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Gabriel's theorems on area ofa circle and regular polygons.

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Abstract

Area of a circle and regular polygon i.e. including inscribed and circumscribed regular polygons are interrelated to each other.The objective is to show how the area of a circle and regular polygons is interrelating. Depend on this; these entire theorems can give an infinite ways to calculate area of a circle and other new ways for regular polygons.The methodology used is the previous known formulas for a circle and regular polygons, to derive new relation formulas. The results obtained are in relationship of a circle with regular polygons of the same perimeter, its inscribed and circumscribed regular polygons.These theorems (results) have overcome some limitations of previous existed formulas; now no need to calculate radius for a circle if only circumference is given and no use of trigonometry (sine, tangent and cosine) for regular polygons in addition to these it also brings us to a simple new way for calculating area of a sector and segment which are present in inscribed regular polygons. Therefore here are new ways for calculating area of a circle, regular polygons, segment and sector.

Key words: Area, circle,Regular polygons, inscribed regular polygons, circumscribedregular polygons,

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Introduction:

For thousands of years, civilized people have used mathematics to investigate size, shape, and the relationships among physical objects. Ancient Egyptians used geometry to solve many practical problems involving boundaries and land areas. David albert (1862—1943) believed that mathematics should have a logical foundation based on two principles, 1. All mathematics follows from a correctly chosen finite set of assumptions or axioms. 2. This set of axioms is not contradictory. The work of Greek scholars such as Thales, Eratosthenes, Pythagoras, and Euclid for centuries provided the basis for the study of geometry in the western world.

Area is the extent part of a surface which is enclosed with in a boundary. Regular polygons are closed figure with congruent sides and congruent interior angles. Larson et al. (2003) explained about general polygons how to calculate the area of them too for the circles.

Circle is the set of points in a plane that are the same distance from a given point called a center of a circle. Lial et al. (2006) studied well about geometry of a circle and in this literature it shows the ratio of diameter to a radius is constant. We can call a circle as the last generation of regular polygons, because as the side of regular polygon increases to infinite then it looks like a circle. In this article it broadly shows that how a circle is related with each regular polygon (inscribed and circumscribed) and how this relation brings a common formula that uses to calculate area of a circle and regular polygons.

This article broadens the ideas present before to calculate area of a circle and regular polygons. These three theorems describe the relationship between a circle and regular polygons (including inscribed and circumscribed).Theorem is any mathematical statement which requires proof in order to be accepted. They are backbone of geometry. John Casson et al. (1996) about circle theorems there is widely explained in relation to interior angles and about chords. Therefore theorem and postulates are the machines through which we simplify life in geometry. Complete understanding of geometry is depending on theorems and postulates. These ideas serve in wide range in calculating both areas of a circle and regular polygons (inscribed and circumscribed) from single formula.

Each theorem in this article describes the relationship between a circle and regular polygons of the same perimeter as a circle and its (inscribed circumscribed) regular polygons. All these relation brings new formulas to calculate area of a circle, regular polygons including (inscribed and circumscribed) regular polygons, sector and segment.

Methodology

There are theorems used to describe the relationship between area of a circle and regular polygons. Then for each regular polygon in relation to a circle, the area difference among them is the product of perimeter square [P^2] for theorem 1 and [r^2] for theorem 2 and 3 and constant of each regular polygon [K] (K_s, K_{i1}, K_{i2} and K_c) these constants are listed in table 1, 2, 3, and 4 respectively below.. K value for each regular polygon is different.

