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Spin determination for the superdeformed bands of some even mass Ce and Nd isotopes

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Abstract The superdeformed bands in the even mass isotopes of the A≈130 mass region characterised by spin alignment and significant band mixing cases. Accordingly, the theoretical and experimental determination of the spin is so difficult. Therefore, in this work a simple method for the spin proposition of rotational superdeformed (SD) bands of some even mass Ce and Nd isotopes has been suggested. It depends on the behaviour of the variation of the kinematic (φ^1) and the dynamic (φ^2) moment of inertia with the angular frequency. Applying this method a good agreement between the calculated and the corresponding experimental values of the gamma-ray transition energies of the SD bands is obtained.

Keywords spin of superdeformed bands, A~130 mass region, Ce and Nd isotopes, moment of ineria.

Introduction

The phenomenon of superdeformation has been studied intensively since the discovery of superdeformed (SD) bands in ^{132}Ce and ^{152}Dy [1,2]. Much effort have been spent attempting to understand the properties of the long cascades of regular spaced γ rays emitted from these highly deformed nuclei at high spin.

In this work, we have studied the SD bands in the A≈130 mass region. In these region, there are: a) a large shell gaps at large deformation caused a deep second minima in the nuclear potential energy, (b) variations in the dynamical moments of inertia with rotational frequency, c) long cascades of $E2$ transitions (up to ≈20 transitions) and (d) complex decay paths from the second to the first minima [3]. The previous properties are similar with SD nuclei in the A≈150 mass region.

Nevertheless, there are also some differences between SD nuclei in the A≈130 and A≈150 mass regions such as: (a) spins of the yrast SD bands are lower ≈25$\hbar$ as compared to≈45$\hbar$, (b) population occurs between collective rotational ND states with nuclear deformations of β≈0.2 in A≈130 and not with noncollective ND states (c) quadrupole deformations are higher in A≈150, β≈0.6 [4], rather than β≈0.4 in A≈130 [5], and (d) the total intensities bands are generally stronger at usually 5% of the reaction channel, facilitating more precise intensity measurements of mass~130 [3].

The decay-out of the SD bands and the determination of excitation energies, spins and parities for the SD states considered the most important concerning the superdeformation phenomenon. These quantities can be determined through the observation of discrete gamma-transitions linking the lowest levels of the SD band to the normally deformed (ND) ones. The decay-out process has been observed firstly in the odd ^{133,135,137}Nd nuclei [6,7]. Recently, such transitions have been achieved in the even-even ^{l32,134}Nd nuclei [8,9].

It is observed_that, most of the even-even mass isotopes in the A≈130 mass region characterised by spin alignment and significant band mixing cases. These cause an obscurity in calculation of the spin theoretically and experimentally. So, in the present work we tried to calculate the spin of the SD bands for some even-even mass isotopes in the A≈130 mass region by a simple and logical method. Moreover, the gamma-ray transition energies of this SD bands have been calculated and compared with the corresponding experimental values.

Methodology

Since the discovery of the SD band in ^{152}Dy [2], several approaches to assign the spins of SD bands have been suggested [10-12]. Most of the previous works are concentrated on the SD bands in the A≈150 and A≈190 mass regions. It is observed that most of the available approaches proceed from the comparison of the calculated transition energies or moment of inertia with the corresponding experimental results and generally are referred to as the best-fit method (BFM) [12].

Bohr and Mottelson [13] pointed out that, under adiabatic approximation, the rotational energy of an axially symmetric nucleus can be expanded as (for $K=0$ band)

$$E_{rot}=AI(I+1)+B[I(I+1)]^2+C[I(I+1)]^3+D[I(I+1)]^4+\ldots\ldots \quad (1)$$

Where $A=\hbar^2/2\varphi$ and B, C, are corresponding to the higher-order inertial parameters. The expression for the $K\neq0$ band takes the form

$$E_{rot}=E_o+A[I(I+1)-K^2]+B[I(I+1)-K^2]^2+C[I(I+1)-K^2]^3+\ldots\ldots \quad (2)$$

