Chapter 9
Capital Budgeting Decision Models

LEARNING OBJECTIVES (Slide 9-2)

1. Explain capital budgeting and differentiate between short-term and long-term budgeting decisions.
2. Explain the payback model and its two significant weaknesses and how the discounted payback period model addresses one of the problems.
3. Understand the net present value (NPV) decision model and appreciate why it is the preferred criterion for evaluating proposed investments.
4. Calculate the most popular capital budgeting alternative to the NPV, the internal rate of return (IRR); and explain how the modified internal rate of return (MIRR) model attempts to address the IRR’s problems.
5. Understand the profitability index (PI) as a modification of the NPV model.
6. Compare and contrast the strengths and weaknesses of each decision model in a holistic way.

IN A NUTSHELL...
In this chapter, the author explains the various capital budgeting techniques that can be used to make informed investment decisions involving productive assets such as plant and equipment, machinery, etc. In particular, 6 alternative evaluations techniques are covered including the payback period, the discounted payback period, the net present value (NPV) model, the internal rate of return (IRR) criterion, the modified internal rate of return model (MIRR), and the profitability index (PI). After illustrating and explaining in detail how each technique is to be used, the strengths and weaknesses of each decision model are summarized. This chapter sets the stage for the material in the next chapter which involves the forecasting and analysis of project cash flows.

LECTURE OUTLINE
9.1 Short-Term and Long-Term Decisions (Slides 9-3 to 9-4)

Long-term decisions, typically involve longer time horizons, cost larger sums of money, and require a lot more information to be collected as part of their analysis, than short-term decisions. The investment of funds into capital or productive assets, which is what capital budgeting entails, meets all three of the above criteria and therefore is considered a long-term decision. The efficacy of capital budgeting decisions can have long-term effects on a firm and are thus to be made with considerable thought and care. Three keys things to remember about capital budgeting decisions include:
1. A capital budgeting decision is typically a go or no-go decision on a product, service, facility, or activity of the firm. That is, we either accept the business proposal or we reject it.

2. A capital budgeting decision will require sound estimates of the timing and amount of cash flow for the proposal.

3. The capital budgeting model has a predetermined accept or reject criterion.

9.2 Payback Period  

(Slides 9-5 to 9-11)

This method, which is the easiest to interpret and use, simply tries to determine the length of time in which an investment pays back its original cost.

If the payback period is less than or equal to the cutoff period, the investment would be acceptable and vice-versa. Thus, its main focus is on cost recovery or liquidity.

The method assumes that all cash outflows occur right at the beginning of the project’s life followed by a stream of inflows. Furthermore, it assumes that that cash inflows occur uniformly over the year. Thus if need to recover $40,000 and we receive 3 cash inflows of $15,000, the method will calculate the payback period as being 2.67 yrs.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Yet to be recovered</th>
<th>Percent of Year Recovered/Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(10,000)</td>
<td>(10,000)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4,000</td>
<td>(6,000)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4,500</td>
<td>(1,500)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>0. (recovered)</td>
<td>15%</td>
</tr>
<tr>
<td>4</td>
<td>8,000</td>
<td>Not used in decision</td>
<td></td>
</tr>
</tbody>
</table>

Payback Period = 2.15yrs.  
Reject,  
≥ 2 years

Let’s say that the owner of Perfect Images Salon is considering the purchase of a new tanning bed, which costs $10,000 and is likely to bring in after-tax cash inflows of $4000 in the first year, $4,500 in the second year, $10,000 in the 3rd year, and $8,000 in the 4th year. The firm has a policy of buying equipment only if the payback period is 2 years or less. Calculate the payback period of the tanning bed and state whether the owner would buy it or not.

The payback period method has two major flaws:

1. It ignores all cash flow after the initial cash outflow has been recovered.
2. It ignores the time value of money.
9.2 (A) Discounted Payback Period: calculates the time it takes to recover the initial investment in current or discounted dollars.

Thus, it takes into account the time value of money by adding up the cash inflows that have been discounted to time 0, using the appropriate hurdle or discount rate and then measuring the payback period.

It is still flawed in that cash flows after the payback are ignored.

Example 2: Calculate Discounted Payback Period

Calculate the discounted payback period of the tanning bed, stated in Example 1 above, by using a discount rate of 10%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Discounted CF</th>
<th>Yet to be recovered</th>
<th>Percent of Year Recovered/Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(10,000)</td>
<td>(10,000)</td>
<td>(10,000)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4,000</td>
<td>3,636</td>
<td>(6,364)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4,500</td>
<td>3,719</td>
<td>(2,645)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>7,513</td>
<td>4,869</td>
<td>35%</td>
</tr>
<tr>
<td>4</td>
<td>8,000</td>
<td>5,464</td>
<td>Not used in decision</td>
<td></td>
</tr>
</tbody>
</table>

Discounted Payback = 2.35 years

9.3 Net Present Value (NPV) (Slides 9-12 to 9-28)

The NPV method is applied by discounting all the cash flows from a project back to time 0 using an appropriate discount rate, \( r \) as shown in Equation 9.1 below:

\[
NPV = -CF_0 + \frac{CF_1}{(1 + r)^1} + \frac{CF_2}{(1 + r)^2} + \frac{CF_3}{(1 + r)^3} + \ldots + \frac{CF_n}{(1 + r)^n}
\]

A positive NPV implies that the project is adding value to the firm’s bottom line and therefore when comparing projects, the higher the NPV the better.

Example 3: Calculating NPV

Using the cash flows for the tanning bed given in Example 2 above, calculate its NPV and indicate whether the investment should be undertaken or not.

NPV of tanning bed  

\[
\text{NPV} = -\$10,000 + \frac{\$4,000}{(1.10)} + \frac{\$4,500}{(1.10)^2} + \frac{\$10,000}{(1.10)^3} + \frac{\$8,000}{(1.10)^4}
\]

\[
= -\$10,000 + \$3636.36 + \$3719.01 + \$7513.15 + \$5464.11
\]

\[
= \$10,332.62
\]

Since the NPV > 0, the tanning bed should be purchased.

9.3 (A) Mutually Exclusive versus Independent Projects

The NPV approach can be used to evaluate independent as well as mutually exclusive projects.

A choice between mutually exclusive projects arises when:

1. There is a need for only one project, and both projects can fulfill that need.
2. There is a scarce resource that both projects need, and by using it in one project, it is not available for the second.

Since the NPV rule considers whether or not cash inflows that have been discounted at the relevant cost of capital outweigh the cash outflows emanating from a project, higher positive NPVs would be preferred to lower or negative NPVs, making the decision clear-cut.

Example 4: Calculate NPV for choosing between mutually exclusive projects

The owner of Perfect Images Salon has a dilemma. She wants to start offering tanning services and has to decide between purchasing a tanning bed and a tanning booth. In either case, she figures that the cost of capital will be 10%. The relevant annual cash flows with each option are listed below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Tanning Bed</th>
<th>Tanning Booth</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td>-12,500</td>
</tr>
<tr>
<td>1</td>
<td>4,000</td>
<td>4,400</td>
</tr>
<tr>
<td>2</td>
<td>4,500</td>
<td>4,800</td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>11,000</td>
</tr>
<tr>
<td>4</td>
<td>8,000</td>
<td>9,500</td>
</tr>
</tbody>
</table>

Can you help her make the right decision?

Since these are mutually exclusive options, the one with the higher NPV would be the best choice.

\[
\text{NPV}_{\text{bed}} = -\$10,000 + \frac{\$4,000}{(1.10)} + \frac{\$4,500}{(1.10)^2} + \frac{\$10,000}{(1.10)^3} + \frac{\$8,000}{(1.10)^4}
\]

\[
= -\$10,000 + \$3636.36 + \$3719.01 + \$7513.15 + \$5464.11
\]

\[
= \$10,332.62
\]
NPV_{booth} = -$12,500 + $4,400/(1.10) + $4,800/(1.10)^2 \\
+ $11,000/(1.10)^3 + $9,500/(1.10)^4 \\
= -$12,500 + $4,000 + $3,966.94 + $8,264.46 + $6,488.63 \\
= $10,220.03

Thus, the less expensive tanning bed with the higher NPV (10,332.62 > 10,220.03) is the better option.

9.3 (B) Unequal Lives of Projects

Firms often have to decide between alternatives that are mutually exclusive, cost different amounts, have different useful lives, and require replacement once their productive lives run out.

In such cases, using the traditional NPV (single life analysis) as the evaluation criterion can lead to incorrect decisions, since the cash flows will change once replacement occurs.

Under the NPV approach, such mutually exclusive projects with unequal lives can be analyzed by using one of the following two modified approaches:

1. We find a common point (lowest common multiple) at which both projects will require replacement at the same time. For example, if Project A lasts for 3 years, while Project B for 4 years; the common point would be year 12, with 4 repetitions of A and 3 repetitions of B. We calculate the required number of NPVs i.e. 4 for A and 3 for B, and then calculate the PV at time 0 of all the future NPVs, choosing the one with the highest total NPV (Replacement Chain Method).

2. We simply convert each project’s NPV into an equivalent annual annuity (EAA) as shown in Equation 9.2 below, by using the PV of an annuity equation or calculator function, and choose the one with the higher EAA.

\[
PVIFA = \frac{1 - [1/(1 + r)^n]}{r}
\]

\[
EAA = \frac{NPV}{PVIFA}
\]

Example 5: Unequal lives

Let’s say that there are two tanning beds available, one lasts for 3 years while the other for 4 years. The owner realizes that she will have to replace either of these two beds with new ones when they are at the end of their productive life, as she plans on being in the business for a long time. Using the cash flows listed below, and a cost of capital is 10%, help the owner decide which of the two tanning beds she should choose.

<table>
<thead>
<tr>
<th>Year</th>
<th>Tanning Bed A</th>
<th>Tanning Bed B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td>-5,750</td>
</tr>
</tbody>
</table>
1. Calculate the NPV of each tanning bed for a single life

\[
\text{NPV}_{\text{bed a}} = -10,000 + 4,000/(1.1) + 4,500/(1.1)^2 + 10,000/(1.1)^3 + 8,000/(1.1)^4
\]
\[
= -10,000 + 3,636.36 + 3719.01 + 7513.15 + 5464.11
\]
\[
= 10,332.62
\]

\[
\text{NPV}_{\text{bed b}} = -5,750 + 4,000/(1.1) + 4,500/(1.1)^2 + 9,000/(1.1)^3
\]
\[
= -5,750 + 3,636.36 + 3719.01 + 6761.83
\]
\[
= 8,367.21
\]

2. Next, calculate the Total NPV of each bed using 3 repetitions for A and 4 for B, i.e. We assume the Bed A will be replaced at the end of Years 4, and 8; lasting 12 years, while Bed B will be replaced in Years 3, 6, and 9, also lasting for 12 years in total.

Note: We will have an NPV in Years 0, 4, and 8 for Bed A (with 3 replications), and in Years 0, 3, 6, and 9 for Bed B (with 4 replications). We assume that the annual cash flows are the same for each replication.

\[
\text{Total NPV}_{\text{bed a}} = 10,332.62 + 10,332.62/(1.1)^3 + 10,332.62/(1.1)^8
\]
\[
= 22,925.20
\]

\[
\text{Total NPV}_{\text{bed b}} = 8,367.21 + 8,367.21/(1.1)^3 + 8,367.21/(1.1)^6 + 8,367.21/(1.1)^9
\]
\[
= 22,925.20
\]

Decision: Bed B with its higher Total NPV should be chosen.

Using the EAA Method.

\[
\text{EAA}_{\text{bed a}} = \text{NPV}_A/(PVIFA,10\%,4) = 10,332.62/(3.1698) = 3,259.56
\]

\[
\text{EAA}_{\text{bed b}} = \text{NPV}_B/(PVIFA,10\%,3) = 8,367.21/(2.48685) = 3,364.58
\]

Decision: Bed B’s EAA = $3,364.58 > Bed A’s EAA = $3,259.56
9.3 (C) Net Present Value Example: Equation and Calculator Function

When solving for the NPV given a series of cash flows, we can use equation 9.1, manually solve for the present values of the cash flows, and sum them up as shown in the examples above; or we can use a financial calculator such as the Texas Instruments Business Analyst II or TI-83 and input the necessary values using either the CF key (BA-II) or the NPV function (TI-83) as shown below:

Example 6: Solving NPV using a calculator and the equation

A company is considering a project which costs $750,000 to start and is expected to generate after-tax cash flows as follows:

- Year 1: $125,000
- Year 2: $175,000
- Year 3: $200,000
- Year 4: $225,000
- Year 5: $250,000

If the cost of capital is 12%, calculate its NPV.

**Equation method:**

\[
NPV = -750,000 + \frac{125,000}{(1 + 0.12)^1} + \frac{175,000}{(1 + 0.12)^2} + \frac{200,000}{(1 + 0.12)^3} + \frac{225,000}{(1 + 0.12)^4} + \frac{250,000}{(1 + 0.12)^5}
\]

\[
NPV = -750,000 + 111,607 + 139,509 + 142,356 + 142,992 + 141,857
\]

\[
NPV = -71,679
\]

**Calculator method:**

**TI-BAII Plus:** We enter the respective cash flows sequentially using the CF key

<table>
<thead>
<tr>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF0</td>
</tr>
<tr>
<td>CF1</td>
</tr>
<tr>
<td>CF2</td>
</tr>
<tr>
<td>CF3</td>
</tr>
<tr>
<td>CF4</td>
</tr>
<tr>
<td>CF5</td>
</tr>
</tbody>
</table>

Then we press the NPV key, enter the discount rate, I, and press the down arrow as follows to get the following result:
TI-83 Method:

We use the NPV function (available under the FINANCE mode) as follows:

\[
\text{NPV(} \text{discount rate, CF}_0, \{\text{CF}_1, \text{CF}_2, \ldots, \text{CF}_n\} \text{ and press the } \text{ENTER} \text{ key}
\]

\[
\text{NPV(12, -750000,}\{125000,175000,200000,225000,250000\}) \text{ ENTER}
\]

Output = -71,679.597

Note: It is important to caution students that the discount rate is entered as a whole number i.e. .12 for 12% and that a comma should separate each of the inputs, with a \{ \} bracket used for cash flows 1 through n.

