A kind of proof about triangles's congruent and a new kind of elimination method

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Abstract: *In this paper, there is a new way to proof the triangles's congruent.And at first we will get a system of equations.Then we will proof the system of equations is only one solution in the range of positive numbers.and This is the key of the paper.we only can proof the situation of the system of linear equations ago.But In this paper I will proof the situation of a kind of system of equations,of course It is not the situation of the system of linear equations.after then we will introduce a new elimination method.*

Keywords: *The system of equations, only one solution,elimination method*

1.Introduction

We know that the Gaussian invented the concept of the triangles's congruent.
When their trilateral are the corresponding same,This two triangles are congruent and so on**[1]**.But we learned that some propositions is difficult to prove.For example when the three bisectors of two triangles are the corresponding same,This two triangles are congruent.In this paper we use a new way to prove the two triangle's congruent.
And through the proof we introduce a new kind of elimination method.

2.Preparatory knowledge

Lemma(1): When the unknown numbers a,b,c meet the following formulas:

$$\frac{b}{a+c},\frac{a}{b+c},\frac{c}{a+b} \qquad (1)$$

And the unknown numbers a,b,c don't double increase.The formulas(1)must have a bigger and a smaller.

The first preparatory knowledge

At first we know the unknown numbers a,b,c have the following six kinds of change.

(1):there will be one becoming bigger and two are unchanged.
(2):there will be one becoming bigger and one becoming smaller and one is unchanged.
(3):there will be one becoming bigger and two becoming smaller.
(4):there will be two becoming bigger and one becoming smaller.

(5):there will be two becoming bigger and one is unchanged.
(6):there will be three becoming bigger(Double increase and Don't double increase).
Because becoming smaller is the same,we will not discuss.
The second we know the formulas(1)have the following two kinds of changes.
(1):there will be one unchanged and two are changed.
(2):there will be three changed.

Proof: The next we will discuss the two kinds of change of the formulas(1).At first we will discuss the first situation.It is one unchanged and two changed.Discuss the following:

We will assume the formula $\frac{b}{a+c}$ is unchanged,the unknown numbers $b,(a+c)$ is bigger at the same k times.If the unknown number a is bigger less than k times,the unknown number c is bigger more than k times.we will know the formula $\frac{a}{b+c}$ is smaller,the formula $\frac{c}{a+b}$ is bigger.So the first situation meet the lemma(1).

The second we will discuss the second situation.It is three change,We will begin from the unknown numbers a,b,c of the following six kinds of change.

1.make the unknown number a become bigger,the unknown numbers b,c is unchanged.We will know the formula $\frac{a}{b+c}$ will become bigger,
the formulas $\frac{b}{a+c},\frac{c}{a+b}$ will become smaller.

2.make the unknown number a become bigger,the unknown number b become smaller,the unknown number c is unchanged.We will know the formula $\frac{a}{b+c}$ will become bigger,the formula $\frac{b}{a+c}$ will become smaller,the formula $\frac{c}{a+b}$ don't know.

3.Make the unknown number a become bigger,the unknown numbers b,c become smaller.the formula $\frac{a}{b+c}$ will become bigger,the formulas $\frac{b}{a+c},\frac{c}{a+b}$ must have a smaller.We knowed the unknown numbers a,b,c have changed.So we will have the following formulas:

$$a_1=(1+z)a,b_1=(1-x)b,c_1=(1-y)c$$

After that we will have the following formulas:

$$\frac{b_1}{a_1+c_1}=\frac{(1-x)b}{[(1+z)a+(1-y)c]}=\frac{(1-x)b}{[(1-x)b+(1-x)c+z\cdot a+x\cdot a+x\cdot c-y\cdot c]}$$

$$\frac{c_1}{a_1+b_1}=\frac{(1-y)c}{[(1+z)a+(1-x)b]}=\frac{(1-y)c}{[(1-y)a+(1-y)b+z\cdot a+x\cdot a+y\cdot b-x\cdot b]}$$

According to the formulas of the $(z\cdot a+x\cdot a+x\cdot c-y\cdot c),(z\cdot a+y\cdot a+y\cdot b-x\cdot b)$

We know the formulas $\frac{b}{a+c},\frac{c}{a+b}$ must have a smaller.

4.make the unknown numbers a,b become bigger,and make the unknown number c become smaller.We know the formula $\frac{c}{a+b}$ will become smaller,the formulas $\frac{a}{b+c},\frac{b}{a+c}$ must have a bigger.In the same way we can get the following formulas:

$$a_1=(1+z)a,b_1=(1+x)b,c_1=(1-y)c$$

So we will have the following formulas:

$$\frac{a_1}{b_1+c_1}=\frac{(1+z)a}{[(1+x)b+(1-y)c]}=\frac{(1+z)a}{[(1+z)b+(1+z)c+x\cdot b-y\cdot c-z\cdot b-z\cdot c]}$$

$$\frac{b_1}{a_1+c_1}=\frac{(1+x)b}{[(1+z)a+(1-y)c]}=\frac{(1+x)b}{[(1+x)a+(1+x)c+z\cdot a-y\cdot c-x\cdot a-x\cdot c]}$$

According to the formulas of the $(x\cdot b-y\cdot c-z\cdot b-z\cdot c),(z\cdot a-y\cdot c-x\cdot a-x\cdot c)$

We know the formulas $\frac{a}{b+c},\frac{b}{a+c}$ must have a bigger.

