

Integrating Language Routines Into Instruction

Compare and Connect

Students need opportunities to reflect on and linguistically respond to comparisons. Using routines that prompt meta-cognitive and meta-linguistic awareness support students in mathematically productive conversations.

Grades 6–8

Language Routine: Compare and Connect

The purpose of this routine is to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.

Routine	
SET-UP	Students are tasked with understanding one another's solution strategies by relating and connecting other students' approaches to their own approach. Ways to set this up so that multiple strategies are likely to be generated by each pair of students include: • <i>I solve it one way, you solve it another</i> • <i>Divide and conquer: you do one and I do another</i> • <i>I have a piece of information, you have a piece of information</i>
WHAT IS SIMILAR, WHAT IS DIFFERENT	Students first identify what is similar and what is different about the approaches. This can also be an initial discussion about what worked well in one approach compared to another and what might make a particular approach more complete or easy to understand.
MATHEMATICAL FOCUS	Students focus on specific mathematical relationships, operations, quantities, and values. Using Talk Moves effectively supports students in making mathematical comparisons and connections. For example: • <i>Who can restate _____'s reasoning in a different way?</i> • <i>Did anyone solve the problem the same way but would explain it differently?</i> • <i>Did anyone solve the problem in a different way?</i> • <i>Does anyone want to add on to _____'s strategy?</i> • <i>Do you agree or disagree? Why?</i>

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From the Classroom

SET-UP	Mr. B: Partner up with another student and decide who will solve problem A and who will solve problem B. Each person will solve their problem on their own paper. [Students are presented with the following two problems:] <table data-bbox="781 447 1302 562"> <tr> <th>Problem A</th><th>Problem B</th></tr> <tr> <td>$\frac{2}{3}x + 7 = 9$</td><td>$4x - 8 = x + 2$</td></tr> </table> Now, switch papers with your partner. Look at how your partner solved their problem. Beneath what they wrote you will solve that same problem but a different way.	Problem A	Problem B	$\frac{2}{3}x + 7 = 9$	$4x - 8 = x + 2$
Problem A	Problem B				
$\frac{2}{3}x + 7 = 9$	$4x - 8 = x + 2$				
WHAT IS SIMILAR, WHAT IS DIFFERENT	Mr. B: Switch papers again; now you're holding your original paper. Discuss with your partner the similarities in and differences between the ways you solved each problem. Think about what worked well in one approach compared to the other, and what might make a particular approach more complete or easy to understand.				
MATHEMATICAL FOCUS	Mr. B: Let's talk about what you are noticing. Angel, how did you solve problem A? Angel: I started by subtracting 7 from both sides. Then I divided by $\frac{2}{3}$ to find x. Mr. B: And what was the different way that your partner solved it? Angel: Konika started by multiplying everything by 3, so then she was working with $2x + 21 = 36$ and then she subtracted 21 from both sides and then divided by 2. Mr. B: And you both found the same answer. Interesting. How can these approaches get us the same answer when we didn't follow the exact same steps? Talk with your partner for a minute and be prepared to share. [Circulates during partner discussions and listens to students.] Alise, please share one of the ideas that came up during your discussion. Alise: We talked about how Konika started by multiplying each term by three, which kept everything equal. Then, ultimately, she followed the same steps as Angel to get the x by itself. Mr. B: Does anyone want to add on to Alise's explanation? [Mr. B continued the whole-class conversation about both problems A and B using Talk Moves to emphasize the mathematical relationships between the different student approaches.]				

Integrating Language Routines Into Instruction

Critique, Correct, and Clarify

Students can start making sense of academic content and negotiate meaning in a language before they understand a language completely.

Grades 6–8

Language Routine: Critique, Correct, and Clarify

The purpose of this routine is to optimize output by giving students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to use an incorrect, incomplete, or ambiguous written argument or explanation that students improve upon by correcting errors and clarifying meaning, thereby advancing their own mathematical thinking and communication.

Routine	
PRESENT	Present a partial/broken argument, explanation, or solution method. The teacher can play the role of the student who produced the response and ask for help in fixing it. • <i>Given response could include a common error</i> • <i>Given response should include an ambiguous term or phrase, or an informal way of expressing a mathematical idea</i>
PROMPT	Prompt students to identify the error(s) or ambiguity, analyze the response in light of their own understanding of the problem, and work both individually and in pairs to propose an improved response.
SHARE	Pairs share out their drafts of the improved response.
REFINE	Students refine their own draft response.

