, as well as connections among areas of mathematical study and between mathematics and the real world.

HMH GO Math! supports students' procedural and conceptual development along coherent learning progressions, as called for by rigorous mathematics standards and by policy and research leaders in the field of K–12 mathematics education. The program supports evidence-based best practices to foster students' success and interest in math with high expectations for all students.

CURRICULUM DESIGN AND STANDARDS

CLEAR, COHESIVE, COMPREHENSIVE ALIGNMENT TO MATHEMATICS STANDARDS

For several decades, the cornerstone of education policy in the United States has centered around the implementation of rigorous standards, along with instruction and accountability measures aligned to those standards. Research demonstrates that standards-based learning environments have a significant positive impact on student achievement in mathematics (Peterson & Ackerman, 2015; Stein et al., 2007; Tarr et al., 2008). In an examination comparing the impact of mathematics curricula on student outcomes, standards-based learning over traditional instruction was determined to contribute more significantly to growth in problem-solving (Cai et al., 2011). Research also shows that high-performing schools have a clear, shared focus, and curriculum, instruction, and assessment closely aligned to state standards (Shannon & Bylsma, 2007).

For students to acquire targeted mathematics skills and perform at proficient levels, it is essential that classrooms identify and clarify what students are expected to learn and understand (NCTM, 2014). Research demonstrates that clarity between teachers and students regarding intentions for what is to be learned and why it is to be learned, as well as criteria for what constitutes success, is one of the most effective teaching practices for yielding targeted outcomes (Almarode & Vandas, 2018; Hattie, 2008; Leahy et al., 2005). "Formulating clear, explicit learning goals sets the stage for everything else" (Hiebert et al., 2007, p. 57). Standards offer a guide for teachers to ensure that they are helping students build the foundations they need to move on to the next grade and, ultimately, be ready for college and careers. Standards can help ensure that teachers are providing effective instruction for all students and can help students set clear goals for learning (Wiliam, 2011).

While standards-based teaching with quality materials is demonstrably effective and standards provide a guide to what is critical to teach, standards alone are insufficient in achieving broad improvement to learning. Within standards-aligned instruction, focus and coherence are critical (Schmidt et al., 2005), particularly as instruction is adapted based on individual students' progress and needs (Pak et al., 2020 Establishing clear priorities from among national, state, or local content standards is an essential component of instructional planning that will ultimately achieve targeted goals.

Standards typically call for more content than can be reasonably and effectively addressed within available instructional time; therefore, teachers must make choices based on the specific needs of their students (Senn et al., 2014; Wiggins & McTighe, 2005). NCTM (2014) urges that curriculum design take into consideration the amount of new content to be introduced in a particular grade or level so that sufficient time will be available to teach concepts and procedures effectively.

To succeed in mathematics, students need a clear, articulated path for learning. "[M]athematics instruction—like any good instruction—must be intentionally designed and carefully orchestrated in the classroom, and should always focus on impacting student learning" (Hattie et al., 2017, pp. 3–4). A coherent math curriculum is sequentially ordered to best reflect the hierarchical and logical structures of mathematics (Schmidt et al., 2005). "A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades" (NCTM, 2014, p.4).

A clear, articulated path toward learning objectives begins with teachers knowing what each student needs to learn each day—and exactly what success looks like for each student (Hattie et al., 2017). Wiggins and McTighe (2005) describe effective instructional design in the classroom as centered on guiding questions such as, *What should students know, understand, and be able to do? How will we know if students have achieved the desired results? How will we support learners as they come to understand important ideas and processes?* The authors propose three stages in their model for designing instruction:

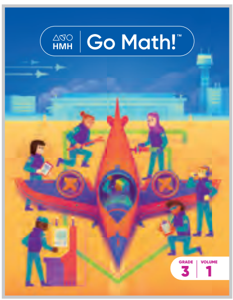
- **Stage 1** clarifies goals, examines content standards, and reviews curriculum expectations with the purpose of establishing priorities.
- **Stage 2** examines the assessment evidence needed to document and validate that the targeted learning has been achieved—a process that further serves to sharpen and focus teaching.
- **Stage 3** requires teachers to consider the most appropriate and effective approaches to assessment-based instruction that yields understanding.

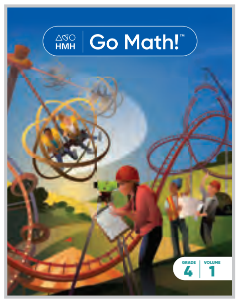
Identifying what students will learn is only one aspect of lesson design. It is critical that classroom experiences also connect to what students need to know and make learning purposeful. Intentional design allows students to recognize, with clarity and intentionality, what is expected of them, including what they are learning and why they are learning it (Kanold, 2018; NCTM, 2014; Wiliam, 2011).

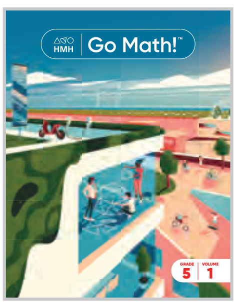
Per Hattie and colleagues (2017), related to intentional design in mathematics is the concept of instructional rigor as viewed as an equally intensive balance among conceptual understanding, procedural skills and fluency, and application. "[M]athematics teaching is most powerful when it starts with appropriately challenging intentions and success criteria" (p. 4). Additionally, promoting self-determination is an important component in classroom instruction aimed at helping all students attain post-academic success and quality of life. Particularly for those with special needs, helping students develop skills associated with self-determination (e.g., planning, self-management, self-awareness, problem-solving, and goal setting) is critical in preparation for experiences within and beyond school (Raley et al., 2018).

HOW *HMH GO MATH!* DELIVERS

Go Math! encourages all students to see themselves and the possibilities for their future success. The interactive Student Edition, available as full-color, write-in printed format or online, is structured to assist students in navigating rigorous assignments on their individual learning pathway. Appropriate intervention options foster high levels of engagement and achievement.





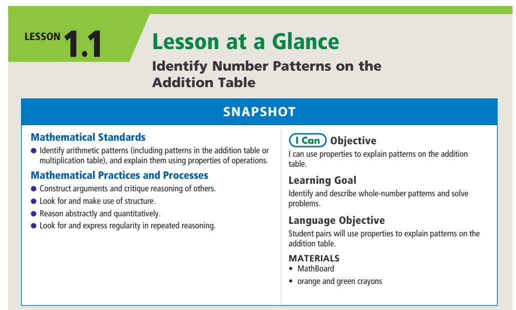


The *Go Math!* Teacher Edition at each grade provides all the support educators need in planning lessons and implementing standards for mathematics via teacher-to-teacher notes, Teacher's Corner, and other professional resources. Teachers can effectively provide standards-based instruction at the whole-class and individual-student level using the following program components:

- Standards Helper, a resource document organized by standard to easily see lessons, supports, and common errors for student learning
- Launch Activity, a focused learning experience with engaging content that introduces new concepts throughout the school year
- Teacher's Corner, an online and interactive resource center for professional support specific to *Go Math!* as well as other research-based supports

The program provides a complete Scope and Sequence across Grades K-5, organized by skill topics and accompanied by a pacing guide, to help teachers plan standards-based instruction.

Ahead of each Launch Activity and lesson, the Teacher Edition includes a Snapshot of targeted mathematical standards, as well as Practices and Processes.


LESSON 1.1

Lesson at a Glance

Identify Number Patterns on the Addition Table

SNAPSHOT

Mathematical Standards
- Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Mathematical Practices and Processes
- Construct arguments and critique reasoning of others.
- Look for and make use of structure.
- Reason abstractly and quantitatively.
- Look for and express regularity in repeated reasoning.

I Can Objective
I can use properties to explain patterns on the addition table.

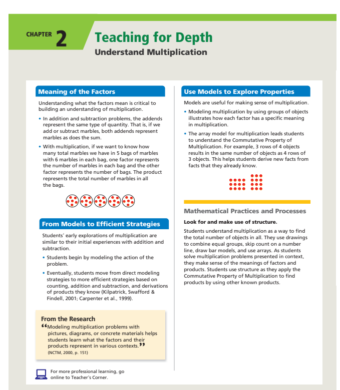
Learning Goal
Identify and describe whole-number patterns and solve problems.

Language Objective
Student pairs will use properties to explain patterns on the addition table.

MATERIALS
- MathBoard
- orange and green crayons

Mathematical Practices and Processes are identified at the task level within the Teacher's Edition, giving teachers point-of-use support and questioning strategies to help students develop mathematical habits during small-group and whole-group instruction.

The program also gives teachers support to facilitate deeper learning and understanding of mathematics, such as through each chapter's Teaching for Depth feature. This guidance provides background about the best practices and research related to targeted concepts and underlying mathematical practices and processes. It also prepares teachers for common errors and misunderstandings that students may have as they begin working on the chapter, and it provides tools and strategies that can be relayed to students to foster proficiency. Teaching for Depth further identifies opportunities for both extension and intervention as needed for individual students as they progress through the chapter.


CHAPTER 2

Teaching for Depth

Understand Multiplication

Meaning of the Factors

Understanding what the factors mean is critical to building an understanding of multiplication.

- In addition and subtraction problems, the addends represent the same type of quantity. That is, if we add or subtract marbles, both addends represent marbles as does the sum.
- With multiplication, if we want to know how many total marbles we have in 5 bags of marbles with 6 marbles in each bag, one factor represents the number of marbles in each bag and the other factor represents the number of bags. The product represents the total number of marbles in all the bags.

From Models to Efficient Strategies

Students' early explorations of multiplication are similar to their initial experiences with addition and subtraction.

- Students begin by modeling the action of the problem.
- Eventually, students move from direct modeling strategies to more efficient strategies based on counting, addition and subtraction, and derivations of products they know (Kilpatrick, Swafford & Findell, 2001; Carpenter et al., 1999).

From the Research

"Modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their products represent in various contexts."
(NCTM, 2000, p. 151)

For more professional learning, go online to Teacher's Corner.

Use Models to Explore Properties

Models are useful for making sense of multiplication.

- Modeling multiplication by using groups of objects illustrates how each factor has a specific meaning in multiplication.
- The array model for multiplication leads students to understand the Commutative Property of Multiplication. For example, 3 rows of 4 objects results in the same number of objects as 4 rows of 3 objects. This helps students derive new facts from facts that they already know.

Mathematical Practices and Processes

Look for and make use of structure.

Students understand multiplication as a way to find the total number of objects in all. They use drawings to combine equal groups, skip count on a number line, draw bar models, and use arrays. As students solve multiplication problems presented in context, they make sense of the meanings of factors and products. Students use structure as they apply the Commutative Property of Multiplication to find products by using other known products.

57E *Go Math!* Grade 3

5E INSTRUCTIONAL MODEL (ENGAGE, EXPLORE, EXPLAIN, ELABORATE, EVALUATE)

With roots in historical education models dating back to the 1900s, including the constructivist "learning cycle" approach, the 5E instructional model was developed by a team of researchers led by Rodger Bybee in collaboration with the Biological Sciences Curriculum Study (BSCS). In ensuing decades, the 5E Instructional Model has become widely used across K–12 and post-secondary levels, as well as supported by a significant research base attesting to its effectiveness (Bybee et al., 2006).

The 5E model consists of five stages of teaching and learning:

- **Engagement:** Students access prior knowledge through learning activities that also spark their interest.
- **Exploration:** Students generate new ideas and consider possibilities.
- **Explanation:** Students demonstrate understanding by explaining what they know, as to guide them toward deeper learning.
- **Elaboration:** Students extend and apply their knowledge.
- **Evaluation:** Students examine their own understanding while teachers monitor and evaluate progress.

Each stage serves a specific function, and together they frame a sequential organization of lessons, units, and programs that contribute to a teacher's more coherent instruction and to a learner's increased understanding of scientific and technological knowledge, attitudes, and skills. Once internalized, this instructional approach also can inform the many ongoing decisions that teachers must make in classroom situations (Bybee et al., 2006).

In a 2006 review, Bybee and colleagues discovered that hundreds of thousands of curriculum materials and lessons had been developed and implemented using the model, which has also been endorsed by state agencies across the country. Bybee's team further found that studies of the 5E model conducted by internal and external evaluators showed positive trends for student mastery of subject matter and interest.

The National Research Council's seminal 1999 publication *How People Learn* supports the core tenets of the 5E approach: "An alternative to simply progressing through exercises that derive from a scope and sequence is to expose students to the major patterns of a subject domain as they arise naturally in problem situations. Activities can be structured so that students are able to explore, explain, extend, and evaluate their progress. Ideas are best introduced when students see a need or reason for their use—this helps them to see relevant uses of the knowledge to make sense of what they are learning" (p. 127).

Research has found that students who are taught with medium to high levels of fidelity to the 5Es demonstrate learning gains double that of students whose teachers implemented the 5E approach with lower levels of fidelity or not at all (Coulson, 2002). Students whose teachers used the 5E approach have also shown increases in scientific reasoning (Boddy et al., 2003).

While, historically, much of the research supporting the 5E approach has been in the area of science teaching, calls for its application within mathematics are increasing (Runisah et al., 2016; Tezer & Cumhur, 2017). "The 5E instructional model can help students move from understanding concrete experiences to the application of principles. The model provides students with opportunities to deeply and meaningfully recall what they already know…the 5E instructional model is considered one of the best approaches recommended for teaching within a constructivist learning approach" (Omotayo & Adeleke, 2017, p. 16).

The 5E instructional model aligns with the research-based practices outlined in NCTM's *Principles to Action* (2014)—in particular, establishing goals to focus learning, implementing tasks that promote reasoning and problem-solving, facilitating meaningful discourse, posing purposeful questions, building procedural fluency from conceptual understanding, and supporting productive struggle. "Researchers recommend that mathematics teaching is effective when students actively participate in the learning process, so mathematics teachers should not use explanatory teaching approaches but should use reconnaissance, manual activities, and interactive group works to encourage students to learn better. One of the approaches where students participate in the active learning process is the 5E instructional model" (Tezer & Cumhur, 2017, p. 4791).

HOW *HMH GO MATH!* DELIVERS

At each grade, the program's Table of Contents builds out the 5E trajectory as an overarching framework for learning. Each *HMH GO Math*! lesson is composed of the corresponding five parts. While every classroom is unique, this framework helps to organize small-group and whole-group instruction for meaningful learning.

Building Procedural Mastery with the 5E Model

Every *Go Math!* lesson follows the same lesson design based on the 5E model of instruction.

Whole Group

Engage

Spark children's curiosity with **Engage** to connect prior knowledge to new mathematical content.

Small and Whole Group

Explore

In **Explore**, children build an understanding of new concepts through discovery and make sense of procedure using Listen and Draw or Unlock the Problem.

Math Talk questions provide entry points for mathematical discourse that explores the new concepts learned.

Small Group

Explain

In **Explain**, children apply their new knowledge of the lesson's mathematical concepts using Share and Show and On Your Own problems, increasing their conceptual understanding as they build towards procedural reliability.

Teachers can assess differentiation needs, providing support with supplemental activities and materials for Intervention, Language Support, and Enrichment.

Whole Group

Elaborate

The real-world problem solving in **Elaborate** gives children the opportunity to develop a deeper understanding of lesson concepts by practicing different solution methods.

Evaluate

Children summarize their thinking in **Evaluate** by connecting the I Can statement to the lesson objective. Practice and Homework problems support children's newly-learned lesson skills in their progression towards procedural fluency and automaticity.

PROMOTING FLUENCY AND AUTOMATICITY THROUGH ONGOING PRACTICE

It is well established that automaticity—the capacity to deliver a correct answer promptly from memory without conscious thought or reliance on calculation (Stickney et al., 2012) is essential for success in mathematics (Baker & Cuevas, 2018; Dahaene, 1999; Hasslebring et al., 1987; National Mathematics Advisory Panel, 2008; Poncy et al., 2007). Because humans' cognitive capacity is limited and working memory has specific constraints on how much information can be processed, the ability to recall information quickly requires less cognitive demand. Developing automaticity with basic mathematical knowledge and skills frees up working memory to enable increased understanding of concepts and boost problem-solving (Pegg et al., 2005). Automaticity further promotes advanced cognitive processes such as discrimination, generalization, and adaptation (Skinner, 1998). "Information-processing theory supports the view that automaticity in math facts is fundamental to success in many areas of higher mathematics. Without the ability to retrieve facts directly or automatically, students are likely to experience a high cognitive load as they perform a range of complex tasks" (Woodward, 2006, p. 269).

Automaticity is closely tied to both reliability and fluency. "Certain procedures and algorithms in mathematics are so basic and have such wide application that they should be practiced to the point of automaticity. Computational fluency in whole number arithmetic is vital. Crucial ingredients of computational fluency are efficiency and accuracy. Ultimately, fluency requires automatic recall of basic number facts" (Ball et al., 2005, p. 3). Fluency also entails speed and accuracy (Lin & Kubina, 2005). "Automaticity is a piece of fluency. Fluency is the end goal and considered true mastery of the concept when reached. Now connect these two terms to mathematics and we develop the idea that students will develop automaticity first, then fluency, and by doing this, they will develop a pattern of sustained success in their mathematics career" (Baker & Cuevas, 2018, p. 13).

Automaticity is a predictor of performance on general mathematics tests (Stickney et al., 2012). As students progress through increasingly complex math learning past single-digit multiplication, calculations such as finding common denominators when adding and subtracting fractions become not just more difficult but also potentially prohibitive to the learning itself. Without the ability to retrieve facts directly, students are more likely to experience excessive cognitive loads and produce work that is inaccurate (Baker & Cuevas, 2018). Automaticity also offers students affective benefits: "Students who can respond automatically may have less math anxiety and be more likely to choose to do assigned mathematics work than those who cannot respond…. This is critical because few if any skill development procedures are likely to enhance skills unless students choose to respond automatically" (Poncy et al., 2007, p. 29).

Academically low-achieving students, as well as those with learning disabilities, experience considerable challenges in developing automaticity in their facts. Such delays and difficulties are apparent from the beginning of elementary school, and students may fail to retrieve facts directly when presented in isolation or when embedded in tasks such as multidigit computations. Research involving primary-level students indicates that students with learning disabilities are more likely to rely on counting strategies than direct retrieval when working single-digit fact problems (Woodward, 2006). To develop fluency and automaticity, as well as conceptual understanding and higher-level mathematical thinking, students need ongoing and varied opportunities to respond to and practice math tasks (Poncy, et al., 2007; Skinner, 1998). The key is for students to have opportunities to rehearse and practice strategies and procedures that solidify their mathematical knowledge—and the type and quantity of practice opportunities matters (NCTM, 2014).

Research has well established that to develop students' procedural fluency, teachers should offer opportunities for meaningful, purposeful, targeted practice that are brief and distributed over time, rather than rote and repetitive (Baroody, 2006; Fuson & Beckmann, 2012/2013; Fuson, Kalchman, & Bransford, 2005; Fuson & Murata, 2007; Issacs & Carroll, 1999; NCTM, 2014; Russell, 2000). After students have established a strong conceptual foundation and the ability to explain the mathematical basis for a procedure or strategy, they should practice with a small number of carefully selected problems and receive ongoing feedback on their progress (Rohrer, 2009). If students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999).

When learning is not meaningful and is disconnected from other knowledge, students have more difficulty absorbing concepts; when students are able to connect procedures and concepts, their retention improves and they are better able to apply what they know in different situations (Fuson et al., 2005). There are some mathematical skills which may be best developed with practice in the context of a meaningful examination of patterns and strategies (Fuson, 2009). "Research indicates that discovering patterns or relations facilitates mastery with fluency…. Focusing on structure, rather than memorizing individual facts by rote, makes the learning, retention, and transfer for any large body of factual knowledge more likely" (Baroody et al., 2009, p. 70).

Practice sessions should also interweave worked example solutions with independent problem-solving, and as students gain proficiency, the number of worked examples should be decreased while the number of independently solved problems increases (Pashler et al., 2007). Worked examples—rather than just a greater number of practice problems—have also been shown to be effective in helping students learn to solve problems faster, perhaps because these worked problems help to reduce students' cognitive loads and allow them to focus on the targeted learning (Booth et al., 2013).

HOW *HMH GO MATH!* DELIVERS

Every lesson within *HMH Go Math!* embeds ample opportunities for students to practice targeted skills and problem-solving strategies, as well as to build fluency and automaticity. Students can use full-color, write-in Student Editions as well as interactive digital practice with learning supports that encourage perseverance and engagement. Games and activities offer additional engaging experience with targeted mathematics content.

Throughout the program, teachers are supported in helping students develop automaticity through strategy-based approaches. Shaee and Show tasks provide opportunities for tudents to improve and demonstrate understanding. Teachers can use the Quick Check to see if students are ready to go on, or to determine how to best differentiate instruction.

On Your Own problems independently build fluency once students understand the lesson's concepts. Also included are Enrich and Reteach activities. Practice and Homework further aids automatic recall of facts to allow for more complex learning and processing.

