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On the description of electromagnetic fields in slow moving media

Rozov Andrey Leonidovich
St. Petersburg State Polytechnic University
Pargolovskaya st., 10-40, St. Petersburg, Russia, 194100
E-mail: rozov20@mail.ru\

Abstract. At present, to describe electromagnetic fields in moving media Minkowski equations obtained on the basis of theory of relativity are used. But important electromagnetic processes run under non relativistic conditions of slow moving media. Therefore, one should carry out its description in terms of classical mechanics. Analysis of Minkowski equations, presented in the paper, revealed a discrepancy between a physical model, which is the base of the equations, and known classical mechanics information: Faraday's experimental findings and Maxwell theory. As a result, Minkowski equations are not an optimal instrument to carry out analysis of electromagnetic processes under non relativistic conditions of slow moving media.

The paper proposes a way of description of electromagnetic fields in slow moving media on the basis of Maxwell theory within the frame of classical mechanics. Received Galilean invariant Maxwell equations lack asymmetry in the description of the reciprocal electrodynamic action of a magnet and a conductor and conform to known experimental data.

Key words: electromagnetic fields, Maxwell's and Minkowski equations, slowly moving media

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1. Introduction

In the present paper the term "electrodynamics of moving media" is used instead of term "electrodynamics of moving bodies", as one of the important applications of electrodynamics is an interaction between electromagnetic fields and plasma.

The study of the interaction processes between electromagnetic fields and moving media becomes increasingly important due to the problems of plasma confinement in thermonuclear fusion reactions.

At present, these processes are described using Minkowski equations, derived on the basis of relativity principle (Minkowski 1908) (similar equations can be obtained from Lorentz electron theory (see, e.g. Pauli 1958)). In installations, which implement thermonuclear fusion reactions, velocities of plasma motion, as a rule, are much lower than the relativistic ones. So, it is possible to describe electromagnetic fields under these conditions in terms of classical mechanics. Le Bellac and L´evy-Leblond (1973) offered a method for analysis of electromagnetic processes based on Galilean limits of Minkowski equations. Now this method is widely used (see, e.g. Brown and Holland 2003; Montigny and Rousseaux 2007; Heras 2010; Steinmetz et al 2011). But there are shortcomings of Galilean limits of Minkowski equations which have been formulated by the authors. Among them there is an important one - the lack of proper relationship between movement of the medium and electromagnetic fields (so the authors had to correct the limits) (Le Bellac and L´evy-Leblond 1973).

Minkowski equations have been used for the simulation of plasma instability under the effect of electromagnetic fields in the processes of thermonuclear fusion during more than fifty years. But until now an effective plasma confinement by magnetic field in these processes has not been solved. So, there are reasons to analyze the applicability of Minkowski equations under non relativistic conditions of slow moving media.

2. Analysis of Minkowski equations

On deriving electrodynamics equations for moving bodies Minkowski followed Poincare-Einstein's mathematical interpretation of the principle of relativity, declaring all inertial systems to be equivalent with respect to physical laws. He chose the coordinate system in which the body was stationary and presented Maxwell equations for stationary body in it. Then he transformed these equations in the laboratory coordinate system, relating to which the body moved, according to the principle of relativity (Minkowski 1908; Pauli 1958; Sommerfeld 1949). In the result Minkowski received equations consisted of Maxwell equations for stationary body and constitutive equations which take into account movement of the body. The essence of Minkowski's method was formulated by Sommerfeld (1949): "the body knows nothing of its motion". But this statement contradicts classical mechanics evidence of the electromagnetic induction phenomena, first of all Faraday's experimental findings (Faraday 1994) and Maxwell theory expressed into equations (1)–(2).

They treat movement of the body (medium) in these phenomena as relevant to the source of magnetic field and, consequently, independent of coordinate systems. More exactly, Faraday and Maxwell used other terms – relevant to the magnetic lines of force – they have absolutely the same sense.

Therefore, one should say within the frame of classical mechanics that the body (medium) does know of its motion under these phenomena.

To understand the cause of this contradiction we have to step aside from Poincare-Einstein's mathematical interpretation of the principle of relativity and move over to its initial Galileo's own physical one (Seeger 1966).