9.4 Internal Rate of Return

The Internal Rate of Return (IRR) is the discount rate which forces the sum of all the discounted cash flows from a project to equal 0, as shown in equation 9.3 below:

\[
0 = \text{CF}_0 + \frac{\text{CF}_1}{(1 + r)^1} + \frac{\text{CF}_2}{(1 + r)^2} + \frac{\text{CF}_3}{(1 + r)^3} + \cdots + \frac{\text{CF}_n}{(1 + r)^n}
\]  

9.3

The decision rule that would be applied is as follows:

- accept if IRR > hurdle rate;
- reject if IRR < hurdle rate

If the project’s IRR is higher than the discount or hurdle rate, it would essentially mean that its NPV would be greater than zero as well. The IRR is measured as a percent while the NPV is measured in dollars.

Example 7: Calculating IRR with a financial calculator

Using the cash flows for the tanning bed given in Example 1 above calculate its IRR and state your decision.

\[
\text{CF}_0 = -10,000; \text{CF}_1 = 4,000; \text{CF}_2 = 4,500; \text{CF}_3 = 10,000; \text{CF}_4 = 8,000
\]

\[
i \text{ or discount rate} = 10%
\]

TI-83 inputs are as follows:

Using the Finance mode, select IRR( function and enter the inputs as follows:

\[
\text{IRR(discout rate,}\{\text{CF}_0,\text{CF}_1,\text{CF}_2,\text{CF}_3,\text{CF}_4\}) \text{ ENTER}
\]
9.4 (A) Appropriate Discount Rate or Hurdle Rate:

A project’s discount rate or hurdle rate is the minimum acceptable rate of return that an investor or firm should earn on a project given its riskiness. For a firm, it would typically be its weighted average cost of capital (covered in later chapters). Sometimes, it helps to draw an NPV profile as shown in Fig. 9.3 below, which a graph is plotting various NPVs for a range of incremental discount rates, showing at which discount rates the project would be acceptable and at which rates it would not.

\[
\text{IRR}(10,\{-10000, 4000, 4500, 10000, 8000\}) \quad \text{ENTER} \Rightarrow 45.02\% = \text{IRR} > 10\% \Rightarrow \text{Accept it!}
\]

9.4 (B) Problems with the Internal Rate of Return:

In most cases, the NPV criterion and the IRR approach lead to consistent decisions. That is, if a project has a positive NPV, its IRR will exceed its hurdle rate, making it acceptable. Similarly, the highest NPV project will also generally have the highest IRR.

However, there are some cases when the IRR method leads to ambiguous decisions or is problematic.

In particular, we can have 2 problems with the IRR approach:

1. Multiple IRRs; and
2. An unrealistic reinvestment rate assumption.

9.4 (C) Multiple IRRs

Projects which have non-normal cash flows (as shown below) i.e. multiple sign changes during their lives often end up with multiple IRRs.

\[
0 = -11,000 + \frac{7,500}{(1+r)^1} + \frac{7,500}{(1+r)^2} + \frac{7,500}{(1+r)^3} + \frac{7,500}{(1+r)^4} + \frac{7,500}{(1+r)^5}
\]
When solving for the IRR for this project using a financial calculator, we get only one of the 2 IRRs, i.e. 5.53%. If the cost of capital was higher than 5.53% we would reject the project since it would not satisfy the IRR decision rule.

But, as Figure 9.4 below shows the NPV profile of this project crosses the X-axis at 2 points (5.62% and 27.78%) meaning it has two IRRs. So, which is the true IRR?

![Graphic of NPV profile with two IRRs: one at 5.62% and one at 27.78%]

This typically happens when a project has non-normal cash flows, i.e. the cash inflows and outflows are not all clustered together i.e. all negative cash flows in early years followed by all positive cash flows later, or vice-versa.

If the cash flows have multiple sign changes during the project’s life, it leads to multiple IRRs and therefore ambiguity as to which one is correct.

In such cases, the best thing to do is to draw an NPV profile and select the project if it has a positive NPV at our required discount rate and vice-versa.

9.4 (D) Reinvestment and Crossover Rates

Another problem with the IRR approach is that it inherently assumes that the cash flows are being reinvested at the IRR, which if unusually high, can be highly unrealistic. In other words, if the IRR was calculated to be 40%, this would mean that we are implying that the cash inflows from a project are being reinvested at a rate of return of 40%, for the IRR to materialize.

A related problem arises when in the case of mutually exclusive projects we have either significant cost differences, and/or significant timing differences, leading to the NPV profiles crossing over as shown in Figure 9.5 below.
Notice that Project B’s IRR is higher than Project A’s IRR, making it the preferred choice based on the IRR approach.

However, at discount rates lower than the crossover rate, Project A has a higher NPV than Project B, making it more acceptable since it is adding more value.

If the discount rate is exactly equal to the crossover rate both projects would have the same NPV.

To the right of the crossover point, both methods would select Project B.

The fact that at certain discount rates, we have conflicting decisions being provided by the IRR method vis-à-vis the NPV method is the problem.

So, when in doubt go with the project with the highest NPV, it will always be correct.

**Example 8: Calculating the crossover rate of two projects**

Listed below are the cash flows associated with two mutually exclusive projects, A and B. Calculate their crossover rate.

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
<th>(A – B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td>-7,000</td>
<td>-3,000</td>
</tr>
<tr>
<td>1</td>
<td>5,000</td>
<td>9000</td>
<td>-4,000</td>
</tr>
<tr>
<td>2</td>
<td>7000</td>
<td>5000</td>
<td>2,000</td>
</tr>
<tr>
<td>3</td>
<td>9000</td>
<td>2000</td>
<td>7,000</td>
</tr>
</tbody>
</table>

| IRR   | 42.98% | 77.79% | 12.04% |

First calculate the yearly differences in the cash flows i.e. (A-B) as shown above,

Next, calculate the IRR of the cash flows in each column, e.g. For $\text{IRR}_{(A-B)}$

$\text{IRR}(10, \{-3000, -4000, 2000, 7000\}) \approx 12.04\%$
\[
\text{IRR}_A = 42.98\% ; \quad \text{IRR}_B = 77.79\% ; \quad \text{IRR}_{(A-B)} = 12.04\%
\]

Now, to check this calculate the NPVs of the two projects at 0%, 10%, 12.04%, 15%, 42.98%, and 77.79%.

<table>
<thead>
<tr>
<th>i</th>
<th>NPVA</th>
<th>NPVB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$11,000.00</td>
<td>$9,000.00</td>
</tr>
<tr>
<td>10%</td>
<td>$7,092.41</td>
<td>$6,816.68</td>
</tr>
<tr>
<td>12.04%</td>
<td>$6,437.69</td>
<td>$6,437.69</td>
</tr>
<tr>
<td>15.00%</td>
<td>$5,558.48</td>
<td>$5,921.84</td>
</tr>
<tr>
<td>42.98%</td>
<td>$0.00</td>
<td>$2,424.51</td>
</tr>
<tr>
<td>77.79%</td>
<td>($3,371.48)</td>
<td>$0.00</td>
</tr>
</tbody>
</table>

Notice how Project A has a higher NPV from 0% to 12.04%, while Project B has the higher NPV at rates above 12.04%. So, the NPV profiles cross-over at 12.04%, and at that rate the two NPVs are equal ($6437.69). Based on the IRRs only, Project B should have been selected, but at discount rates below 12.04%, the decision would have been sub-optimal for the firm.

9.4 (E) Modified Internal Rate of Return

Despite all its shortcomings, as explained above managers like to use the IRR as an evaluation criterion, since it is expressed as a% rather than in dollars.

To get around the criticism of the traditional IRR, regarding it’s unrealistic re-investment rate assumption, the modified IRR (MIRR) was developed.

Under the MIRR, all cash outflows are assumed to be reinvested at the firm’s cost of capital or hurdle rate, which makes it more realistic.

We calculate the future value of all positive cash flows at the terminal year of the project, the present value of the cash outflows at time 0; using the firm’s hurdle rate; and then solve for the relevant rate of return that would be implied using the following equation:

\[
MIRR = \left( \frac{FV}{PV} \right)^{\frac{1}{n}} - 1
\]

Example 9: Calculating MIRR

Using the cash flows given in Example 8 above, and a discount rate of 10%; calculate the MIRRs for Projects A and B. Which project should be accepted? Why?

**Project A:**

PV of cash outflows at time 0 = $10,000

FV of cash inflows at year 3, reinvested at 10% = $5,000*(1.1)^2 + $7,000*(1.1)^1 + $9,000
MIRR_A = (22750/10000)^{1/3} – 1 = 31.52%

**Project B:**

PV of cash outflows at time 0 = $7,000

FV of cash inflows at year 3, reinvested at 10% = $9,000*(1.1)^2 + $5,000*(1.1)^1 + $2,000

\[ \text{MIRR}_B = (15970/7000)^{1/3} – 1 = 31.64\% \]

So, accept Project B since its MIRR is higher.

---

### 9.5 Profitability Index

(Slides 9-46 to 9-47)

When faced with a capital rationing problem, i.e. a constrained budget, it would make sense to choose the projects that give us the best “bang for our buck.”

The Profitability Index can be used to calculate the ratio of the PV of benefits (inflows) to the PV of the cost of a project as follows:

\[
PI = \frac{\text{NPV} + \text{cost}}{\text{cost}}
\]

In essence, it tells us how many dollars we are getting per dollar invested.

**Example 10: PI calculation**

Using the cash flows listed in Example 6, and a discount rate of 10%, calculate the PI of each project. Which one should be accepted, if they are mutually exclusive? Why?

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td>-7,000</td>
</tr>
<tr>
<td>1</td>
<td>5,000</td>
<td>9000</td>
</tr>
<tr>
<td>2</td>
<td>7000</td>
<td>5000</td>
</tr>
<tr>
<td>3</td>
<td>9000</td>
<td>2000</td>
</tr>
<tr>
<td>NPV@10%</td>
<td>$7,092.41</td>
<td>$6,816.68</td>
</tr>
</tbody>
</table>

\[
\text{PI}_A = \frac{\text{NPV} + \text{Cost}}{\text{Cost}} = \frac{($17,092.41/$10,000)} = 1.71
\]

\[
\text{PI}_B = \frac{\text{NPV} + \text{Cost}}{\text{Cost}} = \frac{($13,816.68/$7,000)} = 1.97
\]

If these are mutually exclusive projects, accept Project B, since it returns $1.97 per $1 invested as against Project A’s $1.71 per $1 invested.

---

### 9.6 Overview of Six Decision Models

(Slides 9-48 to 9-53)

Based on the 6 models covered in this chapter, the following observations can be made (as summarized in Table 9.4 shown below):
1. Payback period is simple and fast, but economically unsound. It ignores all cash flow after the cutoff date and it ignores the time value of money.

2. Discounted payback period incorporates the time value of money but still ignores cash flow after the cutoff date.

3. Net present value (NPV) is economically sound and properly ranks projects across various sizes, time horizons, and levels of risk, without exception for all independent projects.

4. Internal rate of return (IRR) provides a single measure (return), but has the potential for errors in ranking projects. It can also lead to an incorrect selection when there are two mutually exclusive projects or incorrect acceptance or rejection of a project with more than a single IRR.

5. Modified internal rate of return (MIRR) in general corrects for most of, but not all, the problems of IRR and gives the solution in terms of a return. The reinvestment rate may or may not be appropriate for the future cash flows, however.

6. Profitability index (PI) incorporates risk and return, but the benefits-to-cost ratio is actually just another way of expressing the NPV.

<table>
<thead>
<tr>
<th>TABLE 9.4 Summary of Six Decision Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Decision criterion</td>
</tr>
<tr>
<td>Complexity of application</td>
</tr>
<tr>
<td>Time value of money</td>
</tr>
<tr>
<td>Risk</td>
</tr>
<tr>
<td>Economic basis and evaluation</td>
</tr>
</tbody>
</table>

9.6 (A) Capital Budgeting Using a Spreadsheet

NPV, MIRR, and IRR can be easily solved once data is entered into a spreadsheet as shown in the exhibits below.

For NPV we enter the following \( \Rightarrow \text{NPV}(\text{rate}, CF_1; CF_n) + CF_0 \)
Note: for the NPV we have to add in the Cash outflow in Year 0 (CF0), at the end, i.e. to the PV of CF1…CFn

For IRR we enter the following \( \text{IRR} (CF0:CF_n) \)

For MIRR \( \text{MIRR}(CF0:CF_n, \text{discount rate}, \text{reinvestment rate}) \); where the discount rate and the reinvestment rate would typically be the same i.e. the cost of capital of the firm.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial investment</td>
<td>(2,000,000)</td>
<td>10.20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Cash flow year 1</td>
<td>40,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Cash flow year 2</td>
<td>130,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Cash flow year 3</td>
<td>320,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Cash flow year 4</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Cash flow year 5</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Cash flow year 6</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Cash flow year 7</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Cash flow year 8</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Cash flow year 9</td>
<td>500,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Discount rate</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Reinvestment rate</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{C1} \quad \boxed{fx} \quad = \text{IRR}(B1:B10)
\]

Use the different functions to solve for IRR, MIRR, and NPV.

\[
\text{D1} \quad \boxed{fx} \quad = \text{MIRR}(B1:B10,B11,B12)
\]

\[
\text{E1} \quad \boxed{fx} \quad = \text{NPV}(0.09,B2:B10)
\]

Questions

1. **How does a business determine if a project (new product or service) is worthwhile?**

   Projects are accepted or rejected based on the use of one of many capital budgeting models. Using the cash flow of a project and a model such as Net Present Value or Internal rate of Return, the business can determine if the project is worthwhile.

2. **What is the difference between a short-term decision and a long-term decision?**

   The obvious answer is time frame that the decision affects. In general, short-term decisions have a shorter length of impact, lower cost, and require less information for a decision compared with a long-term decision.

