Wave-Particle Duality of Gravitational Wave and Designed Experiment

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Abstract
The detection of gravitational waves (GW) demands thorough study of GW. In this article we systematically study both wave behavior and particle nature of GW in framework of Gravitodynamics and derive gravitational counterparts of Electromagnetic wave (EMW). For wave phenomena, we show: (1) intensities of GW quadrupole radiation predicted by either Gravitodynamics or by linearized General Relativity are the same, except by factor of 4; (2) the correlations between redshifts of both GW and EMW and between Hubble Constant and Redshift of GW; (3) formula for relativistic quadrupole radiation. For particle nature, we demonstrate: (1) GW is quantizable; (2) The wave-particle duality of GW exists; (3) Gravitational dipole radiates Gravito-photon; (4) Dirac Sea is generalized to include gravitational charges. An experiment is proposed to detect wave-particle duality of GW. We raise two questions: (1) what is physical mechanism of conversion of mass to GW/Gravito-photons? (2) Does GW/Gravito-photon convert to mass?

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Key words: gravitodynamics, gravitational waves (GW), quantum of GW, wave-particle duality of GW, gravitational experiment
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1. Introduction

Newton’s theory of gravity implies that the gravitational field propagates at infinite speed. In 1905, Henri Poincaré had the idea that gravitational wave (GW) might travel at the speed of light in a manner similar to Electromagnetic Wave (EMW). However, he did not establish a complete theory of gravity [1]. In 1916, Einstein predicted the existence of GW based on General Relativity (GR) [2]. All theories of gravity thereafter predict GW. These include geometric and vector field theories. In 2016, 100 years after Einstein’s prediction, the LIGO and Virgo teams announced their first observations of GW [3]

The observation of GW requires further study of GW. For this aim, we need to resolve the following issues. One of the major issues is the energy problem in GW and gravity, which has been a long-standing challenge to both GR and vector field theories of gravity. Another one is the incompatibility between GR and quantum mechanics. Resolution of these issues and further exploration of the properties of GW will not only be helpful for understanding GW thoroughly but also elucidate the wave-particle duality of GW, quantize gravity, and unify gravity with other interactions in nature.

In GR, the energy-momentum of a gravitational field is: a) not a tensor; b) not conserved; and c) not localizable. These issues of the energy cause conceptual difficulties in GW and in quantization of GR [4, 5, 6, and references wherein]. Schutz (2011) [7] summarized the conceptually fundamental energy issue of GW in GR as “Energy has been one of the most confusing aspects of gravitational wave theory and hence of GR. It caused much controversy, and even Einstein himself took different sides of the controversy at different times in his life. Physicists today have reached a wide consensus. The problem is difficult because of the equivalence principle: in a local frame there are no waves and hence no local definition of energy that can be coordinate-invariant. Moreover, a wave is a time-dependent metric, and in such space-times there is no global energy conservation law”.

One approach to resolve the energy issue in GR is to revisit the older definition of energy and an energy-momentum pseudo-tensor $t^{\mu\nu}$ was proposed [5]. However, this $t^{\mu\nu}$ faces other difficulties [8]. We took a different approach to resolve the energy issue in GR by reinvestigating the validity of the equivalent principle (2015) [9]. We argued that a fast moving test body violates the Universality of Free Fall, which is equivalent to the Weak Equivalence Principle. Moreover, a gravitational synchrotron-type experiment was proposed to detect this violation. The results of the experiment would allow a better
understanding and a more precise expression of the equivalent principle.

At the present time, it is difficult to theoretically resolve the energy issue of GW in GR. Therefore, it is commonly accepted that energy is a useful but not a fundamental concept in GR, i.e., the concept of energy is not necessary to calculate the radiated GW and the effects of GW [7]. However, it is difficult to find geometrical counterparts in GR for all the physical concepts and terms borrowed from physical field theory in order to express GW and gravity. These concepts and terms include “missing mass vanished in gravitational radiation, a conversion of mass to energy”, “spin 2 GW”, and “graviton”, etc. In contrast, energy of waves is an important concept in field theories and was introduced into GW by Einstein because of its strong analogy to EMW [6].

It is inconsistent and confusing to accept some of these physical concepts and reject others in studying GW and gravity.

In vector theories of gravity, the energy issue is that the energy density of static Newtonian gravitational field is negative [10]. By close analogy to electromagnetism, Maxwell and Heaviside proposed the vector theory which has the form same to that of electrodynamics [11]. However they did not go any further since they realized the negative energy issue.

In 2015, in a different direction, based on the Precise Equivalent Principle, the U (1) gauge theory of gravity is proposed, which we denoted as Gravitodynamics [12]. As a vector field theory Gravitodynamics therefore needs to address the negative energy issue. Indeed, it has been shown that the measurable exchanged energy of gravitational fields is always positive, i.e., transported energy in and out of gravitational field is always positive [13]. Therefore the non-measurable total energy of gravitational fields being either negative or positive is only a matter of bookkeeping. There is no negative energy issue in Gravitodynamics.

Let’s consider next issue: the incompatibility between GR and quantum mechanics. In the development of the quantum mechanics, Einstein first noted the wave-particle duality of EMW, he wrote: “It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do” [14].

The detection of GW brings up an old question: does GW exhibit wave-particle duality, i.e., is gravitational field quantizable? This problem needs to be addressed
theoretically as well as empirically. It has been shown that a single spin 2 graviton predicted by GR is not detectable by LIGO [15 and references therein], if it exists.

In contrast, Gravitodynamics shows that gravity is a local physical field like the electromagnetic field, is quantizable, and is renormalizable. Gravitodynamics also predicts the existences of negative gravitational charges and GW. It has been shown that, similar to the dark energy or Einstein’s cosmology constant $\Lambda$, the negative gravitational charges can naturally explain the accelerated expansion of the universe equally well [16]. Moreover, the negative gravitational charges can resolve the fine-tuning problem encountered by Einstein’s cosmology constant $\Lambda$ when explaining the accelerated expansion of the universe. The effects of the dark energy on the propagation of GW have been studied [17].

The benefits of Gravitodynamics are the following: (1) there is no energy issue [13]; (2) GW is quantizable [12]; (3) the existence of negative gravitational charges explains the accelerated expansion of Universe [16]; (4) Gravitodynamics is compatible with Special Relativity (SR). Therefore, in this article we apply Gravitodynamics to systematically study the wave behavior, the particle nature of GW, and radiation of GW. These efforts not only allow a physical understanding of GW but also predict new effects. In addition, this approach makes calculations straightforward. Last but not least, we predict a new phenomenon/experiment that will be able to verify the particle nature of GW, if observed.

### 2. Wave Phenomena of GW in Gravitodynamics

In Newtonian theory of gravity, the gravitational field equation is

$$\nabla \cdot \mathbf{g} = -4\pi \rho_g,$$

where the gravitational field strength $\mathbf{g}$ is time independent and related to potential $V_g$,

$$\mathbf{g} = -\nabla V_g.$$  

Eqs. (1 and 2) imply that the gravitational field propagates at an infinity speed.

In GR, GW is studied in term of $\tilde{h}^{\alpha\beta}$, which is interpreted either as “space-time ripple” in geometric term, or equivalently, as “potentials” in physical term, by equation,

$$\frac{1}{c^2} \frac{\partial^2 \tilde{h}^{\mu\nu}}{\partial t^2} - \nabla^2 \tilde{h}^{\mu\nu} = 0.$$

Although the linearized Einstein equation has been expressed in terms of gravitational and gravito-magnetic field strengths, in tensor form, $G^{\nu\lambda}$, [18].
\[
\frac{\partial G^{\mu\nu}}{\partial x^\alpha} = -4\pi T^{\mu\nu},
\]

the concept of gravitational field strength hasn’t been employed in studying GW in GR.

Note there is only one kind of gravitational charge in both Newtonian gravity and GR, we denote it as the positive gravitational charge. In contrast, there are both positive and negative gravitational charges in Gravitodynamics. The gravitational field equations, thus, have a form identical to those in Electrodynamics [12, 13]. This strong similarity implies that gravity, as electromagnetic field, is a physical field, although the like electric charges are repulsive to each other whereas the like gravitational charges are attractive to each other. Therefore, in this article, we introduce the concepts of EMW into Gravitodynamics to study GW. The propagation of GW is beyond the scope of the article.

2.1. Gravitodynamics Compatible with Special Relativity

The field equations in Gravitodynamics have the tensor form [13],

\[
\frac{\partial F_{gT}^{\mu\nu}}{\partial x^\alpha} = -\frac{4\pi}{c} J_{gT}^\mu,
\]

\[
\frac{\partial F_{gT}^{\mu\nu}}{\partial x^\alpha} + \frac{\partial F_{gT}^{\nu\mu}}{\partial x^\alpha} = 0,
\]

\[
\frac{\partial F_{gT}^{\mu\nu}}{\partial x^\alpha} = \frac{4\pi}{c} J_{gT}^\mu,
\]

\[
\frac{\partial F_{gT}^{\mu\nu}}{\partial x^\alpha} + \frac{\partial F_{gT}^{\nu\mu}}{\partial x^\alpha} = 0,
\]

\[
mc \frac{dV^\mu}{ds} = \frac{Q_{gT}^\pm}{c} F_{gT}^{\mu\nu} V^\nu.
\]

Where \( F_{gT}^{\mu\nu} \equiv \partial^\mu A_{gT}^{\nu\pm} - \partial^\nu A_{gT}^{\mu\pm} \) is the antisymmetric gravitational field four-tensor; \( F_{gT}^{\mu\nu} \equiv F_{gT}^{\mu+} + F_{gT}^{\mu-} \); \( A_{gT}^{\mu\pm} \) is the gravitational four-potential; \( J_{gT}^\mu \) is the positive/negative gravitational four-current; \( J_{gT}^\mu = J_{gT}^\mu + J_{gT}^\mu \); \( V^\mu \) is the four-vector velocity; \( Q_{gT}^\pm \) and \( m \) are the positive/negative gravitational charge and mass of a body, respectively. In this article, letters with subscript “+”, “−” and “±” denote variables related to the positive, negative, and either positive or negative gravitational charges, correspondingly. Letters with subscript “e” and “g” denote variables related to Electrodynamics and Gravitodynamics, respectively. In this tensor form, it is obvious that Gravitodynamics and SR are compatible. We will employ SR in studying GW.

2.2. Transformation Law of Gravitational Fields

In analogy to the transformation of electromagnetic fields in SR, we obtain the same rule for the transformation of gravitational fields, including generalized Newtonian
gravitational and gravitomagnetic fields. In the tensor form, the gravitational fields in one inertial frame $S'$ can be expressed in terms of gravitational fields in another frame $S$,

$$g'_{\mu\nu} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} g_{\alpha\beta}.$$

(7)

In the vector form, the transformation of the gravitational field strengths is,

$$\mathbf{g}'_\pm = \gamma(\mathbf{g}_\pm + \mathbf{\beta} \times \mathbf{B}_g) - \frac{\gamma^2}{1 + \mathbf{\beta} \cdot \mathbf{g}_\pm} \mathbf{\beta} (\mathbf{\beta} \cdot \mathbf{g}_\pm)$$

$$\mathbf{B}'_{g\pm} = \gamma(\mathbf{B}_g - \mathbf{\beta} \times \mathbf{g}_\pm) - \frac{\gamma^2}{1 + \mathbf{\beta} \cdot \mathbf{g}_\pm} \mathbf{\beta} (\mathbf{\beta} \cdot \mathbf{B}_g).$$

(8)

where $\mathbf{\beta} = \mathbf{V}/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$. Let’s consider a special case: a gravitational charge $Q_g$ is at rest in a reference system $S'$ in which the charge $Q_g$ generates a Newtonian field $\mathbf{g}'$, but not gravitomagnetic field $\mathbf{B}_g'$. An observer rests in a system $S$ moving relative to the system $S'$. According to the transformation rule of Eq. (8), the observer will detect a gravitomagnetic field $\mathbf{B}_g$,

$$\mathbf{B}_g = \mathbf{\beta} \times \mathbf{g}_\pm.$$

(9)

This implies that a gravitomagnetic field must accompany with a moving gravitational charge. The existence of gravitomagnetic field is important in studying GW. The existences of gravitomagnetic field and GW are equivalent. There are magnetic or magnetic-type fields in all of four interactions in nature.

2.3. GW Equation and Plane GW

With both positive and negative gravitational charges, Eq. (4) can be written as,

$$\nabla \cdot \mathbf{g}_\pm = - 4\pi \rho_{g\pm},$$

(10)

$$\nabla \cdot \mathbf{B}_{g\pm} = 0,$$

(11)

$$\nabla \times \mathbf{g}_\pm = - \frac{1}{c} \frac{\partial \mathbf{g}_\pm}{\partial t},$$

(12)

$$\nabla \times \mathbf{B}_{g\pm} = - \frac{4\pi}{c} \mathbf{J}_{g\pm} + \frac{1}{c} \frac{\partial \mathbf{B}_{g\pm}}{\partial t}.$$

(13)

Similar to Electrodynamics, we apply the potentials, $V_{g\pm}$ and $A_{g\pm}$, and the generalized Newtonian gravitational field strength, $\mathbf{g}_\pm$, and gravito-magnetic field strengths, $\mathbf{B}_{g\pm}$, to study GW, which are defined as

$$\mathbf{g}_\pm \equiv - \frac{1}{c} \frac{\partial A_{g\pm}}{\partial t} - \nabla V_{g\pm}, \quad \mathbf{B}_{g\pm} \equiv \nabla \times A_{g\pm}.$$

(14)

With the definition of $\mathbf{g}_\pm$ of Eq. (14), which is the generalization of Eq. (2), we call Eq. (10), which has the form same to that of Eq. (1), the generalized Newtonian equation.
All of the four fundamental interactions in nature, as described by Electrodynamics, Yang-Mills theories, GR of gravity [18], and Gravitodynamics [12], correspondingly, contain electrical/electrical-type as well as magnetic/magnetic-type fields. The existence of magnetic-type fields must be an ingredient for a correct theory of gravity and is equivalent to the existence of $G_W$.

We introduce the gravitational Lorentz gauge,

$$\nabla \cdot A_{g} \pm + \frac{1}{c} \frac{\partial V_{g} \pm}{\partial t} = 0, \quad \frac{\partial A_{g}^\mu}{\partial x^\mu} = 0$$ (15)

Eqs. (12 and 13) give GW equations, respectively,

$$\frac{1}{c^2} \frac{\partial^2 V_{g} \pm}{\partial t^2} - \nabla^2 V_{g} \pm = -4\pi p_{g} \pm, \quad \frac{1}{c^2} \frac{\partial^2 A_{g}^\pm}{\partial t^2} - \nabla^2 A_{g}^\pm = -4\pi J_{g} \pm.$$ (16)

Eq. (16) imply that the generalized Newtonian gravitational field/potential, $g_\pm / V_\pm$, and gravito-magnetic field/potential, $B_g / A_g$, propagate as waves with finite speed.