Where E_o is the bandhead energy. By using Eq. (2) one can obtain a formula for the transition energy E_γ and the orbital angular momentum I as follows:

$$E_\gamma(I+2\rightarrow I)=A[4I+6]+B[8I^3+36I^2+I(60-8K^2)-12K^2+36]\ldots \quad (3)$$

Another useful expression is the Harris ω^2 expansion; in particular the two parameter expansion takes the form [14]

$$E(\omega) = \alpha\,\omega^2 + \beta\,\omega^4 + \ldots. \quad (4)$$

It is well known that the most important quantities characterizing the nuclear rotational band is the kinematic moment of inertia

$$\varphi^1=(\hbar I_x/\omega)= \hbar^2 I_x(dE/dI_x)^{-1} \quad (5)$$

and the dynamic moment of inertia

$$\varphi^2=\hbar(dI_x/d\omega)= \hbar^2(d^2E/dI_x^2)^{-1} \quad (6)$$

Then $(dE/dI_x)=\hbar\omega$ (7)

where I_x is the spin projection onto the rotational axis

From Eqs. (5) and (6) we have

$$dE/d\omega =(dE/dI_x)(dI_x/d\omega)= \omega\,\varphi^2 \quad (8)$$

According to Eqs. (4) and (8), the dynamic moment of inertia is

$$\varphi^2 = 2\alpha + 4\beta\omega^2 \qquad (9)$$

by integrating φ^2 w.r.t. ω the spin I_x :

$$I_x = x\,\omega + y\,\omega^3 + i_o \qquad (10)$$

Where $x=2\alpha/\hbar$, $y=4\beta/3\hbar$ and i_0 is the alignment, it was found that it takes the values zero or half in present calculations. Introducing the spin projection onto the rotational axis (K),

$$I_x = [(I+1/2)^2 - K^2]^{1/2} \qquad (11)$$

Where I refers to the midpoint spin of the transition $I+1 \rightarrow I-1$[13].

For a SD cascade the transition energies $E_\gamma(I)=E(I)-E(I-2)$ can be least-squares fit by the previous expression. It was found that, when a correct I_o value is assigned, the calculated energies coincide with the observed results incredibly well. However, if I_o is shifted from the correct value by ± 1, the root-mean-square (rms) deviation σ will increase radically [10,15]:

Where: $$\sigma = [(1/n)\sum_{i=1} | (E_\gamma^{calc.}(I_i) - E_\gamma^{expt.}(I_i)) / E_\gamma^{expt.}(I_i) |^2]^{1/2} \qquad (12)$$

It was observed that the BFM depends on the rms deviation to determine the spin of the SD bands and if a significant band crossing occurs in the transitions involved in the BFM, the σ may display some irregularities and make the assignment of the exact spin more difficult as the case in A≈130 mass region.

In previous works [16,17], a useful method was used for the spin assignment for rotational SD bands in the A≈150 and 190 mass regions. In that method, one can extract the kinematic (φ^1) and the dynamic moments of inertia (φ^2) by using the experimental interband E_γ transition energies as follows:

$$\varphi^1(I-1)/\hbar^2 = (2I-1)/E_\gamma \qquad (13)$$

$$\varphi^2(I)/\hbar^2 = 4/\Delta E_\gamma \qquad (14)$$

Where $\Delta E_\gamma = E_\gamma(I+2 \rightarrow I) - E_\gamma(I \rightarrow I-2)$. It is observed that, while the extracted φ^1 depends on the spin proposition, φ^2 does not. On the other hand, according to the available expressions for rotational bands based on the $I(I+1)$ expansion [13], some properties concerning the variation of φ^1 and φ^2 with angular frequency (or spin) can be found [12]. These properties can be summarised as follows:

a) $\lim_{\omega\to 0} \varphi^1 = \lim_{\omega\to 0} \varphi^2 = \varphi_o$.

b) φ^1 *vs* ω and φ^2 *vs* ω plots never cross with each other at nonzero spins.

c) $\lim_{\omega\to 0} \dfrac{d\varphi^1}{d\omega} = \lim_{\omega\to 0} \dfrac{d\varphi^2}{d\omega} = 0$, i.e., as $\omega \to 0$, φ^1 vs ω and φ^2 vs ω plots become horizontal.

d) φ^1 and φ^2 monotonically increase with ω (for $B<0$) or decrease with ω (for $B>0$).

e) Both φ^1 vs ω and φ^2 vs ω plots are concave upwards (for $B<0$), or downwards (for $B>0$).