Symbols used: $\theta_3 = \frac{360}{n}$, $\theta_2 = \frac{(n-2)90}{n}$, $\theta_1 = \frac{180}{n}$

Formulas used:

For regular polygon: $A = \frac{1}{2}\text{a.P}$

$$\text{a} = \frac{p.tan\theta_2}{2n} \quad or \quad \text{a} = \frac{\text{p}}{2\text{n}.\tan\theta_1}$$

$$A = \frac{p^2}{4n.tan\theta_1} \quad or \quad A = \frac{p^2.tan\theta_2}{4n.}$$

Inscribed regular polygons…

$$\text{A} = \frac{1}{2}\text{nr}^2\sin\frac{360}{n}, \quad A = \frac{P^2}{4ntan\theta_1}, \; A = \frac{P^2tan\theta_2}{4n}$$

Circumscribed regular polygons….

$$A = nr^2tan\theta_1 \quad \text{Or} \quad \text{A} = \frac{1}{2}nr^2\frac{sin\theta_3}{(sin\theta_2)^2}$$

For a circle $A = \pi r^2$

Theorem.1

[Relationship between area of a circle and regular polygon of the same perimeter]

If the perimeter of a regular polygon and the circumference of a circle is the same,then the difference between the area of a circle and a regular polygon is the product of the perimeter (circumference) square and K_s. i.e.[$k_s.P^2$].

Therefore:

Area of circle – Area of regular polygon=$K_s.P^2$

K_s can be calculated as:

1. $[\frac{1}{4\pi} - \frac{\tan\theta_2}{4n}]$ or where $\theta_2 = \frac{(n-2)90}{n}$

2. $[\frac{1}{4\pi} - \frac{1}{4n.\tan\theta_1}]$ where$\theta_1 = \frac{180}{n}$

N.B: The K_s for each regular polygon of n sided can be calculated with the above two ways 1&2. Fork_s look to the table.

Proof

Pre-request:

$A_{circle} = \pi r^2$

$A_{regular\ polygon} = \frac{1}{2} a.p$

$a = \frac{p}{2n.\tan\theta_1}$ or $a = \frac{p.\tan\theta_2}{2n}$

P=2πr

1.$A_{regular\ polygon} = \frac{1}{2} \times \frac{p}{2n.\tan\boldsymbol{\theta_1}} \times p$

$A = \frac{p^2}{4n.\tan\theta_1}$ or

2. $A_{regular\ polygon} = \frac{1}{2} \times \frac{p.\tan\theta_2}{2n} \times$

p$A = \frac{p^2.\tan\theta_2}{4n.}$

Let's now make our proof in two ways with the above given K_s

A. $A_{circle} = A_{regular\ polygon} + K_s.p^2$

$K_s = [\frac{1}{4\pi} - \frac{1}{4n\tan\theta_1}]$

$A_{circle} = \frac{p^2}{4n.\tan\theta_1} + [\frac{1}{4\pi} - \frac{1}{4n.\tan\theta 1}] \times P^2$

$A_{circle} = \frac{p^2}{4n.\tan\theta_1} + [\frac{P^2}{4\pi} - \frac{P^2}{4n.\tan\theta 1}]$

$p^2 = (2\pi r)^2 p^2 = 4\pi^2 r^2$………

$A_{circle} = \frac{4\pi^2 r^2}{4n.\tan\theta_1} + [\frac{4\pi^2 r^2}{4\pi} - \frac{4\pi^2 r^2}{4n.\tan\theta 1}]$

$A_{circle} = \frac{4\pi^2 r^2}{4\pi}$

$A_{circle} = \pi r^2$…true

Therefore the above statement is true.

B. $A_{circle} = A_{regular\ polygon} + K_s.p^2$

$K_s = [\frac{1}{4\pi} - \frac{\tan\theta_2}{4n}]$

$A_{circle} = \frac{p^2.\tan\theta_2}{4n.} + [\frac{1}{4\pi} - \frac{\tan\theta_2}{4n}] \times P^2$

$A_{circle} = \frac{p^2.\tan\theta_2}{4n.} + [\frac{P^2}{4\pi} - \frac{\tan\theta_2 P^2}{4n}]$

$p^2 = 4\pi^2 r^2$…………

$A_{circle} = \frac{p^2.\tan\theta_2}{4n.} + [\frac{4\pi^2 r^2}{4\pi} - \frac{\tan\theta_2 P^2}{4n}]$

$A_{circle} = \frac{4\pi^2 r^2}{4\pi}$

$A_{circle} = \pi r^2$… true

Application

Find the area of a circle whose circumference is 58cm in comparing with a regular polygon of 10 sided.(n=10) P=58cm

