Chapter 6
Bonds and Bond Valuation

LEARNING OBJECTIVES (Slide 6-2)

1. Understand basic bond terminology and apply the time value of money equation in pricing bonds.
2. Understand the difference between annual and semiannual bonds and note the key features of zero-coupon bonds.
3. Explain the relationship between the coupon rate and the yield to maturity.
5. Appreciate bond history and understand the rights and obligations of buyers and sellers of bonds.

IN A NUTSHELL...

Bonds are debt instruments issued by corporations, as well as state, local, and foreign governments to raise funds for growth and financing of public projects. In the six sections within this chapter, the author defines and explains the terminology and methodology used in the analysis and pricing of bonds; differentiates between annual, semi-annual, and zero-coupon bonds; explains how a bond’s coupon rate and yield to maturity are related; clarifies how bond ratings affect their prices; provides some perspective on bond history and the rights and obligations of buyers and sellers of bonds; and shows how treasury bonds, notes, and bills are quoted and priced. The quantitative material in this chapter represents the first practical application of time value techniques (covered in Chapters 3 and 4) in a corporate finance setting and must be well understood by students so as to grasp the important forthcoming topics of cost of capital and capital budgeting.

LECTURE OUTLINE

6.1 Application of the Time Value of Money Tool: Bond Pricing (Slides 6-3 and 6-4)

Since bonds are typically long-term debt instruments which provide periodic interest income along with a return of the principal amount at maturity, their prices can be calculated by using present value techniques i.e. discounting of future cash flows.

6.1 (A) Key Components of a Bond (See Fig. 6.1: Merrill Lynch corporate bond) (Slides 6-5 to 6-6)

*Par value:* The principal or face value of a bond on which interest is paid, typically $1000;
**Coupon rate:** Annual rate of interest paid by issuer.

**Coupon:** The regular interest payment received by buyer. It is calculated as the product of the coupon rate and the par value (and divided by 2, if semi-annual).

**Maturity date:** The expiration date of the bond on which the final coupon and the principal value is paid by the issuer.

**Yield to maturity:** The discount rate or expected rate of return on a bond which is used to determine its price.

### Example 1: Key components of a corporate bond

Let’s say you see the following price quote for a corporate bond

<table>
<thead>
<tr>
<th>Issue</th>
<th>Price</th>
<th>Coupon (%)</th>
<th>Maturity</th>
<th>YTM%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hertz Corp.</td>
<td>91.50</td>
<td>6.35</td>
<td>15-Jun-2010</td>
<td>15.438</td>
</tr>
</tbody>
</table>

This B-rated bond issued by Hertz Corporation is selling at 91.5% of par value, i.e. $915 based on a face value of $1,000. Based on its coupon rate of 6.35%, it will pay $63.50 in coupon interest each year until it matures on June 15, 2010. Based on its price, i.e. $915, an investor is earning a current yield of 6.94% ($63.5/$915) per year and if held to maturity will have earned a yield of 15.438%.

### 6.1 (B) Pricing a Bond in Steps

(Slides 6-7 to 6-10)

Since bonds involve a combination of an annuity (coupons) and a lump sum (par value) its price is best calculated by using the following steps:

1. Lay out the cash flows on a time line;
2. Determine an appropriate discount rate;
3. Calculate the present value of the coupons and the par value;
4. Add up the two present values to calculate the bond price.

(See Figure 6.2: How to price a bond)

### Example 2: Calculating the price of a corporate bond

Calculate the price of an AA-rated, 20-year, 8% coupon (paid annually) corporate bond (Par value = $1,000) which is expected to earn a yield to maturity of 10%.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$80</td>
<td>$80</td>
<td>$80</td>
<td></td>
<td>$80</td>
<td>$80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Method 1: Using TVM equation**

Annual coupon = Coupon rate * Par value = .08 * $1,000 = $80 = PMT
YTM = r = 10%
Maturity = n = 20
Price of bond = Present Value of coupons + Present Value of par value

\[
\text{Present Value of annual coupons} = PMT \times \frac{1 - \frac{1}{(1+r)^n}}{r}
\]

\[
\text{Present Value annual coupons} = $80 \times \frac{1 - \frac{1}{(1+0.10)^{20}}}{0.10}
\]

\[
\text{Present Value of annual coupons} = $80 \times 8.51359 = $681.09
\]

\[
\text{Present Value of Par Value} = \frac{1}{(1+r)^n}
\]

\[
\text{Present Value of Par Value} = $1,000 \times \frac{1}{(1+0.10)^{20}}
\]

\[
\text{Present Value of Par Value} = $1,000 \times 0.14864 = $148.64
\]

Price of bond = $681.09 + $148.64 = $829.73

**Method 2: Using a financial calculator**

Mode: P/Y = 1; C/Y = 1

Input:
\[
N \quad I/Y \quad PV \quad PMT \quad FV
\]
Key: 20 10 ? 80 1000

Output: -829.73

### 6.2 Semiannual Bonds and Zero-Coupon Bonds (Slides 6-11 to 6-14)

Most corporate and government bonds pay coupons on a semiannual basis. Additionally, some companies issue zero-coupon bonds by selling them at a deep discount. To calculate the price of these bonds, the values of the inputs have to be adjusted according to the frequency of the coupons (or absence thereof). For example, for semi-annual bonds, the annual coupon is divided by 2, the number of years is multiplied by 2, and the YTM is divided by 2. The price of the bond can then be calculated by using the TVM equation, a financial calculator, or a spreadsheet.
6.2 (A) Pricing Bonds after Original Issue

The price of a bond is a function of the remaining cash flows (i.e. coupons and par value) that would be paid on it until expiration.

**Example 3: Pricing a semi-annual coupon bond after original issue**

Four years ago, the XYZ Corporation issued an 8% coupon (paid semi-annually), 20-year, AA-rated bond at its par value of $1000. Currently, the yield to maturity on these bonds is 10%. Calculate the price of the bond today.

Remaining number of semi-annual coupons = (20-4)*2 = 32 coupons = \( n \)

Semi-annual coupon = \((0.08*1000)/2 = $40\)

Par value = $1000

Annual YTM = 10% \( \Rightarrow \) YTM/2 \( \Rightarrow \) 5% = \( r \)

**Method 1: Using TVM equations**

\[
\text{Bond Price} = \text{Par Value} \times \frac{1}{(1+r)^n} + \text{Coupon} \times \left( \frac{1}{r} \right)^n
\]

\[
\text{Bond Price} = $1000 \times \frac{1}{(1+0.05)^{32}} + $40 \times \frac{1}{0.05^{32}}
\]

Bond Price = $1000 \times 0.209866 + $40 \times 15.80268

Bond Price = $209.866 + $632.107

Bond Price = $841.97

**Method 2: Using a financial calculator**

Mode: P/Y=2; C/Y = 2

Input:
\[
N \quad I/Y \quad PV \quad PMT \quad FV
\]

Key:
\[
32 \quad 10 \quad ? \quad 40 \quad 1000
\]

Output: -841.97

6.2 (B) Zero-Coupon Bonds

Also known as “pure” discount bonds, zero-coupon bonds are sold at a discount from face value and do not pay any interest over the life of the bond. At maturity, the investor receives the par value, usually $1000. The price of a zero-coupon bond is calculated by merely discounting its par value at the prevailing discount rate or yield to maturity.
6.2 (C) Amortization of a Zero-Coupon Bond.  
(Slides 6-20 to 6-24)

Table 6.2 (page 152) demonstrates how the discount on a zero-coupon bond is amortized over its life. The price appreciation is calculated for each six-month period by multiplying the zero-coupon bond’s beginning price by its semi-annual YTM, and represents the interest earned on the bond. Zero-coupon bond investors are taxed on the annual price appreciation, even though no cash is received from the issuing firm.

Example 4: Price of and taxes due on a zero-coupon bond

John wants to buy a 20-year, AAA-rated, $1000 par value, zero-coupon bond being sold by Diversified Industries Inc. The yield to maturity on similar bonds is estimated to be 9%.

A) How much would he have to pay for it?

Method 1: Using TVM equation

\[ Bond \ Price = \frac{Par \ Value}{(1+r)^n} \]

\[ Bond \ Price = \frac{1,000}{(1 + 0.045)^{40}} \]

Bond Price = $1000 * .1719287 = $171.93

Method 2: Using a financial calculator

Mode: P/Y=2; C/Y = 2
Input: N I/Y PV PMT FV
Key: 40 9 ? 0 1000
Output -171.93

B) How much will he be taxed on the investment after 1 year, if his marginal tax rate is 30%?