For all normal deformed (ND) rotational bands where the spins have been measured experimentally, it is found that φ^1 and φ^2 do exhibit these properties except for K=1/2 bands and significant band mixing cases. Moreover, if the spin are artificially increased or decreased by one or more some of these properties will obviously disappear. So, the previous properties may be used as a very useful guideline for the spin proposition of SD rotational band.

As we have mentioned in the previous section the A≈130 mass region, characterised by spin alignment and significant band mixing cases. So, not all the previous properties will be satisfied properly. The last three properties seem to be sufficient to evaluate the proper band head spins in the A≈130 mass region.

Results and discussions

In this work, the SD bands in the even-even A≈130 mass region have been studied by using the properties (a) to (e). The bandhead spin for each of this SD bands has been assigned from the relation between the angular frequency ω and the extracted moment of inertia (φ^1 and φ^2) as shown in Fig. 1. From this Fig., it is clear that there is a critical spin before which the normal behavior (the properties c, d) and e)) of φ^1 and φ^2 is reversed and the relation becomes impenetrable. So, we can conclude that this critical spin must be the bandhead spin I_o of the SD band. This critical spin can be determined after the disturbed region that represents the strength of the band mixing or spin alignment. After the determination of I_o we have calculated the gamma-ray transition energies of the SD band in Nd and Ce isotopes making use of Eq. (3). The obtained results are in good agreement with the experimental values of E_γ as shown in Table 1 and 2. The fitting parameters are given in Table 3.