5.make the unknown numbers a,b become bigger and make the unknown number c unchanged.We know the formula $\frac{c}{a+b}$ will become smaller,the formulas $\frac{a}{b+c},\frac{b}{a+c}$ must have a bigger.In the same way we can get the following formulas:

$$a_1=(1+z)a,b_1=(1+x)b,c_1=c$$

So we will have the following formulas:

$$\frac{a_1}{b_1+c_1}=\frac{(1+z)a}{[(1+x)b+c]},\frac{b_1}{a_1+c_1}=\frac{(1+x)b}{[(1+z)a+c]}$$

So we know the formulas $\frac{a}{b+c},\frac{b}{a+c}$ must have a bigger.

6.All the unknown numbers a,b,c become bigger.It is the same as the above five kinds of situations.So we will don't discuss.**Summary:**So we have proved the lemma(1).

Lemma(2): In the triangle ABC If the trilateral of the triangle are a,b,c ,It is said:

$BC=a, AB=c, AC=b$ .Angle bisector is AD, BE, CF .We can get the following formulas:

$$AD^2=bc\left[1-\left(\frac{a}{b+c}\right)^2\right]$$

$$BE^2=ac\left[1-\left(\frac{b}{a+c}\right)^2\right]$$

$$CF^2=ab\left[1-\left(\frac{c}{a+b}\right)^2\right]$$

Proof: At first we make the Angle $BAC=\alpha$,We know the Angle $BAD=\frac{\alpha}{2}$ .Make the area of the triangle is S ,So we can get the following formula:

$$\frac{2S}{AD(b+c)}=\sin(\frac{\alpha}{2}) \quad (2)$$

And there was a half Angle formula,We can get the following formula:

$$\sin(\frac{\alpha}{2})=\sqrt{\frac{1-\cos\alpha}{2}} \quad (3)$$

And there was a cosine formula:

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc} \quad (4)$$

And there was a formula(5):

$$S=\sqrt{(P-a)(P-b)(P-c)P}, P=\frac{a+b+c}{2}$$

By the formulas(2)(3)(4)(5),we can get the following formula:

$$AD^2=bc\left[1-\left(\frac{a}{b+c}\right)^2\right] \quad (6)$$

In the same way we can get the following formulas:

$$BE^2=ac\left[1-\left(\frac{b}{a+c}\right)^2\right] \quad (7)$$

$$CF^2=ab\left[1-\left(\frac{c}{a+b}\right)^2\right] \quad (8)$$

Summary: So we have proved the lemma(2).

Lemma(3): In the triangle ABC If the trilateral of triangle are a,b,c .It is said:

$BC=a, AB=c, AC=b$ .Angle bisectors is AD,BE,CF .and the angle bisectors due to the point O .Then we can get the following formula:

$$\frac{AO}{OD}=\frac{b+c}{a}$$

Proof: Assuming that the radius of the inscribed circle of the triangle is r ,Its area is S .We can get the following formula:

$$S=\frac{1}{2}(a+b+c)r \tag{9}$$

Through the point O do the vertical OQ of the BC edge,It due BC to Q .We can get the following formula:

$$OQ=r \tag{10}$$

Through the point A do the BC side's the vertical AF ,It due BC to F .We can get the following formula:

$$AF=\frac{2S}{a} \tag{11}$$

And because the triangle ODQ is similar to the triangle ADF .So we can get the following formula:

$$\frac{OD}{AD}=\frac{OQ}{AF}=\frac{r}{\frac{(a+b+c)r}{a}}=\frac{a}{a+b+c}$$

So we as soon as get the following formula:

$$\frac{AO}{OD}=\frac{b+c}{a} \tag{12}$$

Lemma(4): In the triangle ABC If the trilateral of triangle are a,b,c .It is said:

$BC=a, AB=c, AC=b$ .The angle bisectors is AD,BE,CF .

and the angle bisectors due to the point O .Then we can get the following formula:

$$OD^2=\frac{a^2bc(b+c-a)}{(b+c)^2(a+b+c)} \tag{13}$$

Proof: By the formulas(6)(12)We as soon as can get the above formula.In the same way we can get the following formulas:

$$OE^2=\frac{b^2ac(a+c-b)}{(a+c)^2(a+b+c)} \tag{14}$$

$$OF^2=\frac{abc^2(a+b-c)}{(a+b)^2(a+b+c)} \tag{15}$$

Lemma(5): By the lemma(2)and lemma(3) we can get the following formulas:

$$OA^2=\frac{bc(b+c-a)}{(a+b+c)} \tag{16}$$

$$OB^2=\frac{ac(a+c-b)}{(a+b+c)} \tag{17}$$

$$OC^2=\frac{ab(a+b-c)}{(a+b+c)} \tag{18}$$

3. Definition

The three unknown numbers a,b,c we have the following definition:

If make $x=a:b:c$,and make the unknown number a unchanged.when the unknown numbers b,c are unchanged,there is the unknown number x unchanged.If the unknown numbers a,b,c were to double increase at the same times,then we think the unknown number x is also unchanged.