Galileo pointed to the fact that some mechanical processes on a moving large ship proceeded differently depending on whether or not they occurred in an indoor cabin below deck or on the deck in the open air. In fact, Galileo formulated the relativity principle of mechanical processes only for inertial systems that carry along with themselves a laboratory with all its contents (Seeger 1966).

Thus formulated principle of relativity together with Faraday's experimental findings lead to the physically proved particular coordinate system fixed with the source of magnetic field, relating to which motion of the medium have to be considered.

Let us follow the mechanism of generation of Minkowski equations shortcomings under non relativistic conditions of slow moving media. Consider, for example, Minkowski constitutive equation which has the following form with the accuracy within the terms of order **|v|**/c: $\mathbf{D} = \varepsilon\mathbf{E} + \frac{\mu\varepsilon - 1}{c}\mathbf{v}\times\mathbf{H}$ (see, e.g. Landau et al. 2004; Pauli 1958; Sommerfeld 1949). Substitution of this equation in other Minkowski equations (Maxwell field equations for stationary medium) transforms derivatives of **E** into ones of **E** and $\mathbf{v}\times\mathbf{H}$. As a result, Maxwell induction equation for stationary media (7) transforms into induction equation for moving media (6). But in another Maxwell field equation the term $\frac{\alpha\mu\varepsilon}{c^2}\frac{\partial(\mathbf{v}\times\mathbf{H})}{\partial t}$ appears which doesn't have a physical nature. This can have an effect not only on the accuracy of calculations, but also on the possibility of adequate investigation of plasma instability in the phenomena in question by means of these equations.

So, we see that it is expedient to use alternative models of electrodynamics under non relativistic slow moving media conditions.

According to Krotkov et al. (1999) electrodynamics of slow moving media does not necessitate special relativity. Let us try to obtain an electrodynamic model for slow moving media within the frame of classical mechanics. That's why we have to come back to the original Maxwell theory worked out within the limits of classical mechanics.

3. Derivation of Maxwell equations

The basis of Maxwell electrodynamics is composed of two laws derived experimentally both for stationary and moving media (see, e.g. Sommerfeld 1949):

– Faraday induction law

$$\oint_s \mathbf{E}_s ds = -\frac{1}{c}\frac{d}{dt}\int_\delta \mathbf{B}_n d\delta \,; \tag{1}$$

– Ampere's law

$$\oint_s \mathbf{H}_s ds = -\frac{4\pi}{c}\int_\delta \mathbf{I}_n d\delta \,, \tag{2}$$

where **E, H, B** and **I** are vectors of electric and magnetic field strengths, magnetic induction and current (including displacement current) density, respectively; $\mathbf{E}_s$ and $\mathbf{H}_s$ are tangential components of vectors **E** and **H** to a closed loop *s*; $\mathbf{B}_n$ and $\mathbf{I}_n$ are normal components of vectors **B** and **I** to an arbitrary surface *δ* confined by the loop *s*; *t* – time and *c* – the light speed in vacuum.

The right-hand side (1) generally describes the emergency of electromotive force in the loop both as a result of variation of magnetic field passing through the loop area in time, and also due to the motion of the loop in magnetic field relative to the source of magnetic field. Without loss of generality, we will assume that the source of magnetic field is stationary relative to the laboratory coordinate system. In that case, equations (1)–(2) describe the processes taking place in the moving loop in the laboratory coordinate system relating to which the medium motion is considered. All further transformations of these equations are carried out also in the laboratory coordinate system.

On integrating (1), we apply the Stokes theorem to the left-hand side of the equation, and in the right-hand side we write the total time derivative in terms of its components (in view of the fact that magnetic field is a solenoidal one):

$$\oint_s \mathbf{E}_s ds = \int_\delta \nabla_n \times \mathbf{E} d\delta \,; \tag{3}$$

$$\frac{d}{dt}\int_\delta \mathbf{B}_n d\delta = \int_\delta \left(\frac{\partial \mathbf{B}_n}{\partial t} + \nabla_n \times (\mathbf{B}\times\mathbf{v}) \right) d\delta \,, \tag{4}$$

where **v** is the loop velocity vector.