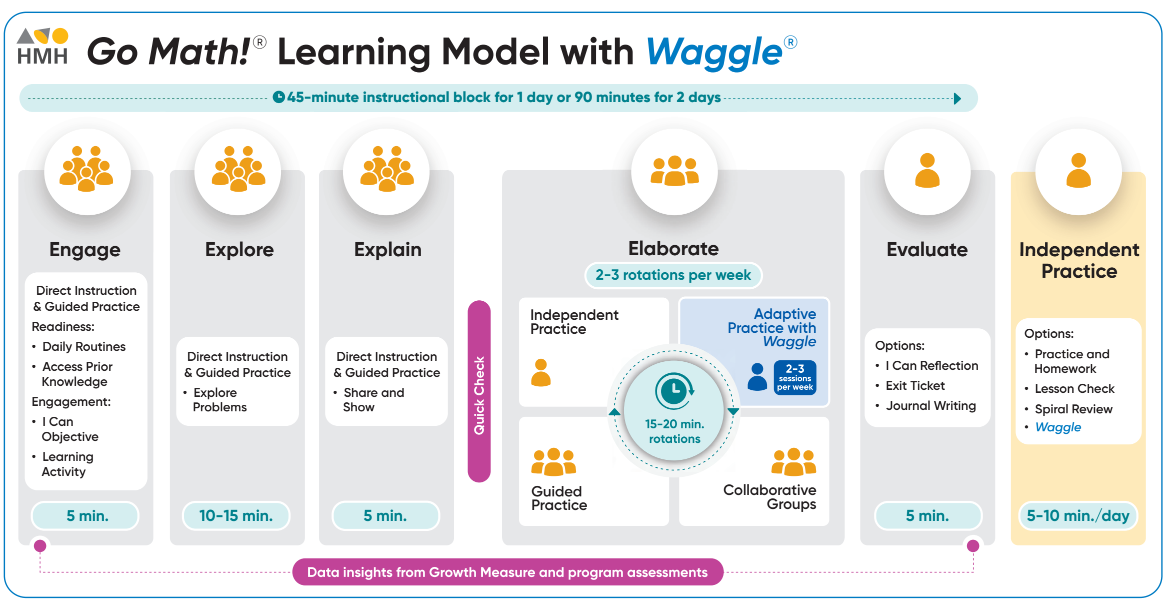
The program's Spiral Review problems, embedded within the student practice, help students recall information taught in the past and solidify long-term retention.

Independent Practice

Options:
- Practice and Homework
- Lesson Check
- Spiral Review
- *Waggle*

Go Math! on Ed®, the HMH online learning platform, combined with *Waggle*® and the HMH Growth Measure® solutions provide comprehensive and customizable skills-based practice that assesses progress in real time. *Waggle* offers flexibility in and out of the classroom, engaging students wherever they are on their learning journey.

Teachers can further leverage HMH Growth Measure to place children on personalized pathways or choose the assignments that extend their instruction.



EMBEDDED LANGUAGE DEVELOPMENT

Multilingual learners are the fastest-growing student population in the United States, representing about 4.5 million or nearly 10% of overall enrollment in public schools during the 2013–14 academic year (Grapin, 2019; NCES, 2016). The description "multilingual learner" applies to all students who regularly interact with languages other than English, including but not limited to those commonly referred to as English language learners (ELLs). "Multilingual learners come from a wide range of cultural, linguistic, educational, and socioeconomic backgrounds and have many physical, social, emotional, experiential, and/or cognitive differences. All bring assets, potential, and resources to schools that educators must leverage to increase equity in standards-based systems. Increasing avenues of access, agency, and equity for all multilingual learners—including newcomers, students with interrupted formal education (SIFE), long-term English learners (L-TELs), students with disabilities, and gifted and talented English learners—requires educators to be knowledgeable, skillful, imaginative, and compassionate" (WIDA, 2020, p. 18).

The 2020 Edition of the *WIDA® English Language Development Standards Framework for K–12* supports the design of standards-based educational experiences that are student-centered, culturally and linguistically sustaining, and responsive to multilingual learners' strengths and needs. Among the overarching principles of the WIDA ELD Standards is the integration of content and language, an approach to language instruction that is made more explicit and specific in the 2020 edition. Historically for WIDA, the integration of content and language has roots in the recognition that multilingual learners develop academic content and language concurrently and in the context of one another. The WIDA 2020 Standards' integration of content and language for multilingual learners promotes students' understanding of connections between content and language, meaning-making within and across content areas or disciplines, and interaction with each other in challenging content activities, as well as coordination of design and delivery of curriculum, instruction, and assessment.

With regard to how language is used in content-area learning, the 2020 edition of the WIDA ELD Standards emphasizes multimodality, the use of multiple means of communication. Effective approaches to this form of language development are intentionally designed and have students generating products from the multimodal process that are imbued with meaning (Grapin, 2019). "Multimodality is inherent to and essential for how students make meaning and engage in disciplinary practices. All students are able to both interpret and express ideas with greater flexibility when using multimodal resources, including multiple languages. Multimodality allows all students to use multiple means to engage, interpret, represent, act, and express their ideas in the classroom. For example, as students read, they also might refer to illustrations or diagrams, and as students write, they might also represent their ideas numerically or graphically" (WIDA, 2020, p. 19).

When students write about and discuss math concepts, they have the chance to think through, defend, and support their ideas. A review of studies conducted by NCTM revealed that "the process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective (Gersten et al., 2007, p. 2). Two National Research Council reports—*Adding It Up* (2001) and *How Students Learn: Mathematics in the Classroom* (2005)—emphasize discussion as a way to increase students' mathematical understanding.

Per NCTM (2000), "Communication is an essential part of mathematics and mathematics education.… Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols" (pp. 59–60).

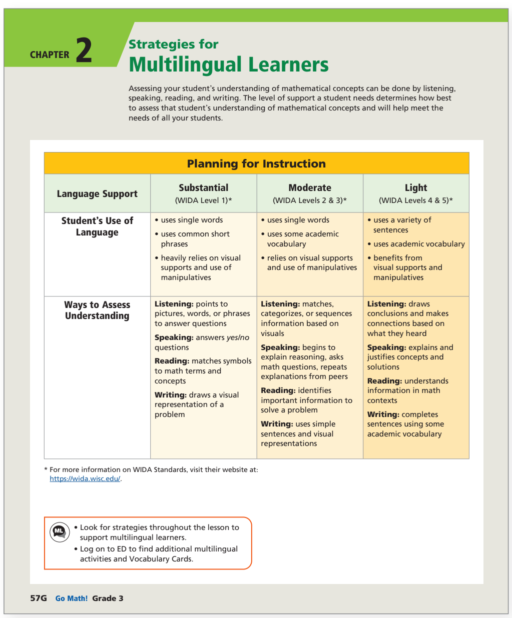
Supporting multilingual learners in developing the language of math is especially critical. Janzen's 2008 review of research on teaching English language learners in mathematics shares that while many perceive that math is easier for ELLs to learn because it involves numbers, mathematics actually presents specific language challenges to this student population. Similarly, working in a transitional language classroom led researchers Bray et al. (2006) to conclude that as students "communicate verbally and in writing about their mathematical ideas, they not only reflect on and clarify those ideas but also begin to become a community of learners" (p. 138). When introducing academic words to English language learners' expressive vocabularies, students respond best to classrooms that offer predictable routines and frequent, comfortable opportunities to express what they have learned (Feldman & Kinsella, 2008).

An effective strategy for fostering mathematical language skills and sense-making is the "Three Reads" protocol, as recommended in *Routines for Reasoning* (Kelemanik et al., 2016). This teaching strategy has students read a complex word problem or task three times, each with a distinct goal. In the first read, the teacher reads the problem stem orally with the purpose of helping students understand meaning and context. In the second, students engage in a choral or partner read with the goal of understanding the math. The third and final read serves to elicit inquiry through question generation and resolution. This process deepens students' linguistic and mathematical development by focusing on authentic text and textual clues, in addition to applying mathematical thinking and strategies.

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! provides print and online resources based on WIDA-aligned support levels to create an inclusive, equitable classroom with point-of-use support for teachers to meet the needs of all students, including multilingual learners. Each Teacher Edition contains strategic planning guides as well as chapter-level, lesson-level, and point-of-use language support.

A Planning for Instruction chart at the start of each chapter helps teachers determine students' current WIDA levels and ways to best assess and meet individual needs.



CHAPTER 2

Strategies for
Multilingual Learners

Assessing your student's understanding of mathematical concepts can be done by listening, speaking, reading, and writing. The level of support a student needs determines how best to assess that student's understanding of mathematical concepts and will help meet the needs of all your students.

Planning for Instruction

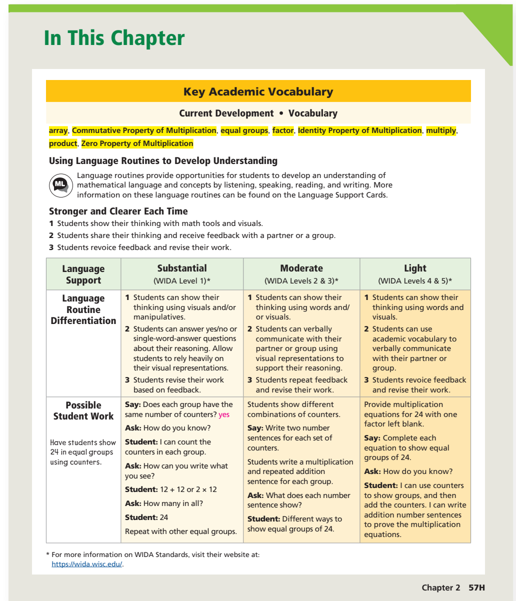
Language Support	Substantial (WIDA Level 1)*	Moderate (WIDA Levels 2 & 3)*	Light (WIDA Levels 4 & 5)*
Student's Use of Language	• uses single words • uses common short phrases • heavily relies on visual supports and use of manipulatives	• uses single words • uses some academic vocabulary • relies on visual supports and use of manipulatives	• uses a variety of sentences • uses academic vocabulary • benefits from visual supports and manipulatives
Ways to Assess Understanding	**Listening:** points to pictures, words, or phrases to answer questions **Speaking:** answers *yes/no* questions **Reading:** matches symbols to math terms and concepts **Writing:** draws a visual representation of a problem	**Listening:** matches, categorizes, or sequences information based on visuals **Speaking:** begins to explain reasoning, asks math questions, repeats explanations from peers **Reading:** identifies important information to solve a problem **Writing:** uses simple sentences and visual representations	**Listening:** draws conclusions and makes connections based on what they heard **Speaking:** explains and justifies concepts and solutions **Reading:** understands information in math contexts **Writing:** completes sentences using some academic vocabulary

* For more information on WIDA Standards, visit their website at: https://wida.wisc.edu/.

- Look for strategies throughout the lesson to support multilingual learners.
- Log on to ED to find additional multilingual activities and Vocabulary Cards.

57G Go Math! Grade 3

Multilingual learners are aided in their academic success and language development when they learn key mathematical vocabulary embedded within lessons. *Go Math!* includes Language Routine Differentiations and Possible Student Work as a guidance for assessing student's understanding of mathematical concepts based on their current language proficiency levels. Language Routines provide opportunities for students to explore and understand mathematical language and concepts through listening, speaking, reading, and writing activities, as well as to engage in mathematical discourse and reasoning. The program's approach to language development also features the Three Reads protocol and emphasizes Critique, Correct, and Clarify, Stronger and Clearer Each Time, and Compare and Connect.



In This Chapter

Key Academic Vocabulary

Current Development • Vocabulary

array, Commutative Property of Multiplication, equal groups, factor, Identity Property of Multiplication, multiply, product, Zero Property of Multiplication

Using Language Routines to Develop Understanding

Language routines provide opportunities for students to develop an understanding of mathematical language and concepts by listening, speaking, reading, and writing. More information on these language routines can be found on the Language Support Cards.

Stronger and Clearer Each Time

1 Students show their thinking with math tools and visuals.
2 Students share their thinking and receive feedback with a partner or a group.
3 Students revoice feedback and revise their work.

Language Support	Substantial (WIDA Level 1)*	Moderate (WIDA Levels 2 & 3)*	Light (WIDA Levels 4 & 5)*
Language Routine Differentiation	1 Students can show their thinking using visuals and/or manipulatives. 2 Students can answer yes/no or single-word-answer questions about their reasoning. Allow students to rely heavily on their visual representations. 3 Students revise their work based on feedback.	1 Students can show their thinking using words and/or visuals. 2 Students can verbally communicate with their partner or group using visual representations to support their reasoning. 3 Students repeat feedback and revise their work.	1 Students can show their thinking using words and visuals. 2 Students can use academic vocabulary to verbally communicate with their partner or group. 3 Students revoice feedback and revise their work.
Possible Student Work Have students show 24 in equal groups using counters.	**Say:** Does each group have the same number of counters? yes **Ask:** How do you know? **Student:** I can count the counters in each group. **Ask:** How can you write what you see? **Student:** 12 + 12 or 2 × 12 **Ask:** How many in all? **Student:** 24 Repeat with other equal groups.	Students show different combinations of counters. **Say:** Write two number sentences for each set of counters. Students write a multiplication and repeated addition sentence for each group. **Ask:** What does each number sentence show? **Student:** Different ways to show equal groups of 24.	Provide multiplication equations for 24 with one factor left blank. **Say:** Complete each equation to show equal groups of 24. **Ask:** How do you know? **Student:** I can use counters to show groups, and then add the counters. I can write addition number sentences to prove the multiplication equations.

* For more information on WIDA Standards, visit their website at: https://wida.wisc.edu/.

Chapter 2 57H

Language support is embedded throughout instruction and is provided via teacher-led small group instruction, which incorporates the program's Tabletop Flipcharts targeted toward multilingual learning. The program also includes a Glossary within Student Editions. Plus, an Interactive Glossary and a 10-language Multilingual Glossary are available online.

ML Multilingual Support

STRATEGY: Frontload

Have students follow in their book as you read a word problem aloud.

- Ask questions to help students figure out how to draw a model to help them with the process of solving a word problem.
- After you have helped students draw a model, put them in pairs and have them discuss their model with a partner.

COHERENT LEARNING PROGRESSIONS

Effective mathematics programs feature curricula that develop important mathematical concepts along coherent, meaningful learning progressions. The curricula should also foster connections among areas of mathematical study and between mathematics and the real world. In its expansive research in mathematics teaching and learning, NCTM (2014, 2020) promotes the idea that "[m]athematics teachers need to have a clear understanding of the curriculum within and across grade levels—in other words, student learning progressions—to effectively teach a particular grade level or course in the sequence" (NCTM, 2014, p. 72).

It is important to note that learning progressions are not standards. Standards are statements about what experts indicate students should know and be able to do. In contrast, learning progressions describe the typical ways students think about a topic and articulate the stages students move through as their understanding develops, which in turn determines the types and levels of tasks that students can be expected to perform at a point in time and what that performance looks like (Kobrin & Panorkou, 2016; Heritage, 2013; Sáez et al., 2013). Learning progressions are a "carefully sequenced set of building blocks that students must master en route to a more distant curricular aim. The building blocks consist of sub skills and bodies of enabling knowledge" (Popham, 2007, p. 83).

Because math learning occurs sequentially by building on previous learning and developing in sophistication, part of a discussion of content in mathematics must address the idea of sequence or progressions that promote for students a view of the curriculum as a broader learning process with defined goals for learning. A coherent, cohesive approach to mathematical concepts and procedures is essential to developing deep understanding (NCTM, 2020). Teachers should support learners as they build on what they know, develop more complex understandings, and realize that mathematics is not a set of discrete parts—it is coherent and connected (Fosnot & Jacob, 2010; Ma, 2010).

"[L]earning progressions can be leveraged in mathematics education as a form of curriculum research that advances a linked understanding of students learning over time through careful articulation of a curricular framework and progression, instructional sequence, assessments, and levels of sophistication in student learning" (Fonger et al., 2018, abstract).

A coherent math curriculum is sequenced within and across grade levels in a way that best reflects the hierarchical and logical structures of mathematics (Schmidt et al., 2005). Beginning in elementary school and continuing throughout their mathematics education, students must develop understanding and use of the "big ideas" that represent overarching concepts, as well as specific mathematical reasoning processes essential across domains (Cross et al., 2009). Charles (2005) defines a "big idea" in mathematics as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). In terms of content, research suggests that for the youngest children, developing a thorough understanding of number and of geometry and spatial measurement are developmentally appropriate and especially crucial to supporting later study (Cross et al., 2009).

An essential element in a focused, coherent progression of mathematics learning is an emphasis on proficiency with key topics (National Mathematics Advisory Panel, 2008). To help students build proficiencies, "[d]epending on the learning goals, and where students are in their learning progression, there is a balance of methods that makes for high impact and effective learning" (Hattie et al., 2017, p. 3).

In addition to improving instruction through the clarification of instructional goals and criteria for success, learning progressions also have the potential to increase the effectiveness of classroom assessment (Moss, 2022). Defined learning progressions allow teachers to pinpoint where on the trajectory toward goals each student falls at a given point in time, optimizing both instruction and assessment (Sáez et al., 2013).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! builds mastery of mathematical standards and provides a focused approaching to developing mathematical understanding, procedural skills, and fluency. An emphasis is places on making connections among concepts and skills as students move through carefully and coherently sequenced learning progressions.

The program articulates learning progressions across grades so that teachers and students recognize the coherence and interconnectedness of topics.

ACROSS THE GRADES

Before	Grade 3	After
• Students add whole numbers with sums up to 100 with procedural reliability. • Students subtract two whole numbers both less than 100 with procedural reliability. • Students explore the addition and subtraction of whole numbers within 1,000.	• Students will add and subtract multi-digit whole numbers including using a standard algorithm with procedural fluency.	• Students will explore the addition and subtraction of multi-digit numbers with decimals to the hundredths.

MATHEMATICAL KNOWLEDGE AND TEACHING

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 7). The most effective instructional programs will build on children's intuitive mathematical thinking and use that initial understanding to help children learn to solve problems, employ strategies, and engage in mathematical thinking (Carpenter et al., 2015).

Effective teaching and its development of students' mathematical knowledge are the driving forces behind powerful mathematics instruction and deep understanding. Research continually demonstrates that mathematics learning should be focused on engaging students in instructional tasks and interactive practices that promote reasoning, problem-solving, and discourse—all with the aim of fostering understanding of mathematical concepts and procedures (NCTM, 2009, 2014; NRC, 2012).

HMH GO Math! empowers teachers by providing them with the tools, resources, and professional learning they need to improve outcomes and create an engaging classroom culture that utilizes evidence-based best practices for instruction.

MATHEMATICAL KNOWLEDGE AND TEACHING

MATHEMATICAL THINKING AND REASONING

"[S]olving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable" (Polya, 1965, p. ix). Problem-solving is an engrained and essential process within human experience. In the discipline of mathematics specifically, activities such as formulating problems and assessing the reasonableness of various approaches to their solutions is central to the development of skills and knowledge (Santos-Trigo, 2020). The most effective instructional programs will build on children's intuitive mathematical thinking and use that initial understanding to help children learn to solve problems, employ strategies, and engage in mathematical thinking (Carpenter et al., 2015). Additionally, early childhood and elementary math education should build a strong foundation of deep understanding, with an emphasis on sense-making and reasoning (NCTM, 2020).

Task selection is a critical aspect of supporting elementary students' reasoning and understanding in mathematics—and among the key features of effective instructional tasks are that they be challenging and connective and as well as open to multiple representations and multiple strategies for solutions. Tasks that consistently encourage high-level student thinking and reasoning (versus those that are routinely procedural) yield the greatest learning; and tasks of higher cognitive demand are necessary when promoting reasoning and problem-solving in the mathematics classroom (Childs & Glenn-White, 2018; Francisco & Maher, 2005; Hiebert et al., 1996; Mueller et al., 2014). "Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014, p. 17).

Students learn best when what they learn is relevant and meaningful. Connecting problem-solving tasks to real-world contexts and applications improves perceptions of the content as interesting and beneficial, thereby increasing motivation to learn (Czerniak et al., 1999; Verschaffel et al., 2020). Students at all levels need to connect the mathematics they are learning to the world around them (Alberti, 2013), but research shows this to be particularly necessary for students historically underrepresented within STEM education and careers (Jackson et al., 2021; Wieselmann et al., 2020) and for advanced learners (VanTassel- Baska & Brown, 2007). "When instruction is anchored in the context of each learner's world, students are more likely to take ownership for…their own learning" (McREL, 2010, p. 7). Further, connecting mathematics to science, social studies, and business topics can increase students' understanding of and ability within mathematics (Russo et al., 2011).

In their study of mathematics learning in early childhood, Cross and colleagues (2009) concluded that to effectively foster students' conceptual understanding, teachers must include four key elements or opportunities within their teaching and learning activities: analyzing and reasoning; creating; integrating; and making real-world connection. "Our findings suggest that if teachers purposefully and persistently practice higher order thinking strategies for example, dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical thinking capabilities" (Miri et al., 2007, p. 353).

Engaging students in problem-solving tasks allows them to actively construct mathematical understandings more deeply and with greater meaning than when teachers present information to students and have them carry out procedural exercises (Masingila et al., 2018). Additionally, having students engage in problem-solving before direct instruction and learn from their failed problem-solving attempts has been linked to significantly greater conceptual understanding as well as transfer of knowledge to novel problems (Kapur, 2010 & 2014). Judging the reasonableness of their computational results is pivotal for students to understand mathematical concepts (Yang & Sianturi, 2019). Mueller and colleagues (2014) studied specific teacher actions that encouraged students to take responsibility for their mathematical problem-solving and assume roles that might otherwise be expected as the teacher's responsibility, such as determining if solutions to a problem are correct, evaluating the reasonableness of arguments, and posing questions.

"Rather than correcting students' errors, the teachers charged the students with considering the reasonableness of solutions. Students were not praised for correct solutions; rather, all solutions were considered and students were afforded the opportunity to defend and/or modify their arguments. A result was that the learners were comfortable judging their own solutions and those of their peers, and learned that they could determine the validity of a mathematical argument" (p. 16–17).