Note we have assumed that the propagation speed of GW in vacuum is equal to that of light, which needs to be experimentally demonstrated though. For the rest of this article, for simplicity, we only consider the positive gravitational charges and their fields, therefore, drop the subscripts “−”, “+”, and “±”.

In vacuum, Eq. (16) gives the gravitational d’Alembert equations:

$$\frac{1}{c^2} \frac{\partial^2 V_{g}}{\partial t^2} - \nabla^2 V_{g} = 0, \quad \frac{1}{c^2} \frac{\partial^2 A_{g}}{\partial t^2} - \nabla^2 A_{g} = 0.$$ (17)

Now Let’s consider plane GW. We choose the potentials so that the scalar potential $V_g = 0$. The gravitational Lorentz gauge becomes the gravitational Coulomb gauge, $\nabla \cdot A_g = 0$. Then Eq. (14) gives,

$$g = -\frac{1}{c} \frac{\partial A_g}{\partial t'}, \quad B_g = -\frac{1}{c} n \times \frac{\partial A_g}{\partial t'}.$$ (18)

Where $n$ is the unit vector along the direction of propagation of GW and $t' = t - \frac{x}{c}$. We have, $B_g \perp g, B_g \perp n$, and $g \perp n$, i.e., GW is transverse. The field strength, $|g| = |B_g|$.

The gravitational Poynting energy flux has been derived in Gravitodynamics [13],

$$S_g = \frac{c}{4\pi} g \times B_g.$$ (19)

For plane GW, the gravitational Poynting energy flux $S_g$, gravitational energy density $W_g$, the flux of momentum $p_g$ of GW become, respectively
\[ S_g = \frac{C}{4\pi} g^2 n = \frac{C}{4\pi} B_g^2 n, \] (20)

\[ W_g = \frac{1}{8\pi} (g^2 + B_g^2) = \frac{1}{4\pi} g^2 = \frac{1}{4\pi} B_g^2. \] (21)

\[ p_g = \frac{W_g}{c} n. \] (22)

The \( S_g \) and \( p_g \) of GW are directed along the direction \( n \) of propagation of GW.

The striking similarities (Table 1) between Electrodynamics and Gravitodynamics suggest that both fields have the same characteristics.

<table>
<thead>
<tr>
<th>Source</th>
<th>Electrodynamics</th>
<th>Gravitodynamics</th>
<th>Linearized Einstein Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Strength</td>
<td>( F_e^{\mu \nu} \equiv \partial^\mu A_e^\nu - \partial^\nu A_e^\mu )</td>
<td>( F_g^{\mu \nu} \equiv \partial^\mu A_g^\nu - \partial^\nu A_g^\mu )</td>
<td>( G^{\mu \nu} \equiv \frac{1}{4} (\partial^\nu F_\mu - \partial^\mu F_\nu) )</td>
</tr>
<tr>
<td>Field Equation</td>
<td>( \frac{\partial F_e^{\mu \nu}}{\partial x^\alpha} = \frac{4\pi}{C} J_e^{\mu} )</td>
<td>( \frac{\partial F_g^{\mu \nu}}{\partial x^\alpha} = \frac{-4\pi}{C} J_g^{\mu} )</td>
<td>( \frac{\partial G^{\mu \nu}}{\partial x^\alpha} = -4\pi T_{+}^{\mu \nu} )</td>
</tr>
<tr>
<td>Wave Equation</td>
<td>( \frac{\partial^2 V_e}{C^2 \partial t^2} - \nabla^2 V_e = 4\pi \rho_e )</td>
<td>( \frac{\partial^2 V_g}{C^2 \partial t^2} - \nabla^2 V_g = -4\pi \rho_g )</td>
<td>( \frac{\partial^2 \tilde{h}_0^{00}}{C^2 \partial t^2} - \nabla^2 \tilde{h}<em>0^{00} = -16\pi \rho</em>{g+} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{\partial^2 A_e}{C^2 \partial t^2} - \nabla^2 A_e = 4\pi I_e )</td>
<td>( \frac{\partial^2 A_g}{C^2 \partial t^2} - \nabla^2 A_g = -4\pi I_g )</td>
<td>( \frac{\partial^2 \tilde{h}_0^{ij}}{C^2 \partial t^2} - \nabla^2 \tilde{h}<em>0^{ij} = -16\pi T</em>{+}^{ij} )</td>
</tr>
<tr>
<td>Poynting flux</td>
<td>( S_e = \frac{C E^2 n}{4\pi} = \frac{C B_g^2 n}{4\pi} )</td>
<td>( S_g = \frac{C g^2 n}{4\pi} = \frac{C B_g^2 n}{4\pi} )</td>
<td>No expression in terms of gravitational field strengths</td>
</tr>
<tr>
<td>Energy density</td>
<td>( W_e = \frac{1}{8\pi} (E^2 + B^2) )</td>
<td>( W_g = \frac{1}{8\pi} (g^2 + B_g^2) )</td>
<td>No expression in terms of gravitational field strengths</td>
</tr>
<tr>
<td>Momentum density</td>
<td>( p_e = \frac{W_e}{c} n )</td>
<td>( p_g = \frac{W_g}{c} n )</td>
<td>No expression in terms of gravitational field strengths</td>
</tr>
</tbody>
</table>

Where \( \tilde{h}^{\mu \nu} \equiv (h^{\mu \nu} - \frac{1}{2} \eta^{\mu \nu} h^{\alpha \beta} g_{\alpha \beta}) \), \( \tilde{h}_0 = (\tilde{h}_0^{01}, \tilde{h}_0^{02}, \tilde{h}_0^{03}) \), and \( J_0^{g+} = (T_0^{01}, T_0^{02}, T_0^{03}) \). We systematically study GW by following the procedure by which EMW is studied [5, 19].
2.4. Monochromatic Plane GW and Polarization

Let’s consider a special case of plane GW, the monochromatic plane GW. The solution of Eq. (17) is
\[ A_g = A_{g0} e^{-i k_g x}, \]
where \( A_{g0} \) is time-independent; GW four-vector is
\[ k^\mu_g = \left( \frac{\omega_g}{c}, k_g \right), \quad k^\mu_g k_g = 0, \quad k_g = \frac{\omega_g}{c} n. \] (24)

Substituting Eq. (23) into Eq. (18), we have gravitational field strengths,
\[ g = i k_g A_g, \quad B_g = i k_g \times A_g. \] (25)

Let’s consider the polarization of GW. Assuming GW propagating along z-axis, we can write the field \( g \) in the following form,
\[ g_x = g_{0x} \cos \left( \omega_g t - k_g \cdot r \right), \quad g_y = \pm g_{0y} \sin \left( \omega_g t - k_g \cdot r \right), \] (26)
where the \( g_x \) and \( g_y \) satisfy the relation:
\[ \frac{g_x^2}{g_{0x}^2} + \frac{g_y^2}{g_{0y}^2} = 1. \] (27)

Eq. (27) implies that this GW is elliptically polarized. For \( g_{0x} = g_{0y} \), the ellipse GW reduces to circularly polarized GW including right and left polarizations. If either \( g_{0x} = 0 \) or \( g_{0y} = 0 \), the ellipse GW wave reduces to linearly polarized GW. An ellipse GW can be a superposition of two plane polarized GW. The motion of a gravitational charge in the field of GW is determined by Eq. (25). The patterns of the motion are different under the influence of different polarized GWs. The motions of a gravitational charge in the field of either a linearly or a circularly polarized GW can be obtained by following the method used in Electrodynamics.

Let’s introduce GW polarization vector \( N^P_g \). Transverse GW is generated in two states of polarizations and propagates along z-axis (Fig.1).

![Fig. 1: Relations between polarization vector \( N^P_g \), angle \( \theta^P_g \), and wave vector \( k_g \)](image-url)
The GW polarization vector $\mathbf{N}_g^p$ is perpendicular to GW wave vector, $\mathbf{N}_g^p \cdot \mathbf{k}_g = 0$, (28)
and can be expressed in terms of GW polarization angle, $\theta_g^p$ ($0^\circ \leq \theta_g^p \leq 90^\circ$),

$$\mathbf{N}_g^p = \cos\theta_g^p \mathbf{x} + \sin\theta_g^p \mathbf{y},$$

(29)

where the $\mathbf{N}_g^p$ defines the plane of vibration of GW. Interesting examples are $\theta_g^p = 0^\circ$ and $45^\circ$, which correspond respectively to $h_+$ and $h_\times$ in GR.

The comparison of the polarizations of GW in both GR and Gravitodynamics is in Table 2.

<table>
<thead>
<tr>
<th>1</th>
<th>Wave Equation</th>
<th>Gravitodynamics</th>
<th>Linearized Einstein Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wave Equation</td>
<td>$\frac{1}{C^2} \frac{\partial^2 \mathbf{V}_g}{\partial t^2} - \nabla^2 \mathbf{V}_g = 0$</td>
<td>$\frac{1}{C^2} \frac{\partial^2 \mathbf{h}^{ij}}{\partial t^2} - \nabla^2 \mathbf{h}^{ij} = 0$</td>
</tr>
<tr>
<td>2</td>
<td>Polarization</td>
<td>$\mathbf{N}_g^p = \cos\theta_g^p \mathbf{x} + \sin\theta_g^p \mathbf{y}$</td>
<td>$h_+$ and $h_\times$</td>
</tr>
</tbody>
</table>

For approximately monochromatic GW, its amplitude $\mathbf{A}_{g0}$ changes with time. Thus its polarization varies with time, which is called partially polarized GW. The gravitational Stokes parameters can be derived in the same way as those in Electrodynamics.

### 2.5. Spectral Resolution

GW comes in a spectrum of frequencies. The two detection techniques, B-modes and laser interferometry, search for GW at different frequencies. We need to know the amplitude in each frequency. The detection of GW in two different frequencies would be strong evidence for the existence of GW.

Any GW can be represented as a superposition of monochromatic GW with different frequencies, discrete or continuously distributed frequencies, as the following,

$$X_{g,\text{disc}} = \sum_{n=\infty}^{\infty} X_{g,n} e^{-i\omega_{g0}t}, \quad X_{g,\text{cont}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{g,\omega} e^{-i\omega_{g}t} d\omega,$$

(30)

where

$$X_{g,n} = \frac{1}{T} \int_{-T/2}^{T/2} X_{g,\text{disc}}(t) e^{i\omega_{g0}t} dt, \quad X_{g,\omega} = \int_{-\infty}^{\infty} X_{g,\text{cont}}(t) e^{i\omega_{g}t} dt,$$  

(31)

Note $\omega_{g0} = \frac{2\pi}{T}$ is the fundamental frequency. The terms $X_{g,\text{disc}}$ or $X_{g,\text{cont}}(t)$ represent any of the quantities of the gravitational field, such as $\mathbf{V}_g$, $\mathbf{A}_g$, $\mathbf{g}$, and $\mathbf{B}_g$.  

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3. Particle Nature of GW in Gravitodynamics

In quantum theory, the concept of wave-particle duality applies to all fields and elementary particles. The discussion in Section 2 focuses on the wave nature of GW. Gravity is a physical field in Gravitodynamics, we now study the particle nature of GW.

3.1 Gravitodynamics is Compatible with Quantum Mechanics: Quantizing GW

For studying the particle nature of GW, following the same procedure demonstrating the particle nature of EMW, we begin with standing GW. The standing GW is confined in a rectangular volume \( V \) with sides \( A, B, C \). The gravitational vector potential can be expanded in Fourier series,

\[
A_g = \sum_{k_g} A_{gk_g} e^{ik_g \cdot r}.
\]  
(32)

The gravitational wave vector \( k_g \) has components,

\[
k_{gx} = \frac{2\pi n_x}{A}, \quad k_{gy} = \frac{2\pi n_y}{B}, \quad k_{gz} = \frac{2\pi n_z}{C},
\]

where \( n_x, n_y, \) and \( n_z \) are integers. From the gravitational Coulomb gauge, \( \nabla \cdot A_g = 0 \),

\[
k_g \cdot A_{gk_g} = 0,
\]  
(33)

where \( A_{gk_g} \) is time dependent, perpendicular to \( k_g \), and satisfies the equation,

\[
\ddot{A}_{gk_g} + \omega_{gk_g}^2 A_{gk_g} = 0, \quad \omega_{gk_g} = C k_g.
\]  
(34)

From Eq. (18), we have the field strengths in the terms of \( A_g \),

\[
g = -\frac{i}{\mathcal{C}} \sum_{k_g} \dot{A}_{gk_g} e^{ik_g \cdot r}, \quad B_g = \nabla \times A_g = i \sum_{k_g} (k_g \times A_{gk_g}) e^{ik_g \cdot r}.
\]  
(35)

Eq. (35) gives the total energy,

\[
E_{gT} = \frac{1}{8\pi} \int (g^2 + B_g^2) \, dV = \frac{V}{8\pi \mathcal{C}^2} \sum_{k_g} (\dot{A}_{gk_g} \cdot \dot{A}_{gk_g}^* + \omega_{gk_g}^2 A_{gk_g} \cdot A_{gk_g}^*).
\]  
(36)

For standing wave, Eq. (32) can be written as,

\[
A_g = \sum_{k_g} (a_{gk_g} e^{ik_g \cdot r} + a_{gk_g}^* e^{-ik_g \cdot r}), \quad a_{gk_g} \sim e^{-i\omega_{gk_g} t}.
\]  
(37)

Then we have

\[
A_{gk_g} = a_{gk_g} + a_{g-k_g}^*.
\]  
(38)

Substituting Eq. (38) into Eq. (36), we obtain the total energy,

\[
E_{gT} = \sum_{k_g} E_{g,k_g}, \quad E_{g,k_g} = \frac{k_g^2 V}{2\pi} a_{gk_g} \cdot a_{gk_g}^*.
\]  
(39)

Eq. (39) shows that the total energy of GW, is the summation of the energy, \( E_{g,k_g} \) of each plane GW, and that GW is expressed in the terms of a series of discrete parameters, \( a_{gk_g} \).