By substituting (3) and (4) into (1), we obtain the equation

$$\int_{\delta}\left(\nabla_n \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}_n}{\partial t} + \frac{1}{c}\nabla_n \times (\mathbf{B}\times\mathbf{v})\right)d\delta = 0\,. \tag{5}$$

Taking into account that the area δ is arbitrary, from (5) one obtains

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{c}\nabla \times (\mathbf{B}\times\mathbf{v})\,. \tag{6}$$

Equation (6), like the initial equation (1), describes both possible variants of electromagnetic induction generation.

In Minkowski equations Maxwell induction equation under conditions of stationary media is used as the equation of electromagnetic induction for the processes in moving media (Minkowski 1908)

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,. \tag{7}$$

Now when integrating equation (1) for the case of moving media, equation (7) is also obtained as the result of use the relativistic relation (see, e.g. Becker 1933; Panofsky and Phillips 1955). In fact, within the frame of classical mechanics it means that the right-hand side of (1) is transformed similarly to (4), which corresponds to its representation in the laboratory coordinate system, and the left-hand side of (1) is treated as an expression presented in the coordinate system moving together with the medium. As a result of incorrect treatment of equation (1) (the left-hand and right-hand sides are considered in different coordinate systems), the resulting equation (7) loses a portion of information contained in equation (1), and the need to use an additional relation which takes into account the induction due to motion (Lorentz force relation) arises.

By introducing vector potential **A**, $\mathbf{B}=\nabla\times\mathbf{A}$ and scalar potential ϕ, we find from (6)

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \frac{1}{c}\mathbf{B}\times\mathbf{v} - \nabla\phi\,. \tag{8}$$

Let us write (8) in the following form;

$$\mathbf{E} = \mathbf{E}_{mot} + \mathbf{E}_{mov}\,, \tag{9}$$

where $\mathbf{E}_{mot}$ is the strength of electric field induced by the variation of magnetic field in time

$$\mathbf{E}_{mot} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi\,, \tag{10}$$

and $\mathbf{E}_{mov}$ – the strength of electric field induced by the loop motion

$$\mathbf{E}_{mov} = -\frac{1}{c}\mathbf{B}\times\mathbf{v}. \tag{11}$$

Faraday investigated the induction phenomenon using conductors (Faraday 1994). According to Wilson and Wilson (1913), in case of dielectrics the strength of electric field induced by the loop motion is equal to $\mathbf{E}=\alpha\mathbf{E}_{mov}$,

where

$$\alpha = \frac{\mu\varepsilon - 1}{\mu\varepsilon}\,, \tag{12}$$

here ε is a dielectric constant and μ – magnetic permeability.

So, for the general case of conductors and dielectrics (9) is written in the following form:

$$\mathbf{E} = \mathbf{E}_{\mathrm{mot}} + \alpha \mathbf{E}_{\mathrm{mov}} \tag{13}$$

(in case of conductors α = 1, because $\mu\varepsilon \to \infty$).

By means of coefficient α it is possible, in particular, to take an approximate account of probable presence of impurities in plasma.

The differential form of (13), being a generalization of (6), looks like

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{c}\nabla \times (\alpha \mathbf{B} \times \mathbf{v})\ . \tag{14}$$

We integrate equation (2) taking into account all possible types of currents. Using the known descriptions of currents (see, e.g. Becker 1933; Panofsky and Phillips 1955; Pauli 1958), as a result of integration of (2) we have in a general case

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j} + \frac{4\pi}{c}\rho \mathbf{v} + \frac{1}{c}\nabla \times (\alpha \mathbf{D} \times \mathbf{v})\ , \tag{15}$$

where **D** is vector of electric induction; **j** – conduction current density vector and ρ –density of free-moving charges.

The last two terms of the right-hand side of (15), that are absent in Maxwell equations in stationary media, consider the magnetic field induction by a convection current of charges moving together with the conductor (Rowland current) and a current that appears at the motion of a dielectric in the electric field (Roentgen current).