Calculate the price of the bond at the end of 1 year.

Mode: P/Y=2; C/Y = 2
Input: N I/Y PV PMT FV
Key: 38 9 ? 0 1000
Output -187.75

Taxable income = $187.75 – $171.93 = $15.82

Taxes due = Tax rate * Taxable income = 0.30*$15.82 = $4.75

Alternately, we can calculate the semi-annual interest earned, for each of the two semi-annual periods during the year.

\( $171.93 \times 0.045 = $7.736 \Rightarrow \) Price after 6 months = $171.93 + $7.736 = $179.667
$179.667 \times 0.045 = 8.084 \Rightarrow \text{Price at end of year} = 179.667 + 8.084 = 187.75$

Total interest income for 1 year = $7.736 + 8.084 = 15.82$

Tax due = $0.30 \times 15.82 = 4.75$

### 6.3 Yields and Coupon Rates (Slide 6-25)

Students must learn how to differentiate between a bond’s coupon rate and its yield to maturity (YTM). This is often confusing for many students. It is important to explain to them that the coupon rate is set by the company at the time of issue and is fixed (except for newer innovations which have variable coupon rates) while its YTM is dependent on market, economic, and company-specific factors and is therefore variable.

#### 6.3 (A) The First Interest Rate: Yield to Maturity (Slide 6-26)

The YTM is the expected rate of return on a bond if held to maturity. The price that willing buyers and sellers settle at determines a bond’s YTM at any given point. Thus, changes in economic conditions and risk factors will cause bond prices and their corresponding YTMs to change. A bond’s YTM can be calculated by entering the bond’s coupon amount ($PMT$), price ($PV$), remaining number of coupons ($n$), and par value ($FV$) into the TVM equation, financial calculator, or spreadsheet.

#### 6.3 (B) The “Other” Interest Rate: Coupon Rate (Slide 6-27)

The coupon rate on a bond is set by the issuing company at the time of issue and represents the annual rate of interest that the firm is committed to pay over the life of the bond. Thus, if the rate is set at 7%, the firm is committing to pay $0.07 \times 1000 = 70$ per year on each bond, either in a single check or two checks of $35 paid six months apart.

#### 6.3 (C) Relationship of Yield to Maturity and Coupon Rate (Slides 6-28 to 6-33)

Typically, an issuing firm gets the bond rated by a rating agency such as Standard & Poor’s or Moody’s. Then, based on the rating and planned maturity of the bond, it sets the coupon rate to equal the expected yield as indicated in the Yield Book available in the capital markets at that time, and sells the bond at par value ($1000). Once issued, if investors expect a higher yield on the bond, its price will go down and the bond will sell below par or as a discount bond and vice-versa. Thus, a bond’s YTM can be equal to (par bond), higher than (discount bond) or lower than (premium bond) its coupon rate.

#### Example 5: Computing YTM

Last year, The ABC Corporation had issued 8% coupon (semi-annual), 20-year, AA-rated bonds (Par value = $1000) to finance its business growth. If investors are currently offering $1200 on each of these bonds, what is their expected yield to maturity on the investment? If you are willing to pay no more than $980 for this bond, what is your expected $YTM$?

Remaining number of coupons = 19*2 = 38

Semi-annual coupon amount = ($0.08 \times 1000)/2 = 40

$PV = 1200$
Mode: P/Y=2; C/Y = 2

Input: \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
Key: 38 ? -1200 40 1000
Output 6.19

Note: This is a premium bond, so it’s YTM < Coupon rate

\[ PV = 980 \]

Mode: P/Y=2; C/Y = 2

Input: \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
Key: 38 ? -980 40 1000
Output 8.21%

Note: This would be a discount bond, so it’s YTM > Coupon rate

### 6.4 Bond Ratings

(Slides 6-34 to 6-35)

Rating agencies such as Moody’s, Standard and Poor’s, and Fitch produce bond ratings ranging from AAA (top-rated) to C (lowest-rated) or D (default). These ratings, which are based on the issuing firm’s riskiness, can help investors assess the likelihood of default and assist issuing companies establish a yield on their newly-issued bonds.

**Junk bonds**: is the label given to bonds that are rated below BBB. These bonds are considered to be speculative in nature and carry higher yields than those rated BBB or above (investment grade).

**Fallen angels**: is the label given to bonds that have had their ratings lowered from investment to speculative grade.

### 6.5 Some Bond History and More Bond Features

(Slides 6-36 to 6-39)

Over the years, corporate bond features have gone through some major changes. The addition of newer features such as call provisions, convertibility, and put options has significantly broadened the array of bond types available.

**Bearer bonds**: original nature of bonds, whereby the bond holder did not have to be registered with the issuing company, and whoever held the bond was entitled to the interest payments and the principal repayment. Currently, firms issue only registered bonds to avoid the problems associated with stolen bonds and for ease of communicating with their creditors.

**Indenture or deed of trust**: a written contract between the bond issuer and the bondholder which spells out the terms of the bond, the number of bonds to be issued, a description of any collateral supporting the bond, any special repayment provisions or call options, and details of protective covenants.

**Collateral, or security of a bond**: refers to the physical and/or financial assets which support the bond in case of issuer default.

**Mortgaged security**: is a security which is backed by real estate.
Debentures: are bonds which are not supported by any assets of the issuing firm.

Senior debt: is unsecured debt which was issued earlier than junior debt and has refunding priority in case of liquidation.

Sinking fund: is a reserve fund set up by some bond issuing companies in which regular payments are made so as to retire the bonds at maturity.

Protective covenants: specify actions which bond issuers are required and/or prohibited from doing, in the interest of bondholder protection.

Callable bond: is a bond which is issued with a call option whereby the issuer can retire the bond prior to its maturity after paying a call premium, which is usually an additional coupon payment in addition to its par value.

Yield to call: is the relevant yield that an investor can expect to earn on a callable bond, based on the number of periods until the bond can be first called and its call price, i.e. par value plus the call premium.

Example 6: Calculating Yield to Call

Two years ago, The Mid-Atlantic Corporation issued a 10% coupon (paid semi-annually), 20-year maturity, bond with a 5-year deferred call feature and a call penalty of one coupon payment in addition to the par value ($1000) if exercised. If the current price on these bonds is $1080, what is its yield to call?

Remaining number of coupons until first call date = 6 = \(n\)

Semi-annual coupon = \(\$50 = PMT\)

Call price = \(\$1050 = FV\)

Bond price = \(\$1080 = PV\)

Mode: \(P/Y=2; C/Y = 2\)

Input: \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)

Key: 6 \(?\) -1080 50 1050

Output 8.43 \(YTC\)

Putable bond: is one which gives the holder the right to sell the bond back to the issuing firm at a pre-determined price at any time prior to maturity. It is especially valuable when the bond’s price is dropping due to rising interest rates or increased riskiness of the issuing firm.

Convertible bond: is one which can be exchanged by the holder for other securities, usually common stock, of the issuer at a pre-determined conversion ratio.

Floating-rate bond: is one that has a variable coupon rate which adjusts to some interest rate benchmark such as the prime rate.

Prime rate is the rate that money-center banks charge their most credit-worthy customers.
Income bonds: are bonds whose coupon amount and payment schedule is tied to the firm’s income.

Exotic bonds are bonds with special features distinct to that particular bond.

6.6 U.S. Government Bonds (Slides 6-40 to 6-41)

U.S. government securities include bills, notes, and bonds sold by the Department of the Treasury, as well as state bonds, issued by state governments, and municipal bonds issued by county, city, or local government agencies. Treasury bills, are zero-coupon, pure discount securities with maturities ranging from 1-, 3-, and 6-months up to 1-year, while Treasury notes have between two to 10 year maturities, and Treasury bonds have greater than 10-year maturities, when first issued.

6.6 (A) Pricing a U.S. Government Note or Bond: is very similar to the method used for pricing corporate bonds and can be done by using TVM equations, a financial calculator or a spreadsheet program. (Slide 6-42)

6.6 (B) Pricing a Treasury Bill: is done by discounting the bill’s face value for the number of days until maturity and at the prevailing bank discount yield.

Bank discount yield: is a special discount rate used in conjunction with treasury bills under a 360 day- per- year convention commonly assumed by bankers.