4.An Example

For example for the unknown numbers OD,OE,OF It meet the following formulas:

$$OD=\frac{b}{a+c},OE=\frac{a}{b+c},OF=\frac{c}{a+b} \tag{19}$$

And we can get the following formula by the definition:

$$x=OD:OE:OF \tag{20}$$

When the numbers a,b,c is certain,the number x whether is a certain,then we can get the following formula by the definition:

$$OD:OE:OF=\frac{b}{a+c}:\frac{a}{b+c}:\frac{c}{a+b} \tag{21}$$

By the lemma(1)and the formula(21)we know:When the unknown numbers a,b,c is certain,the unknown number x is a certain.

5.The proposition to prove

[1]:when the three bisectors of two triangles are the corresponding same,This two triangles are congruent.

[2]: In the triangle ABC If the trilateral of triangle are a,b,c ,It is said:

$BC=a, AB=c, AC=b$ .Angle bisectors is AD,BE,CF .and the angle bisector due to the point O .When the the line segment OD, OE, OF of two triangles are the corresponding same,This two triangles are congruent.

[3]: When the line segment OA, OB, OC of the two triangles are the corresponding same,This two triangles are congruent.

6.The Improved Mothed

At first we will prove the proposition[1]

It is said:when the three bisectors of two triangles are the corresponding same,This two triangles are congruent.

Proof: We can get the following formula by the definition and the formulas(6)(7)(8).

$$AD^2 : BE^2 : CF^2 = \frac{bc(b+c-a)}{(b+c)^2} : \frac{ac(a+c-b)}{(a+c)^2} : \frac{ab(a+b-c)}{(a+b)^2} \quad (22)$$

We assume that the unknown numbers a, b, c meet the following formula:

$$a \le b \le c$$

And no matter how to change,there is always $a < b < c$ .So we know the unknown numbers a, b, c meet the following formula:

$$\frac{bc(b+c-a)}{(b+c)^2} \ge \frac{ac(a+c-b)}{(a+c)^2} \ge \frac{ab(a+b-c)}{(a+b)^2} \quad (23)$$

$$\frac{a}{b+c} > \frac{1}{2}, \frac{c}{a+b} < \frac{1}{2}, \frac{b}{a+c} < \frac{1}{2} \quad (24)$$

Discuss the unknown numbers a, b, c of the six kinds of change.

1.make the unknown number a become bigger,the unknown numbers b, c is unchanged.

We know the formula $\frac{bc(b+c-a)}{(b+c)^2}$ will become smaller,

The formulas $\frac{ac(a+c-b)}{(a+c)^2}, \frac{ab(a+b-c)}{(a+b)^2}$ will become bigger.

And make $x = AD^2 : BE^2 : CF^2$,we know :when the unknown numbers a, b, c is the only one,the x is the only one.

2.make the unknown number a become bigger,the unknown number b become smaller,the unknown number c is unchanged.

We know the formula $\frac{bc(b+c-a)}{(b+c)^2}$ will become smaller, the formula $\frac{ac(a+c-b)}{(a+c)^2}$ will become bigger,

the formula $\frac{ab(a+b-c)}{(a+b)^2}$ is unknown.

And make $x = AD^2 : BE^2 : CF^2$,we know :when the unknown numbers a,b,c is the only one,the x is the only one.

3.Make the unknown number a become bigger,the unknown number b,c become smaller.

We know the formula $\frac{bc(b+c-a)}{(b+c)^2}$ will become smaller.

and because the formulas $\frac{c}{a+b},\frac{b}{a+c}$ must have a bigger and meet the formula(24).

So the formulas $\frac{ac(a+c-b)}{(a+c)^2},\frac{ab(a+b-c)}{(a+b)^2}$ must have a bigger.

And make $x = AD^2 : BE^2 : CF^2$,we know :when the unknown numbers a,b,c is the only one,the x is the only one.

4.make the unknown numbers a,b become bigger,and make the unknown number c become smaller.

We know the formula $\frac{ab(a+b-c)}{(a+b)^2}$ will become bigger,

The formulas $\frac{bc(b+c-a)}{(b+c)^2},\frac{ac(a+c-b)}{(a+c)^2}$ is not unknown.

And make $x = AD^2 : BE^2 : CF^2$,we don’t know if the x is changed.So we can’t prove by using the above methods.

But we know that the formula $ab\left[1-\left(\frac{c}{a+b}\right)^2\right]$ will become bigger.

It is said the unknown number CF will become bigger.So we say this situation is right.

5.make the unknown numbers a,b become bigger,and make the unknown number c unchanged.It is same as above(4),So we will don’t discuss.

6.All the unknown numbers a,b,c become bigger.It is the same as the above five kinds of situations.We will also don’t discuss.

So we can proof the system of equations(6)(7)(8)have only one solution in the range of positive numbers.So the proposition[1] is right.

The second we will prove the[2]

Lemma(6): In the same triangle ABC Angle bisectors is AD, BE, CF .

and the angle bisectors due to the point O .When $OD = OE$,there is $a = b$ or the angle $C = 60$ degrees.