Within Maxwell theory (Maxwell 1891) equations (14)–(15) should be appended with

$$\nabla \cdot \mathbf{D} = 4\pi\rho\ , \quad \nabla \cdot \mathbf{B} = 0\ , \tag{16}$$

as well as the equations of state (constitutive equations), which in media with linear parameters have the form

$$\mathbf{D} = \varepsilon \mathbf{E}\ , \quad \mathbf{B} = \mu \mathbf{H}\ , \quad \mathbf{j} = \sigma \mathbf{E}\ , \tag{17}$$

where σ – conductivity.

Unlike the corresponding Minkowski equations, the right-hand sides of (17) have no members including the velocity. In presented equations the induction due to motion is described in the field equations (14)–(15).

Equations (14)–(17) constitute Maxwell equations of electrodynamics under conditions of slow moving media. There are 17 scalar equations for 16 unknowns. Thus, one of the equations ($\nabla \cdot \mathbf{B}$=0) represents an additional limiting condition.

4. Boundary conditions for Maxwell equations

On the basis of equations (14)–(17) we obtain for a general case of a moving discontinuity surface the boundary conditions which we will write as follows

$$\mathrm{Rot}\,\mathbf{E} = \left(\mathbf{E}_{2\tau} - \mathbf{E}_{1\tau}\right) = \frac{\alpha}{c}\left(\left(\mathbf{v}\times\mathbf{B}\right)_{2\tau} - \left(\mathbf{v}\times\mathbf{B}\right)_{1\tau}\right), \tag{18}$$

$$\mathrm{Rot}\,\mathbf{H} = (\mathbf{H}_{2\tau} - \mathbf{H}_{1\tau}) = \frac{4\pi}{c}(\mathbf{j}_s + \rho_s\mathbf{v}_s) - \frac{\alpha}{c}\left((\mathbf{v}\times\mathbf{D})_{2\tau} - (\mathbf{v}\times\mathbf{D})_{1\tau}\right), \tag{19}$$

$$\mathrm{Div}\,\mathbf{D} = \mathbf{D}_{2n} - \mathbf{D}_{1n} = 4\pi\rho_s, \tag{20}$$

$$\mathrm{Div}\,\mathbf{B} = \mathbf{B}_{2n} - \mathbf{B}_{1n} = 0, \tag{21}$$

where Rot and Div are symbols of surface curl and surface divergence, respectively; indices 1 and 2 belong to different (in relation to normal n) sides of the boundary; τ is the tangent direction of the boundary; $\mathbf{j}_s$ and ρ_s – boundary surface densities of the current vector and charge, respectively, and $\mathbf{v}_s$ – the vector of boundary velocity.

Let us compare boundary conditions (18)–(19) with the boundary conditions resulting from Minkowski equations for slowly moving media (Landau et al. 2004; Tamm 1979)

$$\mathrm{Rot}\,\mathbf{E} = \left(\mathbf{E}_{2\tau} - \mathbf{E}_{1\tau}\right) = \frac{\mathbf{v}_n}{c}\left(\mathbf{B}_2 - \mathbf{B}_1\right), \tag{22}$$

$$\mathrm{Rot}\,\mathbf{H} = \left(\mathbf{H}_{2\tau} - \mathbf{H}_{1\tau}\right) = \frac{4\pi}{c}\mathbf{j}_s - \frac{\mathbf{v}_n}{c}\left(\mathbf{D}_2 - \mathbf{D}_1\right), \tag{23}$$

$$\mathrm{Div}\,\mathbf{D} = \mathbf{D}_{2n} - \mathbf{D}_{1n} = 4\pi\rho_s, \tag{24}$$

$$\mathrm{Div}\,\mathbf{B} = \mathbf{B}_{2n} - \mathbf{B}_{1n} = 0, \tag{25}$$

where $\mathbf{v}_n$ is the projection of the boundary velocity vector on the normal.

The differences of the boundary conditions correspond to distinctions in the equations (boundary conditions (20)–(21) and (24)–(25) coincide together with the corresponding equations).

