The Center for Elementary Mathematics and Science Education (CEMSE) at the University of Chicago is conducting a research and development project called *Number Stories*. (For a full overview of the project, see Number Stories (2015) or Flanders and Rich (2014).) Number stories are mathematics application questions based on real-world contexts supported by factual sources. A major challenge for an author, when writing a number story, is to design a dynamic mathematics activity that engages learners in the problem-solving process.

This challenge naturally raises the question, “What engages the learner?” For this paper, I define an *engaging activity* as one that captures the learners’ interest and supports prolonged mathematical thought or action. What features of a problem or activity lead to this kind of engagement? Are there engagement-related principles we can use to guide future activity development?

To begin exploring these questions, the Number Stories authors developed activities that exhibited a variety of formats and features and began testing them with interested students and adults. In October 2014, a group of undergraduate students became our first set of field testers not affiliated with the project. Out of eight available problems, two problems were the most engaging. Both problems captured the interest of many students, and for each of the two, a handful of students kept working on the problem even as the group moved on.

The first of the engaging problems is called **Buying Tech Stocks**. In this problem, learners are given the opening share prices of four social media stocks and asked to find combinations of shares that can be bought with $5,000. They are challenged to get as close to $5,000 as possible without going over. Learners enter their combinations by dragging sliders to indicate the number of shares for each stock. Learners submit an answer by pressing a check button. The total amount spent on the indicated shares is shown, along with a message about whether the total is over or under $5,000. Figures 1a-c show the starting state of the activity, the result of entering an answer that overspends, and the result of entering an answer that underspends, respectively.
Buying Tech Stocks

Twitter is one of many social media sites. Other publicly traded social media companies include Facebook, LinkedIn, and Sina. Below are the opening prices of these companies' stocks on July 30, 2014. At these prices, what combinations of stocks could you buy with $5,000? Drag the sliders to the number of shares or each stock you want to buy. Try to get as close to a total cost of $5,000 as you can without going over.

<table>
<thead>
<tr>
<th>Price of 1 Share</th>
<th>Number of Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>twitter $47.01</td>
<td>0</td>
</tr>
<tr>
<td>facebook $74.21</td>
<td>0</td>
</tr>
<tr>
<td>LinkedIn $182.10</td>
<td>0</td>
</tr>
<tr>
<td>sina $50.53</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 1a: The starting state of the Buying Tech Stocks problem.

Figure 1b: Feedback on an answer that overspends.
The second problem is called Shortest Route. This problem asks solvers to place a hotel and a museum on a map of Washington, D.C. and then find the shortest route between them along roads. Solvers can measure routes by placing segments on the map and measuring their lengths in centimeters. They use the map scale to convert the distance to miles. When they enter the converted length of their routes, students see feedback that compares that answer to the length of the shortest route found by the problem author. Figures 2a-c show the interface where students draw and measure routes, the feedback students receive for finding a longer route than the problem author, and the feedback students receive for finding a shorter route than the problem author, respectively.

What made these two problems so engaging? Do they have features in common that could be transferred to other problems? When analyzing dynamic activities, there are several categories of features worth examining:

- Features of the task, or the question solvers are trying to answer or the goal they are trying to accomplish.
- Features of the digital environment, or the interactive elements of the activity and tools provided to solvers.
- Features of the feedback, or information provided to solvers about their response in relation to the goal.
Shortest Route

Can you find the shortest route along streets from Hotel Lombardy to the Corcoran Gallery of Art? Use the tools to help you find and measure routes.

Figure 2a: Interface for the Shortest Route problem.

Shortest Route

Your route is 15 centimeters long on the map.

Here is the scale from the top left corner of the map:

Scale:
1 cm = 0.052 mi

How long is your route in miles? 0.78 mile(s)

There is an even shorter route!
Go back and see if you can find it.

Figure 2b: Feedback on a long route.
These categories overlap, but they were useful tools for organizing my thoughts about the features the engaging problems have in common. I made sense of the categories by using a metaphor: If the problem-solving process is a road trip, then the task is the destination, the digital environment is the car driven by the learner, and the feedback is the road signs that provide information about whether the learner is on the right track.

**The Destination: Engaging Features of Tasks**
The tasks associated with each of these problems had two commonalities that may have contributed to engaging students: a clear purpose from the learner’s perspective and a set of optimal solutions.