3. **What question is the payback period model answering? What are the two major drawbacks of payback period? In what situations do businesses still use it?**

   Payback period answers the question, “how soon will I recover my initial investment (money)?” The two major drawbacks are, it ignores all cash flow after the initial cash flow is recovered and it ignores the time value of money. Many companies use payback for small dollar decisions.

4. **If you switch to the discounted payback period from the payback period, what assumption are your making about the timing of the cash flow?**

   We assume that all the cash flow for the period comes at the end of the period. For example if we look at annual cash flows we assume all the cash comes at the end of the year.

5. **What drawback of discounted payback period does the net present value overcome?**

   The Net Present Value model overcomes the problem of ignoring all cash flow after the initial cash flow has been recovered. NPV uses all the discounted cash flows of the project.

6. **Why is it straightforward to compare one project’s NPV with that of another project’s NPV? Why does ranking projects based on the greatest to least NPV make sound financial sense?**

   The final answer for the NPV model is the project’s value in current dollars. So we can compare to projects by simply looking at the one with the larger value in current dollars. The greater the NPV of a project the greater the “bag of money” for doing the project so projects can be ranked from most desirable to least desirable.

7. **Why do different projects have different discount rates in the NPV model?**
Each project has a different level of risk (based on the riskiness of the future cash flows of the project) so each project in the NPV model should receive an appropriate discount rate that is consistent with the level of risk.

8. **When does the internal rate of return model give an inappropriate decision when comparing two mutually exclusive projects?**

With two mutually exclusive projects it is possible to select the one with the lowest net present value by selecting the project with the highest IRR. When two projects have fairly different outflows and timing of inflows, their NPV profiles cross-over at some point called the cross-over rate. Below this rate, the project with the lower IRR has the higher NPV and vice-versa. So, selecting the project with the higher IRR would result in accepting a lower NPV, which is sub-optimal.

9. **If you switch from the internal rate of return to the modified internal rate of return model, what assumption changes with respect to the cash flow of the project?**

When you switch from IRR to Modified IRR you assume that all cash flow is reinvested at the cost of capital for the firm and not the IRR of the project. The IRR model assumes all cash flow is reinvested at the IRR rate.

10. **The Profitability Index produces a ratio between the present value of the benefits and present value of the costs of a project. Is there a time when PI and NPV do not agree on the ranking of projects? If so, under what circumstances would PI and NPV have different rankings of projects?**

The basic problem with PI is that you cannot scale up or down projects and when ranking projects with different PIs, different cost levels can lead to selection problems. For example, if you have a small scale project with a very high PI and a large scale project with a lower PI, the NPV model would select the lower PI project as it has a larger “bag of money” for the firm.

### Prepping for Exams

1. a. 6. d.
2. c. 7. b.
3. b. 8. c.
4. a. 9. b.
5. d. 10. a.

### Problems

1. **Payback period.** Given the cash flows of the four projects, A, B, C, and D, and using the payback period decision model, which projects do you accept and which projects do you reject with a three-year cutoff period for recapturing the initial cash outflow? Assume that the cash flows are equally distributed over the year.
### ANSWER

**Project A:**

Year One: 
\[-10,000 + 4,000 = 6,000 \text{ left to recover}\]

Year Two: 
\[-6,000 + 4,000 = 2,000 \text{ left to recover}\]

Year Three: 
\[-2,000 + 4,000 = \text{ fully recovered}\]

Year Three: 
\[2,000 / 4,000 = \frac{1}{2} \text{ year needed for recovery}\]

Payback Period for Project A: 2 and \(\frac{1}{2}\) years, ACCEPT!

**Project B:**

Year One: 
\[-25,000 + 2,000 = 23,000 \text{ left to recover}\]

Year Two: 
\[-23,000 + 8,000 = 15,000 \text{ left to recover}\]

Year Three: 
\[-15,000 + 14,000 = 1,000 \text{ left to recover}\]

Year Four: 
\[-1,000 + 20,000 = \text{ fully recovered}\]

Year Four: 
\[1,000 / 20,000 = 1/20 \text{ year needed for recovery}\]

Payback Period for Project B: 3 and 1/20 years, REJECT!

**Project C:**

Year One: 
\[-45,000 + 10,000 = 35,000 \text{ left to recover}\]

Year Two: 
\[-35,000 + 15,000 = 20,000 \text{ left to recover}\]

Year Three: 
\[-20,000 + 20,000 = \text{ fully recovered}\]

Year Three: 
\[20,000 / 20,000 = \text{ full year needed}\]

Payback Period for Project B: 3 years, ACCEPT!

**Project D:**

Year One: 
\[-100,000 + 40,000 = 60,000 \text{ left to recover}\]

Year Two: 
\[-60,000 + 30,000 = 30,000 \text{ left to recover}\]

Year Three: 
\[-30,000 + 20,000 = 10,000 \text{ left to recover}\]
Year Four: -$10,000 + $10,000 = fully recovered
Year Four: $10,000 / $10,000 = full year needed
Payback Period for Project B: 4 years, REJECT!

2. **Payback period.** What are the payback periods of Projects E, F, G and H? Assume all the cash flows are evenly spread throughout the year. If the cutoff period is three years, which projects do you accept?

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$40,000</td>
<td>$250,000</td>
<td>$75,000</td>
<td>$100,000</td>
</tr>
<tr>
<td>Cash flow year 1</td>
<td>$10,000</td>
<td>$40,000</td>
<td>$20,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>Cash flow year 2</td>
<td>$10,000</td>
<td>$120,000</td>
<td>$35,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>Cash flow year 3</td>
<td>$10,000</td>
<td>$200,000</td>
<td>$40,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>Cash flow year 4</td>
<td>$10,000</td>
<td>$200,000</td>
<td>$40,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>Cash flow year 5</td>
<td>$10,000</td>
<td>$200,000</td>
<td>$35,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Cash flow year 6</td>
<td>$10,000</td>
<td>$200,000</td>
<td>$20,000</td>
<td>$0</td>
</tr>
</tbody>
</table>

**ANSWER**

**Project E:**
Year One: -$40,000 + $10,000 = $30,000 left to recover
Year Two: -$30,000 + $10,000 = $20,000 left to recover
Year Three: -$20,000 + $10,000 = $10,000 left to recover
Year Four: -$10,000 + $10,000 = fully recovered
Year Four: $10,000 / $10,000 = full year needed
Payback Period for Project E: 4 years

**Project F:**
Year One: -$250,000 + $40,000 = $210,000 left to recover
Year Two: -$210,000 + $120,000 = $90,000 left to recover
Year Three: -$90,000 + $200,000 = fully recovered
Year Three: $90,000 / $200,000 = 0.45 year needed
Payback Period for Project F: 2.45 years

**Project G:**
Year One: -$75,000 + $20,000 = $55,000 left to recover
Year Two: -$55,000 + $35,000 = $20,000 left to recover
Year Three: -$20,000 + $40,000 = fully recovered
Year Three: $20,000 / $40,000 = 0.5 year needed
Payback Period for Project G: 2.5 years

**Project H:**
Year One: -$100,000 + $30,000 = $70,000 left to recover
Year Two: -$70,000 + $30,000 = $40,000 left to recover
Year Three: -$40,000 + $30,000 = $10,000 left to recover
Year Four: -$10,000 + $20,000 = fully recovered
Year Four: $10,000 / $20,000 = 0.5 year needed
Payback Period for Project H: 3.5 years
With a three year cut-off period, ACCEPT F and G, REJECT E and H.

3. **Discounted payback period.** Given the following four projects and their cash flows, calculate the discounted payback period with a 5% discount rate, 10% discount rate, and 20% discount rate. What do you notice about the payback period as the discount rate rises? Explain this relationship.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$10,000</td>
<td></td>
<td>$45,000</td>
<td>$100,000</td>
</tr>
<tr>
<td>Cash flow year 1</td>
<td>$4,000</td>
<td>$2,000</td>
<td>$10,000</td>
<td>$40,000</td>
</tr>
<tr>
<td>Cash flow year 2</td>
<td>$4,000</td>
<td>$8,000</td>
<td>$15,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>Cash flow year 3</td>
<td>$4,000</td>
<td>$14,000</td>
<td>$20,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>Cash flow year 4</td>
<td>$4,000</td>
<td>$20,000</td>
<td>$20,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Cash flow year 5</td>
<td>$4,000</td>
<td>$26,000</td>
<td>$15,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Cash flow year 6</td>
<td>$4,000</td>
<td>$32,000</td>
<td>$10,000</td>
<td>$0</td>
</tr>
</tbody>
</table>

**ANSWER**

Solution at 5% discount rate

**Project A:**

PV Cash flow year one -- $4,000 / 1.05 = $3,809.52
PV Cash flow year two -- $4,000 / 1.05² = $3,628.12
PV Cash flow year three -- $4,000 / 1.05³ = $3,455.35
PV Cash flow year four -- $4,000 / 1.05⁴ = $3,290.81
PV Cash flow year five -- $4,000 / 1.05⁵ = $3,134.10
PV Cash flow year six -- $4,000 / 1.05⁶ = $2,984.86
Discounted Payback Period: -$10,000 + $3,809.52 + $3,628.12 + $3,455.35 = $892.99 and fully recovered
Discounted Payback Period is 3 years.

**Project B:**

PV Cash flow year one -- $2,000 / 1.05 = $1,904.76  
PV Cash flow year two -- $8,000 / 1.05^2 = $7,256.24  
PV Cash flow year three -- $14,000 / 1.05^3 = $12,093.73  
PV Cash flow year four -- $20,000 / 1.05^4 = $16,454.05  
PV Cash flow year five -- $26,000 / 1.05^5 = $20,371.68  
PV Cash flow year six -- $32,000 / 1.05^6 = $23,878.89  
Discounted Payback Period: -$25,000 + $1,904.76 + $7,256.24 + $12,093.73 + $16,454.05 = $12,708.78 and fully recovered  
Discounted Payback Period is 4 years.

**Project C:**

PV Cash flow year one -- $10,000 / 1.05 = $9,523.81  
PV Cash flow year two -- $15,000 / 1.05^2 = $13,605.44  
PV Cash flow year three -- $20,000 / 1.05^3 = $17,276.75  
PV Cash flow year four -- $20,000 / 1.05^4 = $16,454.05  
PV Cash flow year five -- $15,000 / 1.05^5 = $11,752.89  
PV Cash flow year six -- $10,000 / 1.05^6 = $7,462.15  
Discounted Payback Period: -$45,000 + $9,523.81 + $13,605.44 + $17,276.75 + $16,454.05 = $11,860.05 and fully recovered  
Discounted Payback Period is 4 years.

**Project D:**

PV Cash flow year one -- $40,000 / 1.05 = $38,095.24  
PV Cash flow year two -- $30,000 / 1.05^2 = $27,210.88  
PV Cash flow year three -- $20,000 / 1.05^3 = $17,276.75  
PV Cash flow year four -- $10,000 / 1.05^4 = $8,227.02  
PV Cash flow year five -- $10,000 / 1.05^5 = $7,835.26  
PV Cash flow year six -- $0 / 1.05^6 = $0  
Discounted Payback Period: -$100,000 + $38,095.24 + $27,210.88 + $17,276.75 + $8,227.02 + $7,835.26 = -$1,354.84 i.e. This project’s cost is NEVER fully recovered.  
Solution at 10% discount rate
Project A:
PV Cash flow year one -- $4,000 / 1.10 = $3,636.36
PV Cash flow year two -- $4,000 / 1.10² = $3,307.79
PV Cash flow year three -- $4,000 / 1.10³ = $3,005.26
PV Cash flow year four -- $4,000 / 1.10⁴ = $2,732.05
PV Cash flow year five -- $4,000 / 1.10⁵ = $2,483.69
PV Cash flow year six -- $4,000 / 1.10⁶ = $2,257.90
Discounted Payback Period: -$10,000 + $3,636.36 + $3,307.79 + $3,005.26 + $2,732.05 = $2,679.46 and fully recovered
Discounted Payback Period is 4 years.

Project B:
PV Cash flow year one -- $2,000 / 1.10 = $1,818.18
PV Cash flow year two -- $8,000 / 1.10² = $6,611.57
PV Cash flow year three -- $14,000 / 1.10³ = $10,518.41
PV Cash flow year four -- $20,000 / 1.10⁴ = $13,660.27
PV Cash flow year five -- $26,000 / 1.10⁵ = $16,143.95
PV Cash flow year six -- $32,000 / 1.10⁶ = $18,063.17
Discounted Payback Period: -$25,000 + $1,818.18 + $6,611.57 + $10,518.41 + $13,660.27 = $7,608.43 and fully recovered
Discounted Payback Period is 4 years.

Project C:
PV Cash flow year one -- $10,000 / 1.10 = $9,090.91
PV Cash flow year two -- $15,000 / 1.10² = $12,396.69
PV Cash flow year three -- $20,000 / 1.10³ = $15,026.30
PV Cash flow year four -- $20,000 / 1.10⁴ = $13,660.27
PV Cash flow year five -- $15,000 / 1.10⁵ = $9,313.82
PV Cash flow year six -- $10,000 / 1.10⁶ = $5,644.74
Discounted Payback Period: -$45,000 + $9,090.91 + $12,396.69 + $15,026.30 + $13,660.27 = $5174.07 and fully recovered
Discounted Payback Period is 4 years.

Project D:
PV Cash flow year one -- $40,000 / 1.10 = $36,363.64
PV Cash flow year two -- $30,000 / 1.10² = $24,793.39
PV Cash flow year three -- $20,000 / 1.10^3 = $15,026.30
PV Cash flow year four -- $10,000 / 1.10^4 = $6,830.13
PV Cash flow year five -- $10,000 / 1.10^5 = $6,209.21
PV Cash flow year six -- $0 / 1.10^6 = $0
Discounted Payback Period: -$100,000 + $36,363.64 + $24,793.29 + $15,026.30 + $6,830.13 + $6,209.21 = -$10,777.3 and never recovered.
Initial cash outflow is never recovered.
Solution at 20% discount rate

**Project A:**

PV Cash flow year one -- $4,000 / 1.20 = $3,333.33
PV Cash flow year two -- $4,000 / 1.20^2 = $2,777.78
PV Cash flow year three -- $4,000 / 1.20^3 = $2,314.81
PV Cash flow year four -- $4,000 / 1.20^4 = $1,929.01
PV Cash flow year five -- $4,000 / 1.20^5 = $1,607.51
PV Cash flow year six -- $4,000 / 1.20^6 = $1,339.59
Discounted Payback Period: -$10,000 + $3,333.33 + $2,777.78 + $2,314.81 + $1,929.01 + $1,607.51 = $354.93 and fully recovered
Discounted Payback Period is 4 years.