Proof: We can get the following formulas by the formulas(13)(14)

$$\frac{a}{b+c}\left(1-\frac{a}{b+c}\right)=\frac{b}{a+c}\left(1-\frac{b}{a+c}\right)=k \qquad (25)$$

So we can get the following formula:

$$a = b$$

Or

$$\frac{a+b}{c}=\frac{2k+1}{1-k} \qquad (26)$$

How to get the formula(26).The following:

We can get the following formula by the formula(25)

$$\frac{ab+ac-a^2}{(b+c)^2}=\frac{ab+bc-b^2}{(a+c)^2}=k$$

Then we can get the following formula:

$$\frac{(ab+ac-a^2)-(ab+bc-b^2)}{(b+c)^2-(a+c)^2}=k$$

Then we as long as get the following formula:

$$\frac{a+b}{c}=\frac{2k+1}{1-k}$$

We will k plug in the formula(26),we as soon as can get the following formula:

$$c^3+(a+b)c^2-(a^2+b^2-ab)c-(a+b)(a^2+b^2-ab)=0 \qquad (27)$$

Solving the formula(27),we can get that the angle $C = 60$ degrees.So we proved the lemma(6).

The next we can get the following formula by the definition and the formulas(13)(14)(15):

$$OD^2 : OE^2 : OF^2$$

$$=\left[\frac{a}{b+c}\left(1-\frac{a}{b+c}\right)\right]:\left[\frac{b}{a+c}\left(1-\frac{b}{a+c}\right)\right]:\left[\frac{c}{a+b}\left(1-\frac{c}{a+b}\right)\right] \qquad (28)$$

Now we will discuss the formula(28).Discuss the following:

According to the unknown numbers OD, OE, OF and the lemma(6)we will sort.

There is two big situation.The following:

$$\textbf{(1)}\, OD \neq OE \neq OF \;\; \textbf{(2)}\, OD = OE \neq OF$$

Here we begin to prove the proposition[2]

First of all we will assume that the value of the formula $\frac{a}{b+c}$ is unchanged,so we can know the formula $\left[\frac{a}{b+c}\left(1-\frac{a}{b+c}\right)\right]$ is unchanged.Then go to change the unknown numbers a,b,c .And see whether the formula($OD:OE:OF$)is only one.Why do this?That is become there is a theorem:

Theorem(1): In a triangle,the trilateral increase k times at the same time,the numbers OD,OE,OF will also increase k times at the same time,the formula($OD:OE:OF$)is unchanged.

We have assumed the formula $\left[\frac{a}{b+c}\left(1-\frac{a}{b+c}\right)\right]$ is unchanged.So we only discuss the formulas $\left[\frac{b}{a+c}\left(1-\frac{b}{a+c}\right)\right],\left[\frac{c}{a+b}\left(1-\frac{c}{a+b}\right)\right]$ is how to change.

And we have such rules:when $a \neq b \neq c$ and any angle is not equal to 60 degrees,we make $a < b < c$ .And no matter how to change,there is always $a < b < c$,So we know the unknown numbers a,b,c meet the following formula:

$$\frac{a}{b+c} < \frac{b}{a+c} < \frac{c}{a+b} \qquad (28)$$

At first we will discuss the first big case:

It is $OD \neq OE \neq OF$ .When the value of the formula $\frac{a}{b+c}$ is unchanged.the unknown numbers a,b,c can discuss only two situation.

(1)the unknown number a become larger and the formula $(b+c)$ also become larger(Becoming smaller is same).

(2)the unknown number a is unchanged and the formula $(b+c)$ is also unchanged,but the unknown numbers b,c one become larger and one become smaller.

and because there is the lemma(1).So we will only discuss the second situation.Discuss the following:

Make the unknown number a unchanged and make the formula $(b+c)$ also unchanged.when the unknown number b become bigger,the unknown number c become smaller.We know the formula $\frac{b}{a+c}$ will become bigger,the formula $\frac{c}{a+b}$ will become smaller.The formula($OD:OE:OF$)is only one,So the solution is only one.

The second we will discuss the second big case:

It is $OD = OE \neq OF$ .And because the lemma(6)there will be two situation again.

(1)The angle C is equal to 60 degrees(2) $a = b \neq c$

Now we will discuss(1).The same to make a become bigger,make $(b+c)$ also become bigger.The unknown number a is to increase l times,the formula $(b+c)$ is also to increase l time.We know the unknown numbers b,c meet that one increase more than l times and one increase less than l times.So the formulas $\frac{b}{a+c},\frac{c}{a+b}$ will be that one become larger and one become smaller.The formula($OD:OE:OF$)is only one,So the solution is only.

The next we will discuss(2).At this time $OD:OE = 1:1$ .If we increase the numbers a,b at this time,And make the formula $\frac{a}{b+c}$ unchanged.But the formulas $\frac{b}{a+c},\frac{c}{a+b}$ have changed.So the formula of the($OD:OE:OF$)is only one, the solution is only one.