Bond equivalent yield (BEY), is the APR equivalent of the bank discount yield calculated by adjusting it as follows,

\[
BEY = \frac{365 \times \text{Bank discount yield}}{360 - (\text{days to maturity} \times \text{discount yield})}
\]

Example 7: Calculating the price and BEY of a Treasury bill

Calculate the price and BEY of a treasury bill which matures in 105 days, has a face value of $10,000 and is currently being quoted at a bank discount yield of 2.62%.

Price of T-bill = Face value * [1 – (discount yield * days until maturity/360)]

Price of T-bill = $10,000 * [1 – (.0262 * 105/360)] = $10,000*0.9923583

Price of T-bill = $9,923.58

\[
BEY = \frac{\frac{365 \times \text{Bank discount yield}}{360 - (\text{days to maturity} \times \text{discount yield})}}{\frac{365 \times 0.0262}{360 - (105 \times 0.0262)}}
\]

\[BEY = .026768 = \frac{2.68\%}{2 \text{ decimals}}\]

Questions

1. What is a bond? What determines the price of this financial asset?

A bond is a promised set of future payments from the issuer to the buyer of the bond where a formal agreement states the timing and amount of the future cash flow. The price of this financial asset is determined by the timing and amount of the future cash flow and the appropriate discount rate of these payments. The discount rate reflects
the market’s assessment of the required return for investments similar to the bond in terms of risk (default), inflation, maturity, and the current real interest rate.

2. What is the primary difference between an annual bond and a semiannual bond? What changes do you need to make in finding the price of a semiannual bond versus an annual bond?

The primary difference is the timing and the amount of the cash flow of the interest payments. An annual bond pays the annual interest in one payment while a semiannual bond splits the annual interest into two equal payments paid six-months apart. When using the bond pricing equation you need to change the discount rate from the annual yield to the semi-annual or six-month rate by dividing the annual yield by 2. You need to increase the number of periods for n from the number of years to the number of semi-annual periods by multiplying the number of years by 2.

3. When we talk about the yield of a bond, we usually mean the yield to maturity of the bond. Why?

In order to price a bond we need to know how long we will hold the bond and thus the number of coupon payments we will receive. Because each bondholder has potentially a different time horizon we could get many different prices for the same bond. Therefore, it is generally agreed that the price of the bond reflects all remaining coupon payments and the repayment of the principal at maturity. Thus we state the yield on the bond based on holding the bond to maturity and that yield is the yield-to-maturity.

4. Does a zero-coupon bond pay interest?

Yes, it just does not pay annual coupon payments. The price appreciation of the bond is the interest earned on the bond.

5. If a zero-coupon bond does not pay coupons each year, why buy it?

The value of owning a zero-coupon bond is the appreciation in price from period to period. The bond sells for a discount but at maturity pays the par value and therefore a gain is realized on the bond.

6. How does the potential for default of a bond affect the yield of the bond?

The greater is the potential for default, the higher the yield. Investors want to be compensated for taking on more risk and default is one type of risk. So for bonds with higher potential for default the yield goes up and the price goes down.

7. Why are some bonds sold with a premium, some at par value, and some at a discount?

Bonds promise a coupon payment based on the coupon rate of the bond. When this coupon rate is above the yield that the market requires for this type of investment, the potential buyers bid the price above par value. For example, if a company is promising a coupon rate of 10% on a new bond, but similar bonds are paying 7% in the market, the 10% coupon rate provides interest well above the required level. Buyers will compete for the right to purchase the limited supply of these bonds, bidding the price above par value. That is, they will pay a premium to own this 10% coupon bond. The market will bid the price up until the actual yield on the investment falls to the current market rate of 7%. The opposite is true for bonds with coupon rates below the current market yield. Bond buyers will discount the price until the yield on the bond rises to the current market yield for similar investments. Finally, if the
coupon rate is equal to the current yield on similar investments the bond buyer gets the required yield by paying the par value of the bond.

8. **How does collateral impact the price of a bond?**

Collateral reduces the potential loss for a bondholder if the company defaults on the promised bond payment. Because the collateral can be seized as partial or full repayment of the bond if a default should take place, bondholders will pay more for a bond with collateral versus a bond without collateral (a debenture bond).

9. **What role do Moody’s, Standard & Poor’s, or Fitch’s bond ratings play in the pricing of a bond?**

Moody’s and Standard & Poor’s provide reliable information to potential bond buyers about the riskiness of the bond. That is, these rating agencies analyze the firm’s ability to make the future promised payments (potential for default) and therefore provide the appropriate default premium for pricing the bond. The higher the bond rating the lower the required yield (higher the selling price).

10. **What must happen for a bond to be called a “fallen angel”?**

A bond must have been an investment grade bond prior to a downgrade to a speculative bond in order to be called a “fallen angel.”

## Prepping for Exams

1. b.
2. d.
3. a.
4. a.
5. c.
6. a.
7. a.
8. b.

## Problems

Bond Prices: Use the following table for problems 1 through 4.

<table>
<thead>
<tr>
<th>Par Value</th>
<th>Coupon Rate</th>
<th>Years to Maturity</th>
<th>Yield to Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>8%</td>
<td>10</td>
<td>6%</td>
<td>?</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>6%</td>
<td>10</td>
<td>8%</td>
<td>?</td>
</tr>
<tr>
<td>$5,000.00</td>
<td>9%</td>
<td>20</td>
<td>7%</td>
<td>?</td>
</tr>
<tr>
<td>$5,000.00</td>
<td>12%</td>
<td>30</td>
<td>5%</td>
<td>?</td>
</tr>
</tbody>
</table>
1. Price the bonds from the above table with annual coupon payments.

**ANSWER**

\[
\text{Price} = \$1,000.00 \times \frac{1}{(1.06)^{10}} + \$80.00 \left(1 - \frac{1}{(1.06)^{10}}\right)/0.06 \\
\text{Price} = \$1,000.00 \times 0.5584 + \$80.00 \times 7.3601 \\
\text{Price} = \$558.39 + \$588.81 = \$1,147.20 \\
\text{Price} = \$1,000.00 \times \frac{1}{(1.08)^{10}} + \$60.00 \left(1 - \frac{1}{(1.08)^{10}}\right)/0.08 \\
\text{Price} = \$1,000.00 \times 0.4632 + \$60.00 \times 6.7101 \\
\text{Price} = \$463.19 + \$402.60 = \$865.80 \\
\text{Price} = \$5,000.00 \times \frac{1}{(1.07)^{20}} + \$450.00 \left(1 - \frac{1}{(1.07)^{20}}\right)/0.07 \\
\text{Price} = \$5,000.00 \times 0.2584 + \$450.00 \times 10.5940 \\
\text{Price} = \$1,292.10 + \$4,767.30 = \$6,059.40 \\
\text{Price} = \$5,000.00 \times \frac{1}{(1.05)^{30}} + \$600.00 \left(1 - \frac{1}{(1.05)^{30}}\right)/0.05 \\
\text{Price} = \$5,000.00 \times 0.2314 + \$600.00 \times 15.3725 \\
\text{Price} = \$1,156.89 + \$9,223.47 = \$10,380.36
\]

2. Price the bonds from the above table with semiannual coupon payments.

**ANSWER**

\[
\text{Price} = \$1,000.00 \times \frac{1}{(1.03)^{20}} + \$40.00 \left(1 - \frac{1}{(1.03)^{20}}\right)/0.03 \\
\text{Price} = \$1,000.00 \times 0.5537 + \$40.00 \times 14.8775 \\
\text{Price} = \$553.67 + \$595.10 = \$1,148.77 \\
\text{Price} = \$1,000.00 \times \frac{1}{(1.04)^{20}} + \$30.00 \left(1 - \frac{1}{(1.04)^{20}}\right)/0.04 \\
\text{Price} = \$1,000.00 \times 0.4564 + \$30.00 \times 13.5903 \\
\text{Price} = \$456.39 + \$407.71 = \$864.10 \\
\text{Price} = \$5,000.00 \times \frac{1}{(1.0175)^{80}} + \$225.00 \left(1 - \frac{1}{(1.0175)^{80}}\right)/0.0175 \\
\text{Price} = \$5,000.00 \times 0.2314 + \$225.00 \times 30.9087 \\
\text{Price} = \$1,136.41 + \$9,272.60 = \$10,409.01
\]