Solution: first calculate**K_s.**

$K_s = [\frac{1}{4\pi} - \frac{1}{4n.\tan\theta_1}] = 0.002635383116 \quad \theta_1 = \frac{180}{n}$

$K_s = [\frac{1}{4\pi} - \frac{\tan\theta_2}{4n}] = 0.002635383116 \quad \theta_2 = \frac{(n-2)90}{n} = 72$

Let's solve the problem in three ways……………

1. $A_{circle}=\pi r^2$

$R = \frac{58}{2\pi}$ r=9.230986699cm

$A= \pi \times 9.230986699^2$

$A=267.6986143cm^2$

2. $A_{circle}= \frac{p^2}{4n.\tan\theta_1} + K_s.P^2$

$A= \frac{[58cm]^2}{40.\tan 18} + [0.002635383116 \times (58cm)^2]$

$A=267.6986143cm^2$

3. $A_{circle}=\frac{p^2.\tan\theta_2}{4n.} + K_s.P^2$

$A_{circle}=\frac{[58cm]^2.\tan 72}{40} + [0.002635383116 \times (58cm)^2]$

$A_{circle}=267.6986143cm^2$

N.B one advantage of this theorem is you can calculate the area of regular polygons and of a circle from the same formula.

Theorem.2

[Relationship between area of a circle and its inscribedregular polygon]

If a regular polygon is inscribed inside a circle, then the area difference between the area of a circle and of a regular polygon is the product radius square and the constant. **i.e.[$K_{i1}\times r^2$] or [$K_{i2}\times r^2$] where**

r=radius k_{i1} and k_{i2}= are the constants,K_{i2} can be calculate as

K_{i2}= $[\pi-\frac{1}{2}\text{nsin}\theta_3]$ and K_{i1}= $[\pi-\frac{\pi^2}{ntan\theta_1}]$ or K_{i1}= $\left[\pi-\frac{\pi^2 tan\theta_2}{n}\right]$ ….

The K_{i1} that can be calculatingthrough these two ways is the same but $K_{i2}$is different fromK_{i1}.

The previously known formulas to find the area of inscribed regular polygon are

$A=\frac{1}{2}nr^2 sin\frac{360}{n}$ *or* $A=\frac{P^2}{4ntan\theta_1}$ *or* $A=\frac{P^2 tan\theta_1}{4n}$ and

for the area ofa circle $A=\pi r^2$ *P is the given perimeter or circumference of a circle.*

Therefore the three area relationship formulas are:

1. $A_{CIRCLE}=\frac{1}{2}nr^2 sin\theta_3+K_{i2}.r^2$ *i.e.* $A=\frac{1}{2}nr^2 sin\theta_3+[\pi-\frac{1}{2}nsin\theta_3].r^2$ *---only K_{i2}must be used.*
2. $A_{CIRCLE}=\frac{P^2 tan\theta_2}{4n}+K_{i1}.r^2$ *i.e.* $A=\frac{P^2 tan\theta_2}{n}+\left[\pi-\frac{\pi^2 tan\theta_2}{n}\right].r^2$
3. $A_{CIRCLE}=\frac{P^2}{4ntan\theta_1}+K_{i1}.r^2$ *i.e.* $A=\frac{P^2}{ntan\theta_1}+\left[\pi-\frac{\pi^2}{ntan\theta_1}\right].r^2$

N.B $\theta_3=\frac{360}{n}$, $\theta_2=\frac{(n-2)90}{n}$, $\theta_1=\frac{180}{n}$

