Open Science Repository Mathematics

doi: 10.7392/openaccess.70081955 Received: April 23, 2013. Published: April 24, 2013.

Solution *m*-point boundary problem and interpolation with free parameters

Кazbek A.Khasseinov

Kazakh National Technical University named after K.Satpayev, 050013, 22 Satpayev Street, Almaty, dorteh77@mail.ru

Abstract: *In practice, some ways of solution of the problem of approximation, interpolation and forecasting are used based on the idea to represent the function as a private solution of some differential equation n-th order with constant coefficients [1;2;3;4]. But, unfortunately, such problems come down to solution of the Cauchy problem where the additional initial conditions are necessary.*

Keywords: *Boundary value problem ,multipoint flexible interpolation, approximation*

Mathematics Subject Classification 2010: *39-02*

Introduction

Selection of the main system of functions is still open due to the randomness and selection of the coefficients of the linear differential equation. Indefinite coefficients $a_{j-1}(x)$ are found by minimization, i.e. a linear differential operator is added with the known function $y=f(x)$ and its derivatives which existence is supposed, and then the coefficients are selected so that $L_n[f(x)]\equiv 0$. In a similar way, the expression can be limited with some prescribed value $|L_n[f(x)]|<\varepsilon$, where ε is a small number, so the problem is solved approximately. Sometimes, it is comfortable to select coefficients with the constant numbers[3;4].

Let us assume that $y=y(x)\in C^n[x_1,x_m]$, and $\{x_i\}_1^m$ is a partition of the line$[x_1,x_m]$. Function $y=y(x)$ meets ths boundary conditions in m nods of interpolation

$$(T_k y)(x_i) = \sum_{\nu=1}^{n} \rho_{k\nu}(x_i) y^{(\nu-1)}(x_i) = a_{ik}, \qquad (1)$$

where $\rho_{k\nu}(x)\in C[x_1,x_m]$, $k=1,2,\ldots,r_i$; $i=1,2,\ldots,m$, and $\sum_{i=1}^{m} r_i = n$. Coefficients (1) meet a nonsingularity condition in the points x_i

$$\sum_{\nu=1}^{n} \rho_{k\nu}^2(x_i) \neq 0, \quad i=1,2,\ldots,m.$$

Research

Let us consider an arbitrary linear differential operator with the continuous coefficients at $[x_1,x_m]$

. $$Ly = y^{(n)} + \sum_{\nu=1}^{n} a_{\nu-1}(x) y^{(\nu-1)} \,. \tag{2}$$

Let us assume that the following is executed for $\{y_\nu(x)\}_1^n$ - fundamental system of solution of the equation $Ly = 0$ in points $\{x_i\}_1^m$

$$\Delta = \begin{vmatrix} (T_1 y_1)(x_1) & (T_1 y_2)(x_1) & \dots & (T_1 y_n)(x_1) \\ (T_2 y_1)(x_1) & (T_2 y_2)(x_1) & \dots & (T_2 y_n)(x_1) \\ \vdots & \vdots & & \vdots \\ (T_{r_1} y_1)(x_1) & (T_{r_1} y_2)(x_1) & \dots & (T_{r_1} y_n)(x_1) \\ \vdots & \vdots & & \vdots \\ (T_1 y_1)(x_m) & (T_1 y_2)(x_m) & \dots & (T_1 y_n)(x_m) \\ (T_2 y_1)(x_m) & (T_2 y_2)(x_m) & \dots & (T_2 y_n)(x_m) \\ \vdots & \vdots & & \vdots \\ (T_{r_m} y_1)(x_m) & (T_{r_m} y_2)(x_m) & \dots & (T_{r_m} y_n)(x_m) \end{vmatrix} \neq 0 \,. \tag{3}$$

Let us prove the interpolation formula.

Theorem. If function $y(x) \in C^n[x_1, x_m]$ meets the boundary conditions

$$(T_k y)(x_i) = a_{ik}, \; i = 1,2,..., m; \; k = 1,2,..., r_i, \tag{4}$$

so the following relation is valid

$$\begin{aligned} y(x) = \sum_{i=1}^{m} \sum_{k=1}^{r_i} (T_k y)(x_i) \frac{\Delta_{ik}(x)}{\Delta} + \\ + \sum_{i=1}^{m} \sum_{k=1}^{r_i} \frac{\Delta_{ik}(x_i)}{\Delta} \cdot \int_{x_i}^{x} Ly(s) \frac{W_n[(T_k y_l)(x_i), s]}{W(s)} ds. \end{aligned} \tag{5}$$

Here the determinants $\Delta_{ik}(x)$ are produced from Δ by substitution of the elements of the $\left(k + \sum_{\mu=1}^{i-1} r_\mu\right)$- line of the fundamental system of solutions $\{y_\nu(x)\}_1^n$. $W(s)$ - Wronskian for $\{y_l(s)\}_1^n$, and $W_n[(T_k y_l)(x_i), s]$ is produced by substitution of the last line in $W(s)$ with the following expressions

$$(T_k y_l)(x_i), \quad l = 1,2,\dots,n.$$

The proof results from the found solution of the heterogeneous m – point boundary problem [6;10] if we substitute the right part of the differential equation for the left part – linear differential operator (such approach is used in [5]).