*The purpose is clear from the learner’s perspective.*
Problem developers, whether they are classroom teachers, researchers, or curriculum writers, tend to view the purpose of a problem in terms of the mathematical learning goals associated with it. Ainley, Pratt, and Hansen (2006) argue, however, that tasks planned tightly around mathematical objectives often do not lead to meaningful outcomes for the learner, and so a mathematical purpose is not sufficient to engage learners. They suggest that the purpose of a task, as perceived by learners, is “a distinct element that needs to be considered separately from, but in parallel with, learning objectives” (30).
The Buying Tech Stocks and Shortest Route tasks each have a clear purpose from the learner’s perspective. The intended outcomes of the tasks – to find a combination of stock shares with a total price near $5,000, and to find the shortest route along streets between two locations – are stated early and easy to comprehend. Moreover, the outcomes are meaningful to learners. It is not difficult to picture what a solution for each problem might look like or imagine why a solution might be useful. By contrast, a solution to a linear system or a converted map distance, which might have been the desired problem outcomes if the problems were written with attention only to mathematical objectives, are much less meaningful.

In his keynote address at the 2015 OAME Annual Meeting, Dan Meyer (2015) suggested that one strategy for better engaging students is to “turn back the mathiness dial” on tasks. He argued that the mathematics embedded in tasks does not need to be immediately apparent to students; indeed, immediate immersion in mathematics can be intimidating. I see this as related to considering purpose alongside mathematical objectives. If communicating purpose (from the learner’s perspective) is the first goal when presenting a task, it seems that learners interest will be captured more easily—an important and elusive aspect of engagement.

In short, it seems that engaging tasks make it clear to learners where they are going and why. For many people it can be disconcerting to set out on a road trip without a clear idea where they are headed and the reason for the trip.

**There are many solutions, but a few are optimal.**
The second common feature of the two tasks is that they involve optimization. There are many solutions to each task that might be satisfactory. There are a large number of routes, for example, that will get you from the hotel to the museum, and there are many stock combinations you could buy and spend at least 90% of your $5,000. But there is only one shortest possible route (or maybe a handful) and only a few combinations of stocks with a total cost close enough to $5,000 that you can’t buy another share of anything.

Ainley, Pratt, and Hansen (2006) briefly note that optimization tasks seem to be engaging, but offer no specific reasons why this might be. In a review of the extensive body of literature on engagement, Fredricks, Blumenfeld, and Paris (2004) noted that classrooms that allowed learners to make choices were associated with enhanced engagement. Similarly, student perception of task challenge was also associated with engagement. One reason that optimization tasks are engaging might be that they provide a balance between choice and challenge. Since many solutions are satisfactory, there are many places to start working and many avenues of approach. In contrast, the existence of optimal solutions adds an element of challenge and engages learners in prolonged thought and effort.
Our field tests suggest that the optimal solutions need not be clearly defined for students to be engaged in finding them. Indeed, in the Shortest Route problem, Figure 2c shows that the feedback does not state that the author’s solution is necessarily the optimal one. Rather, it suggests that students check whether their route meets the requirement of traveling along roads, and then encourages discussion of routes in the review section of the Number Stories website.

Even when our field testers found that the software would not tell them with certainty when they had found the shortest route, they continued to work on the problem and compare solutions. In the same keynote address referenced above, Meyer (2015) suggested that creating controversy is another possible strategy for promoting engagement. Students become invested in the problem, he said, when they have something to debate. Our field test of the Shortest Route problem seems to support this notion.

In short, it seems that while a clear purpose helps to capture interest, optimization tasks help to sustain interest and effort. Together, these two task features seem to promote engagement and are therefore worthy of investigation as design principles.

The Car: Engaging Features of the Digital Environment
After presenting a task to learners, a Number Stories author must also decide what tools and models to provide in the digital environment. Or, sticking with the road trip metaphor, after presenting the destination to learners, a Number Stories author must decide what kind of vehicle to recommend. What do the cars students drive to the Buying Tech Stock and Shortest Route destinations have in common? There are two common features of the digital environments for these activities that may relate to engagement.

