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Structural Quantum Gravity

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Abstract

This paper explains the concept of Structural Quantum Gravity and how divergent expressions that occur in General Relativity can be avoided by adding additional structural terms. Feynman diagrams with divergent expressions are treated as topological structures with further curvature contributions.

Introduction

In Quantum Field theory particles are assumed as pointlike particles. However, several scattering processes (e.g. Loop contributions) in General Relativity can cause divergent expressions that cannot be absorbed in coupling constants; General Relativity is nonrenormalizable. For fixing the problem of divergent mathematical expressions, there are existing many approaches for Quantum Gravity in Theoretical Physics. For example, the String theory assumes particles as objects with extension [1]. Because the particles are not assumed as objects that are infinitely thin, infinite mathematical expressions in scattering processes are avoided. Other concepts of Quantum Gravity are Loop Quantum Gravity [2], Causal Geometric Triangulation [3] and the theory of Asymptotic Safety [4]. All theories of Quantum Gravity using the concepts that the universe cannot have length and time differences lower than the Planck length

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \tag{1}$$

and lower than the Planck time

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \tag{2}$$

with Gravity constant G, Planck's constant $\hbar$ and the speed of light c. In this paper, an approach with a topological structuration of spacetime curvature (called Structural Quantum Gravity) is treated. Loop Quantum Gravity uses spin foam for a dynamic discretization of spacetime. Structural Quantum Gravity considers the discretization of spacetime as a set of homologic topological spaces C_n of dimension n which obeye the exact sequence (singular complex)

$$\cdots \xrightarrow{d} C_n \xrightarrow{d} C_{n-1} \xrightarrow{d} C_{n-2} \xrightarrow{d} \cdots \tag{3}$$

with the boundary map d and cohomologic topological spaces $Hom(C_n, G)$ which obeye the exact sequence (De Rham complex)

$$\cdots \xleftarrow{\delta} Hom(C_n, G) \xleftarrow{\delta} Hom(C_{n-1}, G) \xleftarrow{\delta} Hom(C_{n-2}, G) \xleftarrow{\delta} \cdots \tag{4}$$

with the coboundary map δ and an algebraic group G. The homological spaces describe the discrete structure of the spacetime, i.e. homological spaces are regions of spacetime which behave geometric since they are topological spaces with boundaries. Cohomological spaces are mappings from spacetime regions to an algebraic group. If the discretized spacetime has no loops (induced by loops in Feynman diagrams) the homological space and the cohomological spaces coincide. Every line in a Feynman diagram represents a homological part of the spacetime where the interlinking vertices represent the boundaries of Feynman diagram lines. The coboundaries of the vertices are lines since the coboundary operator δ takes the "difference" of the vertices. If there are loops, the homological and cohomological spaces are non-isomorphic, because loops enclosing extra spacetime so that there can exist multiple lines from one vertex to another. To solve the problem of divergent mathematical expressions in Feynman path integrals, the concept of structural curvature is introduced for avoiding UV and infrared divergences of loop contributions in Feynman propagators. Structural curvature is the effective curvature resulting from the enclosure of Loop contributions.

Theory

Considering the topological space M that is generated by N elements $m_1, m_2, ..., m_N$. These elements form a module since for the specific coefficients α_i with $i \in \{1, ...N\}$ there is

$$\sum_{i=1}^{N} \alpha_i m_i \in M. \tag{5}$$

These elements which form a module can be considered as the Feynman diagram lines. Hence, the expression dm_i are the two bounding vertices of the line m_i. If P is the set of vertices with elements $p_1, ..., p_M \in P$, the expression δp_i is the set of all lines between the vertices. If A is the set of all areas enclosured by Loop contributions in Feynman diagrams then the following short exact sequence holds:

$$0 \xrightarrow{d} A \xrightarrow{d} M \xrightarrow{d} P \xrightarrow{d} 0 \tag{6}$$