Proof

For the three above mentioned formulas……………

1. Let $K_{i2}=\pi-\frac{1}{2}n\sin\theta_3$

$A_{circle}=\frac{1}{2}nr^2\sin\theta_3 + K_{i2}.r^2$

$A=\frac{1}{2}nr^2\sin\theta_3 + [\pi-\frac{1}{2}n\sin\theta_3].r^2$

$A= \frac{1}{2}nr^2\sin\frac{360}{n} + [\pi-\frac{1}{2}n\sin\frac{360}{n}].r^2$

$A=\frac{1}{2}nr^2\sin\frac{360}{n}+ \pi r^2-\frac{1}{2}nr^2\sin\frac{360}{n}$

$A_{CIRCLE}=\pi r^2$ ………..True

2. Let $K_{i1}=\pi-\frac{\pi^2\tan\theta_2}{n}$

$A_{CIRCLE}=\frac{P^2\tan\theta_2}{4n} + K_{i1}.r^2$

$A=\frac{P^2\tan\theta_2}{4n} + \left[\pi-\frac{\pi^2\tan\theta_2}{n}\right].r^2$

$A=\frac{P^2\tan\theta_2}{4n} + \left[\pi r^2-\frac{\pi^2r^2\tan\theta_2}{n}\right]$

$A=\frac{4\pi^2r^2\tan\theta_2}{4n} + \left[\pi r^2-\frac{\pi^2r^2\tan\theta_2}{n}\right]$

$A= \pi r^2$……..True

3. $K_{i1}=\pi-\frac{\pi^2}{n\tan\theta_1}$

$A_{CIRCLE}=\frac{P^2}{4n\tan\theta_1} + K_{i1}.r^2$

$A=\frac{P^2}{4n\tan\theta_1} + \left[\pi-\frac{\pi^2}{n\tan\theta_1}\right].r^2$

$A=\frac{P^2}{4n\tan\theta_1} + \left[\pi r^2-\frac{r^2\pi^2}{n\tan\theta_1}\right]$

$A= \pi r^2$……………True

N.B for all inscribed regular polygons the value of K_{i1} and K_{i2} is different. Refer to the table 2 and *table* 3 respectively.

Application

1. Find the area of a regular nonagon inscribed in a circle with 16m radius.

Solution: $K_{i2}=0.24904841$ r=16m $\theta_3=40$

$A=\pi r^2 - K_{i2} \times r^2$ $A=16^2\pi - K_{i2} \times 16^2$ $A=740.4913264m^2$	$A=\frac{1}{2}nr^2\sin\theta_3$ $A=\frac{1}{2}nr^2\sin 40$ $A=\frac{1}{2}\times 9\times 16^2\times\sin 40$ $A=740.4913264m^2$

2. Find the circumference of a circle which inscribes a regular Decagon 0f $879m^2$ area.

Given $K_{i2}= 0.202666392$ $A=879m^2$ n=10

Solution…

$A=\pi r^2 - K_{i2} \times r^2$ $A=\pi r^2 - 0.202666392 \times r^2$ $A=(\pi - 0.202666392)\times r^2$ $879m^2=(\pi - 0.202666392)\times r^2$ r=17.29418469m Circumference=2πr Circumference=108.6625672m	$A=\frac{1}{2}nr^2\sin\theta_3$ $A=5\times r^2\times\sin 36$ $879=5\times r^2\times\sin 36$ r=17.29418469m Circumference=2πr Circumference=108.6625672m

N.B: One of the advantages of this theorem is you can calculate area of a segment and sectoreasily. That is $A=\frac{K_{i2}r^2}{n}$ and $A=\frac{1}{2}r^2\sin\theta_3+\frac{K_{i2}r^2}{n}$ respectively.

Theorem.3

[Relationship between area of a circle and its circumscribed regular polygon]

If a regular is circumscribed about a circle then the area difference between the area of the regular polygon and the circle is [$K_c \times r^2$]. i.e. $_{circumscribed}$=A_{Circle}+$K_c.r^2$

r=radius and $K_c = ntan\theta_1 - \pi \quad or\ K_c = \frac{nsin\theta_3}{2(sin\theta_2)^2} - \pi$

Proof

Pre-requisite: $A_{circle} = \pi r^2$ and for circumscribed regular polygons

$$A = nr^2 tan\theta_1 \qquad or \qquad A = \frac{1}{2}nr^2\frac{sin\theta_3}{(sin\theta_2)^2}$$

