Microbes, Apples, Buttons and Bibles

A Dissertation on the Nature of a Mathematical System

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There are questions which loom large in one's mind as he stops to contemplate the nature of truth and the question of epistemology. Much has been written in this area and there has been considerable interest from the standpoint of theology. For example, questions involving proofs for the existence of God, the noetic effects of sin, antithesis theology and indeed constructing a Christian Philosophy of Education all concern themselves with the nature of truth and the capacity of man to attain it.

There has appeared much bombast and blunder from both the Christian point of view and the non-Christian perspective. We claim no authority in the field, nor originality; however, interest has caused us to investigate a very small area of human knowledge and seek to discern its nature. Let us consider the nature of mathematical truth. Is this "purest of the sciences" based upon self-evident truth, eternal principles with creation, or empirical data? Down through the history of thought all of these answers have been given.

Probably the earliest answer to this question was that the truth of mathematics rests upon self-evident truth in contrast to the hypothesis of empirical science. Truths of mathematics demand no empirical verification because they are in another realm - everyone accepts their truth because any questioning of simple mathematical facts would be absurd. 2 + 2 = 4. What foolhardy person could deny it? As this science came into the modern critical era, however, such a vague foundational basis was hardly satisfactory. There was a sense of restlessness and a cloud hung low over the science.

As dissatisfaction grew, a retreat was made from this once dogmatically asserted position to a more defensible ground holding that mathematics is the most general science. John Stuart Mill held that mathematics is an empirical science which differs from the other branches of empirical endeavor in two ways: its subject matter is more general and its testability and confirmation are higher. Because the experience of every succeeding generation adds overwhelming confirmation to the truth of mathematics, we have come to consider its theorems as qualitatively different from those for example of biology, but they are not.

At this point let me add that those who hold that 2 + 2 = 4 is true because the creator has so endowed the creation with a system and an order and this statement of mathematics conforms to that order, must be grouped with the above who hold to an empirical basis for mathematical truth.

It can be stated differently as follows: God created the universe in an orderly fashion and as we reverently contemplate his revelation in nature we are able to discern these eternal principles or order of which mathematical principles are an important group. VanTil would therefore say that there is a Christian mathematics. The regenerate will look differently at mathematics than the unregenerate who looks through colored glasses cemented to his face and therefore cannot see that these principles are innate in the created order finding their source in the Creator.

With all due respect to the reality of the antithesis as an ultimate principle, we would like to examine this view of the source or foundation of mathematical truth. Does it rest upon an empirical base or not? In pursuit of our answer we will follow Carl Hempel in his essay "On the Nature of Mathematical Truth" which appears in *Readings in the Philosophy of Science,* Feigl & Brodbeck. Take a typical example of an empirical hypothesis such as Newton's law of gravitation. From this hypothesis we make predictions – under conditions "a" result "b" will follow. When in actual fact we experience the fulfillment of this hypothesis, i.e., when we see conditions "a" followed by results "b", we say the hypothesis is confirmed. Continual confirmation causes the hypothesis to take on the status of law. However, suppose we find conditions "a" followed not by the results "b" but results "c", we say, the hypothesis is disconfirmed. If repeatedly conditions "a" are not followed by results "b" as was originally hypothesized, we throw out the hypothesis and set out to discover the results of conditions "a" anew. This is the hypothetical deductive method of modern science.

Now let us move into the realm of mathematics. We hypothesize that 3+2 = 5. "If this is actually an empirical generalization of past experiences, then it must be possible to state what kind of evidence would oblige us to concede the hypothesis was not generally true after all." (p. 149) But this we never find in the area of mathematics. Take the following example which Hempel gives:

> "We place some microbes on a slide, putting down first three of them and then another two. Afterwards we count all the microbes to test whether in this instance 3 and 2 actually added up to 5. Suppose now that we counted 6 microbes altogether. Would we consider this as an empirical disconfirmation of the given proposition, or at least as a proof that it does not apply to microbes? Clearly not; rather we would assume we had made a mistake in counting or that one of the microbes had split in two between the first and second count. But under no circumstances could the

> phenomenon just described invalidate the arithmetical proposition in question, for the latter asserts nothing whatever about the behavior of microbes; it merely states that any set consisting of 3 + 2 objects may also be said to consist of 5 objects. And this is so because the symbols "3 + 2" and "5" denote the same number. They are synonymous by virtue of the fact that the symbols "2", "3", "5" and "+" are defined (or tacitly understood) in such a way that the above identity holds as a consequence of the meaning attached to the concepts involved in it. The statement that 3 + 2 = 5, then, is true for similar reasons as, say, the assertion that no sexagenarian is 45 years of age. Both are true simply by virtue of definitions or of similar stipulations which determine the meaning of the key terms involved." (p. 150)

Thus we see that there need be no empirical validation for this statement for the fact that the symbols "3" and "2" added together equal the symbol "5" is true whether or not there are any objects such as apples or buttons or microbes to which to attach the symbols. Indeed we see the mathematical statement 3 + 2 = 5 to be devoid of all empirical content as it stands in itself. Such a statement is called a true *a priori* or an analytic statement, thus indicating "that their truth is logically independent of or logically prior to any experiential evidence." (p. 150)

The above assertions may fall upon deaf ears because one might object that the very method of teaching arithmetic is through empirical objects. The child is not taught 3 + 2 = 5, but rather 3 apples plus 2 apples equals 5 apples. Therefore one might conclude that the empirical data used in teaching is an integral part of the system. However, we would counter that this is to confuse the logical and the psychological basis. Psychologically speaking there may be an empirical basis for arithmetic but as the mind matures the apples are dropped and one thinks abstractly, 3+2 = 5. The fact that 3 + 2 = 5 does not stand in need of empirical validation is because it is a logical statement concerned with manipulating symbols. The analytic statement does not convey factual information. The example used above that no sexagenarian is 45 years of age is an example of this. This statement cannot in any way conflict with any factual data – it conveys no information. It is true simply because its key terms are defined in a certain way.

We might further buttress our position by referring to geometry. That a straight line is the shortest distance between two points was always thought to be self-evident and also it could be demonstrated. Again, however, scholars were not satisfied and finally had to realize that Euclidian Geometry which had always been considered to be a picture of reality as it is, was seen to be but one of many possible pictures of space. Riemann constructed other systems logically consistent and capable of explanation. Here again a discipline had to be purged of empirical content and realize it was dealing with pure relations.

Coming back to mathematics: is there Christian mathematics? We hardly think so. Mathematics is a logical system, a purely relational discipline, a manipulation of symbols which are given definitions and which function as a tool which is applied to empirical content but which itself is devoid of content. How to pour Christian content into second grade arithmetic? Substitute Bibles for apples.