5. Analysis of Maxwell equations in slow moving media

For convenience of the further analysis we will transform the received equations. By substituting (14) and (15) with (16) and (17), after obvious transformations basic field equations will look as follows:

$$\nabla\times\left(\mathbf{E} + \frac{1}{c}\times\left(\alpha\mathbf{B}\times\mathbf{v}\right)\right) = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \tag{26}$$

$$\nabla\times\left(\mathbf{B} - \frac{\mu\varepsilon}{c}\times\left(\alpha\mathbf{E}\times\mathbf{v}\right)\right) = \frac{\mu\varepsilon}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi\mu}{c}\mathbf{j} + \frac{4\pi\mu}{c}\rho\mathbf{v}. \tag{27}$$

One can see that in case of medium movement absence the obtained equations will transform to the known Maxwell equations in stationary media. Let us note necessity of presence of factor α in terms of the equations containing medium velocity.

At medium depression the value of these terms will decrease, thanks to reduction in α, up to zero in emptiness (because $\mu\varepsilon \to 1$) that corresponds to physics of the phenomenon. The member describing Rowland current and containing medium velocity will be disappearing in these conditions due to medium charge density reduction.

Also, it should be noted that the derivation of the Maxwell equations does not require the use of additional relation to account for the induction due to motion — the left part of equation (26) is obtained without use of the Lorentz force equation.

Einstein (1905) wrote that Maxwell electrodynamics when applied to moving bodies, leads to asymmetry in the reciprocal electrodynamic action of a magnet and a conductor which isn't inherent in the phenomenon. It is true concerning Maxwell equations in the stationary medium (considered by Einstein) which do not describe electrodynamic phenomena due to motion. The problem was solved within the theory of relativity. The offered Maxwell equations which are received within the frame of classical mechanics describe electrodynamic phenomena due to motion and one can see that these equations lack asymmetry mentioned by Einstein.

Let us show Galilean invariance of the obtained Maxwell equations. We show the invariance of the equations relating to coordinate system, moving rectilinearly and in regular intervals relating to initial system with certain velocity **w**:

$$\mathrm{x}_{1\mathrm{i}} = \mathrm{x}_\mathrm{i} - \mathrm{w}_\mathrm{i}\,\mathrm{t},\ \mathrm{i} = 1,2,3, \tag{28}$$

where x_i and x_{1i} are coordinates of the initial coordinate system and new coordinate system moving relative to it (axes of both systems are parallel), respectively; w_i are components of vector **w** and t is time in both systems.

In the vector form equation (28) looks like

$$\mathbf{v}_1 = \mathbf{v} - \mathbf{w}, \tag{29}$$

where **v** and $\mathbf{v}_1$ are velocities of medium relating to initial and new coordinate system, respectively.

One has within the limits of the classical mechanics $\mathbf{E} = \mathbf{E}_1,\ \mathbf{B} = \mathbf{B}_1$.

Let us write, for example, equation (26) in new coordinate system

$$\nabla_1 \times \left(\mathbf{E_1} + \frac{1}{c} \times \left(\alpha \mathbf{B}_1 \times (\mathbf{v} - \mathbf{w})\right)\right) = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t}, \tag{30}$$

where the rotor ∇_1 is determined on coordinates of new system x_{1i}.

One can see that (30) and (26) have the same form.

Similarly, equation (27) keeps the form at Galilean transformations.

6. Comparison of the theory with experimental data

It is well-known that Maxwell equations in stationary media agree with corresponding experimental data. Let us analyze an agreement between experimental data and proposed Maxwell equations in slow moving media. Let us consider the following important cases.

1. An unipolar induction: generation of an electromotive force in a radial element of cylindrical permanent magnet evenly rotating round its axis. For these conditions we have formula (11) corresponding to known experimental data (see, e.g. Landau et al. 2004; Tamm 1979);
2. Propagation of a plane electromagnetic wave in a nonmagnetic slow moving medium.