3. Price the bonds from the above table with quarterly coupon payments.

**ANSWER**

\[
\text{Price} = \$1,000.00 \times \frac{1}{(1.015)^{40}} + \$20.00 \left(1 - \frac{1}{(1.015)^{40}}\right)/0.015 \\
\text{Price} = \$1,000.00 \times 0.5584 + \$20.00 \times 7.3601 \\
\text{Price} = \$558.39 + \$588.81 = \$1,085.84 \\
\text{Price} = \$1,000.00 \times \frac{1}{(1.02)^{40}} + \$15.00 \left(1 - \frac{1}{(1.02)^{40}}\right)/0.02 \\
\text{Price} = \$1,000.00 \times 0.4632 + \$15.00 \times 6.7101 \\
\text{Price} = \$463.19 + \$402.60 = \$865.80 \\
\text{Price} = \$5,000.00 \times \frac{1}{(1.0175)^{80}} + \$75.00 \left(1 - \frac{1}{(1.0175)^{80}}\right)/0.0175 \\
\text{Price} = \$5,000.00 \times 0.2314 + \$75.00 \times 30.9087 \\
\text{Price} = \$1,136.41 + \$9,272.60 = \$10,409.01
\]
Price = $5,000.00 \times 0.2584 + 75.00 \times 10.5940
Price = $1,292.10 + $3,178.20 = $4,464

Price = $5,000.00 \times 1/(1.0125)^{120} + 150.00 \times (1-1/(1.0125)^{120})/0.0125
Price = $5,000.00 \times 0.2314 + 150.00 \times 15.3725
Price = $1,156.89 + $9,223.47 = $10,423.50

4. Price the bonds from the above table with monthly coupon payments.

**ANSWER**

Price = $1,000.00 \times 1/(1.005)^{120} + 6.67 \times (1-1/(1.005)^{120})/0.005
Price = $1,000.00 \times 0.5584 + 6.67 \times 7.3601
Price = $558.39 + $860.13 = $1,150.42

Price = $1,000.00 \times 1/(1.0067)^{120} + 5.00 \times (1-1/(1.0067)^{120})/0.0067
Price = $1,000.00 \times 0.4632 + 5.00 \times 6.7101
Price = $463.19 + $50.00 = $513.19

Price = $5,000.00 \times 1/(1.0058)^{240} + 37.50 \times (1-1/(1.0058)^{240})/0.0058
Price = $5,000.00 \times 0.2476 + 37.50 \times 128.98
Price = $1,238.01 + $4,836.84 = $6,074.85

Price = $5,000.00 \times 1/(1.0042)^{360} + 50.00 \times (1-1/(1.0042)^{360})/0.0042
Price = $5,000.00 \times 0.2314 + 50.00 \times 15.3725
Price = $1,156.89 + $9,223.47 = $10,377.65

**Yield-to-Maturity:** Use the following table for problems 5 through 8.

<table>
<thead>
<tr>
<th>Par Value</th>
<th>Coupon Rate</th>
<th>Years to Maturity</th>
<th>Yield to Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>8%</td>
<td>10</td>
<td>?</td>
<td>$1000.00</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>6%</td>
<td>10</td>
<td>?</td>
<td>$850.00</td>
</tr>
<tr>
<td>$5,000.00</td>
<td>9%</td>
<td>20</td>
<td>?</td>
<td>$5,400.00</td>
</tr>
<tr>
<td>$5,000.00</td>
<td>12%</td>
<td>30</td>
<td>?</td>
<td>$4,300.00</td>
</tr>
</tbody>
</table>

5. What is the yield of the above bonds if interest (coupon) is paid annually?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 1 and C/Y = 1

INPUT 10 ? -1000.00 80.00 1000.00
KEYS N I/Y PV PMT FV
CPT 8.0
(TVM Keys) Set Calculator to P/Y = 1 and C/Y = 1

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-850.00</th>
<th>60.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.2619</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 1 and C/Y = 1

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-5400.00</th>
<th>450.00</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.1746</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 1 and C/Y = 1

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-4300.00</th>
<th>600.00</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>13.9991</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. What is the yield of the above bonds if interest (coupon) is paid semiannually?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-1000.00</th>
<th>40.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-850.00</th>
<th>30.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.2300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-5400.00</th>
<th>225.00</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.1807</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-4300.00</th>
<th>300.00</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>13.9936</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. What is the yield of the above bonds if interest (coupon) is paid quarterly?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 4 and C/Y = 4

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-1000.00</th>
<th>20.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td>8.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. What is the yield of the above bonds if interest (coupon) is paid monthly?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 12 and C/Y = 12

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-1000.00</th>
<th>6.67</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 12 and C/Y = 12

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-850.00</th>
<th>30.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td>8.2033</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 12 and C/Y = 12

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-5400.00</th>
<th>37.50</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td>8.1859</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to P/Y = 12 and C/Y = 12

<table>
<thead>
<tr>
<th>INPUT</th>
<th>?</th>
<th>-4300.00</th>
<th>50.00</th>
<th>5000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>CPT</td>
<td>13.9891</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. How long to maturity for the bonds listed below?

<table>
<thead>
<tr>
<th>Par Value</th>
<th>Coupon Rate</th>
<th>Years to Maturity</th>
<th>Yield to Maturity</th>
<th>Price</th>
<th>Coupon Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>8%</td>
<td>?</td>
<td>8.7713%</td>
<td>$950.00</td>
<td>Annual</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>6%</td>
<td>?</td>
<td>7.7038%</td>
<td>$850.00</td>
<td>Semi-Annual</td>
</tr>
</tbody>
</table>
10. **Coupon rates.** What are the coupon rates for the bonds listed below?

<table>
<thead>
<tr>
<th>Par Value</th>
<th>Coupon Rate</th>
<th>Years to Maturity</th>
<th>Yield to Maturity</th>
<th>Price</th>
<th>Coupon Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>?</td>
<td>30</td>
<td>6.0%</td>
<td>$1,412.94</td>
<td>Annual</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>?</td>
<td>25</td>
<td>10.0%</td>
<td>$1,182.56</td>
<td>Semi-Annual</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>?</td>
<td>20</td>
<td>9.0%</td>
<td>$907.63</td>
<td>Quarterly</td>
</tr>
<tr>
<td>$1,000.00</td>
<td>?</td>
<td>10</td>
<td>8.0%</td>
<td>$862.63</td>
<td>Monthly</td>
</tr>
</tbody>
</table>

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 1 and C/Y = 1

INPUT  30  6.0  -1412.94  ?  1000.00
KEYS  N  I/Y  PV  PMT  FV
CPT  90.00
Coupon payments are $90.00 every year so coupon rate is:

\[ 1,000 \times \text{rate} = 90.00 \]

\[ \text{rate} = \frac{90}{1,000} = 0.09 \text{ or } 9\% \]

(TVM Keys) Set Calculator to \(P/Y = 2\) and \(C/Y = 2\)

<table>
<thead>
<tr>
<th>INPUT</th>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>10.0</td>
<td>-1,182.56</td>
<td>?</td>
<td>1000.00</td>
</tr>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60.00</td>
</tr>
</tbody>
</table>

(TVM Keys) Set Calculator to \(P/Y = 2\) and \(C/Y = 2\)

<table>
<thead>
<tr>
<th>INPUT</th>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>10.0</td>
<td>-1,182.56</td>
<td>?</td>
<td>1000.00</td>
</tr>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60.00</td>
</tr>
</tbody>
</table>

Coupon payments are $60.00 every six months so coupon rate is:

\[ 1,000 \times \text{rate} / 2 = 60.00 \]

\[ 1,000 \times \text{rate} = 120.00 \]

\[ \text{rate} = \frac{120}{1,000} = 0.12 \text{ or } 12\% \]

(TVM Keys) Set Calculator to \(P/Y = 4\) and \(C/Y = 4\)

<table>
<thead>
<tr>
<th>INPUT</th>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80</td>
<td>9.0</td>
<td>-907.63</td>
<td>?</td>
<td>1000.00</td>
</tr>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.00</td>
</tr>
</tbody>
</table>

Coupon payments are $20.00 every four months so coupon rate is:

\[ 1,000 \times \text{rate} / 4 = 20.00 \]

\[ 1,000 \times \text{rate} = 80.00 \]

\[ \text{rate} = \frac{80}{1,000} = 0.08 \text{ or } 8\% \]

(TVM Keys) Set Calculator to \(P/Y = 12\) and \(C/Y = 12\)

<table>
<thead>
<tr>
<th>INPUT</th>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>120</td>
<td>8.0</td>
<td>-862.63</td>
<td>?</td>
<td>1000.00</td>
</tr>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.0000</td>
</tr>
</tbody>
</table>

Coupon payments are $5.00 every month so coupon rate is:

\[ 1,000 \times \text{rate} / 12 = 5.00 \]

\[ 1,000 \times \text{rate} = 60.00 \]

\[ \text{rate} = \frac{60}{1,000} = 0.06 \text{ or } 6\% \]

11. **Bond prices and maturity dates.** Moore Company is about to issue a bond with semi-annual coupon payments, a coupon rate of 8%, and par value of $1,000. The yield-to-maturity for this bond is 10%.

a. What is the price of the bond if the bond matures in five, ten, fifteen, or twenty years?

b. What do you notice about the price of the bond in relationship to the maturity of the bond?