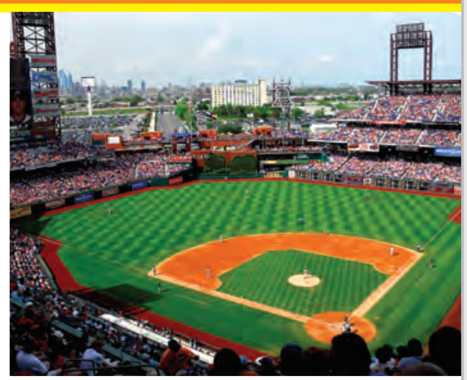
HOW *HMH GO MATH!* DELIVERS

HMH GO Math! cultivates mathematical thinking and reasoning skills and mindsets throughout the program via both engaging student activities and teacher support. This includes emphasis on real-world application of problem-solving as well as on mathematical practices and processes. Students are guided through multi-step problem-solving tasks.

MATH in the Real World

Use the following clues to find the maximum seating capacity of a baseball stadium.

- **The 5-digit number has a 4 in the greatest place-value position and a 1 in the least place-value position.**
- **The digit in the thousands place has a value of 3,000.**
- **The digit in the hundreds place is twice the digit in the thousands place.**
- **There is a 5 in the tens place.**



USING MATHEMATICAL MODELS AND REPRESENTATION TO SUPPORT EXPLORATION

Because mathematics entails the use of signs, symbols, and diagrams to represent abstract notions and study spatial aspects, as well as because the nature of the subject is often invisible and intangible, visualization is integral to learning and teaching mathematics (Presmeg, 2020; Bobis & Way, 2018; Stylianou, 2011). Additionally, mathematics is a tool for understanding present and future real-world problems; this has led to modeling becoming an important part of preparing students for advanced study and careers (Abassian et al., 2020).

A wide body of research supports the use of physical and imagistic models, manipulatives, and other such representations in the mathematics classroom; such representations help make abstract concepts more concrete as well as aid in internalization of procedures for problem-solving, increased creativity, greater metacognition, and students' more active participation in their own learning—all of which contribute key elements for impactful mathematical exploration. Indeed, the abstract nature of mathematics means that people have access to mathematical ideas largely or often only through the representations of those ideas (Carbonneau et al., 2013; Cross et al., 2009; NCTM, 2000 & 2014; NRC, 2001; Stylianou, 2011). "For students to understand such mathematical formalisms, we must help them connect these formalisms with other forms of knowledge, including everyday experience, concrete examples, and visual representations. Such connections form a conceptual framework that holds mathematical knowledge together and facilitates its retrieval and application" (Donovan & Bransford, 2005, p. 364). The positive effects of manipulative use in math instruction extend to digital tools as well as physical objects (Bouck & Park, 2018).

Mathematical representation is commonly thought to be a product—a picture or a set of symbols students make to demonstrate understanding; however, representation in math learning is also an essential process. Students' diagrams and symbolism evolve dynamically over the course of problem-solving and aid thinking and the construction of understanding in highly personal ways (Stylianou, 2011).

When students sketch or visually organize their mathematical thinking, they are able to explore their understanding of concepts, procedures, and processes—and communicate mathematically (Arcavi, 2003; Stylianou & Silver, 2004). Having students then participate in discussions about their representations allows for meaningful learning (Fuson & Murata, 2007).

A trajectory that progresses students through instruction aiming to develop first concrete, then representational or pictorial, and then, ultimately, abstract understanding—also known as CRA or CPA—has long been widely practiced within the field of mathematics education. The theoretical basis of the framework derives from Bruner (1966), who proposed that the learning sequence of a new mathematical concept begins with concrete actions undertaken or with concrete objects (enactive), which is then translated into perceptual images of the experience formed (iconic), subsequently leading to the adoption of the mathematical symbol. The three phases of CRA typically entail use of concrete materials (such as blocks, tiles, cubes, base ten blocks, fraction strips, etc.), then two-dimensional written/pictorial representations and notations (including circles, dots, etc.), and finally abstractions in the form of symbols, numbers, equations, and algorithms; this research-based approach has been shown to generate meaningful connections, foster fluency, and support conceptual understanding of mathematics (Flores, 2010). Empirical research indicates that the CPA model is an effective way for students to develop understanding of mathematical concepts, computations, operations, and reasoning (Leong et al., 2015; Salingay & Tan, 2018). This is particularly true for students who are at-risk or have mathematics difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols (Agrawal & Morin, 2016; Flores, 2010; Sousa, 2008).

A key element within the CPA approach is the effective use of manipulatives within explicit instruction focused on developing conceptual understanding. During exploratory learning, manipulatives offer a systematic way to integrate the use of devices and pictorial representations into explicit instruction designed to teach important concepts and move students through a concrete-representation-abstract teaching sequence (Miller & Hudson, 2007). Interaction with concrete materials such as manipulatives enhances student retention of procedural steps and strategies in mathematical problem-solving. Concrete materials allow students to encode and retrieve information through a variety of sensory experiences—visual, auditory, tactile, and kinesthetic—that reinforce learning (Witzel, 2005). "When used in comprehensive, well-planned, instructional settings, both physical and virtual manipulatives can encourage students to make their knowledge explicit, which helps them build Integrated-Concrete knowledge" (Sarama & Clements, 2016, p. 71).

Visual representation has shown to improve student performance in general mathematics, pre-algebra, word problems, and operations (Gersten et al., 2009). Representations bolster intuition and understanding (Blatto-Vallee et al., 2007) and can help students to communicate, reason, problem-solve, connect, and learn (Hill et al., 2014). Researchers have concluded that visualization is a powerful problem-solving tool and can be helpful in all kinds of mathematical problems, not only geometric problems (Van Garderen, 2006). Representations, models, and manipulatives are important in math learning for all students, but significant research indicates that younger students and those who have special needs or are multilingual or those having difficulty grasping abstract mathematical concepts especially benefit from visual representations of mathematical ideas, including physical objects they use or actions they perform when trying to solve problems (Agrawal & Morin, 2016; Bouck & Park, 2018; Miller & Hudson, 2007; NRC, 2001; Riccomini et al., 2022).

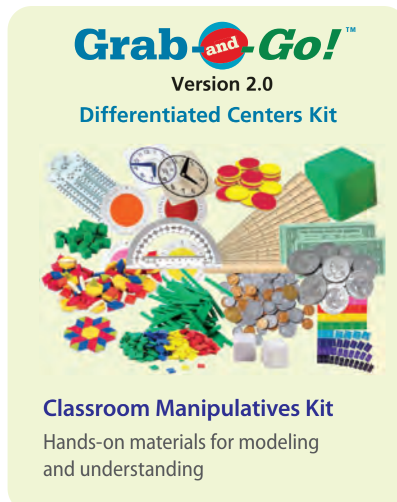
HOW *HMH GO MATH!* DELIVERS

HMH GO Math! provides ongoing opportunities for students to engage with manipulatives and visual representations, including both images provided and drawings constructed by students, to facilitate and stimulate exploration of mathematics as well as to make abstract concepts concrete. The program makes concrete learning available to students at every level on an ongoing basis to support their conceptual understanding wherever it falls on the concrete-representational-abstract progression—and without pushing students toward abstraction before they have a solid foundation.

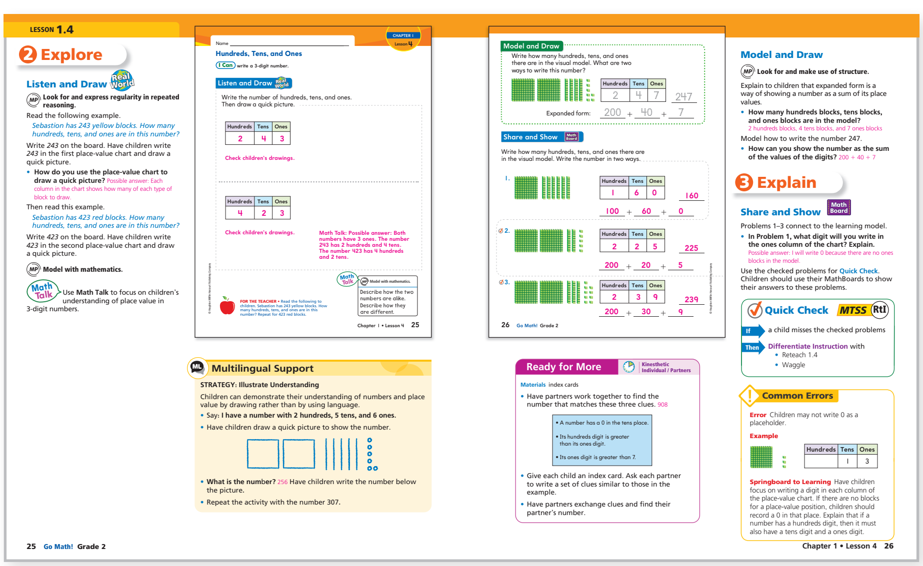
The Listen and Draw and Unlock the Problem tasks are carefully crafted to promote reasoning and problem-solving. Students can solve these tasks using different strategies. These low-floor/high-ceiling tasks give every student an entry point to be successful and build understanding within activities that reflect real-world situations. During these low-floor/high-ceiling tasks, students use their prior learning and choose manipulatives and models. Teachers provide just-in-time support supplied from within program content.

HMH GO Math! provides opportunities for students to choose manipulatives and tools to make sense of mathematics and express relationships. Flexible concrete and digital tools help students connect concepts to procedures and adapt their representations to different mathematical contexts. By seeing how students choose tools and which tools they choose, teachers also gain insight into the connections they're making.

In *HMH GO Math!*, students have access to manipulatives for hands-on exploration, as well as digital manipulatives to supplement and extend that exploration. Both concrete and digital manipulatives support them in making sense of situations, solving problems, and checking their reasoning. Manipulatives are included within Grab-and-Go® kits, which are designed to support differentiated instruction and are tailored to meet the needs of each student as they develop procedural skills and conceptual understanding.



The online experience also includes access to digital iTools to help students build conceptual understanding from a young age, with activities involving counters, base ten blocks, fractions, and more.



PROCEDURAL RELIABILITY

"[P]oor accuracy and fluency in second grade is a harbinger of slowed development of strategies for the complex arithmetic problems children encounter in second, third, and fourth grades and an increased likelihood of poor performance in later elementary school mathematics" (Carr & Alexeev, 2011, p. 629).

According to a skill mastery hierarchy established by Haring and Eaton (1978, in Poncy et al., 2007), the first stage in learning a new skill focuses on enhancing response accuracy. Because developing the ability to respond accurately is the first step to skill mastery, procedures designed to enhance accuracy can impact subsequent stages of skill development, which include fluency, generalization, maintenance, and adaptation. Poncy and colleagues' 2007 research into skill acquisition across various theoretical perspectives, such as cognitive processing, response effort, reinforcement, choice, opportunities to respond, and math anxiety, also suggests that increasing students' accuracy and speed of accurate responding to basic math facts is crucial for developing and mastering more advanced math skills. However, their research also suggests that when students are instructed to rely on using time-consuming multistep strategies for solving basic math facts (e.g., finger counting), the same strategies may interfere and possibly even prevent students from learning to solve basic math facts automatically (Hasselbring et al., 1987; Pellegrino & Goldman, 1987; Woodward, 2006). It appears that even working with manipulatives—commonly and effectively used at the lower elementary level to develop conceptual understandings—does not necessarily yield significant gains in children's accuracy (Grupe & Bray, 1999; Clements & Sarama, 2016).

Students' computational fluency and answer accuracy influence their strategy development and mathematics competency. When solving simple arithmetic problems, insufficient fluency in answer retrieval slows the transition from manipulative-based strategies to cognitive-based strategies (Carr & Alexeev, 2011).

Research by Poncy et al. (2007) shows that several instructional models can increase accuracy and automatic responding to basic mathematics facts (Garnett, 1992; McCallum et al., 2004)—and that these procedures produce high rates of active academic responding that can increase both speed of responding and maintenance, provided that the responses are accurate (Skinner et al., 1989). A key similarity among these procedures is that students should be provided with immediate performance feedback and reinforcement, which can prevent students from practicing errors (Skinner & Smith, 1992).

It is beneficial to embed the enhancement of students' mathematical accuracy within teaching procedure and strategies, as this facilitates both procedural reliability and strategy application; teaching students strategies that can be applied across many problems (such as using a number line to enhance addition accuracy) may be an efficient, effective, recursive means of enhancing accuracy across math facts (Poncy et al., 2007). Such procedures may additionally foster and reinforce students' conceptual understanding of the target task and related concepts (Garnett, 1992; Poncy et al., 2007).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! provides distributed, ongoing practice opportunities that promote reliability in problem-solving throughout the program sequence as well as in Practice and Homework. The program also supports teachers in anticipating common errors students make so that they can effectively redirect learning.

Common Errors

Error Students find the incorrect addition or subtraction value when defining a rule.

Example In Activity 2, students compare the first and second terms, 1 and 4, and define the rule as Add 4.

Springboard to Learning Remind students that the number they need to find is the number you add or subtract to get from one term to the next. Have students circle the first term on the addition table and count the number of squares it takes to get to the next term.

PROCEDURAL FLUENCY

"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems" (NCTM, 2014, p. 42). Conceptual understanding is knowledge of abstract and general principles, whereas procedural understanding is knowledge of the steps or actions between a goal that is then applied with varying degrees of fluency (Rittle-Johnson et al., 2015). "We must emphasize to parents, teachers, counselors, administrators and students that the goals of learning mathematics are multidimensional and balanced: Students must develop a deep conceptual understanding (why), coupled with procedural fluency (how), but in addition they also need the ability to reason and apply mathematics (when), and all while developing a positive mathematics identity and high sense of agency. All four goals are critical components of what it means to be mathematically literate in the 21st century" (Larson, 2017, online).

Procedural fluency is a critical component of mathematical proficiency. One aspect is the "efficient, appropriate, and flexible application of single-digit calculation skills" (Baroody, 2006, p. 22). However procedural fluency is about more than memorizing facts or steps, it entails the following capacities: to apply procedures accurately, efficiently, and flexibly; to build and modify procedures as well as transfer them to different problems and contexts; and to recognize when one strategy or procedure is more appropriate to apply than another. In developing procedural fluency, students need experience integrating concepts and procedures and understanding patterns among them. Students also need to build on familiar procedures in the process of creating their own informal strategies and procedures via opportunities to support and justify their choices of appropriate procedures.

To be mathematically proficient, students need a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations—and the goal for students developing procedural fluency is to acquire a body of known facts and generalizable methods that will allow them to efficiently and accurately solve varied problems (Kling & Bay-Williams, 2014; NCTM, 2014; NRC, 2001, 2005, & 2012; Star, 2005).

Among math educators in the United States, a historical tension has existed between understanding and fluency; on one side is an emphasis on exploration facilitated by sensory experiences with objects and open-ended activity, and on the other is a focus on rote practice and worksheets without attention to the construction of meaning. However, as Fuson urges in addressing this divide (2009), students learning math are best served by a balanced approach that is child-centered within a teacher-guided structure based on individualized pathways driven by students' progress and need and featuring a dual focus on both understanding and fluency. "Procedural knowledge and conceptual understandings must be closely linked" (NRC, 2005, p. 232) and effective mathematics cannot have one without the other, for concepts and procedures develop in tandem and iteratively, with gains in one supporting gains in the other (NCTM, 2014; Rittle-Johnson et al., 2015). When students are able to connect procedures and concepts, and when learning is meaningful, retention improves and students are better able to apply what they know in different situations. If students memorize and practice procedures without conceptual understanding, they lack capacity to apply procedures and the motivation to use them effectively (Fuson et al., 2005; Hiebert, 1999).

To develop students' fluency in procedures, teachers should build on a foundation of conceptual understanding; support students in looking for patterns; allow students to flexibly choose among solution methods; and offer distributed opportunities for purposeful, meaningful practice—not rote, repeated practice (Baroody, 2006; Fuson & Beckmann, 2012/2013; Fuson et al., 2005; Fuson & Murata, 2007; Rohrer, 2009). Research has found that certain instructional routines, many well established and commonly practiced, support the development of mathematical proficiencies, including conceptual understanding, strategic competence, adaptive reasoning, productive disposition, and procedural fluency (Berry, 2018).

Procedural fluency, in order to be meaningful, must also be applied to contexts outside of the mathematics classroom. Application is the purpose for learning math: We learn math so we can use it in situations that require mathematical knowledge. Any meaningful, motivational application of mathematical knowledge draws on both conceptual understanding and procedural fluency and provides a real-world, problem-based context (Alberti, 2013; David & Greene, 2007; Cross et al., 2009; Gaddy et al., 2014; NCTM, 2014).

Rittle-Johnson (2017) recommends three specific cognitive activities within learning tasks that promote the development of conceptual knowledge, procedural knowledge, and procedural flexibility simultaneously. One of these is comparison: comparing alternative processes for solving a problem as well as comparing correct versus incorrect procedures in solving a problem. Another is self-explaining: generating explanations to make sense of new information as well as explanations of solutions to math problems via in part by connecting new information and explanation to background knowledge. Third is exploration before instruction. Students who have opportunity to solve an unfamiliar problem or devise their own formulas and approaches to an unfamiliar problem in advance of receiving teacher-directed instruction typically demonstrate more positive gains and outcomes.

Ongoing, formative assessment is essential for teachers to monitor and develop students' procedural fluency skills; however, the type of assessment is critical. Timed tests commonly used to evaluate fluency have shown to be detrimental to children's fluency skills, and they are likely to induce math anxiety; instead, interviews, observation, and journals are more effective and supportive means of formative assessment to determine students' procedural fluency levels (Kling & Bay-Williams, 2014).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! problem-solving lesson tasks include real-world problems to promote procedural understanding and fluency. Teachers help students understand why the procedures are efficient and how they can be applied to solve similar problem types.

PATH TO FLUENCY • Activity

Identifying Patterns Within Patterns

Materials hundred chart, crayons

This activity provides students with an opportunity to extend their understanding of patterns, and to connect understanding of even and odd numbers with patterns on a hundred chart.

Investigate Students will work with a partner to create and identify patterns. Encourage students to be creative in the patterns they choose.

- One student records the first five numbers of a pattern by shading the boxes of the numbers on a hundred chart. The other partner states the pattern, and then extends the pattern as far as possible on the hundred chart. Partners then analyze the pattern to see the relationship of even and odd numbers to the pattern, and the relationship of the digits within the pattern numbers.
- For example, one partner might record the pattern 5, 16, 27, 38, 49. The second partner might identify the pattern as *add 11* and shade the boxes for 60, 71, 82, and 93. Together, partners would see that numbers in the pattern alternate between even and odd and that the difference between the tens digit and ones digit is first 5 and then increases to 6.
- Students take turns providing the pattern and identifying and extending the pattern. Students might use a different color crayon to record different patterns on the same chart.

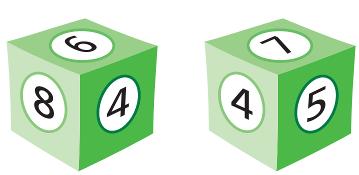
Summarize Ask students how their patterns might change if they started with a different number. Ask how starting with an even or odd number might affect the pattern.

Path to Fluency and Fluency Builder activities appear throughout the program, and fluency and other skills are developed via practice, homework, and games in *Waggle*. *Waggle* provides additional personalized, adaptive practice with standards- or skill-aligned content to build fluency and mastery.

Fluency Builder

Materials blank number cubes

Mental Math Have students work in pairs using two number cubes. Label the sides of one cube with 0, 1, 2, 4, 6, and 8. Label the sides of the other cube with 2, 3, 4, 5, 7 and 9. Have students toss the cubes and quickly name the sum of numbers.



COMMUNICATING MATHEMATICALLY

Research has long demonstrated that mathematical proficiency is about far more than numbers; indeed, having students communicate mathematically is an essential best practice in math learning. Back in 2000, the National Council of Teachers of Mathematics adopted a Communication Standard, which notes that "the communication process also helps build meaning and permanence for ideas and makes them public" (p. 60). "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and argument" (NCTM, 2014, p. 10).

As with all fields of learning, mathematics has its own language and, "like all language skills, learning the language of mathematics is an important goal for all students and can remove barriers to learning mathematical ideas" (Dacey et al., 2013, p. 149). While it is essential that students learn math-specific vocabulary, it is equally critical that students engage with that terminology and broader mathematical concepts through discourse. Mathematical discourse—speaking, writing, or listening about mathematics—is an important way for students to learn and make sense of mathematics; such communicative exchanges provide access to ideas, relationships among those ideas, strategies, procedures, facts, and mathematical history, as well as foster deeper understanding and positive attitudes toward mathematics (Morgan et al., 2014; Leinwand & Fleischman, 2004; Michaels et al., 2008; Smith & Stein, 2011).

"The teacher's role in discussions is critical. Without expert guidance, discussions in mathematics classrooms can easily devolve into the teacher taking over the lesson and providing a "lecture" on the one hand or, on the other, the students presenting an unconnected series of show-and-tell demonstrations" (Smith & Stein, 2018, p.4). Mathematical discourse—speaking, writing, or listening about mathematics via instructional practices such as restating, prompting, and engaging in whole-class discussion, small-group discussion, and paired conversations—is an important and effective way for students to learn and make sense of mathematics. Such communicative exchanges provide access to ideas, relationships among those ideas, strategies, procedures, facts, mathematical history, and more (Chapin et al., 2009) while also promoting positive attitudes toward mathematics (Michaels et al., 2008). "Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical for future success in mathematics" (Carpenter et al., 2003, p. 4).