1. $A_{circumscribed}=A_{Circle}+K_c.r^2$

$K_c = ntan\theta_1 - \pi$

$A_{circum} = \pi r^2 + [ntan\theta_1 - \pi] \times r^2$

$A_{circum} = \pi r^2 + nr^2 tan\theta_1 - \pi r^2$

$A_{circum} = nr^2 tan\theta_1$

2. $A_{circumscribed}=A_{Circle}+K_c.r^2$

$A_{circumscribed}=A_{Circle}+K_c.r^2$

$$K_c = \frac{nsin\theta_3}{2(sin\theta_2)^2} - \pi$$

$A_{circum} = \pi r^2 + [\frac{nsin\theta_3}{2(sin\theta_2)^2} - \pi] \times r^2$

$A_{circum} = \pi r^2 + \frac{1}{2}nr^2\frac{sin\theta_3}{(sin\theta_2)^2} - \pi r^2$

$$A = \frac{1}{2}nr^2\frac{sin\theta_3}{(sin\theta_2)^2}$$

Application

1. Find the area of a regular hexagon circumscribed about a circle with 18m radius. Where K_c= 0.322508961 r=18m

Solution : $A_{circumscribed}=A_{Circle}+K_c.r^2$ $A=\pi r^2+K_c.r^2$ $A=r^2[\pi+K_c]$ $A=18^2[\pi+K_c]$ $A=324[\pi+K_c]$ A= 1122.368923m^2	$A=nr^2\tan\theta_1$ $\theta_1=\frac{180}{n}$ n=6 θ_1=30 $A=6\times 18^2\times\tan 30$ $A=1944\times\tan 30$ A=1122.368923m^2

2. Find the area of a circle that is circumscribed by a square with area of 256m^2.

Solution: here side length of a square is 2r

Therefore r= 8m K_C=0.858407346

Let's solve it in two ways

$A_{circle}=A_{circumscribed.r.p.}-K_c\times r^2$ $A=256m^2-K_c\times 8^2$ A=201.0619298 m^2	$A=\pi r^2$ $A=\pi\times 8^2$ A=201.0619298 m^2	

RESULTS AND DISCUSSION

From the methodology we have seen how a circle relates with its inscribed and circumscribed regular polygons in theorem 2&3 respectively and theorem 1 with a regular polygon which has the same perimeter with a circle. It is proved that these theorems have the same in their application with the already preexisted formulas also with so many advantages over the previous ones.

What advantages do these formulas have over the previous ones? Even though it looks complicated they have advantages like…

1. You never need any trigonometry (sine, cosine and tangent) to calculate area of inscribed, circumscribed and normal regular polygons.
2. Area of a segment and sector can be calculated in easily way from theorem 2 as $A=\frac{K_{i2}\, r^2}{n}$ for segment and $A=\frac{1}{2}r^2\sin\theta_3+\frac{K_{i2} r^2}{n}$ for a sector.
3. Since each regular polygon from equilateral to n—side are has its own K_s, K_{i1}, K_{i2}, and K_c value it gives wide range possible ways to calculate area of a circle.
4. Another last but not least is we can calculate area of a circle and regular polygons including inscribed and circumscribed from the same formula.

In theorem two there are two constants K_{i2} and K_{i1},so we need to take care in which each constant belongs in the methodology part used. K_{i1} is always used when we take a circumference of a circle as the perimeter of the inscribed regular polygon. Each regular polygon has its own constants for each theorem. In the past to calculate area of a segment we were using long method i.e. first we were calculating area of a sector then of a triangle finally we subtract area of a triangle from a sector. Similar for a sector $A = \frac{\theta\pi r^2}{360}$,but now there is a simple method to calculate area of a sector and a new way for a segment. Since all these statements have the same application as the previous ones and for their advantage over that hopefully high school students will use these methods.

Conclusion

Therefore these theorems are applicable for calculating area of a circle, normal regular polygons, (inscribed, circumscribed) regular polygon, sector and segments of inscribed regular polygon. To calculate area of a circle there are infinite possible ways as the number of regular polygons. Every regular polygon has its own K_s, K_{i1}, K_{i2}, and $K_c$value.