The interactive elements are tools for solving the problem.
Because one goal of the Number Stories project is create activities that are not possible in a print environment, many of the activities created during our exploratory period had draggable or otherwise interactive elements. We were interested in finding out what students did with these elements, with a goal of identifying productive uses of the digital environment.

At first glance, the interactive elements in Buying Tech Stocks and Shortest Route seem quite different. The sliders in the Buying Tech Stocks problem (Figure 1a) are the mechanism for entering and adjusting answers. Learners get feedback based on the slider positions. In the Shortest Route problem (Figure 2a), the segment and measuring tools are useful for finding an answer and the marked route is part of the solution, but the feedback is given based on the numbers entered in the input boxes. Ergo, the interactive elements function as stepping-stones, not as mechanisms for entering a final answer.
It is tempting to conclude that these problems contain two different engaging features, rather than a shared one. However, contrasting these problems with one that fell flat in terms of engagement elucidates an important shared feature.

Consider the Height of Floodwater problem shown in Figures 3a-b. In this problem, students are asked to imagine that all the ice that melts off the Greenland Ice Sheet in one year flooded a particular city. The task is to find the height of the floodwater. The red point on the vertical axis is draggable. Students move it to show their prediction for the height of the floodwater before solving the problem numerically (Figure 3a). After finding the correct height, students are shown the actual height of the floodwater, as seen in blue in Figure 3b. As the designer of this problem, I thought that the visual representation of the correct answer, along with the opportunity to compare their predictions to the correct answer, could be powerful for learners.

The student response to this problem was not at all what I imagined it would be. Indeed, students did not find the activity interesting or engaging at all. Many students absent-mindedly dragged the red point up and down without much purpose, and even those who seemed thoughtful about the placement of the red point quickly lost interest once the input box appeared. It was quite a contrast from the way students interacted with Buying Tech Stocks and Shortest Route.

This and similar comparisons made the engaging feature of the successful problems clear to me. The interactive elements in Buying Tech Stocks and Shortest Route are tools for solving the problem. Dragging the sliders and checking the total cost allows learners to zero in on an appropriate combination of shares. Finding route lengths allows students to eliminate routes that are longer than one they have already found. While it is possible to use the tools in ways that are not helpful in solving the problem, in both cases, the tools provide an intuitive and productive way to begin. Keren-Kolb (2013) describes this feature as a use of technology that supports a shift from passive thought to active thought.

In contrast, the draggable red point in the Height of Floodwater problem does nothing to help students solve the problem. The visual representations of the estimate and the correct answer may have some value in terms of promoting understanding. However, there is no reason to believe they add any value in terms of engagement. The draggable point is a tool for recording a prediction. However, when it comes to doing the mathematics to find the actual height of the water, learners are left on their own. The tool on the screen is not a tool for solving the problem.
**Height of Floodwater**

The green square represents an area of 119 square kilometers, which is about the size of Boston, Massachusetts, USA.

Imagine that all the ice that melts off Greenland in 1 year flooded this city. How high would the water be?

Drag the red point up to estimate how high you think the water will be. You can use the black reference points to help.

I'm done: 🔄

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Figure 3a: Prediction Tool in Height of Floodwater problem.

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**Height of Floodwater**

The green square represents an area of 119 square kilometers, which is about the size of Boston, Massachusetts, USA.

Imagine that all the ice that melts off Greenland in 1 year flooded this city. How high would the water be?

Drag the red point up to estimate how high you think the water will be. You can use the black reference points to help.

You estimated the height of the water to be 2.9 km.

Calculate the height to the nearest tenth of a kilometer and enter it here.

2.1 km 🔄

Right! The water would be about 2.1 km high, 0.8 km off from your estimate!

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Figure 3b: Feedback on Height of Floodwater problem.
The tools remove the need for tedious calculation or meticulous procedures.

The tools in Buying Tech Stocks and Shortest Route have another common feature: they remove some tedium. The sliders and displayed totals in the Buying Tech Stocks problem relieve students of the need to perform a large number of decimal multiplication problems by hand or even enter them into a calculator. The measuring tools in the Shortest Route problem remove the need to be meticulous about where to start and end segments or line up the zero mark on a ruler. Misplaced endpoints can easily be dragged to more appropriate positions. Length measurements are found simply by clicking on segments, and the measurements change dynamically when the endpoints are moved. Because the tools free learners of the burdens of carrying out procedures, they are free to focus on deciding what steps to take to solve the problem. I believe that this is part of what kept field testers engaged with these problems.