Discourse within mathematics learning settings, especially when marked by teachers' encouragement that students verbalize their thinking and understanding and their provision of feedback to students on that shared verbalization, has been shown to benefit students across grade levels in their development of reasoning and problem-solving skills (Humphreys & Parker, 2015). Classroom discussions can be organized in ways that have been shown to support the acquisition of mathematics concepts and language development (Smith & Stein, 2018). Asking "why?" and "how do you know?" is one strategy that effective teachers use to encourage students to explain their thinking, solve problems, and share mathematical strategies and ideas with their peers (Clements & Sarama, 2004 & 2007; Thomson et al., 2005).

Discourse also provides teachers with rich assessment. "Mathematical conversations provide opportunities for teachers to hear regularly from their students and to learn about the range of ideas students have about a particular mathematical idea, the details supporting students' ideas, the values students attach to those ideas, and the language students use to express those ideas. The knowledge teachers gain from engaging with their students in conversations is essential for teaching for understanding" (Franke et al., 2007, p. 237).

A classroom in which meaningful communication and discussion are primary vehicles for learning and in which members co-construct and support one another's understanding is known as a "Math-Talk Learning Community;" within effective math-talk communities, teachers shift from the traditional role of directing all learning to one more like a coach or facilitator who promotes greater student agency (Hufferd-Ackles et al., 2004 & 2015; Saylor & Walton, 2018). "Math talk" conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Mercer & Howe, 2012; Zwiers, 2014). The frequency of teachers' math talk has been shown to correlate with students' increased mathematical knowledge (Klibanoff et al., 2006). "The informal and formal representations and experiences need to be continually connected in a nurturing 'math talk' learning community, which provides opportunities for all children to talk about their mathematical thinking and produce and improve their use of mathematical and ordinary language" (Cross et al., 2009, p. 43). "Math Talk" benefits students at different levels of learning and in different contexts, including English language learners (Hufferd-Ackles et al., 2004 & 2015). As students in transitional language classrooms engage in math talk, they "communicate verbally and in writing about their mathematical ideas; they not only reflect on and clarify those ideas but also begin to become a community of learners" (Bray et al., 2006, p. 138).

In a synthesis of empirical research examining writing used in elementary and secondary math instruction conducted over two decades, Powell et al. (2017) concluded that writing should be implemented systematically and explicitly, with appropriate scaffolds to support the development of math communication skills as well as to assess understanding of concepts and procedures. Writing during math instruction has also been found to give students more confidence in their math abilities, create more positive attitudes toward math, and help students to understand complex math concepts (Taylor & McDonald, 2007).

Writing can be effectively incorporated into the mathematics classroom in a wide variety of ways, both formal and informal (Urquhart, 2009). Researchers also cite journal writing as having positive impacts on math achievement and affective experiences and perceptions of math learning (Page & Clarke, 2014), as well as to aid the development of reasoning, sense-making, and discourse (Yow, 2015).

HOW *HMH GO MATH!* DELIVERS

In addition to lesson-level embedded, strategic Language Support, *HMH GO Math!* provides activities throughout that promote mathematical communication through writing, listening, and speaking. These include enrichment opportunities.

The program cultivates a Math Talk community with prompts embedded in each lesson, as well as Language Routine Cards to facilitate communication skill development and meaningful discourse among students. Math Talk activities build proficiency and confidence while promoting mathematical discourse. Math Talk prompts foster mathematical thinking and reasoning skills and habits within each lesson.

Math Talk

MP **Look for and make use of structure.**

Explain why you can use multiplication to check division.

PROFESSIONAL DEVELOPMENT | **MATH TALK IN ACTION**

The class is discussing why 5 is a benchmark for rounding.

Teacher:	Why do you compare the digit to the right of the rounding place to 5?
Mariah:	It tells you if you have to change the digit.
Teacher:	Suppose you want to round 427 to the nearest hundred. What digit do you compare to 5?
Mariah:	Look at the digit to the right of the 4. Compare 2 to 5 to see if the number is closer to 400 or 500.
Teacher:	Is 427 greater than or less than 450?
Melinda:	Less than 450, so 427 is closer to 400. I would round it to 400.
Nic:	Oh, I get it. When rounding to the nearest hundred, deciding if the number to the right is greater or less than 5 is the same as deciding if it is greater or less than the number halfway between. The 5 stands for 50!
Deanna:	If you round to the nearest 100, 5 stands for 50, so you are deciding if the number is more or less than halfway between the two hundreds.
Teacher:	Excellent! So remember, when you compare a number to 5, you are deciding if the number is more or less than halfway to the next ten or hundred.

DEVELOPING MATHEMATICAL HABITS OF MIND

In their seminal article, Cuoco et al. (1996) proposed that "more important than specific mathematical results are the habits of mind used by the people who create those results…. This includes learning to recognize when problems or statements that purport to be mathematical are, in truth, still quite ill-posed or fuzzy; becoming comfortable with and skilled at bringing mathematical meaning to problems and statements through definition, systematization, abstraction, or logical connection making; and seeking and developing new ways of describing situations" (p. 376).

Mathematical habits of mind reflect how mathematicians think about and through complex problems. Proficiency in mathematics requires students to engage in productive mindsets, adopt strategic approaches, and persist toward finding solutions. As Goldenberg et al. (2015) explain:

> The ability to solve new and unforeseen problems requires mastery not just of the results of mathematical thinking (the familiar facts and procedures) but of the ways that mathematically proficient individuals do that thinking. This is especially true as our economy increasingly depends on fields that require mathematics. Mathematical proficiency depends also on other mental habits that dispose one to characterize problems (and solutions) in precise ways, to subdivide and explore problems by posing new and related problems, and to "play" (either concretely or with thought experiments) to gain experience and insights from which some regularity or structure might be derived" (p. 1–2).

Beyond scholastic achievement in mathematics, mathematical habits of mind are essential to critical thought, college and career readiness, access to future opportunities, and productive participation in society (Goldenberg et al., 2015). "If we really want to empower our students for life after school, we need to prepare them to be able to use, understand, control, modify, and make decisions about a class of technology that does not yet exist. That means we have to help them develop genuinely mathematical ways of thinking" (Cross et al., 2009, p. 21).

According to Levasseur & Cuoco (2009), mathematical habits of mind develop as a by-product of teaching mathematics through problem-solving, in a process that entails modeling and reflection so that habits are internalized. Effectively problematizing mathematics has students thinking for themselves and explaining their thinking while also being supported by their teacher, classmates, and math program, as well as struggling productively and ultimately applying their gained knowledge and strategies to new and more complex problems they encounter in the future (Hiebert et al., 1996). Ultimately, problem-solving in the mathematics classroom encourages students to see that their actions can lead to intellectual growth, and this "focus on the potential of students to develop their intellectual capacity provides a host of motivational benefits" (Blackwell et al., 2007, p. 260).

To cultivate mathematical habits of mind, teachers also must create a classroom culture that demonstrates how struggle is a natural part of the learning process (Star, 2015) and allows students to see the benefits of perseverance (Hiebert & Grouws, 2007). Educators and students both must also adopt growth mindsets and positive views on productive struggle. These attitudinal states yield numerous positive affective outcomes and boost academic achievement (Dweck, 2006, 2008; NCTM, 2014).

Research shows that "productive struggle" is necessary to the process of learning mathematics with understanding; when students are given opportunity to grapple with ideas, make mistakes, persist through difficulties, and arrive at solutions, learning outcomes improve (Kapur, 2010 & 2014; Warshauer, 2015). Perseverance through problem-solving also encourages students to think about their own thinking and to discover that learning can happen without rushing to simply find the correct answer (Hiebert & Grouws, 2007). "Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics" (NRC, 2001, p. 131). To effectively foster students' productive struggles and dispositions, teachers must carefully select tasks and provide reassurance and support that students need to complete the tasks—but without diminishing the cognitive demand of the task or giving students too much help or direct answers. Students need sufficient time, not only to persist through productive struggle and devise solutions but also to develop curiosity and stamina (Goldenberg et al., 2015).

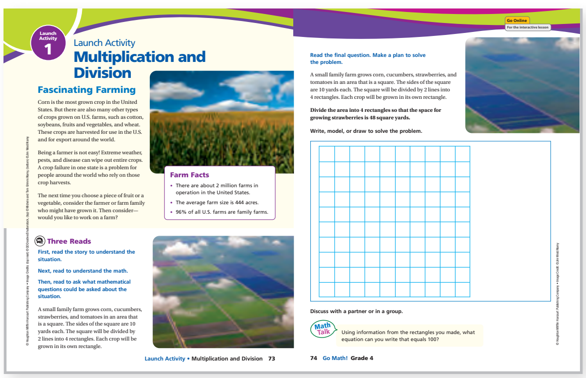
Timing of support also plays a vital role. When scaffolding is given to students before they have the opportunity to make sense of a challenging task independently, they are inhibited in the process of developing productive perseverance. Often within classrooms too much support is provided during initial scaffolding, resulting in a significant decrease in the cognitive demand of the learning task; if an excess of scaffolding is provided upfront for struggling students, then these students are denied access to constructive challenge and equity becomes an issue (Dixon, 2020).

Other practices that support productive perseverance include heterogeneous grouping, effective teacher-directed questioning, setting problems in a setting familiar to students and that draws from their everyday lives, plus goal setting before and reflection after problem-solving (Pasquale, 2016). "Math talk" is also an essential component of mathematical thinking (Cuoco et al. 1996).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! engages students in mathematical thinking and learning with the Launch Into activities that focus on critical areas of emphasis. The program offers immediate relevance and relatability by rooting each Launch Into activity within a real-world context, fostering exploration at each grade level and teaching students about the geography, environment, culture, and other interesting aspects of their home state. Through this grounding of new content learning, students also gain a mathematical perspective on the world around them.

The program also embeds math problem-solving activities within real-world context and connections, which facilitates application of math content learning as well as stimulates interest and provides motivation. Additionally, *GO Math!* fosters mathematical mindsets and flexible thinking via Another Way problem-solving. The program further aids students' awareness of the purpose for learning specific math skills and concepts by providing teachers with About the Math, which provides ready responses to student inquiries into the "whys" of learning math.



1 Engage

with the Interactive Student Edition

I Can Objective

I can use strategies to solve addition problems.

Making Connections

Invite students to tell you what they know about addition.

- **What are clues in a word problem that tell you to add?** Possible answers include sum, together, all, plus
- **What number is in the tens place of 145?** 4
- **What number is in the ones place of 145?** 5

Learning Activity

Tell a story about there being 112 frogs in a pond yesterday and 134 more frogs in the pond today.

What is the problem the students are trying to solve? Connect the story to the problem.

- **How many frogs were in the pond yesterday?** 112 frogs
- **How many frogs are in the pond today?** 134 more frogs
- **What is the problem asking you to do?** find the total number of frogs in the pond for both days

Another Way

MP **Use appropriate tools strategically.**

Have students read the problem. Make sure they understand that this is a two-step problem. As students complete the steps of the problem, ask the following questions:

- **How can you use the bar model in Step 1?** I can use the bar model to find the number of candles sold in September and October.
- **How can you use the information in the problem to label the bar model in Step 1?** I can label the shorter box 132 to show the number of candles sold in September. I can label the longer box 161 to show the number of candles sold in October.
- **What does the bar model in Step 2 show?** The bar model in Step 2 shows the total number of candles sold in September, October, and November.
- **How are the bar models related to the equations that represent them?** Possible answer: The addends are in the boxes, and the sum is the number below the boxes.

MATH in the Real World

The Arctic Lion's Mane Jellyfish is one of the largest known animals. Its tentacles can be as long as 120 feet. Find how this length compares to your height. Round your height to the nearest foot. 120 feet is ______ times as long as ______ feet.

The program also helps students establish cross-curricular connections to the math they are learning, making for richer, more integrated understanding and concept development as well as additional purpose.

CROSS-CURRICULAR

SCIENCE

- Some stars, like the North Star, look brighter than others. The sun is the brightest star that we can see from Earth.
- At night, some planets glow like stars, but they don't twinkle. You can see Venus and Mars from Earth, and sometimes you can see Mercury through a telescope.
- One year on Earth is 365 Earth days long. One year on Venus is about 224 Earth days long. About how much longer is a year on Earth than a year on Venus? Possible answer: about 150 days

SOCIAL STUDIES

- The United States consists of 5 regions—the Northeast, Southeast, Midwest, Southwest, and West.
- Delaware and Rhode Island are the smallest states. They are both located in the Northeast region.
- The highest point in Rhode Island is at an elevation of 812 feet above sea level. The highest point in Delaware is at an elevation of 448 feet above sea level. About how many feet taller is the highest point in Rhode Island than the highest point in Delaware? Possible answer: about 350 feet

TEACHING FOR DEPTH

Elementary-level mathematics programs must build a strong foundation of deep mathematical understanding. "When mathematics instruction goes deep, children are empowered to explore the richness of the mathematical landscape" (NCTM, 2020, p. 77). By emphasizing critical thinking, problem-solving, and mathematical states of mind, educators urged to adopt an approach that prioritizes depth over breadth (Noguera et al., 2015).

Reviews of math curricula suggest that a greater focus on fewer core mathematical ideas at each grade yields a greater depth of understanding that results in higher levels of content mastery (Cobb & Jackson, 2011). "The mathematics curriculum in Grades PreK–8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades" (National Mathematics Advisory Panel, 2008, p. xiii). Examinations of teaching in American mathematics classrooms concurrent with standards reform efforts of the past few decades have shown a lack of depth and rigor, as well as diffused coverage of content (NRC, 2001; Schmidt, 2012). In international comparisons of math and science performance, the countries at the top generally present students with fewer topics but at greater depth and increased coherence (National Mathematics Advisory Panel, 2008; Schmidt et al., 2005). A study exploring correlations between math textbook tasks and achievement on standardized exams found that if a textbook provides opportunity to engage in tasks demanding higher levels of understanding, students using the textbook will have higher scores (Hadar, 2017). A review of U.S. mathematics textbooks found that that among all 4,855 learning tasks included in the sample, only about 40% offered opportunities for students to engage in reasoning and proving activities at least once, and more than 50% provided no opportunity at all for student engagement at that deeper level (Stylianides, 2009). A comparison of how selected middle school-level textbooks from China and the United States represent various types of mathematics problems such as routine problems vs. non-routine problems, open-ended problems vs. close-ended problems, and traditional problems vs. non-traditional problems revealed that, in both countries, more than 96% of problems were routine and traditional, more than 93% were close-ended problems, and more than 92% were irrelevant to real-world situations (Zhu & Fan, 2006).

In the most recent TIMSS results, U.S. fourth graders ranked 15th among the 64 participating education systems in average TIMSS mathematics scores, and U.S. eighth graders ranked 11th among the 46 participating education systems in average TIMSS mathematics scores. The United States had higher average scores than most participating countries in both mathematics and science at both the fourth and eighth grades. However, in 2019, the United States had relatively large score gaps between the top- and bottom-performing students in both TIMSS subjects and grades. In eighth-grade mathematics, only one of the 45 other education systems (Turkey) had a larger score gap between the top-performing (90th percentile) and bottom-performing (10th percentile) students than the United States. (NCES, 2021).

Singapore has scored at the top in results for the Trends in International Mathematics and Science Study (TIMSS) for grade 4 and grade 8 for over 20 years, while the United States has been surpassed internationally in its mathematics performance (Mullis et al., 2016). Experts commonly attribute Singapore's success in mathematics to the country's strong emphasis on problem-solving. In an exploratory study comparing key features of the Singapore and U.S. mathematics systems in the primary grades, Ginsberg et al. (2005) found that a major reason why Singaporean students were more successful than their U.S. counterparts is because their country's mathematics textbooks build deep understanding of concepts through multistep problems and concrete illustrations that demonstrate how abstract mathematical concepts are used to solve problems from different perspectives.

Underpinning deep mathematical understanding are reasoning and sense-making skills (NCTM, 2020). Mastery of mathematics content at greater depth can be indicated by capacity to solve problems of greater complexity (i.e., where the approach is not immediately obvious), demonstration of creativity and imagination, independent exploration and investigation within mathematical contexts and structures, and communication of results that clearly and systematically explain and generalize the mathematics. (Askew et al., 2015). "Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency" (p. 4).

Noguera and colleagues (2015) emphasize that essential to efforts to expand access to deeper learning are equity-based reforms and resources that enable all teachers, particularly those of students from disadvantaged backgrounds to:

- create ambitious and meaningful tasks reflecting how knowledge is used in the field,
- engage students in active learning in which they apply and test what they know,
- draw connections to students' prior knowledge and experiences,
- diagnose understanding so as to sequentially scaffold the learning process,
- assess learning continuously and adapt teaching to student needs,
- encourage strategic and metacognitive thinking so that students evaluate and guide their own learning, and
- provide clear standards, constant feedback, and opportunities for revising work.

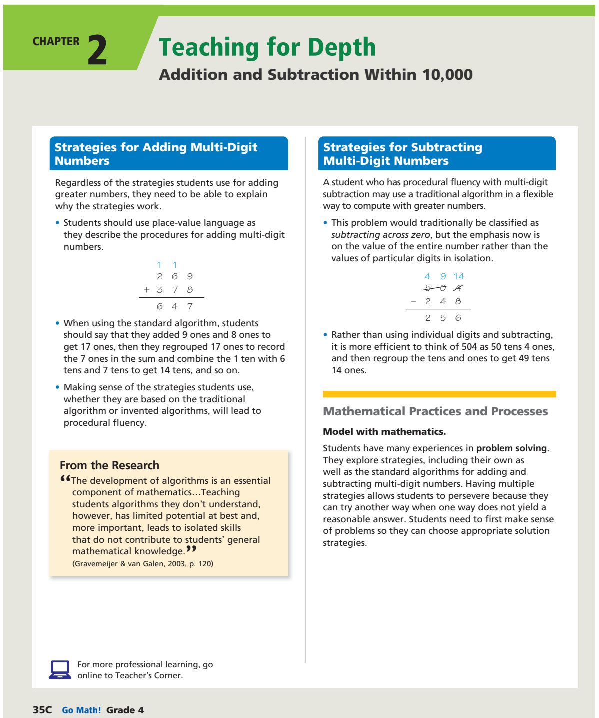
Academic standards should "promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible" (Achieve, 2010, p. 1).

Developing a deep understanding of mathematics results from a solid conceptual understanding with an emphasis on sense-making and reasoning and through a coherent, cohesive instructional approach at the early childhood and elementary level (NCTM, 2020). In the elementary grades, students must develop understanding and make use of the big ideas in mathematics and problem-solving tasks in ways that also contribute to understanding of those big ideas. Mathematics learning requires students to use specific mathematical-reasoning processes, also known as "big ideas," across domains. These big ideas constitute overarching concepts that connect multiple concepts, procedures, or problems within or across domains or topics. They also serve as an important aspect of the process of forming connections and acquiring background knowledge that can be applied to expand later understanding (Cross et al., 2009).

Schmidt and Houang (2002) emphasize that what and how students are taught should reflect the key ideas within an academic discipline that determine how knowledge is organized and generated within that discipline. A coherent set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline, with these deeper structures then serving as a means for connecting the particulars (such as an understanding of the rational number system and its properties).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! supports educators in teaching for deeper, more meaningful learning. Each chapter includes a dedicated Teaching for Depth guide that provides background about the best practices and research related to the targeted concepts.



CHAPTER 2

Teaching for Depth

Addition and Subtraction Within 10,000

Strategies for Adding Multi-Digit Numbers

Regardless of the strategies students use for adding greater numbers, they need to be able to explain why the strategies work.

- Students should use place-value language as they describe the procedures for adding multi-digit numbers.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 2\ 6\ 9 \\ +\ 3\ 7\ 8 \\ \hline 6\ 4\ 7 \end{array}$$

- When using the standard algorithm, students should say that they added 9 ones and 8 ones to get 17 ones, then they regrouped 17 ones to record the 7 ones in the sum and combine the 1 ten with 6 tens and 7 tens to get 14 tens, and so on.
- Making sense of the strategies students use, whether they are based on the traditional algorithm or invented algorithms, will lead to procedural fluency.

From the Research

“The development of algorithms is an essential component of mathematics...Teaching students algorithms they don’t understand, however, has limited potential at best and, more important, leads to isolated skills that do not contribute to students’ general mathematical knowledge.” (Gravemeijer & van Galen, 2003, p. 120)

For more professional learning, go online to Teacher’s Corner.

Strategies for Subtracting Multi-Digit Numbers

A student who has procedural fluency with multi-digit subtraction may use a traditional algorithm in a flexible way to compute with greater numbers.

- This problem would traditionally be classified as *subtracting across zero*, but the emphasis now is on the value of the entire number rather than the values of particular digits in isolation.

$$\begin{array}{r} 4\ \ 9\ 14 \\ \cancel{5}\ \cancel{0}\ \cancel{4} \\ -\ 2\ \ 4\ \ 8 \\ \hline 2\ \ 5\ \ 6 \end{array}$$

- Rather than using individual digits and subtracting, it is more efficient to think of 504 as 50 tens 4 ones, and then regroup the tens and ones to get 49 tens 14 ones.

Mathematical Practices and Processes

Model with mathematics.