To obtain the Hamiltonian of GW, let’s introduce gravitational canonical variables,
\[
q_{g\kappa g} \equiv \sqrt{\frac{V}{4\pi c^2}} \frac{1}{\sqrt{2\omega_{g\kappa g}}} (a_{g\kappa g} + a_{g\kappa g}^*),
\]

(40)

\[
p_{g\kappa g} \equiv -i \sqrt{\frac{V}{4\pi c^2}} \frac{\omega_{g\kappa g}}{2} (a_{g\kappa g} - a_{g\kappa g}^*).
\]

(41)

From Eqs. (40 and 41), we obtain the Hamiltonian of GW,

\[
H_{GW} = \sum_{\kappa g} \frac{1}{2} (p_{g\kappa g}^2 + \omega_{g\kappa g}^2 q_{g\kappa g}^2).
\]

(42)

Both \(q_{g\kappa g}\) and \(p_{g\kappa g}\) are perpendicular to GW vector \(k_g\), and determine the polarization of GW. We denote the two polarization by \(N_{k_{g\delta}}^p\), \(i=1, 2\), \(N_{k_{g\delta}}^p \cdot N_{k_{g\delta}}^p = \delta_{ij}\). Then we have,

\[
a_{g\sigma k_g}^2 = \sum_{\sigma} q_{g\kappa g \sigma}, \quad p_{g\kappa g}^2 = \sum_{\sigma} p_{g\kappa g \sigma}^2.
\]

(43)

This Hamiltonian, Eq. (42), has the form of the “harmonic oscillator”. Now to quantize GW becomes to quantize “harmonic oscillator” of GW. According to quantum mechanics, we introduce the gravitational ladder operators, which satisfy commutation relation,

\[
[a_{g\sigma k_g}, a_{g\sigma k_g}^*] = 1.
\]

(44)

Then we have,

\[
H_{GW} = \sum_{\kappa g} \sum_{\alpha=1}^2 \hbar \omega_{g\kappa g} (N_{\sigma k_g} + \frac{1}{2}),
\]

(45)

\[
[a_{g\sigma k_g}, H] = \hbar \omega_{g\kappa g} a_{g\sigma k_g}, \quad [a_{g\sigma k_g}^*, H] = -\hbar \omega_{g\kappa g} a_{g\sigma k_g}^*,
\]

(46)

where

\[
N_{\sigma k_g} \equiv a_{g\sigma k_g} a_{g\sigma k_g}^*.
\]

(47)

As in the quantum theory, Eq. (45) implies that the energy levels are quantized, i.e., GW is quantized, and that the ground state energy is \(\omega_{g\kappa g}/2 > 0\). Zero energy of quanta of GW is not allowed. Eq. (46) shows that \(a_{g\kappa g}\) and \(a_{g\kappa g}^*\) form a set of creation and annihilation operators. The quantum of GW is spin 1 boson; we denote this boson as “Gravito-photon”, which does not carry the gravitational charge, i.e., no rest mass; therefore gravity is a long-range force. The same conclusion has been reached [12].

We also obtain the total momentum of GW,

\[
P_{GW} = \sum_{\kappa g} \sum_{\alpha=1}^2 \kappa_g N_{\sigma k_g}.
\]

(48)

In this section, we have shown that GW does have particle nature and that Gravitodynamics is compatible with quantum mechanics. It will be interesting to detect the particle nature of GW empirically. In Gravitodynamics the energy is a component of the energy-momentum tensor of GW and is localizable. In GR, in contrast, the energy-momentum of GW is not a tensor and is not localizable (Table 3).
Table 3 Comparison of GW’s Energy in Gravitodynamics and in GR

<table>
<thead>
<tr>
<th></th>
<th>Gravitodynamics</th>
<th>Linearized Einstein Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy of GW</td>
<td>( W_G = \frac{1}{8\pi} \int (g^2 + B_g^2) d\tau )</td>
<td>( t^{\mu\nu} ) (pseudo—tensor)</td>
</tr>
<tr>
<td></td>
<td>( H_{GW} = \sum_{k_g} \sum_{\sigma=1}^{2} \hbar \omega_{g\sigma} N_{\sigma,k_g} \left( \frac{1}{2} + \frac{1}{2} \right) )</td>
<td>Not quantizable</td>
</tr>
</tbody>
</table>

The difficulties in quantization and localization of energy of GW are thus resolved in Gravitodynamics, but not in GR.

### 3.2. Gravito-photon

We have established the wave-particle duality of GW and suggested that the energy and momentum of Gravito-photon can be expressed as, respectively,

\[
E_{g-photon} \equiv h_g v_g, \quad P_{g-photon} \equiv \frac{h_g}{\lambda_g},
\]

where we introduce \( h_g \) as gravitational Planck constant that may or may not be equal to Planck constant, \( v_g \) and \( \lambda_g \) are frequency and wavelength of gravito-photon.

### 3.3. Generalized Dirac Sea

In order to explain the negative energy states, Dirac introduced the concept of Dirac Sea. An interpretation is that the positive operators add a positive energy particle and the negative operators annihilate a positive energy particle. Now we generalize this concept to include the gravitational charges, we denote it as the Generalized Dirac Sea, which states that the positive operator adds a positive energy particle with a positive gravitational charge and the negative operator annihilates a positive energy particle with a positive gravitational charge, i.e., Generalized Dirac Sea exchanges positive gravitational charges with particles during the processes of creation and annihilation.

An example: let’s consider electron-positron annihilation, \( e^- + e^+ \rightarrow \gamma + \gamma \). We divide this process into 3 separate processes relating with mass/energy, electric charge, and gravitational charge, correspondingly,

\[
\begin{align*}
m^- + m^+ & \rightarrow \gamma + \gamma \\
Q_e^- + Q_e^+ & \rightarrow \text{Generalized Dirac Sea} \\
Q_g^- + Q_g^+ & \rightarrow \text{Generalized Dirac Sea}
\end{align*}
\]

\( \text{(50)} \)
where \( m^-, Q_e^-, Q_g^- \) and \( m^+, Q_e^+, Q_g^+ \) are the rest mass, electric charge, and gravitational charge of electron and positron, respectively. In this process, (1) the electron and positron’s rest masses convert to radiation energy of gamma ray; (2) their negative and positive electric charges transport to and store in Generalized Dirac Sea; (3) their gravitational charges transport to and store in Generalized Dirac Sea, which implies that the gravitational charge and inertial rest mass are two different entities.

Another example: the process of the creation of electron-positron pair, which is expressed as \( \gamma \rightarrow e^- + e^+ \). We divide this process into 3 separate processes,

\[
\begin{align*}
\gamma &\rightarrow m^- + m^+ \\
\text{Generalized Dirac Sea} &\rightarrow Q_e^- + Q_e^+ \\
\text{Generalized Dirac Sea} &\rightarrow Q_g^- + Q_g^+ 
\end{align*}
\]

The gamma ray’s energy converts to electron and positron’s rest masses, respectively. The created electron and positron gain electric and gravitational charges from Generalized Dirac Sea, respectively.

### 3.4. Conversion of Mass to Gravito-photons/GW

The detection of GW [3] discloses an important phenomenon, conversion of mass to GW/gravito-photons. Let’s compare the conversion of mass to GW/gravito-photons with the conversion of mass to EMW/photons (Table 4).

<table>
<thead>
<tr>
<th>Table 4 Comparisons of Conversions between Masses and Quanta</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annihilation</strong></td>
</tr>
<tr>
<td>Mass ( \rightarrow ) EMW/Photons</td>
</tr>
<tr>
<td>(Process, e.g., ( e^- + e^+ \rightarrow \gamma + \gamma ))</td>
</tr>
<tr>
<td>( Q_e^- + Q_e^+ \rightarrow \text{Generalized Dirac Sea} )</td>
</tr>
<tr>
<td>( Q_g^- + Q_g^+ \rightarrow \text{Generalized Dirac Sea} )</td>
</tr>
<tr>
<td><strong>Creation</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( \text{Generalized Dirac Sea} \rightarrow Q_e^- + Q_e^+ )</td>
</tr>
<tr>
<td>( \text{Generalized Dirac Sea} \rightarrow Q_g^- + Q_g^+ )</td>
</tr>
<tr>
<td>[ \text{Observed} ]</td>
</tr>
</tbody>
</table>

We raise the following questions: (1) what is the mechanism of conversion of mass to GW/Gravito-photons? (2) Does GW/Gravito-photon convert to mass? (3) Beside the
frequency/energy, what is the difference between photon and gravito-photon? Can we distinguish them experimentally? Note in the process of annihilation, we assume that both positive and negative electric charges are transported into Generalized Dirac Sea, instead of canceling each other. On the other hand in the process of creation, we assume that electron and positron gain negative and positive electric charges from Generalized Dirac Sea, respectively.

3.5. Gravit-photon Mean Free Path and Gravitational Beer-Lambert Law

Let’s introduce new concepts: (1) gravito-superconductor, which is a system of positive gravitational charges and all charges move freely under the influence of gravitational force (turn off other interactions, e.g., electromagnetic force); (2) mean free path; (3) GW opacity; (4) gravito-optical thickness.

During their propagation in a uniform gravito-superconductor, Gravito-photons collide with gravitational charges. Consider a slab with face area XY and thickness Z, containing $n_{gc}$ gravitational charges in unit volume. Probability, $P_{g-\text{photon}}$, per unit length, that a single Gravito-photon interacts with gravitational charges in the slab, is

$$P_{g-\text{photon}} = n_{gc} \sigma_g dz,$$

where $\sigma_g$ is the scattering cross section. For an incident beam density $I_{g,\text{inc}}$ of Gravito-photons travelling through the slab, the remaining density $I_{g,\text{rem}}$ is

$$I_{g,\text{rem}} = I_{g,\text{inc}} - P_{g-\text{photon}}I_{g,\text{inc}},$$

Then we obtain the Gravito-photon transport equation,

$$\frac{dI_{g,\text{rem}}(z)}{dz} = -n_{gc} \sigma_g I_{g,\text{rem}}(z).$$

The solution of Eq. (54) is

$$I_{g,\text{rem}}(z) = I_{g,\text{inc}} e^{-n_{gc} \sigma_g z}.$$

The scattered intensity $I_{g,\text{scat}}$ is given by,

$$I_{g,\text{scat}} = I_{g,\text{inc}} - I_{g,\text{rem}}(z) = I_{g,\text{inc}}(1 - e^{-n_{gc} \sigma_g z}).$$

Since the frequency of GW is low, we ignore the absorption of GW here.

Let’s define the Gravito-photon mean free path,

$$\ell \equiv \frac{1}{\sigma_g n_{gc}}.$$

We obtain the density $I_g$ of the outgoing beam of gravito-photons,

$$I_{g,\text{rem}}(z) = I_{g,\text{inc}} e^{-z/\ell},$$

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where $z$ is the distance traveled by the beam of gravito-photons through the gravito-superconductor. We denote Eq. (58) as the gravitational Beer-Lambert law.

### 3.6. Gravito-photon Opacity

Let’s consider a gravito-superconductor having mass density $\rho_g$ and number density $n_g = \rho_g/\mu$, where $\mu$ is mean proton/neutron weight. For scattering process, the

*Gravito-photon opacity* is defined as

$$\tau \equiv \frac{\sigma_g}{\mu}. \quad (59)$$

Then Eq. (58) can be written as

$$I_{g,\text{rem}}(z) = I_{g,\text{inc}}e^{-\tau \rho_g z}. \quad (60)$$

For general cases, the Gravito-photon opacity is wavelength dependent. If we introduce the *Gravito-photon-optical thickness* $t_g$,

$$t_g \equiv \tau \rho_g z, \quad (61)$$

Eq. (60) gives

$$I_{g,\text{rem}}(z) = I_{g,\text{inc}}e^{-t_g}. \quad (62)$$

The *gravito-optical depth*, $d_{\text{god}}$, is defined as $t_g = 1$. Then we have,

$$d_{\text{god}} = \frac{1}{\tau \rho_g} = \frac{1}{\sigma_g n_g} = \ell, \quad (63)$$

i.e., the *gravito-optical depth* is equivalent to the *Gravito-photon mean free path*. For non-uniform gravito-superconductor, the gravito-optical thickness $t_g$ is

$$t_g \equiv \int \tau \rho_g(z) dz. \quad (64)$$

### 4. Gravitational Potential and Field of Moving Gravitational Charge(s)

Before we study the radiation of GW, let’s consider the gravitational potentials and field strengths of arbitrarily moving gravitational charges.

#### 4.1 Gravitational Retarded Potentials

Let’s consider the gravitational retarded potentials of arbitrary moving gravitational charges, and introduce the *gravitational retarded potential* solutions of Eq. (16),

$$V_g = -\int \frac{\rho}{R} \frac{t-(\frac{R}{c})}{R} d^3x, \quad A_g = -\frac{1}{c} \int \frac{t-(\frac{R}{c})}{R} d^3x, \quad (65)$$

where $d^3x = dx dy dz$, $R$ is the distance from the volume element to the “field point”. 

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For a plane GW, the field strengths, \( \mathbf{g} \) and \( \mathbf{B} \), the gravitational Poynting vector, \( \mathbf{S}_g \), and the density of momentum and angular momentum of the gravitational field, \( \mathbf{p}_g \) and \( \mathbf{l}_g \), are related and completely determined by the vector potential \( \mathbf{A}_g \), where

\[
\mathbf{g} = \frac{1}{c} (\dot{\mathbf{A}}_g \times \mathbf{n}) \times \mathbf{n}, \quad \mathbf{B}_g = \frac{1}{c} (\dot{\mathbf{A}}_g \times \mathbf{n}),
\]

\[
\mathbf{S}_g = \frac{c}{4\pi} \mathbf{B}_g^2 \mathbf{n} = C\mathbf{W}\mathbf{n},
\]

\[
\mathbf{p}_g = \frac{1}{c} \mathbf{S}_g = \frac{1}{4\pi c} \mathbf{B}_g^2 \mathbf{n}, \quad \mathbf{l}_g = \mathbf{r} \times \mathbf{p}_g = \frac{1}{c} \mathbf{W}\mathbf{r} \times \mathbf{n}.
\]

At wave zone, \( R = R_0 \), we obtain the intensity \( dl \) of the gravitational radiation into a solid angle, \( d\Omega \), where

\[
dl = \frac{c}{4\pi} B_g^2 R_0^2 d\Omega.
\]

### 4.2 Gravitational Lienard-Wiechert Potential

The retarded potentials of a point gravitational charge moving arbitrarily are,

\[
V_g = -\frac{Q_g}{R(1-\beta_n)}, \quad A_g = -\frac{Q_g\beta}{R(1-\beta_n)} = \beta V_g.
\]

We denote Eq. (70) the gravitational Lienard-Wiechert potentials. The field strengths are

\[
g = -\frac{Q_g(1-\beta^2)}{(R-\beta R)^3} (R - \beta R) - \frac{Q_g}{C(R-\beta R)^3} R \times [(R - \beta R) \times \dot{\beta}],
\]

\[
\mathbf{B}_g = \frac{1}{R} \mathbf{R} \times \mathbf{g}.
\]

The second part of Eq. (71) causes the radiation of GW. Note \( \mathbf{B}_g \perp \mathbf{g} \) everywhere.

