Chapter 8
Risk and Return

LEARNING OBJECTIVES

1. Calculate profits and returns on an investment and convert holding period returns to annual returns.
2. Define risk and explain how uncertainty relates to risk.
3. Appreciate the historical returns of various investment choices.
4. Calculate standard deviations and variances with historical data.
5. Calculate expected returns and variances with conditional returns and probabilities.
6. Interpret the trade-off between risk and return.
7. Understand when and why diversification works at minimizing risk, and understand the difference between systematic and unsystematic risk.
9. Illustrate how the security market line and the capital asset pricing model represent the two-parameter world of risk and return.

IN A NUTSHELL...

In this chapter, the author discusses various topics related to the risk and return of financial assets. The measurement of holding period return and its conversion into annualized rates of return on investments is covered first, followed by the definition of risk. After providing a bit of perspective on the nature of historical returns characterizing securities markets, the author explains how standard deviation and variance can be used to measure historical or ex-post risk. Next, the calculation of expected or ex-ante risk and return measures is illustrated using a probability distribution framework, along with a discussion of the tradeoff between risk and return. The advantages of diversification that can be derived from the formation of portfolios is covered next, followed by the components of total risk, i.e. systematic and unsystematic risk, and beta, which is the only relevant measure of risk of a well-diversified portfolio. The chapter ends with coverage of the security market line, a type of capital asset pricing model, which was developed as a way of quantifying the relationship between an asset’s risk and return.

LECTURE OUTLINE

8.1 Returns

In order to analyze the performance of an investment it is very important that investors learn how to measure returns over time. Furthermore, since return and risk are intricately
related, the measurement of return also helps in the understanding of the riskiness of an investment.

8.1 (A) Dollar Profits and Percentage Returns: Investment performance can be measured in terms of the profit or loss derived from holding it for a period of time. Some investments, such as stocks and bonds, provide periodic income in the form of dividends or interest, in addition to capital gains (or losses) arising from price changes. Thus, a generic formula for measuring the dollar profit (or loss) on an investment is as follows:

\[ \text{Dollar Profit or Loss} = \text{Ending value} + \text{Distributions} - \text{Original Cost} \] (8.1)

Dollar profits being absolute values do not provide a good gauge of the relative performance of an investment. In other words, is a $2 profit on a $10 investment just as good as a $2 profit on a $100 investment? Obviously not! Hence we calculate the rate of return or percentage return on an investment as follows:

\[ \text{Rate of return} = \frac{\text{Dollar Profit or Loss}}{\text{Original Cost}} \]

Also, since investments can be held for varying periods of time before being disposed off or closed out, an alternative name for overall performance of an investment is holding period return (HPR) and is measured in any one of the 3 following methods:

\[ HPR = \frac{\text{Profit}}{\text{Cost}} \] (8.3a)

\[ HPR = \frac{\text{Ending price} + \text{Distributions} - \text{Beginning price}}{\text{Beginning price}} \] (8.3b)

\[ HPR = \frac{\text{Ending price} + \text{Distributions}}{\text{Beginning price}} - 1 \] (8.3c)

Example 1: Calculating dollar and percentage returns

Joe bought some gold coins for $1000 and sold those 4 months later for $1200. Jane on the other hand bought 100 shares of a stock for $10 and sold those 2 years later for $12 per share after receiving $0.50 per share as dividends for the year. Calculate the dollar profit and percent return earned by each investor over their respective holding periods.

Joe’s Dollar Profit = Ending value – Original cost  
= $1200 – $1000 = $200

Joe’s HPR = Dollar profit/Original cost  
= $200/$1000 = 20%

Jane’s Dollar Profit = Ending value + Distributions – Orig. Cost  
= $12*100 + $0.50*100 – $10*100  
= $1200 + $50 – $1000  
=$250

Jane’s HPR = $250/$1000 = 25%
8.1 (B) Converting Holding Period Returns to Annual Returns: For meaningful comparisons of investment performance, in cases of varying holding periods, it is essential to state HPRs in terms of either simple (annual percentage rate, APR) or compound annual returns (effective annual rate, EAR) by using the following conversion formulas:

\[
\text{Simple annual return or APR} = \frac{\text{HPR}}{n} \quad (8.4)
\]
\[
\text{EAR} = (1 + \text{HPR})^{\frac{1}{n}} - 1 \quad (8.5)
\]

Where \( n \) is the number of years or proportion of a year that the holding period consists of.

Example 2: Comparing HPRs

Given Joe’s HPR of 20% over 4 months and Jane’s HPR of 25% over 2 years, is it correct to conclude that Jane’s investment performance was better than that of Joe?

**ANSWER:** Compute each investor’s APR and EAR and then make the comparison.

Joe’s holding period \((n) = 4/12 = .333 \) years

- Joe’s APR = HPR/n = 20%/0.333 = 60%
- Joe’s EAR = \((1 + \text{HPR})^{\frac{1}{n}} - 1\) = \(1.20^{\frac{1}{.33}} - 1\) = 72.89%

Jane’s holding period = 2 years

- Jane’s APR = HPR/n = 25%/2 = 12.5%
- Jane’s EAR = \((1 + \text{HPR})^{\frac{1}{n}} - 1\) = \((1.25)^{\frac{1}{2}} - 1\) = 11.8%

Clearly, on an annual basis, Joe’s investment far outperformed Jane’s investment.

8.1 (C) Extrapolating Holding Period Returns: It is important to remind students that although the extrapolation of short-term HPRs into APRs and EARs is mathematically correct, it often tends to be highly unrealistic and practically impossible to achieve, especially with very short holding periods.

Extrapolation implies earning the same periodic rate over and over again throughout the relevant number of times per year.

Hence, if the holding period is fairly short, and the HPR fairly high, extrapolation would lead to huge numbers as shown in Example 3 below.

Example 3: Unrealistic nature of APR and EAR

Let’s say you buy a share of stock for $2 and sell it a week later for $2.50. Calculate your HPR, APR, and EAR. How realistic are the numbers?

\[ \text{N} = 1/52 \text{ or } 0.01923 \text{ of 1 year.} \]
\[ \text{Profit} = \$2.50 - \$2.00 = \$0.50 \]
\[ \text{HPR} = \frac{\$0.50}{\$2.00} = 25\% \]
\[ \text{APR} = 25\%/0.01923 = 1300\% \text{ or } 25\% \times 52 \text{ weeks} = 1300\% \]
\[ \text{EAR} = (1 + \text{HPR})^{52} - 1 = (1.25)^{52} - 1 = 109,526.27\% \]

Highly Improbable!
8.2 Risk (Certainty and Uncertainty)  
(Slide 8-14)

Most investments are such that one is not sure about their performance at a future date. If an outcome is known with certainty, such as the value of a treasury bill at maturity, it is considered riskless.

On the other hand, if an investment has a potential for loss, it would be considered risky. Hence, risk can be defined as a measure of the uncertainty in a set of potential outcomes for an event in which there is a chance of some loss.

It is important to measure and analyze the risk potential of an investment, so as to make an informed decision.

8.3 Historical Returns  
(Slides 8-15 to 8-16)

The data in Table 8.1, and the histograms graphed in Figure 8.1, provide a synopsis of the historical record of the returns and variability characterizing 3-month treasury bills, long-term government bonds, large-cap stocks (capitalization > $5 billion) and small-cap stocks (capitalization < $5 billion) in the USA.

Small company stocks earned the highest average return (17.10%) over the 5 decades. However, their annual returns had the greatest variability 29.04%, widest range (103.39% – (-40.54%)) = 143.93%, and were most spread out.

In contrast, 3-month treasury bills earned the lowest average return, 5.23%, but their returns had very low variability (2.98%), a very small range (14.95% – 0.86% = 15.91%) and were much closely clustered around the mean.

Thus, the historical evidence is clear. Returns and risk are positively related.

8.4 Variance and Standard Deviation as a Measure of Risk  
(Slides 8-17 to 8-24)

In statistics, measures of dispersion such as variance and standard deviation can help a researcher determine how spread out or clustered together a set of numbers or outcomes is around their mean or average value. The larger the variance, the greater is the variability and hence the riskiness of the set of values. Equations 8.6 and 8.7 (shown below) can be used to measure variance and standard deviation.

\[
\text{variance}(X) = \frac{\sum (X_i - \text{average})^2}{n-1} = \sigma^2 \\ 8.6
\]

Where \( X_i \) is the return of the asset in period \( i \) and \( n \) is the number of observations in the distribution.

\[
\text{standard deviation} = \sqrt{\text{variance}} = \sqrt{\sigma^2} = \sigma \\ 8.6
\]

Note: Students often mistakenly state the variance measure in percentage terms, while it is actually a squared value. The standard deviation, due to being the square root of the variance, is to be stated in percentage units.
Example 4: Calculating the variance of returns for large-company stocks

Listed below are the annual returns associated with the large-company stock portfolio from 1990 – 1999. Calculate the variance and standard deviation of the returns.

<table>
<thead>
<tr>
<th>Year</th>
<th>Return (R)</th>
<th>(R – Mean)</th>
<th>(R-Mean)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>-3.20%</td>
<td>-22.19%</td>
<td>0.0492396</td>
</tr>
<tr>
<td>1991</td>
<td>30.66%</td>
<td>11.67%</td>
<td>0.0136189</td>
</tr>
<tr>
<td>1992</td>
<td>7.71%</td>
<td>-11.28%</td>
<td>0.0127238</td>
</tr>
<tr>
<td>1993</td>
<td>9.87%</td>
<td>-9.12%</td>
<td>0.0083174</td>
</tr>
<tr>
<td>1994</td>
<td>1.29%</td>
<td>-17.70%</td>
<td>0.031329</td>
</tr>
<tr>
<td>1995</td>
<td>37.71%</td>
<td>18.72%</td>
<td>0.0350439</td>
</tr>
<tr>
<td>1996</td>
<td>23.07%</td>
<td>4.08%</td>
<td>0.0016646</td>
</tr>
<tr>
<td>1997</td>
<td>33.17%</td>
<td>14.18%</td>
<td>0.0201072</td>
</tr>
<tr>
<td>1998</td>
<td>28.58%</td>
<td>9.59%</td>
<td>0.0091968</td>
</tr>
<tr>
<td>1999</td>
<td>21.04%</td>
<td>2.05%</td>
<td>0.0004203</td>
</tr>
<tr>
<td>Total</td>
<td>189.90%</td>
<td>.18166156</td>
<td>0.020184618</td>
</tr>
</tbody>
</table>

Average 18.99%
Variance 0.020184618

Stand. Dev 14.207%

\[
\text{Variance} = \frac{\sum (R - \text{Mean})^2}{N - 1} = \frac{0.18166156}{10 - 1} = 0.020184618
\]

Standard Deviation = \sqrt{\text{Variance}} = \sqrt{0.020184618} = 14.207%

8.4 (A) Normal Distributions:

Figure 8.2 is a standard normal bell-shaped curve with a mean of zero and a standard deviation of one. If data are normally distributed, it lets the researcher or analyst make the following inferences regarding the expected values of the data:
1. About 68% of all observations of the data fall within one standard deviation of the average: the mean plus one or minus one standard deviation (add the 34% on the right of the mean with a corresponding 34% on the left of the mean).

2. About 95% of all observations of the data fall within two standard deviations of the mean: the mean plus two or minus two standard deviations.

3. About 99% of all observations of the data fall within three standard deviations of the mean: the mean plus three or minus three standard deviations.

Thus, if a sample of returns with a mean of 10% and a standard deviation of 12% is normally distributed we could make the following inferences:

- There is a 68% probability that the return in the forthcoming period will lie between 10% + 12% and 10% - 12% i.e. between -2% and 22%.
- There is a 95% probability that the return will lie between 10% + 24% and 10% - 24% i.e. between -14% and 34%
- There is 99% probability that the return will lie between 10% + 36% and 10% - 36% i.e. between -26% and 46%.

These properties imply that financial assets whose returns exhibit smaller variances would be less risky, since there would be less uncertainty about their future performance.

**TABLE 8.2 Returns, Variances, and Standard Deviations of Investment Choices, 1950–1999**

<table>
<thead>
<tr>
<th></th>
<th>Three-Month U.S. Treasury Bills</th>
<th>Long-Term Government Bonds</th>
<th>Large Company Stocks</th>
<th>Small Company Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return</td>
<td>5.23%</td>
<td>5.94%</td>
<td>14.89%</td>
<td>17.10%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0880%</td>
<td>0.9006%</td>
<td>2.7889%</td>
<td>8.4332%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.98%</td>
<td>9.49%</td>
<td>16.70%</td>
<td>29.04%</td>
</tr>
<tr>
<td>Number of negative returns</td>
<td>0</td>
<td>17</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>Number of positive returns</td>
<td>50</td>
<td>33</td>
<td>41</td>
<td>36</td>
</tr>
</tbody>
</table>
Table 8.2 and Figure 8.3 (shown above) show that over the past 5 decades (1950-1999), riskier investment groups have earned higher returns and vice-versa. Thus based on historical evidence, it can be concluded that the higher the return one expects the greater would be the risk (variability of return) that one would have to tolerate.

8.5 Returns in an Uncertain World
(Expectations and Probabilities) (Slides 8-25 to 8-28)

When we are contemplating making an investment, it is the expected or ex-ante returns and risk measures that are more relevant than the ex-post measures that we have covered so far.

To calculate ex-ante measures the various scenarios with their possible outcomes and probabilities are estimated, listed in a probability distribution, and then the expected return and risk measures are estimated using Equation 8.8 and 8.9 (as shown below):

\[
\text{expected payoff} = \sum \text{payoff}_i \times \text{probability}_i \quad 8.8
\]

\[
\sigma^2 = \sum (\text{payoff}_i - \text{expected payoff})^2 \times \text{probability}_i \quad 8.9
\]

8.5 (A) Determining the Probabilities of All Potential Outcomes. When setting up probability distributions the following 2 rules must be followed:

1. The sum of the probabilities must always add up to 1.0 or 100%.
2. Each individual probability estimate must be positive. We cannot have a negative probability value.

Example 5: Expected return and risk measurement

Using the probability distribution shown below, calculate Stock XYZs expected return, \( E(r) \), and standard deviation \( \sigma (r) \).

<table>
<thead>
<tr>
<th>State of the Economy</th>
<th>Probability of Economic State</th>
<th>Return in Economic State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Recession 45% -10%
Steady 35% 12%
Boom 20% 20%

\[ E(r) = \sum \text{Probability of Economic State} \times \text{Return in Economic State} \]
\[ = 45\% \times (-10\%) + 35\% \times (12\%) + 20\% \times (20\%) \]
\[ = -4.5\% + 4.2\% + 4\% = 3.7\% \]

\[ \sigma^2 (r) = \sum [\text{Return in State}_i - E(r)]^2 \times \text{Probability of State}_i \]
\[ = (-10\% - 3.7\%)^2 \times 45\% + (12\% - 3.7\%)^2 \times 35\% + (20\% - 3.7\%)^2 \times 20\% \]
\[ = 84.4605 + 24.1115 + 53.138 = 161.71 \]
\[ \sigma (r) = \sqrt{161.71} = 12.72\% \]

8.6 The Risk-and-Return Trade-off (Slides 8-29 to 8-31)

As described earlier, investments must be analyzed in terms of, both, their return potential as well as their riskiness or variability.

When faced with investment options having varying risk-return profiles, it is important to keep the following 2 investment rules in mind.

8.6 (A) Investment Rules

**Investment rule number 1:** If two investments have the same expected return and different levels of risk, the investment with the lower risk is preferred.

**Investment rule number 2:** If two investments have the same level of risk and different expected returns, the investment with the higher expected return is preferred.

Since investors aim to maximize return and minimize risk, it is obvious that an investment with both a higher expected return and lower level of risk is preferred over another asset.

Typically though, if one asset has a higher expected return than another asset, it also has a higher risk estimate. It is in such cases that the choice is not that clear cut and an investor’s tolerance for and attitude towards risk matters.

Following the principle of diversification or “spreading out” of risk by holding a portfolio of assets, rather than just one or two securities, would serve investors well in a world fraught with uncertainty and risk.

8.7 Diversification: Minimizing Risk or Uncertainty (Slides 8-32 to 8-43)

Diversification is the spreading of wealth over a variety of investment opportunities so as to eliminate some risk.

By dividing up one’s investments across many relatively low-correlated assets, companies, industries, and countries, it is possible to considerably reduce one’s exposure to risk.

Table 8.4 presents a probability distribution of the conditional returns of two firms, Zig and Zag, along with those of a 50-50 portfolio of the two companies.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Return of Zig Company</th>
<th>Return of Zag Company</th>
<th>Return of 50-50 Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.20</td>
<td>25%</td>
<td>5%</td>
<td>15%</td>
</tr>
<tr>
<td>Steady</td>
<td>0.50</td>
<td>17%</td>
<td>13%</td>
<td>15%</td>
</tr>
<tr>
<td>Recession</td>
<td>0.30</td>
<td>5%</td>
<td>25%</td>
<td>15%</td>
</tr>
<tr>
<td>E(r)</td>
<td></td>
<td></td>
<td></td>
<td>15%</td>
</tr>
</tbody>
</table>

The Portfolio’s expected return, E(rp), return can be measured in 2 ways.

