4.1.1 Exercises

In Exercises 1 - 18, for the given rational function \( f \):

- Find the domain of \( f \).
- Identify any vertical asymptotes of the graph of \( y = f(x) \).
- Identify any holes in the graph.
- Find the horizontal asymptote, if it exists.
- Find the slant asymptote, if it exists.
- Graph the function using a graphing utility and describe the behavior near the asymptotes.

1. \( f(x) = \frac{x}{3x - 6} \)
2. \( f(x) = \frac{3 + 7x}{5 - 2x} \)
3. \( f(x) = \frac{x}{x^2 + x - 12} \)
4. \( f(x) = \frac{x}{x^2 + 1} \)
5. \( f(x) = \frac{x + 7}{(x + 3)^2} \)
6. \( f(x) = \frac{x^3 + 1}{x^2 - 1} \)
7. \( f(x) = \frac{4x}{x^2 + 4} \)
8. \( f(x) = \frac{4x}{x^2 - 4} \)
9. \( f(x) = \frac{x^2 - x - 12}{x^2 + x - 6} \)
10. \( f(x) = \frac{3x^2 - 5x - 2}{x^2 - 9} \)
11. \( f(x) = \frac{x^3 + 2x^2 + x}{x^2 - x - 2} \)
12. \( f(x) = \frac{x^3 - 3x + 1}{x^2 + 1} \)
13. \( f(x) = \frac{2x^2 + 5x - 3}{3x + 2} \)
14. \( f(x) = \frac{-x^3 + 4x}{x^2 - 9} \)
15. \( f(x) = \frac{-5x^4 - 3x^3 + x^2 - 10}{x^3 - 3x^2 + 3x - 1} \)
16. \( f(x) = \frac{x^3}{1 - x} \)
17. \( f(x) = \frac{18 - 2x^2}{x^2 - 9} \)
18. \( f(x) = \frac{x^3 - 4x^2 - 4x - 5}{x^2 + x + 1} \)

19. The cost \( C \) in dollars to remove \( p\% \) of the invasive species of Ippizuti fish from Sasquatch Pond is given by

\[
C(p) = \frac{1770p}{100 - p}, \quad 0 \leq p < 100
\]

(a) Find and interpret \( C(25) \) and \( C(95) \).
(b) What does the vertical asymptote at \( x = 100 \) mean within the context of the problem?
(c) What percentage of the Ippizuti fish can you remove for $40000?

20. In Exercise 71 in Section 1.4, the population of Sasquatch in Portage County was modeled by the function

\[
P(t) = \frac{150t}{t + 15},
\]

where \( t = 0 \) represents the year 1803. Find the horizontal asymptote of the graph of \( y = P(t) \) and explain what it means.
21. Recall from Example 1.5.3 that the cost $C$ (in dollars) to make $x$ dOpis media players is 
$C(x) = 100x + 2000$, $x \geq 0$.

(a) Find a formula for the average cost $\bar{C}(x)$. Recall: $\bar{C}(x) = \frac{C(x)}{x}$.
(b) Find and interpret $\bar{C}(1)$ and $\bar{C}(100)$.
(c) How many dOpis need to be produced so that the average cost per dOpi is $200$?
(d) Interpret the behavior of $\bar{C}(x)$ as $x \to 0^+$. (HINT: You may want to find the fixed cost $C(0)$ to help in your interpretation.)
(e) Interpret the behavior of $\bar{C}(x)$ as $x \to \infty$. (HINT: You may want to find the variable cost (defined in Example 2.1.5 in Section 2.1) to help in your interpretation.)

22. In Exercise 35 in Section 3.1, we fit a few polynomial models to the following electric circuit data. (The circuit was built with a variable resistor. For each of the following resistance values (measured in kilo-ohms, $k\Omega$), the corresponding power to the load (measured in milliwatts, $mW$) is given in the table below.)

<table>
<thead>
<tr>
<th>Resistance: ($k\Omega$)</th>
<th>1.012</th>
<th>2.199</th>
<th>3.275</th>
<th>4.676</th>
<th>6.805</th>
<th>9.975</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power: ($mW$)</td>
<td>1.063</td>
<td>1.496</td>
<td>1.610</td>
<td>1.613</td>
<td>1.505</td>
<td>1.314</td>
</tr>
</tbody>
</table>

Using some fundamental laws of circuit analysis mixed with a healthy dose of algebra, we can derive the actual formula relating power to resistance. For this circuit, it is $P(x) = \frac{25x}{(x+3.9)^2}$, where $x$ is the resistance value, $x \geq 0$.

(a) Graph the data along with the function $y = P(x)$ on your calculator.
(b) Use your calculator to approximate the maximum power that can be delivered to the load. What is the corresponding resistance value?
(c) Find and interpret the end behavior of $P(x)$ as $x \to \infty$.

23. In his now famous 1919 dissertation The Learning Curve Equation, Louis Leon Thurstone presents a rational function which models the number of words a person can type in four minutes as a function of the number of pages of practice one has completed. (This paper, which is now in the public domain and can be found [here](#), is from a bygone era when students at business schools took typing classes on manual typewriters.) Using his original notation and original language, we have $Y = \frac{L(X+P)}{(X+P)+R}$ where $L$ is the predicted practice limit in terms of speed units, $X$ is pages written, $Y$ is writing speed in terms of words in four minutes, $P$ is equivalent previous practice in terms of pages and $R$ is the rate of learning. In Figure 5 of the paper, he graphs a scatter plot and the curve $Y = \frac{216(X+19)}{X+148}$. Discuss this equation with your classmates. How would you update the notation? Explain what the horizontal asymptote of the graph means. You should take some time to look at the original paper. Skip over the computations you don’t understand yet and try to get a sense of the time and place in which the study was conducted.

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