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From the Classroom

PRESENT	Ms. L: <i>Let's look at this student's work. This is from one of my students in third period, and he is asking for our help. He knows he made a mistake but isn't quite sure what he needs to do fix it.</i> $(2x - 4)(x + 3) = 2x^2 - 12$ I multiplied $2x$ and x and got $2x^2$. And 4 times 3 is 12. It's minus because there's a minus in the first one.
PROMPT	Ms. L: <i>Look at this work for a minute and make notes, just thinking to yourself, about what you notice. What would you tell him? I'll give you a minute on your own to start.</i> [Ms. L pauses for a full minute while students analyze the student work.] <i>Turn to your elbow partners and talk this through. What are your thoughts? How can we help this student with his work?</i> [Ms. L circulates as students discuss, listening to their conversations, and noting any common misconceptions or common approaches among the small groups. She allows 3–4 minutes for partners to discuss before pausing their discussions.] <i>I'm hearing such detailed and helpful conversations, great job focusing on the math. On a piece of paper write down your thoughts for this student. Include the math and any explanation with words or drawings that you think will help this student understand his mistake and learn how to solve this problem.</i>
SHARE	Ms. L: [after allowing students 5 minutes to record their thoughts] <i>Pause your work for a moment. You are going to join with two other pairs, making groups of six. Each pair will take turns sharing with the others what they would tell my student from third period about his calculation error. Each pair has 2 minutes to explain to the others and then I'll signal when it is time for the next group to start sharing. How long does each group have to share?</i> Whole Class: <i>2 minutes.</i> [Students begin working in their groups. Ms. L circulates, listening to student discussions and noting similarities and differences in student approaches.]
REFINE	Ms. L: <i>I heard so many thoughtful and detailed responses about this error. Let's take a few minutes to gather all our thoughts and decide what makes sense to tell this student about his work. Get back together with your original partner and refine your work to include any of the ideas you heard from others that you think are important. You can add to or change what you have. We want to support this student with words, with pictures, with numbers, anything we possibly can to help make sense of the problem. You have 5 minutes to complete your work.</i> [Ms. L circulates as students are working, prompting students to add diagrams, written explanations, or other representations to deepen their explanations.]

Integrating Language Routines Into Instruction

Stronger and Clearer Each Time

Mathematical understandings and language competence develop interdependently. When students discuss mathematical ideas with one another, they **strengthen** and **clarify** their academic thoughts.

Grades 6–8

Language Routine: Stronger and Clearer Each Time

This routine is well-suited for circumstances that call for students to construct a mathematical argument or defend an idea. Each time students talk with partners, they build from and borrow the ideas and language of previous partners.

Routine							
PRESENT	Teacher poses a problem/question to students						
PREPARE	Students Pre-Write - Students study the problem individually, writing down any questions or ideas/reasoning about how to solve the problem, using complete sentences if possible. - Scaffolding: Provide sentence frames or a skeletal paragraph framework to support students with the language structures, while leaving room for their independent mathematical reasoning.						
	Think Time - Provide a minute for students to think about what they will say to their first partner, considering what they are currently doing (or did) to solve the problem. - Students cannot look at what they wrote during their partner conversations.						
PARTNER	STRUCTURED PAIRING: <table> <tr> <th colspan="2">PARTNER ROTATIONS</th></tr> <tr> <th>Students</th><th>Teacher</th></tr> <tr> <td> - Listener asks clarifying questions, especially related to justifying (Why did you do that?). - Partners switch roles. Each person shares and each person listens, asking clarifying questions. - Rotate to additional partners, strengthening and clarifying their ideas each time </td><td> - Circulates and listens during student discussions - Prompts students at each turn to emphasize strength (focus on math concepts and skills) or clarity (how to describe the math to others) - Removes scaffolds with each successive pairing to build student independence </td></tr> </table>	PARTNER ROTATIONS		Students	Teacher	- Listener asks clarifying questions, especially related to justifying (Why did you do that?). - Partners switch roles. Each person shares and each person listens, asking clarifying questions. - Rotate to additional partners, strengthening and clarifying their ideas each time	- Circulates and listens during student discussions - Prompts students at each turn to emphasize strength (focus on math concepts and skills) or clarity (how to describe the math to others) - Removes scaffolds with each successive pairing to build student independence
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PROCESS	Post-Write Students return to seats and write down their final explanations using sentences or drawings supported with sentences.						
	Compare Students analyze their pre-writes and post-writes, noticing how their ideas were strengthened and/or clarified during partner discussions.						