The finally we will discuss the two situation together:

We know when $OD = OE$,there is $a = b$ or angle $C = 60$ degrees.We can get the following formula by the formula(25).

$$\frac{a+b}{c} = \frac{2k+1}{1-k}$$

When $a = b$,we can get the following formula:

$$\frac{bc}{(b+c)^2} = k \qquad (29)$$

We can get the following formula by the formula(29):

$$b = (1-2k+\sqrt{1-4k})\frac{c}{2k} \text{ or } b = (1-2k-\sqrt{1-4k})\frac{c}{2k}$$

So

$$\frac{c}{a+b} = \frac{c}{2b} = \frac{k}{1-2k-\sqrt{1-4k}} \text{ or } \frac{k}{1-2k+\sqrt{1-4k}}$$

So

$$\frac{c}{a+b}\left(1-\frac{c}{a+b}\right) = \frac{k}{1-2k-\sqrt{1-4k}}\left(1-\frac{k}{1-2k-\sqrt{1-4k}}\right)$$

Or

$$=\frac{k}{1-2k+\sqrt{1-4k}}\left(1-\frac{k}{1-2k+\sqrt{1-4k}}\right)$$

And when the angle $C=60$ degrees,We can get the following formula by the formula(26）

$$\frac{c}{a+b}\left(1-\frac{c}{a+b}\right)=\frac{1-k}{2k-1}\left(1-\frac{1-k}{2k-1}\right)$$

We know this two situation are equal,So we can get an equation about k .

$$16k^2+31k+10=0$$

Solution k is less than zero,So there is no such k value.

So we can proof the system of equations(13)(14)(15)have only one solution in the range of positive numbers.So the proposition[2] is right.

The third we will prove the[3]

It is said:When the line segment OA,OB,OC of the two triangles are the corresponding same,This two triangles are congruent.We can get the following formula by the definition and the formulas(16)(17)(18).

$$OA^2:OB^2:OC^2=[bc(b+c-a)]:[ac(a+c-b)]:[ab(a+b-c)] \quad (30)$$

We will discuss the six kinds of changes of the unknown numbers a,b,c in the same way.

1.make the unknown number a become bigger,the unknown numbers b,c is unchanged.

We know the formula $[bc(b+c-a)]$ will become smaller,

The formulas $[ac(a+c-b)]$,$[ab(a+b-c)]$ will become bigger.

And make $x=OA^2:OB^2:OC^2$,we know :when the unknown numbers a,b,c is the only one,the x is the only one.

2.make the unknown number a become bigger,the unknown number b become smaller,the number c is unchanged.

We know the number $[bc(b+c-a)]$ will become smaller,

The number $[ac(a+c-b)]$ will become bigger.

And make $x=OA^2:OB^2:OC^2$,we know :when the unknown numbers a,b,c is the only one,the x is the only one.

3.Make the unknown number a become bigger,the unknown number b,c become smaller.

We know the formula $\left[bc(b+c-a)\right]$ will become smaller,

The formulas $\left[ac(a+c-b)\right]$, $\left[ab(a+b-c)\right]$ is unknown.

And make $x = OA^2 : OB^2 : OC^2$ .when the unknown numbers a,b,c is the only one,we don't know if the x is only one.We can't prove by using the above methods.

But at this time we know the formula $\frac{bc(b+c-a)}{(a+b+c)}$ will become smaller.

It is said the unknown number OA will become smaller.So we say the situation is right.

4.make the unknown numbers a,b become bigger,and make the unknown number c become smaller.

We know the formula $\frac{ab(a+b-c)}{(a+b+c)}$ will become bigger.

It is said the unknown number OC will become bigger.So we say the situation is also right.

5.make the unknown numbers a,b become bigger,and make the unknown number c unchanged.It is same as above(4),So we will don't discuss.

6.All the unknown numbers a,b,c become bigger.It is the same as the above five kinds of situations.We will also don't discuss.

So we can proof the system of equations(16)(17)(18)have only one solution in the range of positive numbers.So the proposition[3] is right.

7.The main results

[1]:when the three bisectors of the two triangles are the corresponding same,This two triangles are congruent.

[2]: In the triangle ABC If the trilateral of triangle are a,b,c ,It is said:

$BC = a, AB = c, AC = b$ .Angle bisectors is AD, BE, CF .and the angle bisectors due to the point O .When the line segment OD, OE, OF of the two triangles are the corresponding same,This two triangles are congruent.

[3]: When the line segment OA, OB, OC of the two triangles are the corresponding same,This two triangles are congruent.

8.The extension

1.The main ways

We know solving the system of equations is difficult sometimes.There is a question:whether we can judge the numbers of the system of equations's solution.The following we will introduce a way about judging the numbers of the system of equations's solution.

We know the system of equations is usually to have n unknowns numbers and have n formulas.For example the above the system of equations(6)(7)(8)or(13)(14)(15) 0r(16)(17)(18)they have 3 unknowns numbers.For judging the numbers of the set of real Numbers we will sort for a,b,c .The following:

(1) $a>0,b>0,c>0$ (2) $a>0,b>0,c<0$

(3) $a>0,b<0,c>0$ (4) $a>0,b<0,c<0$

(5) $a<0,b>0,c>0$ (6) $a<0,b>0,c<0$

(7) $a<0,b<0,c>0$ (8) $a<0,b<0,c<0$

For example we only discussed(1)for the system of equations(6)(7)(8)or(13)(14)(15) 0r(16)(17)(18).We must discuss the situation of(2)(3)(4)(5)(6)(7)(8)for judging the numbers of the set of real Numbers.We will only discuss(8)for the above classifications and others is same.

Because $a<0,b<0,c<0$,So we make $a=-x,b=-y,c=-z$ .Then plug in the system of equations(6)(7)(8)or(13)(14)(15)0r(16)(17)(18).We get a new system of equations.Then we discuss to use the above methods.In the same way we go to discuss the (2)(3)(4)(5)(6)(7).So that we can judge the numbers of the set of real Numbers.