Analogous, there is existing a short exact sequence for the cohomological spaces

$$0 \xleftarrow{\delta} Hom(A, G) \xleftarrow{\delta} Hom(M, G) \xleftarrow{\delta} Hom(P, G) \xleftarrow{\delta} 0 \tag{7}$$

Considering the mappings

$$f_{AP} : A \to Hom(P, G) \tag{8}$$

and

$$f_{PA} : P \to Hom(A, G). \tag{9}$$

If these mappings are isomorphisms then $M \cong Hom(M, G)$ is implied, because the diagrams

$$\begin{array}{ccc} A & \xrightarrow{\quad d \quad} & M \\ \downarrow{\scriptstyle f_{AP}} & & \downarrow{\scriptstyle i} \\ Hom(P,G) & \xrightarrow{\delta} & Hom(M,G) \end{array} \tag{10}$$

and

$$\begin{array}{ccc} M & \xrightarrow{\quad d \quad} & P \\ \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle f_{PA}} \\ Hom(M,G) & \xrightarrow{\delta} & Hom(A,G) \end{array} \tag{11}$$

commute with the isomorphism i. The mappings f_{AP} and f_{PA} are the restructurizing mappings since these mappings induce another discretization of spacetime. For a Feynman diagram with no loops the restructurizing mappings are an isomorphism because there are no enclosures by loops that can be mapped to new points that cause another discretization scheme of the spacetime. However, if there are loops in Feynman diagrams, the sequences (6) and (7) are not isomorphic. In this case, the i-th extension $Ext^i(-,G)$ has a nonvanishing value. For extensions, there holds an exact sequence

$$0 \longrightarrow Hom(P,G) \longrightarrow Hom(M,G) \longrightarrow Hom(A,G) \longrightarrow$$

$$Ext^1(P,G) \longrightarrow Ext^1(M,G) \longrightarrow Ext^1(A,G) \longrightarrow Ext^2(P,G) \longrightarrow ... \tag{12}$$

The maximal extension order $max(i)$ determines the number of additional restructurizings of the discrete spacetime. May be C a topological space, where for the j subsets $Q_j \subset C$ holds $Ext^i(Q_j,G) = 0$ for every $i > 0$. Hence, the maximal subspace with vanishing extension Q is the union of all Q_j, i.e.

$$Q = \varinjlim Q_j. \tag{13}$$

In General Relativity, the curvature of spacetime is expressed in terms of metric tensors $g_{\mu\nu}$ and its covariant derivatives

$$D_\alpha g_{\mu\nu} := \partial_\alpha g_{\mu\nu} - \Gamma^\beta_{\mu\alpha} g_{\beta\nu}. \tag{14}$$

with Christoffel symbols $\Gamma^\beta_{\mu\alpha}$. If R is the Ricci scalar, the action in General Relativity has the form:

$$S_{GR} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R. \tag{15}$$

In Quantum Field theory, mathematical expressions for scattering amplitudes computed with this action can be divergent, because the Ricci scalar describes the curvature only if the continuous limit of the discretized spacetime is unique, i.e. there is existing only one unique spacetime discretization scheme. For the case of structural curvature the action must be extended by an additional action S_{ext} that can be gained by the transformation of covariant derivatives

$$D_\alpha \mapsto D_\alpha + \Delta_\alpha. \tag{16}$$

For the extensional derivative Δ_α holds the relation

$$\Delta_\alpha g_{\mu\nu} = -\Delta^\beta_{\mu\alpha} g_{\beta\nu} \tag{17}$$

where the coefficients $\Delta^\beta_{\mu\alpha}$ are the metric connection for the spacetime extended by restructurizing. The extensional derivative is independent on the ordinary covariant derivative D_α since the extensional derivative depends only on the form of the Feynman diagram, i.e.