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Table.1 K_s value for each regular polygon.

Regular polygon of n-sides	K_s-value
Triangle	*0.031464948*
Square	*0.017077471*
Pentagon	*0.010758375*
Hexagon	*0.007408688*
Heptagon	*0.005415993*
Octagon	*0.004133298*
Nonagon	*0.003259361*
Decagon	*0.002635383*
11-gon	*0.002175489*
12-gon	*0.001826413*
13-gon	*0.00155174*
14-gon	*0.001346217*
15-gon	*0.00116697*
16-gon	*0.001025292*
17-gon	*0.000907949*
18-gon	*0.000809668*
19-gon	*0.000726531*
20-gon	*0.000655578*
21-gon	0.00059453648
22-gon	0.0005416440483
23-gon	0.000495510976
24-gon	0.00045503233
25-gon	0.00041932012
26-gon	0.00038765413
27-gon	0.0003595425
28-gon	0.000334208
29-gon	0.000311538623
30-gon	0.00029110055
31-gon	0.00027261008
32-gon	0.00025582735
33-gon	0.00024054831
34-gon	0.0002265985
35-gon	0.00021382812
n-side (approaches to a circle)	$K_s=[\frac{1}{4\pi}-\frac{1}{4n\tan\theta}]$ or $K_s=[\frac{1}{4\pi}-\frac{\tan\theta}{4n}]$

Table.2 K_{i1} value for each inscribed regular polygon.

Inscribed regular polygon of n-sides	K_{i1}-value
Triangle	*1.2421864*
Square	*0.67419155*
Pentagon	*0.42472364*
Hexagon	*0.292483274*
Heptagon	*0.213814851*
Octagon	*0.163176053*
Nonagon	*0.128646516*
Decagon	*0.104040755*
11-gon	*0.085884887*
12-gon	*0.072103897*
13-gon	*0.061395798*
14-gon	*0.052909637*
15-gon	*0.046070117*
16-gon	*0.040476905*
17-gon	*0.035844405*
18-gon	*0.031964431*
19-gon	*0.028682306*
20-gon	*0.025881166*
…… n-sided	$K_{i1}=[\pi-\frac{\pi^2}{ntan\theta_1}] or \left[\pi-\frac{\pi^2 tan\theta_2}{n}\right]$

Table.3 K_{i2} value for each inscribed regular polygon.

Inscribed Regular polygon of n-sides	K_{i2}-value
Triangle	*1.84254548*
Square	*1.141592654*
Pentagon	*0.763951362*
Hexagon	*0.543516442*
Heptagon	*0.405182465*
Octagon	*0.313165528*
Nonagon	*0.24904841*
Decagon	*0.202666392*
11-gon	*0.168068157*
12-gon	*0.141592653*
13-gon	*0.120892035*
14-gon	*0.104406479*
15-gon	*0.09106783*
16-gon	*0.080125194*
17-gon	*0.071038491*
18-gon	*0.063411363*
19-gon	*0.056947696*
20-gon	*0.051422709*
……..n-sided	$K_{i2}=[\pi-\frac{1}{2}n\sin\theta_3]$

Table.4 K_c value for each circumscribed regular polygon.

circumscribed Regular polygon of n-sides	K_c-value
Triangle	*2.054559769*
Square	*0.858407346*
Pentagon	*0.491112000*
Hexagon	*0.322508961*
Heptagon	*0.229429678*
Octagon	*0.172115345*
Nonagon	*0.134139454*
Decagon	*0.107604308*
11-gon	*0.088298768*
12-gon	*0.073797655*
13-gon	*0.062619565*
14-gon	*0.053815987*
15-gon	*0.046755771*
16-gon	*0.041005224*
17-gon	*0.036258097*
18-gon	*0.032293000*
19-gon	*0.028946584*
20-gon	*0.026096152*
…n-sided	$K_c = n tan\theta_1 - \pi \ or \ K_c = \frac{n sin\theta_3}{2(sin\theta_2)^2} - \pi.$

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