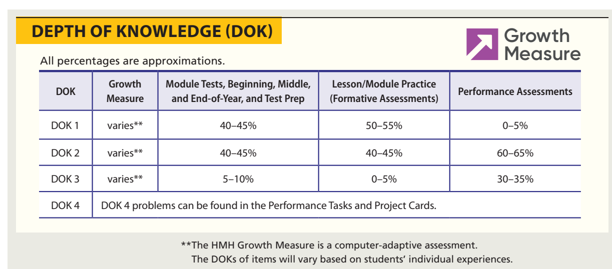
Students have many experiences in **problem solving**. They explore strategies, including their own as well as the standard algorithms for adding and subtracting multi-digit numbers. Having multiple strategies allows students to persevere because they can try another way when one way does not yield a reasonable answer. Students need to first make sense of problems so they can choose appropriate solution strategies.

35C Go Math! Grade 4

The program’s Teaching for Depth feature helps teachers accomplish the following:

- Identify ways the chapter lessons and activities can be extended into opportunities for students to achieve mathematical thinking and reasoning.
- Gain understandings of common errors and misunderstandings students may have as they begin work on the chapter. It then identifies tools and strategies that students can use to become proficient.
- Explore more deeply the research behind the pedagogy and strategies used within the *GO Math!* program.
- Explore specific ways the lessons in the chapter can support mathematical thinking and reasoning.

The *GO Math!* Teacher’s Edition also provides Depth of Knowledge (DOK) indicators for assessment items across the program.



DEPTH OF KNOWLEDGE (DOK)

Growth Measure

All percentages are approximations.

DOK	Growth Measure	Module Tests, Beginning, Middle, and End-of-Year, and Test Prep	Lesson/Module Practice (Formative Assessments)	Performance Assessments
DOK 1	varies**	40–45%	50–55%	0–5%
DOK 2	varies**	40–45%	40–45%	60–65%
DOK 3	varies**	5–10%	0–5%	30–35%
DOK 4	DOK 4 problems can be found in the Performance Tasks and Project Cards.			

**The HMH Growth Measure is a computer-adaptive assessment.
The DOKs of items will vary based on students’ individual experiences.

DEVELOPING THE WHOLE CHILD—AND SUPPORTING ALL CHILDREN

A positive and valued mathematics identity at a young age is key to building a strong mathematical foundation (NCTM, 2020). All students must be empowered, holistically, to participate meaningfully in learning mathematics. While improved access to quality mathematics instruction and college pathways remains essential, other critical factors must also be part of efforts to remedy issues of equity, including ensuring that today's young people enjoy positive mathematical experiences that inspire them to embrace and engage with a math-centric future (Larson, 2018). "The question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all" (NCTM, 2014, p. 61).

Early experiences and performance in mathematics yield effects well beyond classrooms, with consequences affecting economic prosperity, well-being, and quality of life. While mathematics achievement on every scale requires that all students be expected to meet rigorous standards, each student comes to school with a unique background, skill set, perspective, and set of strengths and needs—and therefore must receive effective, individualized support to realize and enjoy success in math learning (Clements & Sarama, 2020; NCTM, 2014; National Mathematics Advisory Panel, 2008; Shapka et al., 2006). Despite the obstacles that some children, particularly those from historically underserved populations, endure, all children are capable of learning and performing in math at high levels. A large body of research has documented that significant positive outcomes are possible when schools and teachers address issues of equity and access (Gutiérrez, 2013; Kisker et al., 2012; Lawrence-Brown, 2004; Lipka et al., 2007; McKenzie et al., 2011). "Providing young children with extensive, high-quality early mathematics instruction can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systematic inequities in educational outcomes" (Cross et al., 2009, p. 2).

"[W]e embrace a perspective on equity that supports teaching practices and reflective tools focused on empowerment of the whole child…. All students, in light of their humanity—their personal experiences, backgrounds, histories, languages, and physical and emotional well-being—must have the opportunity and support to learn rich mathematics that fosters meaning-making, empowers decision-making, and critiques, challenges, and transforms inequities and injustices. Equity does not mean that every student should receive identical instruction. Instead, equity demands that responsive accommodations be made as needed to promote equitable access, attainment, and advancement in mathematics education for each student" (Aguirre et al., 2013, p. 9).

HMH GO Math! supports the whole child, allowing all students to perceive themselves as capable learners—socially, emotionally, and culturally—to become proficient, secure, valuable, and contributing members of diverse communities. Through a wide variety of resources and data points, the program monitors and meets the evolving needs of individual students to help them succeed in math—and life beyond the classroom.

DEVELOPING THE WHOLE CHILD—AND SUPPORTING ALL CHILDREN

PROMOTING EQUITY AND HIGH STANDARDS FOR ALL LEARNERS

"Just, equitable, and inclusive learning opportunities for all students demand change in institutional structures, teaching, and learning environments, and individual beliefs and actions" (NCTM, 2020, p. 25). For over two decades, the National Council of Teachers of Mathematics has advocated for more equitable practices that ensure all students succeed in learning math, as well as for recognition that equity requires diversity of support. From NCTM's Principles & Standards for School Mathematics (2000): "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p. 12).

In its 2015 position statement on Access and Equity in Mathematics Education, NCTM also urges broad attitudinal shifts on the part of educators:

> To close existing learning gaps, educators at all levels must work to achieve equity with respect to student learning outcomes. A firm commitment to this work requires that all educators operate on the belief that all students can learn. To increase opportunities to learn, educators at all levels must focus on ensuring that all students have access to high-quality instruction, challenging curriculum, innovative technology, exciting extracurricular offerings, and the differentiated supports and enrichment necessary to promote students' success at continually advancing levels. Providing all students with access is not enough; educators must have the knowledge, skills, and disposition necessary to support effective, equitable mathematics teaching and learning. (online)

Despite continually growing demands for a STEM-trained workforce (Langdon et al., 2011) as well as shrinking achievement gaps, historically underrepresented groups, which include female, African American, Latinx, and Native American learners, as well as English learners (ELs), students in poverty, and those with disabilities, remain marginalized in STEM education and professions, including specifically in mathematics (Anwar et al., 2019; Jackson et al., 2021; Kang et al., 2019; Sneider & Ravel, 2021). American classrooms today are increasingly diverse; individual students have wide-ranging needs, but they are also best served when their own experiences and backgrounds are valued and leveraged in the course of their learning experiences. All students need to learn mathematics, and with appropriate and differentiated support, all students are capable of success in mathematics. It is vital that educators understand that achievement gaps are not caused by factors such as cultural differences, poverty, and parental education levels, but rather by pervasive inequalities that have historically afforded significantly fewer resources and opportunities to certain groups (Aguirre et al., 2013; Cross et al., 2009; Flores, 2007; Gutiérrez, 2013; Lawrence-Brown, 2004; NCTM, 2014; Ukpokodu, 2011).

Yet, in addressing issues of equity and access, calls are increasing for educators to shift away from perceptions that students from historically disadvantaged backgrounds are deficient; rather, educators are encouraged to adopt a culturally responsive approach in which the distinct cultural, linguistic, and environmental experiences that students bring to school are viewed as assets to be respected, embraced, and leveraged to optimize learning for individual students as well as their peers (Aguirre et al., 2013; Flores, 2007; Gutiérrez, 2013; Lawrence-Brown, 2004; NCTM, 2014 & 2015; Ukpokodu, 2011; Xenofontos, 2019. "[M]any of the critical challenges facing racial and ethnic minority students in the formation of strong, positive mindsets for academic achievement can be alleviated through the careful work of creating supportive contexts that provide consistent and unambiguous messages about minority students' belonging, capability, and value in classrooms and schools" (Farrington et al., 2012, p. 34).

Promoting student engagement by selecting challenging tasks, exerting intense effort and concentration in the implementation of tasks, framing mathematics within the growth mindset, acknowledging student contributions, and attending to culture and language play substantial roles in equalizing mathematics gains between poor and non-poor students (Battey, 2013; Kisker et al., 2012). Additionally, to create an environment in which the barriers that limit comprehensive student access to learning are removed, teaching allows for flexible methods of presentation, expression, and engagement by offering multiple examples, employing multiple media and formats, engaging in supported practices, and allowing flexible opportunities for demonstrating skill (Strangman et al., 2004).

NCTM (2014, 2015) calls for the following practices to address access and equity in mathematics education and thereby improve student outcomes and achievement on a wide range of measures, including assessments, dispositions, and persistence transcendent of students' racial, ethnic, linguistic, gender, and socioeconomic statuses:

- implementing rigorous standards and holding high expectations for all learners who additionally have access to high-quality mathematics curricula and instruction taught by skilled and effective instruction
- allowing adequate time for students to learn
- placing appropriate emphasis on differentiated processes that broaden students' productive engagement with mathematics and that support individual students as needed and appropriate
- monitoring student progress and making needed accommodations accordingly
- leveraging strategic use of human and material resources

HOW *HMH GO MATH!* DELIVERS

GO Math! offers effective instruction tailored to the strengths, interests, and experiences of individual students. The program provides the tools for data-driven instruction and intervention, providing each and every student access to high-quality, relatable learning experiences as well as support needed to meet rigorous mathematics standards.

The program's student-centered lesson model is designed to nurture young mathematicians and the unique assets each brings to the classroom. Each lesson is comprised of the following components to meet the needs of each student in mastering rigorous mathematics learning:

- **Engage:** Students engage in activities and questions that tap into interest and inspire confidence to meet their individual readiness level
- **Explore:** Students make sense of a concept via exploration at greater depth.
- **Explain:** Students share what was learned with peers and their math community, and teachers have opportunities to assess understanding and adjust instruction and establish grouping arrangements to best serve individual students.
- **Elaborate:** Students participate in other activities to further build understanding of new concepts within novel contexts.
- **Evaluate:** Students reflect on their own understanding of new concepts.
- **Independent Practice:** Students reinforce and expand learning.

GO Math! with *Waggle* in tandem provides flexible, child-centered instruction and personalization, immersing students in rigorous learning, skills-based practice, and ongoing assessment—all within a multimedia experience to engage a variety of learning styles. Teachers also have available the HMH Growth Measure® indicators to effectively place children on personalized pathways of learning and practice, or choose the assignments that extend their instruction.

FOCUSING ON THE WHOLE STUDENT

Access Prior Knowledge

Review how to locate numbers on a number line. Draw a number line with intervals of 100 from 4,000 to 5,000. Have students help plot a point on the number line for each of the numbers below.

4,300 4,500 4,700

Ask questions such as:

- Describe how you located 4,300 on the number line. Possible answer: I looked at the thousands place. It shows 4 thousands. So, I start at 4,000 on the number line. I count the tick marks by 100 because that is what each unit stands for. I count to the 3rd tick mark for 300. I made a mark there for 4,300.

The program's Teacher's Edition clearly lays out the relevant prior learning lessons, resources, and tools that can be used for intervention. Other features support teachers in tapping into and celebrating the unique background knowledge each student brings to the classroom.

Productive perseverance and collaboration are supported through fluid student grouping based on ongoing assessment data as well as teacher support. Additionally, the program frames problem-solving within I Can statements, building student confidence.

Prompts for Productive Perseverance

For Launch Activity lessons, the exploration of math concepts is more critical than finding a solution. Children should be encouraged to think about new math ideas in an atmosphere that is conducive to learning, with minimal pressure. They learn to solve the problem in different ways and are able to choose the method that works well for them.

What if children can't start working or can't enter into the conversation for this lesson?
Use one or more of these opening prompts:

- *What information do you know about the problem?*
- *Can you draw a picture that represents what you know?*
- *What numbers are in the problem?*
- *What is given in the problem that might help you answer the question?*

How can I help children who are frustrated?
Ask these leading questions:

- *Think about a starting point. How can you enter into this problem?*
- *What information do you have?*
- *What are you working on? What have you done so far?*
- *What comes next? What are you solving for?*
- *What information do you need to get unstuck? Talk to your partner (or group).*

To increase children's understanding of their own thinking, ask:

- *Do you need to use all the numbers in the problem?*

Name ______________________

CHAPTER 2
Lesson 3

Use Place Value to Subtract

I Can use place value to subtract 3-digit numbers.

EMBEDDED SOCIAL-EMOTIONAL LEARNING

"[Experts] know that effective teachers do more than promote academic learning—they teach the whole child" (Yoder, 2014, p. 1). It is widely acknowledged that cognitive development is inextricably linked to social and emotional development; success in school depends upon students' social-emotional skills, and schools are increasingly adopting the practice of fostering such skills (Osher et al., 2016; Jones & Bouffard, 2012). Systematic social and emotional learning (SEL) is the process of facilitating students' development of knowledge, skills, attitudes, and behaviors that they need to understand and manage emotions, set and achieve positive goals, feel and show empathy for others, establish and maintain positive relationships, make responsible decisions, deal effectively and ethically with daily tasks and challenges, and prepare for success in college and careers (Collaborative for Academic, Social, and Emotional Learning [CASEL], 2020; Elias et al., 1997; Yoder, 2014).

Social and emotional learning has been integral within influential educational practices over the past century, such as those based on the theories of Dewey (1944), Vygotsky (1978), and Gardner (1993). In more recent decades, increased research and interventions have addressed underlying psychological and affective issues that impact academic success for individual students and entire school communities. Studies have linked childhood measures of motivation, time management, self-regulation, communication, and pro-social behaviors to students' later academic achievement. The same has been seen with adult outcomes across multiple domains, including higher education, employment, criminality, substance use, and mental health (Heckman, 2008; Jones et al., 2015). Other research demonstrates that social-emotional traits such as grit and self-discipline are greater predictors of academic achievement in adolescence than cognitive traits, such as IQ (Duckworth & Seligman, 2005; Duckworth et al., 2010). Research also shows that social and emotional skills are malleable and can be intentionally developed (Jones & Bouffard, 2012; Osher et al., 2016; Yeager & Walton, 2011).

"Through systematic instruction, SEL skills may be taught, modeled, practiced and applied to diverse situations so that students can use them as part of their daily repertoire of behaviors" (Durlak et al., 2011, p. 406). Abundant research demonstrates that effective SEL programming yields benefits impacting the trajectories of students' success within school and beyond. In a 2011 meta-review of 213 school-based SEL interventions, Durlak and colleagues found that, across diverse backgrounds and compared to students who did not participate in such programs, students who participated in social-emotional learning demonstrated the following: increased academic achievement, increased social-emotional skills, increased motivation, improved attitudes toward self and school community, improved positive social behaviors, decreased conduct issues, and decreased emotional distress. In a 2017 meta-analysis of 82 SEL interventions in K–12 schools worldwide and representing students of diverse socioeconomic status and ethnic backgrounds, Taylor and colleagues found that SEL program participants fared significantly better than controls in social- emotional skills, attitudes, and indicators of well- being and that the benefits were consistent regardless of students' race, socioeconomic background, or school location. Other research has shown that, within academic settings specifically, students who receive SEL instruction are more motivated to learn and more committed to school and less likely to engage in misconduct or suffer the consequence of behavioral issues such as class disruption, suspension, and grade retention (Zins et al., 2004). Another study showed that SEL leads to students seeking help when needed in managing their own emotions and problem-solving difficult situations (Romasz et al., 2004). Research also indicates that SEL has transformative potential to mitigate issues of equity and promote academic excellence for children of historically underserved racial groups (Jagers et al., 2019).

CASEL (2020) has established a research-based integrated framework that promotes interpersonal, intrapersonal, and cognitive competence, comprised of five core competencies that can be taught in many ways and across many settings:

SELF-AWARENESS

The ability to accurately identify, evaluate, and reflects one's own emotions, thoughts, and values and how they influence behavior. Also includes self-efficacy and the ability to accurately assess one's strength's and limitations, with a well-grounded sense of confidence and a "growth mindset."

SELF-MANAGEMENT

The ability to successfully regulate one's emotions, thoughts, and behaviors in different situations—effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals. Incorporates organizational skills.

SOCIAL AWARENESS

The ability to take the perspective of and empathize with others, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports. Includes respect for others and appreciation of diversity.

RELATIONSHIP SKILLS

The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. The ability to communicate clearly, listen well, cooperate with others, resist inappropriate social pressure, negotiate conflict constructively, and seek and offer help when needed. Also incorporates social engagement and teamwork.

RESPONSIBLE DECISION-MAKING

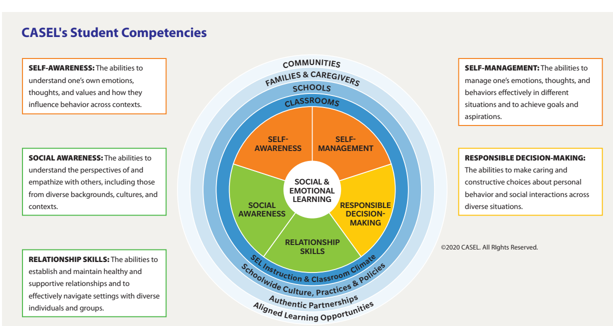
The ability to make constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms. Includes the realistic evaluation of consequences of various actions and consideration of the well-being of oneself and others. Skills entail identifying and solving problems, analyzing situations, evaluating, and reflecting.

Learning is enhanced for students when teachers integrate social-emotional competencies with academic instruction (Elias, 2006) and when students connect with information not just cognitively, such as through memorizing, but socially and emotionally as well (Ensign, 2003). As opposed to teaching social-emotional skills in isolation, an embedded approach optimizes both SEL instruction and academic content instruction. "A systemic approach to SEL intentionally cultivates a caring, participatory, and equitable learning environment and evidence-based practices that actively involve all students in their social, emotional, and academic growth. This approach infuses social and emotional learning into every part of students' daily lives—across all of their classrooms, during all times of the school day, and when they are in their homes and communities" (CASEL, 2020, online). Embedded SEL programming also has the advantage of economizing teachers' limited time by not burdening them with an additional separate initiative (Yoder, 2014).

HOW *HMH GO MATH!* DELIVERS

GO Math! facilitates a safe, supportive, and equitable learning environment that, in accordance with CASEL's principles (2020), helps mathematics teachers to:

- cultivate a sense of belonging and community where students see themselves as mathematicians, collectively striving to develop a deep understanding of mathematics,
- provide structures for physical and emotional safety so that mathematical sense-making is cognitively possible,
- create space for student voice and agency as a means to productively shape students' mathematical identities,
- provide tiered supports that meet the needs of all students to access and experience the joy, wonder, and beauty of mathematics,
- use engaging, relevant, and culturally responsive mathematics instruction built on an understanding of how students grow and develop socially, emotionally, and academically,
- offer frequent opportunities for students to discuss and practice anti-racism and develop collaborative solutions to address inequities using mathematics as a tool, and
- engage in mathematics teaching practices that affirm diverse social, cultural, and linguistic identities.



The program fosters students' development of in each of CASEL's social-emotional competencies as part of school-wide wrap-around support. Teacher Edition prompts encourage awareness, reflection, and practice in individual competencies and help students become confident in their math learning and capabilities.

FOCUSING ON THE WHOLE STUDENT

Social & Emotional Learning

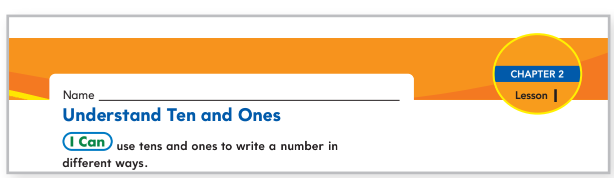
Social Awareness Build students' awareness of the importance of listening to others' reasoning. A learning mindset helps them discover there is more than one way to find an answer (and sometimes more than one correct answer). *When you are comparing strategies with a partner, it is important that you support the other person's learning as well as your own. Do you listen closely to the other person's reasoning? Can you describe how it is the same or different? When you work with a partner, you can learn more than one way to reach a solution.*

FOCUSING ON THE WHOLE STUDENT

Social & Emotional Learning

Self-Management Help students think about how to become unstuck and on their way to a solution without getting frustrated. *When you cannot get to a solution, what can you do to get unstuck? If you cannot get started, maybe reading the directions again will help. Try taking the directions apart to make sure you understand each part. Ask yourself if you understand the question and what you need to find. Another strategy is to talk to a partner about how to get started. In this lesson, does the drawing help you get started?*

GO Math! also frames problem-solving activities as "I Can" statements to help students feel confident from get-go and develop positive self-concepts in which they see themselves as capable learners.


Name ______________________
CHAPTER 2
Lesson 1
Understand Ten and Ones
I Can use tens and ones to write a number in different ways.

CULTURALLY RESPONSIVE MATHEMATICAL TEACHING

"Creating, supporting, and sustaining a culture of access and equity require being responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness. Acknowledging and addressing factors that contribute to differential outcomes among groups of students are critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful. Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement." (NCTM position statement on Access and Equity in Mathematics Education, 2015, online).

Momentum is building for replacing practices and policies that reinforce disparities in education and impede the success of far too many students (Muñiz, 2020). Culturally responsive teaching, also known as culturally relevant pedagogy, provides both understanding and guidance for educators seeking to improve the academic achievement of students from diverse racial, ethnic, cultural, linguistic, and socioeconomic groups. Fundamentally, culturally responsive teaching recognizes patterns of discontinuity between school culture and home and community culture for students of low-income backgrounds and students of color. Culturally responsive teaching also regards the experiences and perspectives that students bring to their classroom as sources of strength and knowledge that enhance academic learning (Gay, 2018). This approach also provides mainstream knowledge through different techniques, but it also involves transforming the actual perspectives, knowledge base, and approaches of a conventional classroom's curriculum and instruction" (Vavrus, 2008, p. 49).