**Project B:**

PV Cash flow year one -- $2,000 / 1.20 = $1,666.67
PV Cash flow year two -- $8,000 / 1.20^2 = $5,555.56
PV Cash flow year three -- $14,000 / 1.20^3 = $8,101.85
PV Cash flow year four -- $20,000 / 1.20^4 = $9,645.06
PV Cash flow year five -- $26,000 / 1.20^5 = $10,448.82
PV Cash flow year six -- $32,000 / 1.20^6 = $10,716.74
Discounted Payback Period: -$25,000 + $1,666.67 + $5,555.56 + $8,101.85 + $9,645.06 + $10,448.82 = $10,417.96 and fully recovered
Discounted Payback Period is 5 years.

**Project C:**

PV Cash flow year one -- $10,000 / 1.20 = $8,333.33
PV Cash flow year two -- $15,000 / 1.20^2 = $10,416.67
PV Cash flow year three -- $20,000 / 1.20^3 = $11,574.07
PV Cash flow year four -- $20,000 / 1.20^4 = $9,645.06
PV Cash flow year five -- $15,000 / 1.20^5 = $6,028.16
PV Cash flow year six -- $10,000 / 1.20^6 = $3,348.97
Discounted Payback Period: -$45,000 + $8,333.33 + $10,416.67 + $11,574.07 + $9,645.06 + $6,028.16 = $997.29 and fully recovered
Discounted Payback Period is 5 years.

Project D:
PV Cash flow year one -- $40,000 / 1.20 = $33,333.33
PV Cash flow year two -- $30,000 / 1.20^2 = $20,833.33
PV Cash flow year three -- $20,000 / 1.20^3 = $11,574.07
PV Cash flow year four -- $10,000 / 1.20^4 = $4,822.53
PV Cash flow year five -- $10,000 / 1.20^5 = $4,018.78
PV Cash flow year six -- $0 / 1.20^6 = $0
Discounted Payback Period: -$100,000 + $33,333.33 + $20,833.33 + $11,574.07 + $4,822.53 + $4,018.78 = -$25,417.95 and initial cost is never recovered.
Discounted Payback Period is infinity.
As the discount rate increases, the Discounted Payback Period also increases. The reason is that the future dollars are worth less in present value as the discount rate increases, requiring more future dollars to recover the present value of the outlay.

4. **Discounted payback period.** Becker Inc. uses discounted payback period for projects under $25,000 and has a cut off period of 4 years for these small value projects. Two projects, R and S, are under consideration. The anticipated cash flows for these two projects are listed below. If Becker Incorporated uses an 8% discount rate on these projects, are they accepted or rejected? If it uses 12% discount rate? A 16% discount rate? Why is it necessary to only look at the first four years of the projects’ cash flows?

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project R</th>
<th>Project S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$24,000</td>
<td>$18,000</td>
</tr>
<tr>
<td>Cash flow year 1</td>
<td>$ 6,000</td>
<td>$ 9,000</td>
</tr>
<tr>
<td>Cash flow year 2</td>
<td>$ 8,000</td>
<td>$ 6,000</td>
</tr>
<tr>
<td>Cash flow year 3</td>
<td>$10,000</td>
<td>$ 6,000</td>
</tr>
<tr>
<td>Cash flow year 4</td>
<td>$12,000</td>
<td>$ 3,000</td>
</tr>
</tbody>
</table>

**ANSWER**

Solution at 8%
Project R:
PV Cash flow year one -- $6,000 / 1.08 = $5,555.56
PV Cash flow year two -- $8,000 / 1.08^2 = $6,858.71
PV Cash flow year three -- $10,000 / 1.08^3 = $7,938.32
PV Cash flow year four -- $12,000 / 1.08^4 = $8,820.36
Discounted Payback Period: -$24,000 + $5,555.56 + $6,858.71 + $7,938.32 + $8,820.36 = $5,172.95 and initial cost is recovered in first four years, project accepted.

Project S:
PV Cash flow year one -- $9,000 / 1.08 = $8,333.33
PV Cash flow year two -- $6,000 / 1.08^2 = $5,144.03
PV Cash flow year three -- $6,000 / 1.08^3 = $4,762.99
PV Cash flow year four -- $3,000 / 1.08^4 = $2,205.09
Discounted Payback Period: -$18,000 + $8,333.33 + $5,144.03 + $4,762.99 = $240.36 and initial cost is recovered in first three years, project accepted.

Solution at 12%

Project R:
PV Cash flow year one -- $6,000 / 1.12 = $5,357.14
PV Cash flow year two -- $8,000 / 1.12^2 = $6,377.55
PV Cash flow year three -- $10,000 / 1.12^3 = $7,117.80
PV Cash flow year four -- $12,000 / 1.12^4 = $7,626.22
Discounted Payback Period: -$24,000 + $5,357.14 + $6,377.55 + $7,117.80 + $7,626.22 = $2,478.71 and initial cost is recovered in first four years, project accepted.

Project S:
PV Cash flow year one -- $9,000 / 1.12 = $8,035.71
PV Cash flow year two -- $6,000 / 1.12^2 = $4,783.16
PV Cash flow year three -- $6,000 / 1.12^3 = $4,270.68
PV Cash flow year four -- $3,000 / 1.12^4 = $1,906.55
Discounted Payback Period: -$18,000 + $8,035.71 + $4,783.16 + $4,270.68 + $1,906.55 = $996.10 and initial cost is recovered in first four years, project accepted.

Solution at 16%

Project R:
PV Cash flow year one -- $6,000 / 1.16 = $5,172.41
PV Cash flow year two -- $8,000 / 1.16^2 = $5,945.30
PV Cash flow year three -- $10,000 / 1.16^3 = $6,406.58
PV Cash flow year four -- $12,000 / 1.16^4 = $6,627.49

Discounted Payback Period: -$24,000 + $5,172.41 + $5,945.30 + $6,406.58 + $6,627.49
= $151.78 and initial cost is recovered in first four years, project accepted.

**Project S:**

PV Cash flow year one -- $9,000 / 1.16 = $7,758.62
PV Cash flow year two -- $6,000 / 1.16^2 = $4,458.98
PV Cash flow year three -- $6,000 / 1.16^3 = $3,843.95
PV Cash flow year four -- $3,000 / 1.16^4 = $1,656.87
Discounted Payback Period: -$18,000 + $7,758.62 + $4,458.98 + $3,843.95 + $1,656.87
= -$281.58 and initial cost is not recovered in first four years, project rejected. Because Graham Incorporated is using a four year cut-off period, only the first four years of cash flow matter. If the first four years of anticipated cash flows are insufficient to cover the initial outlay of cash, the project is rejected regardless of the cash flows in years five and forward.

5. **Comparing payback period and discounted payback period.** Mathew Inc. is debating using the payback period versus the discounted payback period for small-dollar projects. The company's information officer has submitted a new computer project with a $15,000 cost. The cash flow will be $5,000 each year for the next five years. The cutoff period used by the company is three years. The information officer states that it doesn't matter which model the company uses for the decision; the project is clearly acceptable. Demonstrate for the information officer that the selection of the model does matter.

**ANSWER**

Calculate the Payback Period for the project:

Payback Period = -$15,000 + $5,000 + $5,000 + $5,000 = 0

So the payback period is 3 years and the project is a go!

Calculate the Discounted Payback Period for the project at any positive discount rate, say 1%...

Present Value of cash flow year one = $5,000 / 1.01 = $4,950.50
Present Value of cash flow year two = $5,000 / 1.01^2 = $4,901.48
Present Value of cash flow year three = $5,000 / 1.01^3 = $4,852.95
Discounted Payback Period = -$15,000 + $4,950.50 + $4,901.48 + $4,852.95 = -$295.04
so the payback period is over 3 years and the project is a no-go!

6. **Comparing payback period and discounted payback period.** Nielsen Inc. is switching from the payback period to the discounted payback period for small-dollar projects. The cutoff period will remain at three years. Given the following four
projects' cash flows and using a 10% discount rate, determine which projects it would have accepted under the payback period and which it will now reject under the discounted payback period.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project 1</th>
<th>Project 2</th>
<th>Project 3</th>
<th>Project 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$10,000</td>
<td>$15,000</td>
<td>$8,000</td>
<td>$18,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>$4,000</td>
<td>$7,000</td>
<td>$3,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>$4,000</td>
<td>$5,500</td>
<td>$3,500</td>
<td>$11,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$0</td>
</tr>
</tbody>
</table>

**ANSWER**

Calculate the Discounted Payback Periods of each project at 10% discount rate:

**Project One**

Present Value of cash flow year one = $4,000 / 1.10 = $3,636.36

Present Value of cash flow year two = $4,000 / 1.10^2 = $3,305.78

Present Value of cash flow year three = $4,000 / 1.10^3 = $3,005.26

Discounted Payback Period = -$10,000 + $3,636.36 + $3,305.78 + $3,005.26 = -$52.60

so the discount payback period is over 3 years and the project is a no-go!

**Project Two**

Present Value of cash flow year one = $7,000 / 1.10 = $6,363.64

Present Value of cash flow year two = $5,500 / 1.10^2 = $4,545.46

Present Value of cash flow year three = $4,000 / 1.10^3 = $3,005.26

Discounted Payback Period = -$15,000 + $6,363.64 + $4,545.46 + $3,005.26 = -$1,085.64

so the discount payback period is over 3 years and the project is a no-go!

**Project Three**

Present Value of cash flow year one = $3,000 / 1.10 = $2,272.73

Present Value of cash flow year two = $3,500 / 1.10^2 = $2,892.56

Present Value of cash flow year three = $4,000 / 1.10^3 = $3,005.26

Discounted Payback Period = -$8,000 + $2,272.73 + $2,892.56 + $3,005.26 = $625.55

so the discount payback period is under 3 years and the project is a go!

**Project Four**

Present Value of cash flow year one = $10,000 / 1.10 = $9,090.91

Present Value of cash flow year two = $11,000 / 1.10^2 = $9,090.91

Present Value of cash flow year three = $0 / 1.10^3 = $0

Discounted Payback Period = -$18,000 + $9,090.91 + $9,090.91 + $0 = $181.82

so the discount payback period is under 3 years and the project is a go!
Projects one and two will now be rejected using discounted payback period with a discount rate of 10%.

7. **Net present value.** Quark Industries has a project with the following projected cash flows:

   - Initial Cost, Year 0: $240,000
   - Cash flow year one: $25,000
   - Cash flow year two: $75,000
   - Cash flow year three: $150,000
   - Cash flow year four: $150,000

   a. Using a 10% discount rate for this project and the NPV model, determine whether the company should accept or reject this project?
   b. Should the company accept or reject it using a 15% discount rate?
   c. Should the company accept or reject it using a 20% discount rate?

**ANSWER**

(a) 
NPV = -$240,000 + $25,000/1.10 + $75,000/1.10^2 + $150,000/1.10^3 + $150,000/1.10^4 
NPV = -$240,000 + $22,727.27 + $61,983.47 + $112,697.22 + $102,452.02 
NPV = $59,859.98 and accept the project.

(b) 
NPV = -$240,000 + $25,000/1.15 + $75,000/1.15^2 + $150,000/1.15^3 + $150,000/1.15^4 
NPV = -$240,000 + $21,739.13 + $56,710.76 + $98,627.43 + $85,762.99 
NPV = $22,840.31 and accept the project.

(c) 
NPV = -$240,000 + $25,000/1.20 + $75,000/1.20^2 + $150,000/1.20^3 + $150,000/1.20^4 
NPV = -$240,000 + $20,833.33 + $52,083.33 + $86,805.56 + $72,337.96 
NPV = -$7,939.82 and reject the project.

8. **Net present value.** Lepton Industries has a project with the following projected cash flows:

   - Initial Cost, Year 0: $468,000
   - Cash flow year one: $135,000
   - Cash flow year two: $240,000
   - Cash flow year three: $185,000
Cash flow year four: $135,000

a. Using an 8% discount rate for this project and the NPV model, determine whether the company should accept or reject this project?
b. Should the company accept or reject it using a 14% discount rate?
c. Should the company accept or reject it using a 20% discount rate?