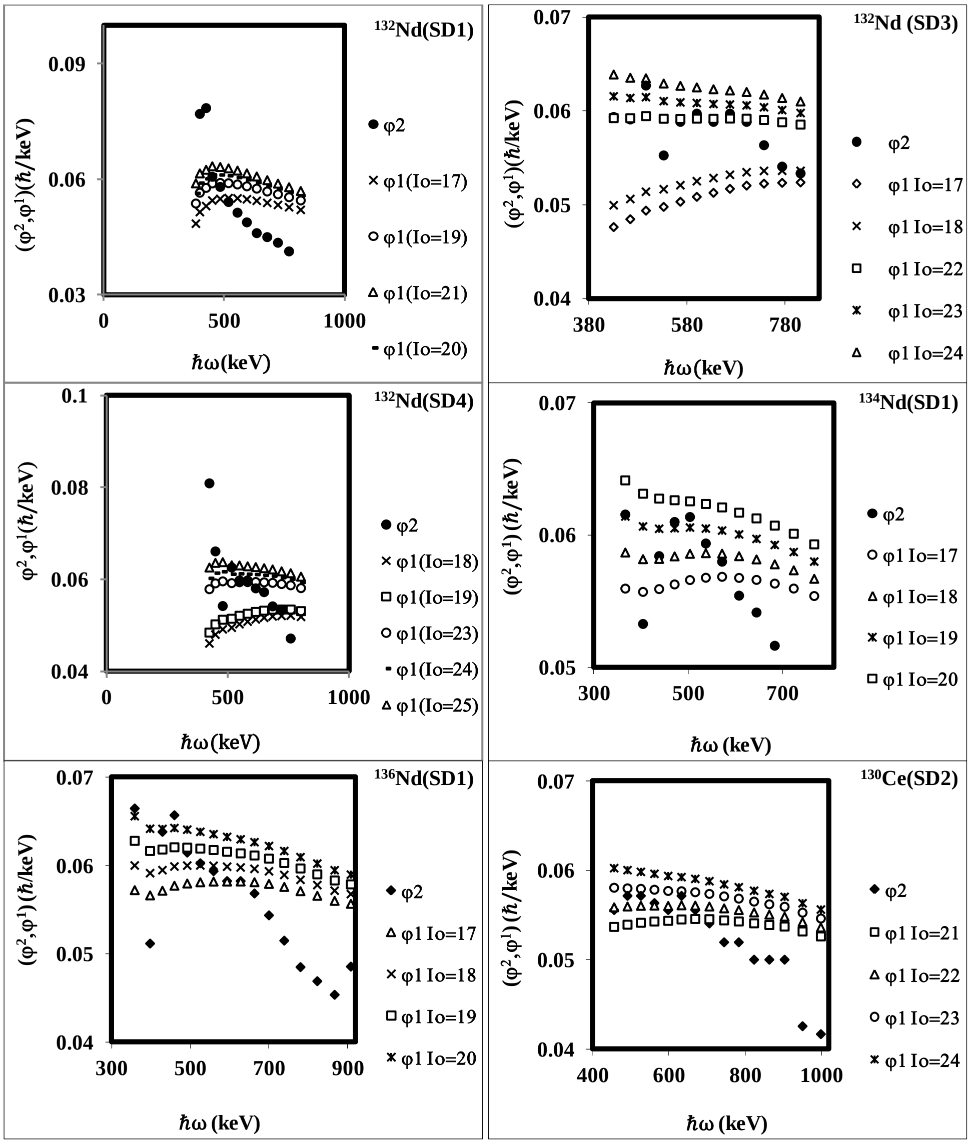
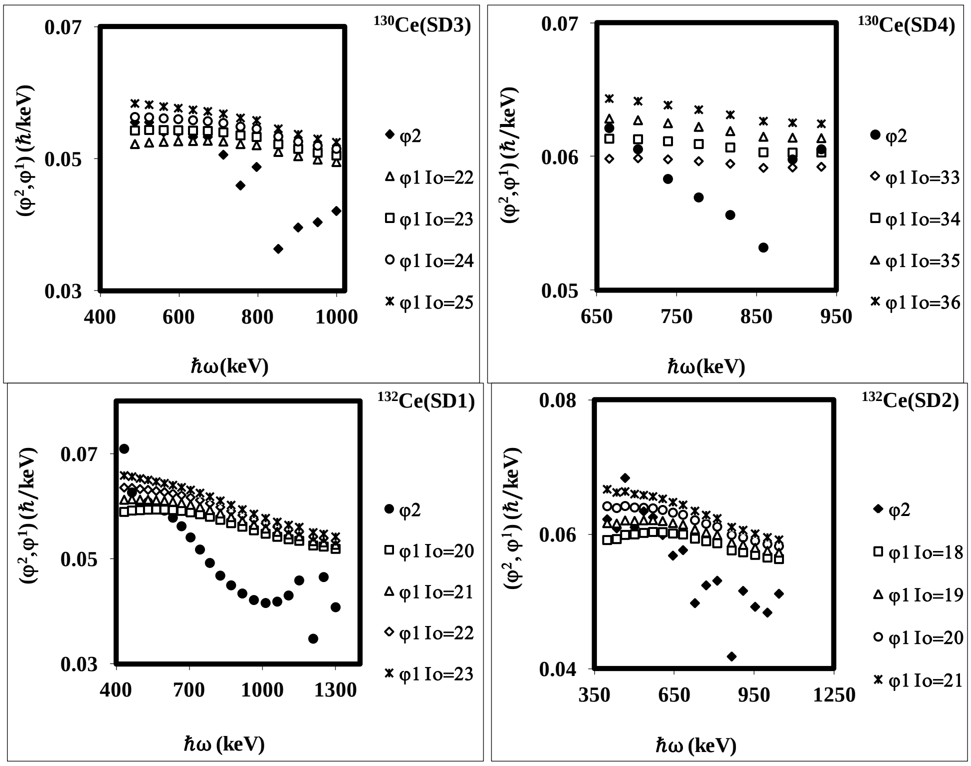


Fig.1 The relation between (φ^1, φ^2) and ($\hbar\omega$) with different I_o for the SD bands in some Nd and Ce isotopes.