Ainley, Pratt, and Hansen (2006) explain the value of such tools by discussing utility as a dimension of understanding that should be addressed along with procedural and relational understanding. Students may not be developing proficiency with procedures when solving problems with tools that remove procedural burdens. However, they are building another equally important kind of understanding about why particular mathematical skills and procedures are useful. Ainley, Pratt, and Hansen point out that understanding of procedures and relationships between mathematical concepts has traditionally been taught first, with the utility of concepts taught last, if at all. They argue that the advent of technology tools allows learners to develop an appreciation of the utility of a particular mathematical technique before tackling its related concepts and procedures.

According to Ainley, Pratt, and Hansen (2006), activities that focus on the utility of a mathematical idea are more engaging than those that focus on procedures. Our field tests seem to support this notion. I speculate further that starting with a problem the embeds calculation tools and allows learners to appreciate the utility of a mathematical concept might lead to better engagement with later activities that are aimed at developing procedures. In the same keynote address referenced above, Meyer (2015) suggested that another way to engage students is to create a headache for which mathematics is the aspirin. Using his metaphor, one might say that posing problems like Buying Tech Stocks and Shortest Route create a headache, and allowing students to use tools to administer the mathematical aspirin may motivate students to learn how the aspirin works.

The Road Signs: Engaging Features of Feedback

One of the most powerful features of a digital environment is the ability to provide multiple types of personalized feedback in real time. Useful feedback is like a road sign: It tells you something about where you are and where you are headed. We have experimented with different types of feedback in number stories. Field tests and literature on feedback suggest that two particular features of feedback are related to better engagement.
There is an easy way to track progress.
In a review of the extensive body of research on feedback, Hattie and Timperley (2007) say that learners are best able to use feedback when it provides information about the gap between the goal and the current state. This means that effective feedback provides more information than just whether a learner’s answer is correct or incorrect. It tells learners something about how far away they are from a correct answer.

The feedback in both the Buying Tech Stocks and Shortest Route problems provides a comparison of the submitted answer with a benchmark. In the case of Buying Tech Stocks, the benchmark is the $5,000 spending cap. In Shortest Route, the benchmark is the length of the shortest route found by the problem author. Although these benchmarks are not the goals of the tasks in and of themselves—the tasks are not to spend exactly $5,000 or to match the author’s route length—they are reasonable targets to aim for. The information about the gap between the learner’s answer and the benchmark can be used to track progress toward the goal.

Hattie and Timperley (2007) also point out that students are more likely to increase their effort—and therefore, I’d add, stay engaged—when they believe that they will eventually succeed in reaching the goal. Being able to track progress toward a goal has the potential to support that belief. If learners can deduce that they are getting closer to the goal, it may help them stay engaged with the problem. Learners are more likely to continue the road trip when posted signs tell them that they’re headed the right way.

The feedback allows for purposeful experimentation.
As noted above, tasks with a clear purpose from the learner’s perspective are often more engaging than those without a clear purpose. However, a task with a clear purpose doesn’t necessarily suggest a clear path toward achieving that purpose. Effective feedback can illuminate possible solution strategies, and the most engaging problems contain feedback that helps learners develop their own strategies.

For example, a learner who is unsure how to solve the Buying Tech Stocks problem could begin by dragging the sliders to any position, checking the total amount spent, and adjusting the numbers of shares accordingly. Similarly, a learner who has no intuitive sense for what may be the Shortest Route can begin by trying a few routes, finding their lengths, and attempting to deduce what the shorter routes have in common. In both cases, the feedback allows students to experiment with different answers and use the results to develop strategies for solving the problem.

Anderson (2007) relates strategy development to engagement not only with specific problems, but also with mathematics as a discipline. He notes, “when students are able to develop their own strategies and meanings for solving mathematics problems, they learn to view themselves as capable members of a community engaged in mathematics learning” (9).
Conclusion
Although more research is needed to make definitive claims, it seems clear that some problems are more engaging than others and that the more engaging problems have features in common. Further investigation of how intentional use of these features leads to a more engaging experience for learners will be a worthwhile pursuit for the Number Stories team and the mathematics education community as a whole.

References


