Jim Flanders and Katie Rich are curriculum developers representing the Digital Curriculum Group at the University of Chicago Center for Elementary Mathematics and Science Education (CEMSE). This paper presents their 9 October 2014 report on the group’s Number Stories Project given at the joint CabriWorld IV and IberoAmerican Cabri VII Conference held at the University of Medellin in Medellin, Colombia.

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The *Number Stories* Project is an exploration of how to build and organize dynamic mathematics materials on the web. Number stories are real-world questions based on real-world contexts supported by factual sources. Both contexts and questions are written for, and by, a wide variety of individual users such as school students at any level, teachers, teacher-educators, home-schoolers, district supervisors, curriculum developers, and others who are simply interested in how mathematics can be used to solve problems or model situations in their lives. The public home page is shown at right.
A premise of the project is that an individual’s actual curriculum is the set of problems he or she attempts to solve. To make such personalized curricula, users can write their own stories, or edit or use other people’s stories, in the online Number Stories environment. Traditional, print-based curricula understandably have to present material to an idealized “typical” student and are often based on an imposed scope and sequence of objectives. Personalized Number Stories problem sets, or clusters, can be built by sequencing problems from the collection and/or new problems written by the user to meet individual interests and needs. The screenshot at right shows an individual user’s home page with 3 clusters: Algebra Problems (opened to show its 3 problems), Favorites, and Sports Questions.
A digital collection of mathematics resources is not a unique idea. The online Kahn Academy (Kahn 2014), for example, is a collection of demonstrations of mathematical skills and properties supported by problems for students to practice those skills. It also helps students navigate between related skills, as shown in the screenshot of a concept map at right. We at the Number Stories Project applaud this work, along with some of the better apps and other notable websites that help students learn and practice skills.

However, we believe that an individual’s mathematical understanding is greatly improved by also applying those skills to solve relevant problems that s/he encounters in everyday life. Thus we are focusing on collecting and writing high-quality real-world problems that dynamically engage users in finding solutions. The screenshot at right shows a variety of arithmetic number stories available in the database.

When particular skills are used in a number story solution, we plan to link the user to places such as Kahn Academy to learn more about those skills. However, the main purpose of the Number Stories Project is to promote understanding about how mathematics is used in daily life, not simply abstract skill development.
Along with capitalizing on the dynamic digital environment, focusing on applications of mathematics is not a unique idea either. The University of Chicago School Mathematics Project Grades 6-12 textbooks (UCSMP 2014), Everyday Mathematics (EM 2014), and the many fine resources from the Consortium for Mathematics and Its Applications (COMAP 2001) have based mathematics curricula on applications for decades. And although they have had a variety of digital resources connected to their curricula during much of that time, the mathematical and pedagogical content of the curricula have been limited by the need to deliver the content in print form. Thus the digital resources of these projects have tended to remain ancillary to the content and relegated to enrichment or “if we have time” use.


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The *Number Stories* problems are web based. There is no expectation that they be downloaded, printed, and solved with pencil-and-paper techniques. In fact, problems that can be solved with only pencil-and-paper approaches are not good candidates for the *Number Stories* collection. This does not mean that problems have to include elaborate animations, for example, but it does mean that our project is committed to making each problem take advantage of a dynamic mathematics environment. Advantages include, but are by no means limited to, use of dynamic geometry and algebra tools, links to the source material of contexts and questions, intelligent feedback to user’s responses, and the opportunity for each user to rate and review other people’s contexts, questions, or solutions.
Yet even these goals of writing dynamic mathematics applications are not unique. The Building Better Math Project has developed a collection of applications of mathematics for tenth-graders in Canada (BBMP 2014). Unlike the Number Stories Project, the BBMP is aimed at supplementing existing curricula and standards and supporting classroom implementation of its lessons. However, the two projects are similar in that all problems are solved online. BBMP problems are built using the Maple engine, a venerable computer algebra system with a growing number of visual enhancements.