2.Summary

Because the system of equations is usually to have n unknown numbers and have n formulas.So there are 2^n kinds of classifications.For judging the numbers of the set of real Numbers we need discuss 2^n kinds of situations.

3.Discussion

For a system of equations whether we have a kind of elimination method to judge the numbers of the set of real Numbers.The following we will introduce a new kind of elimination method.At first We are going to start from the simplest.For example The following equations:

$$2x+3y=4$$
$$4x+y=5$$

We introduce an unknown quantity l ,and make $x=ly$,and then plug in the above equations,We can get the following formula:

$$(2l+3)y=4$$
$$(4l+1)y=5$$

The two formulas divided each other,we can get the following formula:

$$\frac{2l+3}{4l+1}=\frac{4}{5}$$

Then we can get: $l=\frac{11}{6}$

$$x=\frac{11}{6}y$$

Plug in the original equations,We can get:

$$y=\frac{3}{5}$$
$$x=\frac{11}{10}$$

For example(2):

$$x^3+y^3+xy^2+y+x=2$$

$$2x^2+3y^2+x+y=1$$

In the same way we introduce an unknown quantity l ,and make $x=ly$,and then plug in the above equations,We can get the following formula:

$$l^3y^3+y^3+ly^3+y+ly=2 \qquad (31)$$

$$2l^2y^2+3y^2+ly+y=1 \qquad (32)$$

The two formulas(31)(32) divided each other,we can get the following formula:

$$\frac{(l^3+1+l)y^2+1+l}{(2l^2+3)y+1+l}=2 \qquad (33)$$

We can get the following formula by the formula(33).

$$(l^3+1+l)y^2-2(2l^2+3)y=1+l \qquad (34)$$

The two formulas(32)(34) divided each other,we can get the following formula:

$$\frac{(l^3+1+l)y-2(2l^2+3)}{(2l^2+3)y+1+l}=1+l \qquad (35)$$

We can get a formula about y by the formula(35).Then we will y plug in the formula(31)or the formula(32).We will get a formula about l .

For example(3):

$$x^2+y^2+x+3y+z=1 \qquad (36)$$

$$3x^3 + y^2 + z^2 + x + y + z = 3 \qquad (37)$$

$$2x^2 + y^3 + z + 3y = 4 \qquad (38)$$

At this time we will introduce two unknown quantities l, k ,and make $x = ly, z = ky$. and then plug in the above equations,We can get the following formula:

$$l^2 y^2 + y^2 + ly + 3y + ky = 1 \qquad (39)$$

$$3l^3 y^3 + y^2 + k^2 y^2 + ly + y + ky = 3 \qquad (40)$$

$$2l^2 y^2 + y^3 + ky + 3y = 4 \qquad (41)$$

We can get a formula about y by the formula(39)(40),then we will y plug in the formula(39)or the formula(40). we can get a formula(42) about l, k . In the same way we can also get a formula about l, k by the formula(40)(41).After that we will get a system of equations about l, k .Then we will get a new formula. Its solution reflects the original equations solution to some degree.

For the system of equations of n unknowns numbers,We need introduce $n-1$ unknowns quantity,then get a new system of equations.After that we need introduce $n-2$ unknowns quantity.Until it is an unknown quantity.The last We get a equation containing only one unknown.Its solution reflects the original equations solution's situations to some degree.

9.Acknowledgements

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A kind of way that get the Evans triangles

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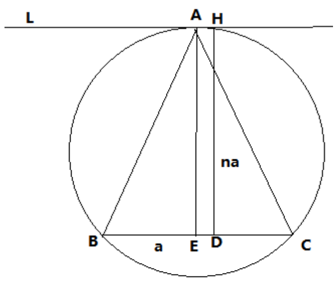
Abstract: In this paper,We will get a diophantine equation.we will solve the diophantine equation to get the Evans triangle.

Keywords: Evans triangle,the diophantine equation,Integer,rational

1.Introduction

In the American Mathematical Monthly In 1977 R.g Evans asked such a question:
Found all the integer sides triangles that made the ratio of its a high and the bottom edge was an integer.those triangle were only $n = 3$.At the same time Evans had given such a triangle,the length of the sides was4,13,15**[4]**.And It was popularized by Richard K.Guy in his famous book Unsolved Problems in Number Theory as $D19$ **[5]**.BianXin constructed a kind of these triangles and said those triangles had infinite in 2009**[3]**.
And It was to not have a way that get the Evans triangles ago.In this paper we will introduce a kind of way to get the Evans triangles.



2.The Method

As show in figure:In this circle Its radius is R .The center of the circle is O .There is a triangle ABC .It is inscribed in the circle O .We make the $BC = a$ .The high of the BC side is been as $n \cdot a$ .Through point O do straight line OD and make OD be perpendicular to the BC and due to BC in D .By the pythagorean theorem we can get the following formula:

$$OD = \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$$

Through the point A do straight line L and make the straight line L is parallel to BC .Next we will extend the straight line DO and make it due to L in H .We know the OH is perpendicular to the BC .Through the point A do straight line AE and make AE is perpendicular to BC and duo to BC in E .