For the specified conditions in the absence of charges and currents, from (14)–(17) we have

$$(1)\quad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} - \frac{1}{c}\nabla\times(\alpha\mathbf{B}\times\mathbf{v}),$$

$$(2)\quad \nabla\times\mathrm{B} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{1}{c}\nabla\times(\alpha\mathbf{B}\times\mathbf{v}),\tag{31}$$

$$(3)\quad \nabla\cdot\mathbf{D} = 0,\quad \nabla\cdot\mathbf{B} = 0,$$

$$(4)\quad \mathbf{D} = \varepsilon\mathbf{E}.$$

Because we are interested only in terms that are linear by the velocity, then for plane waves propagating in the $\boldsymbol{n}_0$ direction along the ξ coordinate it is possible to assume

$$\mathbf{v}\cdot\nabla = (\mathbf{v}\cdot\boldsymbol{n}_0)\frac{\partial}{\partial\xi} = -\frac{\mathbf{v}\cdot\boldsymbol{n}_0}{u_0}\frac{\partial}{\partial t},\tag{32}$$

where u_0 is the light speed in the stationary media, $u_0 = c/n_m$ and $n_m = \sqrt{\varepsilon}$ – the refraction index of the medium.

Taking into account (31)(3), we have

$$\nabla\times(\alpha\mathbf{B}\times\mathbf{v}) = (\alpha\mathbf{v}\cdot\nabla)\cdot\mathbf{B},\qquad \nabla\times(\alpha\mathbf{D}\times\mathbf{v}) = (\alpha\mathbf{v}\cdot\nabla)\cdot\mathbf{D}.\tag{33}$$

Substituting (32) and (33) into (31) (1)–(31) (2) and taking into account (31) (4), we find

$$\nabla\times\mathbf{E} = -\frac{1}{c}\left(1-\frac{\alpha}{u_0}\cdot\boldsymbol{n}_0\cdot\mathbf{v}\right)\frac{\partial\mathbf{B}}{\partial t},\qquad \nabla\times\mathbf{B} = \frac{1}{c}\varepsilon\left(1-\frac{\alpha}{u_0}\cdot\boldsymbol{n}_0\cdot\mathbf{v}\right)\frac{\partial\mathbf{E}}{\partial t}.\tag{34}$$

Calculating a curl from the first equation (34) and using the second equation (34), we obtain

$$\nabla^2\mathbf{E} = \frac{1}{u_0^2}\left(1-\frac{\alpha}{u_0}\cdot\boldsymbol{n}_0\cdot\mathbf{v}\right)^2\frac{\partial^2\mathbf{E}}{\partial t^2} \approx \frac{1}{u_0^2}\left(1-\frac{2\alpha}{u_0}\cdot\boldsymbol{n}_0\cdot\mathbf{v}\right)\frac{\partial^2\mathbf{E}}{\partial t^2}.\tag{35}$$

This wave equation corresponds to the propagation velocity

$$u = u_0\left(1-\frac{2\alpha}{u_0}\cdot\boldsymbol{n}_0\cdot\mathbf{v}\right)^{-1/2} \approx u_0 + \alpha\cdot\boldsymbol{n}_0\cdot\mathbf{v},\tag{36}$$

which to the accuracy within the terms of order $|\mathbf{v}|/c$ coincides with known experimental data (Panofsky and Phillips 1955; Tamm 1979; Tonnelat 1966).

7. Conclusion

The Galilean invariant equations of electrodynamics in slow moving media and the corresponding boundary conditions are derived on the basis of Maxwell theory within the frame of classical mechanics.

In case of medium movement absence the obtained equations transform to the known Maxwell equations for stationary media.

The derivation of the Maxwell equations in slow moving media does not require the use of additional relation to account for the induction due to motion – Lorentz force relation.

The received Maxwell equations lack asymmetry in the description of the reciprocal electrodynamic action of a magnet and a conductor.

A discrepancy is revealed between a physical model, which is the base of Minkowski equations, and known classical mechanics information: Faraday's experimental findings and Maxwell theory.

Minkowski equations describe phenomena of electromagnetic induction for slow moving media within the frame of theory of relativity correctly, but at the same time in Minkowski equations the term $\frac{\alpha\mu\varepsilon}{c^2}\frac{\partial(\mathbf{v}\times H)}{\partial t}$ appears, which does not have a physical nature. As a result, Minkowski equations are not an optimal instrument to carry out analysis of electromagnetic processes under non relativistic conditions of slow moving media.

Both the analysis and the comparison of the received Maxwell equations with experimental data prove the applicability of the offered equations in practice.

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