### 4.3 Spectral Resolution of Gravitational Retarded Potential

The gravitational retarded potential generated by moving gravitational charges can be expressed by its monochromatic components, where

\[
V_{g\omega} = -\int \frac{g_{\omega R}}{R} e^{i k g R} \, d^3 x, \quad A_{g\omega} = -\int \frac{J_{\omega R}}{C R} e^{i k g R} \, d^3 x.
\]

In the case of a single point gravitational charge \( Q_g \), Eq. (73) gives

\[
V_{g\omega} = -Q_g \int_{-\infty}^{\infty} e^{i \omega t + \frac{R(t)}{C}} \, dt, \quad A_{g\omega} = -Q_g \int_{-\infty}^{\infty} \frac{\beta(t)}{R(t)} e^{i \omega t + \frac{R(t)}{C}} \, dt,
\]

where \( V(t) \) is the velocity of the point gravitational charge.

For discrete series of frequencies, \( \omega = n\omega_0 \), \( \omega_0 = \frac{2\pi}{T} \), the gravitational retarded potential generated by a moving gravitational charge can be expressed as
\[ V_{gn} = -\frac{Q_g}{r} \int_0^T e^{i\omega \cdot (t + \frac{R(t)}{c})} dt, \quad A_{gn} = -\frac{Q_g}{cT} \int_0^T \frac{V(t)}{R(t)} e^{i\omega \cdot (t + \frac{R(t)}{c})} dt. \] (75)

### 4.4. Gravitational Darwin Lagrangian

After obtaining the gravitational retarded potential, we can rewrite the equation of motion for a gravitational charge in external fields. At the “zero approximation”, the Newtonian Lagrangian is,

\[ L^{(0)}_{gi} = \frac{1}{2} m_i V_i^2 - Q_{gi} V_g, \] (76)

where \( V_i \) is the velocity of the \( i \)th gravitational charge of a system of gravitational charges; \( V_g \) is the potential of the external field. Since the existence of gravito-magnetic field \( A_g \) in Gravitodynamics, the relativistic Lagrangian is

\[ L_{gi} = -m_i c^2 \sqrt{1 - \beta_i^2} - Q_{gi} V_g + \frac{1}{c} Q_{gi} A_g \cdot V_i. \] (77)

Expanding the term \( m_i c^2 \sqrt{1 - \beta_i^2} \) in power of \( V_i/C \) and keeping the terms of second order, and substituting the retarded potentials, we obtain the gravitational Darwin Lagrangian for the whole system, from Eq. (77),

\[ L_g = \sum_i \left( \frac{1}{2} m_i V_i^2 + \frac{1}{8c^2} m_i V_i^4 \right) - \sum_{i>j} \left( \frac{Q_{gi} Q_{gj}}{R_{ij}} - \frac{Q_{gi} Q_{gj}}{2c^2 R_{ij}} [V_i \cdot V_j + (V_i \cdot n_{ij})(V_j \cdot n_{ij})] \right), \] (78)

where \( A_g = \frac{Q_{gi}(V + (V \cdot n)n)}{2cR} \) has been used.

### 5. Radiation of Gravitational Wave

In this section, we study GW generated by a single moving gravitational charge, a gravitational dipole, and a gravitational multi-pole, correspondingly. At a long distance from the source, GW can be considered as a plane wave.

#### 5.1. A Non-Relativistic Point Source: Gravitational Larmor Radiation

Let’s consider the radiation of a single moving gravitational charge \( Q_g \). It is convenient to use the retarded potential. At a long distance, \( R = R_0 \), Eq. (70) gives

\[ A_g = -\frac{Q_g \beta}{R_0 (1 - \beta \cdot n)}, \] (79)

When \( V_i \ll C \) Eq. (79) gives,
\[ A_g = -\frac{Q_0 \beta}{R_0}. \]  

(80)

Substituting Eq. (80) into Eqs. (66 and 69), the total radiation of a non-relativistic single gravitational charge is proportional to the square of the acceleration of the charge,

\[ I = \frac{2Gm^2}{3C^3} a^2. \]  

(81)

Where the non-radiation term, \( \sim 1/R^2 \), have been ignored. We denote Eq. (81) as the gravitational Larmor formula.

**5.2. A Relativistic Point Source: Gravitational Lienard Radiation**

Now let’s consider a relativistic gravitational charge moving with velocity, \( \mathbf{V} \sim C \), and acceleration, \( \mathbf{a} \), relative to an observer. In a reference system in which the charge is at rest at a given moment, we have the intensity of radiation given by the gravitational Larmor formula, Eq. (81). Next, according to SR, transferring to the observer’s system, we obtain the energy of radiation,

\[ W = \frac{2Gm^2}{3C^3} \int_{-\infty}^{\infty} \frac{1}{(1-\beta^2)^3} [a^2 - (\mathbf{\beta} \times \mathbf{a})^2] dt. \]  

(82)

We denote Eq. (82) as the gravitational Lienard formula. In terms of the gravitational field strength, we obtain,

\[ W = \frac{2G^2m^2}{3C^3} \int_{-\infty}^{\infty} \frac{1}{(1-\beta^2) \beta^2} \left[ (\mathbf{g} + \mathbf{\beta} \times \mathbf{B}_g)^2 - (\mathbf{\beta} \cdot \mathbf{g})^2 \right] dt. \]  

(83)

For the angular distribution of the radiation energy at a long distance \( R \), by keeping only the lowest term in \( 1/R \) we obtain the intensity radiated into the solid angle \( d\Omega \),

\[ dI = \frac{Gm^2}{4\pi C^3} \left\{ \frac{a^2}{(1-\beta n)^4} + \frac{2(n \cdot a)(\mathbf{\beta} \cdot a)}{(1-\beta n)^5} - \frac{(1-\beta^2)(n \cdot a)^2}{(1-\beta n)^6} \right\} d\Omega. \]  

(84)

For the situation where \( \mathbf{a} / \mathbf{V} \), Eq. (84) gives the intensity,

\[ dI = \frac{Gm^2}{4\pi C^3} \left\{ \frac{a^2 \sin^2 \theta}{(1-\beta \cos \theta)^5} \right\} d\Omega, \]  

(85)

where \( \theta \) is the angle between \( \mathbf{n} \) and \( \mathbf{V} \), \( \mathbf{n} = \mathbf{R} / R \). The radiation energy of a fast moving gravitational charge vanishes along the direction \( \theta \to 0 \) or \( \pi \). It goes to maximum when \( \theta \to \sqrt{1-\beta^2} \).

For the situation \( \mathbf{a} \perp \mathbf{V} \) Eq. (84) gives the intensity,
\begin{equation}
  dl = \frac{Gm^2a^2}{4\pi C^3} \left\{ \frac{1}{(1-\beta \cos \theta)^3} - \frac{(1-\beta^2)\sin^2 \theta \cos^2 \phi}{(1-\beta \cos \theta)^5} \right\} d\Omega,
\end{equation}

where $\phi$ is the azimuthal angle between $\mathbf{n}$ and $\mathbf{a}$-$\mathbf{V}$ plane.

5.3. A Relativistic Point Source: Gravitational Bremsstrahlung

When the velocity and acceleration of a relativistic gravitational charge are collinear, i.e., $\mathbf{V} \times \mathbf{a} = 0$, from the gravitational Lienard formula, Eq. (82), we obtain the gravitational bremsstrahlung,

\begin{equation}
  I = \frac{2Gm^2a^2}{3C^3(1-\beta^2)^3},
\end{equation}

The radiation of gravitational bremsstrahlung can also be treated as the scattering of virtual quanta.

5.4. A Relativistic Point Source: Gravitational Synchrotron Radiation

Let’s consider GW radiation of a star orbiting around a rotating massive neutron star or a rotating black hole that generates a gravito-magnetic field $\mathbf{B}_g$ around itself. The radius $r$ of the orbit, the cyclic frequency $\omega_s$ of the motion, the gravito-magnetic field $\mathbf{B}_g$, and the velocity $\mathbf{V}$ of the star are related by,

\begin{equation}
  r = \frac{C V}{\sqrt{G B_g V (1-\beta^2)}}, \quad \omega_{gs} = \frac{\sqrt{G B_g}}{C \sqrt{1-\beta^2}} \tag{88}
\end{equation}

By rewriting Eq. (83) in terms of the field strengths while ignoring the time integral for simplicity as well as setting $\mathbf{g} = 0$ and $\mathbf{B}_g \perp \mathbf{V}$, we have the total intensity of radiation,

\begin{equation}
  I = \frac{2G^2B_g^2p^2}{3C^5}, \tag{89}
\end{equation}

where $p = mV/\sqrt{1-\beta^2}$ is the momentum of the star. The total intensity of radiation is proportional to the square of momentum $p$. This radiation is the gravitational synchrotron radiation or gravito-magnetic bremsstrahlung.

The synchrotron GW is elliptically polarized. In the ultra relativistic situation, there are two different results. The elliptical polarization degenerates to linear polarization when $\theta = 0$ whereas it becomes circular when $\theta$ is large.

5.5. Non-Relativistic Dipole Radiation: Conventional/Wave Approach

Now, let’s apply the conventional method to treat the radiation of GW by a system of
point gravitational charges. For the situation where each point charge moves with low velocities, \( \mathbf{V} \ll C \), Eq. (70) gives the vector potential at the wave zone,

\[
\mathbf{A}_g = -\frac{1}{C R_0} \mathbf{d}_{g,T}, \quad \mathbf{d}_{g,T} = \sum_i Q_{g,i} \mathbf{r}_i, \tag{90}
\]

where \( \mathbf{d}_T \) is the *conventional gravitational dipole moment*. Then Eq. (66) gives,

\[
\mathbf{B}_g = -\frac{1}{C^2 R_0} \mathbf{d}_{g,T} \times \mathbf{n}, \quad \mathbf{g} = -\frac{1}{C^2 R_0} (\mathbf{d}_{g,T} \times \mathbf{n}) \times \mathbf{n}. \tag{91}
\]

From Eq. (69) the intensity of the dipole radiation passing through the element of the spherical surface with its center at the origin and with a radius \( R_0 \) is

\[
dI_T = \frac{1}{4\pi C^2} (\mathbf{d}_{g,T} \times \mathbf{n})^2 \, d\Omega, \quad I_T = \frac{2}{3C^3} \mathbf{d}_{g,T}^2. \tag{92}
\]

For a system of gravitational charges, \( \mathbf{d}_T = 0 \). Therefore, from wave approach, we have \( dI_T = 0 \), i.e., there is not gravitational dipole radiation.

### 5.6. Non-Relativistic Dipole Radiation: Alternative/Particle Approach

Since wave-particle duality, a point source emits a beam of gravito-photons,

\[
H_{GW} = \sum_k \sum_{\sigma=1}^2 \hbar \omega_{g,k} \left(N_{\sigma,k} + \frac{1}{2}\right). \tag{45}
\]

Therefore every point source emits a beam of gravito-photons independently. Based on this particle nature of GW, the total beam intensity of a system of point sources is the summation of the beam intensities of gravito-photons emitted by each of point sources,

\[
I_{g,\text{beam},T} = \sum_k I_{g,\text{beam},k}. \tag{93}
\]

Moreover, based on wave-particle duality of GW, the beam intensity \( I_{g,\text{beam},k} \) of gravito-photons and the radiated energy intensity \( I_{g,\text{wave},k} \) of GW of the same point source are equal,

\[
I_{g,\text{beam},k} = I_{g,\text{wave},k}. \tag{94}
\]

Thus to find the total beam density turns out to be to find the total radiated energy intensity of each of point sources,

\[
I_{g,\text{beam},T} = \sum_k I_{g,\text{beam},k} = \sum_k I_{g,\text{wave},k}, \tag{95}
\]

\[
I_{g,\text{wave},k} = \frac{2}{3C^3} \mathbf{d}_{g,k}^2.
\]

Note the \( \mathbf{d}_{g,k} \) of a point source is not zero,

\[
\mathbf{d}_{g,k} \equiv Q_{g,i} \mathbf{r}_k \neq 0, \quad I_{g,\text{wave},k} \neq 0.
\]

Therefore
The total beam intensity is equal to the summation of the radiated energy intensity of each of point sources, which is not equal to zero. Based on particle nature of GW, the gravitational dipole indeed radiates Gravito-photons.

**5.7. Non-Relativistic Gravitational Quadrupole Radiation**

Now let’s expand the vector potential of Eq. (65) for a system of point gravitational charges in powers of $\mathbf{r} \cdot \mathbf{n}/C$,

$$
A_g = -\frac{\sum \mathbf{Q}_g}{CR_0} - \frac{1}{C^2 R_0} \frac{\partial}{\partial t} \sum \mathbf{Q}_g \mathbf{V} (\mathbf{r} \cdot \mathbf{n}),
$$

(97)

where we have applied $\mathbf{J}_g = \rho_g \mathbf{V}$. Eq. (97) can be put in multipole formula,

$$
A_g = -\frac{\mathbf{d}_{g,dT}}{CR_0} - \frac{1}{2C^2 R_0} \mathbf{B} - \frac{1}{CR_0} \mathbf{d}_{g,m} \times \mathbf{n}.
$$

(98)

We call vector $\mathbf{d}_{g,dT}$, $\mathbf{D}$ and $\mathbf{d}_{g,m}$ the gravitational dipole, the *gravitational quadrupole*, and *gravito-magnetic dipole moments*, respectively. The component of $\mathbf{D}$ is $D_{g\alpha} = D_{g\alpha \beta} n_{\beta}$. The gravitational charge quadrupole $D_{g\alpha \beta}$ is related to the mechanical quadrupole $D_{\text{me} \alpha \beta}$,

$$
D_{g\alpha \beta} \equiv \sqrt{G} D_{\text{me} \alpha \beta},
$$

(99)

$$
D_{\text{me} \alpha \beta} = \sum m \left( x_{\alpha} x_{\beta} - \frac{1}{3} \delta_{\alpha \beta} r^2 \right).
$$

The gravito-magnetic dipole moment is defined as,

$$
\mathbf{d}_{g,m} \equiv \frac{1}{2C} \sum \mathbf{Q}_g \mathbf{r} \times \mathbf{V}.
$$

(100)

Substituting Eq. (98) into Eq. (66), we obtain the field strengths,

$$
\mathbf{B}_g = -\frac{1}{C^2 R_0} \left( \mathbf{d}_{g,dT} + \frac{1}{2C} \mathbf{B} + \mathbf{d}_{g,m} \times \mathbf{n} \right) \times \mathbf{n},
$$

(101)

$$
\mathbf{g} = -\frac{1}{C^2 R_0} \left( \mathbf{d}_{g,dT} \times \mathbf{n} + \frac{1}{2C} \mathbf{B} \times \mathbf{n} - \mathbf{d}_{g,m} \right) \times \mathbf{n}.
$$

(102)

For small velocity, substituting Eq. (101) into Eq. (69), we obtain the total radiation

$$
I = \frac{2}{3C^3} \mathbf{d}_{g,dT}^2 + \frac{G}{20C^3} D_{\text{me} \alpha \beta}^2 + \frac{2}{3C^3} \mathbf{d}_{g,m}^2.
$$

(103)

Note: Eq. (103) implies that, for a system of gravitational charges, the dipole, the quadrupole, and gravito-magnetic dipole radiations co-exist. Under certain situations, one of these three radiations is dominant.