1. \[ E(rp) = \text{Weight in Zig} \times E(r_{ZIG}) + \text{Weight in Zag} \times E(r_{ZAG}) = 0.50 \times 15\% + 0.50 \times 15\% = 15\% \]

OR

2. (a) First calculate the state-dependent returns for the portfolio (Rps) as follows:
   \[ R_{ps} = \text{Weight in Zig} \times R_{ZIG,s} + \text{Weight in Zag} \times R_{ZAG,s} \]
   Portfolio return in Boom economy = \(.5 \times 25\% + .5 \times 5\% = 15\%\)
   Portfolio return in Steady economy = \(.5 \times 17\% + .5 \times 13\% = 15\%\)
   Portfolio return in Recession economy = \(.5 \times 5\% + .5 \times 25\% = 15\%\)

(b) Then, calculate the Portfolio’s expected return using Equation 8.8 as follows:
   \[ E(rp) = \sum \text{Probability of Economic State} \times \text{Portfolio Return in Economic State} \]
   \[ = 0.2 \times (15\%) + 0.5 \times (15\%) + 0.3 \times (15\%) \]
   \[ = 3\% + 7.5\% + 4.5\% = 15\% \]

The portfolio’s expected variance and standard deviation can be measured by using the following equations:
\[ \sigma^2 (rp) = \sum [(\text{Return in State}_i - E(rp))^2 \times \text{Probability of State}_i] \]
\[ = [(15\% - 15\%)^2 \times 0.20 + (15\% - 15\%)^2 \times 0.50 + (15\% - 15\%)^2 \times 0.30] \]
\[ = 0 + 0 + 0 = 0 \]
\[ \sigma (rp) = \sqrt{0} = 0\% \]

Note: Remind students that the squared differences are multiplied by the probability of the economic state and then added across all economic states.

8.7 (A) When Diversification Works:

The benefits of diversification are best realized by combining stocks that are not perfectly positively correlated with each other.

The more negatively correlated a stock is with the other stocks in an investment portfolio, the greater will be the reduction in risk achieved by adding it to the portfolio.

Table 5 presents a probability distribution of returns for 2 stocks, i.e. The Zig Co., and The Peat Co., and for an equally-weighted portfolio of the two stocks as well.
Using the data from Table 8.5 we get the following risk-return measures:

<table>
<thead>
<tr>
<th></th>
<th>Zig</th>
<th>Peat</th>
<th>50-50 Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r)$</td>
<td>12.5%</td>
<td>10.70%</td>
<td>11.60%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>15.6%</td>
<td>10.00%</td>
<td>12.44%</td>
</tr>
</tbody>
</table>

Figure 8.10 plots the risk-return profiles of the 3 investments and illustrates the diversification benefit derived from combining these 2 stocks into a portfolio.

The portfolio has an expected return that is equivalent to the weighted average returns of the 2 stocks, but its standard deviation or risk (12.44%) is lower than the weighted average of the 2 standard deviations (12.8%).

The portfolio’s risk-return combination plots above the line joining the risk-return combinations of the two stocks, indicating that the portfolio’s expected return is higher than what would have been expected based on the weighted average risk of the two stocks.

8.7 (B) Adding More Stocks to the Portfolio: Systematic and Unsystematic Risk

The total risk of an investment can be broken down into two parts: unsystematic or diversifiable risk and systematic or non-diversifiable risk.

*Unsystematic risk* or company-specific risk can be diversified away by efficient portfolio formation and diversification into investments that have low correlation with each other. Examples include product problems, labor problems, etc. that plague individual companies or sectors.
Systematic risk or market risk is non-diversifiable risk in that it permeates throughout the system and affects everyone, albeit some more than others. For example, recession or inflation.

A Well-diversified portfolio is one whose unsystematic risk has been completely eliminated. For example, large mutual fund companies.

Figure 8.11 depicts the breakdown of total risk into its two components, i.e. systematic risk and unsystematic risk and shows how as the number of stocks in a portfolio approaches around 25, almost all of the unsystematic risk is eliminated, leaving behind only systematic risk.

8.8 Beta: The Measure of Risk in a Well-Diversified Portfolio (Slides 8-44 to 8-47)

Beta is a statistical measure of the volatility of an individual security compared with the market as a whole. It is the relative tendency of a security’s returns to respond to overall market fluctuations.

The average beta is 1.0, and a stock with a beta of 1.0 is said to have the same level of risk as that of the market in general.

Securities with a beta less than 1.0 are considered less risky than the average stock and the market in general, for example, utility stocks.

Securities with a beta greater than 1.0 are considered more risky than the average stock and the market in general, for example technology stocks.

A zero-beta, such as a Treasury bill, is uncorrelated or independent of the market in general.

Betas are estimated by running a regression of the returns (typically weekly returns) on a stock (dependent variable) with those on a market index (independent variable), such as the Standard and Poor’s 500. The slope of the regression line (coefficient of the independent variable) measures beta or the systematic risk estimate of the stock.

Once individual stock betas are determined, the portfolio beta is easily calculated as the weighted average by using equation 8.10.
Example 6: Calculating a portfolio beta

Jonathan has invested $25,000 in Stock X, $30,000 in stock Y, $45,000 in Stock Z, and $50,000 in stock K. Stock X’s beta is 1.5, Stock Y’s beta is 1.3, Stock Z’s beta is 0.8, and stock K’s beta is -0.6. Calculate Jonathan’s portfolio beta.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Investment</th>
<th>Weight</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>$25,000</td>
<td>0.1667</td>
<td>1.5</td>
</tr>
<tr>
<td>Y</td>
<td>$30,000</td>
<td>0.2000</td>
<td>1.3</td>
</tr>
<tr>
<td>Z</td>
<td>$45,000</td>
<td>0.3000</td>
<td>0.8</td>
</tr>
<tr>
<td>K</td>
<td>$50,000</td>
<td>0.3333</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>$150,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\beta_p = \sum_{i=1}^{n} w_i \times \beta_i
\]

Portofolio Beta = \(0.1667 \times 1.5 + 0.20 \times 1.3 + 0.30 \times 0.8 + 0.3333 \times -0.6\)
= \(0.25005 + 0.26 + 0.24 + -0.19998 = 0.55007\)

We have introduced two different measures of risk related to financial assets; standard deviation (or variance) and beta.

The standard deviation is a measure of the total risk of an asset, both its systematic and unsystematic risk.

Beta is a measure of an asset’s systematic risk. When we view any one of our assets as part of a well-diversified portfolio, it is proper to use beta as the measure of risk for the asset.

If we do not have a well-diversified portfolio, it is more prudent to use standard deviation as the measure of risk for our asset.

8.9 The Capital Asset Pricing Model and the Security Market Line (Slides 8-48 to 8-60)

The Security Market Line, shown below in Figure 8.12, is a graphical depiction of the relationship between an asset’s required rate of return and its systematic risk measure, i.e. beta. It is based on 3 assumptions.

**Assumption 1**: There is a basic reward for waiting: the risk-free rate. This means that an investor could earn the risk-free rate by delaying consumption.

**Assumption 2**: The greater the risk, the greater the expected reward. Investors expect to be proportionately compensated for bearing risk.

**Assumption 3**: There is a consistent trade-off between risk and reward at all levels of risk. As risk doubles, so does the required rate of return, and vice-versa.

These three assumptions imply that the SML is upward sloping, has a constant slope (linear), and has the risk-free rate as its Y-intercept.
8.9 (A) The Capital Asset Pricing Model (CAPM) is the equation form of the SML and is used to quantify the relationship between the expected rate of return and the systematic risk of individual securities as well as portfolios.

It states that the expected return of an investment is a function of:

1. The time value of money (the reward for waiting)
2. A reward for taking on risk
3. The amount of risk

The equation representing the CAPM (Equation 8.11 as shown below) is in effect a straight line equation of the form:

\[ y = a + b \times x \]

Where, \( y \) is the value of the function, \( a \) is the intercept of the function, \( b \) is the slope of the line, and \( x \) is the value of the random variable on the x-axis.

By substituting expected return \( E(r_i) \) for the \( y \) variable, the risk-free rate \( r_f \) for the intercept \( a \), the market risk premium, \( (E(r_m) - r_f) \) for the slope \( b \), and the systematic risk measure, \( \beta \), for the random variable on the x-axis, we have the formal equation for the SML.

\[ E(r_i) = r_f + [E(r_m) - r_f] \times \beta_i \]  \hspace{1cm} 8.11

Note: Students often assume that beta is the slope of the SML. Emphasize the point that the slope of the SML is the market risk premium i.e. \( (E(r_m) - r_f) \) and not beta, which is on the X-axis. If investors demand a higher risk premium to bear average risk, the slope of the SML will increase and vice-versa.

Example 7: Finding expected returns for a company with known beta

The New Ideas Corporation’s recent strategic moves have resulted in its beta going from 0.8 to 1.2. If the risk-free rate is currently at 4% and the market risk premium is being estimated at 7%, calculate its expected rate of return.

Using the CAPM equation we have:
Where;
Rf = 4%; E(r_m) – r_f = 7%; and β = 1.2
Expected rate of return = 4% + 7% × 1.2 = 4% + 8.4 = 12.4%
Note: Students often forget their rules of operation. Remind them to first multiply 7% × 1.2 = 8.4; and then add 4% = 12.4%.

8.9 (B) Application of the SML

Although the SML was primarily developed as a way of explaining the relationship between an asset’s expected return and risk, it has many practical applications. Some such applications include:

1) To determine the prevailing market or average risk premium, given the expected returns of a couple of stocks and their betas

Example 8: Determining the market risk premium

Stocks × and Y seem to be selling at their equilibrium values as per the opinions of the majority of analysts. If Stock × has a beta of 1.5 and an expected return of 14.5%, and Stock Y has a beta of 0.8 and an expected return of 9.6% calculate the prevailing market risk premium and the risk-free rate.

Since the market risk premium is the slope of the SML i.e. \([E(r_m) – r_f]\) we can solve for it as follows:

\[
\text{slope of line} = \frac{\Delta Y}{\Delta X}
\]

Where \(\Delta Y\) is the change in expected return = 14.5% – 9.6% = 4.9%, and
\(\Delta X\) is the change in beta = 1.5-0.8 = 0.7

So, slope of the SML = 4.9%/0.7 = 7% = \([E(r_m) – r_f]\)

To calculate the risk-free rate we use the SML equation by plugging in the expected rate for any of the stocks along with its beta and the market risk premium of 7% and solve.

Using Stock X’s information we have:

14.5% = r_f + 7% × 1.5 \(\Rightarrow\) r_f = 14.5 - 10.5 = 4%

2) To determine the investment attractiveness of stocks given their betas and expected return (based on analysts’ forecasts)

Example 9: Assessing market attractiveness

Let’s say that you are looking at investing in 2 stocks A and B. A has a beta of 1.3 and based on your best estimates is expected to have a return of 15%, B has a beta of 0.9 and
is expected to earn 9%. If the risk-free rate is currently 4% and the expected return on the market is 11%, determine whether these stocks are worth investing in.

Using the SML we have

\[ E(r_i) = r_f + [E(r_m) - r_f] \times \beta_i \]  \hspace{1cm} 8.11

Stock A’s expected return = 4% + (11% – 4%) × 1.3 = 13.1%

Stock B’s expected return = 4% + (11% – 4%) × 0.9 = 10.3%

So, Stock A would plot above the SML, since 15%>13.1% and would be considered undervalued, while stock B would plot below the SML (9%<10.3%) and would be considered overvalued.

3) To determine portfolio allocation weights and expected return given a desired portfolio beta and individual stock betas.

**Example 10: Calculating portfolio expected return and allocation using 2 stocks**

Andrew has decided that given the current economic conditions he wants to have a portfolio with a beta of 0.9, and is considering Stock R with a beta of 1.3 and Stock S with a beta of 0.7 as the only 2 candidates for inclusion. If the risk-free rate is 4% and the market risk premium is 7%, what will his portfolio’s expected return be and how should he allocate his money among the two stocks?

Determine portfolio expected return using the SML

\[ E(r_p) = r_f + [E(r_m) - r_f] \times \beta_p \]  \hspace{1cm} 8.11

= 4% + 7% × 0.9 = 4%+6.3%=10.3%

Next, using the two stock betas and the desired portfolio beta, infer the allocation weights as follows: Let Stock R’s weight = X%; Stock S’s weight = (1-X)%.

Portfolio Beta = 0.9 = X% × 1.3 + (1 – X)% × 0.7=1.3X+0.7 – 0.7X \rightarrow 0.6X+0.7 \rightarrow 0.9

=0.6X+0.7 \rightarrow 0.2=0.6X \rightarrow x = 0.2/0.6 = 1/3 \rightarrow 1 – X = 2/3

To check: 1/3 × 1.3 + 2/3 × 0.7 = 0.4333+0.4667 = 0.9 = Portfolio Beta

**Questions**

1. **What are the two parameters for selecting investments in the finance world?**
   How do investors try to get the most out of their investment with regard to these two parameters?

   The two parameters are risk and return. Investors try to maximize return and minimize risk.

2. **What are the two ways to measure performance in the finance world?**

   Two ways to measure performance are dollar profits and percentage return.
3. Why is it not practical to convert holding period returns from very short periods to annual returns?

Extrapolating small period returns to a much longer period requires that the investment is reinvested in all subsequent periods for the same return. This is often not attainable.

4. How do we define risk?

Risk can be defined in terms of uncertainty. Uncertainty is the absence of exact knowledge about an outcome prior to the event.

5. What type of investment has had the highest return on average and the largest variance from 1950–1999? How much has this investment varied over that fifty-year period?

From the table in the text Small Company Stocks had the highest average annual return and the highest standard deviation. This investment has varied, on average, by about 29.04% per year from its mean over the 50-year period from 1950-1999.

6. What is one of the problems in dealing with an event that has a large number of potential outcomes?

An event with a large number of potential outcomes requires that each outcome be assigned a probability. The greater the number of potential outcomes the more probabilities that must be assigned which usually will be more difficult as the outcomes increase.

7. What are the two investment rules and how do they influence choices when considering a pair of potential investments?

Investment rule #1 says that given two assets with identical returns, you select the one with the least amount of risk. Investment rule #2 says that given two investments with the same amount of risk, you select the one with the higher return.

8. Why might two different investors select two different potential investments if one investment had the highest return and the highest risk over the other investment?

Each investor has his or her own tolerance for risk. So one investor may be more risk averse than another investor and select a lower risk investment versus someone with a higher tolerance for risk.

9. What does it mean to diversify your portfolio, and what are you trying to gain by so doing?

Diversification is a means to lower risk without giving up substantial return for that level of risk reduction. By selecting a series of investments over a small number of investments the portfolio provides a better return for that level of risk than individual assets.

10. What is a positive correlation between two assets’ returns? What is a negative correlation between two assets’ returns? Which correlation is better for reducing the variance of a portfolio made up of two assets?

A positive correlation is when two random variables (investments) move in the same direction when an underlying feature (the economy) changes. A negative correlation
is when two random variables move in opposite directions when an underlying factor changes. For portfolio diversification, a negative correlation offers more risk reduction.

11. What is the difference between unsystematic and systematic risk? Which risk can you avoid? Which risk can you not avoid?

Unsystematic risk is firm specific risk while systematic risk is risk that varies with changes in the economy. You can avoid unsystematic risk by diversification. You cannot avoid systematic risk.

12. What is beta in the financial world? What is standard deviation in the financial world? What type of risk does each measure? What assumption do you make about the stock when you use beta as a measure of its risk?

Beta is the systematic risk of an asset in a well-diversified portfolio or a well-diversified portfolio’s total risk measure. Standard deviation is the total risk of an asset. Beta measures systematic risk, standard deviation measures both systematic risk and unsystematic risk. When using beta for an individual stock you assume the stock is part of a well-diversified portfolio.

Prepping for Exams

1. a.
2. b.
3. d.
4. a.
5. b.
6. d.
7. d.
8. a.
9. c.
10. b.

Problems

1. **Profits.** What are the profits on the following investments?

<table>
<thead>
<tr>
<th>Investment</th>
<th>Original Cost or Invested $</th>
<th>Selling Price of Investment</th>
<th>Distributions Received $</th>
<th>Dollar Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>$500.00</td>
<td>$540.00</td>
<td>$0.00</td>
<td></td>
</tr>
<tr>
<td>Stock</td>
<td>$23.00</td>
<td>$34.00</td>
<td>$2.00</td>
<td></td>
</tr>
<tr>
<td>Bond</td>
<td>$1,040.00</td>
<td>$980.00</td>
<td>$80.00</td>
<td></td>
</tr>
<tr>
<td>Bike</td>
<td>$400.00</td>
<td>$220.00</td>
<td>$0.00</td>
<td></td>
</tr>
</tbody>
</table>
### ANSWER

<table>
<thead>
<tr>
<th>Investment</th>
<th>Original Cost or Invested $</th>
<th>Selling Price of Investment</th>
<th>Distributions Received $</th>
<th>Dollar Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>$500.00</td>
<td>$540.00</td>
<td>$0.00</td>
<td>$40</td>
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<tr>
<td>Stock</td>
<td>$23.00</td>
<td>$34.00</td>
<td>$2.00</td>
<td>$13</td>
</tr>
<tr>
<td>Bond</td>
<td>$1,040.00</td>
<td>$980.00</td>
<td>$80.00</td>
<td>$20</td>
</tr>
<tr>
<td>Bike</td>
<td>$400.00</td>
<td>$220.00</td>
<td>$0.00</td>
<td>$-180</td>
</tr>
</tbody>
</table>

CD Dollar Return = $540 + $0 – $500 = $40
Stock Dollar Return = $34 + $2 – $23 = $13
Bond Dollar Return = $980 + $80 – $1,040 = $20
Bike Dollar Return = $220 + $0 – $400 = $-180

2. **Profits.** What are the profits on the following investments?