Although the *Number Stories* Project differs from the BBMP approach to curriculum development, we applaud their efforts to improve the digital problem-solving experience via the *Maple* engine. We are partnering with Cabrilog S.A.S., authors of the *New Cabri* dynamic-mathematics authoring application, to make a rich problem-solving environment that exceeds many of the capabilities of *Maple*, especially for younger users. *New Cabri* allows problem authors to manipulate objects in 2- and 3-dimensional environments and act on those manipulations within a comprehensive Boolean logic framework. For example, in the problem shown at right, a user can move copies of the animals in the group picture to the boxes on the lower part of the screen. S/he can click the green check button at any time to see which of the animals are in the proper sequence. Other buttons allow the user to reset the problem to its initial state or to link to information and image sources. These sources are included for all users to see the real-world connections for each problem.
In summary, the *Number Stories* Project is focused on addressing the four major needs at right that we think must be met in order to build quality digital curriculum. The remainder of this presentation is about our progress to date in addressing these needs.

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**NUMBER STORIES: Needs to be met**

A. *There is a need for an organized, searchable collection of interactive, digital problems for learning mathematics.*

B. *There is a need to develop personalized digital curricula from the ground up by building private collections, or clusters, of digital problems organized for and by individual users.*

C. *There is a need for quality dynamic applications of mathematics to complement the resources available for learning and practicing mathematical skills and properties.*

D. *There is a need for problems to address 21st-century mathematics made newly available, or more important than ever, by dynamic mathematical tools.*
### Phase 1 development

From roughly December 2013 through June 2014, Phase 1 of the Number Stories Project pursued all these aims simultaneously through the establishment of three teams: Platform developers, problem authors, and New Cabri activity developers. The main priority for Phase 1 was a proof-of-concept development of the platform in sufficient detail and with sufficient flexibility to support all four goals. This platform was built on schedule and rolled-out to all the teams in July 2014.

On the content development fronts, the main goals were to allow the authors and activity developers the time to explore what problem creation entails in the Number Stories environment. To that end there were minimal rules for problem development – that is, authors and activity developers alike were invited to do what they wanted within the general bounds of the goals described previously. Then, as exploration led to questions about policies and principles that might be shared across problems, sets of global guidelines began to evolve – a process that continues in Phase 2. Along with guidelines, specific desires of the teams also were identified, leading to some of the conclusions to follow, along with suggestions for more focused development efforts.

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**Phase 2 development**

From July 2014 through July 2015, Phase 2 of the *Number Stories* Project will continue development toward all the four primary goals but will turn its main attention to content development goals C and D: developing high-quality applications and problems that address 21st century mathematics. Priorities are initially set to deal with limits and challenges identified in Phase 1, but we expect that they will evolve to reflect new challenges as they are identified. The first beta-test of selected problems of the *Number Stories* platform and selected problems was at this 2014 CabriWorld and IberoAmerican Cabri conference.
Goal A progress

The Number Stories platform is now virtually complete. A user can search the database, launch problems, propose solutions, and propose new contexts and questions via rich text or PDF upload. Users can also review and rate problems. For example, the screenshot at right shows search results for contexts related to the theme of sports and fitness. The user can select any of the contexts to see the associated questions.
A problem view page for “Pool Positions” is shown at right. The user can launch the context and question, add a solution, write a review, or even add a new question to the context.

Limits to the platform currently lie in how the New Cabri engine is launched and how users might develop their own new dynamic problems via New Cabri templates in teacher mode and/or via original New Cabri activities in author mode. Dealing with these limitations requires negotiating with Cabrilog to further customize the CEMSE-Cabri version of the engine.
**Goal A next steps**

The features of the platform are being alpha-tested in preparation for field-testing and proposal development during Phase 2. We have begun adding additional features for the staff to help with content development and platform administration. It is possible that some field-testing will be aimed toward platform improvement. In fact, participants in the CabriWorld workshops were some of our first independent testers and provided welcome constructive criticism and advice.