The dipole radiation part, $\frac{2}{3C^3} \mathbf{d}_{g,dT}^2 = \frac{2}{3C^3} \sum \mathbf{d}_{g,dT}^2$, is the summation of the
gravitational Larmor radiation, Eq. (81), for each of point charges of the source.

The quadrupole radiation,

$$I_{\text{quadrupole}} = \frac{G}{20C^5} \mathbf{J}_m^2 \mathbf{n}_a \mathbf{n}_\beta,$$

is the same as that of GR [7, 20],

$$I = \frac{G}{5C^5} \mathbf{J}_m^2 \mathbf{n}_a \mathbf{n}_\beta,$$

except a factor of 4. Therefore Gravitodynamics predicts the GW phenomena which are the same as that predicted by linearized GR, except a factor 4 that will distinguish both theories. Note the factor 4 can be absorbed into the mass of a source and/or the distance from the source to an observer. Let’s compare the quadrupole radiation of GW calculated by both linear GR and Gravitodynamics (Table 5).

### Table 5

<table>
<thead>
<tr>
<th></th>
<th>Gravitodynamics</th>
<th>Linearized Einstein Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wave Equation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{C^2} \frac{\partial^2 V_g}{\partial t^2} - \nabla^2 V_g = -4\pi \rho_g$</td>
<td>$\frac{1}{C^2} \frac{\partial^2 h^{\mu\nu}}{\partial t^2} - \nabla^2 h^{\mu\nu}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{C^2} \frac{\partial^2 A_g}{\partial t^2} - \nabla^2 A_g = -4\pi J_g$</td>
<td>$= -16\pi T^{\mu\nu}$</td>
</tr>
<tr>
<td>2</td>
<td>Quadrupole Solution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathbf{A}_g \sim -\frac{1}{R} \mathbf{B}$</td>
<td>$\mathbf{h}^{T_{ij}} \sim \frac{1}{R} \mathbf{B}$</td>
</tr>
<tr>
<td>3</td>
<td>Field strength</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathbf{B}_g \sim -\frac{\partial \mathbf{A}_g}{\partial t} \sim -\frac{1}{R} \mathbf{B}$</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>Intensity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathbf{S}_g \sim B_g^2 \sim \left(\frac{\partial A_g}{\partial t}\right)^2 \sim B^2$</td>
<td>$T^{GW}<em>{00} \sim \left(\frac{\partial h^{T</em>{ij}}}{\partial t}\right)^2 \sim B^2$</td>
</tr>
<tr>
<td></td>
<td>$I = \int \mathbf{S}_g \cdot d\mathbf{a} = \frac{G}{20C^5} \mathbf{J}_m^2 \mathbf{n}<em>a \mathbf{n}</em>\beta$</td>
<td>$I = \frac{G}{5C^5} \mathbf{J}_m^2 \mathbf{n}<em>a \mathbf{n}</em>\beta$</td>
</tr>
</tbody>
</table>

Line 1 shows that GW equation in both theories have the same form, but the difference is that GW is spin 1 and spin 2 wave in Gravitodynamics and GR, respectively.

Line 2 shows that GW potential in both theories has the same quadrupole moment.

Line 3 shows that there are field strengths in Gravitodynamics, but not in GR.

Line 4 shows that the intensity of GW in Gravitodynamics is defined as the square of field strength, $\mathbf{B}_g$, and, for plane wave, as the square of time derivative of potentials, $\frac{\partial A_g}{\partial t}$.
in GR, however, the intensity of GW is directly expressed as the square of time derivative of “potentials”, $\frac{\partial R^{\text{Rij}}}{\partial t}$, without introducing the concept of the field strength.

Therefore it is not surprise that the intensity in both theories has the same expression.

5.8. GW Radiation Damping: Non-Relativistic Source

Following Electrodynamics [5, 19], expanding the vector potential to the second order in $1/C$ and choosing a proper gauge such that the third expansion of the scalar potential vanishes, we obtain the reaction force of the radiation acting on a single gravitational charge,

$$ F = - \frac{2Gm^2}{3C^3} \dot{a}. \quad (105) $$

We denote it as the gravitational Abraham-Lorentz force. Averaging over time, the work done by this force is,

$$ F \cdot V = \frac{2Gm^2}{3C^3} \dot{a}^2. \quad (106) $$

This work is equal to the intensity of radiation of the gravitational charge, Eq. (81), therefore this force is also called the gravitational radiation damping.

We can expressed the radiation reaction force in terms of field strengths,

$$ F = - \frac{2\sqrt{6}m^2}{3C^3} \dot{g} - \frac{2G^2m^2}{3C^4} g \times B_g. \quad (107) $$

For a system of gravitational charges, the work done by the reaction force of the dipole radiation acting on the system is

$$ W_{\text{dipole}} = \sum_k F_k \cdot V_k = \frac{2}{3C^3} \sum_k G_m^2 \dot{a}_k^2. \quad (108) $$

The work done by the reaction force of the quadrupole radiation acting on the system is

$$ W_{\text{quadrupole}} = F \cdot V = \frac{G}{20C^5} \bar{V}^2_{\text{ma} \beta}. \quad (109) $$

The total energy loss due to the dipole and quadrupole radiation is the summation,

$$ W_T = \frac{2}{3C^3} \sum_k G_m^2 \dot{a}_k^2 + \frac{G}{20C^5} \bar{D}^2_{\text{ma} \beta}. \quad (110) $$

Due to the energy loss of the radiation, its angular momentum, $\tilde{M}$, is decreased as

$$ \tilde{M} = - \frac{2G}{3C^3} \sum_k m_k^2 \dot{V}_k \times \dot{a}_k. \quad (111) $$

5.9. GW Radiation Damping: Relativistic Source

The equation of motion of a relativistic gravitational charge becomes, from Eq. (6),
\[ \text{mc} \frac{du^\mu}{ds} = \frac{Q_\gamma}{c} F^{\mu \nu} u_\nu + F_{\text{damp}}^\mu, \]  

where \( F_{\text{damp}}^\mu \) is the 4-dimension form of the GW radiation damping force. Its 3-dimension form is,

\[ F_{\text{damp}} = \frac{2G^2 m^2}{3c^5} \left( F_{g \alpha \beta} u^\beta \right) \left( F_{g \alpha \gamma} u_\gamma \right) n. \]  

Let the direction of the velocity of the charge as the z axis, Eq. (13) gives,

\[ F_{z,\text{damp}} = -\frac{2G^2 m^2}{3c^4(1-\beta^2)} \left[ (g_x - B_y)^2 + (g_y + B_x)^2 \right]. \]  

6. GW Interacting with Gravitational Charges: Absorption

At the classical level, GW of arbitrary frequency will interact with gravitational charges, i.e., the energy of GW transfers to the kinematic energy of gravitational charges. It is conceptually clear and mathematically simple to employ the concept of field strengths in study.

6.1. Absorption of Energy of GW

We apply Gravitodynamics to describe GW absorption by gravitational charges. Let’s consider a gravitational charge being influenced by an oscillation force with frequency \( \omega_0 \) and driven by an incoming GW, the equation of motion of the charge is

\[ \ddot{r} + \omega_0^2 r = \frac{Q_\gamma}{m} g_0 e^{-i\omega t} = \sqrt{G} g. \]  

Then we have the solution,

\[ r = \frac{\sqrt{G}}{\omega_0^2 - \omega^2} g, \quad \dot{r} = -\frac{i\omega \sqrt{G}}{\omega_0^2 - \omega^2} g. \]  

Now let’s study the penetration of GW into a gravito-superconductor. From Eq. (16), we obtain a formula for the gravitational current; call it the gravitational Ohm’s law,

\[ J_g = \rho_g g = \rho_g \dot{r} = -i \frac{\rho_g \omega \sqrt{G}}{\omega_0^2 - \omega^2} g. \]  

Substituting Eq. (17) into Eq. (13), we have

\[ \nabla \times B_g = i \frac{4\pi \rho_g \omega \sqrt{G}}{c(\omega_0^2 - \omega^2)} g + \frac{1}{c} \frac{\partial g}{\partial t}. \]  

Taking the operation “\( \nabla \times \)” and applying Eq. (12), we have
\[ \nabla^2 \mathbf{B}_g = \frac{1}{c^2} \frac{\partial^2 \mathbf{B}_g}{\partial t^2} - i \frac{4\pi}{c^2} \frac{\rho_g \omega \sqrt{G}}{\left(\omega_0^2 - \omega^2\right)} \frac{\partial \mathbf{B}_g}{\partial t} = 0. \]  

(119)

Similarly, we have,

\[ \nabla^2 \mathbf{g} = \frac{1}{c^2} \frac{\partial^2 \mathbf{g}}{\partial t^2} - i \frac{4\pi}{c^2} \frac{\rho_g \omega \sqrt{G}}{\left(\omega_0^2 - \omega^2\right)} \frac{\partial \mathbf{g}}{\partial t} = 0. \]  

(120)

The solutions are

\[ \mathbf{B}_g = \mathbf{B}_0 e^{i(k_{GW}z - \omega t)}, \quad \mathbf{g} = \mathbf{g}_0 e^{i(k_{GW}z - \omega t)}. \]  

(121)

Where the GW number \( k_{GW} \), phase velocity \( C_p \) of GW, GW wavelength, and \textit{gravitational refractive index} \( n_g \) of the gravito-superconductor are related as

\[ k_{GW} = \frac{\omega}{C_p}, \quad \lambda_p = \frac{2\pi}{k_{GW}}, \]  

(122)

\[ C_p \equiv \frac{C}{\sqrt{1 - \frac{4\pi \sqrt{G} \rho_g}{\omega_0^2 - \omega^2}}}, \quad n_g = \frac{C}{C_p} = \sqrt{1 - \frac{4\pi \sqrt{G} \rho_g}{\omega_0^2 - \omega^2}}. \]  

(123)

There are several situations as the following:

(A) First case: \( \omega_0^2 = 0 \). We have

\[ k_{GW} = \frac{\omega}{C_p}, \quad C_p = \frac{C}{\sqrt{1 + \frac{4\pi \sqrt{G} \rho_g}{\omega^2}}}, \quad n_g = \sqrt{1 + \frac{4\pi \sqrt{G} \rho_g}{\omega^2}}. \]  

(124)

(B) Second case: \( \omega_0^2 \neq 0 \) and \( \omega_0^2 < \omega^2 \). We have

\[ k_{GW} = \frac{\omega}{C_p}, \quad C_p \equiv \frac{C}{\sqrt{1 + \frac{4\pi \sqrt{G} \rho_g}{\omega^2 - \omega_0^2}}}, \quad n_g = \sqrt{1 + \frac{4\pi \sqrt{G} \rho_g}{\omega^2 - \omega_0^2}}. \]  

(125)

For case (A) and (B), the phase velocity \( C_p \) of GW propagated in a gravito-superconductor is less than \( C \); which makes the gravitational Cherenkov radiation possible. GW will be refracted in the gravito-superconductor.

(C) Third case: \( \omega_0^2 \neq 0, \, \omega_0^2 > \omega^2, \) and \( 1 > \frac{4\pi \sqrt{G} \rho_g}{\omega_0^2 - \omega^2} \). Then, we have

\[ k_{GW} = \frac{\omega}{C_p}, \quad C_p = \sqrt{\frac{C}{1 - \frac{4\pi \sqrt{G} \rho_g}{\omega_0^2 - \omega^2}}} > C, \quad n_g = \sqrt{1 - \frac{4\pi \sqrt{G} \rho_g}{\omega_0^2 - \omega^2}} < 1. \]  

(126)

For this case, the phase velocity of GW propagated in a gravito-superconductor is greater than \( C \), and the refractive index is less than \( 1 \). We won’t study this case here.
For case: \( \omega_0^2 \neq 0, \omega_0^2 > \omega^2, \) and \( 1 < \frac{4\pi \sqrt{\rho c}}{\omega_0^2 - \omega^2} \). Then we have

\[
k_{GW} \equiv i \kappa, \quad \kappa \equiv \frac{\omega}{c} \sqrt{\frac{4\pi \sqrt{\rho c}}{\omega_0^2 - \omega^2} - 1}. \tag{127}\]

Let’s define the gravitational penetration depth, or skin depth, \( d_g \),

\[
d_g \equiv \frac{1}{\kappa} = \frac{c}{\omega} \sqrt{\frac{4\pi \sqrt{\rho c}}{\omega_0^2 - \omega^2} - 1} = \frac{\lambda}{2\pi} \frac{1}{\sqrt{\frac{4\pi \sqrt{\rho c}}{\omega_0^2 - \omega^2} - 1}}. \tag{128}\]

Like EMW, for case (D), GW will penetrate into a gravito-superconductor and its amplitude will be reduced. Then, the solution, Eq. (121), becomes

\[
B_g = B_{g0} e^{-\kappa z} e^{-i\omega t}, \quad g = g_0 e^{-\kappa z} e^{-i\omega t}. \tag{129}\]

### 6.2. Absorption of Energy of GW with Radiation Damping

Taking into account the radiation damping, Eq. (107), the equation of motion of a gravitational charge is,

\[
\dot{r} + \omega_0^2 r = \frac{Q_g}{m} g_0 e^{-i\omega t} - \frac{2\sqrt{\gamma m}}{3c^2} g = (\sqrt{\gamma} + i\xi) g, \tag{130}\]

\[
\frac{2\sqrt{\gamma m} \omega}{3c^2} \equiv \xi. \tag{131}\]

Then we have the solution,

\[
r = \frac{\sqrt{\gamma} + i\xi}{\omega_0^2 - \omega^2} g, \quad \dot{r} = \left( \frac{\omega \xi}{\omega_0^2 - \omega^2} - i \frac{\omega \sqrt{\gamma}}{\omega_0^2 - \omega^2} \right) g. \tag{132}\]