So we know:

$$DH = n \cdot a \, , AE = n \cdot a \, .$$

Next we can get:

$$OH = n \cdot a - OD = n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$$

And because the straight line DH through the circle O ,we can get:

$$CE = \left| HA - \frac{a}{2} \right| = \left| \sqrt{R^2 - \left[n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2} \right]^2} - \frac{a}{2} \right|$$

Or:

$$CE = \left| HA + \frac{a}{2} \right| = \left| \sqrt{R^2 - \left[n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2} \right]^2} + \frac{a}{2} \right|$$

And by the pythagorean theorem we can get:

$$AC = \sqrt{CE^2 + AE^2}$$

$$= \sqrt{\left[\sqrt{R^2 - \left(n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2} \right)^2} - \left(\frac{a}{2}\right)^2 \right]^2 + (n \cdot a)^2} \qquad (1)$$

Or:

$$AC = \sqrt{CE^2 + AE^2}$$

$$= \sqrt{\left[R^2 - \left(n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2} \right)^2 + \left(\frac{a}{2}\right)^2 \right]^2 + (n \cdot a)^2} \qquad (2)$$

The same available:

$$BA = \sqrt{\left[\sqrt{R^2 - \left(n \cdot a - \sqrt{R^2 - \left(\frac{a}{2}\right)^2} \right)^2} + \left(\frac{a}{2}\right)^2 \right]^2 + (n \cdot a)^2} \qquad (3)$$

Or:

$$BA=\sqrt{\left[\sqrt{R^2-\left(n\cdot a-\sqrt{R^2-\left(\frac{a}{2}\right)^2}\right)^2}-\left(\frac{a}{2}\right)^2\right]^2+(n\cdot a)^2} \qquad (4)$$

Because this two situation is the same,we only discuss a situation.

At this time we only need to meet the trilateral is rational and the number n is an integer for the triangle.And because when the number n is a certain,the trilateral no mater how to change,the number n is unchanged.So we only need change the sides a, AC, BA and make the sides a, AC, BA be integers.

Lemma(1): If a, AC, BA are rational numbers,then $\frac{AC}{a}$, $\frac{BA}{a}$ also are the rational numbers. If $y=\frac{AC}{a}+\frac{BA}{a}$, y will also be a rational numbers.

By the formula(1)(3)or(2)(4)and lemma(1)we can get:

$$\frac{AC}{a}=\sqrt{\left[\left(\frac{R}{a}\right)^2-\left(n-\sqrt{\left(\frac{R}{a}\right)^2-\left(\frac{1}{2}\right)^2}\right)^2-\left(\frac{1}{2}\right)^2\right]^2+n^2}$$

$$\frac{BA}{a}=\sqrt{\left[\left(\frac{R}{a}\right)^2-\left(n-\sqrt{\left(\frac{R}{a}\right)^2-\left(\frac{1}{2}\right)^2}\right)^2+\left(\frac{1}{2}\right)^2\right]^2+n^2}$$

Make $\frac{R}{a}=x$, So

$$\frac{AC}{a}=\sqrt{\left[\sqrt{x^2-\left(n-\sqrt{x^2-\left(\frac{1}{2}\right)^2}\right)^2}-\left(\frac{1}{2}\right)^2\right]+n^2} \qquad (5)$$

$$\frac{BA}{a}=\sqrt{\left[\sqrt{x^2-\left(n-\sqrt{x^2-\left(\frac{1}{2}\right)^2}\right)^2}+\left(\frac{1}{2}\right)^2\right]^2+n^2} \qquad (6)$$

We know the numbers R, a become the number x .This is the key to solve the problem.And because the following formula:

$$y=\frac{AC}{a}+\frac{BA}{a}$$

So we will(5)(6)plug in this formula.then we will get a formula about x .this formula is as follows:

$$x=\frac{1}{2}\cdot\left[\frac{y^2-1}{4n}+\frac{n}{y^2-1}\right],x>\frac{n^2+\frac{1}{4}}{4n},y>\sqrt{4n^2+1}$$

Solution to this.We think this problem has been solved.So we make $n=4$ and give y a numerical.Get the value of x .After that get the value of the $\frac{AC}{a}$, $\frac{BA}{a}$ .Founding the numbers $\frac{AC}{a}$, $\frac{BA}{a}$ is not rational,So we think about plugging x in $\frac{AC}{a}$, $\frac{BA}{a}$ .After plugging we get:

$$\frac{AC}{a}=\frac{y}{2}-\frac{1}{2}\cdot\sqrt{1-\frac{4n^2}{y^2-1}},\frac{BA}{a}=\frac{y}{2}+\frac{1}{2}\cdot\sqrt{1-\frac{4n^2}{y^2-1}}$$

through the above formula,We know that what place is wrong.We learned that we must ensure $\sqrt{1-\frac{4n^2}{y^2-1}}$ is a rational.To make this formula is a rational,we make this formula is equal to the number k .We can get this formula**[2]**:

$$4n^2=(1-k^2)(y^2-1)$$

In this formula we only need ensure the numbers k,y are rational,the number n is a integer.We as soon as get the Evans triangle.Now Let me give a kind of solution's way.We know the numbers k,y are rational,So We make:

$$k=\frac{b_1}{a_1},y=\frac{b_2}{a_2}.$$

Of course the numbers of a_1,a_2,b_1,b_2 is integers and the numbers a_1,b_1 are coprime, the numbers a_2,b_2 are also coprime.So we can get the following formula:

$$4n^2=\left(\frac{a_1^{\ 2}-b_1^{\ 2}}{a_1^{\ 2}}\right)\cdot\left(\frac{b_2^{\ 2}-a_2^{\ 2}}{a_2^{\ 2}}\right)$$