<table>
<thead>
<tr>
<th>Investment</th>
<th>Original Cost or Invested $</th>
<th>Selling Price of Investment</th>
<th>Distributions Received $</th>
<th>Dollar Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>$500.00</td>
<td>$525.00</td>
<td>$0.00</td>
<td>$25</td>
</tr>
<tr>
<td>Stock</td>
<td>$34.00</td>
<td>$26.00</td>
<td>$2.00</td>
<td>-$6</td>
</tr>
<tr>
<td>Bond</td>
<td>$955.00</td>
<td>$1000.00</td>
<td>$240.00</td>
<td>$285</td>
</tr>
<tr>
<td>Car</td>
<td>$42,000.00</td>
<td>$3,220.00</td>
<td>$0.00</td>
<td>-$38,780</td>
</tr>
</tbody>
</table>

CD Dollar Return = $525 + $0 – $500 = $25
Stock Dollar Return = $26 + $2 – $34 = -$6
Bond Dollar Return = $1,000 + $240 – $955 = $285
Car Dollar Return = $3,220 + $0 – $42,000 = -$38,780

3. **Returns.** What are the returns on the following investments?
### Investment Summary

<table>
<thead>
<tr>
<th>Investment</th>
<th>Original Cost or Invested $</th>
<th>Selling Price of Investment</th>
<th>Distributions Received $</th>
<th>Percent Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>$500.00</td>
<td>$540.00</td>
<td>$0.00</td>
<td>8.00%</td>
</tr>
<tr>
<td>Stock</td>
<td>$23.00</td>
<td>$34.00</td>
<td>$2.00</td>
<td>56.52%</td>
</tr>
<tr>
<td>Bond</td>
<td>$1,040.00</td>
<td>$980.00</td>
<td>$80.00</td>
<td>1.92%</td>
</tr>
<tr>
<td>Bike</td>
<td>$400.00</td>
<td>$220.00</td>
<td>$0.00</td>
<td>-45.00%</td>
</tr>
</tbody>
</table>

**CD Percent Return** = \( \frac{($540 + $0 - $500)}{$500} = 0.0500 \text{ or } 5.00\% \)

**Stock Percent Return** = \( \frac{($34 + $2 - $23)}{$23} = 0.565217 \text{ or } 56.52\% \)

**Bond Percent Return** = \( \frac{($980 + $80 - $1040)}{$1040} = 0.01923 \text{ or } 1.92\% \)

**Bike Percent Return** = \( \frac{($220 + $0 - $400)}{$400} = -0.45 \text{ or } -45\% \)

### Returns

4. **Returns.** What are the returns on the following investments?

<table>
<thead>
<tr>
<th>Investment</th>
<th>Original Cost or Invested $</th>
<th>Selling Price of Investment</th>
<th>Distributions Received $</th>
<th>Percent Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>$500.00</td>
<td>$525.00</td>
<td>$0.00</td>
<td>5.00%</td>
</tr>
<tr>
<td>Stock</td>
<td>$34.00</td>
<td>$2600</td>
<td>$2.00</td>
<td>-17.65%</td>
</tr>
<tr>
<td>Bond</td>
<td>$955.00</td>
<td>$1000.00</td>
<td>$240.00</td>
<td>29.84%</td>
</tr>
<tr>
<td>Car</td>
<td>$42,000.00</td>
<td>$3,220.00</td>
<td>$0.00</td>
<td>-92.33%</td>
</tr>
</tbody>
</table>

**CD Percent Return** = \( \frac{($525 + $0 - $500)}{$500} = 0.0500 \text{ or } 5.00\% \)

**Stock Percent Return** = \( \frac{($2600 + $2 - $34)}{$34} = 77.65 \text{ or } 77.65\% \)

**Bond Percent Return** = \( \frac{($1000 + $240 - $955)}{$955} = 0.298397 \text{ or } 29.84\% \)

**Car Percent Return** = \( \frac{($3,220 + $0 - $42,000)}{$42,000} = -0.923305 \text{ or } -92.33\% \)
CD Percent Return = ($525 + $0 − $500) / $500 = 0.0500 or 5.00%
Stock Percent Return = ($26 + $2 − $34) / $34 = -0.1765 or -17.65%
Bond Percent Return = ($1,000 + $240 − $955) / $955 = 0.2984 or 29.84%
Car Percent Return = ($3,220 + $0 − $42,000) / $42,000 = -0.9233 or -92.33%

5. **Holding period and annual (investment) returns.** Baker Baseball Cards Inc. originally purchased the rookie card of Hammerin’ Hank Aaron for $35.00. After holding the card for five years, Baker auctioned off the card for $180.00. What are the holding period return and the annual return on this investment?

**ANSWER**

Holding Period Return = ($180 − $35) / $35 = 4.1429 or 414.29%
Annual Percentage Return = HPR/n = 414.29%/5 = 82.86%
EAR = (1 + 4.1429)^1/5 − 1 = 1.3875 − 1 = 0.3875 or 38.75%

OR

Using a financial calculator:
PV = -35; FV = 180; N = 5; PMT = 0; I = 38.75% =>

6. **Holding Period and Annual (Investment) Returns.** Bohenick Classic Automobiles restores and rebuilds old classic cars. The company purchased and restored a classic 1957 Thunderbird convertible six years ago for $8,500. Today at auction, the car sold for $50,000. What are the holding period return and the annual return on this investment?

**ANSWER**

Holding Period Return = ($50,000 − $8,500) / $8,500 = 4.8824 or 488.24%
APR = HPR/n = 488.24%/6 = 81.37%
EAR = (1 + 4.8824)^1/6 − 1 = 1.3436 − 1 = 0.3436 or 34.36%

7. **Comparison of returns.** Looking back at Problems 5 and 6, which investment had the higher holding period return? Which had the higher annual return?

**ANSWER**

Holding Period Return for Trading Card = ($180 − $35) / $35 = 4.1429 or 414.29%
Holding Period Return for Classic Car = ($50,000 − $8,500) / $8,500 = 4.8824 or 488.24%

Trading Card HPR < Classic Car HPR
Trading Card APR = HPR/n = 414.29%/5 = 82.86%
Classic Car APR = HPR/n = 488.24%/6 = 81.37%
Trading Card APR > Classic Car APR ➔ 82.86% > 81.37%
Trading Card EAR = \( (1 + 4.1429)^{1/5} - 1 = 1.3875 - 1 = 0.3875 \) or 38.75%
Classic Car Annual Return = \( (1 + 4.8824)^{1/6} - 1 = 1.3436 - 1 = 0.3436 \) or 34.36%
Trading Card EAR > Classic Car EAR.

8. **Comparison of returns.** WG Investors are looking at three different investment opportunities. Investment One is a five-year investment with a cost of $125 and a promised payout of $250 at maturity. Investment Two is a seven-year investment with a cost of $125 and a promised payout of $350. Investment Three is a ten-year investment with a cost of $125 and a promised payout of $550. WG Investors can only take on one of the three investments. Assuming all three investment opportunities have the same level of risk, calculate the annual return for each investment and select the best investment choice.

**ANSWER**

Holding Period Return for Investment One = \( \frac{($250 - $125)}{125} = 1.00 \) or 100.00%
EAR-- Investment One = \( (1 + 1.00)^{1/5} - 1 = 1.1487 - 1 = 0.1487 \) or 14.87%
Holding Period Return for Investment Two = \( \frac{($350 - $125)}{125} = 1.80 \) or 180.00%
EAR-- Investment Two = \( (1 + 1.80)^{1/7} - 1 = 1.1585 - 1 = 0.1585 \) or 15.85%
Holding Period Return for Investment Three = \( \frac{($550 - $125)}{125} = 3.40 \) or 340.00%
EAR-- Investment Three = \( (1 + 3.40)^{1/10} - 1 = 1.15969 - 1 = 0.15967 \) or 15.97%

Investment Three has the highest annual return rate of the three choices. If all choices have the same level of risk, choose Investment Three.

9. **Historical returns.** Calculate the average return of the U.S. Treasury bills, long-term government bonds, and large company stocks for 1990–1998 from Table 8.1. Which had the highest and which had the lowest return?

**ANSWER**

Average Return U.S. Treasury Bill for 90s: 5.02%
Average Return U.S. Long-Term Government Bonds for 90s: 9.23%
Average Return U.S. Large Company Stocks for 90s: 18.99%
Highest was Large Company Stocks, Lowest was 3 Month T-Bills

10. **Historical returns.** Calculate the average return of the U.S. Treasury bills, long-term government bonds, and large company stocks for the 1950 to 1959, 1960 to 1969, 1970 to 1979, and 1980 to 1989 from Table 8.1. Which had the highest return? Which had the lowest return?
ANSWER

Answer from data is:
Average Return U.S. Treasury Bill for 50s: 1.87%
Average Return U.S. Long-Term Government Bonds for 50s: 0.35%
Average Return U.S. Large Company Stocks for 50s: 20.94%

Answer from data is:
Average Return U.S. Treasury Bill for 60s: 3.90%
Average Return U.S. Long-Term Government Bonds for 60s: 1.31%
Average Return U.S. Large Company Stocks for 60s: 8.74%

Answer from data is:
Average Return U.S. Treasury Bill for 70s: 6.31%
Average Return U.S. Long-Term Government Bonds for 70s: 6.80%
Average Return U.S. Large Company Stocks for 70s: 7.55%

Answer from data is:
Average Return U.S. Treasury Bill for 80s: 9.04%
Average Return U.S. Long-Term Government Bonds for 80s: 11.99%
Average Return U.S. Large Company Stocks for 80s: 18.24%

Highest Return was 20.94% in the 50s for Large Company Stocks and the lowest return was 0.35% for Long-Term Government Bonds in the 50s.

11. **Standard deviation.** Calculate the standard deviation of the U.S. Treasury bills, long-term government bonds, and large company stocks for 1990 to 1999 from Table 8.1. Which had the highest variance? Which had the lowest variance?

ANSWER


\[ \text{variance } (X) = \frac{\sum (X_i - \text{average})^2}{n - 1} = \sigma^2 \]

\[ \text{standard deviation} = \sqrt{\text{variance}} = \sqrt{\sigma^2} = \sigma \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Return</th>
<th>((R - \text{Average}))</th>
<th>((R - \text{Average})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>7.86%</td>
<td>2.84%</td>
<td>0.00080429</td>
</tr>
<tr>
<td>1991</td>
<td>5.65%</td>
<td>0.63%</td>
<td>3.9188E–05</td>
</tr>
<tr>
<td>1992</td>
<td>3.54%</td>
<td>–1.48%</td>
<td>0.00022023</td>
</tr>
<tr>
<td>1993</td>
<td>2.97%</td>
<td>–2.05%</td>
<td>0.00042189</td>
</tr>
<tr>
<td>1994</td>
<td>3.91%</td>
<td>–1.11%</td>
<td>0.0001241</td>
</tr>
<tr>
<td>1995</td>
<td>5.58%</td>
<td>0.56%</td>
<td>3.0914E–05</td>
</tr>
<tr>
<td>1996</td>
<td>5.50%</td>
<td>0.48%</td>
<td>2.2658E–05</td>
</tr>
</tbody>
</table>

### Long-Term Government Bonds (1990–1999)

<table>
<thead>
<tr>
<th>Year</th>
<th>Return</th>
<th>(R – Mean)</th>
<th>(R – Mean)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>7.13%</td>
<td>−2.10%</td>
<td>0.000442</td>
</tr>
<tr>
<td>1991</td>
<td>18.39%</td>
<td>9.16%</td>
<td>0.008385</td>
</tr>
<tr>
<td>1992</td>
<td>7.79%</td>
<td>−1.44%</td>
<td>0.000208</td>
</tr>
<tr>
<td>1993</td>
<td>15.48%</td>
<td>6.25%</td>
<td>0.003903</td>
</tr>
<tr>
<td>1994</td>
<td>−7.18%</td>
<td>−16.41%</td>
<td>0.026939</td>
</tr>
<tr>
<td>1995</td>
<td>31.67%</td>
<td>22.44%</td>
<td>0.050342</td>
</tr>
<tr>
<td>1996</td>
<td>−0.81%</td>
<td>−10.04%</td>
<td>0.010086</td>
</tr>
<tr>
<td>1997</td>
<td>15.08%</td>
<td>5.85%</td>
<td>0.003419</td>
</tr>
<tr>
<td>1998</td>
<td>13.52%</td>
<td>4.29%</td>
<td>0.001838</td>
</tr>
<tr>
<td>1999</td>
<td>−8.74%</td>
<td>−17.97%</td>
<td>0.032303</td>
</tr>
<tr>
<td>Total</td>
<td>92.33%</td>
<td></td>
<td>0.137864</td>
</tr>
<tr>
<td>Mean</td>
<td>9.23%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>0.015318</td>
<td>=.137864/(10 − 1)</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>12.38%</td>
<td>=(.015318)^{1/2}</td>
<td></td>
</tr>
</tbody>
</table>
### U.S. Large Company Stocks

<table>
<thead>
<tr>
<th>Year</th>
<th>Return</th>
<th>(R – Mean)</th>
<th>(R – Mean)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>-3.20%</td>
<td>-22.19%</td>
<td>0.04924</td>
</tr>
<tr>
<td>1991</td>
<td>30.66%</td>
<td>11.67%</td>
<td>0.013619</td>
</tr>
<tr>
<td>1992</td>
<td>7.71%</td>
<td>-11.28%</td>
<td>0.012724</td>
</tr>
<tr>
<td>1993</td>
<td>9.87%</td>
<td>-9.12%</td>
<td>0.008317</td>
</tr>
<tr>
<td>1994</td>
<td>1.29%</td>
<td>-17.70%</td>
<td>0.031329</td>
</tr>
<tr>
<td>1995</td>
<td>37.71%</td>
<td>18.72%</td>
<td>0.035044</td>
</tr>
<tr>
<td>1996</td>
<td>23.07%</td>
<td>4.08%</td>
<td>0.001665</td>
</tr>
<tr>
<td>1997</td>
<td>33.17%</td>
<td>14.18%</td>
<td>0.020107</td>
</tr>
<tr>
<td>1998</td>
<td>28.58%</td>
<td>9.59%</td>
<td>0.009197</td>
</tr>
<tr>
<td>1999</td>
<td>21.04%</td>
<td>2.05%</td>
<td>0.00042</td>
</tr>
<tr>
<td>Total</td>
<td>189.90%</td>
<td></td>
<td>0.181662</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>18.99%</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>0.020185</td>
<td>=0.181662/(10-1)</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>14.21%</td>
<td>=(.020185)^{1/2}</td>
<td></td>
</tr>
</tbody>
</table>

Variance of U.S. Treasury Bills Returns for 90s \( \rightarrow \) .000186 (Lowest)  
Variance of U.S. Long-Term Government Bond Returns for 90s \( \rightarrow \) 0.015318  
Variance of U.S. Large Company Stocks Returns for 90s \( \rightarrow \) .020185 (Highest)

12. **Variance and standard deviation.** Calculate the variance and standard deviation of the U.S. Treasury bills, long-term government bonds, and small-company stocks for the 1950 to 1959, 1960 to 1969, 1970 to 1979, and 1980 to 1989 from Table 8.1. Which had the highest variance? Which had the lowest variance?
### 1950s

<table>
<thead>
<tr>
<th>Year</th>
<th>T-Bills</th>
<th>L-T Gov. Bonds</th>
<th>Large Stocks</th>
<th>Small Co. Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>1.20%</td>
<td>-0.96%</td>
<td>32.68%</td>
<td>48.45%</td>
</tr>
<tr>
<td>1951</td>
<td>1.49%</td>
<td>-1.95%</td>
<td>23.47%</td>
<td>9.41%</td>
</tr>
<tr>
<td>1952</td>
<td>1.66%</td>
<td>1.93%</td>
<td>18.91%</td>
<td>6.36%</td>
</tr>
<tr>
<td>1953</td>
<td>1.82%</td>
<td>3.83%</td>
<td>-1.74%</td>
<td>-5.66%</td>
</tr>
<tr>
<td>1954</td>
<td>0.86%</td>
<td>4.88%</td>
<td>52.55%</td>
<td>65.13%</td>
</tr>
<tr>
<td>1955</td>
<td>1.57%</td>
<td>-1.34%</td>
<td>31.44%</td>
<td>21.84%</td>
</tr>
<tr>
<td>1956</td>
<td>2.46%</td>
<td>-5.12%</td>
<td>6.45%</td>
<td>3.82%</td>
</tr>
<tr>
<td>1957</td>
<td>3.14%</td>
<td>9.46%</td>
<td>-11.14%</td>
<td>-15.03%</td>
</tr>
<tr>
<td>1958</td>
<td>1.54%</td>
<td>-3.71%</td>
<td>43.78%</td>
<td>70.63%</td>
</tr>
<tr>
<td>1959</td>
<td>2.95%</td>
<td>-3.55%</td>
<td>12.95%</td>
<td>17.82%</td>
</tr>
<tr>
<td>Average</td>
<td>1.87%</td>
<td>0.35%</td>
<td>20.94%</td>
<td>22.28%</td>
</tr>
<tr>
<td>Var</td>
<td>0.000055</td>
<td>0.0021066</td>
<td>0.03996</td>
<td>0.086793</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.74%</td>
<td>4.59%</td>
<td>19.99%</td>
<td>29.46%</td>
</tr>
</tbody>
</table>

