The Effectiveness of Cooperative Learning in Teaching Quantitative Reasoning With Ternary Diagrams in a Science Class

By Man-Yin Tsang, Lisa Tutty, and Carl-Georg Bank

Quantitative reasoning, although included in most science courses, can be challenging to teach. In this article, we explore whether cooperative learning may help instructors teach quantitative reasoning and enhance students’ understanding and learning experience. Our lesson was taught in a large introductory geoscience course. The lesson required the undergraduate students to process geological data, represent the processed data graphically in a ternary diagram, and interpret the results in terms of geological environments. Students were assigned to groups in which they were asked to either work in pairs (experimental group) or individually (control group) on the tasks.

Students’ performance on questions related to ternary diagrams on the test and their feedback in the evaluation survey indicate that the cooperative approach enhances the ability of freshmen and sophomores to apply the quantitative reasoning they learned to new problems. Most participants prefer learning in a cooperative setting rather than the individual approach. We suggest that cooperative learning can help develop quantitative reasoning in undergraduate science classes.
The study of ternary diagrams develops skills in calculations, plotting, and interpretation, making them a common and useful tool for teaching science students. However, we have noticed, as have other studies (Smosna & Bruner, 2007), that students generally do not perform well on ternary diagram–related questions compared with questions on other topics in tests. Therefore, we explore methods to enhance students’ understanding of ternary diagrams during their learning process.

Cooperative learning

Merits of cooperative learning include that students can obtain immediate feedback from their peers (Cortright et al., 2003); students can check their understanding by explaining their thoughts to one another (McKeachie & Svinicki, 2006) and are thus empowered to identify difficulties and ask questions to the instructor during class time; discussion, explanation, summarization, and exploration among students can lead to higher-level reasoning, conceptual understanding, and the use of new approaches (Hamm & Adams, 1992; Lunetta, 1990); and social interactions can enhance students’ motivation and participation (Lunetta, 1990; Showers & Cantor, 1985). Knierim and colleagues (2015) applied cooperative learning in geoscience classes and found improvement in students’ exam scores, especially for lower-achieving students. In addition to boosting understanding of the learning topic, cooperative learning possesses other advantages. Springer and colleagues (1999) studied students across science, technology, engineering, and mathematics (STEM) and found that cooperative learning increases students’ self-esteem. With students communicating and interacting, cooperative learning also trains students to collaborate productively (Jacob, 1999). This ability to work within teams matches modern businesses’ hiring preferences (Mandel, 2003; Marasi, 2019).

Although cooperative learning is widely applied, most studies about its effects on teaching quantitative reasoning are limited to learning by younger children (e.g., Kramarski & Mevarech, 2003; Slavin, 1989, 1995). We found few studies targeting college students (e.g., Abdullah & Shariff, 2008; Magel, 1998). The effect of cooperative learning on gaining quantitative reasoning seems understudied at the undergraduate level, likely because peer interaction is not usually an explicit learning outcome when instructors are designing lessons on quantitative reasoning at the college level. There are also concerns over lower-performing students relying on students who perform better instead of thoroughly understanding the skills themselves (Lunetta, 1990; Slavin, 1983). Our study is an attempt to address this gap in studies.

Teaching ternary diagrams: Challenges and the cooperative approach

Geoscience students in our department commented on previous course surveys that they found exercises using ternary diagrams difficult. We also noticed that students’ performance on exam questions related to ternary diagrams was noticeably poorer than their performance on other questions. This problem is not unique to our students. Smosna and Bruner (2007) also reported consistently lower grades on
assignments using ternary diagrams, mainly because students find the data confusing, misplot data, or cannot see a trend from the diagram. Factors that may contribute to these issues are that (i) students are unaware of the mistakes in their calculations, (ii) so they do not ask questions or seek help, and (iii) after the problem sets are marked, few students check the answer key to identify where they made mistakes. To address these issues, we incorporated cooperative learning into a practical session teaching ternary diagrams. This encourages students to cross-check methods with peers, notice mistakes during class time, and promptly seek help from peers or teachers. To avoid the problem of lower-performing students overly relying on students who perform better (Lunetta, 1990; Slavin, 1983), we designed the class exercise carefully so each student practices individually despite learning in a cooperative setting.