Continued



The experimental determination of I_o for [132]Nd (SD1) band was suggested to be equal (17)[18]. In Fig. 1 we observe that if I_o takes the values from 17 to 20 most of the five properties (a-e) withdraw. Conversely, if I_o=21 the curves of $\varphi^1$and φ^2 vs ħω is more logical and the calculated E_γ's energies are close to the experimental data as shown in Table 1.

Fig. 1, shows the $\varphi^1$and $\varphi^2$of the [132]Nd(SD3) band assigned to be plotted as function of rotational frequency for different I_0. It is clear that there is a hump of the φ^2 curve at $\hbar\omega$≈500 keV. This increase of the φ^2 is due to the alignment of the $i_{13/2}$ intruder orbital [19-21]. There is a doubted experimental determination of I_0. It is previously suggested to be 17 or 18 [18]. From Fig. 1, one can observe that at I_0=17 and 18 the value of the φ^1 vs. $\hbar\omega$ increases and for I_0=22 it remains almost constant, and some of the five properties (a-e) disappear obviously. On the other hand, for I_0=23 the extracted φ^1 and φ^2 do exhibit the five properties (a-e) after $\hbar\omega$≈500 keV. Taking into consideration that I_o=23, it is found that the calculated gamma-ray transition energies are in fair agreement with the experimental results as shown in Table 1.

Table 1. A sample of experimental and calculated E_γ's energies(in keV) of some SD bands for ^{132}Nd, ^{134}Nd and ^{136}Nd isotopes.

^{132}Nd(SD1)			^{132}Nd(SD3)			^{132}Nd(SD4)		
$E_{\gamma(exp.)}$ [22]	$E_{\gamma(cal.)}$ For I_o=17	$E_{\gamma(cal.)}$ For I_o=21	$E_{\gamma exp.)}$ [22]	$E_{\gamma(cal.)}$ For I_o=18	$E_{\gamma(cal.)}$ For I_o=23	$E_{\gamma exp.)}$ [22]	$E_{\gamma(cal.)}$ For I_o=18	$E_{\gamma(cal.)}$ For I_o=25
764	676.1	714.1	793.7	789	797.4	848	814.1	839.5
797	746.4	778.7	861.1	861	861.1	897.5	887.31	902.5
849	818.1	844.8	929.2	932	925.5	958.1	959.7	966.4
900	891.0	912.4	992.2	1003	990.7	1032	1031.4	1031.2
966	965.4	981.8	1065	1074	1056.8	1096	1102.3	1097.0
1035	1041.3	1053.0	1133	1143	1123.8	1163	1172.2	1163.7
1109	1119.0	1126.1	1200	1212	1191.8	1230	1241.3	1231.6
1187	1198.4	1201.3	1268	1281	1260.8	1299	1309.4	1300.5
1269	1279.7	1278.6	1335	1349	1330.8	1369	1376.4	1370.6
1356	1362.9	1358.1	1403	1416	1402	1443	1442.3	1442.0
1445	1448.3	1440.1	1474	1482	1474.4	1518	1507.1	1514.6
1537	1535.9	1524.5	1548	1548	1548	1603	1570.6	1571.1
1634	1625.9	1611.5	1623	1613	1622.9			
	σ = 0.038	σ = 0.021		σ = 0.0078	σ = 0.0044		σ = 0.014	σ = 0.0071

Continued

^{134}Nd(SD1)			^{136}Nd(SD1)		
$E_{\gamma exp.)}$ [22,23]	$E_{\gamma(cal.)}$ For I_o=17	$E_{\gamma(cal.)}$ For I_o=18	$E_{\gamma(exp.)}$ [22,24]	$E_{\gamma(cal.)}$ For I_o=17	$E_{\gamma(cal.)}$ For I_o=19
668	674	668	656.6	651.3	666
733	741	733	716.8	717	728.2
808.1	809	800	795	783.3	791.2
876.6	877	867	857.9	850.2	855
942.2	945	935	918.4	917.8	919.7
1007.4	1014	1005	983.7	986.2	985.4
1074.8	1083	1075	1050.1	1055.3	1052.1
1143.8	1154	1147	1117.5	1125.4	1119.9
1216	1224	1220	1186.2	1196.3	1188.8
1289.9	1295	1295	1254.9	1268.2	1258.9
1367.4	1367	1370	1325.3	1341.1	1330.3
1448	1440	1448	1398.9	1415.1	1403.1
1535	1513	1527	1476.6	1490.2	1477.2
			1559.1	1566.5	1552.9
			1644.4	1644	1630
			1732.6	1722.8	1708.7
			1815	1802.9	1789.1
	$\sigma =$ 0.0072	$\sigma =$ 0.0052		$\sigma =$ 0.0080	$\sigma =$ 0.0077