Standard Deviation for U.S. Treasury Bill for 50s: 0.74%; Variance=.000055  
Standard Deviation for U.S. Long-Term Government Bonds for 50s: 4.59%  
Standard Deviation for U.S. Small Company Stocks for 50s: 29.46%

### 1960s

<table>
<thead>
<tr>
<th>Year</th>
<th>T-Bills</th>
<th>L-T Gov. Bonds</th>
<th>Large Stocks</th>
<th>Small Co. Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>2.66%</td>
<td>13.78%</td>
<td>0.19%</td>
<td>-5.16%</td>
</tr>
<tr>
<td>1961</td>
<td>2.13%</td>
<td>0.19%</td>
<td>27.63%</td>
<td>30.48%</td>
</tr>
<tr>
<td>1962</td>
<td>2.72%</td>
<td>6.81%</td>
<td>-8.79%</td>
<td>-16.41%</td>
</tr>
<tr>
<td>1963</td>
<td>3.12%</td>
<td>-0.49%</td>
<td>22.63%</td>
<td>12.20%</td>
</tr>
<tr>
<td>1964</td>
<td>3.54%</td>
<td>4.51%</td>
<td>16.67%</td>
<td>18.75%</td>
</tr>
<tr>
<td>1965</td>
<td>3.94%</td>
<td>-0.27%</td>
<td>12.50%</td>
<td>37.67%</td>
</tr>
<tr>
<td>1966</td>
<td>4.77%</td>
<td>3.70%</td>
<td>-10.25%</td>
<td>-8.08%</td>
</tr>
<tr>
<td>1967</td>
<td>4.24%</td>
<td>-7.41%</td>
<td>24.11%</td>
<td>103.39%</td>
</tr>
<tr>
<td>1968</td>
<td>5.24%</td>
<td>-1.20%</td>
<td>11.00%</td>
<td>50.61%</td>
</tr>
<tr>
<td>1969</td>
<td>6.59%</td>
<td>-6.52%</td>
<td>-8.33%</td>
<td>-32.27%</td>
</tr>
<tr>
<td>Average</td>
<td>3.90%</td>
<td>1.31%</td>
<td>8.74%</td>
<td>19.12%</td>
</tr>
<tr>
<td>Var</td>
<td>0.000186</td>
<td>0.003915</td>
<td>0.021117</td>
<td>0.153854</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>1.36%</td>
<td>6.26%</td>
<td>14.53%</td>
<td>39.22%</td>
</tr>
</tbody>
</table>

Standard Deviation for U.S. Treasury Bill for 60s: 1.36%
Standard Deviation for U.S. Long-Term Government Bonds for 60s: 6.26%
Standard Deviation for U.S. Small Company Stocks for 60s: 39.22%

### 1970s

<table>
<thead>
<tr>
<th>Year</th>
<th>T-Bills</th>
<th>L-T Gov. Bonds</th>
<th>Large Stocks</th>
<th>Small Co. Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>6.50%</td>
<td>12.69%</td>
<td>4.10%</td>
<td>–16.54%</td>
</tr>
<tr>
<td>1971</td>
<td>4.34%</td>
<td>17.47%</td>
<td>14.17%</td>
<td>18.44%</td>
</tr>
<tr>
<td>1972</td>
<td>3.81%</td>
<td>5.55%</td>
<td>19.14%</td>
<td>–0.62%</td>
</tr>
<tr>
<td>1973</td>
<td>6.91%</td>
<td>1.40%</td>
<td>–14.75%</td>
<td>–40.54%</td>
</tr>
<tr>
<td>1974</td>
<td>7.93%</td>
<td>5.53%</td>
<td>–26.40%</td>
<td>–29.74%</td>
</tr>
<tr>
<td>1975</td>
<td>5.80%</td>
<td>8.50%</td>
<td>37.26%</td>
<td>69.54%</td>
</tr>
<tr>
<td>1976</td>
<td>5.06%</td>
<td>11.07%</td>
<td>23.98%</td>
<td>54.81%</td>
</tr>
<tr>
<td>1977</td>
<td>5.10%</td>
<td>0.90%</td>
<td>–7.26%</td>
<td>22.02%</td>
</tr>
<tr>
<td>1978</td>
<td>7.15%</td>
<td>–4.16%</td>
<td>6.50%</td>
<td>22.29%</td>
</tr>
<tr>
<td>1979</td>
<td>10.45%</td>
<td>9.02%</td>
<td>18.77%</td>
<td>43.99%</td>
</tr>
<tr>
<td>Average</td>
<td>6.31%</td>
<td>6.80%</td>
<td>7.55%</td>
<td>14.37%</td>
</tr>
<tr>
<td>Var</td>
<td>0.000381</td>
<td>0.0040207</td>
<td>0.037099</td>
<td>0.131502</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>1.95%</td>
<td>6.34%</td>
<td>19.26%</td>
<td>36.26%</td>
</tr>
</tbody>
</table>

Standard Deviation for U.S. Treasury Bill for 70s: 1.95%
Standard Deviation for U.S. Long-Term Government Bonds for 70s: 6.34%
Standard Deviation for U.S. Small Company Stocks for 70s: 36.26%
1980s

<table>
<thead>
<tr>
<th>Year</th>
<th>T-Bills</th>
<th>L-T Gov. Bonds</th>
<th>Large Stocks</th>
<th>Small Co. Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>11.57%</td>
<td>13.17%</td>
<td>32.48%</td>
<td>35.34%</td>
</tr>
<tr>
<td>1981</td>
<td>14.95%</td>
<td>3.61%</td>
<td>–4.98%</td>
<td>7.79%</td>
</tr>
<tr>
<td>1982</td>
<td>10.71%</td>
<td>6.52%</td>
<td>22.09%</td>
<td>27.44%</td>
</tr>
<tr>
<td>1983</td>
<td>8.85%</td>
<td>–0.53%</td>
<td>22.37%</td>
<td>34.49%</td>
</tr>
<tr>
<td>1984</td>
<td>10.02%</td>
<td>15.29%</td>
<td>6.46%</td>
<td>–14.02%</td>
</tr>
<tr>
<td>1985</td>
<td>7.83%</td>
<td>32.68%</td>
<td>32.00%</td>
<td>28.21%</td>
</tr>
<tr>
<td>1986</td>
<td>6.18%</td>
<td>23.96%</td>
<td>18.40%</td>
<td>3.40%</td>
</tr>
<tr>
<td>1987</td>
<td>5.50%</td>
<td>–2.65%</td>
<td>5.34%</td>
<td>–13.95%</td>
</tr>
<tr>
<td>1988</td>
<td>6.44%</td>
<td>8.40%</td>
<td>16.86%</td>
<td>21.72%</td>
</tr>
<tr>
<td>1989</td>
<td>8.32%</td>
<td>19.49%</td>
<td>31.34%</td>
<td>8.37%</td>
</tr>
<tr>
<td>Average</td>
<td>9.04%</td>
<td>11.99%</td>
<td>18.24%</td>
<td>13.88%</td>
</tr>
<tr>
<td>Var</td>
<td>0.0008</td>
<td>0.0124889</td>
<td>0.016021</td>
<td>0.034069</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>2.88%</td>
<td>11.18%</td>
<td>12.66%</td>
<td>18.46%</td>
</tr>
</tbody>
</table>

Standard Deviation for U.S. Treasury Bill for 80s: 2.88%
Standard Deviation for U.S. Long-Term Government Bonds for 80s: 11.18%
Standard Deviation for U.S. Small Company Stocks for 80s: 18.46%
Highest variance was 0.153854 in the 60s for Small Company Stocks and the lowest variance was .000055 for 3 month T-Bills in the 50s.
13. **Internet exercise.** Find the thirteen-week Treasury bill rates for the years 2000 to present. Go to Treasury Direct (www.treasurydirect.gov) and in the “Institutions” section, click “Find Historical Auction Data.” Select the thirteen-week Treasury bill historical data from January 1, 2000, to present. Record the first auction of each year from 2000 to present using the investment rate. What was the average from 2000 to present? What was the standard deviation of this sample of auction rates? How does it compare with the data presented in Table 8.1?

**ANSWER:**

The following investment rates come from the first auction of each year:

<table>
<thead>
<tr>
<th>Year</th>
<th>T-Bill rate</th>
<th>(R – Avg)</th>
<th>(R – Avg)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>5.53%</td>
<td>2.19%</td>
<td>0.000478</td>
</tr>
<tr>
<td>2001</td>
<td>5.86%</td>
<td>2.53%</td>
<td>0.000638</td>
</tr>
<tr>
<td>2002</td>
<td>1.74%</td>
<td>–1.60%</td>
<td>0.000256</td>
</tr>
<tr>
<td>2003</td>
<td>1.21%</td>
<td>–2.13%</td>
<td>0.000454</td>
</tr>
<tr>
<td>2004</td>
<td>0.90%</td>
<td>–2.44%</td>
<td>0.000594</td>
</tr>
<tr>
<td>2005</td>
<td>2.32%</td>
<td>–1.02%</td>
<td>0.000104</td>
</tr>
<tr>
<td>2006</td>
<td>4.17%</td>
<td>0.83%</td>
<td>6.9E-05</td>
</tr>
<tr>
<td>2007</td>
<td>5.06%</td>
<td>1.72%</td>
<td>0.000297</td>
</tr>
<tr>
<td>2008</td>
<td>3.26%</td>
<td>–0.08%</td>
<td>6.33E-07</td>
</tr>
<tr>
<td>2009</td>
<td>1.52%</td>
<td>–1.31%</td>
<td>.00017161</td>
</tr>
<tr>
<td>2010</td>
<td>0.81%</td>
<td>–2.02%</td>
<td>4.0804E-4</td>
</tr>
<tr>
<td>2011</td>
<td>1.52%</td>
<td>–1.31%</td>
<td>.00017161</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>33.9%</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>2.83%</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td>0.000352</td>
<td></td>
</tr>
<tr>
<td>S.Dev</td>
<td>1.88%</td>
<td>=(.000352)^{1/2}</td>
<td></td>
</tr>
</tbody>
</table>

These investment rates are lower than the prior decade i.e 1990s (Average = 5.02%) and the 50 year history of the Treasury Bill (Average = 5.23%) and closer to the rates of the 1960s (Average = 3.9%).

14. **Internet exercise.** Find the Standard & Poor's 500 annual returns for 2000 to the present. Go to Yahoo! Finance (www.finance.yahoo.com), and in the "Get Quotes" search field, enter SPY (the ticker symbol for the Standard & Poor's 500 electronically traded fund). Select historical prices and find the year-end prices for the fund and all dividends from 2000 through the end of the most recent year. Find each year’s return (remember to add in the dividend distributions for the year). What was
the average annual return? What was the standard deviation? How does it compare with the data presented in Table 8.1?

ANSWER

Year End Price of SPY (1999-2010) and annual Dividends and annual Return

<table>
<thead>
<tr>
<th>Year</th>
<th>Price</th>
<th>Dividends (annual)</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999--</td>
<td>146.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000--</td>
<td>131.19</td>
<td>1.454</td>
<td>– 9.69%</td>
</tr>
<tr>
<td>2001--</td>
<td>114.30</td>
<td>1.032</td>
<td>–12.09%</td>
</tr>
<tr>
<td>2002--</td>
<td>88.23</td>
<td>1.498</td>
<td>–21.50%</td>
</tr>
<tr>
<td>2003--</td>
<td>111.28</td>
<td>1.630</td>
<td>27.97%</td>
</tr>
<tr>
<td>2004--</td>
<td>120.87</td>
<td>2.197</td>
<td>10.59%</td>
</tr>
<tr>
<td>2005--</td>
<td>124.51</td>
<td>2.149</td>
<td>4.79%</td>
</tr>
<tr>
<td>2006--</td>
<td>141.62</td>
<td>2.446</td>
<td>15.71%</td>
</tr>
<tr>
<td>2007--</td>
<td>146.21</td>
<td>2.701</td>
<td>5.15%</td>
</tr>
<tr>
<td>2008--</td>
<td>90.24</td>
<td>2.721</td>
<td>–36.42%</td>
</tr>
<tr>
<td>2009--</td>
<td>111.44</td>
<td>2.177</td>
<td>25.91%</td>
</tr>
<tr>
<td>2010--</td>
<td>125.75</td>
<td>1.786</td>
<td>14.44%</td>
</tr>
</tbody>
</table>

Average 2.26%
Standard Deviation 20.11%

The early years of the start of this decade had negative returns for three consecutive years, and 2008 was a terrible year as well. The average return is very low (2.26%), and below every previous decade of the last fifty years. The standard deviation is higher than the fifty year standard deviation of large company stocks i.e. 20.11% versus 16.7%.

15. Expected return. Hull Consultants, a famous think tank in the Midwest, has provided probability estimates for the four potential economic states for the coming year. The probability of a boom economy is 10%, the probability of a stable growth economy is 15%, the probability of a stagnant economy is 50%, and the probability of a recession is 25%. Estimate the expected returns on the following individual investments for the coming year.

<table>
<thead>
<tr>
<th>INVESTMENT</th>
<th>Forecasted Returns for Each Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boom</td>
</tr>
<tr>
<td>Stock</td>
<td>25%</td>
</tr>
<tr>
<td>Corporate Bond</td>
<td>9%</td>
</tr>
<tr>
<td>Government Bond</td>
<td>8%</td>
</tr>
</tbody>
</table>
Expected Return on Stock = 0.10 \times 0.25 + 0.15 \times 0.12 + 0.50 \times 0.04 + 0.25 \times (-0.12)
= 0.0250 + 0.0180 + 0.0200 – 0.0300 = 0.0330 or 3.3%

Expected Return on Corp. Bond = 0.10 \times 0.09 + 0.15 \times 0.07 + 0.50 \times 0.05 + 0.25 \times 0.03
= 0.0090 + 0.0105 + 0.0250 + 0.0075 = 0.0520 or 5.2%

Expected Return Gov. Bond = 0.10 \times 0.08 + 0.15 \times 0.06 + 0.50 \times 0.04 + 0.25 \times 0.02
= 0.0080 + 0.0090 + 0.0200 + 0.0050 = 0.0420 or 4.2%

16. **Variance and standard deviation (expected).** Using the data from Problem 15, calculate the variance and standard deviation of the three investments, stock, corporate bond, and government bond. If the estimates for both the probabilities of the economy and the returns in each state of the economy are correct, which investment would you choose, considering both risk and return? Why?

**ANSWER**

Variance of Stock = 0.10 \times (0.25 - 0.033)^2 + 0.15 \times (0.12 - 0.033)^2 +
0.50 \times (0.04 - 0.033)^2 + 0.25 \times (-0.12 - 0.033)^2
= 0.10 \times 0.0471 + 0.15 \times 0.0076 + 0.50 \times 0.0000 +
0.25 \times 0.0234
= 0.0047 + 0.0011 + 0.0000 + 0.0059 = 0.0117

Standard Deviation of Stock = (0.0117)^{1/2} = 0.1083 or 10.83%

Variance of Corp. Bond = 0.10 \times (0.09 - 0.052)^2 + 0.15 \times (0.07 - 0.052)^2 +
0.50 \times (0.05 - 0.052)^2 + 0.25 \times (0.03 - 0.052)^2
= 0.10 \times 0.0014 + 0.15 \times 0.0003 + 0.50 \times 0.0000 +
0.25 \times 0.0005
= 0.0001 + 0.0000 + 0.0000 + 0.0001 = 0.000316

Standard Deviation of Corp. Bond = (0.0004)^{1/2} = 0.01776 or 1.78%

Variance of Gov. Bond = 0.10 \times (0.08 - 0.042)^2 + 0.15 \times (0.06 - 0.042)^2 +
0.50 \times (0.04 - 0.042)^2 + 0.25 \times (0.02 - 0.042)^2
= 0.10 \times 0.0014 + 0.15 \times 0.0003 + 0.50 \times 0.0000 +
0.25 \times 0.0005
= 0.0001 + 0.0000 + 0.0000 + 0.0001 = 0.000316

Standard Deviation of Gov. Bond = (0.000316)^{1/2} = 0.01776 or 1.78%

The best choice is the corporate bond. First, comparing the corporate bond and the stock, the corporate bond has a higher expected return and a lower variance (standard deviation). Second comparing the corporate bond and the government bond the corporate
bond has a higher return and the same variance (standard deviation). This result is due to the low probabilities of “good” economic states where the stock performs best.

17. **Expected return.** Bacon and Associates, a famous Northwest think tank, has provided probability estimates for the four potential economic states for the coming year. The probability of a boom economy is 20%, the probability of a stable growth economy is 45%, the probability of a stagnant economy is 20%, and the probability of a recession is 15%. Estimate the expected return on the following individual investments for the coming year.