We incorporated cooperative learning in teaching ternary diagrams in the geoscience course ESS105H1, Our Home Planet, offered by the Department of Earth Sciences at the University of Toronto. A total of 341 students were enrolled in the course, and 199 agreed to participate in this study (Table 1). Among the students who agreed to disclose their majors, 80 students were from STEM majors and 75 were not (Table 1). Students were assigned (alphabetically according to their first name) to eight practical sessions that took place the same morning, covered the same topic, used the same teaching slides, and required students to finish a problem set. We carefully designed two versions of problem sets that covered similar tasks and calculations. Four practical sessions used a problem set that students could complete individually (the individual classes), and the other four sessions used a problem set that required students to work in pairs and combine data to complete (the cooperative classes). The problem set for Student A in the cooperative classes can be found in the Online Appendix (Figures A.1 and A.2). Four teaching assistants taught the eight practical sessions. To ensure the quality of this study, all teaching assistants attended the same briefing beforehand, followed the same lesson plan, and used the same teaching slides; two teaching assistants taught the individual classes before the cooperative classes, and the other two assistants taught the cooperative classes before the individual classes. Both classes taught by the same teaching assistant were similar in size.

In both the individual classes and the cooperative classes, students were taught to construct a ternary diagram based on the numbers of microplanktons, spores, and pollens from sediment samples at one location (Questions 1 through 3 in Online Appendix Figure A.2) and then use the diagram to interpret environmental changes (Questions 4 and 5 in Online Appendix Figure A.2). With the raw numbers given, students needed to normalize the data and plot points representing each sediment sample onto the microplankton-spore-pollen ternary diagram (Figure 1). We then showed them information from published research on how samples from different marine environments generally fall onto different parts of the ternary diagram (bottom of Online Appendix Figure A.1). For instance, a deltaic sample likely has a higher percentage of spores than microplankton and pollens, so it is plotted close to the spore corner of the diagram; an offshore sample likely has more microplanktons than spores or pollens, so it is plotted close to the microplankton corner of the diagram. With the completed ternary diagram, students needed to identify that the studied location turned from an offshore to a deltaic environment, implying that the water level dropped over time (Question 4, Online Appendix Figure A.2). Students then needed to use basic geological knowledge to answer that such changes were likely

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<th>TABLE 1: The numbers of student participants in the study.</th>
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<td>Number of students</td>
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<td>Participants disclosed majors in STEM disciplines</td>
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Note: Students with STEM majors included those in science and engineering disciplines, actuarial science, environmental studies, and kinesiology. Students with non-STEM majors included those in architecture, arts, cinema studies, philosophy, and social sciences. Students could choose not to disclose their majors or years of study for our statistical analyses.
accompanied by changes in the mineral and plant contents of the sediments (Question 5, Online Appendix Figure A.2).

Sample #1 was shown to all students as an example in the problem set. In the cooperative classes, students were advised to do Sample #2 with their partners (Question 1, Online Appendix Figure A.2). This step was intended to familiarize students with the calculation method and allow them to discuss their understanding and disagreements with their partners. Then Student A should plot Samples #3 through #5, and Student B should plot Samples #6 through #8 onto the ternary diagram (Question 2, Online Appendix Figure A.2). We deliberately designed the problem sets so Students A and B needed to work on different samples, which would prevent the issue of the lower-performing student solely relying on the higher-performing student solely relying on cooperative learning. Students A and B had to combine all eight points on the ternary diagram (Question 3, Online Appendix Figure A.2) to see the trend of data points moving from the microplankton corner toward the spore corner, representing the change of the coastal environment. They then needed to discuss further to classify their samples and comment on the sedimentary history based on their findings (Questions 4 and 5, Online Appendix Figure A.2).

In addition to teaching students the geological implications, this step also served as another peer check-in. It encouraged students to explain to peers their rationale and strengthen their ability to use the ternary diagram to make geological interpretations. In this exercise, students had opportunities to identify problems and misunderstandings with peers, explain their ideas to peers, settle disagreements, and promptly ask for clarification from the teaching assistant if needed.

The individual classes completed the problem set with a more traditional approach. After seeing Sample #1 as the example, individual students needed to plot four data points that showed the same trend toward the spore corner on the ternary diagram. Students had the option of discussing with their adjacent classmates during the practical session as they would in ordinary class settings.

**Evaluating students’ understanding**

During the practical sessions, teaching assistants discussed the contents of the problem sets thoroughly, and students could seek help. Consequently, students generally performed well. We do not see significant differences in students’ performance between the individual classes and the cooperative classes (both classes achieved an average score of 11 out of 12). To accurately evaluate students’ individual understanding of ternary diagrams, we had a test with six questions (Questions 31 through 36, Online Appendix Figures A.3 and A.4) based on ternary diagrams 1 week after the practical session. Questions 31 through 34 (Online Appendix Figure A.3) showed students the microplankton-spore-pollen diagram that they had seen in the practical session. Questions 31 and 33 required students to carry out the proper calculations and plotting, while Questions 32 and 34 tested students’ geological understanding. Questions 35 and 36 (Online Appendix Figure A.4) also tested students’ calculation and understanding, respectively, but students were shown a quartz-feldspar-plagioclase diagram that they had not seen before in the course.

