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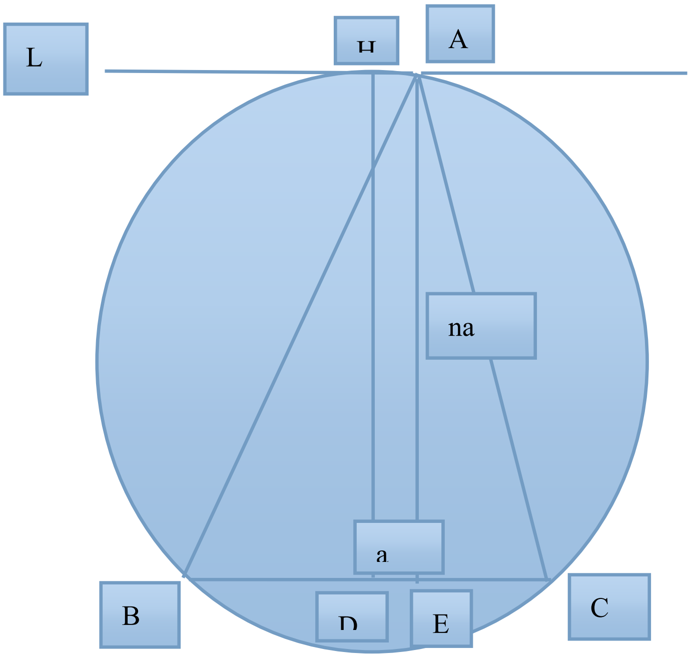
A mathematical formula obtained by Evans triangle

Zhou Ya Nan
East China Institute Of Technology

Abstract: R.J Evans in 1977 in the American Mathematical Monthly ask such a question: Find all the integer side of the triangle ,Make it a high and the bottom edge of the ratio is an integer, This triangle is only $d=3$. At the same time, Evans has given such a triangle, The length of 3,8,53. In 2009 the side Hin constructs a kind of the triangle ,And that such a triangle has infinite .In this paper, a formula are obtained by Evans, the definition of the triangle, By this formula obtained Evans triangle.

Key words: Integers, rational Numbers.



The Improved Method

As shown in figure: In the O circle with a R, Triangle ABC for round O inscribed triangle lines, The $BC=a$. At the edge of the BC high as $n\cdot a$. O do OD is perpendicular to the BC , duo to BC in D . By the Pythagorean theorem $OD=\sqrt{CE^2+AE^2}$, Point A do BC of parallel lines BC. Delay DO Hand in L in H. The OH is perpendicular to the BC . Point A do AE vertical BC due BC to E。Then : $AE=n\cdot a$.

SO: $DH=n\cdot a$, $OH=n\cdot a-OD=n\cdot a-\sqrt{CE^2+AE^2}$,

$$CE = | HA - \frac{a}{2} |$$

$$= \left| \sqrt{R^2 - (n \cdot a - \sqrt{R^2 - (\frac{a}{2})^2})^2} - \frac{a}{2} \right|$$

then: $AC = \sqrt{CE^2 + AE^2}$

$$= \sqrt{\left[\sqrt{R^2 - (n \cdot a - \sqrt{R^2 - (\frac{a}{2})^2})^2} - (\frac{a}{2})^2\right]^2 + (n \cdot a)^2}$$

The same available:

$$BA = \sqrt{\left[\sqrt{R^2 - (n \cdot a - \sqrt{R^2 - (\frac{a}{2})^2})^2} + (\frac{a}{2})^2\right]^2 + (n \cdot a)^2}$$

For the triangle, We only need to meet the trilateral is rational, And for the rational number can. That is when $n = (3, 4, 5, 6, 7, 8, 9 . . . . . .)$. Side a, AC, BA is rational. Then the triangle line of nature, At the same time expand k times, Until meet the trilateral all rational Numbers for integer.

Lemma: If a, AC, BA are rational, Then $\frac{AC}{a}$, $\frac{BA}{a}$ Also for the rational Numbers, If $y = \frac{AC}{a} + \frac{BA}{a}$, y will be as rational Numbers.

Then:

$$\frac{AC}{a} = \sqrt{\left[\sqrt{(\frac{R}{a})^2 - (n - \sqrt{(\frac{R}{a})^2 - (\frac{1}{2})^2})^2} - (\frac{1}{2})^2\right]^2 + n^2}$$

$$\frac{BA}{a} = \sqrt{\left[\sqrt{(\frac{R}{a})^2 - (n - \sqrt{(\frac{R}{a})^2 - (\frac{1}{2})^2})^2} + (\frac{1}{2})^2\right]^2 + n^2}$$

Make $\frac{R}{a} = x$, Then

$$\frac{AC}{a} = \sqrt{\left[\sqrt{x^2 - (n - \sqrt{x^2 - (\frac{1}{2})^2})^2} - (\frac{1}{2})^2\right]^2 + n^2}$$

$$\frac{BA}{a}=\sqrt{\left[\sqrt{x^2-(n-\sqrt{x^2-(\frac{1}{2})^2})^2+(\frac{1}{2})^2}\right]^2+n^2}$$

Known R,a into x, This is the key to the problem solving,

The following order:

$$y=\frac{AC}{a}+\frac{BA}{a}$$

Then, start to equation. After finishing:

$$x=\frac{1}{2}\cdot\left[\frac{y^2-1}{4n}+\frac{n}{y^2-1}\right],\quad x>\frac{n^2+\frac{1}{4}}{4n}\quad,\quad y>\sqrt{4n^2+1}$$

Solution here. We will think of this problem has been solved. So we make $n=4$. y assignment is given. Get the x value, then get the value of the $\frac{AC}{a}$, $\frac{BA}{a}$. Found that it is not rational. So we will thinking about x generation into the $\frac{AC}{a}$, $\frac{BA}{a}$. the generations:

$$\frac{AC}{a}=\frac{y}{2}-\frac{1}{2}\sqrt{1-\frac{4n^2}{y^2-1}}\;,\quad \frac{BA}{a}=\frac{y}{2}+\frac{1}{2}\sqrt{1-\frac{4n^2}{y^2-1}}$$

We through the above formula, We can know that we need only ensure $\sqrt{1-\frac{4n^2}{y^2-1}}$ for mileage. To make this type is equal to the number k,

The {$4n^2=(1-k^2)(y^2-1)$}' s formula can be obtained。So through the $4n^2=(1-k^2)(y^2-1)$ formula, We just make sure k,y to have the mileage, n as an integer.

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