<table>
<thead>
<tr>
<th>INVESTMENT</th>
<th>Forecasted Returns for Each Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boom</td>
</tr>
<tr>
<td>Stock</td>
<td>25%</td>
</tr>
<tr>
<td>Corporate Bond</td>
<td>9%</td>
</tr>
<tr>
<td>Government Bond</td>
<td>8%</td>
</tr>
</tbody>
</table>

**ANSWER**

Expected Return Stock = 0.20 × 0.25 + 0.45 × 0.12 + 0.20 × 0.04 + 0.15 × (-0.12)
= 0.0050 + 0.0540 + 0.0080 - 0.0180 = 0.0940 or 9.4%

Expected Return Corp. Bond = 0.20 × 0.09 + 0.45 × 0.07 + 0.20 × 0.05 + 0.15 × 0.03
= 0.0180 + 0.0315 + 0.0100 + 0.0045 = 0.0640 or 6.4%

Expected Return Gov. Bond = 0.20 × 0.08 + 0.45 × 0.06 + 0.20 × 0.04 + 0.15 × 0.02
= 0.0160 + 0.0270 + 0.0080 + 0.0030 = 0.0540 or 5.4%

18. **Variance and standard deviation (expected).** Using the data from Problem 17, calculate the variance and standard deviation of the three investments, stock: corporate bond, and government bond. If the estimates for both the probabilities of the economy and the returns in each state of the economy are correct, which investment would you choose, considering both risk and return? Why?

**ANSWER**

Variance of Stock = 0.20 × (0.25 - 0.094)^2 + 0.45 × (0.12 - 0.094)^2 + 0.20 × (0.04 - 0.094)^2 + 0.15 × (-0.12 - 0.094)^2
= 0.20 × 0.0243 + 0.45 × 0.0007 + 0.20 × 0.0029 + 0.15 × 0.0458
First comparing the corporate bond and the government bond the corporate bond has a higher return and the same variance (standard deviation) so the corporate bond is preferred over the government bond. Second comparing the corporate bond and the stock we find that the stock has a higher expected return but a higher standard deviation. Therefore we cannot say one investment is a better choice than the other. Each investor would look at the reward compared to the risk and determine if the additional reward (return) for the stock is worth the additional risk (standard deviation).

19. Expected return and standard deviation. Use the information in the following to answer the questions below.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Return on A in State</th>
<th>Return on B in State</th>
<th>Return on C in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.35</td>
<td>0.040</td>
<td>0.210</td>
<td>0.300</td>
</tr>
<tr>
<td>Normal</td>
<td>.50</td>
<td>0.040</td>
<td>0.080</td>
<td>0.200</td>
</tr>
<tr>
<td>Recession</td>
<td>.15</td>
<td>0.040</td>
<td>-0.010</td>
<td>-0.260</td>
</tr>
</tbody>
</table>

a. What is the expected return of each asset?

**ANSWER (a)**

Expected Return A = 0.35 \times 0.04 + 0.50 \times 0.04 + 0.15 \times 0.04

= 0.0140 + 0.0200 + 0.0060 = 0.0040 or 4.0%

Expected Return B = 0.35 \times 0.21 + 0.50 \times 0.08 + 0.15 \times (-0.01)

= 0.0735 + 0.0400 – 0.0015 = 0.1120 or 11.2%

Expected Return C = 0.35 \times 0.30 + 0.50 \times 0.20 + 0.15 \times (-0.26)

= 0.1050 + 0.1000 – 0.0390 = 0.1660 or 16.6%

b. What is the variance of each asset?
ANSWER (b)

Variance of A = 0.35 \times (0.04 - 0.04)^2 + 0.50 \times (0.04 - 0.04)^2 + 0.15 \times (0.04 - 0.04)^2
= 0.35 \times 0.0000 + 0.50 \times 0.0000 + 0.15 \times 0.0000
= 0.0000 + 0.0000 + 0.0000 = 0.0000

Variance of B = 0.35 \times (0.21 - 0.112)^2 + 0.50 \times (0.08 - 0.112)^2 + 0.15 \times (-0.01 - 0.112)^2
= 0.35 \times 0.0096 + 0.50 \times 0.0010 + 0.15 \times 0.0149
= 0.0034 + 0.0005 + 0.0022 = 0.0061

Variance of C = 0.35 \times (0.30 - 0.166)^2 + 0.50 \times (0.20 - 0.166)^2 + 0.15 \times (-0.26 - 0.166)^2
= 0.35 \times 0.0180 + 0.50 \times 0.0012 + 0.15 \times 0.1815
= 0.0063 + 0.0006 + 0.0272 = 0.0341

(c. What is the standard deviation of each asset?

ANSWER (c)

Standard Deviation of A = (0.0000)^{1/2} = 0.0000 or 0.00%
Standard Deviation of B = (0.0061)^{1/2} = 0.0781 or 7.81%
Standard Deviation of B = (0.0341)^{1/2} = 0.1846 or 18.46%

20. **Expected return and standard deviation.** Use the information in the following to answer the questions below.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Return on D in State</th>
<th>Return on E in State</th>
<th>Return on F in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.35</td>
<td>.060</td>
<td>.310</td>
<td>.150</td>
</tr>
<tr>
<td>Normal</td>
<td>.50</td>
<td>.060</td>
<td>.180</td>
<td>.120</td>
</tr>
<tr>
<td>Recession</td>
<td>.15</td>
<td>.060</td>
<td>-.210</td>
<td>-.060</td>
</tr>
</tbody>
</table>

a. What is the expected returns of each asset?

b. What is the variance of each asset?

c. What is the standard deviation of each asset?

ANSWER (a)

Expected Return D = 0.35 \times 0.06 + 0.50 \times 0.06 + 0.15 \times 0.06
= 0.021 + 0.03 + 0.009 = 0.06 or 6.0%

Expected Return E = 0.35 \times 0.31 + 0.50 \times 0.18 + 0.15 \times (-0.21)
= 0.1085 + 0.09 - 0.0315 = 0.167 or 16.7%

Expected Return F = 0.35 \times 0.15 + 0.50 \times 0.12 + 0.15 \times (-0.06)
= 0.0525 + 0.06 - 0.0090 = 0.1035 or 10.35%
Variance of D = 0.35 \times (0.06 - 0.06)^2 + 0.50 \times (0.06 - 0.06)^2 + 0.15 \times (0.06 - 0.06)^2
= 0.35 \times 0.0000 + 0.50 \times 0.0000 + 0.15 \times 0.0000
= 0.0000 + 0.0000 + 0.0000 = 0.0000

Variance of E = 0.35 \times (0.31 - 0.167)^2 + 0.50 \times (0.18 - 0.167)^2 + 0.15 \times (-0.21 - 0.167)^2
= 0.35 \times 0.020449 + 0.50 \times 0.0001 + 0.15 \times 0.1421
= 0.007157 + 0.0000 + 0.021319 = 0.028561

Variance of F = 0.35 \times (0.15 - 0.1035)^2 + 0.50 \times (0.12 - 0.1035)^2 + 0.15 \times (-0.06 - 0.1035)^2
= 0.35 \times 0.002162 + 0.50 \times 0.0144 + 0.15 \times 0.0036
= 0.000757 + 0.000136 + 0.00401 = 0.004903

Standard Deviation of D = (0.0000)^{1/2} = 0.0000 or 0.00%
Standard Deviation of E = (0.028561)^{1/2} = 0.169 or 16.9%
Standard Deviation of F = (0.004903)^{1/2} = 0.0700 or 7%

21. Expected return and standard deviation. Use the information in the following to answer the questions below.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Return on J in State</th>
<th>Return on K in State</th>
<th>Return on L in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.30</td>
<td>0.050</td>
<td>0.240</td>
<td>0.300</td>
</tr>
<tr>
<td>Growth</td>
<td>.40</td>
<td>0.050</td>
<td>0.120</td>
<td>0.200</td>
</tr>
<tr>
<td>Stagnant</td>
<td>.20</td>
<td>0.050</td>
<td>0.040</td>
<td>0.060</td>
</tr>
<tr>
<td>Recession</td>
<td>.10</td>
<td>0.050</td>
<td>-0.100</td>
<td>-0.200</td>
</tr>
</tbody>
</table>

a. What is the expected return of each asset?

**ANSWER (a)**

Expected Return J = 0.30 \times 0.05 + 0.40 \times 0.05 + 0.20 \times 0.05 + 0.10 \times 0.05
= 0.0150 + 0.0200 + 0.0100 + 0.0050 = 0.050 or 5.0%

Expected Return K = 0.30 \times 0.24 + 0.40 \times 0.12 + 0.20 \times 0.04 + 0.10 \times (-0.10)
= 0.0720 + 0.0480 + 0.0080 - 0.0100 = 0.1180 or 11.80%

Expected Return L = 0.30 \times 0.30 + 0.40 \times 0.20 + 0.20 \times 0.06 + 0.10 \times (-0.20)
= 0.0900 + 0.0800 + 0.0120 + 0.0200 = 0.1620 or 16.20%

b. What is the variance and standard deviation of each asset?

**ANSWER (b)**

\[ \sigma^2 (J) = 0.30 \times (0.05 - 0.05)^2 + 0.40 \times (0.05 - 0.05)^2 + 0.20 \times (0.05 - 0.05)^2 + 0.10 \times (0.05 - 0.05)^2 \]

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= 0.30 × 0.0000 + 0.40 × 0.0000 + 0.20 × 0.0000 + 0.10 × 0.0000
= 0.0000 + 0.0000 + 0.0000 + 0.0000 = 0.0000

Standard Deviation of J = (0.0000)\(^{1/2}\) = 0.0000 or 0.00%

\[ \sigma^2 (K) = 0.30 \times (0.24 - 0.1180)^2 + 0.40 \times (0.12 - 0.1180)^2 + 0.20 \times (0.04 - 0.1180)^2 + 0.10 \times (-0.10 - 0.1180)^2 \]
= 0.30 × 0.0149 + 0.40 × 0.0000 + 0.20 × 0.0061 + 0.10 × 0.0475
= 0.0045 + 0.0000 + 0.0012 + 0.0048 = 0.0104

Standard Deviation of J = 0.0104\(^{1/2}\) = 0.1022 or 10.22%

\[ \sigma^2 (L) = 0.30 \times (0.30 - 0.1620)^2 + 0.40 \times (0.20 - 0.1620)^2 + 0.20 \times (0.06 - 0.1620)^2 + 0.10 \times (-0.20 - 0.1620)^2 \]
= 0.30 × 0.0190 + 0.40 × 0.0014 + 0.20 × 0.0104 + 0.10 × 0.1310
= 0.0057 + 0.0006 + 0.0021 + 0.0131 = 0.0215

Standard Deviation of L = 0.0215\(^{1/2}\) = 0.1465 or 14.65%

\[ \sigma_c = \sqrt{0.0215} = 0.1465 \]

c. What is the expected return of a portfolio with 10% in Asset J, 50% in Asset K, and 40% in Asset L?

ANSWER (c)

\[ \text{Expected Return Portfolio} = 0.10 \times 0.05 + 0.50 \times 0.118 + 0.40 \times 0.162 \]
= 0.0050 + 0.0590 + 0.0648 = 0.1288 or 12.88%

OR

First determine the portfolio’s return in each state of the economy with the allocation of assets at 10% in J, 50% in K, and 40% in L.

Expected Return Portfolio in Boom = 0.10 × 0.05 + 0.50 × 0.24 + 0.40 × 0.30
= 0.0050 + 0.1200 + 0.1200 = 0.2450 or 24.50%

Expected Return Portfolio in Growth = 0.10 × 0.05 + 0.50 × 0.12 + 0.40 × 0.20
= 0.0050 + 0.0600 + 0.0800 = 0.1450 or 14.50%

Expected Return Portfolio in Stagnant = 0.10 × 0.05 + 0.50 × 0.04 + 0.40 × 0.06
= 0.0050 + 0.0200 + 0.0240 = 0.0490 or 4.90%

Expected Return Portfolio in Recession = 0.10 × 0.05 + 0.50 × (-0.10) + 0.40 × (-0.20)
= 0.0050 – 0.0500 – 0.0800 = -0.1250 or -12.50%

Now take the probability of each state times the portfolio outcome in that state:

Expected Return Portfolio = 0.30 × 0.2450 + 0.40 × 0.1450 + 0.20 × 0.0490 + 0.10 × (-0.1250)
= 0.0735 + 0.0580 + 0.0098 – 0.0125 = 0.1288 or 12.88%

Note that either way produces the same expected return but that for the variance calculation the portfolio returns in the three economic states are needed.
d. What are the portfolio’s variance and standard deviation using the same asset weights from part (c)?

**ANSWER (d)**

Variance of Portfolio = \(0.30 \times (0.2450 - 0.1288)^2 + 0.40 \times (0.1450 - 0.1288)^2 + 0.20 \times (0.0490 - 0.1288)^2 + 0.10 \times (-0.1250 - 0.1288)^2\)

\[= 0.30 \times 0.0135 + 0.40 \times 0.0003 + 0.20 \times 0.0064 + 0.10 \times 0.0644\]

\[= 0.0041 + 0.0001 + 0.0013 + 0.0064 = 0.0119\]

Standard Deviation of Portfolio = \(0.0119^{\frac{1}{2}} = 0.1090\) or 10.90%

---

22. **Expected return and standard deviation.** Use the information in the following to answer the questions below.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Return on R in State</th>
<th>Return on S in State</th>
<th>Return on T in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.15</td>
<td>0.040</td>
<td>0.280</td>
<td>0.450</td>
</tr>
<tr>
<td>Growth</td>
<td>.25</td>
<td>0.040</td>
<td>0.140</td>
<td>0.275</td>
</tr>
<tr>
<td>Stagnant</td>
<td>.35</td>
<td>0.040</td>
<td>0.070</td>
<td>0.025</td>
</tr>
<tr>
<td>Recession</td>
<td>.25</td>
<td>0.040</td>
<td>-0.035</td>
<td>-0.175</td>
</tr>
</tbody>
</table>

a. What is the expected return of each asset?

**ANSWER (a)**

Expected Return R = \(0.15 \times 0.04 + 0.25 \times 0.04 + 0.35 \times 0.04 + 0.25 \times 0.04\)

\[= 0.0060 + 0.0100 + 0.0140 + 0.0100 = 0.0040\] or 4.0%

Expected Return S = \(0.15 \times 0.28 + 0.25 \times 0.14 + 0.35 \times 0.07 + 0.25 \times -0.035\)

\[= 0.0420 + 0.0350 + 0.0245 + 0.0088 = 0.0928\] or 9.28%

Expected Return T = \(0.15 \times 0.45 + 0.25 \times 0.275 + 0.35 \times 0.025 + 0.25 \times -0.175\)

\[= 0.0675 + 0.0688 + 0.0088 - 0.0438 = 0.1013\] or 10.13%

b. What are the variances and standard deviations of each asset?

**ANSWER (b)**

\[\sigma^2 (R) = 0.15 \times (0.04 - 0.04)^2 + 0.25 \times (0.04 - 0.04)^2 + 0.35 \times (0.04 - 0.04)^2 + 0.25 \times (0.04 - 0.04)^2\]

\[= 0.15 \times 0.0000 + 0.25 \times 0.0000 + 0.35 \times 0.0000 + 0.25 \times 0.0000\]

\[= 0.0000 + 0.0000 + 0.0000 + 0.0000 = 0.0000\]

\[\text{Standard Deviation of R} = (0.0000)^{\frac{1}{2}} = 0.0000\] or 0.00%

\[\sigma^2 (S) = 0.15 \times (0.28 - 0.0928)^2 + 0.25 \times (0.14 - 0.0928)^2 + 0.35 \times (0.07 - 0.0928)^2 + 0.25 \times (-0.035 - 0.0928)^2\]
c. What is the expected return of a portfolio with equal investment in all three assets?

ANSWER (c)

Expected Return Portfolio = 0.3333 \times 0.04 + 0.3333 \times 0.0928 + 0.3333 \times 0.1013

= 0.0133 + 0.0309 + 0.0338 = 0.0780 or 7.80%

OR

First determine the portfolio’s return in each state of the economy with the allocation of assets at 1/3 in R, 1/3 in S, and 1/3 in T.

Expected Return Portfolio in Boom = 0.3333 \times 0.04 + 0.3333 \times 0.28 + 0.3333 \times 0.45

= 0.0133 + 0.0933 + 0.1500 = 0.2567 or 25.67%

Expected Return Portfolio in Growth = 0.3333 \times 0.04 + 0.3333 \times 0.14 + 0.3333 \times 0.275

= 0.0133 + 0.0467 + 0.0917 = 0.1517 or 15.17%

Expected Return Portfolio in Stagnant = 0.3333 \times 0.04 + 0.3333 \times 0.07 + 0.3333 \times 0.025

= 0.0133 + 0.0233 + 0.0083 = 0.0450 or 4.50%

Expected Return Portfolio in Recession = 0.3333 \times 0.04 + 0.3333 \times (-0.035) + 0.3333 \times (-0.175)

= 0.0133 - 0.0117 - 0.0583 = -0.0567 or -5.67%

Now take the probability of each state times the portfolio outcome in that state:

Expected Return Portfolio = 0.15 \times 0.2567 + 0.25 \times 0.1517 + 0.35 \times 0.0450 + 0.25 \times -0.0567

= 0.0385 + 0.0379 + 0.0158 - 0.0142 = 0.0780 or 7.80%

Note that either way produces the same expected return but that for the variance calculation the portfolio returns in the three economic states are needed.
d. What is the portfolio’s variance and standard deviation using the same asset weights in part c?