Now let’s study the penetration of GW into a gravito-superconductor. We have a formula for the gravitational current, call it \textit{generalized gravitational Ohm’s law},

\[
J_g = \rho_g \dot{r} = \rho_g \left( \frac{\omega \xi}{\omega_0^2 - \omega^2} - i \frac{\omega \sqrt{\gamma}}{\omega_0^2 - \omega^2} \right) g. \tag{133}\]

Substituting Eq. (133) into Eq. (13), we have

\[
\nabla \times B_g = -\frac{4\pi \rho c}{\omega} \left( \frac{\omega \xi}{\omega_0^2 - \omega^2} - i \frac{\omega \sqrt{\gamma}}{\omega_0^2 - \omega^2} \right) g + \frac{1}{c^2} \frac{\partial g}{\partial t}. \tag{134}\]

Applying Eq. (12), we have

\[
\nabla^2 B_g = -\frac{1}{c^2} \frac{\partial^2 B_g}{\partial t^2} + \frac{\omega}{c^2} (\delta - i\tau) \frac{\partial B_g}{\partial t} = 0, \tag{135}\]

\[
\nabla^2 g = -\frac{1}{c^2} \frac{\partial^2 g}{\partial t^2} + \frac{\omega}{c^2} (\delta - i\tau) \frac{\partial g}{\partial t} = 0, \tag{136}\]

29
\[ \delta \equiv \frac{4\pi \rho g c}{\omega^2} \] 
\[ \tau \equiv \frac{\sqrt{64\pi \rho g}}{\omega^2}. \] (137)

The solutions are
\[ \mathbf{B}_g = \mathbf{B}_0 e^{i(k_{GW} z - \omega t)}, \quad \mathbf{g} = \mathbf{g}_0 e^{i(k_{GW} z - \omega t)}, \] (138)
where
\[ K_{GW}^2 = \frac{\omega^2}{c^2} (1 - \tau) - i \frac{\omega^2}{c^2} \delta. \] (139)

We obtain,
\[ K_{GW} = K + ik, \] (140)
\[ K = \frac{\omega}{c} \sqrt{\frac{(1 - \tau) + \sqrt{(1 - \tau)^2 + (\delta)^2}}{2}}, \quad \kappa = \frac{\omega}{c} \sqrt{-\frac{(1 - \tau) + \sqrt{(1 - \tau)^2 + (\delta)^2}}{2}}. \] (141)

When \( \delta \ll 1 - \tau \), we have
\[ K = \frac{\omega}{c_p}, \quad C_p = \frac{c}{\sqrt{1 - \tau}}, \quad \kappa = 0. \] (142)

The radiation attenuation can be ignored. Note \((1 - \tau)\) may be either positive or negative.

Where GW wavelength \( \lambda_p \) and refractive index \( n_g \) of the gravito-superconductor are
\[ \lambda_p = \frac{2\pi}{k_{GW}}, \quad n_g = \frac{c}{c_p}. \] (143)

When \( \delta \gg 1 - \tau \), there is the radiation attenuation. We have
\[ K = \frac{\omega}{c} \sqrt{\delta/2}, \quad \kappa = \frac{\omega}{c} \sqrt{\delta/2}. \] (144)

Where GW wavelength \( \lambda_p \) and refractive index \( n_g \) of the gravito-superconductor are
\[ \lambda_p = \frac{2\pi}{K}, \quad n_g = \frac{cK}{\omega}. \] (145)

Let’s define the gravitational skin depth,
\[ d_g \equiv \frac{1}{\kappa}. \] (146)

Eq. (146) implies that GW will penetrate into a gravito-superconductor and its amplitude will be reduced. In other words, GW loses energy that becomes the thermal energy of the gravito-superconductor. Then, the solution, Eq. (138), becomes
\[ \mathbf{B}_g = \mathbf{B}_0 e^{-\kappa z} e^{i(Kz - \omega t)}, \quad \mathbf{g} = \mathbf{g}_0 e^{-\kappa z} e^{i(Kz - \omega t)} \]
\[ \mathbf{B}_g = \mathbf{B}_0 e^{-z/d_g} e^{i(Kz - \omega t)}, \quad \mathbf{g} = \mathbf{g}_0 e^{-z/d_g} e^{i(Kz - \omega t)} \] (147)

6.3. Refraction of GW

There is an interface between two linear media with gravitational refractive index
\( n_{g1} \) and \( n_{g2} \), respectively. GW approaches from media 1, through the interface, enter into media 2. If, \( n_{g1} \neq n_{g2} \), we obtain the gravitational Snell’s law for GW,

\[
\frac{\sin \theta_T}{\sin \theta_i} = \frac{n_{g1}}{n_{g2}},
\]

(148)

where \( \theta_T \) and \( \theta_i \) are the transmission and incident angle, respectively.

### 6.4. Cherenkov Radiation

Section 6.2 shows that the phase velocity \( C_p \) of GW of frequency \( \omega \) propagated in a gravito-superconductor may be less than \( C \). When the speed \( V \) of a relativistic gravitational charge moving in the gravito-superconductor is greater than the phase velocity \( C_p \) of GW, the gravitational charge emits gravitational Cherenkov radiation with the same \( \omega \). The lost energy per unit distance along the path of the gravitational charge is the following: for case (A) [Eq. (124)],

\[
\frac{dW_c}{dx} = \left( \frac{\rho_g}{C} \right)^2 \int \omega \left( 1 - \frac{1}{\beta^2 \left( \frac{4\pi \sqrt{\rho_g \omega}}{\omega^2} \right)} \right) d\omega,
\]

(149)

for case (B) [Eq. (125)],

\[
\frac{dW_c}{dx} = \left( \frac{\rho_g}{C} \right)^2 \int \omega \left( 1 - \frac{1}{\beta^2 \left( \frac{4\pi \sqrt{\rho_g \omega}}{\omega^2 - \omega_0^2} \right)} \right) d\omega,
\]

(150)

and for the case of Eq. (142), when \( (1 - \tau) > 1 \),

\[
\frac{dW_c}{dx} = \left( \frac{\rho_g}{C} \right)^2 \int \omega \left( 1 - \frac{1}{\beta^2 (1 - \tau)} \right) d\omega.
\]

(151)

Where \( \rho_g \) is the charge density in the gravito-superconductor, \( \beta = V/C \). We denote Eqs. (149, 150, and 151) as the gravitational Tamm-Frank formula for different situations, correspondingly. For the far fields, the emission angles of gravitational Cherenkov radiation for case (A), case (B), and case of Eq. (142) are, correspondingly,

\[
\cos \theta_C = \frac{1}{\beta \sqrt{\frac{4\pi \sqrt{\rho_g \omega}}{\omega^2}}}, \quad \cos \theta_C = \frac{1}{\beta \sqrt{1 + \frac{4\pi \sqrt{\rho_g \omega}}{\omega^2 - \omega_0^2}}}, \quad \cos \theta_C = \frac{1}{\beta \sqrt{1 - \tau}}.
\]

(152)

### 7. Gravito-Photon Interacting with Gravitational Charges: Absorption

The absorption of photons by an electron is the photoelectron effect described by

\[
K_{\text{max}} = \hbar \omega_{\text{photon}} - \phi.
\]

(153)
The absorption of gravito-photons by an electron is the gravitational counterpart of photoelectron effect; we denote it as the \textit{gravito-photo-electron effect}. We suggest that this effect is described by

\[ K_{\text{max}} = h_g \omega_{\text{gravito-photon}} - \phi, \tag{154} \]

where \( K_{\text{max}} \) is the maximum kinematic energy of the emitted electron, \( h_g (=h) \) is the Gravitational Planck Constant, \( \omega_{\text{gravito-photon}} \) is the frequency of absorbed gravito-photon, \( \phi \) is the work function of the electron-emissive surface.

Comparing Eqs. (153 and 154), we argue that the threshold for gravito-photoelectron effect to take place is the same as that for photoelectron effect, i.e., dependent only on frequency of incident waves. In other word, an electron can’t distinguish whether it is a photon, a gravito-photon, or a graviton transporting energy to it. Table 6 shows the comparison between spin 1 photon and spin 1 gravito-photon.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & Photon & Gravito-Photon \\
\hline
Rest mass & 0 & 0 \\
\hline
Charge & Electric charge: 0 \hspace{1cm} & Gravitational charge: 0 \hspace{1cm} \\
\hline
Wave nature & Detected & Detected* \hspace{1cm} \\
\hline
Particle nature & Detected & Not yet \\
\hline
\end{tabular}
\caption{Comparison between spin 1 photon and spin 1 gravito-photon.}
\end{table}

* We argue that the detection of GW cannot distinguish between Gravito-photon and graviton.

It has been shown that the present LIGO/Virgo cannot detect a single graviton [15]. Thus the LIGO/Virgo cannot detect a \textit{single gravito-photon}. However we will show later that LIGO is capable to detect a \textit{beam of gravito-photons}, which can be an evidence of the existence of gravito-photons.

In atomic gravito-photo-effect, the total gravito-photelectric cross section, \( \sigma_{\text{gpe}} \), is,

\[ \sigma_{\text{gpe}} \sim \begin{cases} \frac{Z^4}{(h_g \omega_{\text{gravito-photon}})^2} \\ \frac{Z^5}{h_g \omega_{\text{gravito-photon}}} \end{cases} \begin{cases} \text{low energy} \\ \text{high energy} \end{cases} \right\}. \tag{155} \]

8. \textbf{GW Interacting with Gravitational Charges: Scattering}

We adopt the wave language to study GW scattering in this section.

8.1. \textbf{GW Scattering Cross-section}
Let’s define the GW scattering cross-section,

$$d\sigma_{GW} \equiv \frac{dI}{S_g}$$ (156)

Where \(dI\) is the intensity of energy emitted by a scattering system in a given direction per unit time. \(S_g\) is the gravitational Poynting vector of incident GW. Eq. (156) implies that the incident GW loses energy, \(\sim S_g\sigma_{GW}\). The dash over the letters denotes the time average.

8.2. GW Scattered by Free Gravitational Charge

Let’s consider a simple situation where a gravitational scattering charge is under influence of an incident linearly polarized plane GW, \(\mathbf{g} = \mathbf{g}_0 \cos(\omega t)\). For the small velocity of the scattering charge, we can ignore the gravito-magnetic field of GW, and assumed the field of GW acts on the scattering charge is the same. The dipole of this charge is \(\mathbf{d} = Q\mathbf{r}\). The scattering charge experiences the \(\mathbf{g}\) field, \(m\mathbf{F} = Q\mathbf{g}\), and we have

$$\ddot{\mathbf{d}} = Q\mathbf{g} = Gm\mathbf{g}.$$ (157)

The radiated GW has the same frequency as that of the incident GW, we called it the gravitational coherent scattering.

Substituting Eq. (157) into Eqs. (91, 92, and 156), we obtain, the radiated intensity, Poynting vector, scattering cross-section, respectively,

$$dI = \frac{G^2m^2}{4\pi c^3} (\mathbf{g} \times \mathbf{n'})^2 d\Omega, \quad \mathbf{S} = \frac{c}{4\pi} g^2 \mathbf{n},$$ (158)

$$d\sigma_{GW} = \left(\frac{Gm}{c^2}\right)^2 \sin^2\theta d\Omega, \quad \sigma_{GW} = \frac{8\pi}{3} \left(\frac{Gm}{c^2}\right)^2.$$ (159)

We denote \(\sigma_{GW}\) as the gravitational Thomson cross-section. The existence of the non-zero scattering cross-section implies that a part of incident GW will be scattered.

Note, unlike in the gravito-photon-electron effect in Section 7, the scattering cross-section \(\sigma_{GW}\) is independent with the frequency of GW.

For unpolarized GW, the scattering cross-section is,

$$d\sigma_{GW} = \frac{1}{2} \left(\frac{Gm}{c^2}\right)^2 (1 + \cos^2\theta) d\Omega.$$ (160)

Where \(\theta\) is the scattering angle. Eqs. (159 and 160) imply that GW transports its energy to scattering gravitational charge on the path of propagation.
8.3. GW Scattered by Vibrating Gravitational Charges

Let’s consider a situation where a gravitational scattering charge being influenced by an oscillation force with frequency $\omega_0$ and driven by an incoming GW, the equation of motion of the charge is

$$\ddot{r} + \omega_0^2 r = \frac{Qg}{m} g_0 e^{-i\omega t} = \sqrt{G} g.$$  \hfill (161)

Then we have the solution,

$$r = \frac{\sqrt{G}}{(\omega_0^2 - \omega^2)} g, \quad \dot{r} = -\frac{i\omega \sqrt{G}}{(\omega_0^2 - \omega^2)} g. \quad \hfill (162)$$

Following the procedure of Electrodynamics and applying Eqs. (158 and 92), we obtain the cross-section,

$$d\sigma_{GW} = \left(\frac{\hat{\alpha}m}{c^2}\right)^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2} \sin^2\theta d\Omega. \quad \hfill (163)$$

We denote it as the generalized gravitational Thomson cross-section.

8.4. GW Scattered by Vibrating Gravitational Charge with Radiation Damping

Taking into account the radiation damping, in a general expression, $\gamma \dot{r}$, the equation of motion of a scattering gravitational charge is,

$$\ddot{r} + \gamma \dot{r} + \omega_0^2 r = \frac{Qg}{m} g_0 e^{-i\omega t}. \quad \hfill (164)$$

The solution of Eq. (164) is,

$$r = \frac{\sqrt{G} g}{\omega_0^2 - \omega^2 - i\gamma \omega}, \quad \dot{r} = -\frac{i\sqrt{G} \omega}{\omega_0^2 - \omega^2 - i\gamma \omega} g, \quad \hfill (165)$$

Following the procedure of Electrodynamics, we obtain the cross-section,

$$d\sigma_{GW} = \left(\frac{\hat{\alpha}m}{c^2}\right)^2 \frac{\omega^4 \sin^2\theta d\Omega}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2}. \quad \hfill (166)$$

We denote it as the generalized gravitational Thomson cross-section, which depends on the frequency of GW.

8.5. Frequency Change of Scattered Radiation

As mentioned above, Gravitodynamics obey SR. We have the transformation law of the frequency from a charge rest system to an observer system, in which the charge is moving with velocity $V$,.
\[ \omega' = \omega \frac{1-\beta \cos \theta}{1-\beta \cos \theta'} \]  
\[ \text{(167)} \]

where \( \theta \) and \( \theta' \) are angles of the incident and scattered GWs with the direction of \( \mathbf{V} \), respectively.

### 9. Gravito-Photon Interacting with Gravitational Charges: Scattering

In this section we study the Gravito-photon scattering.