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| *There is a need for an organized, searchable collection of interactive, digital problems for learning mathematics.* | • Alpha-test platform features  
• Add additional features for staff to manage content development and platform administration  
• Field-test platform |
**Goal B progress**

The *Number Stories* platform allows a user to make collections of contexts and/or problems called *clusters* that appear on the user’s homepage, called My Page. Every user has a “Favorites” cluster that can be used to collect favorite problems. New clusters can also be made from scratch. For example, the user with My Page at right created clusters of Algebra Problems and of Sports Questions. S/he could then search for more sports questions and add them to the latter cluster, or any other cluster for that matter.
Cluster items can be dragged to any desired order in My Page. The user here has added “Who is Stronger for Their Size?” to the Sports Questions cluster and is dragging it to second position in the cluster.
By clicking “Launch Cluster” on My Page, the items are presented to the user in a problem view without excessive distractions. For example, the options to add solutions or questions, manage clusters, or write reviews are not visible in problem view.

The problem that the user can launch and solve is at the top of the screen (“Who is Stronger for Their Size?” is shown at right), and the rest of the problems in the cluster are shown in the carousel below. The problems appear in the order that the creator of the cluster chose. However, solvers can work on them in any order, simply by clicking in the carousel.
Clusters have potential beyond individual use on My Page. Our first step to using the Number Stories platform for curriculum development is a Library of author-made clusters available to all users as shown at right. Here we have begun drafts of clusters that:

- Mimic a traditional lesson focused on adding and subtracting decimals.
- Explore the context of cork harvesting.
- Present a varied collection of problems aimed at the 15-18-year-old audience.
- Present preservice or inservice teachers with problems designed to help them learn about writing their own problems.
- Explore a sports theme, similar to the cluster described previously.

Our goal in the library is to demonstrate as wide a variety of cluster types as we can to show the versatility and broad appeal of the Number Stories platform.
**Goal B next steps**

We will make clusters to help deliver activities for field-testing and to demonstrate the platform to potential project supporters. We may also explore the feasibility of deconstructing existing textbook lessons as New Cabri activities in the Number Stories platform.
**Goal C progress**

In Phase 1 we assembled a group of authors, all experienced curriculum developers and/or teachers working part time, who began exploring how to develop *Number Stories* based on our context/question model – a model that has proven to be quite challenging.

Even given their limited schedules and geographic dispersion, the group is becoming a highly collaborative team able to both critique and support each other’s problem development and identify global issues of writing in the *Number Stories* environment. To date we have working drafts of several dozen rich problems and have made significant progress turning them into dynamic activities. The screen at right shows an example of early manuscript development of the cork context. Much like in journalism, authors are asked to clearly document the real-world basis for all their contributions.

**Goal C**  
There is a need for quality dynamic applications of mathematics to complement the resources available for learning and practicing mathematical skills and properties.
One of the bigger challenges to writing problems is incorporating open-ended problems into the single-user, online *Number Stories* environment. Because of a well-intentioned desire not to lead the problem solver to a specific solution, a common approach early in Phase 1 was to pose an open-ended question and only ask for a free response, as in the parrot problem at right. Two unfortunate consequences of this approach are that constructive feedback is not possible and it requires the solver to self motivate and attempt to solve the problem *before* reading the author’s solution. With luck the solver is motivated to make a solution attempt, but this same scenario could happen in a textbook with a solution manual and so does not justify the use of elaborate technology. And, perhaps, the solver may be even more likely to give up without even looking at the solution than in a classroom environment—a prediction that may be worth studying in field tests.

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**Do Parrots Have Names?**

Parrots use “contact calls” to reach other members of their group when they’re out of sight. Contact calls from different parrots sound different. Researchers at Universität Hamburg wanted to know whether parrots use different contact calls for different members of their flock.

They recorded the calls of 17 spectacled parrots “talking” to different members of their families, and made spectrograms of the calls. A spectrogram is a graph of the frequency of a call over time.

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**Different-Sounding Calls**

These spectrograms show Eddy’s calls to three members of his family. What differences do you observe between Eddy’s calls to the three other parrots?