**ANSWER**

(a)

\[
\text{NPV} = -468,000 + \frac{135,000}{1.08} + \frac{240,000}{1.08^2} + \frac{185,000}{1.08^3} + \frac{135,000}{1.08^4}
\]

\[
\text{NPV} = -468,000 + 125,000.00 + 205,761.32 + 146,858.96 + 99,229.03
\]

\[
\text{NPV} = 108,849.31 \text{ and accept the project.}
\]

(b)

\[
\text{NPV} = -468,000 + \frac{135,000}{1.14} + \frac{240,000}{1.14^2} + \frac{185,000}{1.14^3} + \frac{135,000}{1.14^4}
\]

\[
\text{NPV} = -468,000 + 118,421.05 + 184,672.21 + 124,869.73 + 79,930.84
\]

\[
\text{NPV} = 39,893.83 \text{ and accept the project.}
\]

(c)

\[
\text{NPV} = -468,000 + \frac{135,000}{1.20} + \frac{240,000}{1.20^2} + \frac{185,000}{1.20^3} + \frac{135,000}{1.20^4}
\]

\[
\text{NPV} = -468,000 + 112,500.00 + 166,666.67 + 107,060.19 + 65,104.17
\]

\[
\text{NPV} = -16,668.97 \text{ and reject the project.}
\]

9. **Net present value.** Quark Industries has four potential projects, all with an initial cost of $2,000,000. The capital budget for the year will allow Quark Industries to accept only one of the four projects. Given the discount rates and the future cash flows of each project, determine which project Quark should accept.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project M</th>
<th>Project N</th>
<th>Project O</th>
<th>Project P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$1,000,000</td>
<td>$300,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$800,000</td>
<td>$500,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$600,000</td>
<td>$700,000</td>
</tr>
<tr>
<td>Year 4</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$400,000</td>
<td>$900,000</td>
</tr>
<tr>
<td>Year 5</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$200,000</td>
<td>$1,100,000</td>
</tr>
<tr>
<td>Discount rate</td>
<td>6%</td>
<td>9%</td>
<td>15%</td>
<td>22%</td>
</tr>
</tbody>
</table>

**ANSWER**

Find the NPV of each project and compare the NPVs.
Project M’s NPV = \(-2,000,000 + \frac{500,000}{1.06} + \frac{500,000}{1.06^2} + \frac{500,000}{1.06^3} + \frac{500,000}{1.06^4} + \frac{500,000}{1.06^5}\)

Project M’s NPV = \(-2,000,000 + 471,698.1 + 444,998.2 + 419,809.60 + 396,046.8 + 373,629.1\)

Project N’s NPV = 106,181.9

Project N’s NPV = \(-2,000,000 + \frac{600,000}{1.09} + \frac{600,000}{1.09^2} + \frac{600,000}{1.09^3} + \frac{600,000}{1.09^4} + \frac{600,000}{1.09^5}\)

Project N’s NPV = \(-2,000,000 + 550,458.72 + 505,008.00 + 463,331.09 + 425,055.13 + 389,958.83\)

Project N’s NPV = 333,790.77

Project O’s NPV = \(-2,000,000 + \frac{1,000,000}{1.15} + \frac{800,000}{1.15^2} + \frac{600,000}{1.15^3} + \frac{400,000}{1.15^4} + \frac{200,000}{1.15^5}\)

Project O’s NPV = \(-2,000,000 + 869,565.22 + 604,914.93 + 394,509.74 + 228,701.30 + 99,435.34\)

Project O’s NPV = 197,126.53

Project P’s NPV = \(-2,000,000 + \frac{300,000}{1.22} + \frac{500,000}{1.22^2} + \frac{700,000}{1.22^3} + \frac{900,000}{1.22^4} + \frac{1,100,000}{1.22^5}\)

Project P’s NPV = \(-2,000,000 + 245,901.64 + 335,931.20 + 385,494.82 + 406,259.18 + 406,999.18\)

Project P’s NPV = \(-219,413.98\) (would reject project regardless of budget)

And the ranking order based on NPVs is,

Project N – NPV of 333,790.77
Project O – NPV of 197,126.53
Project M – NPV of 164,738.34
Project P – NPV of \(-219,413.98\)

Swanson Industries should pick Project N.

10. **Net present value.** Lepton Industries has four potential projects, all with an initial cost of $1,500,000. The capital budget for the year will allow Lepton to accept only one of the four projects. Given the discount rates and the future cash flows of each project, determine which project Lepton should accept.
Find the NPV of each project and compare the NPVs.

Project Q’s NPV = \(-$1,500,000 + \frac{350,000}{1.04} + \frac{350,000}{1.04^2} + \frac{350,000}{1.04^3} + \frac{350,000}{1.04^4} + \frac{350,000}{1.04^5}\)

Project Q’s NPV = \(-$1,500,000 + $336,538.46 + $323,594.67 + $311,148.73 + $299,181.47 + $287,674.49\)

Project Q’s NPV = $58,137.84

Project R’s NPV = \(-$1,500,000 + \frac{400,000}{1.08} + \frac{400,000}{1.08^2} + \frac{400,000}{1.08^3} + \frac{400,000}{1.08^4} + \frac{400,000}{1.08^5}\)

Project R’s NPV = \(-$2,000,000 + $370,370.37 + $342,935.53 + $317,532.90 + $294,011.94 + $272,233.28\)

Project R’s NPV = $97,084.02

Project S’s NPV = \(-$1,500,000 + \frac{700,000}{1.13} + \frac{600,000}{1.13^2} + \frac{500,000}{1.13^3} + \frac{400,000}{1.13^4} + \frac{300,000}{1.13^5}\)

Project S’s NPV = \(-$1,500,000 + $619,469.03 + $469,888.01 + $346,525.08 + $245,327.49 + $162,827.98\)

Project S’s NPV = $344,037.59

Project T’s NPV = \(-$1,500,000 + \frac{200,000}{1.18} + \frac{400,000}{1.18^2} + \frac{600,000}{1.18^3} + \frac{800,000}{1.18^4} + \frac{1,000,000}{1.18^5}\)

Project T’s NPV = \(-$1,500,000 + $169,491.53 + $287,273.77 + $365,178.52 + $412,631.10 + $437,109.22\)

Project T’s NPV = $171,684.14

And the ranking order based on NPVs is,

Project S – NPV of $344,037.59
Project T – NPV of $171,684.14
Project R – NPV of $97,084.02
Project Q – NPV of $58,137.84
Campbell Industries should pick Project S.

11. **NPV unequal lives.** Grady Enterprises is looking at two project opportunities for a parcel of land that the company currently owns. The first project is a restaurant, and the second project is a sports facility. The projected cash flow of the restaurant is an initial cost of $1,500,000 with cash flows over the next six years of $200,000 (Year one), $250,000 (Year two), $300,000 (Years three through five), and $1,750,000 in Year six, when Grady plans on selling the restaurant. The sports facility has the following cash outflow: initial cost of $2,400,000 with cash flows over the next three years of $400,000 (Years one to three) and $3,000,000 in Year four, when Grady plans on selling the facility. If the appropriate discount rate for the restaurant is 11% and the appropriate discount rate for the sports facility is 13%, using NPV, determine which project Grady should choose for the parcel of land. Adjust the NPV for unequal lives with the equivalent annual annuity. Does the decision change?

### ANSWER

Find the NPV of both projects and then solve for EAA with respective discount rates.

NPV (Restaurant) = 

\[-1,500,000 + \frac{200,000}{1.11} + \frac{250,000}{1.11^2} + \frac{300,000}{1.11^3} + \frac{300,000}{1.11^4} + \frac{300,000}{1.11^5} + \frac{1,750,000}{1.11^6} = 413,719.36\]

**EAA Restaurant**

P/Y = 1 and C/Y = 1

Input 6 11.0 -413,719.36 ? 0

Keys N I/Y PV PMT FV

CPT 97,793.56

NPV (Sports Facility) = 

\[-2,400,000 + \frac{400,000}{1.13} + \frac{400,000}{1.13^2} + \frac{3,000,000}{1.13^3} + \frac{3,000,000}{1.13^4} = 384,417.22\]

**EAA Sports Facility**

P/Y = 1 and C/Y = 1

Input 4 13.0 -384,417.22 ? 0

Keys N I/Y PV PMT FV

CPT 129,238.84

The decision changes from the higher NPV of the restaurant to the higher EAA of the sports facility.

12. **NPV unequal lives.** Singing Fish Fine Foods has $2,000,000 for capital investments this year and is considering two potential projects for the funds. Project one is updating the deli section of the store for additional food service. The estimated annual after-tax cash flow of this project is $600,000 per year for the next five years. Project
two is updating the wine section of the store. Estimated annual after-tax cash flow for this project is $530,000 for the next six years. If the appropriate discount rate for the deli expansion is 9.5% and the appropriate discount rate for the wine section is 9.0%, using NPV, determine which project Singing Fish should choose for the parcel of land. Adjust the NPV for unequal lives with the equivalent annual annuity. Does the decision change?

**ANSWER**

Find the NPV of both projects and then solve for EAA with respective discount rates.

NPV (Deli Section) = -$2,000,000 + $600,000 / (1.095)^1 + $600,000 / (1.095)^2 + $600,000 / (1.095)^3 + $600,000 / (1.095)^4 + $600,000 / (1.095)^5 = $303,825.27

EAA Deli Section

P/Y = 1 and C/Y = 1

Input 5 9.5 -303,825.27 ? 0

Keys N I/Y PV PMT FV

CPT 79,127.16

NPV (Wine Section) = -$2,000,000 + $530,000 / (1.09)^1 + $530,000 / (1.09)^2 + $530,000 / (1.09)^3 + $530,000 / (1.09)^4 + $530,000 / (1.09)^5 = $377,536.85

EAA Wine Section

P/Y = 1 and C/Y = 1

Input 6 9.0 -377,536.85 ? 0

Keys N I/Y PV PMT FV

CPT 84,160.43

The decision does not change as the higher NPV of the wine section also has the higher EAA.

13. **Internal rate of return and modified internal rate of return.** What are the IRRs and MIRRs of the four projects for Quark Industries in Problem 9?

**ANSWER**

This is an iterative process but can be solved quickly on a calculator or spreadsheet.

<table>
<thead>
<tr>
<th>Cash Flows</th>
<th>Project M</th>
<th>Project N</th>
<th>Project O</th>
<th>Project P</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF0</td>
<td>($2,000,000)</td>
<td>($2,000,000)</td>
<td>($2,000,000)</td>
<td>($2,000,000)</td>
</tr>
<tr>
<td>CF1</td>
<td>$500,000</td>
<td>$600,000</td>
<td>$1,000,000</td>
<td>$300,000</td>
</tr>
</tbody>
</table>

14. **Internal rate of return and modified internal rate of return.** What are the IRRs and MIRRs of the four projects for Lepton Industries in Problem 10?

**ANSWER**

This is an iterative process but can be solved quickly on a calculator or spreadsheet.

<table>
<thead>
<tr>
<th>Cash Flows</th>
<th>Project Q</th>
<th>Project R</th>
<th>Project S</th>
<th>Project T</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF0</td>
<td>($1,500,000)</td>
<td>($1,500,000)</td>
<td>($1,500,000)</td>
<td>($1,500,000)</td>
</tr>
<tr>
<td>CF1</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$700,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>CF2</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$600,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>CF3</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$500,000</td>
<td>$600,000</td>
</tr>
<tr>
<td>CF4</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$400,000</td>
<td>$800,000</td>
</tr>
<tr>
<td>CF5</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$300,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Disc.rate</td>
<td>4%</td>
<td>8%</td>
<td>13%</td>
<td>18%</td>
</tr>
<tr>
<td><strong>IRR</strong></td>
<td>5.37%</td>
<td>10.42%</td>
<td>23.57%</td>
<td>21.86%</td>
</tr>
<tr>
<td><strong>MIRR</strong></td>
<td>4.79%</td>
<td>9.36%</td>
<td>17.76%</td>
<td>20.59%</td>
</tr>
</tbody>
</table>

15. **MIRR unequal lives.** What is the MIRR for Grady Enterprises in Problem 11? What is the MIRR when you adjust for the unequal lives? Does the adjusted MIRR for unequal lives change the decision based on MIRR? *Hint:* Take all cash flows to the same ending period as the longest project.

<table>
<thead>
<tr>
<th>Year</th>
<th>Restaurant</th>
<th>Sports Facility</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1500000</td>
<td>-2400000</td>
</tr>
<tr>
<td>1</td>
<td>200000</td>
<td>400000</td>
</tr>
<tr>
<td>2</td>
<td>250000</td>
<td>400000</td>
</tr>
<tr>
<td>Year</td>
<td>Deli Section</td>
<td>Wine Section</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
<td>--------------</td>
</tr>
<tr>
<td>0</td>
<td>-2000000</td>
<td>-2000000</td>
</tr>
<tr>
<td>1</td>
<td>600000</td>
<td>530000</td>
</tr>
<tr>
<td>2</td>
<td>600000</td>
<td>530000</td>
</tr>
<tr>
<td>3</td>
<td>600000</td>
<td>530000</td>
</tr>
<tr>
<td>4</td>
<td>600000</td>
<td>530000</td>
</tr>
<tr>
<td>5</td>
<td>600000</td>
<td>530000</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>530000</td>
</tr>
</tbody>
</table>

Disc. 9.5% 9%

ANSWER

FV (Restaurant) = $200,000 \times (1.11)^5 + $250,000 \times (1.11)^4 + $300,000 \times (1.11)^3 + $300,000 \times (1.11)^2 + $300,000 \times (1.11) + $1,750,000 \times (1.11)^0 = $3,579,448.53

MIRR = ($3,579,448.53 / $1,500,000)^{1/6} - 1 = 15.6%

FV (Sports Facility) = $400,000 \times (1.13)^3 + $400,000 \times (1.13)^2 + $400,000 \times (1.13)^1 + $3,000,000 \times (1.13)^0 = $4,539,918.80

MIRR = ($4,539,918.80 / $2,400,000)^{1/4} - 1 = 17.3%

Adjust the shorter project to the longer project's life:

FV (Sports Facility) = $400,000 \times (1.13)^5 + $400,000 \times (1.13)^4 + $400,000 \times (1.13)^3 + $3,000,000 \times (1.13)^2 = $5,797,022.32

MIRR = ($5,797,022.32 / $2,400,000)^{1/6} - 1 = 15.8%

Adjusting for unequal lives does not change the decision as the sports facility still has a higher MIRR but the rates are almost similar with the adjustment.

16. MIRR unequal lives. What is the MIRR for Singing Fish Fine Foods in Problem 12? What is the MIRR when you adjust for the unequal lives? Does the adjusted MIRR for unequal lives change the decision based on MIRR? 

Hint: Take all cash flows to the same ending period as the longest project.
Rate

ANSWER

Take the cash flows out to each project’s ending point and calculate the MIRR.