Relation (5) is actually an identity valid at the arbitrary continuous coefficients $a_{j-1}(x)$, $j=1,2,\ldots,n$. Therefore, it is an interpolation formula to represent the prescribed function $y=y(x)$. Fundamental system of solutions $\{y_l(x)\}_1^n$ of the arbitrary linear differential equation $L_n y(x)=0$ serves as the main system of functions. The main system of the continuous functions which approximate function $y(x)$ is usually built based on the priory selection connected with the initial problem. Let us consider two problems.

Problem 1. Let us assume that $y=f(x)\in C^n[x_1,x_m]$ is known and the boundary conditions in the nods x_i are given or found

$$(T_k y)(x_i)=a_{ik},\ i=1,2,\ldots,m;\ k=1,2,\ldots,r_i,$$

which function $y=f(x)$ comply with.
It is necessary to approximate the analytically prescribed function with the smooth functions.

n linearly independent functions $y_1(x), y_2(x),..., y_n(x)$ are selected arbitrarily that $\Delta\neq 0$. Based on these functions, the free coefficients of the linear differential equation $L_n y(x)=0$ are found, which fundamental system of solutions is $\{y_k(x)\}_1^n$. Adding these functions $(T_k y)(x_i)$ and $y=f(x)$ into the identity (5) we produce representation of the prescribed function.

Problem 2. It is necessary to find an unknown function $y=\varphi(x)\in C^n[x_1,x_m]$ complying with the boundary condition in m nods of interpolation

$$(T_k y)(x_i)=a_{ik},\ i=1,2,\ldots,m;\ k=1,2,\ldots,r_i.$$

Let us consider n linearly independent arbitrary functions $y_1(x), y_2(x),..., y_n(x)$ where $\Delta\neq 0$. Supposed that $y=\varphi(x)$ is a solution of the homogeneous equation $Ly(x)=0$, based on the interpolation formula (5) we can find the sought function in the following form

$$y=\sum_{i=1}^{m}\sum_{k=1}^{r_i}(T_k y)(x_i)\frac{\Delta_{ik}(x)}{\Delta},$$

because the integral member is transformed into zero.

Conclusion

Thus, the interpolated function is represented in a form of the solution of the homogeneous or heterogeneous linear differential equation with the arbitrary variable coefficients (it is possible with the constant coefficients as well) which are found under the selected fundamental system of solutions by minimization of the integral member. Actually, the free parameters are included into the construction of the function $y(x)$ through the boundary conditions (1), and selection of the free parameters can help us to manage its behavior [7;8;9].

References

1.Kulikov N. Engineering method of solution and research of the usual differential equations. M.: Higher School, 1964, 224 p.

2.Kulikov N. Mathematical modeling of the experiment results and forecasting on the base of the function with flexible structure. M.: MTIPP, 1974,173 p.

3.Samoilenko A., Ronto N. Numeric and analytic methods of solution of the boundary problem. Kiev: Higher School,1985. 223 p.

4.Theory of forecasting and making decisions / Edited by Sarkisyan S. M.: Higher School,1977. 352 p.

5. Householder A. Principles of Numerical Analysis. M.: IL, 1956, 319 p.

6.Khasseinov K.A.Initial and multi-point problems for LDE and characteristic of Riccati type.Synopsis of thesis for degree of a candidate of physic-mathematical sciences.Moscow,1984,114 p.

7.Khasseinov K.A. Flexible interpolation with the degrees of fleribility//Prog.XII Inter.conf.on nonlinear oscill.Cracow, 1990.

8.Trenogin V.A.,Khasseinov K.A. Dual Problems to Abstract Nonlocal Problems.Science conference of DE,Turkey,Fetchie,16-23 June 2001.

9.Zhuosheng Lu,Fuding Xie.Explicit bi-soliton-like solutions for a generalized KP equation with variable Coefficients.Mathematical and Computer Modelling,2010

10. Kazbek A. Khasseinov . Multipoint BoundaryValue Problem for the Adjoint Equation and Its Green's Function.J.of Mathematics Research, Toronto,Canada,Vol.5,No.2,2013,p.15-31.