For the ^{132}Nd (SD4) band, Fig. 1 shows that there is a hump of the φ^2 curve at $\hbar\omega$≈550 keV due to the alignment of the $i_{13/2}$ intruder orbital [19-21]. The experimental value of I_o was suggested to be 18 or 19 [18]. From Fig. 1, it is clear that at I_o=18 and 19 the value of φ^1 increases monotonically with ω, which seems hard to understand and for I_o=24 it has almost a constant value, and the five properties (a-e) disappear. In contrary, for I_o=25 the extracted φ^1 and φ^2 do exhibit the five properties (a-e) after $\hbar\omega$≈550 keV. Therefore, I_o=25 seems to be the best choice for bandhead spin value. The calculated gamma-ray transition energies based on I_o=25 are very close to the corresponding experimental data as shown in Table 1.

As shown in Fig. 1, the dynamical moment of inertia $\mathcal{J}^2$ of the SD(1) in ^{134}Nd exhibits a broad hump centered at $\hbar\omega \approx 500$ keV that has been attributed to the rotational alignment of a pair of $h_{11/2}$ protons [25]. The bandhead spin I_o of this band is expected experimentally to be 17 [18,22]. It is clearly seen that for I_o=18, the extracted $\mathcal{J}^1$ and $\mathcal{J}^2$ in ($\mathcal{J}^1$,$\mathcal{J}^2$-$\hbar\omega$) plots do have the five properties (a-e) beyond the region of the hump. On the contrary, if the bandhead spin is decreased by 1, some of the five properties no longer exist. Also, from Table 1, it is clear that at I_o=18 the calculated gamma-ray transition energies are more close to the experimental data, so the suggestion that I_o=18 is more logical.

The $\mathcal{J}^1$ and $\mathcal{J}^2$ plots for SD(1) band of ^{136}Nd shows that there is a hump of the $\mathcal{J}^2$at $\hbar\omega \approx 460$ keV as shown in Fig. 1. This behavior of the $\mathcal{J}^2$ is due to the crossing associated with the alignment of a pair of $i_{13/2}$ neutrons [23]. The bandhead spin I_o of this band is suggested experimentally to be 17 [18,26]. It is found that at I_o=19, the variation of $\mathcal{J}^1$ and $\mathcal{J}^2$ plots do have the five properties (a-e) after $\hbar\omega \approx 460$ keV. In contrast, if I_o is decreased, some of the five properties disappeared. From Table 1, it is clear that at I_o=19 the calculated gamma-ray transition energies are more close to the experimental data. So the most plausible choice of the bandhead spin for this SD band is 19.

Table 2. A sample of experimental and calculated E_γ's energies (in keV) of some SD bands for ^{132}Ce and ^{130}Ce isotopes.