**ANSWER (A)**

At five years to maturity

\[ \text{Price} = 1,000.00 \times \frac{1}{(1.05)^{10}} + 40.00 \times \frac{1 - 1/(1.05)^{10}}{0.05} \]

\[ \text{Price} = 1,000.00 \times 0.6139 + 40.00 \times 7.7217 \]

\[ \text{Price} = 613.91 + 308.87 = 922.78 \]
At ten years to maturity
Price = $1,000.00 \times \frac{1}{(1.05)^{20}} + \$40.00 \left(1 - \frac{1}{(1.05)^{20}} \right)/ 0.05
Price = $1,000.00 \times 0.3769 + \$40.00 \times 12.4622
Price = $376.89 + \$498.49 = $875.38

At fifteen years to maturity
Price = $1,000.00 \times \frac{1}{(1.05)^{30}} + \$40.00 \left(1 - \frac{1}{(1.05)^{30}} \right)/ 0.05
Price = $1,000.00 \times 0.2314 + \$40.00 \times 15.3725
Price = $231.38 + \$614.90 = $846.28

At twenty years to maturity
Price = $1,000.00 \times \frac{1}{(1.05)^{40}} + \$40.00 \left(1 - \frac{1}{(1.05)^{40}} \right)/ 0.05
Price = $1,000.00 \times 0.1420 + \$40.00 \times 17.1591
Price = $142.05 + \$686.36 = $828.41

ANSWER (B)

The longer the maturity of a bond selling at a discount, all else held constant, the lower the price of the bond!

12. **Bond Prices and Maturity Dates.** Les Company is about to issue a bond with semiannual coupon payments, a coupon rate of 10%, and par value of $1,000. The yield-to-maturity for this bond is 8%.

a. What is the price of the bond if the bond matures in five, ten, fifteen, or twenty years?

b. What do you notice about the price of the bond in relationship to the maturity of the bond?
ANSWER (A)

At five years to maturity
Price = $1,000.00 \times 1/(1.04)^{10} + 50.00 \times (1 - 1/(1.04)^{10}) / 0.04
Price = $1,000.00 \times 0.6756 + 50.00 \times 8.1109
Price = $675.56 + $405.55 = $1,081.11

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2
INPUT 10 8.0 ? 50.00 1000.00
KEYS N I/Y PV PMT FV
CPT -1,081.11

At ten years to maturity
Price = $1,000.00 \times 1/(1.04)^{20} + 50.00 \times (1 - 1/(1.04)^{20}) / 0.04
Price = $1,000.00 \times 0.2083 + 50.00 \times 17.2920
Price = $208.32 + $864.60 = $1,172.92

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2
INPUT 20 8.0 ? 50.00 1000.00
KEYS N I/Y PV PMT FV
CPT -1,172.92

At fifteen years to maturity
Price = $1,000.00 \times 1/(1.04)^{30} + 50.00 \times (1 - 1/(1.04)^{30}) / 0.04
Price = $1,000.00 \times 0.1083 + 50.00 \times 19.7928
Price = $108.32 + $989.64 = $1,197.93

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2
INPUT 30 8.0 ? 50.00 1000.00
KEYS N I/Y PV PMT FV
CPT -1,197.93

ANSWER (B)

The longer the maturity of a bond selling for a premium, all else held constant, the higher the price of the bond!

13. Zero-coupon bond. Addison Company will issue a zero coupon bond this coming month. The projected yield for the bond is 7%. If the par value of the bond is $1,000, what is the price of the bond using a semiannual convention if
a. The maturity is 20 years?
b. The maturity is 30 years?
c. The maturity is 50 years?
d. The maturity is 100 years?

**ANSWER (A)**

Price = $1,000 \times \frac{1}{(1.035)^{40}} = $1,000 \times 0.2526 = $252.57

**ANSWER (B)**

Price = $1,000 \times \frac{1}{(1.035)^{60}} = $1,000 \times 0.1269 = $126.93

**ANSWER (C)**

Price = $1,000 \times \frac{1}{(1.035)^{100}} = $1,000 \times 0.0321 = $32.06

**ANSWER (D)**

Price = $1,000 \times \frac{1}{(1.035)^{200}} = $1,000 \times 0.0010 = $1.03

14. **Zero-coupon bond.** Wesley Company will issue a zero-coupon bond this coming month. The projected yield for the bond is 5%. If the par value of the bond is $1,000, what is the price of the bond using a semiannual convention if

a. The maturity is 20 years?
b. The maturity is 30 years?
c. The maturity is 50 years?
d. The maturity is 100 years?

**ANSWER (A)**

Price = $1,000 \times \frac{1}{(1.025)^{40}} = $1,000 \times 0.3724 = $372.43

**ANSWER (B)**

Price = $1,000 \times \frac{1}{(1.025)^{60}} = $1,000 \times 0.2273 = $227.28

**ANSWER (C)**

Price = $1,000 \times \frac{1}{(1.025)^{100}} = $1,000 \times 0.0846 = $84.65

**ANSWER (D)**

Price = $1,000 \times \frac{1}{(1.025)^{200}} = $1,000 \times 0.0072 = $7.17

15. **Zero-coupon bond.** What is the annual implied interest of a five-year zero-coupon bond (using the semiannual pricing convention) with a current yield of 12% and a par value of $1,000.00?

**ANSWER**

Step One is to find the price of the zero coupon bond:
Price = $1,000 \times 1 / (1.06)^{10} = $1,000 \times 0.5584 = $558.39

Interest each year is the change in price which is the capitalized interest of the bond.

Note on a semi-annual bond there are two interest payments per year and “two” prices to figure each year.

**Amortization of Zero-Coupon Bond**

<table>
<thead>
<tr>
<th>Period (six-months)</th>
<th>Beginning Balance</th>
<th>Beginning Balance \times 0.06 = Interest</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$558.39</td>
<td>$558.39 \times 0.06 = $33.51</td>
<td>$591.90</td>
</tr>
<tr>
<td>2</td>
<td>$591.90</td>
<td>$591.90 \times 0.06 = $35.51</td>
<td>$627.41</td>
</tr>
<tr>
<td>3</td>
<td>$627.41</td>
<td>$627.41 \times 0.06 = $37.65</td>
<td>$665.06</td>
</tr>
<tr>
<td>4</td>
<td>$665.06</td>
<td>$665.06 \times 0.06 = $39.90</td>
<td>$704.96</td>
</tr>
<tr>
<td>5</td>
<td>$704.96</td>
<td>$704.96 \times 0.06 = $42.30</td>
<td>$747.26</td>
</tr>
<tr>
<td>6</td>
<td>$747.26</td>
<td>$747.26 \times 0.06 = $44.83</td>
<td>$792.09</td>
</tr>
<tr>
<td>7</td>
<td>$792.09</td>
<td>$792.09 \times 0.06 = $47.53</td>
<td>$839.62</td>
</tr>
<tr>
<td>8</td>
<td>$839.62</td>
<td>$839.62 \times 0.06 = $50.38</td>
<td>$890.00</td>
</tr>
<tr>
<td>9</td>
<td>$890.00</td>
<td>$890.00 \times 0.06 = $53.40</td>
<td>$943.40</td>
</tr>
<tr>
<td>10</td>
<td>$943.40</td>
<td>$943.40 \times 0.06 = $56.60</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>

The first year’s interest is: $33.51 + $35.51 or ($627.41 – $558.39) = $69.02

The second year’s interest is: $37.65 + $39.90 or ($704.96 – $627.41) = $77.55

The third year’s interest is: $42.30 + $44.83 or ($747.26 – $665.06) = $87.13

The fourth year’s interest is: $47.53 + $50.38 or ($839.62 – $792.09) = $97.91

The fifth year’s interest is: $53.40 + $56.60 or ($1,000 – $890.00) = $110.00

Total interest is $1,000 – $558.39 = $441.61

16. **Callable bond.** Corso Books has just sold a callable bond. The bond is a thirty year semi-annual bond with a coupon rate of 6%. Investors, however, can call the bond starting at the end of ten years. If the yield-to-call on this bond is 8% and the call requires Corso Books to pay one year of additional interest at the call (two coupon payments), what is the price of this bond if priced with the assumption that it will be called on the first available call date?