FV (Deli Section) = $600,000 \times (1.095)^4 + $600,000 \times (1.095)^3 + $600,000 \times (1.095)^2 + $600,000 \times (1.095)^1 + $600,000 \times (1.095)^0 = $3,626,771

MIRR = ($3,626,771 / $2,000,000)^{1/5} – 1 = 12.64%

FV (Wine Section) = $530,000 \times (1.09)^5 + $530,000 \times (1.09)^4 + $530,000 \times (1.09)^3 + $530,000 \times (1.09)^2 + $530,000 \times (1.09)^1 + $530,000 \times (1.09)^0 = $3,457,367.32

MIRR = ($3,457,367.32 / $2,000,000)^{1/6} – 1 = 9.55%

Adjust the shorter project (deli section) to the longer projects life:

FV (Deli Section) = $600,000 \times (1.095)^5 + $600,000 \times (1.095)^4 + $600,000 \times (1.095)^3 + $600,000 \times (1.095)^2 + $600,000 \times (1.095)^1 = $3,971,314.24

MIRR = ($3,971,314.24 / $2,000,000)^{1/6} – 1 = 12.11%

Adjusting for unequal lives does not change the decision as the deli section still has a higher MIRR.

17. Comparing NPV and IRR. Chandler and Joey were having a discussion about which financial model to use for their new business. Chandler supports NPV and Joey supports IRR. The discussion starts to get heated when Ross steps in and states, “Gentlemen, it doesn’t matter which method we choose, they give the same answer on all projects.” Is Ross correct? Under what conditions will IRR and NPV be consistent when accepting or rejecting projects?

ANSWER

Ross is partially right as NPV and IRR both reject or both accept the same projects under the following conditions:

- The projects have standard cash flows
- The hurdle rate for IRR is the same as the discount rate for NPV
- All projects are available for acceptance regardless of the decision made on another project (projects are not mutually exclusive)

18. Comparing NPR and IRR. Monica and Rachel are having a discussion about IRR and NPV as a decision model for Monica’s new restaurant. Monica wants to use IRR because it gives a very simple and intuitive answer. Rachel states that IRR can cause errors, unlike NPV. Is Rachel correct? Show one type of error can be made with IRR and not with NPV.
ANSWER

The most typical example here is with two mutually exclusive projects where the IRR of one project is higher than the IRR of the other project but the NPV of the second project is higher than the NPV of the first project. When comparing two projects using only IRR this method fails to account for the level of risk of the project cash flows. When the discount rate is below the cross-over rate one project is better under NPV while the other project is better if the discount rate is above the cross-over rate and still below the IRR.

19. Profitability index. Given the discount rates and the future cash flows of each project, which projects should they accept using profitability index?

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project U</th>
<th>Project V</th>
<th>Project W</th>
<th>Project X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-$2,000,000</td>
<td>-$2,500,000</td>
<td>-$2,400,000</td>
<td>-$1,750,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>$ 500,000</td>
<td>$ 600,000</td>
<td>$1,000,000</td>
<td>$ 300,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>$ 500,000</td>
<td>$ 600,000</td>
<td>$ 800,000</td>
<td>$ 500,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>$ 500,000</td>
<td>$ 600,000</td>
<td>$ 600,000</td>
<td>$ 700,000</td>
</tr>
<tr>
<td>Year 4</td>
<td>$ 500,000</td>
<td>$ 600,000</td>
<td>$ 400,000</td>
<td>$ 900,000</td>
</tr>
<tr>
<td>Year 5</td>
<td>$ 500,000</td>
<td>$ 600,000</td>
<td>$ 200,000</td>
<td>$1,100,000</td>
</tr>
<tr>
<td>Discount rate</td>
<td>6%</td>
<td>9%</td>
<td>15%</td>
<td>22%</td>
</tr>
</tbody>
</table>

ANSWER

Find the present value of benefits and divide by the present value of the costs for each project.

Project U’s PV Benefits = $500,000/1.05 + $500,000/1.05² + $500,000/1.05³ + $500,000/1.05⁴ + $500,000/1.05⁵
Project U’s PV Benefits = $476,190.48 + $453,514.74 + $431,918.80 + $411,351.24 + $391,763.08 = $2,164,738.34
Project U’s PV Costs = $2,000,000
Project U’s PI = $2,106,182 / $2,000,000 = $1.08 accept project.

Project V’s PV Benefits = $600,000/1.09 + $600,000/1.09² + $600,000/1.09³ + $600,000/1.09⁴ + $600,000/1.09⁵
Project V’s PV Benefits = -$2,000,000 + $550,458.72 + $505,008.00 + $463,331.09 + $425,055.13 + $389,958.83 = $2,333,790.77
Project V’s PV Costs = $2,500,000
Project V’s PI = $2,333,790.77 / $2,500,000 = 0.9335 and reject project.

Project W’s PV Benefits = $1,000,000/1.15 + $800,000/1.15² + $600,000/1.15³ + $400,000/1.15⁴ + $200,000/1.15⁵

Project W’s PV Benefits = $869,565.22 + $604,914.93 + $394,509.74 + $228,701.30 + 
$99,435.34 = $2,197,126.53

Project W’s PV Costs = $2,400,000

Project W’s PI = $2,197,126.53 / $2,400,000 = 0.9155 and reject project.

Project X’s PV Benefits = $300,000/1.22 + $500,000/1.22^2 + $700,000/1.22^3 + 
$900,000/1.22^4 + $1,100,000/1.22^5

Project X’s PV Benefits = $245,901.64 + $335,931.20 + $385,494.82 + $406,259.18 + 
$406,999.18 = $1,780,586.02

Project X’s PV Cost = $1,750,000

Project X’s PI = $1,780,586.02 / $1,750,000 = 1.0175 and accept project.

20. Profitability index. Given the discount rates and the future cash flow of each project 
listed, use the PI to determine which projects the company should accept.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project A</th>
<th>Project B</th>
<th>Project C</th>
<th>Project D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>$-1,500,000</td>
<td>$-1,500,000</td>
<td>$-2,000,000</td>
<td>$-2,000,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$700,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$600,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$500,000</td>
<td>$600,000</td>
</tr>
<tr>
<td>Year 4</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$400,000</td>
<td>$800,000</td>
</tr>
<tr>
<td>Year 5</td>
<td>$350,000</td>
<td>$400,000</td>
<td>$300,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Discount rate</td>
<td>4%</td>
<td>8%</td>
<td>13%</td>
<td>18%</td>
</tr>
</tbody>
</table>

ANSWER

Find the present value of benefits and divide by the present value of the costs for each project.

Project A’s PV Benefits = $350,000/1.04 + $350,000/1.04^2 + $350,000/1.04^3 + 
$350,000/1.04^4 + $350,000/1.04^5

Project A’s PV Benefits = $336,538.46 + $323,594.67 + $311,148.73 + $299,181.47 + 
$287,674.49 = $1,558,137.84

Project A’s PV Costs = $1,500,000

Project A’s PI = $1,558,137.84 / $1,500,000 = 1.0388 and accept project.

Project B’s PV Benefits = $400,000/1.08 + $400,000/1.08^2 + $400,000/1.08^3 + 
$400,000/1.08^4 + $400,000/1.08^5

Project B’s PV Benefits = -$2,000,000 + $370,370.37 + $342,935.53 + $317,532.90 + 
$294,011.94 + $272,233.28 = $1,597,084.02

Project B’s PV Costs = $1,500,000
Project B’s PI = $1,597,084.02 / $1,500,000 = 1.0647 and accept project.

Project C’s PV Benefits = $700,000 / 1.13 + $600,000 / 1.13^2 + $500,000 / 1.13^3 + $400,000 / 1.13^4 + $300,000 / 1.13^5

Project C’s PV Benefits = $619,469.03 + $469,888.01 + $346,525.08 + $245,327.49 + $162,827.98 = $1,844,037.59

Project C’s PV Costs = $2,000,000

Project C’s PI = $1,844,037.59 / $2,000,000 = 0.9220 and reject project.

Project D’s PV Benefits = $200,000 / 1.18 + $400,000 / 1.18^2 + $600,000 / 1.18^3 + $800,000 / 1.18^4 + $1,000,000 / 1.18^5

Project D’s PV Benefits = $169,491.53 + $287,273.77 + $365,178.52 + $412,631.10 + $437,109.22 = $1,671,684.14

Project D’s PV Costs = $2,000,000

Project D’s PI = $1,671,684.14 / $2,000,000 = 0.8358 and reject project.

21. Comparing all methods. Given the following after-tax cash flows on a new toy for Tyler's Toys, find the project's payback period, NPV, and IRR. The appropriate discount rate for the project is 12%. If the cutoff period is six years for major projects, determine whether management will accept or reject the project under the three different decision models.

Year 0 cash outflow: $10,400,000

Years 1 to 4 cash inflow: $2,600,000 each year

Year 5 cash outflow: $1,200,000

Years 6 to 8 cash inflow: $750,000 each year

ANSWER

Payback Period: $10,400,000 + $2,600,000 + $2,600,000 + $2,600,000 + $2,600,000 + $1,200,000 + $750,000 + $750,000 = $10,400,000 + $2,600,000 + $2,600,000 + $2,600,000 + $2,600,000 + $1,200,000 + $750,000 + $750,000 = $21,600,000

0.6 so total Payback is 6.6 years and project is rejected with six year cut-off.

Net Present Value: $10,400,000 + $2,600,000 / 1.12 + $2,600,000 / 1.12^2 + $2,600,000 / 1.12^3 + $2,600,000 / 1.12^4 - $1,200,000 / 1.12^5 + $750,000 / 1.12^6 + $750,000 / 1.12^7 + $750,000 / 1.12^8

NPV = $10,400,000 + $2,321,428.57 + $2,072,704.08 + $1,850,628.64 + $1,652,347.00 - $680,912.23 + $379,973.34 + $339,261.91 + $302,912.42

NPV = $2,161,656.25 and reject project under NPV rules.

IRR = (discount rate where NPV = 0) = -$10,400,000 + $2,600,000 / (1+r) + $2,600,000 / (1+r)^2 + $2,600,000 / (1+r)^3 + $2,600,000 / (1+r)^4 - $1,200,000 / (1+r)^5 + $750,000 / (1+r)^6 + $750,000 / (1+r)^7 + $750,000 / (1+r)^8
In calculator solve for r, \( CF_0 = -10,400,000 \)

- \( C_{01} = 2,600,000 \) and \( F_{01} = 4 \)
- \( C_{02} = -1,200,000 \) and \( F_{02} = 2 \)
- \( C_{03} = 750,000 \) and \( F_{03} = 3 \)

\[ CPT\ IRR = 3.1955\% \]

\[ \Rightarrow \text{Reject project as IRR is less than 12\%} \]

**Present Value of Benefits**

\[
\frac{2,600,000}{1.12} + \frac{2,600,000}{1.12^2} + \frac{2,600,000}{1.12^3} + \frac{750,000}{1.12^4} - \frac{2,321,428.57}{1.12^5} - \frac{2,072,704.08}{1.12^6} - \frac{1,850,628.64}{1.12^7} + \frac{1,652,347.00}{1.12^8} + \frac{379,973.34}{1.12^9} + \frac{339,261.91}{1.12^{10}} + \frac{302,912.42}{1.12^{11}} = 8,919,255.73
\]

**Present Value of Costs**

\[
\frac{10,400,000}{1.12} + \frac{1,200,000}{1.12^2} + \frac{750,000}{1.12^3} = $11,080,912.23
\]

**Profitability Index**

\[
\frac{8,919,255.73}{11,080,912.23} = 0.8049 \text{ and reject.}
\]

22. **Comparing all methods.** Risky Business is looking at a project with the estimated cash flows as follows:

- **Initial Investment at start of project:** $3,600,000
- **Cash Flow at end of Year 1:** $500,000
- **Cash Flow at end of Years 2 through 6:** $625,000 each year
- **Cash Flow at end of Year 7 through 9:** $530,000 each year
- **Cash Flow at end of Year 10:** $385,000

Risky Business wants to know payback period, NPV, IRR, MIRR, and PI of this project. The appropriate discount rate for the project is 14\%. If the cutoff period is six years for major projects, determine whether management at Risky Business will accept or reject the project under the five different decision models.

**ANSWER**

\[
\text{Payback Period} = -$3,600,000 + $500,000 + $625,000 + $625,000 + $625,000 + $625,000 + $625,000 = $25,000 \text{ and we only need part of year 6 so,}
\]

\[
$600,000 / $625,000 = 0.96 \text{ and Payback Period is 5.96 years and project is accepted.}
\]

\[
\text{NPV} = -$3,600,000 + $500,000 / 1.14 + $625,000 / 1.14^2 + $625,000 / 1.14^3 + $625,000 / 1.14^4 + $625,000 / 1.14^5 + $530,000 / 1.14^6 + $530,000 / 1.14^7 + $385,000 / 1.14^{10}
\]

\[
\text{NPV} = -$3,600,000 + $438,596.49 + $480,917.21 + $421,857.20 + $370,050.17 + $324,605.42 + $284,741.59 + $211,807.78 + $185,796.30 + $162,979.21 + $103,851.37 = -$614,797.27 \text{ and project is rejected using NPV rules.}
\]
Northern Illinois University
Department of Business Administration

Chapter 9 ■ Capital Budgeting Decision Models 317

IRR In calculator solve for r, CF0 = -3,600,000
   C01 = 500,000 and F01 = 1
   C02 = 625,000 and F02 = 5
   C03 = 530,000 and F03 = 3
   C04 = $385,000 and F04 = 1
   CPT IRR = 9.3349%

Reject project as IRR is less than 14%

MIRR solution with the 14% cost of capital as the investment rate for the annual cash
flows.

Take all positive cash flows to year 10 and find the future value of the cash flow at the
end of year ten.

$500,000 \times 1.14^9 = $1,625,974
$625,000 \times 1.14^8 = $1,782,866
$625,000 \times 1.14^7 = $1,563,918
$625,000 \times 1.14^6 = $1,371,858
$625,000 \times 1.14^5 = $1,203,384
$625,000 \times 1.14^4 = $1,055,600
$530,000 \times 1.14^3 = $ 785,218
$530,000 \times 1.14^2 = $ 688,788
$530,000 \times 1.14^1 = $ 604,200
$385,000 \times 1.14^0 = $ 385,000

Future Cash Flow Value $11,066,806

Now find the discount rate that equates the future cash flow value to the original
investment:

MIRR = ($11,066,806 /$3,600,000)\(^{1/10}\) – 1 = 11.885%

Present Value of Benefits = $500,000 / 1.14 + $625,000/1.14^2 + $625,000/1.14^3 +
   $625,000/1.14^4 + $625,000/1.14^5 + $625,000/1.14^6 +
   $530,000/1.14^7 + $530,000 /1.14^8 + $530,000/1.14^9 +
   $385,000/1.14\(^10\)

Present Value of Benefits = $438,596.49 + $480,917.21 + $421,857.20 + $370,050.17 +
   $324,605.42 + $284,741.59 + $211,807.78 + $185,796.30 +
   $162,979.21+$103,851.37 = $2,985,202.73

Profitability Index = $2,985,202.73 / $3,600,000 = 0.8292 and reject.
23. **NPV profile of a project.** Given the following cash flows of Project L-2, draw the NPV profile. *Hint:* use a discount rate of zero for one intercept (y-axis) and solve for the IRR for the other intercept (x-axis).