$$D_\beta \Delta_\alpha = \Delta_\beta D_\alpha. \tag{18}$$

May be X a metric space and $X_{;\alpha} := D_\alpha X$ the covariant derivative of the metric space. If $Y := \Delta_\alpha X$ is the structural derivative of the metric space the following diagram commutes:

$$\begin{array}{ccc} X & \xrightarrow{D_\alpha} & X_{;\alpha} \\ \downarrow{\scriptstyle \Delta_\alpha} & & \downarrow{\scriptstyle \Delta_\beta} \\ Y & \xrightarrow{D_\beta} & Y_{;\beta} \end{array} \tag{19}$$

If there are non-vanishing extensional derivatives, the mappings f_{AP}, f_{PA} and i in diagrams (10) and (11) are not isomorphisms. By setting $Hom(P,G) = Hom_\times(P,G) \oplus Ext^1(P,G)$, $Hom(M,G) = Hom_\times(M,G) \oplus Ext^1(M,G)$ and $Hom(A,G) = Hom_\times(A,G) \oplus Ext^1(A,G)$ with $Hom_\times(A,G) \cong P$,
$Hom_\times(M,G) \cong M$ and $Hom_\times(P,G) \cong A$, there can be assumed that the metric space X is set isomorphic to M (because the metric spaces is a set of Feynman diagram lines). The covariant derivative of the space M is the set of vertices P. Diagram (11) can be written as

$$\begin{array}{ccc} M & \xrightarrow{\qquad\qquad D_\alpha \qquad\qquad} & P \\ \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle f_{PA}} \\ Hom_\times(M,G) \oplus Ext^1(M,G) & \xrightarrow{D_\beta} & Hom_\times(A,G) \oplus Ext^1(A,G) \end{array} \tag{20}$$

since d and δ are derivatives according to the directions α and β. Comparing (19) with (20) and using the fact that the extensional derivative is independent under restructurizing (i.e. $Hom_\times(M,G)$ and $Hom_\times(A,G)$ have no contributions to the extensional derivative), it follows that the extensional derivative is defined by:

$$\Delta_\alpha : M \to Ext^1(M,G). \tag{21}$$

The extensional derivative maps the topological space M to all extension contributions projected in the direction α. Substituting (16) and (18) into (15) yields:

$$S = S_{GR} + S_{ext} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + \Delta^{\mu\nu}_{\mu} \Delta^{\lambda}_{\lambda\nu} - \Delta^{\mu\nu}_{\lambda} \Delta^{\lambda}_{\mu\nu}). \tag{22}$$

For the computation of $\Delta^\nu_{\alpha\mu}$ the extensional derivative is assumed as an unique constant for a given form of a Feynman diagram. By (21), the value of $\Delta^\nu_{\alpha\mu}$ is determined by the number of topological extensions. May be $r^\nu_{\alpha\mu}$ the number of extensions associated with the directions α, μ and ν and Λ^* a coupling constant. Then, there can be assumed:

$$\Delta^\nu_{\alpha\mu} = \Lambda^* r^\nu_{\alpha\mu}. \tag{23}$$

The Feynman propagator for Structural Quantum Gravity has the form

$$K = \frac{1}{Z} \int d[g_{\mu\nu}] e^{iS_{GR} + iS_{ext}} \tag{24}$$

where the expression $e^{iS_{ext}}$ is the structural curvature factor and with normalizing constant Z. The integral in the action (24) can be replaced by a sum by performing the transformation $\int d^4x \mapsto \sum l_p^3 t_p$.

Conclusions

The action of Structural Quantum Gravity (22) is the same action as the Einstein-Hilbert action of General Relativity weighted with a structural curvature factor. This structural curvature factor prevents the diverging of Feynman diagrams with loops, because this factor becomes smaller for larger structural curvatures. Hence, the theory is a renormalizable quantum field theory for all kind of scattering processes. By combining (23) with (22) and taking the classical limit the Structural Quantum Gravity tends to Einstein's theory of General Relativity. If the loop contributions become significant, Structural Quantum Gravity tends to Einstein's field equations with cosmological constant.

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