**ANSWER**

Determine the cash flows until the call date.

The coupons are 0.06 \times $1,000 / 2 = $30.00

Final payment is the par value plus one year of interest or two coupon payments: Par Value + extra interest = $1,000 + 2 \times $30.00 = $1,060
If the yield to call is 8.0% (4.0% semi-annual rate) and the call is ten years away (20 semi-annual periods) the price is:

\[(TVM \text{ Keys}) \text{ Set Calculator to } P/Y = 2 \text{ and } C/Y = 2\]
\[
\begin{array}{cccccc}
\text{INPUT} & 20 & 8.0 & ? & 30.00 & 1060.00 \\
\text{KEYS} & N & I/Y & PV & PMT & FV \\
\text{CPT} & -891.48 \\
\end{array}
\]

17. **Callable bond.** McCarty Manufacturing Company makes baseball equipment. The company decides to issue a callable bond that it expects to sell for $840 per bond. If the bond is a twenty-year semiannual bond with a 6% coupon rate and a current yield to maturity of 7%, what is the cost of the option attached to the bond? (Assume $1000 par value). Hint: find the price of an equivalent bond without the call option.

**ANSWER**

First determine the price of the bond without the option attached.

\[(TVM \text{ Keys}) \text{ Set Calculator to } P/Y = 2 \text{ and } C/Y = 2\]
\[
\begin{array}{cccccc}
\text{INPUT} & 40 & 7.0 & ? & 30.00 & 1000.00 \\
\text{KEYS} & N & I/Y & PV & PMT & FV \\
\text{CPT} & -893.22 \\
\end{array}
\]

Subtract the cost of the callable bond from the straight bond to get the price of the call option.

\[\text{Call Option} = \$893.22 - \$840.00 = \$53.22\]

18. **Missing information on a bond.** Your broker faxed you the following information on two semiannual coupon bonds that you are considering as a potential investment. Unfortunately, your fax machine is blurring some of the items and all you can read from the fax on the two different bonds is the following information:

<table>
<thead>
<tr>
<th>IBM Coupon Bond</th>
<th>AOL Coupon Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face Value (Par)</td>
<td>$1,000</td>
</tr>
<tr>
<td>Coupon Rate</td>
<td>9.5%</td>
</tr>
<tr>
<td>Yield to Maturity</td>
<td>7.5%</td>
</tr>
<tr>
<td>Years to Maturity</td>
<td>10</td>
</tr>
<tr>
<td>Price</td>
<td>$689.15</td>
</tr>
</tbody>
</table>

Fill in the missing data from the information sent by the broker.

**ANSWER**

**IBM’s Variables**

\[n = 10 \times 2 = 20\]
\[R = 0.075 / 2 = 0.0375\]
Coupon = $1000 × 0.095 / 2 = $47.50  
Par value = $1000

IBM’s Price = $1,000 × \frac{1}{(1 + 0.0375)^{20}} + $47.50 × \frac{1}{0.0375} \left[ 1 - \frac{1}{(1 + 0.0375)^{20}} \right]

IBM’s Price = $1000 × 0.4789 + $47.50 × 13.8962 = $1,138.96

**AOL’s Coupon Rate?** This will be a little more difficult, but we can solve first for the coupon and then backtrack to the coupon rate.

**AOL’s Variables:**
\[ n = 20 × 2 = 40 \]
\[ r = \frac{0.095}{2} = 0.0475 \]
Par Value = $1000  
Price = $689.15

$689.15 = $1,000 × \frac{1}{(1 + 0.0475)^{40}} + \text{Coupon} × \frac{1}{0.0475} \left[ 1 - \frac{1}{(1 + 0.0475)^{40}} \right]

Now we need to isolate the coupon amount on the left-hand side of the equation and we have:

\[ \text{Coupon} × \left( \frac{1}{(1 + 0.0475)^{40}} \right) = $689.15 − $1,000 × \frac{1}{(1 + 0.0475)^{40}} \]

\[ \text{Coupon} × (17.7630) = $689.15 − $1000 × 0.15626 = $532.89 \]

Coupon = $532.89 / 17.7630 = $30.00

So if the coupon is $30.00 every six months, the annual interest is $60.00 (2 × $30) and the coupon rate is the annual interest divided by the par value:

Coupon rate = $60.00 / $1000 = 0.06 or 6%

---

**Treasury notes and bonds. For Questions 19 through 23 use the following information:**

**Today is February 15, 2008**

<table>
<thead>
<tr>
<th>Type</th>
<th>Issue Date</th>
<th>Price</th>
<th>Coupon Rate</th>
<th>Maturity Date</th>
<th>YTM</th>
<th>Current Yield</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note</td>
<td>Feb 2000</td>
<td>---</td>
<td>6.50%</td>
<td>2-15-2010</td>
<td>3.952%</td>
<td>6.199%</td>
<td>AAA</td>
</tr>
</tbody>
</table>
19. What is the price in dollars of the February 2000 Treasury note if its par value is $100,000? Verify the current yield of this note.

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>4</th>
<th>3.952</th>
<th>?</th>
<th>3250</th>
<th>100000</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-104,853.88</td>
</tr>
</tbody>
</table>

Current Yield is: $6500 / $104,853.88 = 6.199%

20. What is the yield to maturity of the August 2005 Treasury bond? Compare the yield to maturity and the current yield. How do you explain this relationship?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>21</th>
<th>?</th>
<th>100000</th>
<th>2125</th>
<th>100000</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.25</td>
</tr>
</tbody>
</table>

Current Yield is: 4.25% = YTM

If a note (or bond) sells for its par value then the current yield is equal to the yield-to-maturity.

21. What is the price of the August 2003 Treasury bond (assume a $100,000 par value) with the yield to maturity from the table? Verify the current yield. Why is the current yield higher than the yield to maturity?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

<table>
<thead>
<tr>
<th>INPUT</th>
<th>31</th>
<th>4.830</th>
<th>?</th>
<th>3625</th>
<th>100000</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEYS</td>
<td>N</td>
<td>I/Y</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
</tr>
<tr>
<td>CPT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-126,192.53</td>
</tr>
</tbody>
</table>

Current Yield is: $7250 / $126,192.53 = 5.745%

The current yield is higher than the YTM since the bond is selling at a premium. The lower the price of a bond, the higher its yield to maturity. If the bond sells below its par value it yield to maturity would be higher than its current yield and vice-versa.
22. What is the yield to maturity of the February 1995 Treasury bond with the yield from the table? Verify the current yield. Why is the current yield higher than the yield-to-maturity?

**ANSWER**

(TVM Keys) Set Calculator to P/Y = 2 and C/Y = 2

\[
\begin{array}{cccc}
\text{INPUT} & 14 & ? & -126190 \\
\text{KEYS} & N & I/Y & PV \\
\text{CPT} & 4.150 & PMT & FV \\
\end{array}
\]

Current Yield is:

\[
\frac{8,500}{126,190.00} = 6.736\%
\]

If a note (or bond) sells for its par value then the current yield is equal to the yield-to-maturity. This bond is selling at a premium so its yield-to-maturity is lower than its current yield.

23. What pattern do you see in the yield-to-maturity of these Treasury notes and bonds?

**ANSWER**

The yield to maturity of these Treasury notes and bonds increases as maturity increases implying that they are positively related.

*Treasury bills.* Use the information in the following table for Questions 24 through 28. Note: The face value of the Treasury bill is assumed to be $10,000.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Days to Maturity</th>
<th>Bank Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar 30</td>
<td>28</td>
<td>1.20</td>
</tr>
<tr>
<td>Apr 30</td>
<td>59</td>
<td>2.00</td>
</tr>
<tr>
<td>Jun 30</td>
<td>120</td>
<td>2.45</td>
</tr>
<tr>
<td>Aug 30</td>
<td>181</td>
<td>?</td>
</tr>
</tbody>
</table>

24. What is the price for the March 30 Treasury bill?

**ANSWER**

\[
\text{Price} = 10,000 \times \left(1 - \left[ 0.0120 \times \frac{28}{360} \right] \right) = 9,990.67
\]

Or

Discount = Face Value × Discount Rate × (Days to Maturity / 360)

Discount = $10,000 × 0.0120 × (28/360) = $9.33

Price = $10,000 − $9.33 = $9,990.67
25. What is the price for the April 30 Treasury bill?