#### 9.1. Gravito-photon Transporting Momentum to Scattering Charge

Gravito-photons as particles will lose the momentum, \( \sim W \sigma_{\text{GW}} \), during the scattering. All the lost momentum of the incident Gravito-photons is transported to the scattering gravitational charges. The average force acting on the scattering gravitational charge is equal to the average momentum absorbed per unit time,

\[
f = \sigma_{\text{GW}} \mathbf{n}_{\text{inc}}. \quad \text{(168)}
\]

The \( \mathbf{n}_{\text{inc}} \) is the unit vector in the direction of propagation of the incident Gravito-photons.

Note Eq. (168) implies that Gravito-photons put average force on the charge and pushes it to accelerate \emph{along the same direction} of the propagation of Gravito-photons, like bullets hit a target. Eq. (168) indicates the particle nature of GW.

#### 9.2. An Indirect Evidence of Wave-Particle Duality of GW

We have shown that when GW with the incident intensity \( I_{\text{g,inc}} \) penetrates into a gravito-superconductor, its intensity decreases to \( I_{\text{g,rem}} \). Eqs. (129, 146) give,

\[
\frac{I_{\text{g,rem}}}{I_{\text{g,inc}}} = \frac{\mathbf{R}_{\text{rem}}^z}{\mathbf{R}_{\text{inc}}^z} = e^{-2\kappa z} = e^{-2\pi/d_g}. \quad \text{(169)}
\]

Let’s define \emph{GW absorption coefficient} or \emph{GW attenuation constant}, \( \alpha \),

\[ \alpha \equiv 2\kappa = 2/d_g. \quad \text{(170)} \]

Then Eq. (169) becomes

\[
\frac{I_{\text{g,rem}}}{I_{\text{g,inc}}} = e^{-\alpha z}. \quad \text{(171)}
\]

Eq. (171) is derived from wave nature of GW.

For comparison, we recall the \emph{gravitational Beer-Lambert law}, Eq. (58), here again,

\[
\frac{I_{\text{g,rem}}}{I_{\text{g,inc}}} = e^{-z/\ell}. \quad \text{(58)}
\]
Eq. (58) is derived from particle nature of GW. If we let the GW absorption coefficient, $\alpha$, equals to the reciprocal of the gravito-photon mean free path, $\ell$,

$$\ell = \frac{1}{\alpha} = \frac{d_g}{z},$$  \hspace{1cm} (172)

then, Eq. (171) and Eq. (58) are identical (Table 7).

<table>
<thead>
<tr>
<th>GW Interacting with gravito-superconductor</th>
<th>Wave Nature of GW $\frac{I_{g,rem}}{I_{g,inc}} = e^{-\alpha z}$</th>
<th>Particle Nature of GW $\frac{I_{g,rem}}{I_{g,inc}} = e^{-z/\ell}$</th>
</tr>
</thead>
</table>

In other words, GW indeed has wave-particle duality. Also the gravitational mean free path is equal to half of the skin depth.

9.3. Gravito-photon Compton Scattering:

Based on the particle nature of GW, let’s consider a situation where gravito-photon with energy $E_{g0}$ and wavelength $\lambda_{g0}$ is scattered by a rest electron, loses some of its energy, and is deflected from its original propagation direction. By SR and conservations of both energy and momentum, we obtain the energy, $E_g$, of the outgoing gravito-photon,

$$E_g = \frac{h \gamma c}{\lambda_g}, \hspace{1cm} \lambda_g = \lambda_{g0} + \lambda_{GC}(1 - \cos \theta), \hspace{1cm} \lambda_{GC} \equiv \frac{h_g}{m_e c},$$  \hspace{1cm} (173)

where we denote $\lambda_{GC}$ as the gravitational Compton wavelength of the electron, $\theta$ is the scattering angle of gravito-photons.

10. Applications and Predictions

Gravitodynamics is consistent with SR. We apply the formulas obtained above and the method in SR to study astronomical objects and predict new effects.

10.1. Transformation Law of Energy Density

From SR, the transformation law of the energy density $W_{GW}$ of a plane GW from one inertial reference system $K$ to another system $K'$ is

$$W_{GW} = W'_{GW} \frac{(1 + \beta \cos \alpha')^2}{1 - \beta^2}.$$  \hspace{1cm} (174)

The $\alpha'$ is the angle in the $K'$ system between the direction of velocity $V$ and the direction of propagation of GW.
10.2. Transformation Law of Intensity of GW

The transformation law of the intensity/energy of the radiation of waves from a source-rest reference system to a moving reference system is applicable to GW. The intensity $dI'$ in the $K'$ system in which the system of gravitational charge is at rest as a whole is,

$$dI' = \mathcal{F}(\cos\theta', \phi')d\Omega'.$$

The energies, $dE_g'$ and $dE_g$, in the $K'$ system and the observer’s system $K$, are related by

$$dE_g' = \frac{1 - \beta \cos\theta}{\sqrt{1 - \beta^2}} dE_g.$$

(175)

Applying the transformation rules,

$$\sin\theta = \frac{\sin\theta'\sqrt{1 - \beta^2}}{1 + \beta \cos\theta'}, \quad \cos\theta = \frac{\cos\theta' + \beta}{1 + \beta \cos\theta'}, \quad dt = \frac{dt'}{\sqrt{1 - \beta^2}}.$$

(176)

we obtain a general transformation law for the intensity of the radiation of GW,

$$dl = \frac{(1 - \beta^2)^2}{(1 - \beta \cos\theta)^2} \mathcal{F}\left(\frac{\cos\theta - \beta}{1 - \beta \cos\theta}, \phi\right) d\Omega.$$

(177)

10.3. Transformation Law of Frequency: Gravitational Doppler Effect

Now let’s derive the *Gravitational Doppler effect*. As mentioned above, Gravitodynamics is compatible with SR. An important conclusion is that the redshifts of GW follow the rule same as that of EMW. According to the rule of transformation of the four-vector from a source-rest system $K_s$ to an observer system $K_o$ moving with velocity $-V$ relative to the system $K_s$, we obtain the *Gravitational Doppler effect*,

$$\omega_{go} = \omega_{gs} \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos\alpha}, \quad \lambda_{go} = \lambda_{gs} \frac{1 - \beta \cos\alpha}{\sqrt{1 - \beta^2}},$$

(178)

where $\omega_{gs}(\lambda_{gs})$ and $\omega_{go}(\lambda_{go})$ are, respectively, the frequencies (wavelength) of GW in the source-rest system $K_s$ and in the observer system $K_o$, respectively; $\alpha$ is the angle, in the $K_o$ system, between the direction of emission of GW and the direction of motion of the source. Eq. (178) is the gravitational counterpart of the Doppler effect of EMW.

Now we can find the frequency/wavelength of emitted GW from observed GW.

The formulas describing both EMW Doppler effect and the GW Doppler effect have the same form. The redshifts of EMW and GW radiated by the same source carrying both electrical and gravitational charges are the same. Therefore by measuring the redshift,
$Z_{EMW}$, of EMW, we obtain the “redshift velocity” of the source of both EMW and GW from the following formula,

$$Z_{GW} = Z_{EMW} \equiv \frac{1 - \beta \cos \alpha}{\sqrt{1 - \beta^2}} - 1.$$  \hspace{1cm} (179)

Returning back to GW, substituting Eq. (179) into Eq. (178), we obtain

$$\omega_{gs} = \omega_{g0}(1 + Z_{EMW}), \quad \lambda_{gs} = \frac{\lambda_{g0}}{1 + Z_{EMW}}.$$  \hspace{1cm} (180)

By Eq. (180), we can obtain the emitted frequency and wavelength of GW from the redshift of EMW.

For $\alpha = 90^0$, Eq. (180) gives the gravitational transverse Doppler effect of GW,

$$\lambda_{gs} = \lambda_{g0}\sqrt{1 - \beta^2}, \quad \omega_{gs} = \omega_{g0}\frac{1}{\sqrt{1 - \beta^2}}.$$  \hspace{1cm} (181)

For the case of $V \ll C$ and $\alpha$ not close to $90^0$, Eq. (180) becomes

$$\omega_{gs} \approx \omega_{g0}(1 - \beta \cos \alpha), \quad \lambda_{gs} \approx \frac{\lambda_{g0}}{(1 - \beta \cos \alpha)}.$$  \hspace{1cm} (182)

Eqs. (180, 181 and 182) give the relations between wavelength/frequency and the velocity of the source of GW.

### 10.4. Hubble Constant and Redshift of GW

For a source radiating only GW, how can we find redshift of GW? For this aim, we need to find the relations between frequency/wavelength and the distance of the source of GW. For the time scale measuring GW, we only consider the constant Hubble parameter. An expression for Hubble's Law is,

$$V = H_0 R_0,$$  \hspace{1cm} (183)

where $H_0$ is Hubble's constant, $R_0$ is the proper distance. Then we obtain the redshift,

$$Z_{GW} \equiv \frac{1 - \frac{H_0 R_0 \cos \alpha}{c}}{\sqrt{1 - \left(\frac{H_0 R_0}{c}\right)^2}} - 1.$$  \hspace{1cm} (184)

Substituting Eq. (184) into Eq. (180), we obtain the emitted frequency $\omega_{gs}$ and wavelength $\lambda_{gs}$,

$$\omega_{gs} = \omega_{g0}\frac{1 - \frac{H_0 R_0 \cos \alpha}{c}}{\sqrt{1 - \left(\frac{H_0 R_0}{c}\right)^2}}, \quad \lambda_{gs} = \frac{\lambda_{g0}}{\sqrt{1 - \left(\frac{H_0 R_0}{c}\right)^2}}.$$  \hspace{1cm} (185)

Eq. (185) is a useful tool to determine the emitted GW's frequency and wavelength.
once we know the distance of a GW source in the universe.

We need to distinguish emitted frequency/wavelength of GW from measured frequency/wavelength on Earth. The calculations of detected GW’s parameters related with frequency, e.g., those in the pocket formulae, such as Scaling amplitude, Chirp/Chirp waveform, Chirp mass, GW phase, GW form, and Luminosity distance, etc., need to include the effects of redshift of GW. The detail discussion is out of scope of this article.

10.5. Perihelion Precession of Mercury

Based on Eq. (77), Borodikhin has shown that the gravitational Darwin Lagrangian for a binary system,

\[ L_g = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{Q_{g1} Q_{g2}}{R_{12}} + \frac{Q_{g1} Q_{g2} V^2}{c^2 R_{12}}, \]  \hspace{1cm} (186)

can explains the perihelion precession of Mercury [21].

10.6. Gravitational Tully-Fisher law

The gravitational Larmor formula, Eq. (81), can be written as

\[ I = \frac{2 G m^2}{3 c^2 r^2} V^4. \] \hspace{1cm} (187)

Where \( V = \omega r \) and \( a = \omega^2 r \) have been used.

Eq. (187) shows that the intensity of GW is proportional to the \( V^4 \). We denote Eq. (187) as the gravitational Tully-Fisher law [22]. The gravitational Tully-Fisher relation is a correlation for spiral galaxies between their total energy of GW emitted by the spiral galaxies per unit time and how fast they are rotating.

10.7. Dipole Radiation of Non-Relativistic Binary System: Wave Approach

As an example, let’s consider a non-relativistic binary system. For a system of two point gravitational charges, e.g., a system of binary stars, moving with low velocities, \( V_i \ll C \), we choose the origin of the reference system at the center of mass. Then the gravitational dipole moment has the form,

\[ \mathbf{d}_T = Q_{g1} \mathbf{r}_1 + Q_{g2} \mathbf{r}_2 = \mu \left( \frac{Q_{g1}}{m_1} - \frac{Q_{g2}}{m_2} \right) \mathbf{r}. \] \hspace{1cm} (188)

Where \( \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \) and \( \mu = \frac{m_1 m_2}{m_1 + m_2} \). Substituting the definition of positive gravitational charge, \( Q_{g+} = \sqrt{G} m \), into Eq. (188), we have \( \mathbf{d}_T = 0 \), i.e., the gravitational dipole
moment vanishes. Substituting $\mathbf{d_T}$ into Eq. (92), the gravitational dipole moment does not radiate energy.

Note: even with the conventional wave approach, we still have the following cases: Considering a gravitational dipole constituted with one positive and one negative gravitational charges, then $\mathbf{d_T} \neq 0$, we have GW radiated by this gravitational dipole.

**10.8. Dipole Radiation of Non-Relativistic Binary System: Particle Approach**

Let’s image a situation where an observer is at the center of a coordinate system and a first gravitational charge moves in the system with acceleration. The radiation of the first charge is given by Eq. (81). Then a second gravitational charge is dropped into this system. The particle nature of GW maintains that the first charge still radiates. The second charge radiates as well. Moreover, as these two charges radiates independently, the total intensity of radiation of this system is the summation of the radiation intensities of these two charges, i.e.,

$$I_T = \sum_{i} I_i = \frac{2G}{3C^3} \left( m_1^2 a_1^2 + m_2^2 a_2^2 \right) = \frac{16G^3 m_1^2 m_2^2}{3C^3 R^4}. \quad (189)$$

In wave language, this system of two gravitational charges is called gravitational dipole. Eq. (189) implies that a gravitational dipole does radiate gravito-photons or GW.

Comparison of the different conclusions drawn from Wave and Particle Approaches is listed in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>Dipole Radiation: Particle</th>
<th>Dipole Radiation: Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dipole</strong></td>
<td>$\mathbf{d_{gi}} = Q_{gi} \mathbf{r_i} \neq 0$, $i = 1, 2$</td>
<td>$\mathbf{d_{gT}} = \sum_i Q_{gi} \mathbf{r_i} = Q_{g1} \mathbf{r_1} + Q_{g2} \mathbf{r_2} = 0$</td>
</tr>
<tr>
<td><strong>Vector potential</strong></td>
<td>$\mathbf{A_{gi}} = -\frac{\mathbf{d_{gi}}}{C \mathbf{R_0}} \neq 0$</td>
<td>$\mathbf{A_{gT}} = -\frac{\mathbf{d_{gT}}}{C \mathbf{R_0}} = 0$</td>
</tr>
<tr>
<td><strong>Field Strength</strong></td>
<td>$\mathbf{B_{gi}} = -\frac{\mathbf{d_{gi}} \times \mathbf{n}}{C^2 \mathbf{R_0}} \neq 0$</td>
<td>$\mathbf{B_{gT}} = -\frac{\mathbf{d_{gT}} \times \mathbf{n}}{C^2 \mathbf{R_0}} = 0$</td>
</tr>
<tr>
<td><strong>Intensity</strong></td>
<td>$I_T = \sum_i I_i = \frac{2}{3C^3} \sum \mathbf{d_{gi}}^2 \neq 0,$</td>
<td>$I_T = \frac{2}{3C^3} \mathbf{d_{gT}}^2 = 0$</td>
</tr>
</tbody>
</table>

If we treat the beam of gravito-photons radiated by each of sources of a system individually, as every source radiates, the total beam of gravito-photons radiated by the system is not zero. However if we treat the system of sources as a whole, then as the
gravitational dipole moment of the system is zero, so is the total radiation of the system. This appears to suggest that a source would radiate when it does not ‘know’ the existence of other sources in its system and it would not radiate otherwise.