^{132}Ce (SD1)			^{132}Ce (SD2)			^{130}Ce (SD2)	
$E_{\gamma exp.)}$ [27]	$E_{\gamma(cal.)}$ For I_o=20	$E_{\gamma(cal.)}$ For I_o=22	$E_{\gamma(exp.)}$ [27]	$E_{\gamma(cal.)}$ For I_o=19	$E_{\gamma(cal.)}$ For I_o=20	$E_{\gamma(exp.)}$ [20]	$E_{\gamma(cal.)}$ For I_o=23
770.8	733	736.1	724.4	724.4	724.4	841	843
809.3	798	798.8	794.3	793	791.2	914	911
865.71	865	862.7	865.89	862.7	859.2	983	980
929.6	932	927.8	929.01	933.5	928.6	1052	1050
995.9	1000	994.2	1000.7	1005.6	999.5	1124	1120
1061.71	1069	1062	1068.5	1079.1	1071.9	1196	1192
1128.78	1139	1131.3	1138.4	1154	1145.9	1266	1265
1196.4	1210	1202.1	1211.3	1230.4	1221.7	1338	1340
1265.6	1283	1274.5	1288.5	1308.3	1299.4	1412	1415
1336.8	1356	1341.7	1364.5	1388	1379	1489	1492
1410.8	1431	1417.3	1453.9	1469.5	1460.6	1566	1570
1488.1	1507	1494.8	1538.3	1552.8	1544.4	1646	1650
1569.4	1585	1574.3	1621.5	1638.1	1630.4	1726	1730
1654.9	1664	1655.8	1730.2	1725.5	1718.8	1806	1813
1743.9	1744	1748	1816.1	1814.9	1809.6	1900	1899
1836.1	1827	1834.1	1906.6	1906.6	1902.9	1996	1989
1931	1910	1923	1998.9	2000.6	1998.9		
2027.2	1996	2013.4	2085.6	2097	2097.6		
2122.8	2083	2106.7					
2215.8	2172	2202.5					
2303	2263	2301					
2418	2356	2402					
2504	2451	2506					
	σ = 0.0188	σ = 0.0116		σ = 0.009	σ = 0.0055		σ = 0.0027

Continued

^{130}Ce (SD3)		^{130}Ce (SD4)	
$E_{\gamma exp.)}$ [20]	$E_{\gamma(cal.)}$ For I_o=24	$E_{\gamma exp.)}$ [22]	$E_{\gamma(cal.)}$ For I_o=35
904	900	1261	1261
976	972	1331	1333
1048	1046	1403	1407
1122	1122	1478	1481
1196	1199	1555	1556
1271	1278	1634	1633
1346	1359	1717	1711
1425	1442	1790	1790
1512	1527	1862	1870
1594	1614		
1704	1704		
1805	1796		
1904	1890		
1999	1987		
	σ =0.0070		σ =0.0023

For the yrast SD(1) band of ^{132}Ce, the bandhead spin I_o has been measured experimentally to be equal 20 [24] or 22 [27] . In Fig. 1, there is a sharp rise of φ^2at $\hbar\omega$≈430 keV due to the alignment of two $i_{13/2}$ neutrons [28-30]. After this point if I_o=20 some of the five properties (a-e) disappear obviousl ỹBut it is attractive to note that for the spin suggestion I_o=22, the extracted φ^1 and φ^2do exhibit the properties (a-e). Making use of this assignment the gamma-ray transition energies for this SD bands can be reproduced nicely by the $I(I+1)$ expression (Eq. (3)) as shown in Table 2. So the most reliable choice is I_o=22 rather than I_o=20, which is in agreement with the spin assignment by Liu Y X *et al* [27].

Table 3 The fitting parameters of the present model (Eq.(3)).

Nucleus	I_o	A(keV^{-1})	B×10^{-4} (keV^{-3})
^{132}Nd(SD1)	17[22]	8.1712	2.85
	21*	7.1011	3.32
^{132}Nd(SD3)	17[22]	9.5515	-1.9361
	18[22]	9.2058	-0.9853
	23*	7.4749	1.3827
^{132}Nd(SD4)	18[22]	9.5110	-1.5412
	25*	7.3112	1.4408
^{134}Nd(SD1)	17[22,23]	8.2135	0.6805
	18*	7.7136	1.9385
^{136}Nd(SD1)	17[22,24]	7.9067	1.3748
	19*	7.3519	1.6378
^{132}Ce(SD1)	20[27]	7.7462	1.6016
	22*[28]	7.1446	2.1509
^{132}Ce(SD2)	19[27]	7.9820	2.2892
	20*	7.6279	2.5666
^{130}Ce(SD2)	23*	7.8990	1.6660
^{130}Ce(SD3)	24*	8.0859	2.6247
^{130}Ce(SD4)	35*	8.1204	1.3167

*The values of the bandhead spins adopted in the present work.