Culturally responsive teaching is from an assets-based perspective that recognizes the unique and valuable strengths and experiences each child brings to the classroom. The success for all students requires that their unique experiences be validated and leveraged so as to enhance their learning (Lawrence-Brown, 2004). Growing research indicates that a crisis in mathematics learning among minority and low-income children is attributable to teaching practices that do not engage these students, and school policies and curricula that instead marginalize them—but CRT reform efforts have corrective outcomes (Ukpokodu, 2011). Aguirre and colleagues (2013) urge a holistic perspective on equity that has educators attending to and understanding the cultural identities of children of color and non-majority backgrounds in the context of local and broader social realities in which those children live.

Culturally responsive teaching connects students' cultural backgrounds and personal experiences, along with their performance styles to their academic knowledge and intellectual tools, in ways that value and leverage what students already know. In their curriculum and practices, culturally responsive teachers embrace the sociocultural histories and realities of their students, and transcend personal and institutional biases to establish and support broader perspectives and learning environments that are inclusive, engaging, and effective for all students (Gay, 2018; Kozleski, 2010; Lawrence-Brown, 2004; Vavrus, 2008).

To foster such shifts in education policy and practices, New America offers Culturally Responsive Teaching: A Reflection Guide (Muñiz, 2020), which draws on insights from research on culturally relevant, responsive, and sustaining pedagogies to foster culturally responsive education. The guide advocates for the following eight competencies:

- reflect on One's Cultural Lens
- recognize and Redress Bias in the System
- draw on Students' Culture to Shape Curriculum and Instruction
- bring Real-World Issues Into the Classroom
- model High Expectations for All Students
- promote Respect for Student Differences
- collaborate With Families and the Local Community
- communicate in Linguistically and Culturally Responsive Ways

The following are research-based practices for equitable, culturally responsive teaching:

- setting clear, rigorous expectations for all learners while also attending to each student's distinct cultural, cognitive, emotional, and psychological well-being and needs
- providing a range of high-quality, effective, and equitably distributed resources to support students
- drawing on students' unique funds of knowledge, recognizing diverse forms of culture, perspectives, language, and discourse are assets for learning and within a classroom environment
- allowing adequate time for students to learn
- establishing protocols and norms for broad participation in individual classroom activities and the learning process as a whole
- implementing differentiated processes for instruction that foster students' mathematical thinking and broaden students' productive engagement with mathematics in ways that also support individual students as needed, meeting them at their developmental level with a positive, appropriate level of challenge
- positioning students as capable, defiant of stereotypes, as well as agents in their own learning, and building a classroom culture in which students view their peers that way
- attending to race and culture and other differences and experiences
- monitoring student progress through fair and accurate assessment and making needed accommodations accordingly

"When classrooms are organized into communities that are designed to encourage academic and cultural excellence, students learn to facilitate their own learning as well as that of their fellow students. This kind of classroom requires careful planning and explicit teaching around social interactions so that students learn to assume leadership for learning, feel comfortable exploring differences of opinion, and accept that they may need help from their classmates in order to be successful. Along the way, students learn to see the classroom and their interactions from more than one perspective so that they can identify potential difficulties that come from assumptions of privilege, the distribution of power (who gets to make the rules), and the assessment of performance and competence." (Kozleski, 2010, p. 3)

HOW *HMH GO MATH!* DELIVERS

GO Math! fosters Culturally Responsive Mathematics Teaching (CRMT) practices that honor and leverage students' mathematical thinking, cultures, languages, identities, and backgrounds, as called for by New America's Eight Competencies for Culturally Responsive Teaching. In conjunction with SEL supports, the program's CRMT features focus on the wellness and academic success of all students.

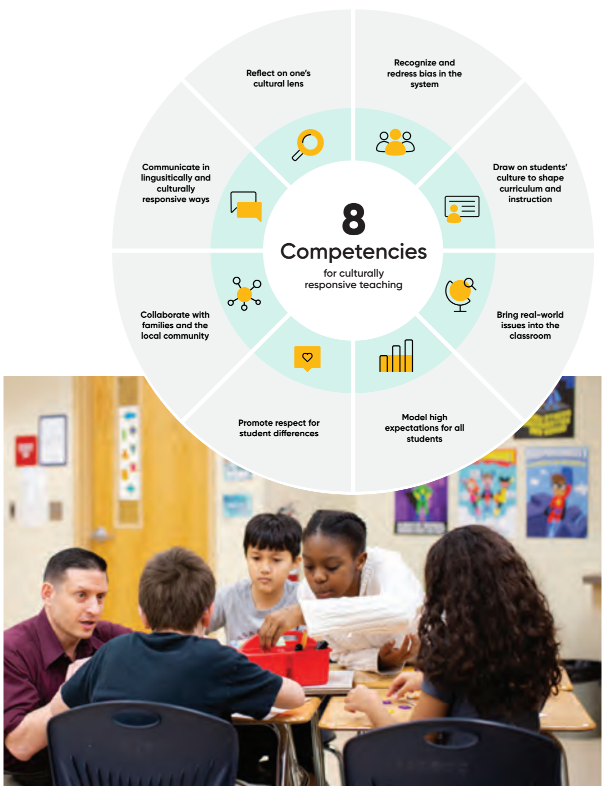
Student Editions are inclusive and diverse in the people and cultures reflected within math learning and problems they contain. Teacher Editions include guidance that encourages teachers to practice culturally responsive mathematics teaching and reflection, as well as prompts to facilitate engagement with the cultural backgrounds students bring to the classroom.

Consider the active and participatory communication styles of linguistically diverse children and how they engage in Math Talk.

SUPPORTING ALL LEARNERS

Embrace and honor the linguistic diversity of children by engaging them in relevant mathematics tasks with multiple paths for success.

Image Credit: © HMH



GO Math! offers effective instruction tailored to the strengths, interests, and experiences of individual students. The program provides the tools for data-driven instruction and intervention, providing each and every student access to high-quality, relatable learning experiences, as well as support needed to meet rigorous mathematics standards.

FOCUSING ON THE WHOLE STUDENT

Culturally Responsive Education

The relationships that you build with the students in your class are instrumental to student success. It is important to show students that you genuinely care for their academic and overall well-being, particularly when it comes to students with disabilities and students who are neurodiverse or who are culturally or linguistically diverse. When students see themselves as different in some way, they are more prone to feel ostracized; when students feel included, they are better prepared to learn.

Access Prior Knowledge

Use *i*Tools: Counters to review basic multiplication facts. After students give a correct product, ask:

- **What division equation could the counters represent?** Possible answer for 3 × 5 = 15: 15 ÷ 3 = 5

After the last basic fact you review, ask:

- **Why can the same model be used for both multiplication and division?** Possible answer: Because you are using the same equal groups for both. For multiplication, you find the total number in the equal groups. For division, you separate into equal groups.

DIFFERENTIATION: INTERVENTION AND ENRICHMENT

In classrooms around the United States, a significant number of students have some academic, social, or behavioral difficulties due to a variety of reasons, ranging from issues in the school or home environment or atypical cognitive, emotional, or psychological functioning. But regardless of the source of issues, schools must intervene as needed to effectively solve problems prohibiting a student's progress (Riley-Tillman et al., 2020).

A widely implemented framework in K–12 schools, a multi-tiered, data-driven system of supports (MTSS) is utilized to address the academic, social, and behavioral needs of all students based on their specific competencies and needs and the premise that, with adequate support and empirical approaches to interventions, all students are capable of grade-level learning (Harlacher et al., 2014; Riley-Tillman et al., 2020; Ziomek-Daigle et al., 2016).

Cognitive development and academic success are inextricably linked to social and emotional development (Osher et al., 2016; Jones & Bouffard, 2012), and early interventions that promote children's social and emotional behaviors can have lasting positive effects on well-being, employment, mental health, and quality of life in adulthood (Heckman, 2008; Jones et al., 2015).

Teachers today face the challenge of meeting the needs of an increasingly diverse student population, representing a wide array of cultural and linguistic backgrounds, prior knowledge, readiness, interests, motivations, home situations, and learning styles. While it is critical that all students have high expectations for learning as well as access to high-quality instruction, it is also necessary that all students receive the supports and differentiation they need, regardless of their socioeconomic contexts, if successful outcomes are to be achieved (Gutiérrez, 2013; NCTM, 2014; Tomlinson, 1997, 2005).

As Vygotsky (1978) noted in his seminal research on learning, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level" (p. 86). Research continues to support the notion that, for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000). In addition to meeting students at their present level of learning, to provide instruction that is effective and engaging to all students, teachers must incorporate flexible methods of presentation, employ multiple media formats, adopt supportive practices, and allow varied, multiple opportunities for demonstrating skill (Hall, Vue, Strangman, & Meyer, 2004).

Students struggling with mathematics benefit from early identification as well as from resolutions that may prevent subsequent difficulties (Gersten et al., 2009). The MTSS approach can be used in determinations and implementations of differentiated support and Response to Intervention (RtI). Differentiated instruction and RtI are complementary approaches and share a goal of modifying instruction until it meets the needs of all learners (Allan & Goddard, 2010). Differentiated instruction provides a way to respond to the needs of diverse learners in the classroom and remedy issues, whereas RtI provides a structured system for prevention, identifying difficulties and needs early on and offering tiers of support with which to intervene.

DIFFERENTIATED INSTRUCTION

Differentiated instruction is a well-established, evidence-based, organized approach to flexibly alter teaching that maximizes learning for all students and yields positive outcomes across students'

achievement levels (National Mathematics Advisory Panel, 2008; Stetson et al., 2007; Tomlinson, 1999). Differentiated instruction is also necessary in order to meet the diverse needs of learners (Tomlinson, 2000), particularly within mathematics classrooms (Chamberlin & Powers, 2010). A differentiated approach to instruction recognizes and supports the classroom as an inclusive community where students are nourished as individual learners and provided with an appropriate, motivating balance of challenge and success. In effective differentiated environments, all learners—those struggling and those advanced—can be successful (Lawrence-Brown, 2004). Differentiated classrooms are "responsive to students' varying readiness levels, varying interests, and varying learning profiles" (Kalbfleisch & Tomlinson, 1998, p. 54), and they offer students varying levels of expectations for task completion within a lesson or a unit based on their specific needs (McLeskey et al., 2001).

RESPONSE TO INTERVENTION (RTI)

Response to Intervention (RtI) is an early detection, prevention, and support system used to identify struggling students and provide assistance before they fall behind (Gersten et al., 2009). Effective Response to Intervention programs include systematic, ongoing assessment and collection of data to identify student needs and the use of effective interventions in response to the assessment data, which then are continually evaluated to determine their effectiveness and any future need of intervention (Griffiths et al., 2006).

Response to Intervention integrates instruction, intervention, and assessment to create a cohesive program that results in higher student achievement (Mellard & Johnson, 2008). "At the heart of the RtI model is personalized instruction, during which each students' unique needs are evaluated and appropriate instruction is provided, so that students will succeed" (McREL, 2010, p. 15). RtI comprises three levels of support:

- **Tier 1** represents the general curriculum and universal programming in terms of teaching and testing, which should meet the needs of roughly 80% of students.
- **Tier 2** includes some targeted supplementary support in the form of small-group instruction and progress monitoring.
- **Tier 3** encompasses intensive, specialized intervention or special education services at the individual student level and frequent progress monitoring (Riley-Tillman et al., 2020; Smith & Johnson, 2011).

In a What Works Clearinghouse review of RtI practices, Gersten and colleagues (2009) found strongest evidence to recommend the following: Interventional instruction should be explicit and systematic, to include providing models of proficient problem-solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review; and interventions should include instruction on solving word problems based on common underlying structures.

Studies examining the effectiveness of integrated systems such as RtI indicate that such approaches to interventions can lead to improvement in mathematics performance on various achievement measures when used to intervene with students who are under-performing in mathematics (Burns et al., 2005; Ketterlin-Geller et al., 2008). Fuchs and colleagues (2007) found that multiple tiers of intervention, "designed strategically to work in supplementary and coordinated fashion, may operate synergistically to decrease math problem-solving difficulties for children who are otherwise at risk for poor outcomes" (p. 19).

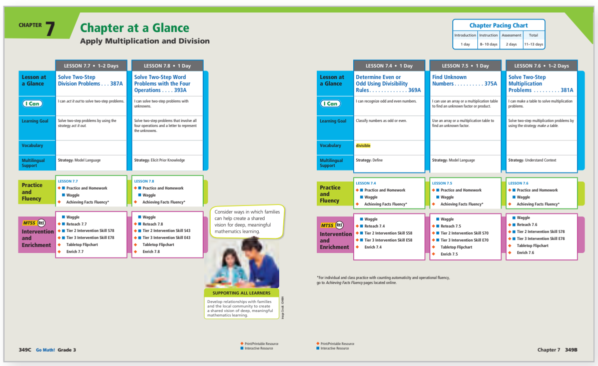
HOW *HMH GO MATH!* DELIVERS

Effective instruction begins with knowing students' strengths and challenges in real time. *HMH GO Math!* generates assessment data points through multiple sources and provides a continuum of print and digital support that allows for robust, targeted intervention and differentiation through each chapter.

Quick Check MTSS (RtI)

If a student misses the checked problems

Then **Differentiate Instruction** with
- Reteach 1.5
- Waggle



The program includes flexible options for differentiating instruction. Differentiation resources include Reteach, Challenge, and Additional Practice. These can be assigned to students who finish early (Enrich), or to students who are generally on grade level but who may need additional support (Reteach), or to provide further independent practice.

Interactive Lesson Reteach options aid in interpreting student performance and identifying follow-up support or intervention. Interactive Lesson Challenge provides enrichment opportunities. Ready for More allows for deeper exploration and learning for on- and above-level students.

Within the Teacher's Edition, Tier II and Tier III tools are clearly recommended and available online.

The program frames problem-solving activities as "I Can" statements to help students feel confident from the get-go and develop positive self-concepts in which they see themselves as capable learners.

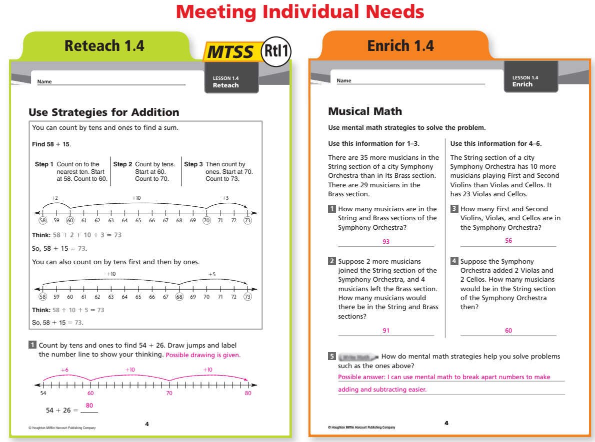
Intervention Options MTSS (RtI) **Response to Intervention**

Use Show What You Know, Lesson Quick Check, and Assessments to diagnose students' intervention levels.

TIER 1	TIER 2	TIER 3	ENRICHMENT
On-Level Intervention	**Strategic Intervention**	**Intensive Intervention**	**Independent Activities**
For students who are generally at grade level but need early intervention with the lesson concepts, use: • Reteach • Tabletop Flipchart Mini Lesson • Waggle Tier 1 Activity	For students who need small-group instruction to review concepts and skills needed for the chapter, use: Prerequisite Skills Activities Tier 2 Activity	For students who need one-on-one instruction to build foundational skills for the chapter, use: Prerequisite Skills Activities Tier 3 Activity	For students who successfully complete lessons, use: • Waggle Practice and Games Grab-and-Go! Version 2.0 Differentiated Centers Kit • Ready for More Activity for every lesson • Enrich

Show What You Know provides diagnostic assessments at chapter openers to zero in on students' prior and prerequisite knowledge about the topic, identifying gaps and guiding planning and decision-making grouping arrangements and RtI approaches.

Meeting Individual Needs



Ready for More

Visual Partners

Materials index cards

- Give each student 3 index cards.
- Have each partner write multiplication comparisons in words such as "Bing has 5 trophies. Su has 6 times as many trophies as Bing." on each of their index cards.
- Have students trade index cards with their partner. Students draw a model and write a multiplication expression for each problem on the back of the index card.
- Partners trade index cards again and check each other's work.

DYNAMIC LEARNING

Effective, equitable mathematics instruction nurtures within children a positive mathematical identity and a strong sense of agency; it allows children to see themselves as capable doers of math who construct their own mathematical understandings, as well as to experience firsthand the wonder, joy, and beauty of math (NCTM, 2020).

Research demonstrates that non-academic factors such as implicit math self-concepts and stereotypes are linked to students' actual math achievement and positively correlated to stronger—or weaker—outcomes at the individual student level (Cvencek et al., 2015). A learner's confidence in their own knowledge is rooted in the perception of being an active agent (rather than passive participant) working upon the world (Lawler, 2012). "The dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (Aguirre et al., 2013, p. 14).

Learning is an active process of engagement. Recursively, engagement leads to motivation, which leads to learning. When students are interested in what they are learning, they will spend the time and energy needed for learning to occur. Effective teachers know that students must be engaged by the content and activities presented to them to be motivated to persist in the learning process and, ultimately, to succeed in achieving learning targets (Eccles et al., 1998; Guthrie & Humenick, 2004). When students are actively engaged in the process of observing, reasoning, and making connections through hands-on learning, they acquire necessary skills and ways of thinking (Stewart et al., 2005). Teaching for depth entails active learning methods that allow students to apply and test what they know within authentic, meaningful tasks reflecting how knowledge is used in its corresponding field (Noguera et al., 2015).

Hands-on learning is critical to mathematical development (Ojose, 2008). Especially during the elementary level, children need to be able to use their senses and hands-on experimentation in order to test their thinking and find creative solutions (Thuneberg et al., 2017). As noted by Thuneberg and colleagues (2018), an interactive, hands-on approach to learning draws on Dewey's (1938) learning-by-doing principle and corresponds with Piaget's concrete operations stage of development. The concrete operations stage of development spans ages 7 to 11, representing most of the elementary years of school, and it is characterized by organizational and rational thinking—and remarkable cognitive growth as children's development of language and acquisition of basic skills accelerate dramatically, and yet their senses are still also utilized for learning about the world around them. "[H]ands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage [of concrete operational development]" (Burns & Silbey, 2000, p. 55). Whether using traditional activities, such as counting with beans or coins, or more sophisticated manipulatives, such as geoboards, pattern blocks, and tangrams, hands-on learning aids students in more readily understanding concepts and boosts their self-confidence (DeGeorge & Santoro, 2004). Manipulatives (both material and digital), visual models, and representations, which also to help make abstract mathematical concepts more concrete as well as aid in the internalization of procedures for problem-solving, increased creativity, led to greater metacognition, and promoted students' more active participation in their own learning—all of which contribute key elements for impactful mathematical exploration (Cross et al., 2009; NCTM, 2000 & 2014; NRC, 2001; Ojose, 2008).

Research shows that effective STEM education capitalizes on students' interests and experiences, identifies and build on what students know, and provides experiences to actively engage students in STEM-related practices and sustain their interest (NRC, 2011). Students can more effectively develop STEM concepts via interactions with digital models, simulations, and dynamic representations of mathematical, scientific, and engineering systems (U.S. Department of Education, 2019).

In STEM-learning contexts, active learning has been shown to yield to significant increases in assessment achievement and raise course grades over traditional lecturing approaches. "The analysis supports theory claiming that calls to increase the number of students receiving STEM degrees could be answered, at least in part, by abandoning traditional lecturing in favor of active learning" (Freeman et al., 2014, abstract).

Mathematics can be an impactful means of creating for students a lens to understand, critique, and create solutions for the world when students are actively engaged in the process of authentically doing mathematics (NCTM, 2020).

HOW *HMH GO MATH!* DELIVERS

The instructional journey provided to students within HMH *GO Math!* and its 5E approach ensure engagement in active learning within dynamic grouping and is tailored to differentiated individual needs.

Whole Group
Engage

5 minutes

Readiness
- Problem of the Day
- Fluency Builder or Vocabulary Builder
- Access Prior Knowledge

Engagement
- I Can
- Making Connections
- Learning Activity

Small and Whole Group
Explore

15–20 minutes

Exploration
- Investigate, Unlock the Problem
- Multilingual Support and Strategy
- Common Errors

Small Group
Explain

15–20 minutes

Quick Check
- Share and Show

Differentiated Instruction
Grab-and-Go!™ Version 2.0

Intervention
- Waggle
- Reteach
- Tier 2 and Tier 3 Resources

Language Support
- Vocabulary Activities
- Language Routines
- Multilingual Glossary

Enrichment
- Waggle Games
- Ready for More
- Enrich

Whole Group
Elaborate

5 minutes
- Math on the Spot Videos
- Higher-Order Thinking Problems

Evaluate
- I Can Reflection
- Exit Ticket
- Practice and Homework
- Fluency Practice
- Waggle

Active learning is further supported through ongoing opportunities for manipulative use and the construction of visual representations as well as peer-to-peer discourse, independent practice, and games. These learning activities are embedded within lessons and are further enhanced through the Grab-and-Go Differentiation Kit.

DIFFERENTIATED INSTRUCTION • Independent Activities

Grab-and-Go!™
Version 2.0
Differentiated Centers Kit

Tabletop Flipchart
Mini-lessons for reteaching to targeted small groups

Readers
Supports key math skills and concepts in real-world situations.