An example. For a binary system rotating in an x-y plane with a constant frequency $\omega$, Eq. (189) gives the total radiation,

$$I = \frac{4Gm^2r_0^3\omega^4}{3C^3}.$$  \hspace{1cm} (190)

Where $r_0$ is the distance between the charge and the center of the binary system.

10.9. Comparison: Dipole and Quadrupole Radiations of Non-Relativistic Binary System

As shown above, there are dipole and quadrupole radiations for a non-relativistic binary system. We now compare the dipole and quadrupole radiations. For simplicity, let’s consider a binary system of two equal gravitational charges, $Q_{g1} = Q_{g2}$, i.e. $m_1 = m_2 = m$. Treating the two stars as point charge and orbiting in an x-y plane. The averaged intensity of quadrupole radiation of this system, from Eq. (103), is

$$\langle I \rangle = \frac{G}{20C^5} \left( \frac{P^2_{\mu\nu}}{m^5} \right) = \frac{16G^4m^5}{5C^5R^5},$$ \hspace{1cm} (191)

where Kepler third law, $\omega^2 = \frac{2Gm}{R^3}$, is applied.

For the same system we have dipole radiation, from Eq. (189), $I_{dipole} = \frac{16G^3m^4}{3C^2R^4}$. Then the ratio of dipole to quadrupole radiations for this system is,

$$\frac{I_{dipole}}{I_{quadrupole}} = \frac{5C^2R}{3Gm} = \frac{10R}{3R_s}.$$ \hspace{1cm} (192)

where $R_s = \frac{2GM}{C^2}$ is the Schwarzschild radius. Eq. (192) implies that when

$$R < 0.3R_s \quad \text{or} \quad R > 0.3R_s,$$ \hspace{1cm} (193)

the quadrupole or the dipole radiation is dominant, respectively. We have ignored gravito-magnetic dipole moment, and have, for non-relativistic binary system,

$$\frac{I_{quadrupole}}{I_{total}} = \frac{I_{quadrupole} + I_{dipole}}{3R_s} = \frac{3R_s}{3R_s + 10R}.$$ \hspace{1cm} (194)

Since the radiation energy loss, the radius of the orbit decreases.
10.10. Dipole Radiation of Relativistic Binary System:

For a system of two relativistic gravitational charges, $V_i \sim C$, in a procedure similar to that in Section 5.2, we can obtain the generalized gravitational Lienard equation,

$$I_T = \sum_k I_k = \sum_k \frac{2Gm_k^2}{3C^3(1-\beta_k^2)^3} \left[ \dot{a}_k^2 - (\beta_k \times \dot{a}_k)^2 \right]. \quad (195)$$

For a binary system, we have,

$$I_T = \frac{2G}{3C^3} \left[ \frac{m_1^2 \dot{a}_1^2}{(1-\beta_1^2)^3} + \frac{m_2^2 \dot{a}_2^2}{(1-\beta_2^2)^3} - \frac{m_1^2 |\beta_1 \times \dot{a}_1|^2}{(1-\beta_1^2)^3} - \frac{m_2^2 |\beta_2 \times \dot{a}_2|^2}{(1-\beta_2^2)^3} \right]. \quad (196)$$

For a special situation where two charges are equal, then $m_1 = m_2$, $a_1 = a_2$, $\beta_1 = \beta_2$, we have the intensity of GW radiated by the dipole momentum of the binary system,

$$I_T = \frac{4Gm^2}{3C^3(1-\beta^2)^3} \left[ \dot{a}^2 - |\beta \times \dot{a}|^2 \right]. \quad (197)$$

10.11. Quadrupole Radiation of Relativistic Binary System

Let’s consider a binary system of gravitational charges having equal charges, $Q_{g1} = Q_{g2}$, and moving circularly. At a starting time, the velocities of two charges are slow. Eq. (191) gives

$$\langle I \rangle = \frac{G}{20C^5} \left( B_{m\alpha\beta}^2 \right) = \frac{2Gm^2}{5C^5} \omega^6 R^4 = \frac{2Gm^2 a^4}{5C^5 \omega^2} = \frac{2G}{5C^5 m^2 \omega^2} \left( \frac{dP}{dt} \cdot \frac{dP}{dt} \right)^2, \quad (198)$$

where $a (= \omega^2 R)$ is the acceleration of the charge.

Since two charges lose energy by GW radiation, their orbits shrink and speeds become faster and closer to that of light. To deriving the intensity of the GW quadruple radiation of this relativistic binary system, we write the Lorentz invariant generalization of Eq. (198),

$$\langle I \rangle = \frac{G}{20C^5} \left( B_{m\alpha\beta}^2 \right) = \frac{2G}{5C^5 m^2 \omega^2} \left( \frac{dP}{dt} \cdot \frac{dP}{dt} \right)^2, \quad (199)$$

where $d\tau = dt/\gamma$. We obtain the intensity of GW radiated by the gravitational quadrupole momentum of the relativistic binary system,

$$\langle I \rangle = \frac{G}{20C^5} \left( B_{m\alpha\beta}^2 \right) = \frac{2Gm^2}{5C^5 \omega^2} \left( \frac{1}{(1-\beta^2)^3} \right)^2 \left[ \dot{a}^2 - (\beta \times \dot{a})^2 \right]^2. \quad (200)$$

10.12. Comparison: Dipole and Quadrupole Radiations of Relativistic Binary System
For a relativistic binary system, let’s compare the dipole radiation,

\[ I_{\text{dipole, binary}} = \frac{4Gm^2}{3C^3(1-\beta^2)^3} \left[ a^2 - |\beta \times a|^2 \right], \tag{197} \]

with Quadrupole Radiations,

\[ (I)_{\text{quadrupole, binary}} = \frac{2Gm^2}{5C^5\alpha^2(1-\beta^2)^4} \left[ a^2 - (\beta \times a)^2 \right]^2. \tag{200} \]

The ratio is,

\[ \frac{I_{\text{dipole, binary}}}{(I)_{\text{quadrupole, binary}}} = \frac{10\omega^2C^2(1-\beta^2)^3}{3[a^2 - |\beta \times a|^2]}. \tag{201} \]

For a special case of \( \beta \times a = \beta a \), Eq. (201) becomes,

\[ \frac{I_{\text{dipole, binary}}}{(I)_{\text{quadrupole, binary}}} = \frac{10\omega^2C^2(1-\beta^2)^2}{3a^2}. \tag{202} \]

For \( \beta \to 0 \), Eq. (202) becomes,

\[ \frac{I_{\text{dipole, binary}}}{(I)_{\text{quadrupole, binary}}} = \frac{10C^2(1-\beta^2)^2}{3\omega^2R^2} = \frac{5C^2R}{3Gm}, \tag{203} \]

which agrees with Eq. (192).

10.13. Gravitational Bremsstrahlung of Two Relativistic Stars Collision

For a special case of head-on collision of two relativistic gravitational charges, e.g., two relativistic stars, carrying similar gravitational charges, the generalized gravitational bremsstrahlung is,

\[ I_T = \sum K I_K = \frac{2G}{3C^3} \left[ \frac{m_1^2a_1^2}{(1-\beta_1^2)^3} + \frac{m_2^2a_2^2}{(1-\beta_2^2)^3} \right]. \tag{204} \]

In the case of two relativistic star merge, we need to take into account the generalized gravitational bremsstrahlung.


Let’s consider a simple example of the radiation damping, a system of two equal gravitational charges attracting each other and slowly moving circularly. Here we are not use the conventional dipole method, since it will give zero dipole radiation incorrectly. Instead, we use the particle approach developed in this article, i.e., treat the dipole radiation of each gravitational charge independently, and the quadrupole radiation of the system as a whole. The total energy loss due to the dipole and quadrupole radiation is the
summation,

\[
W_T = \frac{2}{3c^3} \sum K Q_g^2 a_k^2 + \frac{G}{20c^5} D_{m\alpha \beta}.
\]  

(205)

For a binary non-relativistic system of equal charges, then the total lost energy is obtained,

\[
W_T = W_{\text{dipole}} + W_{\text{quadrupole}} = \frac{4Gm^2}{3c^3} a^2 + \frac{16G^4m^5}{5c^5R^5}.
\]  

(206)

10.15. Energy Loss by Radiation: Relativistic Binary System

For a binary relativistic system of equal charges, the total lost energy is,

\[
W_T = W_{\text{dipole}} + W_{\text{quadrupole}} = \frac{4Gm^2}{3c^3} \frac{a^2 - (\beta \times a)^2}{(1 - \beta^2)^3} + \frac{2Gm^2}{5c^5\omega^2} \frac{a^2 - (\beta \times a)^2}{(1 - \beta^2)^6}.
\]  

(207)

11. Experiment Designed to Detect Wave-Particle Duality of GW

It has been shown that LIGO can’t be used to detect a single spin-2 graviton, the quantum of GW in GR [15]. However, the particle nature of GW implies that spin-1 gravito-photons will transport its energy to gravitational charges encountered on the way of propagation. Adopting the concept detecting neutrinos and based on Eqs. (57 and 58) or Eqs. (60 and 61), we propose an experiment, as shown bellow (Fig. 2), to explore the particle nature of gravito-photons by detecting a beam of gravito-photons, instead of detecting a single gravito-photon.

Experiment set up. GW comes from a source and hits the first detector, such as a LIGO detector, with the beam density \(I_{g\text{inc}}\). The beam density of gravito-photons is proportional to the intensity of radiation energy of GW. We want to measure the remaining beam density \(I_{g\text{rem}}\), or the intensity of GW, on the detector 2, such as another LIGO detector, located on the opposite of a gravitational-conductor, such as Earth. The results of observation allow us to estimate the cross section and gravitational mean free path of gravito-photons.

Fig. 2: Gravito-photons passing through Earth

Detector 1    Detector 2

Earth

GW
Calculated result based on Particle nature of GW.

Eq. (55), $I_{\text{g,rem}}(z) = I_{\text{g,inc}} e^{-n_{gc} \sigma_g z}$, gives

$$\sigma_{g-p} = \frac{1}{zn_{gc}} \ln \left( \frac{I_{\text{g,inc}}}{I_{\text{g,rem}}} \right).$$

(208)

For Earth, its diameter $z \approx 1.3 \times 10^9 \text{cm}$, $n_{gc} = \frac{\text{average density of Earth}}{\text{mass of neutron}} \approx 3.3 \times 10^{24} / \text{cm}^3$.

Then we have

$$\sigma_{g-p} \approx 2.5 \times 10^{-34} \ln \left( \frac{I_{\text{g,inc}}}{I_{\text{g,rem}}} \right) \text{cm}^2.$$

(209)

For the detected $I_{\text{g,rem}}$, if it is less than $I_{\text{g,inc}}$, the experimental result would be an evidence of the particle nature of GW. This detection is similar to detect neutrino. Here we have ignored the cross-section of absorption of gravito-photons.

Once we determined $\sigma_{g-p}$, we have gravitational mean free path of gravito-photons through a gravito-superconductor with known density. Eq. (209) will help us to calculate the intensity of GW at the source, $I_{\text{g,source}}(z)$, by,

$$I_{\text{g,source}}(z) = I_{\text{g,earth}} e^{n_{\text{g,universe}} \sigma_{g-p} z_{\text{source-earth}}},$$

(210)

where $I_{\text{g,earth}}$ is the intensity detected on Earth; $n_{\text{g,universe}}$ is the number density of the universe; $\sigma_{g-p}$ is the gravito-photon cross-section; $z_{\text{source-earth}}$ is the distance between the source and Earth.

12. Discussions and Remark

In this article, we have systematically study both wave phenomena and particle nature of GW in the framework of Gravitodynamics theoretically and experimentally and shown the following:

(1) Gravitodynamics is compatible with SR.

(2) Gravitodynamics is compatible with quantum mechanics. GW has been quantized and has the wave-particle duality. An experiment to detect the particle nature of GW has been proposed.

(3) Creation and annihilation operators take gravitational charges from and send gravitational charges to the Generalized Dirac Sea, respectively.

(4) Gravito-photons penetrate into a gravito-superconductor with attenuation.

(5) Gravitodynamics has no energy issue because it is based on the Precise Equivalence Principle [12,13] and does not require the Weak Equivalence Principle.

(6) The physical characteristics of GW have been studied by following
Electrodynamics and, especially, in terms of the field strengths, \( \mathbf{g} \) and \( \mathbf{B}_g \); thus the directions of looking for new effects of GW are conceptually clear, and the related calculations are simple.

(7) The concept of gravitational field strength has been introduced into GR. For the components of \( T^{00} \) and \( T^{0i} \), the linearized Einstein equation has been written as [18],

\[
\nabla \cdot \mathbf{g} = - 4\pi \rho_g, \quad (211) \\
\nabla \cdot \mathbf{B}_g = 0, \quad (212) \\
\n\nabla \times \mathbf{g} = - \frac{1}{c} \frac{\partial \mathbf{B}_g}{\partial t}, \quad (213) \\
\n\nabla \times \mathbf{B}_g = - \frac{4\pi}{c} \mathbf{J}_g + \frac{1}{c} \frac{\partial \mathbf{g}}{\partial t}, \quad (214) \\
\mathbf{F} = Q_g \mathbf{g} + 4Q_g \nabla \times \mathbf{B}_g. \quad (215)
\]

This set of linearized Einstein equations has the form same to that of Gravitodyanamics. All of conclusions obtained in this article are derived from Gravitodyanamics. Eqs. (211-215), therefore, will give the same results, except a factor of 4 in the expression of gravitational force. This factor will distinguish these two theories experimentally.

(8) The gravitational dipole momentum indeed radiates GW. The total radiation energy of GW is the summation of the dipole and quadrupole radiations. For a binary system, when the distance between two stars is large, the dipole radiation is dominant. Due to the radiation damping, the orbit is shrinking. Shrinking to a certain size, the quadrupole radiation becomes dominant.

(9) The gravitational Doppler effect, the correlation between the redshifts of both GW and EMW, and the correlation between Hubble constant and redshifts of GW are derived, which need to be considered in the detection of GW, especially for a faraway source.

(10) A formula describing GW quadrupole radiation of relativistic binary system is derived.

(11) There is gravitational synchrotron radiation from a relativistic star moving in the gravitomagnetic field of a rotating massive object, such as black hole.

(12) There is gravitational Bremsstrahlung from a head-on collision of two relativistic stars.

(13) The gravitational charge and inertial rest mass are different entities.

Reference


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