Fig. 1 shows the φ^1 and φ^2plots for the SD(2) band of ^{132}Ce. In this band there is a hump of the φ^2curve at $\hbar\omega$≈450 keV due to the effect of $i_{13/2}$ neutron intruders [25]. It is clearly seen that when the measured bandhead spin (I_o=19) [24] is used, some of the five properties (a-e) no longer exist. On the contrary, if the bandhead spin is increased by 1, the extracted φ^1 and φ^2do exhibit the properties (a-e). From Table 2 at I_o =20 the transition energies are more close to the experimental results. So, the suggestion that I_o=20 for this band is more reasonable.

To our knowledge there is no previous experimental work concerning the detection of the bandhead spins of ^{130}Ce (SD2), ^{130}Ce (SD3) and ^{130}Ce (SD4). The extracted φ^1 and φ^2vs. $\hbar\omega$ plot shown in figure 2 leads us to expect the bandhead spins for the aforementioned SD bands to be 23 and 24 and 35 respectively. The theoretical predictions of the gamma-ray transition energies based on these expected bandhead spins for these SD bands are in good agreement with the experimental values of the gamma-ray transition energies as shown in Table 2. This result gives a further support for the right detection of the aforementioned bandhead spins.

Another simple approach to test the validity of the quantitative method applied in the present work is the Harris ω^2 expansion [13], whose convergence is believed [12] to be superior to the I(I+1) expansion (Eq. (2)) and particularly the Harris two parameter expansion (Eq. (4)) that was widely used in the high-spin nuclear physics [31]. Therefore, the bandhead spins of the previous bands have been calculated via least square fit procedure to the experimental data in the spirit of Harris two parameter expansion for φ^2(Eq. (9)). Accordingly, the bandhead spin I_o could be calculated making use of Eq. (10). The fitting parameters along with both experimental and calculated I_o are given in Table 4.

Table 4 The fitting parameters of Harris two parameter expansion (Eq. (10)).

Nucleus	I_o Experimentally	I_o by $\varphi^1$and φ^2 vs ħω	I_o from least square fit	$x\times 10^{-2}$ (keV)	$y\times 10^{-8}$ (keV^3)
^{132}Nd(SD1)	17[22]	21	21	6.62	-1.33
^{132}Nd(SD3)	17 or 18[22]	23	24	9.20	-13.33
^{132}Nd(SD4)	18 or 19[22]	25	25	6.66	-0.86
^{134}Nd(SD1)	17[22,23]	18	20	6.90	-1.33
^{136}Nd(SD1)	17[22,24]	19	18	7.4	-2.00
^{132}Ce(SD1)	20[27] or 22[28]	22	22	6.55	-1.01
^{132}Ce(SD2)	19[27]	20	22	6.8	-1.66
^{130}Ce(SD2)		23	22	5.6	0.266

^{130}Ce(SD3)		24	17	3.6	3.33
^{130}Ce(SD4)		35	41	7.5	-1.66

From the systematic analysis of the determined I_o in the four cases by the two methods, it was drawn that there exist an obvious agreement in three cases and the deviation does not exceed $\Delta I=2$ in the remaining ^{134}Nd(SD1) band.

Conclusion

In this work, the base line spins I_o of SD bands in some even-even Ce and Nd isotopes have been determined by an accurate method. In this method and under certain circumstances mentioned in details in the text the relation between the moments of inertia (φ^1& φ^2) and the angular frequency ($\hbar\omega$) could be used as a very useful guideline for the bandhead spins I_o prediction of the SD bands.

Making use of the determined I_o, the gamma-ray transition energies of the SD bands for some even-mass Ce and Nd isotopes have been calculated by the available expression for rotational bands (Bohr-Mottelson's $I(I+1)$ expansion). The obtained results are compared with the corresponding experimental data and a good agreement has been obtained which supports the present proposed method.

To give another support to the quantitative method of this work, a least square fit procedure has been applied to the experimental data of four SD bands making use of Harris two parameter formula. The predicted I_o of the fit is in fair agreement with those obtained by the present applied method.

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