Games
Reinforce math content and vocabulary

Activities
Meaningful and fun math practice

CONNECTING WITH FAMILIES AND COMMUNITIES

2020's unprecedented shutdowns of school buildings due to the pandemic not only pushed digital learning into new ground but also reinforced what well-established research had previously indicated: Family engagement is an effective means of boosting student achievement, particularly for schools serving historically disadvantaged communities (Bansak & Starr, 2021; Barnard, 2004; Crosnoe, 2013; Dearing et al., 2007; Harper et al., 2021).

Abundant research demonstrates that, across geographic and socioeconomic factors, families have significant impacts on children's academic success, and increased communication and collaboration between teachers and caregivers is beneficial on multiple measures, to both individual students and to whole school communities (Barnard, 2004; Barton & Coley, 1992; Bryk et al., 2009; Clements et al., 2004; Hampden-Thompson et al., 2013; Henderson & Mapp, 2002; Jeynes, 2005; Mayer et al., 2000; Reynolds, 2000; Reynolds & Clements, 2005; Reynolds et al., 2002). In a large-scale survey of U.S. teachers, researchers found that "educators across all subgroups identify family involvement as the most critical factor of student success, followed closely by high expectations for all students" (Scholastic & Bill & Melinda Gates Foundation, 2012, p. 9). Another review of research suggests that family engagement is "beneficial across all levels of academic achievement for all minority groups, and particularly for Latino populations" (Park & McHugh, 2014, p. 2). "When parents are explicitly invited to engage in school mathematics, research suggests that they do so, regardless of race/ethnicity or education attainment level" (Harper et al., 2021, p. 17).

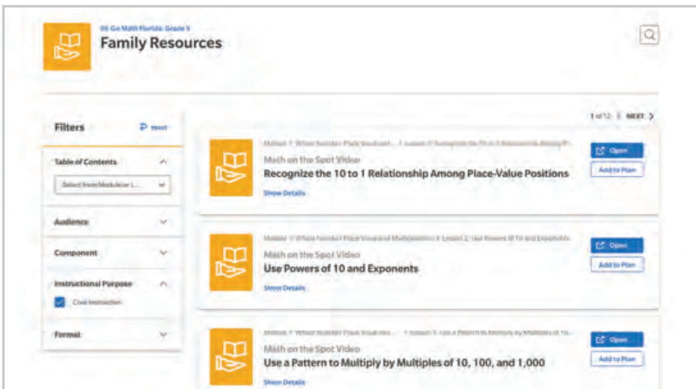
Family involvement is particularly important for multilingual learners and is additionally beneficial when the efforts feature cultural sensitivity (Chrispeels & González, 2004; Marschall, 2006; National Hispanic Caucus of State Legislators [NHCSL] & the Tomás Rivera Policy Institute, 2010). Schools' initiatives to boost at-home literacy experiences for multilingual learners via collaboration with parents has been shown to yield positive outcomes in language learning and general academic achievement, especially when parents receive training and supports (August & Shanahan, 2006; Rodriguez-Brown et al., 1999).

Research also suggests family involvement for children of lower socioeconomic backgrounds aids student understanding and achievement in mathematics particularly, and when it incorporates training in mathematical concepts for students' caregivers, the efforts can yield additional benefits (Berger & Riojas-Cortez, 2021; Harper et al., 2021; Knapp et al., 2017). In a study of parents' efforts to sustain and support their children's learning of mathematics during COVID-19 disruptions, Harper and colleagues urge schools to continue collaboration with parents, especially those in marginalized groups, by engaging parents explicitly—less as task managers for home learning and more as allies provided with the support they need (e.g., via materials and tutorial videos in specific concepts) to effectively support their children. "The mathematics education community must take more seriously commitments to include parents in the process of evolving and even revolutionizing school mathematics. Renewed efforts are needed to bridge parents' and children's experiences with mathematics education" (p. 18).

HOW *HMH GO MATH!* DELIVERS

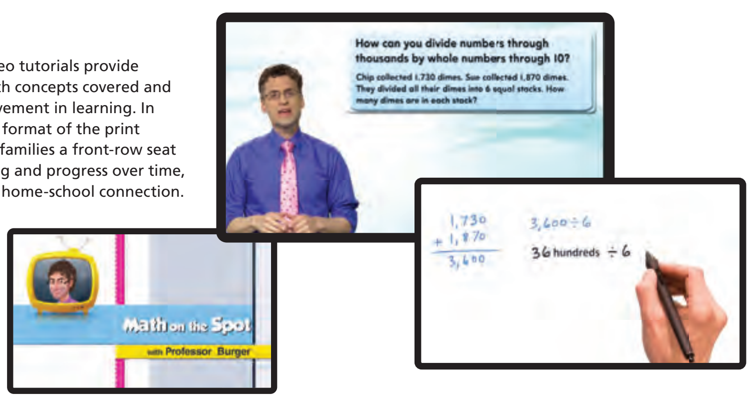
GO Math! provides evidence-based resources teachers can use to engage families throughout the school year.

Embedded through the Student Edition are Take Home Activities that involve parents and caregivers in ways that relate directly to what students are learning so that they can actively participate in their children's education, monitor their children's progress, and nurture their children's interests.



In addition, the write-in format of the print Student Edition gives families a front-row seat to their child's thinking and progress over time, encouraging a strong home-school connection.

Math on the Spot video tutorials provide instruction of the math concepts covered and allow for family involvement in learning. In addition, the write-in format of the print Student Edition gives families a front-row seat to their child's thinking and progress over time, encouraging a strong home-school connection.



School-Home Letters inform families about the skills, strategies, and topics students are encountering at school, extending rich dialogue beyond the classroom. The School-Home Letters are available in English, Spanish, Portuguese, and Haitian Creole.

Family Room® supports diverse learning environments, facilitates remote learning, and ultimately connects families with Ed, the HMH learning platform. There, caregivers and family members can find equitable, on-demand resources to help support their children.

ASSESSMENT, DATA, AND REPORTS

"The results of large-scale mathematics assessments should not be used as the sole source of information to make high-stakes decisions about schools, teachers, and students. High-stakes decisions should also take into account relevant and valid data on classroom-based performance, such as formative and summative assessments of high quality that offer students a range of opportunities to demonstrate their mathematical knowledge. Moreover, educational systems—states, districts, and schools—should be held accountable for providing essential support for high-quality mathematics teaching and learning before teachers and students are held accountable for assessment results" (NCTM, 2016, online).

Research continually demonstrates that data-driven approaches to mathematics instruction, entailing comprehensive, ongoing monitoring of student progress toward meeting standards and other goals for learning and using that information to guide instruction, is key to developing students' math skills. "An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement" (NCTM, 2014, p. 5).

HMH GO Math! provides a variety of options for ongoing assessment and aids to monitor student progress and to flexibly adjust instruction based on data about class and individual needs. These resources are available at the module and lesson level.

ONGOING, INTEGRATED, FLEXIBLE ASSESSMENTS FOR DATA-DRIVEN INSTRUCTION

Assessment is an integral part of effective instruction and is a process by which teachers can continuously monitor student understanding and use data to optimize student growth toward learning goals. A wealth of studies indicates that regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning outcomes for all students (Black & Wiliam, 1998a, 1998b; Clarke & Shinn, 2004; Kingston & Nash, 2011; Klute et al., 2017; Lee et al., 2020; Roschelle et al., 2016; Sondergeld et al., 2021; Stecker et al., 2005; Wiliam, 2010, 2011).

Teachers can collect a variety of evidence before, during, and after instruction to evaluate progress and adjust instruction with the aim of best supporting each student. While timing of administration throughout the school year is important, it is also critical that a broad range of measures and tasks be utilized diagnostically, formatively, and summatively to compile a comprehensive picture of a student's growth and track that growth over time (Hattie et al., 2017; National Mathematics Advisory Panel, 2008; NCTM, 2000, 2014). In addition to improving instruction through the clarification of instructional goals and determination of what constitutes success, learning progressions also have the potential to increase the effectiveness of classroom assessment (Moss, 2022; Sáez et al., 2013).

According to NCTM (2014), key aspects of effective mathematics assessment practices include a focus on evidence that identifies indicators of students' mathematical thinking along learning progressions that show how their mathematical thinking develops over time as well as reliance on ongoing, integrated formative assessment to monitor progress and guide instructional decision-making. "Teachers using assessment for learning continually look for ways in which they can generate evidence of student learning, and they use this evidence to adapt their instruction to better meet their students' learning needs" (Leahy et al., 2005, p. 23).

Effective assessment tools allow teachers to collect data about what is working and what is not so that they can take precise, swift, and effective action to better serve students. "Assessment should not merely be done to students; it should also be done for students, to guide and enhance their learning" (NCTM, 2000, p. 22).

DIAGNOSTIC ASSESSMENT

To make effective decisions about students' instructional needs, teachers rely on diagnostic assessment. Tailoring instruction and supplemental practice based on the results of valid diagnostic assessment improves learning outcomes (Mayes et al., 2008). Diagnostic assessments provide data about students' prior knowledge and current skill levels within a domain as well as preconceptions or misunderstandings regarding learning material (Ketterlin-Geller & Yovanoff, 2009). A screening tool given to students at the opening of the school year can help identify those who are at-risk or need additional support (Fuchs & Fuchs, 2006).

FORMATIVE ASSESSMENT

The phrase "formative assessment" encompasses the wide variety of activities—formal and informal—that teachers employ throughout the learning process to gather instructional data to assess student understanding and to make and adapt instructional decisions. Formative assessment moves testing from the end into the middle of instruction, to guide teaching and learning as it occurs. Effective teachers use formal tools (such as quizzes or homework assignments) and informal tools (such as discussion and observation) to regularly monitor student learning and check student progress (Hattie & Clarke, 2018; Hattie & Timperley, 2007; Heritage, 2013).

Curricula designed and developed for 21st-century learning should use formative assessment to "(a) make learning goals clear to students; (b) continuously monitor, provide feedback, and respond to students' learning progress; and (c) involve students in self- and peer assessment" (Committee on Defining Deeper Learning and 21st Century Skills, 2012, p. 182). Prompt feedback to students is a key component in effective formative assessment (Roschelle et al., 2016). In its review of studies examining formative assessment, the National Mathematics Advisory Panel (2008) concluded that "use of formative assessments benefited students at all ability levels" (p. 46). However, formative assessment is especially beneficial for lower-performing and at-risk students, including those historically underserved due to ethnicity, poverty, and disabilities and those enrolled in special education programs; monitoring student progress and directly involving students in the classroom assessment process shrinks achievement gaps and improves overall achievement (Black & Wiliam, 1998a & 1998b; NCTM, 2020; Tibbitt, 2020; Xenofontos, 2019).

SUMMATIVE ASSESSMENT

Summative assessment differs from those that are formative or diagnostic in nature because the purpose of summative assessment is to determine the student's overall achievement in a specific area of learning at a particular time. Teachers can effectively use summative assessments as another measure, in another point in time, and with another means by which to best evaluate student understanding. As part of an integrated assessment system, summative measures can also help teachers shape instruction and differentiate to personalize learning. Summative assessments are also useful as accountability measures for grading and gauging student learning against a set of standards or expectations. Summative assessments provide evaluative information to teachers about the effectiveness of their instructional program. Research indicates that classroom summative assessments also have the potential to positively impact learning (Harlen, 2005; Moss, 2013; NCTM, 2016).

PERFORMANCE-BASED ASSESSMENT

Performance tasks allow teachers to engage students in real-world activities and model "what is important to teach and… what is important to learn" (Lane, 2013, p. 313). Assessment systems in high-performing nations "emphasize deep knowledge of core concepts within and across the disciplines, problem-solving, collaboration, analysis, synthesis, and critical thinking. As a large and increasing part of their examination systems, high-achieving nations use open-ended performance tasks… to give students opportunities to develop and demonstrate higher order thinking skills" (Darling-Hammond, 2010, p. 3). Performance-based tasks may take different forms, require different types of performances, and be used for different purposes (formative or summative), but they are typically couched in an authentic or real-life scenario and require high-level thinking.

Research has established the benefits of performance-based assessment. A review of classroom assessment practices in an age of high-stakes testing led Schneider and colleagues (2013) to conclude that "the value of high-quality performance tasks should not be diminished and should be encouraged as an important tool" (p. 66). Performance-based assessments in STEM subjects specifically provide an alternative and complement to standardized achievement tests because they enable a holistic evaluation of the performance of an individual student. For reasons of equity and accurate representation of individual students' knowledge and skills, performance-based assessment appropriate for students from low SES levels are essential (Zimmerman et al., 2020).

DATA-DRIVEN INSTRUCTIONAL DECISIONS

Data-driven instructional decision-making is the systematic collection, analysis, and application of many forms of data from multiple sources in order to identify students' strengths and weaknesses regarding learning objectives and subsequently address student learning needs and optimize performance in future instruction. Rigorous, ongoing formative assessment that yields meaningful data is a fundamental component with an effective data-driven decision-making system. Research indicates that when it is implemented well, data-driven instruction has the potential to dramatically improve student achievement (Bambrick-Santoyo, 2014; Dunn et al., 2013; Marsh et al., 2006; Schifter et al., 2014).

HOW *HMH GO MATH!* DELIVERS

HMH GO Math! includes a wide range of diagnostic, formative, and summative assessments to measure students' understanding as they develop mastery of math standards. The program allows teachers to gather data from multiple sources to monitor student progress and drive decision-making. Assessment tools are provided in varied formats, in print and digitally, and offer flexibility as well as readily actionable results. The program offers a range of Online Assessments to benchmark student progress and inform instruction. Measures also include Prerequisite Skills Inventories, Benchmark Tests and Readiness Reviews, Chapter Reviews and Tests, and Performance Tasks.

Learning Readiness and Show What You Know are diagnostic tools that assess prior knowledge needed in each chapter and provide recommendations for intervention and individualized instruction.

Ongoing formative assessment opportunities are embedded throughout the program. These include Quick Check, a quick formative assessment to determine student mastery of lesson content and identify which differentiation resources will be most useful for each student.

Reports allow immediate review of data with a class-level breakdown of performance. Teachers can auto-sort students into performance groups with the Grouping Report. Standards Reports, Assessment Reports, and Suggested Resources are also available.

Yearly progress can be tracked with HMH Growth Measure, a valid and reliable student growth measure administered digitally three times annually and designed to monitor student growth and determine grade-level expectation. Detailed data reports guide instructional decisions and help provide individualized learning opportunities.

DIGITAL LEARNING EXPERIENCE

Over the past decade, policies and practices regarding technology use in classrooms around the country have shifted incrementally to widespread—and widely varying—application. Concurrent with such trends, there has been an emergence of growing evidence attesting to the positive impacts of technology in education as well as profound advances and innovations within the technology itself. No longer a question of whether technology can improve learning, the issues became how to enable technology to deliver improved learning outcomes for all students. Since the start of the 21st century, educators in the United States have broadly adopted the understanding that "technology can be a powerful tool for transforming learning. It can help affirm and advance relationships between educators and students, reinvent our approaches to learning and collaboration, shrink long-standing equity and accessibility gaps, and adapt learning experiences to meet the needs of all learners" (U.S. Department of Education, 2018, p. 3).

But when the global pandemic hit in 2020, digital learning suddenly and profoundly became—rather than a means of improving education—a critical mission, the only way of providing instruction to students remotely. As Fisher and colleagues (2020) noted, teaching in 2020 wasn't so much distance learning as crisis teaching. Educational technology, in modes and methods, advanced leaps and bounds over the past several years. Yet, also, the digital divide along socioeconomic lines was exposed to alarming degrees. Issues of access need to be rectified if all of our nation's students are to be equitably served by schools (Li, 2022). An additional barrier to broader adoption and implementation of educational technologies exposed by pandemic necessity: lack of teacher training in both emergency and intentional digital instruction (Johnson et al., 2022). While the long-term impacts of COVID-19 will continue to present unprecedented challenges and uncertainties for schools, one point of clarity is that the future of education will rely, in some significant part, on technology, which requires that educators have available to them resources that support effective digital and blended hybrid instruction.

HMH GO Math! leverages cutting-edge technology to provide interactive, adaptive, personalized instruction, practice, and assessment solutions aligned to state standards and addressing individual students' ongoing needs.

DIGITAL LEARNING EXPERIENCE

BEST PRACTICES IN DIGITAL LEARNING

Before COVID-19 drove educators around the United States and the world to suddenly switch to remote teaching in early 2020, the number of students receiving instruction in online and blended learning environments had been steadily growing (Gemin & Pape, 2017; Graham et al., 2019). While this area of inquiry is relatively new, findings that emerged over the past two decades indicate that digital learning has enormous potential to positively transform education for diverse groups of students when evidence-based practices are incorporated into instructional design (Chen et al., 2018; Johnson et al., 2022; Patrick & Powell, 2009; U.S. Department of Education, 2010, 2016). Improvements in student-centered, cooperative, and higher-order learning, as well as problem-solving and writing skills, have been found within computer-intensive classroom settings (Ross et al., 2010). In 2016, the U.S. Department of Education reported that technology-intensive instruction can make education more equitable by closing the digital-use divide and making transformative learning opportunities available to all students.

Blended learning utilizes both device-driven, technology-based instruction and face-to-face instruction in a conventional classroom context, with the objective to maximize the advantages of each. Research findings on the effects of blended learning are strikingly positive (Delgado et al., 2015; Tamim et al., 2011). In a meta-analysis examining online and traditional face-to-face instruction with mixes of both, blended instruction emerged as the most effective of the three approaches (U.S. Department of Education, 2010). Likely because blended learning teaches students through engaging media and modes that fit with their daily practices and experiences, students tend to view blended learning favorably (U- ur et al., 2011). Blended learning opportunities specifically expand the possibility of growth for all students while affording historically disadvantaged students' greater equity of access to high-quality education, in the form of both enhanced, instructionally effective content and more personalized learning (Molnar, 2014) that affords students greater control over path, pace, time, and place (O'Byrne & Pytash, 2015).

An established body of evidence indicates that instruction improves when technology-based multimedia is incorporated; the effect is enhanced student engagement and motivation as well as reduced cognitive load, factors that facilitate learning (Abdoolatiff & Narod, 2009; Chen et al., 2018; Johnson et al., 2022; Mayer, 2013, 2017). "Advances in computer and communication technologies now allow instructors to supplement verbal modes of instruction with visual modes of instruction, including dazzling graphics that students can interact with. Research on multimedia learning provides encouraging evidence that under appropriate circumstances, students learn better from words and pictures than from words alone" (Mayer, 2013, p. 396).

Effective digital learning environments utilize technology to improve differentiation of instruction, which by necessity includes real-time assessment and prompt feedback (Curtis & Werth, 2015; Johnson et al., 2022; Turley & Graham, 2019). Digital programming offers an additional benefit of increased automation, which can significantly simplify educators' lives by eliminating low-value manual tasks such as attendance records and student assessment data entry. The further impact of allowing digital platforms to capture student achievement data is a freeing up of resources so that educators can "take advantage of the things that leading brick-and-mortar schools do well, such as creating a strong, supportive culture that promotes rigor and high expectations for all students, as well as providing healthy, supportive relationships and mentorship" (Horn & Staker, 2011, p. 7).

Research suggests that the best practices in blended learning are largely the same as those in traditional classrooms but with some critical adaptations within the digital environment (Anthony, 2019; Borup & Archambault, 2018) since successful technology-based learning demands specific skills and must meet individual developmental needs of students (Johnson et al., 2022; Pulham & Graham, 2018; O'Byrne & Pytash, 2015). To achieve optimal growth, blended learning should support teachers in being flexible and responsive to students, to integrate multiple data sources into their constant stream of formative assessment, and to deliberately incorporate more rigorous learning activities (Anthony, 2019). In a large-scale study, Kwon and colleagues (2019) found that for online learning to be successful, it is important that teaching is structured so that students make steady attempts to complete learning tasks, ideally with students' own self-regulated learning scaffolded by course pacing guides.

As Fisher et al. (2020) emphasize, it is the choice of task that matters in advancing learning—not the medium. Teachers should use technology as the means and starting point, not the core of teaching. Same principles of effective instruction that apply in conventional classroom settings apply in digital instruction. As Hattie's (2018, with Clarke) ongoing findings about best practices with technology continue to affirm, these principles include: fostering student self-regulation to help them move toward deeper learning, increasing student agency, including a diversity of instructional approaches (not just some direct instruction and then some offline independent work), including well-designed peer learning, and providing feedback within a high-trust environment integrated into the learning cycle.

Virtual reality simulations in learning environments that imitate a real-life process or situation and which allow learners to test effects of their hypotheses on intended outcomes have been shown to boost learning outcomes (Castaneda, 2008; Dani & Koenig, 2008; Johnson et al., 2022; Merchant et al., 2012). "Technology can help learning move beyond the classroom and take advantage of learning opportunities available in museums, libraries, and other out-of-school settings" (U.S. Department of Education, 2016, p. 12).