**ANSWER (d)**

Variance of Portfolio = \(0.15 \times (0.2567 - 0.0780)^2 + 0.25 \times (0.1517 - 0.0780)^2 + 0.35 \times (0.045 - 0.0780)^2 + 0.25 \times (-0.0567 - 0.0780)^2\)

\[
= 0.15 \times 0.0319 + 0.25 \times 0.0054 + 0.35 \times 0.0011 + 0.25 \times 0.0181
\]

\[
= 0.0048 + 0.0014 + 0.0004 + 0.0045 = 0.0111
\]

Standard Deviation of Portfolio = \((0.0111)^{1/2} = 0.1052\) or 10.52%

23. **Benefits of diversification.** Sally Rogers has decided to invest her wealth equally across the three following assets. What are her expected returns and the risk by investing in the three assets? How does this compare to investing in Asset M alone?

_Hint:_ Find the standard deviation of Asset M and of the portfolio equally invested in Asset M, N, and O.

<table>
<thead>
<tr>
<th>States</th>
<th>Probability</th>
<th>Asset M Return</th>
<th>Asset N Return</th>
<th>Asset O Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>30%</td>
<td>12%</td>
<td>19%</td>
<td>2%</td>
</tr>
<tr>
<td>Normal</td>
<td>50%</td>
<td>8%</td>
<td>11%</td>
<td>8%</td>
</tr>
<tr>
<td>Recession</td>
<td>20%</td>
<td>2%</td>
<td>-2%</td>
<td>12%</td>
</tr>
</tbody>
</table>

**ANSWER**

First find the expected return of the equally weighted portfolio in the three economic states:

Return of Portfolio in Boom = \(1/3 \times (12\%) + 1/3 \times (19\%) + 1/3 \times (2\%) = 11.00\%\)

Return of Portfolio in Normal = \(1/3 \times (8\%) + 1/3 \times (11\%) + 1/3 \times (8\%) = 9.00\%\)

Return of Portfolio in Recession = \(1/3 \times (2\%) + 1/3 \times (-2\%) + 1/3 \times (12\%) = 4.00\%\)

Next find the expected returns of Asset M and the Portfolio.

\[E(r_M) = 3.6\% + 4.0\% + 0.4\% = 8\%\]

\[E(r_p) = 3.3\% + 4.5\% + 0.8\% = 8.6\%\]

Then find the standard deviation of Asset M and the Portfolio.

\[\text{Standard Deviation of Asset M} = [0.30 \times (0.12 - 0.08)^2 + 0.50 \times (0.08 - 0.08)^2 + 0.20 \times (0.02 - 0.08)^2]^{1/2}\]

\[= [0.30 \times 0.0016 + 0.20 \times 0.0036]^{1/2}\]

\[= [0.00048 + 0.00072]^{1/2} = [0.0012]^{1/2} = 0.0346\] or 3.46%

\[\text{Standard Deviation of Portfolio} = [0.30 \times (0.11 - 0.086)^2 + 0.50 \times (0.09 - 0.086)^2 + 0.20 \times (0.4 - 0.086)^2]^{1/2}\]

\[= [0.30 \times 0.0006 + 0.50 \times 0.0000 + 0.20 \times 0.0021]^{1/2}\]
The benefit of the portfolio over Asset M alone is an increase in return of 0.6% and a simultaneous reduction in total risk of 1%.

24. **Benefits of diversification.** Use the same assets in Problem 23. Could Sally reduce her total risk even more by using Assets M and N only, Assets M and O only, or Assets N and O only? Use a 50/50 split between the asset pairs and find the standard deviation of the asset pairs.

**ANSWER**

First find the return of each 50/50 portfolio in the different states of the economy:

First find the expected return of the equally weighted portfolio in the three economic states:

Return of Portfolio M and N in Boom = 1/2 (12%) + 1/2 (19%) = 15.50%
Return of Portfolio M and N in Normal = 1/2 (8%) + 1/2 (11%) = 9.50%
Return of Portfolio M and N in Recession = 1/2 (2%) + 1/2 (-2%) = 0.00%
Return of Portfolio M and O in Boom = 1/2 (12%) + 1/2 (2%) = 7.00%
Return of Portfolio M and O in Normal = 1/2 (8%) + 1/2 (8%) = 8.00%
Return of Portfolio M and O in Recession = 1/2 (2%) + 1/2 (12%) = 7.00%
Return of Portfolio N and O in Boom = 1/2 (19%) + 1/2 (2%) = 10.50%
Return of Portfolio N and O in Normal = 1/2 (11%) + 1/2 (8%) = 9.50%
Return of Portfolio N and O in Recession = 1/2 (-2%) + 1/2 (12%) = 5.00%

Second find the expected returns of each individual asset and each 50/50 combination.

Expected Return Asset M = 0.30 \times (12\%) + 0.50 \times (8\%) + 0.20 \times (2\%)
E(r_M) = 3.6\% + 4.0\% + 0.4\% = 8\%

Expected Return Asset N = 0.30 \times (19\%) + 0.50 \times (11\%) + 0.20 \times (-2\%)
E(r_N) = 5.7\% + 5.5\% - 0.4\% = 10.8\%

Expected Return Asset O = 0.30 \times (2\%) + 0.50 \times (8\%) + 0.20 \times (12\%)
E(r_O) = 0.6\% + 4.0\% + 2.4\% = 7\%

Expected Return Portfolio MN = 0.30 \times (15.5\%) + 0.50 \times (9.5\%) + 0.20 \times (0\%)
E(r_{MN}) = 4.65\% + 4.75\% + 0.0\% = 9.4\%

Expected Return Portfolio MO = 0.30 \times (7\%) + 0.50 \times (8\%) + 0.20 \times (7\%)
E(r_{MO}) = 2.1\% + 4.0\% + 1.4\% = 7.5\%

Expected Return Portfolio NO = 0.30 \times (10.5\%) + 0.50 \times (9.5\%) + 0.20 \times (5\%)
E(r_{NO}) = 3.15\% + 4.75\% + 1.0\% = 8.9\%

Finally find the standard deviation of each asset and each 50/50 portfolio:

\[
\text{Standard Deviation of Asset M} = \sqrt{[0.30 \times (0.12 - 0.08)^2 + 0.50 \times (0.08 - 0.08)^2 + 0.20 \times (0.02 - 0.08)^2]} = 0.0246 \text{ or } 2.46\%
\]
= \[0.30 \times 0.0016 + 0.50 \times 0.0000 + 0.20 \times 0.0036\]^{1/2} = [0.00048 + 0.00072]^{1/2} = [0.0012]^{1/2} = 0.0346 or 3.46%

Standard Deviation of Asset N = \[0.30 \times (0.19 - 0.108)^2 + 0.50 \times (0.11 - 0.108)^2 + 0.20 \times (-0.02 - 0.108)^2\]^{1/2} = \[0.30 \times 0.0067 + 0.50 \times 0.0000 + 0.20 \times 0.0164\]^{1/2} = [0.0020 + 0.0000 + 0.0033]^{1/2} = 0.0728 or 7.28%

Standard Deviation of Asset O = \[0.30 \times (0.02 - 0.07)^2 + 0.50 \times (0.08 - 0.07)^2 + 0.20 \times (0.12 - 0.07)^2\]^{1/2} = \[0.30 \times 0.0025 + 0.50 \times 0.0001 + 0.20 \times 0.0025\]^{1/2} = [0.0008 + 0.0001 + 0.0005]^{1/2} = 0.0361 or 3.61%

Standard Deviation of Portfolio MN = \[0.30 \times (0.155 - 0.094)^2 + 0.50 \times (0.095 - 0.094)^2 + 0.20 \times (0.0 - 0.094)^2\]^{1/2} = \[0.30 \times 0.0037 + 0.50 \times 0.0000 + 0.20 \times 0.0088\]^{1/2} = [0.0011 + 0.0000 + 0.0018]^{1/2} = [0.0029]^{1/2} = 0.0537 or 5.37%

Standard Deviation of Portfolio MO = \[0.30 \times (0.7 - 0.075)^2 + 0.50 \times (0.08 - 0.075)^2 + 0.20 \times (0.7 - 0.075)^2\]^{1/2} = \[0.30 \times 0.0000 + 0.50 \times 0.0000 + 0.20 \times 0.0000\]^{1/2} = [0.0000 + 0.0000 + 0.0000]^{1/2} = [0.0000]^{1/2} = 0.0050 or 0.50%

Standard Deviation of Portfolio NO = \[0.30 \times (0.105 - 0.089)^2 + 0.50 \times (0.095 - 0.089)^2 + 0.20 \times (0.05 - 0.089)^2\]^{1/2} = \[0.30 \times 0.0003 + 0.50 \times 0.0000 + 0.20 \times 0.0015\]^{1/2} = [0.0001 + 0.0000 + 0.0003]^{1/2} = [0.0004]^{1/2} = 0.02 or 2%

If Sally chose a 50/50 split between Asset M and O the benefit is a decrease in total risk to only a half percent (0.5%).

25. Beta of a portfolio. The beta of four stocks—G, H, I, and J—are 0.45, 0.8, 1.15, and 1.6, respectively. What is the beta of a portfolio with the following weights in each asset?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Weight in G</th>
<th>Weight in H</th>
<th>Weight in I</th>
<th>Weight in J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1</td>
<td>25%</td>
<td>25%</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>Portfolio 2</td>
<td>30%</td>
<td>40%</td>
<td>20%</td>
<td>10%</td>
</tr>
</tbody>
</table>

26. Expected return of a portfolio using beta. Use the same four assets from Problem 25 in the same three portfolios. What are the expected returns of the four individual assets and the three portfolios, if the current SML is plotting with an intercept of 4% (risk-free rate) and a market premium of 10% (slope of the line)?

**ANSWER**

- **Expected Return of Asset G**
  \[ \text{Expected Return of Asset G} = 4\% + 0.45 \times 10\% = 8.5\% \]

- **Expected Return of Asset H**
  \[ \text{Expected Return of Asset H} = 4\% + 0.8 \times 10\% = 12\% \]

- **Expected Return of Asset I**
  \[ \text{Expected Return of Asset I} = 4\% + 1.15 \times 10\% = 15.5\% \]

- **Expected Return of Asset J**
  \[ \text{Expected Return of Asset J} = 4\% + 1.6 \times 10\% = 20\% \]

- **Expected Return of Portfolio 1**
  \[ \text{Expected Return of Portfolio 1} = 4\% + 1.0 \times 10\% = 14\% \]

- **Expected Return of Portfolio 2**
  \[ \text{Expected Return of Portfolio 2} = 4\% + 0.845 \times 10\% = 12.45\% \]

- **Expected Return of Portfolio 3**
  \[ \text{Expected Return of Portfolio 2} = 4\% + 1.145 \times 10\% = 15.45\% \]

27. Beta of a portfolio. The beta of four stocks—P, Q, R, and S—are respectively 0.6, 0.85, 1.2, and 1.35. What is the beta of a portfolio with the following weights in each asset?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Weight in P</th>
<th>Weight in Q</th>
<th>Weight in R</th>
<th>Weight in S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25%</td>
<td>25%</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>2</td>
<td>30%</td>
<td>40%</td>
<td>20%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
<td>20%</td>
<td>40%</td>
<td>30%</td>
</tr>
</tbody>
</table>

**ANSWER**

- **Beta of Portfolio 1**
  \[ \text{Beta of Portfolio 1} = 0.25 \times 0.6 + 0.25 \times 0.85 + 0.25 \times 1.2 + 0.25 \times 1.35 \]
  \[ \beta_{\text{portfolio - 1}} = 0.15 + 0.2125 + 0.3 + 0.3375 = 1.0 \]

- **Beta of Portfolio 2**
  \[ \text{Beta of Portfolio 2} = 0.30 \times 0.6 + 0.40 \times 0.85 + 0.20 \times 1.2 + 0.10 \times 1.35 \]
  \[ \beta_{\text{portfolio - 2}} = 0.18 + 0.34 + 0.24 + 0.135 = 0.895 \]

- **Beta of Portfolio 3**
  \[ \text{Beta of Portfolio 3} = 0.10 \times 0.6 + 0.20 \times 0.85 + 0.40 \times 1.2 + 0.30 \times 1.35 \]
  \[ \beta_{\text{portfolio - 3}} = 0.06 + 0.17 + 0.48 + 0.405 = 1.115 \]
28. **Expected return of a portfolio using beta.** Use the same four assets from Problem 27 in the same three portfolios. What are the expected returns of the four individual assets and the three portfolios, if the current SML is plotting with an intercept of 3% (risk-free rate) and a market premium of 11% (slope of the line)?

**ANSWER**

Expected Return of Asset P = 3% + 0.6 (11%) = 9.6%
Expected Return of Asset Q = 3% + 0.85 (11%) = 12.35%
Expected Return of Asset R = 3% + 1.2 (11%) = 16.2%
Expected Return of Asset S = 3% + 1.35 (11%) = 17.85%
Expected Return of Portfolio 1 = 3% + 1.0 (11%) = 14%
Expected Return of Portfolio 2 = 3% + 0.895 (11%) = 12.845%
Expected Return of Portfolio 3 = 3% + 1.115 (11%) = 15.265%

29. **Changing risk level.** Mr. Malone wants to change the overall risk of his portfolio. Currently his portfolio is a combination of risky assets with a beta of 1.25 and an expected return of 14%. Mr. Malone will add a risk-free asset (U.S. Treasury bill) to his portfolio. If he wants a beta of 1.0, what percent of his wealth should be in the risky portfolio and what percent should be in the risk-free asset? If he wants a beta of 0.75? If he wants a beta of 0.50? If he wants a beta of 0.25? Is there a pattern here?

**ANSWER**

The weight in the risk-free asset is 1-w, and the weight in the risky portfolio is w and the total of the two reflects 1 or 100% of his wealth.

\[
\text{Beta of 1.0} = (1-w) \times (0) + (w) \times (1.25) \\
1.0 = w (1.25) \\
w = 1 / 1.25 = 0.80 \\
\]
Thus, 80% of wealth in the risky portfolio and 20% of wealth in risk-free asset.

\[
\text{Beta of 0.75} = (1-w) \times (0) + (w) \times (1.25) \\
0.75 = w (1.25) \\
w = 0.75 / 1.25 = 0.60 \\
\]
Thus, 60% of wealth is in the risky portfolio and 40% of wealth in risk-free asset.

\[
\text{Beta of 0.50} = (1-w) \times (0) + (w) \times (1.25) \\
0.50 = w (1.25) \\
w = 0.50 / 1.25 = 0.40 \\
\]
Thus, 40% of wealth in risky portfolio and 60% of wealth in risk-free asset.

\[
\text{Beta of 0.25} = (1-w) \times (0) + (w) \times (1.25) \\
0.25 = w (1.25) \\
w = 0.25 / 1.25 = 0.20 \\
\]
Thus, 20% of wealth in risky portfolio and 80% of wealth in risk-free asset.

The pattern is for every beta change of 0.25 Sam will need to switch 20% of his wealth out of the risky portfolio and into the risk-free asset. This constant ratio means that there is a linear relationship between portfolio weights and beta.
30. Changing risk level. Ms. Chambers wants to change the expected return of her portfolio. Currently Ms. Chambers has all her money in U.S. Treasury Bills with a return of 3%. She can switch some of her money into a risky portfolio with an expected return of 15%. What percent of her wealth will she need to invest in the risky portfolio to get an expected return of 5%? Of 7%? Of 9%? Of 11%? Of 13%? Of 15%? Is there a pattern here?

**ANSWER**

The weight in the risk-free asset is 1- w, and the weight in the risky portfolio is w and the total of the two reflects 1 or 100% of his wealth.

- **Expected Return 5% = (1-w) × (3%) + (w) × (15%)**
  
  \[0.05 = 1-w (0.03) + w (0.15)\]
  
  \[0.05 = 0.03 + w (0.15 - 0.03)\]
  
  \[0.05 – 0.03 = w (0.12)\]
  
  \[w = 0.02 / 0.12 = 0.1667 = 16.67\%\]

Thus 1/6 (16.67%) of the wealth is invested in the risky portfolio and 5/6 in the risk-free asset.

- **Expected Return 7% = (1-w) × (3%) + (w) × (15%)**
  
  \[0.07 = 1-w (0.03) + w (0.15)\]
  
  \[0.07 = 0.03 + w (0.15 - 0.03)\]
  
  \[0.07 – 0.03 = w (0.12)\]
  
  \[w = 0.04 / 0.12 = 0.3333 = 33.33\%\]

Thus 1/3 (33.33%) of the wealth is invested in the risky portfolio and 2/3 in the risk-free asset.

- **Expected Return 9% = (1-w) × (3%) + (w) × (15%)**
  
  \[0.09 = 1-w (0.03) + w (0.15)\]
  
  \[0.09 = 0.03 + w (0.15 – 0.03)\]
  
  \[0.09 – 0.03 = w (0.12)\]
  
  \[w = 0.06 / 0.12 = 0.50 = 50.0\%\]

Thus 1/2 (50.0%) of the wealth is invested in the risky portfolio and 1/2 in the risk-free asset.