Because the numbers a_1,b_1 are coprime,So the numbers $(a_1^{\ 2}-b_1^{\ 2}),a_1^{\ 2}$ are also coprime and the numbers $(b_2^{\ 2}-a_2^{\ 2}),a_2^{\ 2}$ are also coprime.With this condition we can get the following these formulas again.

$$a_1^{\ 2}-b_1^{\ 2}=k_1\cdot x_1^{\ 2}\cdot a_2^{\ 2} \qquad (7)$$

$$b_2^{\ 2}-a_2^{\ 2}=k_1\cdot x_2^{\ 2}\cdot a_1^{\ 2} \qquad (8)$$

$$2n = k_1 \cdot x_1 \cdot x_2 \qquad (9)$$

The key is how to solve these formulas.The following we will introduce a kind of way to solve these formulas.For these formulas we can give n a numerical.For example we make $n = 4$ .Then we can get:

$$k_1 \cdot x_1 \cdot x_2 = 8$$

We know the number 8 can be decomposed into the following situations:

$$8 = 2\times 2\times 2, 8 = 2\times 4\times 1, 8 = 8\times 1\times 1$$

So we can get the following formulas:

(1) $k_1 = 2, x_1 = 2, x_2 = 2$

(2) $k_1 = 2, x_1 = 4, x_2 = 1$. or $k_1 = 2, x_1 = 1, x_2 = 4$ .or $k_1 = 4, x_1 = 2, x_2 = 1$

or $k_1 = 4, x_1 = 1, x_2 = 4$ .or $k_1 = 1, x_1 = 2, x_2 = 4$ .or $k_1 = 1, x_1 = 4, x_2 = 2$.

(3) $k_1 = 8, x_1 = 1, x_2 = 1$ .or $k_1 = 1, x_1 = 8, x_2 = 1$ .or $k_1 = 1, x_1 = 1, x_2 = 8$.

For these situations we discuss only a situation.We will discuss the situation(1).
We know these numbers meet the following relationships by the situation(1)

$$k_1 = 2, x_1 = 2, x_2 = 2$$

So will these numbers plug in(7)(8).We can get the following formulas:

$$\begin{aligned} a_1^2 - b_1^2 &= 8a_2^2 \\ b_2^2 - a_2^2 &= 8a_1^2 \end{aligned} \qquad (10)$$

At this moment we need solve the formulas(10).And when the formulas(10) are to have solution,We will get such a triangle that the ratio of its a high and the bottom edge was only $n = 4$ .In order to get the solution of the formulas(10)we first solve one formulas.For example we solve the following formulas:

$$a_1^2 - b_1^2 = 8a_2^2$$

We can get the numerical of the unknown numbers a_1, b_1, a_2 .Then we will the numbers a_1, a_2 plug in the following formula:

$$b_2^2 - a_2^2 = 8a_1^2$$

When the number b_2 is integer,we as long as get such a triangle.At this time we get a Evans triangle that the ratio of its a high and the bottom edge was only $n = 4$ .Because those triangle had infinite,we can use this way to get Evans triangle.For finding all these triangles we need make $n = (5,6,7,...)$ and go to solve the formulas(7)(8)(9)by using the above way.

3.The Main Result

In this paper its result is not a conclusion,and Its result is a way that get the Evans striangle.we have given the ways how to solve the following formula.

$$4n^2 = (1-k^2)(y^2-1) \qquad (11)$$

Now we will have a summary.we know if we will get the Evans triangle,we must solve the formula(11).Its solution summarized below:
The first we should assume the number n is a known number.and give n a numerical. After that we can get the numbers of k_1, x_1, x_2 by the number n .later we can get a system of equations about(7)(8).As long as the system of equation about(7)(8)is to have solutions,we as long as get the Evans triangle.And how to solve the system of equations about(7)(8).we first solve one formulas,solve the formula(7)or solve the formula(8).After that we will this solutions plug in (8)or(7)and get the integer solutions of(a_1, a_2, b_1, b_2),we as long as get the Evans triangles.

Appendix A. instances and table

The following we will illustrate ten these triangles.

The unknown numbers a, b, c is the triangle's trilateral and the unknown number n is the ratio of a high and the bottom edge.

Table.A.

n	a	b	c
3	4	13	15
8	6	50	52
15	8	123	125
24	10	244	246
35	12	425	427
48	14	678	680
63	16	1015	1017
80	18	1448	1450
99	20	1989	1991
120	22	2650	2652

4.The questions

(1)We know it is difficult to get the solution by above ways.So we guess if there is a easy way to get the solution.And if we can get a general solution for the diophantine equation formula(11).

(2)We can get all the solutions by above ways.But there is a question for the following formulas:

$$a_1^{\ 2} - b_1^{\ 2} = k_1 \cdot x_1^{\ 2} \cdot a_2^{\ 2}$$

$$b_2^{\ 2} - a_2^{\ 2} = k_1 \cdot x_2^{\ 2} \cdot a_1^{\ 2}$$

When the numbers k_1, x_1, x_2 is what numbers,The system of question is no solution.

(3)If we can use the mathematical approximation to judge that the formula(11)have infinite solutions.

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Appendix Chinese Resume

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