The calculation and interpretation methods for the quartz-feldspar-plagioclase diagram were similar to the microplankton-spore-pollen diagram to test if students could adapt what they learned from the microplankton-spore-pollen diagram to other types of ternary diagrams.

We evaluated all participants’ performance statistically with the two-sided chi-squared test and Fisher’s exact test. It may not be suitable to perform the chi-squared test if 20% of expected numbers are smaller than 5. Therefore, whenever this situation applied, we referred to Fisher’s exact test to confirm the statistical significance of the results. We also divided participants into different categories according to their years (Years 1 and 2 or Years 3 and higher) and majors of study to identify category-specific observations. Students with STEM majors were in conventional majors in science and engineering (e.g., chemistry, civil engineering), as well as actuarial science, criminology, environmental studies, and kinesiology; students with non-STEM majors were in conventional majors in arts and social sciences (e.g., history, sociology, visual studies), as well as architecture, cinema studies, and philosophy. Participants in business disciplines had a variable degree of training in quantitative skills, so they are included in neither the STEM nor non-STEM categories.

We observed that participants from the individual classes and cooperative classes performed similarly on the test (Online Appendix Table A.1), suggesting that cooperative learning did not significantly enhance or lessen students’ understanding of the topic. However, if we look at students from Years 1 and 2 only, participants from cooperative
classes performed significantly better than those in individual classes on Question 36. Questions 35 and 36 challenged students to apply their skills to a ternary diagram they had not seen before in the course. Based on the calculation in Question 35, Question 36 asked students to make a classification with the ternary diagram. All first- and second-year students from cooperative classes who had correct calculations for Question 35 answered Question 36 correctly. However, only 86% of their counterparts from individual classes could answer Question 36 correctly (p value from Fisher’s exact test = 0.014). Such a difference is not observed in students in Year 3 or above. This suggests that the cooperative setting can benefit students earlier in their college careers when learning ternary diagrams. With a cooperative setting, freshmen and sophomores can better adopt the quantitative reasoning they learned for new topics and make diagram-based interpretations accordingly.

**Students’ perspectives**

After the practical sessions, we also issued students a survey for their comments on the learning mode. Previous studies suggest that cooperative learning boosts students’ self-esteem (Springer et al., 1999). When we applied cooperative learning to teaching ternary diagrams, the enhancement of students’ self-esteem was not obvious, whether we consider all participants or participants with STEM majors. Participants from the individual classes and cooperative classes felt similarly about the ease of the practical session and their confidence in using ternary diagrams in the future (Online Appendix Table A.2). An exception with marginal statistical significance is for students with non-STEM majors: A higher percentage of participants in individual classes felt confident in plotting ternary diagrams in the future than did those students in cooperative classes (p value from Fisher’s exact test = 0.048; Question 2 in Online Appendix Table A.2).

There were different responses for students in the individual classes and cooperative classes for the survey question asking whether students prefer working individually or as a group (Question 4 in Online Appendix Table A.2). Although the majority of both types of students preferred working as a group (57% of individual classes and 73% of cooperative classes), the individual classes had a significantly higher percentage of students who favored the individual setting (31%) compared with students in the cooperative classes who favored the individual setting (14%). This suggests that more students from the cooperative classes are open to the idea of using ternary diagrams in a group setting. This is especially noticeable among students with a major in STEM disciplines. Among students with STEM majors, 41% preferred to work individually and 50% preferred to work as a group in the individual classes. In comparison, only 9% preferred to work individually and 68% preferred to work as a group in the cooperative classes. Non-STEM students, by contrast, did not show significant differences between the preferences for individual classes and cooperative classes. This suggests that the learning environment may affect the openness toward cooperative work for STEM students more than for non-STEM students. Consequently, cooperative learning may help prepare STEM students for careers that require teamwork. However, drawing such inferences based on the survey question without knowing more about the students’ perceptions of cooperative settings and their prior experience working in teams can be overreaching. More studies are needed to test these hypotheses.

Students could also comment openly in the evaluation survey. Their comments help explain the majority’s preference for working with peers. Aligning with one of our goals of using cooperative learning, students think that working with peers helps their understanding:

*Working in a group helps me reassure that I’m learning information correctly and there are no mistakes in my work.*

*It’s helpful to discuss each question and find some mistakes earlier.*

*[It is] useful to check and confirm answers.*

*[We can] discuss answers and possibilities.*

*[Working with peers allows us] to benefit from each other’s knowledge.*

*[It is a] new concept, [so] easier if you are allowed to work with others.*

*[You] can work together, share ideas, make sure you are doing it correctly.*

*[This format] allows comparative analysis and reinforcement … allows group problem-solving and questions.*

*[There is] more support and guidance.*

Some participants prefer the group setting due to the level of ease they perceived, as these comments suggest:
Among par-...we designed the exercise: based on areas we considered when participants made their preference on working individually or cooperatively. One student in an individual class also highlighted the merit of cooperation among students across disciplines: “As a breadth course for some non-science students, it might benefit them to work with those who already know more of the concepts in this course.”