- **Expected Return 11% = (1-w) × (3%) + (w) × (15%)**
  
  \[0.11 = 1-w (0.03) + w (0.15)\]
  
  \[0.11 = 0.03 + w (0.15 – 0.03)\]
  
  \[0.11 – 0.03 = w (0.12)\]
  
  \[w = 0.08 / 0.12 = 0.6667 = 66.67\%\]

Thus 2/3 (66.67%) of the wealth is invested in the risky portfolio and 1/3 in the risk-free asset.

- **Expected Return 13% = (1-w) × (3%) + (w) × (15%)**
  
  \[0.13 = 1-w (0.03) + w (0.15)\]
  
  \[0.13 = 0.03 + w (0.15 – 0.03)\]
  
  \[0.13 – 0.03 = w (0.12)\]
  
  \[w = 0.10 / 0.12 = 0.8333 = 83.33\%\]

Thus 5/6 (83.33%) of the wealth is invested in the risky portfolio and 1/6 in the risk-free asset.

- **Expected Return 15% = (1-w) × (3%) + (w) × (15%)**
\[
0.15 = 1-w (0.03) + w (0.15) \\
0.15 = 0.03 + w (0.15 - 0.03) \\
0.15 - 0.03 = w (0.12) \\
w = 0.12 / 0.12 = 1.0 = 100%
\]
Thus all (100%) of the wealth is invested in the risky portfolio and none in the risk-free asset.

The pattern is linear in the change in expected return and the percent invested in the risky portfolio.

31. **Reward-to-risk ratio.** Royal Seattle Investment Club has $100,000 to invest in the equity market. Frasier advocates investing the funds in KSEA Radio with a beta of 1.3 and an expected return of 16%. Niles advocates investing the funds in Northwest Medical with a beta of 1.1 and an expected return of 14%. The club is split 50/50 on the two stocks. You are the deciding vote, and you cannot pick a split of $50,000 for each stock. Before you vote, you look up the current risk-free rate (the 1 year U.S. Treasury bill with a yield of 3.75%). Which stock do you select?

**ANSWER**

You should pick the stock with the highest reward-to-risk ratio. To solve for reward-to-risk ratio take the risk-free asset with a beta of 0 (implied) and return of 3.75%, and the individual assets under consideration and find the slope of the line from the risk-free asset and the individual risky asset.

Slope for KSEA Radio = \((0.16 - 0.0375) / 1.3 = 0.1225 / 1.3 = 0.0942\) or 9.42%
Slope for NW Med = \((0.14 - 0.0375) / 1.1 = 0.1025 / 1.1 = 0.0932\) or 9.32%

While very close, the choice is KSEA Radio based on the higher reward-to-risk ratio.

32. **Reward-to-risk ratio.** Uptown Investment Club has $50,000 to invest in the equity market. Chandler advocates investing the funds in Monica’s restaurant with a beta of 1.8 and an expected return of 22%. Ross advocates investing the funds in Rachel’s clothing store with a beta of 0.9 and an expected return of 11%. The club is split 50/50 on the two stocks. You are the deciding vote, and you cannot pick a split of $25,000 for each stock. Before you vote, you look up the current risk-free rate (the 1 year U.S. Treasury bill with a yield of 2.45%). Which stock do you select?

**ANSWER**

You should pick the stock with the highest reward-to-risk ratio. To solve for reward-to-risk ratio take the risk-free asset with a beta of 0 (implied) and return of 2.45%, and the individual assets under consideration and find the slope of the line from the risk-free asset and the individual risky asset.

Slope for Monica’s restaurant = \((0.22 - 0.0245) / 1.8 = 0.1960 / 1.8 = 010.86\) or 10.86%
Slope for Rachel’s clothing store = \((0.11 - 0.0245) / 0.9 = 0.0860 / 0.9 = 095\) or 9.5%
The choice is Monica’s restaurant based on the higher reward-to-risk ratio.

33. **Different investor weights.** Two risky portfolios exist for investing: one is a bond portfolio with a beta of 0.5 and an expected return of 8%, and the other is an equity portfolio with a beta of 1.2 and an expected return of 15%. If these portfolios are the only two available assets for investing, what combination of these two assets will give the following investors their desired level of expected return? What are the betas of each investor’s combination of the bond and equity portfolio?

a. Bart: desired expected return 14%

b. Lisa: desired expected return 12%

c. Maggie: desired expected return 10%

**ANSWER**

a. Bart’s \( E(r) = 14\% = (w) \times 8\% + (1-w) 15\% \)

\[
14\% = w \times 8\% + 15\% - w \times 15\%
\]

\[
w \times 7\% = 15\% - 14\%
\]

\[
w = \frac{1\%}{7\%} = \frac{1}{7} = 0.1429 \text{ or } 14.29\%
\]

Bart should invest \( \frac{1}{7} \)th of his wealth in bonds and \( \frac{6}{7} \)th of his wealth in equity. Beta of Bart’s portfolio = \( \frac{1}{7} \times 0.5 + \frac{6}{7} \times 1.2 = 1.1 \)

b. Lisa \( E(r) = 12\% = (w) \times 8\% + (1-w) 15\% \)

\[
12\% = w \times 8\% + 15\% - w \times 15\%
\]

\[
w \times 7\% = 15\% - 12\%
\]

\[
w = \frac{3\%}{7\%} = \frac{3}{7} = 0.4286 \text{ or } 42.86\%
\]

Lisa should invest \( \frac{3}{7} \)th of her wealth in bonds and \( \frac{4}{7} \)th of her wealth in equity. Beta of Lisa’s portfolio = \( \frac{3}{7} \times 0.5 + \frac{4}{7} \times 1.2 = 0.9 \)

c. Maggie’s \( E(r) = 10\% = (w) \times 8\% + (1-w) 15\% \)

\[
10\% = w \times 8\% + 15\% - w \times 15\%
\]

\[
w \times 7\% = 15\% - 10\%
\]

\[
w = \frac{5\%}{7\%} = \frac{5}{7} = 0.7142857 \text{ or } 71.43\%
\]

Maggie should invest \( \frac{5}{7} \)th of her wealth in bonds and \( \frac{2}{7} \)th of her wealth in equity. Beta of Maggie’s portfolio = \( \frac{5}{7} \times 0.5 + \frac{2}{7} \times 1.2 = 0.7 \)
34. Different investor weights. Two risky portfolios exist for investing: one is a bond portfolio with a beta of 0.7 and an expected return of 9%, and another is an equity portfolio with a beta of 1.5 and an expected return of 17%. If these portfolios are the only two available assets for investing, what combination of these two assets will give the following investors their desired level of expected return? What are the betas of each investor’s combination of the bond and equity portfolio?

a. Jerry: desired expected return 16%

b. Elaine: desired expected return 13%

c. Cosmo: desired expected return 10%

**ANSWER**

a. Jerry’s E(r) = 16% = (w) × 9% + (1-w) 17%

16% = w × 9% + 17% - w × 17%

w × 8% = 17% - 16%

w = 1% / 8% = 1/8 = 0.125 or 12.5%

Jerry should invest 1/8th of his wealth in bonds and 7/8th of his wealth in equity. Beta of Jerry’s portfolio = 1/8 × 0.7 + 7/8 × 1.5 = 1.4

b. Elaine’s E(r) = 13% = (w) × 9% + (1-w) 17%

13% = w × 9% + 17% - w × 17%

w × 8% = 17% - 13%

w = 4% / 8% = 4/8 = 0.50 or 50%

Elaine should invest 1/2 of his wealth in bonds and 1/2 of his wealth in equity. Beta of Elaine’s portfolio = 1/2 × 0.7 + 1/2 × 1.5 = 1.1

c. Cosmo’s E(r) = 10% = (w) × 9% + (1-w) 17%

10% = w × 9% + 17% - w × 17%

w × 6% = 17% - 10%

w = 7% / 8% = 7/8 = 0.875 or 87.5%

Cosmo should invest 7/8th of his wealth in bonds and 1/8th of his wealth in equity. Beta of Cosmo’s portfolio = 7/8 × 0.7 + 1/8 × 1.5 = 0.8

Solutions to Advanced Problems for Spreadsheet Application

1. Returns and variances in the period 2000-2009

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>5.8640%</td>
<td>5.8650%</td>
<td>-10.6822%</td>
<td>-4.2041%</td>
<td>146.88</td>
<td>504.75</td>
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<tr>
<td>2000</td>
<td>1.7400%</td>
<td>4.2200%</td>
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<td>131.19</td>
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<td>-22.8084%</td>
<td>-21.5783%</td>
<td>114.3</td>
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<td>4.3650%</td>
<td>26.1249%</td>
<td>45.3731%</td>
<td>111.28</td>
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<td>2003</td>
<td>2.2690%</td>
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<tr>
<td>2006</td>
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<td>146.21</td>
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<td>2.6700%</td>
<td>-38.2806%</td>
<td>-34.8002%</td>
<td>90.24</td>
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<tr>
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<td>625.39</td>
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<tr>
<td>2009</td>
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<td>23.4929%</td>
<td>25.2157%</td>
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AVERAGE

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<td>2.4750%</td>
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VARIANCE

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<td>0.0420%</td>
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<td>4.1983%</td>
<td>5.3094%</td>
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STD. DEV.

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<td>23.0420%</td>
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2. Portfolio of assets with expected returns.

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<td>30.00%</td>
<td>24.00%</td>
<td>15.00%</td>
<td>5.00%</td>
<td>-20.00%</td>
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<tr>
<td>Normal</td>
<td>45.00%</td>
<td>15.00%</td>
<td>12.00%</td>
<td>12.00%</td>
<td>9.00%</td>
<td>2.00%</td>
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<tr>
<td>Recession</td>
<td>20.00%</td>
<td>5.00%</td>
<td>0.00%</td>
<td>6.00%</td>
<td>14.00%</td>
<td>10.00%</td>
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<tr>
<td>Bust</td>
<td>10.00%</td>
<td>-35.00%</td>
<td>-20.00%</td>
<td>2.00%</td>
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<tr>
<td>Expected Return</td>
<td></td>
<td>11.75%</td>
<td>9.40%</td>
<td>10.55%</td>
<td>10.10%</td>
<td>0.90%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td>17.77%</td>
<td>12.67%</td>
<td>4.17%</td>
<td>4.46%</td>
<td>14.53%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Portfolio Weights</th>
<th>Percent In A</th>
<th>Percent In B</th>
<th>Percent In C</th>
<th>Percent In D</th>
<th>Percent In E</th>
<th>Total Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>20.0%</td>
<td>20.0%</td>
<td>20.0%</td>
<td>20.0%</td>
<td>20.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Two</td>
<td>35.0%</td>
<td>30.0%</td>
<td>20.0%</td>
<td>10.0%</td>
<td>5.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Three</td>
<td>20.0%</td>
<td>30.0%</td>
<td>30.0%</td>
<td>10.0%</td>
<td>10.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Four</td>
<td>10.0%</td>
<td>15.0%</td>
<td>25.0%</td>
<td>35.0%</td>
<td>15.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expect Returns</th>
<th>Return of Portfolio in Each State</th>
<th>Portfolio One</th>
<th>Portfolio Two</th>
<th>Portfolio Three</th>
<th>Portfolio Four</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio One</td>
<td>8.54%</td>
<td>Boom</td>
<td>10.800%</td>
<td>20.200%</td>
<td>16.200%</td>
</tr>
<tr>
<td>Portfolio Two</td>
<td>10.10%</td>
<td>Normal</td>
<td>10.000%</td>
<td>12.250%</td>
<td>11.300%</td>
</tr>
<tr>
<td>Portfolio Three</td>
<td>9.44%</td>
<td>Recession</td>
<td>7.000%</td>
<td>4.850%</td>
<td>5.200%</td>
</tr>
<tr>
<td>Portfolio Four</td>
<td>8.89%</td>
<td>Bust</td>
<td>-0.600%</td>
<td>-14.350%</td>
<td>-7.400%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Deviations</th>
<th>Weighted Average</th>
<th>Standard Dev</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio One</td>
<td>3.326%</td>
<td>8.704%</td>
<td>5.377%</td>
</tr>
<tr>
<td>Portfolio Two</td>
<td>9.637%</td>
<td>11.521%</td>
<td>1.883%</td>
</tr>
<tr>
<td>Portfolio Three</td>
<td>6.703%</td>
<td>9.495%</td>
<td>2.792%</td>
</tr>
<tr>
<td>Portfolio Four</td>
<td>1.241%</td>
<td>6.948%</td>
<td>5.707%</td>
</tr>
</tbody>
</table>

Solutions to Mini-Case

Lawrence’s Legacy: Part 2

This mini-case provides a comprehensive review of risk and return including mean variance computations, Capital Asset Pricing Model and capital market history.

1. How do we measure the returns on our portfolio?

Kraska will answer this question by assuming that a $1,000,000 portfolio in a given year earns $30,000 in dividends and either gains or loses $100,000 in market value. Compute the HPR in dollars and percentage terms. Compute the APR and EAR. Be prepared to answer a follow-up question about the value of the portfolio after 5% has been distributed in grants. Kraska will also compute the annualized return (EAR) earned by Lawrence on his investment in Google to illustrate a multi-year perspective. Lawrence purchased the Google stock for $200,000 and held it for three years before he died.
Given the purpose of the portfolio, to fund annual grants, it is convenient to think in terms of a 1 year holding period return which includes both distributions and capital gains or losses. If the portfolio gains $100,000, the dollar HPR is $130,000, the percentage HPR is

\[
\frac{($30,000 + $100,000)}{$1,000,000} = 13\%.
\]

If it loses $100,000, the dollar HPR is \((- $100,000 + $30,000) = $(70,000), the percentage HPR is:

\[
\frac{($(70,000) )}{$1,000,000} = -7\%.
\]

With a 1 year holding period, HPR, APR, and EAR are all the same.

The amount of the grants will be either $1,130,000 \times 0.05 = $56,500, leaving $1,073,500 or $930,000 \times 0.05 = $46,500, leaving $883,500.

To illustrate how to compute annualized returns (EAR) over a longer period, we can look at how Lawrence did on his investment in Google.

He paid $200,000 for the stock in 2004. Three years later it was worth $1,250,000. ($1,250,000/$200,000 )\(^{1/3}\) -1 = or 173.80% per year! Such opportunities do not come along every day, but they do explain the appeal of investing in stock.

2. How can we assess the risk of an individual stock?

In financial terms, an investment is risky if the outcome is uncertain and some possible outcomes are unfavorable. We can understand this better by looking at some examples.

a. Kraska will first address this question by looking at recent returns on Amazon and on Coca Cola. Compute the mean and standard deviation for each and explain what they mean. He has collected the following data:

<table>
<thead>
<tr>
<th>Year</th>
<th>AMAZON</th>
<th>COKE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>134.77%</td>
<td>33.35%</td>
</tr>
<tr>
<td>2006</td>
<td>-16.31%</td>
<td>26.35%</td>
</tr>
<tr>
<td>2005</td>
<td>6.46%</td>
<td>2.24%</td>
</tr>
<tr>
<td>2004</td>
<td>-15.83%</td>
<td>13.93%</td>
</tr>
<tr>
<td>2003</td>
<td>178.56%</td>
<td>21.94%</td>
</tr>
</tbody>
</table>

The average return is the sum of the returns shown above divided by 5.

For Amazon 287.65/5 = 57.53%

For Coca Cola 69.95/5 = 13.99%

Amazon’s variance is

\[
\frac{[(134.77-57.53)^2+(-16.31-57.53)^2+(6.46-57.53)^2+(-15.83-57.53)^2+(178.56-57.53)^2]}{(5-1)}=8,514.17 \text{ and the standard deviation is } (8,514.17)^{1/2}=97.27%.
\]
Coca Cola’s variance is
and the standard deviation is \( (377.25)^{1/2}=19.42\% \)

It is obvious that the average return on Amazon has been much higher, but that year to year returns are widely scattered and quite unpredictable. Investors who held the stock through 2004 or 2006 would have lost money. The average return on Coca Cola was lower, but there were no negative years, which is not to say that there could not be, and returns are clustered within a much tighter range.

The standard deviation implies that about 2/3 of the time, returns on Amazon would be somewhere between 57.53%+97.27% =154.76% and 57.53% – 97.27% =-39.70%. In other words, one would not be very sure about very much.

For Coke, 2/3 of the returns should be between 13.99%+19.42%=33.41% and 13.99% – 19.42%=-5.43%.