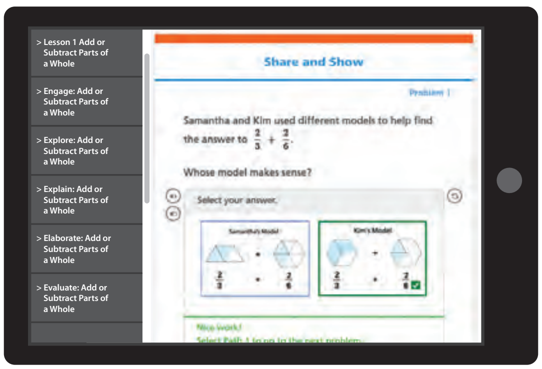
The U.S. Department of Education (2019) stresses how technology plays a central role in STEM education in terms of both its role within the STEM professions today's students are being trained for as well as the potential that technology has to significantly improve both experiences and outcomes for students as they learn STEM concepts and build STEM knowledge throughout their K–12 educations. The dimensions of digital instruction that support powerful STEM learning generally, and math specifically, include the following:

- **Dynamic representations:** Students can more effectively develop STEM concepts via interactions with digital models, simulations, and dynamic representations.
- **Collaborative reasoning:** Technology platforms support students' collaborative discussion and shared construction of STEM concepts, fostering engagement and equalizing participation.
- **Immediate and individualized feedback:** Digital tools provide students with prompt and customized feedback as they practice or demonstrate their STEM skills, yielding faster and improved learning outcomes.
- **Computational thinking:** Students can use technology to engage in formulation, analysis, and solving of problems using algorithms, data, and simulations to investigate questions and build new understandings about phenomena.
- **Project-based interdisciplinary learning:** Both process and product are enriched when students utilize technology tools in the context of authentic project- or challenge-based learning activities that integrate multiple STEM fields.
- **Embedded assessments:** Assessments aligned to ongoing STEM instruction and delivered digitally provide opportunities for students to reflect on and demonstrate and for teachers to evaluate their learning.
- **Evidence-based models:** Students use technology to reference or create models based on data and evidence.

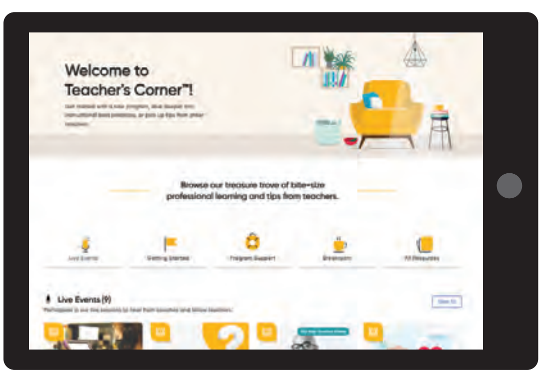
HOW *HMH GO MATH!* DELIVERS

HMH GO Math! provides a full, dynamic digital experience for both students and teachers, and a wide variety of additional resources and support through program content available on Ed, HMH's online learning platform

The program's Online Student Experience provides interactive practice with engaging Launch Activity lessons, hints, and corrective feedback.



The program's Online Teacher Experience provides complete support for lesson planning, assigning resources, viewing reports, and grouping. It also gives teachers flexible multi-grade access to both prerequisite and challenge content for standards progressions.



GO Math! with *Waggle* is part of the HMH suite of digital solutions connected by HMH Growth Measure on Ed and connected to the program at point-of-use, allowing for seamless integration.

Waggle propels the *GO Math!* digital experience to the cutting edge, offering highly individualized, interactive, engaging instruction and ongoing, targeted assessment. Lessons within *Waggle* provide instruction, reinforcement, and guided independent practice with embedded, dynamic multimedia content. The platform is also adaptive, providing students with ongoing practice in skills and fluency customized to their specific needs in meeting rigorous math standards. The digital features include a text-to-speech button that provides multimodality, links to background information on related topics to spark curiosity and interest, and hints for extra support.

Waggle also offers language support in the form of fully adapted lesson content in Spanish.

HMH Growth Measure can be used to track yearly progress and provide further personalized pathways of skills-based instruction and practice. This valid and reliable student growth measure is administered digitally three times per year and is designed to monitor student growth and determine grade-level expectations. Detailed data reports are used to guide instructional decisions and help provide individualized learning opportunities.

HMH Go™ is an app that gives students the ability to download their core digital resources for later offline use.

INCREASED AGENCY AND A MORE PERSONALIZED APPROACH TO INSTRUCTION

Digital learning opportunities expand the possibility of growth for all students in the form of enhanced, instructionally effective, and engaging content, as well as more personalized learning with preferred modalities; agency over the pace of their own learning; and more frequent and timely feedback—while affording historically disadvantaged students additional benefits via greater equity of access to high-quality education (Johnson et al., 2022; Molnar, 2014; O'Byrne & Pytash, 2015; U.S. Department of Education, 2016). "Digital learning has the capacity to transform schools into new models for education that are student-centric, highly personalized for each learner, and more productive, as it delivers dramatically better results at the same or lower cost" (Horn & Staker, 2011, p. 2).

Research shows that effective technology use in the classroom motivates students to take charge of their own learning, and that digital learning itself is enhanced when students are given more control over their interaction with media (Horn & Staker, 2011; O'Byrne & Pytash, 2015; U.S. Department of Education, 2010). Technology is increasingly being utilized in the United States to personalize learning and give students more choice over what and how they learn—and at what pace; this will better prepare students to organize and direct their learning in their lives even after formal schooling (U.S. Department of Education, 2016). "Online learning has the potential to transform teaching and learning by redesigning traditional classroom instructional approaches, personalizing instruction, and enhancing the quality of learning experiences." (Patrick & Powell, 2009, p. 9).

Other researchers have indicated that multimedia learning leads to increased student motivation because of the responsiveness and student control these environments allow and the subsequent engagement in active learning (Chen et al., 2018; Johnson et al., 2022; Schunk et al., 2008; Sims et al., 2002). Multimedia learning can further reduce cognitive load (Mayer, 2017). Zhang (2005) found students in a full interactive multimedia-based e-learning environment achieved better performance and higher levels of satisfaction than those in a traditional classroom and those in a less interactive e-learning environment, with a lack of control over content diminishing potential benefits.

"This study implies that to create effective learning, e-learning environments should provide interactive instructional content that learners can view on a personalized self-directed basis" (p. 160). However, a student's individual capacity to self-regulate, important in all educational settings, is a critical factor in digital learning in particular; self-efficacy and developmental needs must be carefully considered in the delivery of online instruction (Johnson et al., 2022).

A blended learning approach specifically offers a more consistent and personalized pedagogy that helps each child feel and be successful at school. Digital learning tools can provide more flexibility and support for individual students by modifying content and complexity; additionally, advances in software technology have increased adaptive learning and improved feedback. By providing a diverse array of online and other digital resources, technology supports learning drawn from real-world challenges and students' personal interests and passions while also aiding the organization of a project-based curriculum (U.S. Department of Education, 2016). Further, current digital learning platforms afford opportunities for timely progress monitoring and assessment; teachers can use such technologies to meet instructional needs of individual students as well as provide prompt, direct feedback to guide specific learning (Kerton & Cervato, 2014; Johnson et al., 2022; Pulham & Graham, 2018; Roschelle et al., 2016; Turley & Graham, 2019).

Digital learning can also increase the capacity for students to work together. Computer-based collaborative tools allow for online interactions that can create and strengthen a community of learners while fostering students' communication and collaboration skills (Dikkers, 2018). "What makes blended learning particularly effective is its ability to facilitate a community of inquiry" (Garrison & Kanuka, 2004, p. 97).

HOW *HMH GO MATH!* DELIVERS

GO Math! with *Waggle* features child-centered, active, engaging instruction and ongoing, targeted assessment with powerful personalization. This platform provides students with rigorous practice in skills and fluency that rigorous math standards require. The technology is also adaptive, so students' individual strengths and needs are met continuously, and their progress is monitored in real time. *Waggle* allows flexibility in and out of the classroom.

Waggle offers teachers effective monitoring of student progress continuously in real time and at both the individual and class level to aid instructional planning and optimize learning. As a bonus feature, *Waggle* also allows students to personalize their learning experience via their own dashboards and with the selection of avatars and environments through which they progress through the interactive programming on the platform. To bolster engagement—by injecting more fun—the platform also features familiar, age-appropriate gaming elements, including gathering and redeeming rewards.

Teachers can leverage the HMH Growth Measure to place children on personalized pathways of skills-based instruction and practice, or choose the assignments that extend their instruction. *GO Math!* and *Waggle* are part of HMH's suite of digital solutions connected by HMH Growth Measure on *Ed*.

PROFESSIONAL LEARNING AND SERVICES

HMH GO Math! features practical approaches to professional learning that support teachers in becoming developers of high-impact learning experiences for their students. Comprehensive blended professional learning solutions are data and evidence driven, mapped to instructional goals, and centered on students. HMH allows teachers to achieve agency in their professional growth through effective instructional strategies, embedded teacher support, and ongoing blended professional learning relevant to everyday teaching.

PROFESSIONAL LEARNING AND SERVICES

CONNECTED PROFESSIONAL AND PERSONALIZED LEARNING

Effective, curriculum-based professional learning (PL) consists of ongoing, active experiences that focus on improving the rigor and impact of instructional practices and ideally replicate the learner-centered approaches that teachers are expected to provide for their students. Elements of effective, curriculum-based PL include high-quality educative curriculum materials; transformative learning experiences that shift teachers' attitudes, beliefs, and practices; and a prioritization of equity to ensure all students meet high expectations. Functional design elements include learning designs that model inquiry-based instruction, experiences to shift teachers' beliefs, opportunities for reflection and feedback, and change-management strategies that address individual concerns and group challenges. Finally, structural design features include collective participation in which teachers practice and reflect on the curriculum, models of learning that evolve from initial use to ongoing support to building capacity, and a considered use of time. These elements of effective, curriculum-based PL must exist in a system with strong leadership, adequate resources, and coherence toward common goals (Short & Hirsh, 2020).

How professional learning is delivered has an impact on its effectiveness. PL programs with teacher-to-teacher collaboration focused on instructional improvement—whether in professional learning communities (PLCs), teacher teams, or group work in PL sessions—have demonstrated improvement in teachers' instructional skills. Another effective practice is conducting follow-up meetings or coaching sessions after the initial implementation of a program so that teachers can share their experiences and receive feedback. The content of the PL is equally important. It should focus on subject-specific instructional practices (not merely content knowledge), prioritize specific supportive materials over general principles, and help teachers build stronger relationships with students (Hill & Papay, 2022).

A systematic review of effective professional learning for STEM teachers found that it is necessary to have discipline-specific content highly relevant to participating teachers' classroom contexts that is delivered to teachers via inquiry-based learning experiences. This approach both engages teachers as learners and models effective strategies for teachers to deliver science content to their students (Bancroft & Nyirenda, 2020).

A recent meta-analysis of 95 studies investigating STEM professional development programs found an average weighted impact estimate of +0.21 standard deviations. Programs saw stronger outcomes when they helped teachers learn to use curriculum materials; focused on improving teachers' content knowledge, pedagogical content knowledge, and/or understanding of how students learn; incorporated summer workshops; and included teacher meetings to troubleshoot and discuss classroom implementation (Lynch et al., 2019).

Current reform efforts across disciplines require significant shifts in teachers' roles from traditional, rote, fact-based approaches to fostering students' deeper engagement, critical thinking, and problem-solving. For schools to support these standards and instructional practices, effective professional learning during the implementation stage, when teachers are learning and committing to an instructional approach, is critical (Gulamhussein, 2013). While technology transforms the teacher's role, this does not mean that evidence-based teaching practices should be discarded. In fact, effective instruction results when teachers purposefully combine these tools with proven instructional approaches (Kieschnick, 2017).

The Professional Learning Models (PLMs) for Success in Mathematics project was designed to support mathematics educators with evidence-based professional learning through modeling of effective instructional practices, job-embedded collaboration, and opportunities for feedback and reflection (Araoz et al., 2019). The PLMs are focused on developing evidence-based teaching practices, including facilitating meaningful mathematical discourse and implementing tasks that promote reasoning. The PLMs are currently being implemented and studied (Araoz et al., 2019).

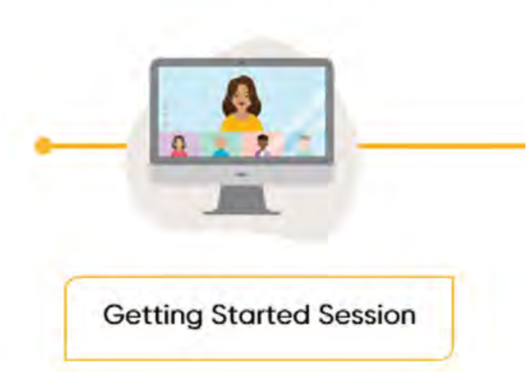
Effective professional learning, whether in-person, online, or blended, offers teachers coherent experiences so that their learning is connected to their work in the classroom, and it builds proficiency. This approach includes alignment between the study of theory and practice, observation of practice, individual coaching, and further practice and reflection through collaboration. Each of these components is essential to support and build on the content and pedagogy that is learned, observed, and practiced in each of the other components (Rock, 2019).

Many school districts and providers of teachers' professional development are moving toward a more personalized model of professional development, taking a cue from the movement toward personalized learning for students. This approach often focuses on short modules, which teachers can choose and then complete on their own time. The modules can incorporate aspects of gamification, micro-credentialing, and online professional development communities. By allowing teachers to choose their own professional development courses and activities and complete them in their own place at their own pace, the professional development will be better matched to their needs. Teachers will be able to set goals, find resources to help them meet those goals, track their progress, and get feedback from supervisors and colleagues (Gamrat et al., 2014; Meeuwse & Mason, 2018).

HOW *HMH GO MATH!* DELIVERS

HMH provides a continuum of connected professional learning designed to foster teacher agency, promote collaboration, and build collective efficacy and capacity to support teachers' role as designers of quality instruction. Through strategic planning, guided implementation support, and blended coaching, HMH helps schools and districts achieve measurable gains with professional learning centered on research-based practices and student outcomes.

HMH GO Math! Includes Guided Implementation Support.



Guided Implementation Support for *HMH GO Math!* helps educators build confidence and success with their new math program. A Getting Started session provides learner-centered foundational program knowledge through exploration and hands-on activities and is the first step toward a successful implementation.

Ongoing training and support resources are also provided through the Teacher Success Pathways on Ed, HMH's learning platform. There, teachers access program-specific guided learning pathway(s) focused on subject-specific instructional practices. A recommended sequence of topics, which includes live sessions, videos, interactive media, and related resources, helps teachers plan, teach, utilize teaching materials, and assess student learning using their new HMH program.

HMH continues to provide a personalized model for professional development by engaging teachers throughout the school year and adoption via Teacher's Corner on *Ed*. Teachers have access to a searchable library including classroom videos, tips from other teachers, teacher-to-teacher collaboration through live events, and additional content and support from HMH's experienced Instructional Coaches. Please visit us at https://www.hmhco.com/programs/teachers-corner for a quick video tour. Teacher's Corner includes the following:

- **Live Events:** Educators learn from HMH coaches, thought leaders, and fellow educators in live online sessions with active participation and feedback.
- **Getting Started:** Teachers can get their classes up and running with interactive guides that provide step-by-step instructions, videos, and tips from other teachers and HMH coaches.
- **Program Support:** This provides on-demand teaching resources and professional learning tools, including model lesson videos, teacher tips, interactive support, and more.
- **The Breakroom:** Teachers can be inspired with ideas from other educators, new lesson resources, and reflection opportunities.
- **Leader's Corner:** Teachers can have district and site-based leader access to program support and resources to assist with program implementation.
- **Family Room:** This feature dedicates a space with personalized, easily accessible, and on-demand resources to parents and students.

HMH Professional Learning is recognized as a provider of effective and relevant professional learning by the Professional Learning Partners Guide. HMH Professional Learning received a "high-quality" rating in three key areas: Launching Instructional Materials, Ongoing Professional Learning for Teachers, and Ongoing Professional Learning for Leaders.

COACHING TO STRENGTHEN TEACHING AND LEARNING

Research has demonstrated that sustained, job-embedded coaching is the most effective form of professional learning, whether it is delivered in person or online. Coaching delivered in person is most effective when coaches are highly expert and focus their work with teachers on a clearly specified instructional model or program. Other opportunities for teachers to develop their knowledge of the targeted instructional model (e.g., in courses, workshops, or coach-led learning groups) are also an important component of successful coaching programs. Online coaching shows promise for being at least as effective as in-person coaching for improving outcomes, though the research base comparing delivery systems is thin. The balance of evidence to date, however, suggests that the medium through which coaching is delivered is less important than the quality and substance of the learning opportunities provided to teachers (Matsumara et al., 2019).

A recent meta-analysis of coaching programs found overall effect sizes of 0.49 SD on instructional practices and 0.18 SD on student achievement. Encouragingly, teachers who received virtual coaching performed similarly to teachers who received in-person coaching for improving both instructional practices and student achievement. The authors identified several aspects of online coaching as potential strengths: increasing the number of teachers with whom a high-quality coach can work, reducing educators' concern about being evaluated by their coach, and lowering costs while increasing scalability (Kraft et al., 2018).

The best evidence for coaching is found for one-to-one coaching, where coaches observe and offer feedback on teachers' practice. There is not a great deal of research on specific coaching practices, but effective coaches might engage in co-planning, modeling, or guiding teacher reflection. Effective coaching is time-intensive, and the most successful programs have invested in the selection, training, and ongoing support of their coaches (Hill & Papay, 2022).

Online coaching programs can be operated at relatively lower costs, are scalable, and make it more feasible to pair teachers with coaches who have expertise in their content area and grade level. Results from a randomized field trial documented sizable and sustained effects on both teachers' ability to analyze instruction and on their instructional practice, as measured by the Mathematical Quality of Instruction instrument and student surveys. However, these improvements in instruction did not result in corresponding increases in math test scores as measured by state standardized tests or interim assessments (Kraft and Hill, 2020).

The International Society for Technology in Education (ISTE) embraces a professional development model that includes effective coaching, collaborative communities, and a technology-rich environment. Effective coaching is contextual, relevant, and ongoing. ISTE recommends that school districts chose a coaching model that best fits the needs of their teachers, whether it is cognitive coaching, instructional coaching, or peer coaching (Beglau et al., 2011). Effective professional learning programs provide continued follow-up and support from coaches (Sweeney, 2010).

Virtual coaching can provide a framework for a shared leadership structure that focuses on facilitating teachers' autonomy, self-management, empowerment, and cooperation. Because the coach and the teacher jointly pursue the goal of increased student achievement, virtual coaching provides social support for both parties, leading to enhanced emotional and psychological strength. Any coaching relationship—traditional or virtual—builds on several underlying qualities of both teacher and coach. Chief among them are a willingness to change, a trusting relationship, a high level of initiative, and a personal and organizational commitment to the workplace (Blackman, 2010).

HOW *HMH GO MATH!* DELIVERS

HMH Coaching for *HMH GO Math!* offers individuals or teams of teachers sustainable, data-driven, and personalized support aligned to each teacher's learning goals. Our research-based blended coaching model is student focused and proven to help teachers improve their practice and raise student achievement.

Expert coaches support teachers' success as they work together over the year to plan instruction and implement *HMH GO Math!* in the classroom. HMH offers blended coaching to provide teachers with personalized support to best integrate *HMH GO Math!* with their everyday instructional practices while also facilitating teachers' autonomy and confidence. HMH coaches build strong relationships by helping teachers select, monitor, and achieve their goals. Coaching sessions may include 1–20 teachers and are centered on evidence of student learning. HMH Coaches may model high-impact instructional strategies, answering program and practice questions, offering observation and feedback, or leading cohort-based professional learning.

The Coaching Membership experience includes access to the award-winning HMH Coaching Studio. In this online community, participants can access additional resources and collaborate with their coach and colleagues to address their students' most pressing learning needs. Through Coaching Studio, teachers have access to:

- Goal Tracker, which allows teachers to have access to growth goals personalized to them,
- Model Lesson Library, which has hundreds of HMH classroom and expert videos of best practices,
- Collaboration Hub, where teachers can find discussion forums, resource sharing, and video-based reflection to drive collaboration with their coach and peers, and
- Video-Powered Coaching, which allows teachers to upload video of their instruction for reflection or to share with their coach and peers.

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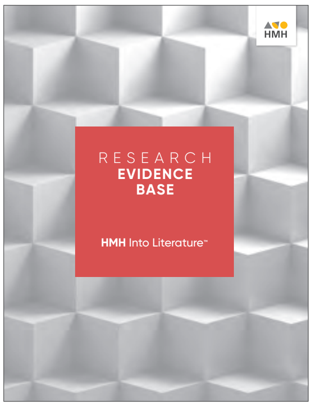
NOTES

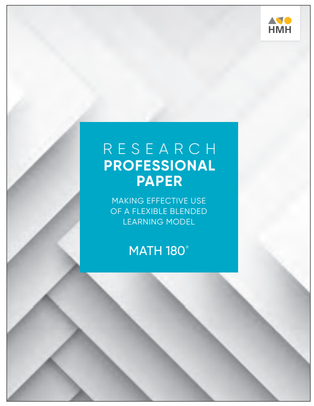
HMH RESEARCH PUBLICATIONS

Research Into Practice Into Results



RESEARCH EVIDENCE BASE PAPERS

Research Evidence Base papers provide an in-depth account of the theoretical underpinnings, evidence base, and expert opinions that guide the design and development of new and revised programs. These papers map known research and design principles to practical applications of the program.



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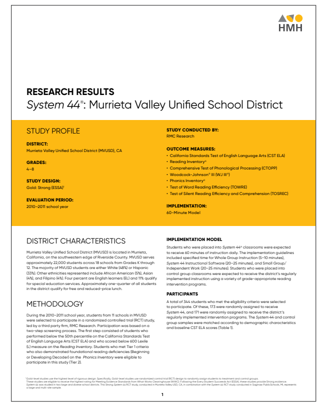
Research Professional papers highlight an important theoretical construct, practical application, program component, or other topic related to learning in the context of HMH programs. They are authored by experts in the field, researchers, and thought leaders within the industry.



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