Cash flows: Year 0 = -$250,000  
Year 1 = $45,000  
Year 2 = $75,000

Year 3 = $115,000  
Year 4 = $135,000

**ANSWER**

NPV (discount rate = 0) = -$250,000 + $45,000 + $75,000 + $115,000 + $135,000  
= $120,000 (y-axis intercept)

NPV (discount rate = 5%) = -$250,000 + $45,000/1.05 + $75,000/1.05^2 + $115,000/1.05^3  
+ $135,000/1.05^4  
= $71,290.51

NPV (discount rate = 10%) = -$250,000 + $45,000/1.10 + $75,000/1.10^2  
+ $115,000/1.10^3  
+ $135,000/1.10^4  
= $31,500.58

NPV (discount rate = 15%) = -$250,000 + $45,000/1.15 + $75,000/1.15^2  
+ $115,000/1.15^3  
+ $135,000/1.15^4  
= -$1,357.74

NPV (discount rate = 20%) = -$250,000 + $45,000/1.20 + $75,000/1.20^2  
+ $115,000/1.20^3  
+ $135,000/1.20^4  
= -$28,761.57

IRR = 14.77%

NPV Dollars

NPV Profile  
Of Project L-2

Discount Rates
24. **NPV profile of two mutually exclusive projects.** Moulton Industries has two potential projects for the coming year, Project B-12 and Project F-4. The two projects are mutually exclusive. The cash flows are listed below. Draw the NPV profile of each project and determine the cross-over rate of the two projects. If the appropriate hurdle rate is 10% for both projects, which project, does Moulton Industries choose?

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Project B-12</th>
<th>Project F-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-$4,250,000</td>
<td>-$3,800,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>$2,000,000</td>
<td>$0</td>
</tr>
<tr>
<td>Year 2</td>
<td>$2,000,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>$2,000,000</td>
<td>$1,500,000</td>
</tr>
<tr>
<td>Year 4</td>
<td>$0</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>Year 5</td>
<td>$0</td>
<td>$2,500,000</td>
</tr>
</tbody>
</table>

**ANSWER**

Draw the NPV profile select different discount rates such as 0% for y-axis intercept and then 5%, 10%, 15%, and 20%. Also find the IRR of the projects for the x-axis intercept.

**Project B-12 NPVs at different discount rates:**

At 0% discount rate, NPV = -$4,250,000 + $2,000,000 + $2,000,000 + $2,000,000 = $1,750,000

At 5% discount rate, NPV = -$4,250,000 + $2,000,000 / 1.05 + $2,000,000 / 1.05^2 + $2,000,000 / 1.05^3 = $1,196,496

At 10% discount rate, NPV = -$4,250,000 + $2,000,000 / 1.10 + $2,000,000 / 1.10^2 + $2,000,000 / 1.10^3 = $723,704

At 15% discount rate, NPV = -$4,250,000 + $2,000,000 / 1.15 + $2,000,000 / 1.15^2 + $2,000,000 / 1.15^3 = $316,450

At 20% discount rate, NPV = -$4,250,000 + $2,000,000 / 1.20 + $2,000,000 / 1.20^2 + $2,000,000 / 1.20^3 = -$37,037

**IRR of Project B-12 = 19.44%**

**Project F-4 NPVs at different discount rates :**

At 0% discount rate, NPV = -$3,800,000 + $1,000,000 + $1,500,000 + $2,000,000 + $2,500,000 = $3,200,000

At 5% discount rate, NPV = -$3,800,000 + $1,000,000 / 1.05 + $1,500,000 / 1.05^2 + $2,000,000 / 1.05^3 + $2,500,000 / 1.05^4 = $2,007,006

At 10% discount rate, NPV = -$3,800,000 + $1,000,000 / 1.10 + $1,500,000 / 1.10^2 + $2,000,000 / 1.10^3 + $2,500,000 / 1.10^4 = $1,071,749

At 15% discount rate, NPV = -$3,800,000 + $1,000,000 / 1.15 + $1,500,000 / 1.15^2 + $2,000,000 / 1.15^3 + $2,500,000 / 1.15^4 = $328,866
At 20% discount rate, NPV = -$3,800,000 + $1,000,000 / 1.20^2 + $1,500,000/1.20^3 + $2,000,000 / 1.20^4 + $2,500,000 / 1.20^5 = -$268,300

IRR of Project F-4 = 17.62%

Cross-over rate of the two projects is found by setting the cash flows equal to each other (as the NPV is same at this discount rate) and solving for r. The simple way is to move all cash flows to one side of the equation and solve for the IRR with the difference in cash flows for each year.

<table>
<thead>
<tr>
<th>Project</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-12</td>
<td>-$4.250</td>
<td>$2.000</td>
<td>$2.000</td>
<td>$2.000</td>
<td>$0.000</td>
<td>$0.000</td>
</tr>
<tr>
<td>F-4</td>
<td>-$3.800</td>
<td>$0.000</td>
<td>$1.000</td>
<td>$1.500</td>
<td>$2.000</td>
<td>$2.500</td>
</tr>
<tr>
<td>Difference</td>
<td>-$0.450</td>
<td>$2.000</td>
<td>$1.000</td>
<td>$0.500</td>
<td>-$2.000</td>
<td>-$2.500</td>
</tr>
</tbody>
</table>

Dollars in Millions

IRR of the differences in cash flows is:

$0 = -$450,000 + $2,000,000 / (1+r) + $1,000,000 / (1+r)^2 + $500,000 / (1+r)^3 - $2,000,000 / (1+r)^4 - $2,500,000 / (1+r)^5

And solving for r, r = 15.2195%

To verify this answer substitute 15.2195% in the NPV calculations for both projects:

At 15.2195% discount rate, NPV = -$4,250,000 + $2,000,000 / 1.152195^1 + $2,000,000 / 1.152195^2 + $2,000,000 / 1.152195^3 = $299,879

At 15.2195% discount rate, NPV = -$3,800,000 + $1,000,000 / 1.152195^1 + $1,500,000/1.152195^2 + $1,500,000/1.152195^3 + $2,000,000 / 1.152195^4 + $2,500,000 / 1.152195^5 = $299,879

NPV Profiles of B-12 and F-4

At 10% Discount Rate F-4 is above B-12 and is the better of the two projects.

Solutions for Advanced Problems for Spreadsheet Application

1. **NPV Profile:**

   ![NPV Profile Chart](chart.png)
### 2. IRR and MIRR:

<table>
<thead>
<tr>
<th>Year</th>
<th>CF ($ millions)</th>
<th>IRR by Function</th>
<th>NPV at 24.270% discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(35.0500)</td>
<td>24.270%</td>
<td>$0.00</td>
</tr>
<tr>
<td>1</td>
<td>$3.4400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$3.4400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$5.7900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$9.2300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$14.6800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$18.3900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$21.0700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$16.4200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$11.6800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rates</th>
<th>MIRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0%</td>
<td>14.89%</td>
</tr>
<tr>
<td>4.0%</td>
<td>15.69%</td>
</tr>
<tr>
<td>6.0%</td>
<td>16.50%</td>
</tr>
<tr>
<td>8.0%</td>
<td>17.32%</td>
</tr>
<tr>
<td>10.0%</td>
<td>18.14%</td>
</tr>
<tr>
<td>12.0%</td>
<td>18.98%</td>
</tr>
<tr>
<td>14.0%</td>
<td>19.82%</td>
</tr>
<tr>
<td>16.0%</td>
<td>20.67%</td>
</tr>
<tr>
<td>18.0%</td>
<td>21.53%</td>
</tr>
<tr>
<td>20.0%</td>
<td>22.40%</td>
</tr>
<tr>
<td>22.0%</td>
<td>23.27%</td>
</tr>
<tr>
<td>24.0%</td>
<td>24.15%</td>
</tr>
<tr>
<td>26.0%</td>
<td>25.04%</td>
</tr>
<tr>
<td>28.0%</td>
<td>25.93%</td>
</tr>
<tr>
<td>30.0%</td>
<td>26.84%</td>
</tr>
</tbody>
</table>
Solutions to Mini-Case

BioCom Inc: Part 1

This mini-case provides a review of the methodology, and rationale associated with the various capital budgeting evaluation methods such as Payback Period, Discounted PPB, NPV, IRR, MIRR, and PI.

1. Compute the payback period for each project.

<table>
<thead>
<tr>
<th>Time of Cash Flow</th>
<th>Nano Test Tubes</th>
<th>Microsurgery Kit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>−$11,000</td>
<td>−$11,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>2,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>3,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Year 3</td>
<td>4,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Year 4</td>
<td>5,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Year 5</td>
<td>7,000</td>
<td>4,000</td>
</tr>
</tbody>
</table>

Payback for Nano: cash flows for the first 3 years total $9,000. \((11,000-9000)/4,000=.5\), so payback for Nano is 3.5 years. For Microsurgery: cash flows for the first 2 years total $8,000. \((11,000-8,000)/4000 = .75\), so payback for Microsurgery is 2.75 years.

a. Explain the rationale behind the payback method.

The payback simply computes the break-even point for a project in terms of time rather than units or dollars. It is the amount of time required for a project to recover the initial investment.

b. State and explain the decision rule for the payback method.

The payback method implies that the sooner a project recovers the initial investment, the better. The choice of an acceptable payback period is arbitrary.

c. Explain how the payback method would be used to rank mutually exclusive projects.

For acceptable mutually exclusive projects, the one with the shortest payback period would be chosen.

d. Comment on the advantages and shortcomings of this method.

Payback is simple to compute and the logic is obvious, even to people with no background in finance. The method does not formally recognize the time value of money, it ignores cash flows that occur after the payback period, it does not necessarily select projects that add value to the company, or discriminate between projects that add more or less value.
2. Compute the discounted payback period for each project using a discount rate of 10%.

<table>
<thead>
<tr>
<th>Nano Test Tubes</th>
<th>Year</th>
<th>PV of CF at 10%</th>
<th>Remaining cost to recover</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>$ (11,000.00)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2,000/1.10¹=</td>
<td>1,818.18</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3,000/1.10²=</td>
<td>2,479.34</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4,000/1.10³=</td>
<td>3,005.26</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5,000/1.10⁴=</td>
<td>3,415.07</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7,000/1.10⁵=</td>
<td>4,346.45</td>
</tr>
</tbody>
</table>

In (000’s)

<table>
<thead>
<tr>
<th>Microsurgery Kit</th>
<th>Year</th>
<th>PV of CF at 10%</th>
<th>Remaining cost to recover</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>($11,000.00)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4,000/1.10¹=</td>
<td>3,636.36</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4,000/1.10²=</td>
<td>3,305.79</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4,000/1.10³=</td>
<td>3,005.26</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4,000/1.10⁴=</td>
<td>2,732.05</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4,000/1.10⁵=</td>
<td>2,483.69</td>
</tr>
</tbody>
</table>

With the discounted payback method, the Nano Test Tube project does not break even until the 5th year. The Microsurgery Kit project breaks even in the 4th year.

a. Explain the rationale behind the discounted payback method.

The discounted payback method tells us how long it will take for a project to break even if we include the time value of money.
b. **Comment on the advantages and shortcomings of this method.**

By including the time value of money, the discounted payback method indicates that any project with a payback period shorter than its useful life will add value. For this reason, the method could be helpful if the useful life of the project is uncertain. Other than that, the disadvantages are the same as for the simple payback method, and of course the simplicity of the basic method is lost.

3. **Compute the net present value (NPV) for each project. BioCom uses a discount rate of 9% for projects of average risk.**

NPV(Nano Test Tubes) @9% = $4,540.28

NPV(Microsurgery Kit) @9% = $4,558.61

a. **Explain the rationale behind the NPV method.**

The net present value method discounts all cash flows so that they can be treated as if they were all received at the same point in time. For example, the Microsurgery Kit project is equivalent to spending $11 million immediately in order to receive $15.56 million immediately.

b. **State and explain the decision rule behind the NPV method.**

The decision rule for the NPV method is fairly obvious. Any NPV greater than $0.00 means the project is acceptable.

c. **Explain how the NPV method would be used to rank mutually exclusive projects.**

For mutually exclusive projects, one should simply choose the one with the largest NPV.

d. **Comment on the advantages and shortcomings of this method.**

Because the NPV method considers all cash flows through the life of the project and specifically incorporates the time value of money, it is considered to be the most reliable capital budgeting method.

e. **Without performing any calculations, explain what happens to NPV if the discount rate is adjusted upward for projects of higher risk or downward for projects of lower risk.**

The NPV computation requires cash flows to be divided by (1+r). Therefore, so as long as negative cash flows are followed only by positive cash flows, higher values of r will lead to lower NPVs and vice versa.

4. **Compute the internal rate of return (IRR) for each project.**

For Nano Test Tubes, the IRR is $(11,000) + 2,000/1.2097^1 + 3,000/1.2097^2 + 4,000/1.2097^3 + 5,000/1.2097^4 + 7,000/1.2097^5 = $0.00

For the Microsurgery Kit, the IRR is $(11,000) + 4,000/1.2392^1 + 4,000/1.2392^2 + 4,000/1.2392^3 + 4,000/1.2392^4 + 4,000/1.2392^5 = $0.00

By trial and error, the IRR is found to be 20.97% for Nano Test Tubes, and 23.92% for the Microsurgery Kit. The solutions can be found effortlessly with a financial calculator or a spreadsheet such as EXCEL.
a. Explain the rationale behind the IRR method.