The sample is very small, and past returns have no causal effect on future returns, so the numbers should not be taken too seriously, but they do clearly show the relationship between uncertainty, risk and returns.

b. Kraska will also suggest that is good to assess risk by looking forward to how we expect stocks to react to a particular set of circumstances or states of nature. Use the following set of assumptions for the coming year to compute the expected rate of return and standard deviation for Amazon, Coca Cola, and a portfolio with equal dollar amounts invested in Amazon and Coca Cola. Explain briefly what they mean.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State</th>
<th>Conditional Return</th>
<th>Conditional Return</th>
<th>Conditional Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>30.00%</td>
<td>-25.00%</td>
<td>5.00%</td>
<td>-10.00%</td>
</tr>
<tr>
<td>Average</td>
<td>50.00%</td>
<td>30.00%</td>
<td>12.00%</td>
<td>21.00%</td>
</tr>
<tr>
<td>Boom</td>
<td>20.00%</td>
<td>50.00%</td>
<td>20.00%</td>
<td>35.00%</td>
</tr>
</tbody>
</table>

**Amazon**
Expected return = \( .30(-25)+.50(30)+.20(50)=17.5\% \)
Standard Deviation = \[ .30(-25-17.5)^2+.50(30-17.5)^2+.20(50-17.5)^2 \] \( \frac{1}{2} = 28.83\% \)

**Coca Cola**
Expected return = \( .30(5)+.50(12)+.20(20)=11.5\% \)
Standard Deviation = \[ .30(5-11.5)^2+.50(12-11.5)^2+.20(20-11.5)^2 \] \( \frac{1}{2} = 5.22\% \)

**Portfolio**
Expected return = \( .30(-10)+.50(21)+.20(35)=14.5\% \)
Standard Deviation = \([.30(−10−14.5)^2+.50(21−14.5)^2+.20(32.50−14.5)^2]^{1/2} = 16.89\%\)

The mean and standard deviation can be interpreted in the same way as for the sample data in question a. Note that in this exercise both the probabilities and the returns are somewhat subjective. Thus, the lower expected rate of return for Amazon reflects our judgment that, even though they happened in the past, triple digit returns are unlikely in the future because they imply growth rates that simply are not sustainable over the long run. We also assigned a higher probability to a recession than to a boom economy.

Kraska will point out that the expected return on the portfolio is exactly the weighted average of the returns on the two stocks, but the standard deviation (16.89\% is less than the average of the standard deviations of the two stocks (17.03\% \(\Rightarrow\frac{(28.83\%+5.22\%)}{2}\)). This means that combining stocks in a portfolio offers some reduction in risk without reducing expected returns.

3. **What kinds of investments are safe and earn a high rate of return?**

Kraska responds that unfortunately there is no such thing. If there were, everyone would want to buy it; demand would drive the price up, and returns would soon drop to expected rates.

4. **Google seems to be a great company. Why did Lawrence require the town to sell the Google stocks and reinvest the money in a diversified portfolio?**

Lawrence was reluctant to sell Google because taxes on his profits would have reduced the amount he was able to donate, but he knew that it is wise not to put all our eggs in one basket. Even the best companies run the risks of competition, technological change, loss of key personnel, adverse legal rulings on intellectual property rights, and many others. These types of risk are known as unsystematic risk because they arise from factors within the company or the industry, rather than overall market conditions. At best, it is unlikely that Google’s value could continue to increase at its present pace for very much longer.

5. **How many stocks should we have in our portfolio?**

Kraska would explain that returns on most stocks, like Amazon and Coca Cola, are positively correlated, but not perfectly. By combining 20 to 30 stocks, most of the unsystematic risk has been removed and the benefits of diversification nearly exhausted. It is important to know that systematic risk, the risk that cannot be avoided by diversification, is by no means unimportant. Over a period of many years, returns on well diversified portfolios of large company stocks have averaged around 12\% with a standard deviation of 20\%. We should not expect to match these results each and every year, but if we can do it over the long run, we should be able to distribute 5\% of the portfolio each year in grants, while the portfolio and the grants will grow at the rate of 7\% per year.

6. **How much risk will the portfolio carry?**

a. Kraska will answer this question by explaining the Capital Asset Pricing Model in the most straightforward terms possible.
We can see from the previous questions that the relevant risk of a single stock is not so much how it behaves by itself, but how it will affect a well-diversified portfolio. We should expect no additional return for bearing risk that everyone knows how to avoid. We only earn higher rates of return for bearing unavoidable risk. Professional investors use a statistic called beta to estimate how sensitive, on average, returns on a particular stock are to the returns on the market as a whole (often represented by a stock index such as the S&P 500.) For the quantitative people on the committee, beta is the slope of the regression line when returns on a particular stock are regressed against market returns. By definition, the beta of the market is 1 (the market does what the market does). If we bought the whole market, we would expect to receive the risk free rate plus a premium for bearing average risk. A stock with a beta of 2 has double the average risk, while a stock with a beta of .5 has half the average risk. Beta allows us to compute the required rate of return on any stock, or other investment using a very simple formula

Expected rate of return on stock “i” = the risk-free rate + beta (market risk premium). The market risk premium is simply the difference between the expected return on the market and the risk-free rate.

b. He will illustrate how we use CAPM to compute the expected rate of return on a stock. Use an expected market return of 12%, a risk free rate of 5%, and the betas for Amazon (3.02), Coca Cola (.62), and Merck Pharmaceuticals (1.11) to compute the expected rate of return on these stocks.

\[ E(r_{amazon}) = 5\% + 3.02(12\% - 5\%) = 26.14\% \]
\[ E(r_{coke}) = 5\% + 0.62(12\% - 5\%) = 9.34\% \]
\[ E(r_{merck}) = 5\% + 1.11(12\% - 5\%) = 12.77\% \]

c. He will illustrate the concept of portfolio beta using the same three stocks. Compute the beta for a portfolio composed of $20,000 invested in Amazon, $50,000 in Coca Cola, and $35,000 in Merck.

The beta of a portfolio is just the weighted average of the betas of the stocks in the portfolio, so

<table>
<thead>
<tr>
<th>stock</th>
<th>weight</th>
<th>beta</th>
<th>weight × beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon</td>
<td>20/105</td>
<td>3.02</td>
<td>0.58</td>
</tr>
<tr>
<td>Coke</td>
<td>50/105</td>
<td>0.62</td>
<td>0.30</td>
</tr>
<tr>
<td>Merck</td>
<td>35/105</td>
<td>1.11</td>
<td>0.37</td>
</tr>
</tbody>
</table>

\[ \text{Portfolio beta} = 1.24 \]

Kraska would attempt to keep the portfolio beta close to 1.

Additional Problems with Solutions

1. **Comparing HPRs, APRs and EARs.** Two years ago, Jim bought 100 shares of IBM stock at $50 per share, and just sold them for $65 per share after receiving dividends worth $3 per share over the two year holding period. Mary, bought 5 ounces of gold
at $800 per ounce, three months ago, and just sold it for $1000 per ounce. Calculate each investor’s HPR, APR, and EAR and comment on your findings.

**ANSWER**

<table>
<thead>
<tr>
<th>Jim’s holding period (n)</th>
<th>= 2 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim’s HPR</td>
<td>$(\text{Selling Price} + \text{Distributions-Purchase Price})/\text{Purchase Price} = [$65(100) + $3(100) - $50(100)]/$50(100) = [$6500 + $300 - $5000]/$5000 = $1800/$5000 = 36%</td>
</tr>
<tr>
<td>Jim’s APR</td>
<td>$\text{HPR}/n = 36%/2 = 18%</td>
</tr>
<tr>
<td>Jim’s EAR</td>
<td>$\left(1 + \text{HPR}\right)^{1/n} - 1 = (1.36)^{1/2} - 1 = 16.62%</td>
</tr>
</tbody>
</table>

Mary’s holding period = 3/12 = 0.25 of a year

| Mary’s HPR | $(\text{Selling Price} - \text{Purchase Price})/\text{Purchase Price} = ($1000 \times 5 - \$800 \times 5)/\$800 \times 5 = ($5000 - \$4000)/\$4000 = \$1000/\$4000 = 25\% |
| Mary’s APR | $\text{HPR}/n = 25\%/0.25 = 100\% |
| Mary’s EAR | $\left(1 + \text{HPR}\right)^{1/n} - 1 = (1.25)^{1/0.25} - 1 = 144.14\% |

Clearly, Mary had a higher HPR, APR, and EAR than Jim. However, the APR and HPR seem unrealistic because of her short holding period. It implies that Mary would make 3 additional trades of 25\% profit over the next 3 quarters.

2. **Calculate ex-post risk measures.** Listed below are the annual rates of return earned on Stock X and Stock Y over the past 6 years. Which stock was riskier and why?

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock X</th>
<th>Stock Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>20%</td>
<td>16%</td>
</tr>
<tr>
<td>2005</td>
<td>15%</td>
<td>17%</td>
</tr>
<tr>
<td>2006</td>
<td>-10%</td>
<td>20%</td>
</tr>
<tr>
<td>2007</td>
<td>30%</td>
<td>24%</td>
</tr>
<tr>
<td>2008</td>
<td>25%</td>
<td>23%</td>
</tr>
<tr>
<td>2009</td>
<td>14%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

**ANSWER**

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock X</th>
<th>Stock Y</th>
<th>(X-Mean)$^2$</th>
<th>(Y-Mean)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>20%</td>
<td>16%</td>
<td>0.001877778</td>
<td>0.0001</td>
</tr>
<tr>
<td>2005</td>
<td>15%</td>
<td>17%</td>
<td>4.44444E-05</td>
<td>0.0004</td>
</tr>
<tr>
<td>2006</td>
<td>-10%</td>
<td>20%</td>
<td>0.065877778</td>
<td>0.0025</td>
</tr>
<tr>
<td>2007</td>
<td>30%</td>
<td>24%</td>
<td>0.020544444</td>
<td>0.0081</td>
</tr>
<tr>
<td>2008</td>
<td>25%</td>
<td>23%</td>
<td>0.008711111</td>
<td>0.0064</td>
</tr>
<tr>
<td>2009</td>
<td>14%</td>
<td>-10%</td>
<td>0.000277778</td>
<td>0.0625</td>
</tr>
<tr>
<td>Average</td>
<td>16%</td>
<td>15%</td>
<td>0.019466667</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Variance 13.95\% 12.65\% Std. dev.
We calculate each stock’s average return, variance, and standard deviation over the past 6 years and compare their risk per unit of return i.e. $\sigma$/Average

<table>
<thead>
<tr>
<th></th>
<th>Stock X</th>
<th>Stock Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return</td>
<td>16%</td>
<td>15%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>13.95%</td>
<td>12.65%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.8718%</td>
<td>0.8433%</td>
</tr>
</tbody>
</table>

Stock $X$ was riskier than Stock $Y$ since it had the higher Standard Deviation of the two, and its average return was not much higher than Stock $Y$’s average return resulting in $0.872 \%$ risk per unit of return versus Stock $Y$’s $0.843 \%$ risk per unit of return.

3. **Calculating ex-ante risk and return measures.** Using the probability distribution shown below, calculate the expected risk and return estimates of each stock and of a portfolio comprised of 40% of Stock A and 60% of Stock B.

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State occurring</th>
<th>Stock A's Conditional return</th>
<th>Stock B's Conditional return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.3</td>
<td>-12%</td>
<td>20%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.5</td>
<td>14%</td>
<td>12%</td>
</tr>
<tr>
<td>Boom</td>
<td>0.2</td>
<td>25%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

**ANSWER** (Slides 8-67 to 8-69)

Stock A’s expected return $= 0.3 \times (-12\%) + 0.5 \times (14\%) + 0.2 \times (25\%) = 8.4\%$

Stock B’s expected return $= 0.3 \times (20\%) + 0.5 \times (12\%) + 0.2 \times (-10\%) = 10\%$

Stock A’s expected variance $= 0.3 \times (-12-8.4)^2 + 0.5 \times (14-8.4)^2 + 0.2 \times (25-8.4)^2$

$= 124.848 + 15.68 + 55.112$

$= 195.64$

Stock A’s expected std. dev. $= \sqrt{195.64} = 13.99\%$

Stock B’s expected variance $= 0.3 \times (20-10)^2 + 0.5 \times (12-10)^2 + 0.2 \times (-10-10)^2$

$= 30 + 2 + 80$

$= 112$

Stock B’s expected std. dev. $= \sqrt{112} = 10.58\%$

Portfolio AB’s expected return $= \text{Wt. in A} \times E(R_A) + \text{Wt. in B} \times E(R_B)$

$= .4 \times 8.4\% + .6 \times 10\% = 9.36\%$

OR

We can calculate the portfolio’s conditional returns and then compute the expected return and standard deviation/variance.

Portfolio AB’s recession return $= .4 \times (-12) + .6 \times (20) = 7.2\%$

Portfolio AB’s normal return $= .4 \times (14) + .6 \times (12) = 12.8\%$

Portfolio AB’s boom return $= .4 \times (25) + .6 \times (-10) = 4\%$

Portfolio AB’s expected return $= 0.3 \times 7.2 + 0.5 \times 12.8 + 0.2 \times 4 = 9.36\%$

Portfolio AB’s expected variance $= 0.3 \times (7.2-9.36)^2 + 0.5 \times (12.8-9.36)^2 + 0.2 \times (4-9.36)^2$

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\[ = 1.39968 + 5.9168 + 5.74592 = 13.0624 \]

Portfolio AB’s expected std. dev. \( = \sqrt{13.0624} = 3.61\% \)

4. **Calculate a portfolio’s expected rate of return using the CAPM.** Annie is curious to know what her portfolio’s CAPM-based expected rate of return should be. After doing some research she figures out the market values and betas of each of her 5 stocks (shown below) and is told by her consultant that the risk-free rate is 3% and the market risk premium is 8%. Help Annie calculate her portfolio’s expected rate of return.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Value</th>
<th>Weight</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$35,000</td>
<td>0.1400</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>$40,000</td>
<td>0.1600</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>$45,000</td>
<td>0.1800</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>$50,000</td>
<td>0.2000</td>
<td>-0.8</td>
</tr>
<tr>
<td>5</td>
<td>$80,000</td>
<td>0.3200</td>
<td>0.8</td>
</tr>
</tbody>
</table>

First determine the portfolio beta using the following formula:

\[ \beta_p = \sum_{i=1}^{n} w_i \times \beta_i \]

8.10

Portfolio Beta = \(0.14 \times 1.6 + 0.16 \times 1.2 + 0.18 \times 1.0 + (-0.8) \times 0.2 + 0.32 \times 0.8\)

\(= 0.224 + 0.192 + 0.18 + (-1.6) + 0.256 = 0.692\)

Next, using the CAPM equation and \(r_f = 3\%\), \(E(r_m-r_f) = 8\%\); calculate the portfolio’s expected rate.

\(E(r_p) = 3\% + 8\% \times (0.692) = 8.54\%\)

5. **Applying the CAPM to determine market attractiveness.**

a. Annie is curious to know whether the following 5 stocks are appropriately valued in the market. Accordingly, she creates a table (shown below) listing the betas of each stock along with their ex-ante expected return values that have been calculated using a probability distribution. She also lists the current risk-free rate and the expected rate of return on the broad market index. Help her out and state your steps.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Expected Return</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22%</td>
<td>1.8</td>
</tr>
<tr>
<td>2</td>
<td>8%</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>14%</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>1.1</td>
</tr>
<tr>
<td>5</td>
<td>16%</td>
<td>1.4</td>
</tr>
<tr>
<td>(R_f)</td>
<td>3.5%</td>
<td>----</td>
</tr>
</tbody>
</table>

ANSWER (a)  
(Slides 8-73 to 8-76)

**Step 1.** Using the CAPM equation calculate the risk-based return of each stock

<table>
<thead>
<tr>
<th>Stock</th>
<th>Expected Return</th>
<th>Beta</th>
<th>CAPM E(R)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26%</td>
<td>1.8</td>
<td>24.20%</td>
<td>Undervalued</td>
</tr>
<tr>
<td>2</td>
<td>16%</td>
<td>0.9</td>
<td>13.85%</td>
<td>Undervalued</td>
</tr>
<tr>
<td>3</td>
<td>14%</td>
<td>1.2</td>
<td>17.30%</td>
<td>Overvalued</td>
</tr>
<tr>
<td>4</td>
<td>16.15%</td>
<td>1.1</td>
<td>16.15%</td>
<td>Correctly valued</td>
</tr>
<tr>
<td>5</td>
<td>20%</td>
<td>1.4</td>
<td>19.60%</td>
<td>Undervalued</td>
</tr>
<tr>
<td>R_f</td>
<td>3.50%</td>
<td>----</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>R_m</td>
<td>15%</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2.** If CAPM-based E(R) is less than the ex-ante return listed, the stock is undervalued, i.e. it is expected to earn a higher rate than it should, based on its beta. Hence, Stocks 1, 2, and 5 are undervalued, while Stock3 is overvalued, and Stock 4 is correctly valued.

b. If Annie wants to form a 2-stock portfolio of the most undervalued stocks with a beta of 1.3, how much will she have to weight each of the stocks by?

ANSWER (b)

Based on the results in (A), Stocks 1 and 2 are most undervalued and would be chosen by Annie to form the 2-stock portfolio with a beta = 1.3.

Stock 1’s beta = 1.8; Stock 2’s beta = 0.9; Desired Portfolio beta = 1.3

Since the portfolio beta = weighted average of individual stock betas

Let Stock 1’s Weight be X%; Thus Stock 2’s Weight would be (1-X)%

\[ 1.8 \times X\% + 0.9 \times (1 - X)\% = 1.3 \]
\[ 1.8X + 0.9 - 0.9X = 1.3 \]
\[ 0.9X = 0.4 \]
\[ X = 0.4/0.9 = 0.4444 \text{ or } 44.44\% = \text{Stock 1’s Weight} \]
\[ 1 - X = 0.5556 \text{ or } 55.56\% = \text{Stock 2’s Weight} \]

Check…\(0.4444 \times 1.8 + 0.5556 \times 0.9 = 0.79992+ 0.50004=1.3\)