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From the Classroom	
PRESENT	Ms. G: <i>Look up here [directing students to look at the board where she has written: $x^2 > x$]. Do you agree or disagree? Study this individually, writing down any questions, ideas, or reasoning about this inequality. Use complete sentences if possible.</i>
PREPARE	Students Pre-Write [Ms. G pauses for a full minute while students write their thoughts.]
	Think Time Ms. G: <i>In a moment you will partner up to discuss if you agree or disagree and why. You won't take your papers with you, so pause now and study your notes.</i>
PARTNER	Structured Pairing Ms. G: <i>To start you will partner up. When you talk with your partner, explain your ideas like a mathematician would. Partners, ask questions to clarify. It is important to be clear in your explanations. The purpose is to borrow ideas from your partner to make your argument stronger and clearer.</i>
	In Pairs Ms. G: <i>Find your first partner and start your discussion.</i> [Ms. G circulates while students discuss and listens for common justifications. After one minute she signals students to switch listener/speaker roles.]
	Rotate Partners Ms. G: <i>Rotate to a new partner and repeat the sharing and listening. Incorporate ideas you heard as you make your arguments stronger and clearer.</i> [Ms. G circulates again while students discuss and listens for common justifications. She has students switch one more time for a total of three partner discussions.]
PROCESS	Post-Write Ms. G: <i>Head back to your seats. You have 2 minutes to revise your original argument. Take things you heard from your partners and strengthen and clarify your original thoughts about whether x^2 is greater than x.</i>
	Post-Write Ms. G: <i>Now that you've refined your arguments, look at what you first wrote and your final draft. What do you notice?</i> Raaqim: <i>In my first draft I forgot about the word variable. When I was talking with my second partner, she said that word and so I added that in my explanation. It sounds more like a mathematician with the actual word.</i> [Ms. G continues to facilitate the class discussion. Students share how their arguments are improved after their partner discussions.]

Integrating Language Routines Into Instruction

Three Reads

Language is a tool that not only allows students to communicate their math understanding to others, but also to organize their own experience, ideas, and learning for themselves.

Grades 6–8

Language Routine: Three Reads

This routine supports reading comprehension, sense-making, and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner in mathematical conversation. Students are supported in reading a situation/problem three times—each time with a particular focus.

Routine	
First Read	Students read or listen to the problem (without a question stem) with the goal of comprehending the text .
Second Read	Students read or listen to the situation with the goal of comprehending the mathematics .
Third Read	Students read the situation and brainstorm possible mathematical questions pertaining to the context of the problem.

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From the Classroom

First Read	Mr. A: <i>Listen carefully. Listen for what is going on in this problem.</i> <i>Mariana and her sister Amelie are running a 10K race. Mariana runs at a rate of 4 mph and starts her race at 10 am. Amelie starts her race at 10:15 am and runs at a rate of 4.5 mph.</i> <i>Turn and talk to each other and tell each other what the problem is about. I expect each pair to have at least one sentence to share.</i> [Students turn and talk together. Mr. A circulates during discussions and listens to students.] Mr. A: <i>Let's start with Yesenia. What did you and your partner discuss?</i> Yesenia: <i>We think this problem is about running. There's a girl and her sister and they're running in the same race.</i>
Second Read	Mr. A: <i>Alright, this time when I read, listen for important information like the numbers and the relationship between the numbers.</i> [Mr. A reads the problem a second time.] <i>Turn and talk with your partner again, thinking about the information you have about the numbers.</i> [Students turn and talk together. Mr. A circulates during discussions and listens to students.] Mr. A: <i>Who can start us off? What numbers are important?</i> Sophia: <i>The rate that each girl is running matters. So the 4 and the 4.5 are important.</i> Mr. A: <i>OK, is there anything else that is important to think about?</i> Juan: <i>I think it matters what time each girl started the race. Mariana started at 10 am but her sister started 15 minutes later.</i>
Third Read	Mr. A: <i>This time when I read the problem, think about what possible questions we could ask.</i> [Mr. A reads the problem a third time.] <i>What questions can we ask? Turn and talk with your partner and be ready to share at least one question in two minutes.</i> [Students turn and talk together. Mr. A circulates during discussions and listens to student discussions.] Mr. A: <i>I listened while you were talking. Take a look up here, I wrote down what I heard:</i> • <i>Who wins the race?</i> • <i>When are the two girls at the same place?</i> • <i>Will Amelie catch up to Mariana? When?</i> • <i>Who is running faster?</i> <i>Let's think about these different questions and how we can find the answers.</i> [Mr. A encourages students to create a model that represents the times in the problem, and to use their models to answer the two questions. When students have completed their representations, he encourages them to compare and contrast different ways of thinking. How are these models the same? How are they different?]