By computing the highest discount rate at which a project will have a positive NPV, the IRR method is supposed to assure that the actual rate of return on an accepted project is higher than the required rate of return.

b. State and explain the decision rule behind the IRR method. Assume a hurdle rate of 9%.

If the IRR exceeds the required rate of return (9%), the project should be accepted. Otherwise, it should be rejected.

c. Explain how the IRR method would be used to rank mutually exclusive projects.

When choosing between projects with acceptable IRRs, the one with the highest IRR should be chosen.

d. Comment on the advantages and shortcomings of this method.

IRR uses all cash flows and incorporates the time value of money. When evaluating independent projects, IRR will always lead to the same decision as NPV.

Because IRR assumes that cash flows will be reinvested at the internal rate of return, which is not always or even usually the case, it can rank mutually exclusive projects incorrectly. With certain patterns of cash flows, the IRR equation has more than one solution, which confuses the decision rule. IRR is slightly more difficult to compute than NPV, but this consideration is irrelevant in the age of specialized calculators and electronic spreadsheets.

5. Compute the modified internal rate of return (MIRR) for each project.

For Nano Test Tubes, we find the MIRR by first reinvesting all cash flows at the reinvestment rate, which we assume to be the same as the required rate of return, 9%.

\[
2,000 \times 1.09^4 + 3,000 \times 1.09^3 + 4,000 \times 1.09^2 + 5,000 \times 1.09^1 + 7,000 \times 1.09^0 = $23,910.65
\]

MIRR = \((\frac{23,910.65}{11,000})^{1/5} - 1\) = 16.80%

For the Microsurgery Kit, the IRR is

\[
4,000 \times 1.09^4 + 4,000 \times 1.09^3 + 4,000 \times 1.09^2 + 4,000 \times 1.09^1 + 4,000 \times 1.09^0 = $23,938.84
\]

MIRR = \((\frac{23,938.84}{11,000})^{1/5} - 1\) = 16.83%

a. Explain the rationale behind the MIRR method.

The MIRR method computes the rate of return on invested capital when intermediate cash flows are reinvested at a predetermined rate, which is typically the same as the required rate of return.

b. State and explain the decision rule behind the MIRR method. Assume a hurdle rate of 9%.

The decision rule for MIRR is the same as for IRR: If the IRR exceeds the required rate of return (9%), the project should be accepted. Otherwise, it should be rejected.
c. Explain how the MIRR method would be used to choose between mutually exclusive projects.

As with IRR, choose the project with the highest MIRR.

d. Explain how this method corrects for some of the problems inherent in the IRR method.

The MIRR method corrects the two major disadvantages of IRR, the reinvestment rate problem and the multiple solution problem. When projects are adjusted for a common economic life, MIRR should lead to the same decision as NPV, but the criterion is stated as a percentage rate rather than in dollars (or other currency).

6. Explain to the R & D staff why BioCom uses the NPV method as its primary project selection criterion.

Bi-Com uses the NPV method because it wishes to choose those projects which most increase the value of the company. NPV provides a decision criterion stated directly in terms of value in present dollars.

7. Challenge question. Construct NPV profiles for both projects using discount rates of 1% through 15% at one percentage point intervals. At approximately what discount rate does the Nano test tube project become superior to the microsurgery kits? This problem is best solved using an electronic spreadsheet.

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>NPV NANO</th>
<th>NPV MICRO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>$9,268.61</td>
<td>$8,413.72</td>
</tr>
<tr>
<td>2%</td>
<td>$8,572.92</td>
<td>$7,853.84</td>
</tr>
<tr>
<td>3%</td>
<td>$7,910.80</td>
<td>$7,318.83</td>
</tr>
<tr>
<td>4%</td>
<td>$7,280.24</td>
<td>$6,807.29</td>
</tr>
<tr>
<td>5%</td>
<td>$6,679.40</td>
<td>$6,317.91</td>
</tr>
<tr>
<td>6%</td>
<td>$6,106.53</td>
<td>$5,849.46</td>
</tr>
<tr>
<td>7%</td>
<td>$5,560.05</td>
<td>$5,400.79</td>
</tr>
<tr>
<td>8%</td>
<td>$5,038.43</td>
<td>$4,970.84</td>
</tr>
<tr>
<td>9%</td>
<td>$4,540.28</td>
<td>$4,558.61</td>
</tr>
<tr>
<td>10%</td>
<td>$4,064.30</td>
<td>$4,163.15</td>
</tr>
<tr>
<td>11%</td>
<td>$3,609.25</td>
<td>$3,783.59</td>
</tr>
<tr>
<td>12%</td>
<td>$3,174.00</td>
<td>$3,419.10</td>
</tr>
<tr>
<td>13%</td>
<td>$2,757.47</td>
<td>$3,068.93</td>
</tr>
<tr>
<td>14%</td>
<td>$2,358.66</td>
<td>$2,732.32</td>
</tr>
<tr>
<td>15%</td>
<td>$1,976.63</td>
<td>$2,408.62</td>
</tr>
</tbody>
</table>
Because the Nano project brings in a greater total amount of money, it has a higher NPV at lower discount rates. Because the Microsurgery Kit project brings the money in faster, it has a higher NPV at higher discount rates. The crossover rate is just under 9%.

### Additional Problems with Solutions  
(Slides 9-54 to 9-67)

1. **Computing Payback Period and Discounted Payback Period.**

Regions Bank is debating between two the purchase of two software systems; the initial costs and annual savings of which are listed below. Most of the directors are convinced that given the short lifespan of software technology, the best way to decide between the two options is on the basis of a payback period of 2 years or less. Compute the payback period of each option and state which one should be purchased. One of the directors states, “I object! Given our hurdle rate of 10%, we should be using a discounted payback period of 2 years or less.” Accordingly, evaluate the projects on the basis of the DPP and state your decision.

**ANSWER**

<table>
<thead>
<tr>
<th>Year</th>
<th>Software Option A</th>
<th>PVCF@10%</th>
<th>Software Option B</th>
<th>PVCF@10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($)1,875,000</td>
<td>$1,875,000</td>
<td>($)2,000,000</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>1</td>
<td>$1,050,000</td>
<td>$954,545.45</td>
<td>$1,250,000</td>
<td>$1,136,363.64</td>
</tr>
<tr>
<td>2</td>
<td>$900,000</td>
<td>$743,801.65</td>
<td>$800,000</td>
<td>$661,157.02</td>
</tr>
<tr>
<td>3</td>
<td>$450,000</td>
<td>$338,091.66</td>
<td>$600,000</td>
<td>$450,788.88</td>
</tr>
</tbody>
</table>
Payback period of Option A = 1 year + (1,875,000 - 1,050,000)/900,000 = 1.92 years
Payback period of Option B = 1 year + (2,000,000 - 1,250,000)/800,000 = 1.9375 years.

**Based on the Payback Period, Option A should be chosen.**

For the discounted payback period, we first discount the cash flows at 10% for the respective number of years and then add them up to see when we recover the investment.

DPP A = -1,875,000 + 954,545.45 + 743,801.65 = -176652.9 \( \Rightarrow \) still to be recovered in Year 3

\[ DPP = 2 + \left( \frac{-176652.9}{338091.66} \right) = 2.52 \text{ years} \]

DPP B = -2,000,000 + 1,136,363.64 + 661,157.02 = -202479.34 \( \Rightarrow \) still to be recovered in Year 3

\[ DPPB = 2 + \left( \frac{-202479.34}{450788.88} \right) = 2.45 \text{ years} \]

Based on the Discounted Payback Period and a 2 year cutoff, neither option is acceptable.


Locey Hardware Products is expanding its product line and its production capacity. The costs and expected cash flows of the two projects are given below. The firm typically uses a discount rate of 15.4 percent.

**a. What are the NPVs of the two projects?**

**b. Which of the two projects should be accepted (if any) and why?**

<table>
<thead>
<tr>
<th>Year</th>
<th>Product Line Expansion</th>
<th>Production Capacity Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(2,450,000)</td>
<td>(8,137,250)</td>
</tr>
<tr>
<td>1</td>
<td>$ 500,000</td>
<td>$ 1,250,000</td>
</tr>
<tr>
<td>2</td>
<td>$ 825,000</td>
<td>$ 2,700,000</td>
</tr>
<tr>
<td>3</td>
<td>$ 850,000</td>
<td>$ 2,500,000</td>
</tr>
<tr>
<td>4</td>
<td>$ 875,000</td>
<td>$ 3,250,000</td>
</tr>
<tr>
<td>5</td>
<td>$ 895,000</td>
<td>$ 3,250,000</td>
</tr>
</tbody>
</table>

**ANSWER**

NPV @15.4% = $86,572.61; $20,736.91

Decision: Both NPVs are positive, and the projects are independent, so assuming that Locey Hardware has the required capital, both projects are acceptable.
3. Computing IRR.

KLS Excavating needs a new crane. It has received two proposals from suppliers. Proposal A costs $900,000 and generates cost savings of $325,000 per year for 3 years, followed by savings of $200,000 for an additional 2 years. Proposal B costs $1,500,000 and generates cost savings of $400,000 for 5 years. If KLS has a discount rate of 12%, and prefers using the IRR criterion to make investment decisions, which proposal should it accept?

**ANSWER** (Slides 9-59 to 9-60)

<table>
<thead>
<tr>
<th>Year</th>
<th>Crane A</th>
<th>Crane B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(900,000)</td>
<td>$(1,500,000)</td>
</tr>
<tr>
<td>1</td>
<td>$325,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>2</td>
<td>$325,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>3</td>
<td>$325,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>4</td>
<td>$200,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>5</td>
<td>$200,000</td>
<td>$400,000</td>
</tr>
</tbody>
</table>

Required rate of return: 12%

<table>
<thead>
<tr>
<th>IRR</th>
<th>17.85%</th>
<th>10.42%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision</td>
<td>Accept Crane A</td>
<td>$IRR &gt; 12%$</td>
</tr>
</tbody>
</table>

Decision: Accept Crane A

4. Using MIRR.

The New Performance Studio is looking to put on a new opera. They figure that the set-up and publicity will cost $400,000. The show will go on for 3 years and bring in after-tax net cash flows of $200,000 in Year 1; $350,000 in Year 2; -$50,000 in Year 3. If the firm has a required rate of return of 9% on its investments, evaluate whether the show should go on using the MIRR approach.

**ANSWER** (Slides 9-61 to 9-63)

The forecasted after-tax net cash flows are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>After–tax cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$400,000</td>
</tr>
<tr>
<td>1</td>
<td>200,000</td>
</tr>
</tbody>
</table>

The formula for MIRR is as follows:

\[ MIRR = \left( \frac{FV}{PV} \right)^{\frac{1}{n}} - 1 \]

Where

- \( FV \) = Compounded value of cash inflows at end of project’s life (Year 3) using realistic reinvestment rate (9%);
- \( PV \) = Discounted value of all cash outflows at Year 0;
- \( N \) = number of years until the end of the project’s life = 3.

\[ FV_3 = 200,000 \times (1.09)^2 + 350,000 \times (1.09)^1 = 237,620 + 381,500 = 619,120 \]

\[ PV_0 = 400,000 + 50,000 \div (1.09)^3 = 438,609.17 \]

\[ MIRR = \left( \frac{619,120}{438,609.17} \right)^{\frac{1}{3}} - 1 = (1.411552)^{\frac{1}{3}} - 1 = 12.18\% \]

The show must go on, since the MIRR = 12.18% > Hurdle rate = 9%.

5. Using multiple methods with mutually exclusive projects.

The Upstart Corporation is looking to invest one of 2 mutually exclusive projects, the cash flows for which are listed below. Their director is really not sure about the hurdle rate that he should use when evaluating them and wants you to look at the projects’ NPV profiles to better assess the situation and make the right decision.

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-454,000</td>
<td>($582,000)</td>
</tr>
<tr>
<td>1</td>
<td>$130,000</td>
<td>$143,333</td>
</tr>
<tr>
<td>2</td>
<td>$126,000</td>
<td>$168,000</td>
</tr>
<tr>
<td>3</td>
<td>$125,000</td>
<td>$164,000</td>
</tr>
<tr>
<td>4</td>
<td>$120,000</td>
<td>$172,000</td>
</tr>
<tr>
<td>5</td>
<td>$120,000</td>
<td>$122,000</td>
</tr>
</tbody>
</table>

To get some idea of the range of discount rates we should include in the NPV profile, it is a good idea to first compute each project’s IRR and the crossover rate, i.e., the IRR of the cash flows of Project B-A as shown below:

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
<th>B-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(454,000)</td>
<td>($582,000)</td>
<td>($128,000)</td>
</tr>
</tbody>
</table>
So, it’s clear that the NPV profiles will cross-over at a discount rate of 5.2%.

Project A has a higher IRR than Project B, so at discount rates higher than 5.2%, it would be the better investment, and vice-versa (higher NPV and IRR), but if the firm can raise funds at a rate lower than 5.2%, then Project B will be better, since its NPV would be higher.

To check this let’s compute the NPVs of the 2 projects at 0%, 3%, 5.24%, 8%, 10.2%, and 11.6%...

<table>
<thead>
<tr>
<th>Rate</th>
<th>NPV(A)</th>
<th>NPV(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00%</td>
<td>167,000</td>
<td>187,333</td>
</tr>
<tr>
<td>3.00%</td>
<td>115,505</td>
<td>123,656</td>
</tr>
<tr>
<td>5.24%</td>
<td>81,353</td>
<td>81,353</td>
</tr>
<tr>
<td>8.00%</td>
<td>43,498</td>
<td>34,393</td>
</tr>
<tr>
<td>10.2%</td>
<td>15,810</td>
<td>0</td>
</tr>
<tr>
<td>11.6%</td>
<td>0</td>
<td>-19,658</td>
</tr>
</tbody>
</table>

Note that the two projects have equal NPVs at the cross-over rate of 5.24%. At rates below 5.24%, Project B’s NPVs are higher; whereas at rates higher than 5.24%, Project A has the higher NPV.