GRADES 6–8 SCIENCE ACTIVITY

EXPONENTIAL GROWTH AND THE SPREADING OF DISEASE

There is an old story about a peasant doing a favor for the king. So thankful was the king, that he said he would grant the subject any wish.

"My needs are simple, my lord. Just place one grain of rice on the first square of a chessboard. Place two on the next square. Four on the next. Continue doubling that amount until all of the squares are full."

"No problem," replied the king–until he realized the power of exponential growth.

Most likely you have seen graphs that represent both linear and exponential increases. Linear relationships are graphed by straight-line plots. Mathematically, they can be represented by the familiar form $y=mx+b$.

In the above equation, "m" represents the slope of the line. It is a fixed number that describes what some refer to as "rise over run," or mathematically stated as $\Delta y/\Delta x$. In all linear relationships, the slope stays the same. That means the rate of change is fixed.

In contrast, the slope of an exponential increase is not linear. Instead, it continually changes, resulting in a curve. At first, the bend may be hard to observe. However, as you plot successive points, the amount of curvature increases. This change in the incline of the slope reflects a change in rate.

Epidemiologists are well aware of exponential growth. They study the distribution and possible control of diseases. Exponential growth is a type of rate increase that can be observed in the spread of disease.

In this activity, you too will construct an understanding and appreciation of exponential growth. Then, you'll apply what you've learned to the dynamics of a spreading disease.

Materials

- Chessboard or hand-made 8x8 grid
- Rice grains, beads, confetti, or any small objects

Procedure

1. Find a chessboard you can use. If one isn't available, you can use a pencil, ruler, and sheet of paper to make a model of one. A chessboard has 64 squares, aligned in an 8x8 grid.

2. Use grains of rice, beads, confetti, or any small objects to populate the squares of the chessboard.

3. Begin at square 1, by placing one object in its center. Place two objects in square 2, four objects in square 3. Keep going until you reach a practical limit.

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Analysis

4 What did you learn about exponential growth in this exercise?

5 Make a graph that shows what you just did. What values will you place on the x-axis? What values will you place on the y-axis? Explain your choice in terms of dependent and independent variables. Describe the plotted form.

6 How does the quantity of objects placed on one square compare to the total quantity of objects in all the squares before it?

7 Can you come up with an equation that can be used to calculate the number of objects for a given square?

8 Calculate how many grains of rice the king would have had to supply on Day 16, Day 32, Day 48, and Day 64.

9 Suppose there are about 30,000 grains of rice in one pound of rice. Estimate the number of pounds of rice that will be placed on that 64th square? How does this compare to the yearly global production of rice, if that harvest is about 500 million tons?

Transferring Knowledge

Apply what you have learned to explain the spread of COVID-19.

1 Why is it critical to address an outbreak of a new, contagious disease as soon as possible?

2 What factors can limit the exponential rise in cases of a contagious disease? Explain.

Claims, Evidence, and Reasoning

1 Make a claim about the spread and infection rate of COVID-19. What evidence would you need to support this claim? Once you've gathered the data, use reasoning strategies to determine if your claim was valid.

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PARENT/TEACHER RESOURCES

EXPONENTIAL GROWTH AND THE SPREADING OF DISEASE

Time Required
45 minutes

Objective
Students will construct an understanding and appreciation of exponential growth and apply what they've learned to the dynamics of a spreading disease.

Materials
- Chessboard or hand-made 8x8 grid
- Rice grains, beads, confetti, or any small objects

Setup and Procedure

1. More time may be required to complete the activity if you or your student need to make the 8x8 grid.

2. Any small objects can be used to populate the squares of the chessboard or grid. The smaller the better to see the pattern. A practical limit will likely fall in the range of 31 objects (Square 5) to 127 objects (Square 7), depending on the size of the objects.

SCORING RUBRIC

Analyze (Answers to Each Number in Analysis)

4. Students should recognize and be able to describe a pattern. For example, the number of objects added to each square doubles the value of the objects on the previous square. Students may be astonished at the incredible increase that begins about midway through the exercise. It's almost incomprehensible to imagine what the 64th square will look like when filled according to this doubling pattern! And let's not forget to add the number of objects occupying the sixty-three squares leading up to the 64th. This is the power of exponential growth, a rate of increase that can be applied to the growth of an epidemic.

5. The *x*-axis will identify the specific square that was being filled. It would start at zero, and increase by one, as you moved from left to right. The value of *x* represents the independent variable because it is the variable that is changed or controlled. The values for *x* are placed on the *x*-axis.

   The *y*-axis will identify the number of objects placed on a specific square. The value of *y* represents the dependent variable because, as the value of *x* changes, the value of *y* changes. The values for *y* are placed on the *y*-axis.

   Student's graphs should indicate a curved line based on the calculated values for *x* and *y*.

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Calculate

6 Students may recognize that each successive square has twice the quantity of objects as was placed on the square just before it. In other words, as you advance to the next square, you double the number of objects.

7 The number of objects on any given square is equal to two raised to the power of that square's position in the sequence of 0–63 (where the first square is allocated the position zero). The resultant equation is Number of Objects = 2^n, again, where n = the the square's position in the sequence of 0–63.

8 The first square starts with one grain of rice, so

- Square 1 = 20 = 1

Note that any non-zero number to the 0 power equals 1.

- Square 2 = 2^1 = 2×1 = 2
- Square 3 = 2^2 = 2×2 = 4

And so on. So,

- Square 16 = 32,768
- Square 32 = 2,147,483,648
- Square 48 = 140,737,488,355,328
- Square 64 = 9,223,372,036,854,775,808

Note that with 64 squares, we only get to the power of 63, not 64.

9 Square 64 contains 9,223,372,036,854,775,808 grains of rice, so

- 9,223,372,036,854,775,808 ÷ 30,000 = 3.07×10^{14} or about three trillion pounds of rice would need to be placed on Square 64.

One ton is equal to 2,000 pounds, so

- 3.07×10^{14} ÷ 2,000 = 1.5×10^{11} tons of rice compared to 500 million or 5×10^8 tons of rice harvested globally every year. That is (1.5×10^{11}) – (5×10^8) = about 300 times the amount of rice harvested globally in one year.

Transferring Knowledge

Apply what you have learned to explain the spread of COVID-19.

1 Students should consider the pattern of exponential growth that they calculated above and apply that to the growth of a pathogen, for example, the COVID-19 virus. The sooner an outbreak is addressed, the easier it can be contained. If you wait too long, the numbers could overwhelm any attempt at containment.

2 Students might suggest a number of factors such as quarantine, social distancing, handwashing, covering a cough or sneeze, and keeping things clean, which can all reduce spreading of a contagious disease.

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Claims, Evidence, and Reasoning

Students' claims should state that the spread and infection rate of COVID-19 is exponential.

Support for claims should include the following:

- In places where social distancing was put into effect early in the epidemic, the spread of COVID-19 was contained.
- The evidence can be found in the number of reported cases. When more testing became available, reported cases increased due to positive test confirmations. Also, learning more about COVID-19 changed the definition of COVID-19 infection, which added to the number of reported cases.

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