**ANSWER**

\[
\text{Price} = 10,000 \times \left(1 - 0.0200 \times \frac{59}{360}\right) = 9,967.22
\]

Or

\[
\text{Discount} = \text{Face Value} \times \text{Discount Rate} \times \left(\frac{\text{Days to Maturity}}{360}\right)
\]

\[
\text{Discount} = 10,000 \times 0.0200 \times \left(\frac{59}{360}\right) = 32.78
\]

\[
\text{Price} = 10,000 - 32.78 = 9,967.22
\]

26. What is the price for the June 30 Treasury bill?

**ANSWER**

\[
\text{Price} = 10,000 \times \left(1 - 0.0245 \times \frac{120}{360}\right) = 9,918.33
\]

Or

\[
\text{Discount} = \text{Face Value} \times \text{Discount Rate} \times \left(\frac{\text{Days to Maturity}}{360}\right)
\]

\[
\text{Discount} = 10,000 \times 0.0245 \times \left(\frac{120}{360}\right) = 81.67
\]

\[
\text{Price} = 10,000 - 81.67 = 9,918.33
\]

27. Determine the bank discount rate of the August 30 Treasury bill if it is currently selling for $9,841.625. What is the bond equivalent yield?

**ANSWER**

Using the formula for pricing a T-bill we get:

\[
9,841.625 = 10,000 \times \left(1 - \text{discount} \times \frac{181}{360}\right)
\]

Or

\[
\text{Discount Rate} = \frac{\text{Discount}}{\text{Face Value}} \times \left(\frac{360}{\text{Days to Maturity}}\right)
\]

\[
\text{Discount} = 10,000 - 9,841.625 = 158.375
\]

\[
\text{Discount Rate} = \frac{158.375}{10,000} \times \left(\frac{360}{181}\right) = 0.0315 \text{ or } 3.15\%
\]

\[
\text{Bond Equivalent Yield is:}
\]

\[
\text{BEY} = \frac{365 \times 0.0315}{360 - 181 \times 0.0351} = 0.03245145
\]

\[
\approx 3.245\%
\]
To calculate the Effective Annual Rate we first compute the HPR:

$$HPR = \frac{\left(10,000 - 9,841.625\right)}{9,841.625} = 0.0016092363$$

Then we find the annual compounded rate of return:

$$EAR = \left(1 + 0.0016092363\right)^{365/181} - 1 = 0.032716911 \approx 3.27\%.$$  
Thus, the bank discount yield is 3.15%, BEY is 3.245% and EAR is 3.27%.

28. What are the bond equivalent yields of the March 30, April 30, and June 30 T-Bills?

**ANSWER**

Bond Equivalent Yield for the March 30 T-Bill is:

$$BEY = \frac{365 \times 0.0120}{360 - 28 \times 0.0120} = 0.012178$$

$\approx 1.22\%$

Bond Equivalent Yield for the Apr 30 T-Bill is:

$$BEY = \frac{365 \times 0.020}{360 - 59 \times 0.020} = 0.020344462$$

$\approx 2.03\%$

Bond Equivalent Yield for the Jun 30 T-Bill is:

$$BEY = \frac{365 \times 0.0245}{360 - 120 \times 0.0245} = 0.02504481$$

$\approx 2.50\%$

---

**Solutions to Advanced Problems for Spreadsheet Applications**

1. Bond Ladder:
### Bond Data

<table>
<thead>
<tr>
<th>Years to Maturity</th>
<th>Coupon Rate</th>
<th>Yield to Maturity</th>
<th>Par Value</th>
<th>Current Price</th>
<th>Cost of Ten Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.5000%</td>
<td>3.2500%</td>
<td>$1,000.00</td>
<td>$1,021.96</td>
<td>$10,219.63</td>
</tr>
<tr>
<td>2</td>
<td>6.2500%</td>
<td>3.5000%</td>
<td>$1,000.00</td>
<td>$1,052.68</td>
<td>$10,526.75</td>
</tr>
<tr>
<td>3</td>
<td>4.7500%</td>
<td>3.5700%</td>
<td>$1,000.00</td>
<td>$1,033.29</td>
<td>$10,332.90</td>
</tr>
<tr>
<td>4</td>
<td>7.0000%</td>
<td>4.0000%</td>
<td>$1,000.00</td>
<td>$1,109.88</td>
<td>$11,098.82</td>
</tr>
<tr>
<td>5</td>
<td>6.5000%</td>
<td>4.1250%</td>
<td>$1,000.00</td>
<td>$1,106.32</td>
<td>$11,063.20</td>
</tr>
<tr>
<td>6</td>
<td>8.2500%</td>
<td>4.2500%</td>
<td>$1,000.00</td>
<td>$1,209.89</td>
<td>$12,098.92</td>
</tr>
<tr>
<td>7</td>
<td>8.0000%</td>
<td>4.3750%</td>
<td>$1,000.00</td>
<td>$1,216.56</td>
<td>$12,165.58</td>
</tr>
<tr>
<td>8</td>
<td>7.2500%</td>
<td>4.5000%</td>
<td>$1,000.00</td>
<td>$1,183.05</td>
<td>$11,830.49</td>
</tr>
<tr>
<td>9</td>
<td>6.5000%</td>
<td>4.6250%</td>
<td>$1,000.00</td>
<td>$1,136.76</td>
<td>$11,367.64</td>
</tr>
<tr>
<td>10</td>
<td>5.5000%</td>
<td>4.7500%</td>
<td>$1,000.00</td>
<td>$1,059.16</td>
<td>$10,591.56</td>
</tr>
<tr>
<td>11</td>
<td>5.2500%</td>
<td>4.8750%</td>
<td>$1,000.00</td>
<td>$1,031.64</td>
<td>$10,316.38</td>
</tr>
<tr>
<td>12</td>
<td>4.7500%</td>
<td>5.0000%</td>
<td>$1,000.00</td>
<td>$977.64</td>
<td>$9,776.44</td>
</tr>
<tr>
<td>13</td>
<td>4.0000%</td>
<td>5.0625%</td>
<td>$1,000.00</td>
<td>$899.70</td>
<td>$8,996.96</td>
</tr>
<tr>
<td>14</td>
<td>4.5000%</td>
<td>5.1250%</td>
<td>$1,000.00</td>
<td>$938.10</td>
<td>$9,380.98</td>
</tr>
<tr>
<td>15</td>
<td>5.2500%</td>
<td>5.1875%</td>
<td>$1,000.00</td>
<td>$1,006.46</td>
<td>$10,064.60</td>
</tr>
<tr>
<td>16</td>
<td>6.0000%</td>
<td>5.2500%</td>
<td>$1,000.00</td>
<td>$1,080.51</td>
<td>$10,805.12</td>
</tr>
<tr>
<td>17</td>
<td>6.5000%</td>
<td>5.3125%</td>
<td>$1,000.00</td>
<td>$1,131.86</td>
<td>$11,318.59</td>
</tr>
<tr>
<td>18</td>
<td>6.7500%</td>
<td>5.3750%</td>
<td>$1,000.00</td>
<td>$1,157.35</td>
<td>$11,573.47</td>
</tr>
<tr>
<td>19</td>
<td>7.5000%</td>
<td>5.4325%</td>
<td>$1,000.00</td>
<td>$1,243.13</td>
<td>$12,431.26</td>
</tr>
<tr>
<td>20</td>
<td>8.0000%</td>
<td>5.0000%</td>
<td>$1,000.00</td>
<td>$1,376.54</td>
<td>$13,765.42</td>
</tr>
</tbody>
</table>

Cost of Portfolio: $219,724.69
### Payoff Table

<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon Payments</th>
<th>Principal Payments</th>
<th>Total Payments</th>
<th>Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$6,200.00</td>
<td>$-</td>
<td>$6,200.00</td>
<td>June</td>
</tr>
<tr>
<td>1</td>
<td>$6,200.00</td>
<td>$10,000.00</td>
<td>$16,200.00</td>
<td>December</td>
</tr>
<tr>
<td>1.5</td>
<td>$5,925.00</td>
<td>$-</td>
<td>$5,925.00</td>
<td>June</td>
</tr>
<tr>
<td>2</td>
<td>$5,925.00</td>
<td>$10,000.00</td>
<td>$15,925.00</td>
<td>December</td>
</tr>
<tr>
<td>2.5</td>
<td>$5,612.50</td>
<td>$-</td>
<td>$5,612.50</td>
<td>June</td>
</tr>
<tr>
<td>3</td>
<td>$5,612.50</td>
<td>$10,000.00</td>
<td>$15,612.50</td>
<td>December</td>
</tr>
<tr>
<td>3.5</td>
<td>$5,375.00</td>
<td>$-</td>
<td>$5,375.00</td>
<td>June</td>
</tr>
<tr>
<td>4</td>
<td>$5,375.00</td>
<td>$10,000.00</td>
<td>$15,375.00</td>
<td>December</td>
</tr>
<tr>
<td>4.5</td>
<td>$5,025.00</td>
<td>$-</td>
<td>$5,025.00</td>
<td>June</td>
</tr>
<tr>
<td>5</td>
<td>$5,025.00</td>
<td>$10,000.00</td>
<td>$15,025.00</td>
<td>December</td>
</tr>
<tr>
<td>5.5</td>
<td>$4,700.00</td>
<td>$-</td>
<td>$4,700.00</td>
<td>June</td>
</tr>
<tr>
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### Solutions to Mini-Case

**Bay Path Cranberry Products**

This case reviews basic calculations for bond prices and yields and examines various bond features and ratings from the point of view of decisions that must be made by the issuing company.