Next, we review comments from students in different categories. A higher percentage of non-STEM participants in individual classes felt confident in plotting ternary diagrams in the future than did those in cooperative classes (Question 2 in Online Appendix Table A.2). Some open comments from non-STEM students in individual classes suggest a possible reason for this confidence: Within the fixed time of the practical session, students in individual classes had more time to contemplate the exercise and plot the diagram, as they were not required to discuss and compromise with peers. Non-STEM students in individual classes possibly felt that they had adequate time to digest the plotting skills so they could confidently apply the skills again in the future. Here are some comments from non-STEM participants:

[I] needed some time to think through the problems.
[Working individually] gives me time to figure out where to plot the points on my own without my partner just showing me. I can go at my own pace.

I like to take my time in figuring things out—usually in a group, one person will tell everyone how to do it, and that doesn’t allow me to learn it on my own.

The difference in confidence between students in individual classes and those in cooperative classes was not observed among STEM students, possibly because participants from STEM majors required less time to feel confident with the plotting skills, so allocating some time to discussion did not hinder the buildup of their confidence. As we did not directly ask students about the elements that changed their confidence in plotting, further studies are needed to confirm the relationship between class time and confidence in plotting ternary diagrams.

We pay particular attention to the reasons why some students prefer the individual setting over the cooperative setting (Question 4 in Online Appendix Table A.2). Among participants in STEM majors who were in individual classes and preferred to complete similar tasks alone, three commented that they found the problem easy enough, so there was no need to discuss it with others. Other participants made their preference based on areas we considered when we designed the exercise:

I like having my own ideas and making sure I understand the content.

Doing the activity individually allows you to think of answers yourself.

It is more about individual learning.

It’s easier to understand plotting if you actively do it yourself.

These concerns from students can be handled with a careful design of the exercise. As mentioned in the introduction, we encourage idea sharing in cooperative learning, include sections for individual practices to ensure individual students’ understanding, and prevent lower-performing students from solely relying on students with stronger performance (see the section “Teaching Ternary Diagrams: Challenges and the Cooperative Approach”). Some students have a preference due to reluctance or personal conditions:

I can concentrate better alone.
[I] don’t like pairing up.
I prefer to work alone.
I feel like I work better individually.

Among students with STEM majors in cooperative classes, only four (9%) preferred working on similar tasks individually, and only one wrote down a detailed reason: “I work faster by myself.”

Interestingly, participants from various categories commented that they had a preference based on efficiency in completing the tasks:

It is more efficient when plotting the diagram. (student in cooperative classes who preferred the cooperative setting)

It is more efficient to do it on my own. (student in individual classes who preferred the individual setting)

[It] was quicker to split up work. (student in individual classes who preferred the cooperative setting)
Whether cooperative learning changes students’ efficiency in completing quantitative tasks is beyond the scope of the current study. The students’ feedback suggests that this would be a compelling topic for future research.

Our study looks at the effectiveness of cooperative learning for undergraduate students who are learning about ternary diagrams as a skill of quantitative reasoning. We hope to stimulate further discussions on whether we can teach quantitative skills without the traditional individualistic approach and include diverse competencies (e.g., collaboration, communication) as learning outcomes when teaching quantitative reasoning. Most businesses today prefer workers who can work both independently and within a group (Marasi, 2019). Collaboration is also common in scholarly research activities. College education fosters students who have both quantitative literacy and good cooperative skills. Instead of seeing these two strengths as separate learning goals to be covered in different teaching environments, there are compelling reasons to combine the two and create courses that can have a greater impact.

**Conclusion**

This study introduced cooperative learning when teaching quantitative reasoning with ternary diagrams in a university geoscience class. Our results suggest that cooperative learning enhances the ability of freshmen and sophomores to apply the skills they learned to new topics. In addition, the majority of student participants prefer working in a cooperative setting rather than an individual one.

We demonstrate that with carefully designed course materials, potential problems related to using cooperative learning for teaching quantitative skills (e.g., lower-performing students relying too much on students who perform better) can be avoided. This study encourages educators to consider explicitly addressing “soft” skills (including collaboration and communication) as supplementary learning outcomes alongside quantitative reasoning in science classes. Our study demonstrates that this does not compromise students’ understanding of quantitative reasoning. Cooperative learning can be a vehicle for students to learn and interact, thus expanding the impact of lessons on quantitative reasoning.

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**References**


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