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Goal C next steps

We need to keep exploring how to deal with the challenges identified in Phase 1 – specifically, how to engage the user in problem solving as part of the question. Overall, we plan to balance development of closed-ended and open-ended problems.

To address this objective of designing engaging problems the authors have tentatively outlined three approaches to problem design that we will investigate during Phase 2 and eventually study in field tests. To illustrate these approaches consider the context of the Peanuts comic strip at right.
A first approach is shown in “Chain, Chain, Chain” at right. This is a single, generally worded problem with distinct solutions provided. The solutions that each may require different mathematics experience to understand; for example, one solution might require knowledge of geometric series while another may require only basic knowledge of how to use a calculator. A benefit of this approach is that it may minimize the need to lead a problem solver down some specific solution path. A detriment is that it requires tagging the problem as suitable for a much wider target age range and so makes searching less effective.

A second approach is to design separate problems with different target group solutions in mind. This has the opposite benefit/detriment possibilities than the first approach; to make the problem focused on a particular solution, and thus a narrower target audience, the problem itself needs to in some way steer the user toward that solution. An example is the “How Many Letters?” question at right that leads the solver to use a geometric series formula – a solution probably out of reach for younger solvers.
A third approach is to write a single problem with distinct solution pathways embedded within the activity. This is by far the most sophisticated approach and certainly within the capabilities of New Cabri, but writing these activities is much more ambitious than either of the other approaches. The “Counting Letters” problem at right shows how this approach might be implemented by giving a solver the option to use a geometric series approach like the one in “How Many Letters?” above, an approach based on recognizing a pattern as hinted at in the drawing, or the option to be guided through a simpler problem.

To study these and other approaches to engaging problem solvers we are currently preparing a proposal for the U.S. National Science Foundation directorate for Advancing Informal STEM Learning.
**Goal D progress**

In Phase 1 a creative team of three activity developers began ascending the rather steep *New Cabri* learning curve, joined in late spring by Kate Mackrell, an educator with many years of experience using all the Cabrilog apps who has designed the remarkable activity shown at right. The globe can be rotated to see any continent, and the yellow plane can be made tangent to it at a choice of several cities then dragged through the globe to show the maximum distance a plane might travel from the city on a given amount of fuel.

Two CEMSE authors are also making significant progress learning to program and a couple of others are interested. Some *New Cabri* activity developers are also venturing into problem writing. Overall, it is encouraging to see the cooperation between authors and the *New Cabri* team continually improving.

**Goal D** There is a need for problems to address 21st-century mathematics made newly available, or more important than ever, by dynamic mathematical tools.
Goal D next steps

Limitations to New Cabri progress seem to be related to a mixture of technical difficulties inherent in this beta software (both the app and the web-launched plugin) and conceptual difficulties related to experience with the dynamic geometry processes that underlie all the New Cabri features. As dedicated as we are toward using New Cabri as our problem development engine, we must realistically continue to evaluate its place as the primary problem-writing engine in the Number Stories platform. Toward this end we are currently discussing with Cabrilog a prototype of how context and question can be embedded in the web page instead of launched with a plugin. Until we arrange funding to solve some of the technical problems we are willing to adjust to the limits they impose. For conceptual difficulties, we will continue to support our authors and activity developers as they learn to use New Cabri to its full potential.

We remain committed during Phase 2 to exploring how New Cabri can help us meet the research and development needs we have outlined in this presentation, focusing on engaging the problem solver through well-designed feedback and making more sophisticated use of the New Cabri dynamic mathematics tools. Your suggestions and other constructive criticism are warmly encouraged.

Goal D

There is a need for problems to address 21st-century mathematics made newly available, or more important than ever, by dynamic mathematical tools.

Next Steps

- Continue to explore New Cabri’s remarkable features for providing different types of feedback and dynamic geometry and mathematics tools for problem solving
- Evaluate New Cabri’s long-term place within the Number Stories platform

Jim Flanders/Katie Rich
Bibliography