For all questions, assume par value is $1,000 and semiannual bond interest payment.

1. **A company in a line of business similar to Bay Path’s recently issued at par noncallable bonds with a coupon rate of 5.8% and a maturity of 20 years. The bonds were rated Aa1 by Moody’s and AA by Standard & Poor’s. What rate of return (yield to maturity) did investors require on these bonds if the bonds sold at par value?**

   If the bonds sold at par, then the yield to maturity had to be equal to the coupon rate of 5.8%.

2. **Bay Path has one outstanding bond issue with a coupon of 8% which will mature in 5 years. The bond now sells for $1,141.69. What is the yield to maturity on this bond?**

   **Formula:**
   \[ N = 5 \text{ years} \times 2 = 10, \text{ coupon payment} = 1,000 \times .08/2 = 40 \]
   \[ 1,141.69 = 1000(1/(1+r)^{10}) + 40[(1-(1/1+r)^{10})/r = .0239], \]
   \[ \text{Yield to maturity} = .0239 \times 2 = 4.78\% \]

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Unfortunately, r must be found by trial and error. Use of a financial calculator or EXCEL is recommended.

Calculator:

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Yield to maturity = 2.39 × 2 = 4.78%

The EXCEL formula is \( \text{rate}(N, \text{PMT}, -\text{PV}, \text{FV}) \)

3. Based on your answers to questions 1 and 2 above, what coupon rate should Bay Path offer if it wants to realize $50,000,000 from the bond issue and to sell the bonds as close to par value as possible? (Ignore the cost of selling the bonds.)

The yields on bonds of different companies with the same maturity and the same rating will be more similar than bonds of the same company with different maturities because maturity and default risk premiums increase with time. If Bay Path wishes to sell the bonds at or near par value, the coupon should be about 5.8%.

4. Suppose Bay Path actually offers a coupon rate of 6% on its 20 year bonds, expecting to sell the bonds at par. What will happen to the price of a single bond with a par value of $1,000 if the required yield on the bonds unexpectedly falls to 5% or rises to 7%?

\[ N=20 \text{ years} \times 2 = 40, \text{ coupon payment} = 1,000 \times .06/2 = 30, \text{ r} = .05/2 = .025, \text{ and .07/2 = .035} \]

\[ 1,125.51 = 1000(1/(1.025)^{40} + 30[(1-(1/1.025)^{40})/.025] \]

\[ 893.22 = 1000(1/(1.035)^{40} + 30[(1-(1/1.035)^{40})/.035] \]

Calculator:

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5. How much money will Bay Path realize from its $50 million bond issue if the actual yield is either 5% or 7%? (Refer to your answers to question 4. Ignore selling costs.)

If rates fall just before the bonds are issued, the bonds will sell at a premium and the company will realize $1,125.51 × 50,000 = $56,275,500. If rates rise and the bonds are issued at a discount, Bay Path will realize only $893.22 × 50,000 = $44,661,000. The premiums or discounts have the effect of reducing or increasing, respectively, the true rate of interest Bay Path must pay on its bonds.

6. How would the following affect the yield on Bay Path’s newly issued bonds?

a. The bonds are callable.

b. The bonds are subordinated to Bay Path’s existing bond issue.
c. The bond rating is better or worse than the Moody’s Aa1 that Bay Path anticipates.
   a. The call feature increases investor risk and will raise the yield to maturity (which from Bay Path’s point of view is the cost of borrowing).
   b. Subordination increases default risk and will raise the yield to maturity.
   c. If the bond rating is better than anticipated, it will lower the required yield, and vice versa.

Additional Problems with Solutions

1. **Pricing a semi-annual bond.** Last year, The Harvest Time Corporation sold $40,000,000 worth of 7.5% coupon, 15-year maturity, $1000 par value, AA-rated; non-callable bonds to finance its business expansion. Currently, investors are demanding a yield of 8.5% on similar bonds. If you own one of these bonds and want to sell it, how much money can you expect to receive on it?

   **ANSWER** *(Slides 6-46 to 6-47)*

   **Using a financial calculator**
   
   Mode: P/Y=2; C/Y = 2
   Input: N I/Y PV PMT FV
   Key: 28 8.5 ? 37.5 1000
   Output -919.03

   2. **Yield-to-Maturity.** Joe Carter is looking to invest in a four-year bond that pays semiannual coupons at a coupon rate of 5.6 percent and has a par value of $1,000. If these bonds have a market price of $1035, what yield to maturity is being implied in the pricing?

   **ANSWER** *(Slides 6-48 to 6-49)*

   **Using a financial calculator**
   
   Mode: P/Y=2; C/Y = 2
   Input: N I/Y PV PMT FV
   Key: 8 ? -1035 28 1000
   Output 4.63

   The expected YTM is 4.63%

3. **Price of a zero-coupon bond.** Krypton Inc. wants to raise $3 million by issuing 10-year zero coupon bonds with a face value of $1,000. Their investment banker informs them that investors would use a 9.25% percent discount rate on such bonds. At what price would these bonds sell in the market place assuming semi-annual compounding? How many bonds would the firm have to issue to raise $3 million?
Using a financial calculator: Price of zero-coupon bond

Mode: P/Y=2; C/Y = 2
Input: N I/Y PV PMT FV
Key: 20 9.25 ? 0 1000
Output -404.85

The zero-coupon bond would sell for $404.85

To raise $3,000,000, the company would have to sell:
$3,000,000/$404.85 = 7411 bonds

4. Tax on zero-coupon bond income. Let’s say that you buy 100 of the 7411 bonds that were issued by Krypton Inc. as described in Problem 3 above for $404.85. At the end of the year, how much money will the bond be worth, and how much tax will you be assessed assuming that you have a marginal tax rate of 35%?

ANSWER

Calculate the price of the bond at the end of each semi-annual period during the next year.

The change in price for each semi-annual period represents the implied interest income on a zero which is taxed at 35%

Price of Zero after 6 months assuming YTM of 9.25%:

Mode: P/Y=2; C/Y = 2
Input: N I/Y PV PMT FV
Key: 19 9.25 ? 0 1000
Output -423.57

Price of Zero at the end of 2 semi-annual periods assuming YTM of 9.25%

Mode: P/Y=2; C/Y = 2
Input: N I/Y PV PMT FV
Key: 18 9.25 ? 0 1000
Output -443.16

Implied interest earned on zero = $443.16 – $404.85 = $38.31
Taxes due = Tax rate * Taxable income = 0.35 * $38.31 = $13.41

5. Price, and BEY, on a Treasury bill. Calculate the price, and BEY of a treasury bill which matures in 181 days, has a face value of $10,000, and is currently being quoted at a bank discount yield of 2.32%.

ANSWER

Price of T-bill = Face value * [1 – (discount yield * days until maturity/360)]
Price of T-bill = $10,000 * [ 1 – (.0232 * 181/360)] = $10,000*0.98833555
Price of T-bill = $9,883.36

\[ BEY = \frac{365 \times \text{Bank discount yield}}{360 - (\text{days to maturity} \times \text{discount yield})} = \frac{365 \times 0.0232}{360 - (181 \times 0.0232)} \]

\[ BEY = 0.023799 = 2.38\% \text{ (rounded to 2 decimals)} \]