Specific Objectives

Students will understand

- terminology such as gross income, adjusted income, taxable income.
- the distinction between variables and constants.
- the distinction between expressions, equations, and inequalities.
- equations represent relationships between unknown quantities.

Students will be able to

- interpret information from a tax table.
- apply concept of percentage in given context.
- simplify algebraic expressions and equations using the distributive property.
- evaluate expressions.
- construct equations to represent relationships between unknown quantities.

In the United States, people pay income tax on any income they earn during the year. The total amount of money they earn through work, investments, and other means is called total income. From this, people are able to deduct certain things, like retirement contributions and tuition payments, reducing their income to what is called their adjusted gross income. From this, certain expenses can be deducted resulting in the taxable income that tax is actually calculated on.

Taxes can be, as you've probably heard, fairly complicated, but for this lesson we are going to focus on a simple case.

- A married couple with no kids, filing jointly. No one can claim either as a dependent.
- Together, they earned $93,000 of wage income, and had no interest or other income

1. Because their situation is pretty simple, this couple can fill out a 1040EZ tax form. As the name implies, this is the easy form for simple cases. Fill out the income portion of the form to find the couple’s taxable income.
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2. Usually tax owed is looked up from a tax table.
   a. You would use the first line of the table if the taxable income from line 6 is at least 72,000 but less than 72,050. Write this as a compound inequality, using \( t \) to represent the income from line 6.

   \[ 72,000 \leq t < 72,050 \]

   b. Use the table provided to determine the tax the couple owes.

   Alternatively, the couple could calculate the tax based on tax rate information. A portion of the rate table from 2014 is shown.

<table>
<thead>
<tr>
<th>If Taxable Income Is:</th>
<th>The Tax Is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not over $18,150</td>
<td>10% of the taxable income</td>
</tr>
<tr>
<td>Over $18,150 but not over $73,800</td>
<td>$1,815 plus 15% of the excess over $18,150</td>
</tr>
<tr>
<td>Over $73,800 but not over $148,850</td>
<td>$10,162.50 plus 25% of the excess over $73,800</td>
</tr>
<tr>
<td>Over $148,850 but not over $226,850</td>
<td>$28,925 plus 28% of the excess over $148,850</td>
</tr>
</tbody>
</table>

   The rates you see on the right (10%, 15%, 25%, 28%) are called **marginal tax rates**, but most people just call them **tax brackets**.

3. Use the tax rates above to calculate the couple's tax. Does it match the tax table?
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4. A commonly held belief is that marginal tax rate applies to all of a person's taxable income.
   a. Does the couple owe 15% of their taxable income?
   b. If not, what percent of their taxable income are they paying?

Sometimes people have been hesitant to accept raises, worried that it will push them into a new tax bracket and they'll end up losing money.

5. Suppose the couple got an unexpected $1,500 bonus at the end of the year, pushing them into the 25% marginal rate tax bracket.
   a. How much tax will they pay?
   b. After paying taxes, will the couple end up with less take-home pay because the bonus pushed them into a higher tax bracket?

While we can easily calculate an individual’s taxes using the tax form, it is very tedious to have to fill out the form over and over again if you have a large number of people that all fit the same conditions. This is somewhere where an equation with variables would be helpful.

You’ve seen equations and formulas before. For example, the area equation for a circle, \( A = \pi r^2 \), is a an equation that relates two variables, \( r \) and \( A \). Variables are letters used to represent unknown or changeable quantities. A formula or equation relates two or more variables, usually showing how one variable can be calculated from the other variables.

An expression consists of numbers and variables. It does not contain an equals sign, whereas an equation always does include an equal sign. An inequality is like an equation, but involves an inequality sign.
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Examples:

\[ 3x + 2y + 4 \quad 2x^2 + 5 \quad \text{expressions} \]
\[ y = 2x^2 + 5 \quad 3x - 2y = 4 \quad \text{equations} \]
\[ y < 2t + 3 \quad a + 2b \geq 4 \quad \text{inequalities} \]

In math, the things we typically do depends on what kind of thing we’re working with. With expressions, we often simplify, by combining like terms and distributing. Sometimes we might evaluate an expression by replacing the variables with values and calculating the value.

- Combining like terms: Terms can only be added only if they contain the same power of the same variables. For example, \(3x\) and \(5x\) are like terms, but \(2\) and \(x^2\) are not.

- Distributing: \(a(b + c) = ab + ac\)

6. Simplify these expressions:

   a. \(3x + 4 - 5x + 6\)
   b. \(3(x - 2)\)
   c. \(5(2x - 3) - 4(3x + 2)\)

7. Evaluate these expressions using the given values for the variables:

   a. Evaluate \(3x\) if \(x = 6\)
   b. Evaluate \(4x - 2\) if \(x = -5\)

   c. Evaluate \(3x^2 - 5x + 3\) if \(x = -2\)
   d. Evaluate \(LW\) if \(L = 5\) and \(W = 4\)

   e. Evaluate \(\pi r^2\) if \(r = 6\)
While tax tables and the tax rates work well for calculating tax for a single individual, a business might want to create a spreadsheet that automatically calculates the tax for an individual based on their taxable income. To do this, they need to write a formula. Let $T$ be amount of tax owed, $x$ be their taxable income, and $w$ be their wage income.

8. Assume that we're only interested in couples with taxable incomes between $18,150 and $73,800.
   a. Write an equation that will take taxable income, $x$, and compute the tax, $T$, they owe.
   b. Simplify the formula using distribution and combining like terms, as needed.

9. Assume also that we're only interested in couples who fill out a 1040EZ and only have wage income.
   a. Write an equation that will take their wage income, $w$, and compute the tax, $T$, they owe.
   b. Simplify the formula using distribution and combining like terms, as needed.